NATIONAL TAIPEI UNIVERSITY

The Analysis of Partisan Conflict Index Time Series Model

Course: Time series (2019.09.01~2020.01.18)

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Abstract

In recent years, the conflicts of political parties are increasing in various countries such as Taiwan, the United Kingdom, the U.S. ...etc. Political parties have huge influence on the development of the countries. It raised our curiosity to study the effects of Partisan Conflict Index of the United States, which indicates the frequency of newspaper articles reporting disagreement in a given month.

In the era of the information explosion, people can receive current news and search for information anytime and anywhere. Due to this reason, we think of Google Trends. Everyone cares a lot about the economic development of the country so we choose the keyword “Economy” as the second observation. In addition, we want to know whether the Partisan Conflict Index will increase people's concern about crimes, so we choose the keyword "Crime" as our last observation.

In this research, we fit models for both single-dimension and multi-dimension time series data. For single-dimension time series data, we fit a $ARMA(m,n) + GARCH(p,q)$ model [1] and make predictions with it. As for multi-dimension data, we fit it with a VAR model. From Granger Causality test, we can find out that the three data will Granger affect each other.

1. Data Description

In this research, we combined three data into a multivariate time series dataset. One of the variables in this dataset is from Stock AI, and the others are from Google Trends. Google Trends is a search trends feature that shows how frequently a given search term is entered into Google's search engine relative to the site's total search volume over a given period of time. We picked out the keywords "Economy" and "Crime" on Google Trends, and named the data as "ECO" and "CRIME". The data downloaded from Stock AI is Partisan Conflict Index of the U.S. (PARTY). The Partisan Conflict Index tracks the degree of political disagreement among the U.S. Higher index values indicate greater conflict among political parties, Congress, and the President.

2. Analysis Method

2.1 Establishment of Time Series Model

The establishment of a time series model requires the following four steps:

(I)Model identification

ARMA Model (The Auto-Regressive and Moving Average Model):

The ARMA model is composed of the Autoregressive Model (AR model) and the Moving Average Model (MA model). The AR model establishes that a realization at time t is related to the p previous realization. The MA model indicates the current variable (y_t) and the residual term is related to the previous q period ($\epsilon_{t-1}, \dots, \epsilon_{t-q}$). An ARMA (p, q) model is given by

$$y_t = \phi_1 y_{t-1} + \cdots + \phi_p y_{t-p} + \epsilon_t + \theta_1 \epsilon_{t-1} + \cdots + \theta_q \epsilon_{t-q} \qquad (1)$$

where $\phi_1, \dots, \phi_p$ are the coefficients of the autoregressive model; $\theta_1 + \cdots + \theta_q$ are the coefficients of the moving average model; ϵ_t is the error term; p and q are the order of the model. Let $B(L) = 1 - \varphi_1 L - \varphi_2 L^2 - \cdots - \varphi_p L^p$, $\theta(L) = 1 + \theta_1 L + \theta_2 L^2 + \cdots + \theta_p L^p$, where L is the lag operator, then the $ARMA\ (p, q)$ model can also be represented as

$$\phi_p(L) y_t = \theta_q(L) \epsilon_t \qquad (2)$$

If there is seasonality in the $ARMA(p,q)$ model, the model can be written as

$$y_t = \phi_s y_{t-s} + \cdots + \phi_{ps} y_{t-ps} + \epsilon_t + \theta_s \epsilon_{t-s} + \cdots + \theta_{qs} \epsilon_{t-qs} \tag{3}$$

or

$$\phi_p(L^s) y_t = \theta_q(L^s) \epsilon_t \tag{4}$$

The equation in (4) is called the seasonal $ARMA(p,q)_s$ model, where s defines the number of time periods.

The observations in consecutive months from our daily life often depend on each other. Under this condition, we can use the multiplicative model:

$$\Phi_p(L^s)\phi_p(L)(1-L^s)^D(1-L^s)^d y_t = C + \Theta_Q(L^s)\theta_q(L)\epsilon_t \tag{5}$$

The constant C in (5) indicates the stationary time series $(1-L^s)^D(1-L^s)^d y_t$ may have a non-zero mean. The model (5) is called $ARIMA(p,q,d)(P,D,Q)_s$ multiplicative seasonal model.

For the selection of p and q, we need to examine the autocorrelation function (ACF) and partial autocorrelation function (PACF). ACF can be used to determine the number of Lag q of the MA model; PACF is to determine the number of Lag p of the AR model. The criteria for discriminating ACF and PACF are shown in Table 1 below.

Table 1 The criteria for discriminating ACF and PACF

ACF	PACF	Model
Exponential die-down / Positive & negative alternative decreasing	Cutoff after p period	$AR(p)$
Cutoff after q period	Exponential die-down / Positive & negative alternative decreasing	$MA(q)$
Exponential die-down / Positive & negative alternative decreasing	Exponential die-down / Positive & negative alternative decreasing	$ARMA(p,q)$

In addition, the AIC [2] or BIC [3] criterion can also be used to determine the order of the model. The criterion for selecting the model is to choose the smallest value of the AIC or BIC. The equation (6) and (7) are the definitions of AIC and BIC, respectively.

$$AIC = -2logL + 2(p + q + 1) \tag{6}$$
$$BIC = -2logL + (p + q + 1)\log(T) \tag{7}$$

The L represents the probability of the estimation of the model, and T represents the length of the data.

ARIMA model (Autoregressive Integrated Moving Average model)

Given a time series data X_t where t is an integer index and the X_t are real numbers, an $ARMA(p', q)$ model is given by

$$X_t - \alpha_1 X_{t-1} - \cdots - \alpha_{p'} X_{t-p'} = \epsilon_t + \theta_1 \epsilon_{t-1} + \cdots + \theta_q \epsilon_{t-q}$$

or equivalently by

$$\left(1 - \Sigma_{i=1}^{p'} \alpha_i L^i\right) X_t = \left(1 + \Sigma_{i=1}^{q} \theta_i L^i\right) \epsilon_t$$

where L is the lag operator, the $\left(1 - \Sigma_{i=1}^{p'} \alpha_i L^i\right)$ are the parameters of the autoregressive part of the model, the θ_i are the parameters of the moving average part and the ϵ_t are error terms. The error terms ϵ_t are generally assumed to be independent, identically distributed variables sampled from a normal distribution with zero mean.

Assume now that the polynomial $\left(1 - \Sigma_{i=1}^{p'} \alpha_i L^I\right)$ has a unit root (a factor $(1 - L)$) of multiplicity d Then it can be rewritten as

$$\left(1 - \Sigma_{i=1}^{p'} \alpha_i L^i\right) = \left(1 + \Sigma_{i=1}^{p'-d} \phi_i L^i\right)(1 - L)^d \tag{8}$$

An *ARIMA(p, d, q)* process expresses this polynomial factorization property with $p = p' - d$, and is given by

$$\left(1 - \Sigma_{i=1}^{p} \phi_i L^i\right)(1 - L)^d X_t = \left(1 + \Sigma_{i=1}^{q} \theta_i L^i\right) \epsilon_t \tag{9}$$

and thus can be thought as a particular case of an $ARMA(p + d, q)$ process having the autoregressive polynomial with d unit roots. (For this reason, no $ARIMA$ model with $d > 0$ is wide sense stationary.)

The above can be generalized as follows.

$$\left(1 - \Sigma_{i=1}^{p} \phi_i L^i\right)(1-L)^d X_t = \delta + \left(1 + \Sigma_{i=1}^{q} \theta_i L^i\right)\epsilon_t \tag{10}$$

This defines an *ARIMA(p, d, q)* process with drift $\frac{\delta}{1-\Sigma\phi_i}$.

GARCH Model

If an ARMA model is assumed for the error variance, the model is a Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model.

In that case, the $GARCH\ (p, q)$ model (where p is the order of the GARCH terms and q is the order of the ARCH terms σ^2) is given by

$$\mathrm{y_t} = x_t' b + \epsilon_t \tag{11}$$

$$\epsilon_\mathrm{t} | \psi_{t-1} \sim N(0,\ \sigma_t) \tag{12}$$

$$\begin{aligned} \sigma_\mathrm{t}^2 &= \omega + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 \epsilon_{t-2}^2 + \cdots + \alpha_q \epsilon_{t-q}^2 + \\ &\quad \beta_1 \sigma_{t-1}^2 + \beta_2 \sigma_{t-2}^2 + \cdots + \beta_p \sigma_{t-p}^2 \\ &= \omega + \Sigma_{\mathrm{i}=1}^{\mathrm{q}} \alpha_\mathrm{i} \epsilon_{t-i}^2 + \Sigma_{\mathrm{j}=1}^{p} \beta_j \sigma_{t-j}^2 \end{aligned} \tag{13}$$

Generally, when testing for heteroskedasticity in econometric models, the best test is the White test. However, when dealing with time series data, this means to test for ARCH and GARCH errors.

VAR Model

A VAR model [4] describes the evolution of a set of k variables (called endogenous variables) over the same sample period $(t = 1, \ldots, T)$ as a linear function of only their past values.

A p^{th} order VAR, denoted $VAR(p)$, is

$$y_t = c + A_1 y_{t-1} + A_2 y_{t-2} + \cdots + A_p y_{t-p} + e_t \tag{14}$$

Where the observation y_{t-1} is called the i^{th} lag of y, c is a k-vector of constants, A_i is a time-invariant (k × k)-matrix and e_t is a k-vector of error terms satisfying

1. $E(e_t) = 0$, every error term has mean zero
2. $E(e_t e_t') = \Omega$, the contemporaneous covariance matrix of error terms is *Ω*
3. $E(e_t e_{t-k}') = 0$, no serial correlation in individual error terms

(II) Estimation of unknown parameters

After determining p, d and q, now we can estimate the parameters $\varphi_1, \ldots, \varphi_p$, $\theta_1, \ldots, \theta_q$. The commonly used estimation methods will differ depending on what model we select. In AR model, we often use Yule-Walker Estimation, Burg Estimation, Maximum Likelihood Estimation (MLE) and Least Squares Estimation (LSE); while the common estimation methods in MA and ARMA models are Hannan-Rissanen Estimates and Innovations Algorithm. If there exist ARCH up to order q in the residual ϵ_t, use GARCH to estimate the parameters.

(III) Diagnostic test

After estimating the model parameters, we need to check whether the residual of the model follow WN (WN). If it follows WN, the residual of the model is independent and normally distributed. If the residual doesn't follow WN, the model must be further modified.

There are many ways to test for WN. In this study, we use the Q statistic [5] to perform the self-correlation test on the residual terms. The test assumptions and statistics are (15) and (16).

$$\begin{aligned} &H_0: \rho_1 = \rho_2 = \cdots = \rho_k = 0 \ \text{(residual follow WN)} \\ &H_a: \ at\ least\ one\ of\ the\ \rho_1, \rho_2, \ldots, \rho_k \neq 0 \ (\text{not } \mathrm{H}_0) \end{aligned} \tag{15}$$

$$Q = T(T+2)\Sigma_{i=1}^{k} \frac{\widehat{\rho_i}^2}{(T-i)} \tag{16}$$

T is the total number of residual. The Q statistic will follow the chi-square distribution with degrees of freedom $(k-p-q)$. If the test statistics are not significant, the null hypothesis will not be rejected, that is, the residual of the model follow WN. Otherwise, the residual will not follow WN, and the model must be refitted.

(IV) Model Predicting

This study evaluates the best model through out-of-sample predictions, that is, the total number of samples is $T + N$. The last N observations are out-of-sample data, and the beginning T observations are assumed to be in-sample data to estimate it.

Granger Causality test

The Granger Causality test [6] is a statistical hypothesis test for determining whether one time series is useful in forecasting another, first proposed in 1969 by Clive WJ Granger. A time series X is said to Granger-cause Y if it can be shown, usually through a series of t-tests and F-tests on lagged values of X, that those X values provide statistically significant information about future values of Y. The null hypothesis for the test is that lagged x-values do not explain the variation in y. In other words, it assumes that X_t doesn't Granger-cause Y_t.

The assumption for the Granger Causality test is that the time series must be stationary, otherwise spurious regression may occur. Therefore, before performing the Granger Causality test, a unit root test should be performed on the stability of each index time series.

3. Processing of Non-stationary Data

3.1 Unit Root Test

Time series data can often be divided into stationary time series and non-stationary time series. The stationary time series are divided into strong stationary and weak stationary. A strong stationary time series data $\{y_t\}$ is a stochastic process, and the probability distribution of this stochastic process does not change as the time pass by. The analysis of common weak stationary must guarantee the following three characteristics:

(1) $E(y_t) = u,\ \forall t$

(2) $Var(y_t) = \sigma^2 < \infty, \forall t$

(3) $cov(y_t, y_{t-k}) = \gamma(k)$

Most economic variables have a stochastic trend, which is a non-stationary time series. If we conduct the regression analysis of these non-stationary time series, it will cause spurious regression [7]. Thus, it is necessary to ensure that the variable is a stationary time series.

A unit root test is the process of testing whether a time series is stationary. The Augmented Dickey-Fuller (ADF) unit root test [8] divides the data into three forms:

(1) With intercept and time trend

(2) With intercept term but no time trend

(3) No intercept term and time trend term.

The statistical assumption for these models is

$$\begin{matrix} H_0: \tau = 0 \\ H_a: \tau < 0 \end{matrix},\ where\ \ \tau = 1 - \beta_1 - \cdots - \beta_p. \tag{17}$$

Under the hypothesis of ADF, if the null hypothesis is rejected, the time series data is stationary. Otherwise, it is a non-stationary time series data, and we can treat it by d-order difference in order to make it stationary.

4. Analysis of Data

4.1 Data description and processing

The data used in this study are monthly data, and the time range is from 2010 to 2019. There are 120 variables. In empirical analysis, EVIEWS statistical software was used to carry out single-root test, construction model and Granger Causality analysis.

4.2 ADF TEST: Check for stationary

According to the results of ADF test, we decided to treat $ECO,\ log(PARTY)$ and $CRIME$ by first difference in order to make them station for VAR modeling.

From Figure 1, Figure 3 and, Figure 5we can see that there exist trends in the original data. After the first order differencing (Figure 2, Figure 4, and Figure 6), there is no trend in the differenced data.

Table 2 unit root test of ECO

test for ADF	ECO		$dECO$, first difference of ECO	
	t-statistics	p-value	t-statistics	p-value
test statistic	-0.6439	00.8546	-8.31710	0
test critical values of 5% level	-2.8187		-2.89187	
conclusion	non-stationary		stationary	

Table 3 unit root test of CRIME

test for ADF	$CRIME$		$dCRIME$, first difference of $CRIME$	
	t-statistics	p-value	t-statistics	p-value
test statistic	-0.86554	0.7951	-11.4438	0
test critical values of 5% level	-2.8922		-2.8922	
conclusion	non-stationary		stationary	

Table 4 unit root test of log(PARTY)

Test for ADF	$log(PARTY)$		$dlog(PARTY)$, first difference of $log(PARTY)$	
	t-statistics	p-value	t-statistics	p-value
test statistic	-2.6011	0.096	-13.304	0
test critical values of 5% level	-2.8895		-2.8895	
conclusion	non-stationary		stationary	

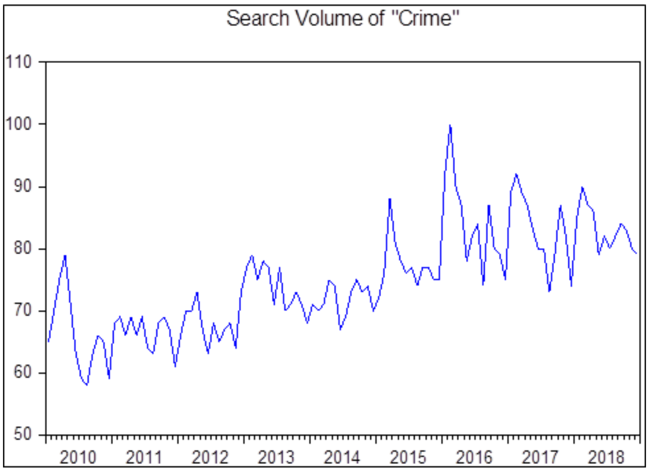

Figure 1 Plot of $CRIME$

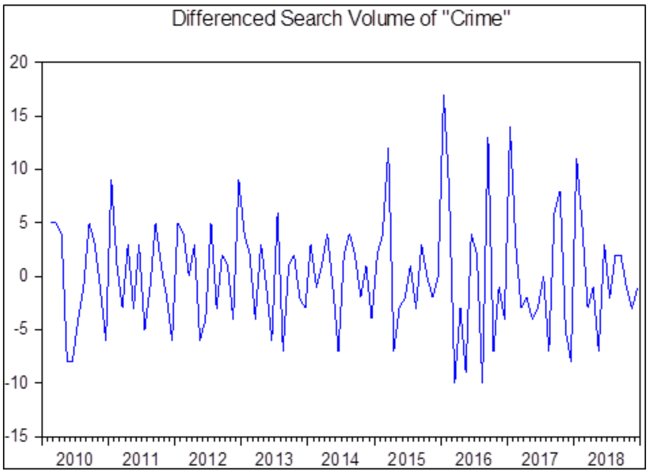

Figure 2 Plot of 1st differenced $CRIME$

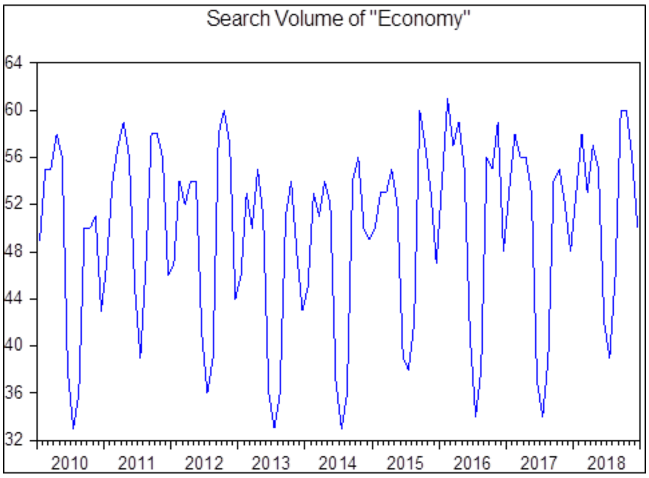

Figure 3 Plot of ECO

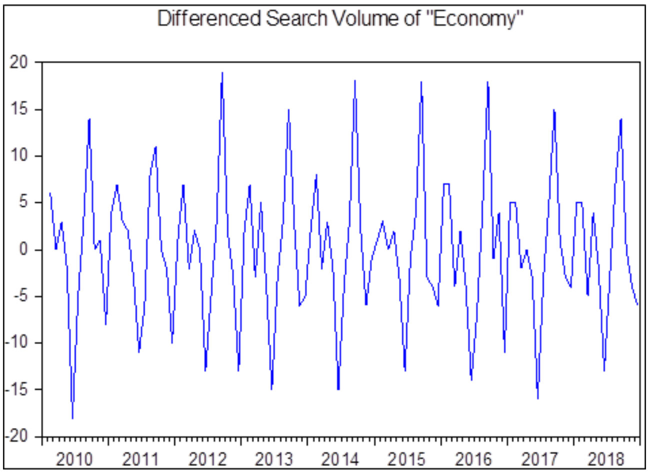

Figure 4 Plot of 1st differenced ECO

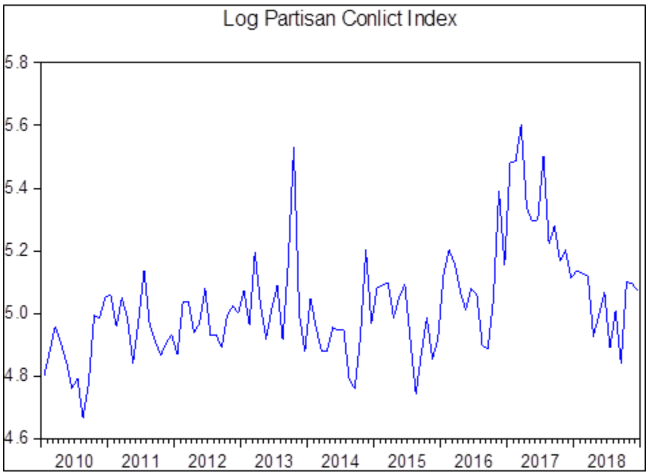

Figure 5 Plot of $log(PARTY)$

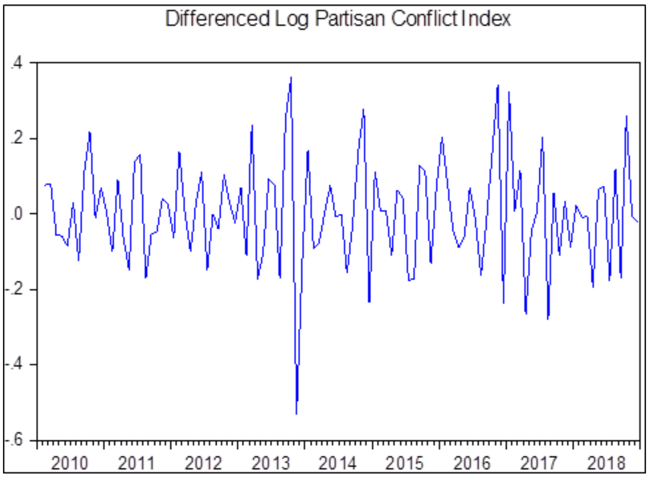

Figure 6 Plot of 1st differenced $log(PARTY)$

4.3 Model Fitting

Single-dimensional time series data

The hypothesis for testing GARCH is

$$H_0 : there\ is\ no\ existing\ ARCH\ up\ to\ order\ q\ in\ the\ residuals\ \epsilon_t$$
$$H_a : not\ H_0$$

Since the Heteroskedasticity Test of $ARMA(1,1)(2,1)$ from $dECO$ does not reject H_0, ARCH doesn't exist up to order q in the residual. There is no need to fit a GARCH model.

The Heteroskedasticity Test of $ARMA(1,1)(1,1)$ from $dCRIME$ and $ARMA(1,1)(2,1)$ from $dlog(PARTY)$ both reject H_0, ARCH exists up to order q in the residual. It is necessary to fit a GARCH model.

Table 5 ARIMA model selected

	$dECO$	$dCRIME$	$dlog(PARTY)$
Selected ARMA model	(1,1)(2,1)	(1,1)(1,1)	(1,1)(2,1)
AIC value:	5.879	5.6482	-1.2601

Table 6 Heteroskedasticity Test for ARCH of ARMA

	$dECO$	$dCRIME$	$dlog(PARTY)$
F-statistic	0.201295	6.787206	12.19837
Obs*R-squared	0.20477	6.493925	11.12776
Prob. F(1,104)	0.6546	0.0105	0.0007
Prob. Chi-Square(1)	0.6509	0.0108	0.0009

Table 7 AIC of ARCH compared with GARCH

	$ARCH(q)$	$GARCH(p)$	AIC
$dlog(PARTY)$	0	1	-1.132
	1	0	-1.371
	1	1	-1.379
	1	2	-1.367
	2	1	-1.36
	2	2	-1.49*
$dCRIME$	0	1	5.714
	1	0	5.689
	1	1	5.649
	1	2	5.672
	2	1	5.626*
	2	2	5.685

We choose the model that minimizes AIC. Using this criterion, we chose *GARCH(1, 2)* for $dCRIME$, and $GARGH(2,2)$ for $dlog(PARTY)$.

The following equations are the expressions of the fitted models:

$\boldsymbol{ECO\ :\ ARIMA(1,1,1)(2,0,1)_{12}}$

$$(1-0.6907L)(1-1.1418L^{12}+0.1418L^{24})dECO_t = (1-L)(1-0.9999L^{12})\epsilon_t$$
$$\epsilon_t\ \ is\ \ WN{\sim}N(0,\sigma = 2.182033)$$

$\boldsymbol{CRIME\ :\ ARIMA(1,1,1)(1,0,1)_{12} - GARCH(2,1)}$

$$(1-0.21197L)(1-1.009L^{12})dCRIME_t = (1-0.8253L)(1-0.90739L^{12})\epsilon_t,$$
$$\sigma_t^2 = 3.5242 - 0.1458\epsilon_{t-1}^2 - 0.1490\epsilon_{t-2}^2 + 0.7595\sigma_{t-1}^2$$

$\boldsymbol{log(PARTY)\ :\ ARIMA(1,1,1)(1,0,1)_{12} - GARCH(2,2)}$

$$(1-0.1138L)(1-0.6612L^{12}-0.2488L^{24})dlog(PARTY)_t$$
$$= (1-0.5479L)(1-0.9042L^{12})\epsilon_t,$$
$$\sigma_t^2 = 0.0004 + 0.18\epsilon_{t-1}^2 - 0.28166\epsilon_{t-2}^2 + 1.3981\sigma_{t-1}^2 - 0.3121\sigma_{t-2}^2$$

4.4 Residual Test of Q statistic

If the pattern of Autocorrelation and Partial correlation from the correlogram of standardized residual squared table are within the confidence interval, we can conclude that the distribution of residual follows WN. The hypothesis for the Q test is

$$H_0\!: \rho_1 = \rho_2 = \cdots = \rho_k = 0\ \ (residuals\ follow\ WN)$$
$$H_a\!:\ at\ least\ one\ of\ the\ \rho_1, \rho_2, \ldots, \rho_k \neq 0\ \ (not\ H_0)$$

Since the result of Q test shows that all the P-Value are larger than 0.05, we failed to reject the null hypothesis. Thus, our final model for $dCRIME$ and $dlog(PARTY)$ are $\boldsymbol{ARIMA(1,1,1)(1,0,1)_{12} - GARCH(1,2)}$ and $\boldsymbol{ARIMA(1,0\ 1)(1,0,1)_{12} - GARCH(2,2)}$**.**

4.5 Multi-dimensional time series model (VAR MODEL)

$$
\begin{aligned}
dCRIME_t = 4.606 &- 0.56dCRIME_{t-1} - 0.55dCRIME_{t-2} - 0.45dCRIME_{t-3} \\
&- 0.37dCRIME_{t-4} - 0.35dCRIME_{t-5} + 0.27dCRIME_{t-6} \\
&- 0.01dCRIME_{t-7} - 0.006dlog(PARTY)_{t-1} \\
&- 0.026dlog(PARTY)_{t-2} + 0.006dlog(PARTY)_{t-3} \\
&- 0.018dlog(PARTY)_{t-4} - 0.088dlog(PARTY)_{t-5} \\
&+ 0.015dlog(PARTY)_{t-6} + 0.041dlog(PARTY)_{t-7} \\
&+ 0.008dECO_{t-1} + 0.186dECO_{t-2} + 0.094dECO_{t-3} \\
&+ 0.130dECO_{t-4} + 0.274dECO_{t-5} + 0.195dECO_{t-6} \\
&+ 0.048dECO_{t-7}
\end{aligned}
$$

$$
\begin{aligned}
dlog(PARTY)_t& \\
&= 34.86 - 0.31dCRIME_{t-1} + 0.40dCRIME_{t-2} - 0.12dCRIME_{t-3} \\
&- 0.31dCRIME_{t-4} - 0.04dCRIME_{t-5} + 0.06dCRIME_{t-6} \\
&+ 0.05dCRIME_{t-7} + 0.55dlog(PARTY)_{t-1} + 0.11dlog(PARTY)_{t-2} \\
&+ 0.13dlog(PARTY)_{t-3} + 0.07dlog(PARTY)_{t-4} \\
&- dlog(PARTY)_{t-5} + 0.04dlog(PARTY)_{t-6} \\
&+ 0.06dlog(PARTY)_{t-7} + 0.21DECO_{t-1} + 0.54dECO_{t-2} \\
&- 0.17dECO_{t-3} + 0.16dECO_{t-4} + 0.11dECO_{t-5} + 0.24dECO_{t-6} \\
&- 0.74dECO_{t-7}
\end{aligned}
$$

$$
\begin{aligned}
dCEO_t = 5.58 &+ 0.04dCRIME_{t-1} - 0.26dCRIME_{t-2} - 0.41dCRIME_{t-3} \\
&- 0.53dCRIME_{t-4} - 1.08dCRIME_{t-5} - 0.65dCRIME_{t-6} \\
&- 0.17dCRIME_{t-7} - 0.01dlog(PARTY)_{t-1} + 0.05dlog(PARTY)_{t-2} \\
&+ 0.005dlog(PARTY)_{t-3} - 0.001dlog(PARTY)_{t-4} \\
&- 0.13dlog(PARTY)_{t-5} + 0.07dlog(PARTY)_{t-6} \\
&- 0.01dlog(PARTY)_{t-7} - 0.16dECO_{t-1} - 0.298dECO_{t-2} \\
&- 0.41dECO_{t-3} - 0.05dECO_{t-4} + 0.19dECO_{t-5} + 0.24dECO_{t-6} \\
&- 0.74dECO_{t-7}
\end{aligned}
$$

4.6 Granger Causality Test

The result of VAR Granger Causality suggests that there is causal relationship among $dlog(PARTY)$, $dECO$ and $dCRIME$. The result that $dlog(PARTY)$ affects $dECO$ coincides with the result in [9] at significant level of 0.1.

Table 8 VAR Granger Causality/Block Exogeneity Wald Tests

Dependent variable	Excluded	Chi-sq	df	Prob.
$dlog(PARTY)$	DECO	22.34629	7	0.0022
	DCRIME	6.801401	7	0.4498
	All	33.51607	14	0.0024
$dECO$	DLPARTY	12.50228	7	0.0852
	DCRIME	25.08091	7	0.0007
	All	44.50895	14	0
$dCRIME$	DLPARTY	24.6167	7	0.0009
	DECO	44.40865	7	0
	All	74.0791	14	0

The columns $dlog(PARTY)$, $dCRIME$ and $dECO$ in Table 9 represent the Granger effects of the respective independent variables on the dependent variables, and the numbers represent the lags. The signs in the effect column shows whether the effects on the dependent variables are positive or negative.

Table 9 Effects on the dependent variables

dependent variables	effect	Lags of independent variables		
		$\boldsymbol{logPARTY}$	$\boldsymbol{dCRIME}$	$\boldsymbol{dECO}$
$\boldsymbol{logPARTY}$	+	1, 2, 3, 4, 6, 7	2, 6, 7	1, 2, 4, 5, 6
	-	5,	1, 3, 4, 5	3, 7
$\boldsymbol{dCRIME}$	+	3, 6, 7	6,	1, 2, 3, 4, 5, 6, 7
	-	1, 2, 4, 5	1, 2, 3, 4,5, 7	
$\boldsymbol{dECO}$	+	2, 3, 6	1	5, 6
	-	1, 4, 5, 7	2, 3, 4, 5, 6, 7	1, 2, 3, 4, 7

5. Forecast Result

We use data from 2010M1 to 2018M12 as training data to predict testing data which is from 2019M1 to 2019M12.

From Figure 7, prediction of $ARIMA(1,1,1)(2,0,1)_{12}$ of ECO, we can see most of the standardized residual are within -2 to 2, which follow Normal distribution. The Fitted line and the Actual line almost coincide with each other, indicating the model of ECO predicted well. From Figure 8, prediction of $ARIMA(1,1\ 1)(1,0,1)_{12} - GARCH(1,2)$ of $CRIME$, we can see the standardized residual are roughly within -2 to 2, which follow Normal distribution. The Fitted line overlaps the Actual line, indicating the model of $CRIME$ did a fine job on forecasting. From Figure 9, prediction of $ARIMA(1,1,1)(1,0,1)_{12} - GARCH(2,2)$ of $log(PARTY)$, we can see many of the standardized residual are out of the region from -2 to 2. It seems that the prediction of the model of $log(PARTY)$ is not good enough.

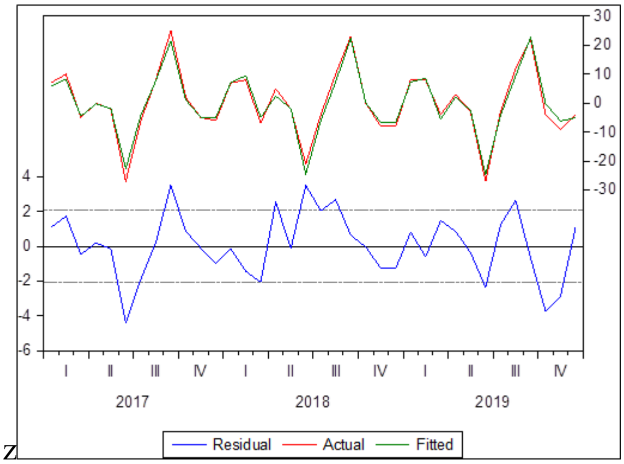

Figure 7 Prediction of $ARIMA(1,1,1)(2,0,1)_{12}$ of ECO

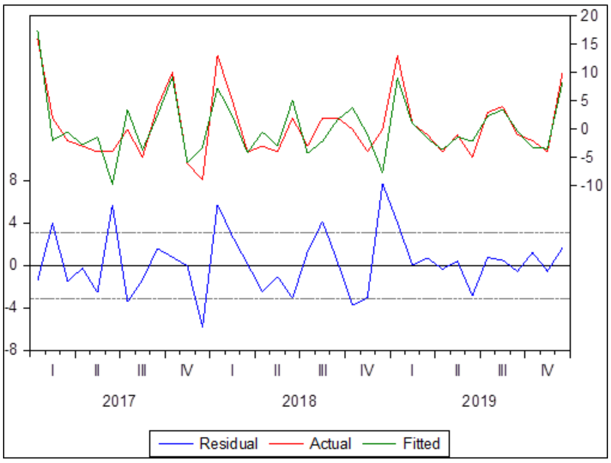

Figure 8 Prediction of $ARIMA - GARCH$ of $CRIME$

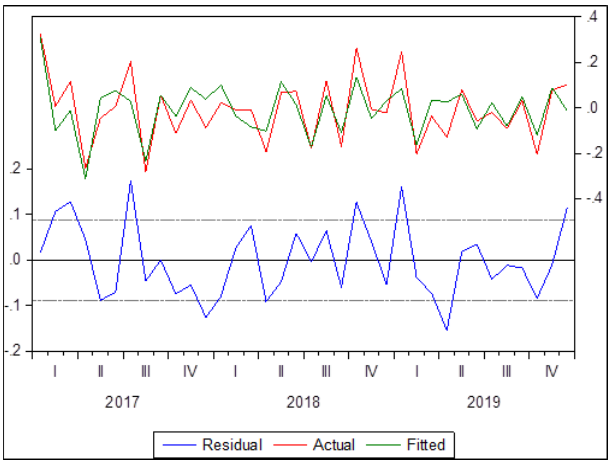

Figure 9 Prediction of AR$IMA - ARCH$ of $log(PARTY)$

6. Reference

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7. Appendix

Table 10 $ARIMA(1,1,1)(2,0,1)_{12}$ of $dECO$

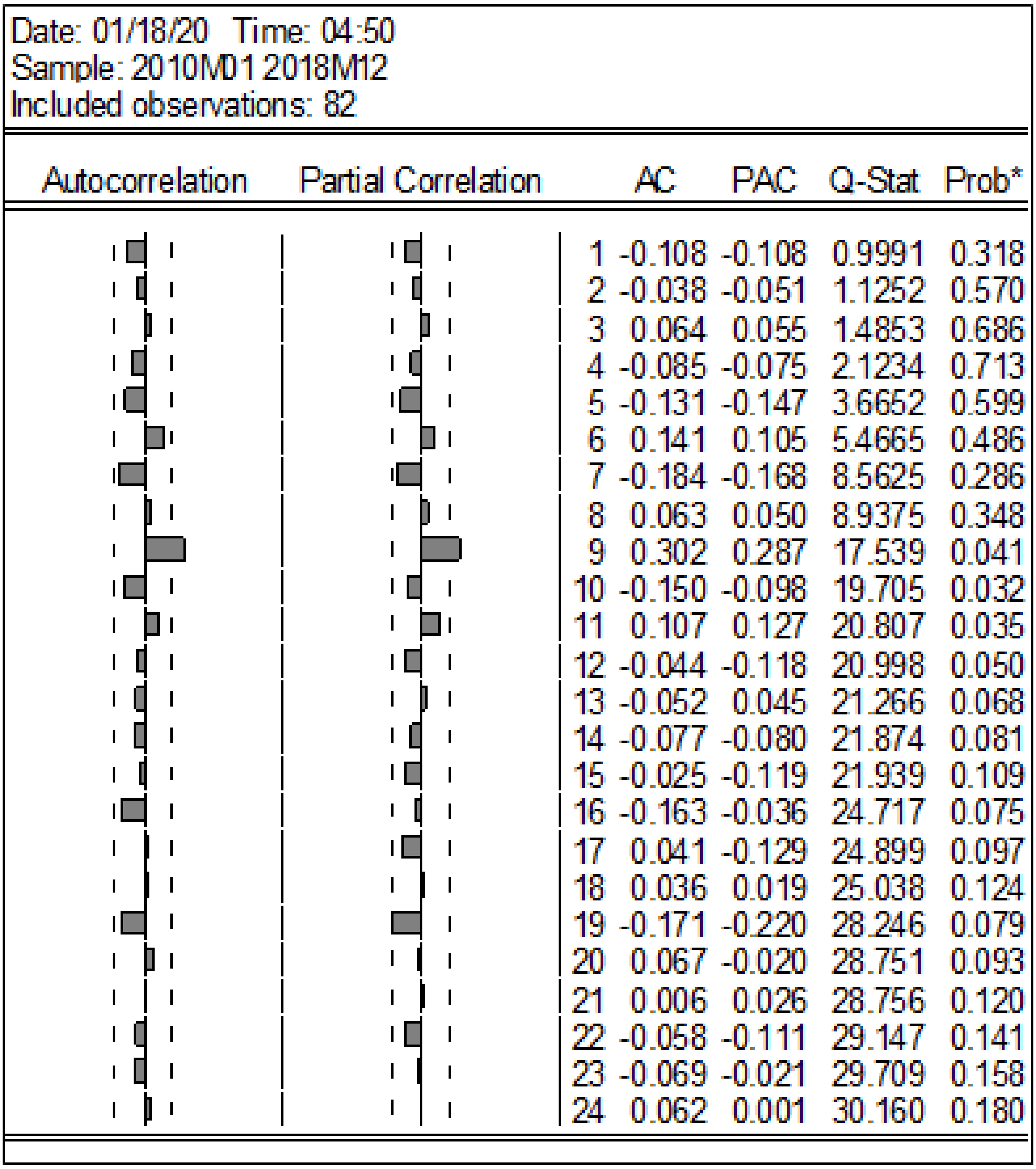
Variable	Coefficient	Std.Error	t-Statistic	Prob
C	0.029348	0.052983	0.553911	0.5809
AR(1)	0.690693	0.087227	7.918383	0
SAR(12)	1.141835	0.004105	278.1765	0
SAR(24)	-0.14184	0.000599	-236.867	0
MA(1)	-1	13.39754	-0.07464	0.9406
SMA(12)	-0.99942	1.20E-04	-8333.69	0
SIGMASQ	4.449785	2.035318	2.186285	0.0311
S.E. of regression		2.182033 Akaike info criterion		4.921407
F-statistic		187.6667 Durbin-Watson stat		2.068446

Table 11 Estimate of $ARIMA(1,0\ 1)(1,0,1)_{12}$ from PARTY

Variable	Coefficient	Std. Error	z-Statistic	Prob
C	-0.04487	0.045099	-0.99488	0.3198
AR(1)	0.113829	0.23797	0.478332	0.6324
SAR(12)	0.661242	0.075708	8.734108	0
SAR(24)	0.248776	0.075109	3.312189	0.0009
MA(1)	-0.54794	0.190734	-2.87279	0.0041
SMA(12)	-0.9042	0.000476	-1900.79	0
Variance Equation				
C	0.000399	0.000465	0.85753	0.3912
RESID(-1)^2	0.180008	0.23951	0.751568	0.4523
RESID(-2)^2	-0.28165	0.284864	-0.9887	0.3228
GARCH(-1)	1.398128	0.291912	4.789555	0
GARCH(-2)	-0.31206	0.350297	-0.89084	0.373
S.E. of regression		0.120607 Akaike info criterion		-1.49283
Durbin-Watson stat				1.828981

Table 12 Estimate of $\boldsymbol{ARIMA}(\mathbf{1},\mathbf{1},\mathbf{1})(\mathbf{1},\mathbf{0},\mathbf{1})_{\mathbf{12}} - \boldsymbol{GARCH}(\mathbf{1},\mathbf{2})$ from dcrime

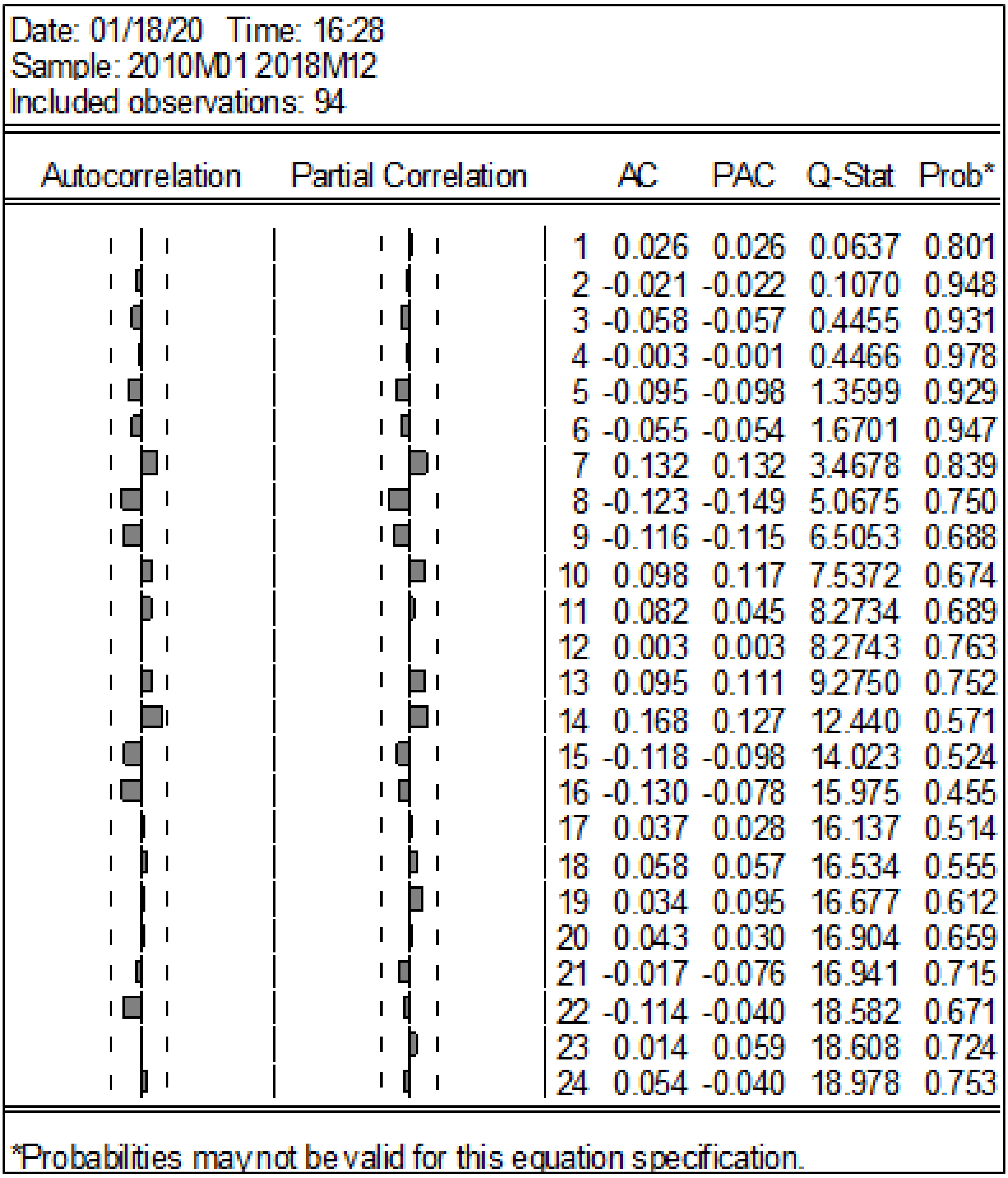
Variable	Coefficient	Std. Error	z-Statistic	Prob
C	1.589074	13.7938	0.115202	0.9083
AR(1)	0.211968	0.192348	1.102003	0.2705
SAR(12)	1.00916	0.074193	13.60174	0
MA(1)	-0.8253	0.080569	-10.2434	0
SMA(12)	-0.90732	0.032291	-28.0979	0
Variance Equation				
C	3.524287	8.188923	0.430373	0.6669
RESID(-1)^2	0.145769	0.152476	0.956018	0.3391
RESID(-2)^2	-0.14896	0.125955	-1.1826	0.237
GARCH(-1)	0.75952	0.621173	1.222719	0.2214
S.E. of regression	3.906438	Akaike info criterion		5.659143
Durbin-Watson stat				1.752551



Date: 01/18/20 Time: 04:50
Sample: 2010M01 2018M12
Included observations: 82

	AC	PAC	Q-Stat	Prob*
1	-0.108	-0.108	0.9991	0.318
2	-0.038	-0.051	1.1252	0.570
3	0.064	0.055	1.4853	0.686
4	-0.085	-0.075	2.1234	0.713
5	-0.131	-0.147	3.6652	0.599
6	0.141	0.105	5.4665	0.486
7	-0.184	-0.168	8.5625	0.286
8	0.063	0.050	8.9375	0.348
9	0.302	0.287	17.539	0.041
10	-0.150	-0.098	19.705	0.032
11	0.107	0.127	20.807	0.035
12	-0.044	-0.118	20.998	0.050
13	-0.052	0.045	21.266	0.068
14	-0.077	-0.080	21.874	0.081
15	-0.025	-0.119	21.939	0.109
16	-0.163	-0.036	24.717	0.075
17	0.041	-0.129	24.899	0.097
18	0.036	0.019	25.038	0.124
19	-0.171	-0.220	28.246	0.079
20	0.067	-0.020	28.751	0.093
21	0.006	0.026	28.756	0.120
22	-0.058	-0.111	29.147	0.141
23	-0.069	-0.021	29.709	0.158
24	0.062	0.001	30.160	0.180

Figure 10 Correlogram of $log(PARTY)$

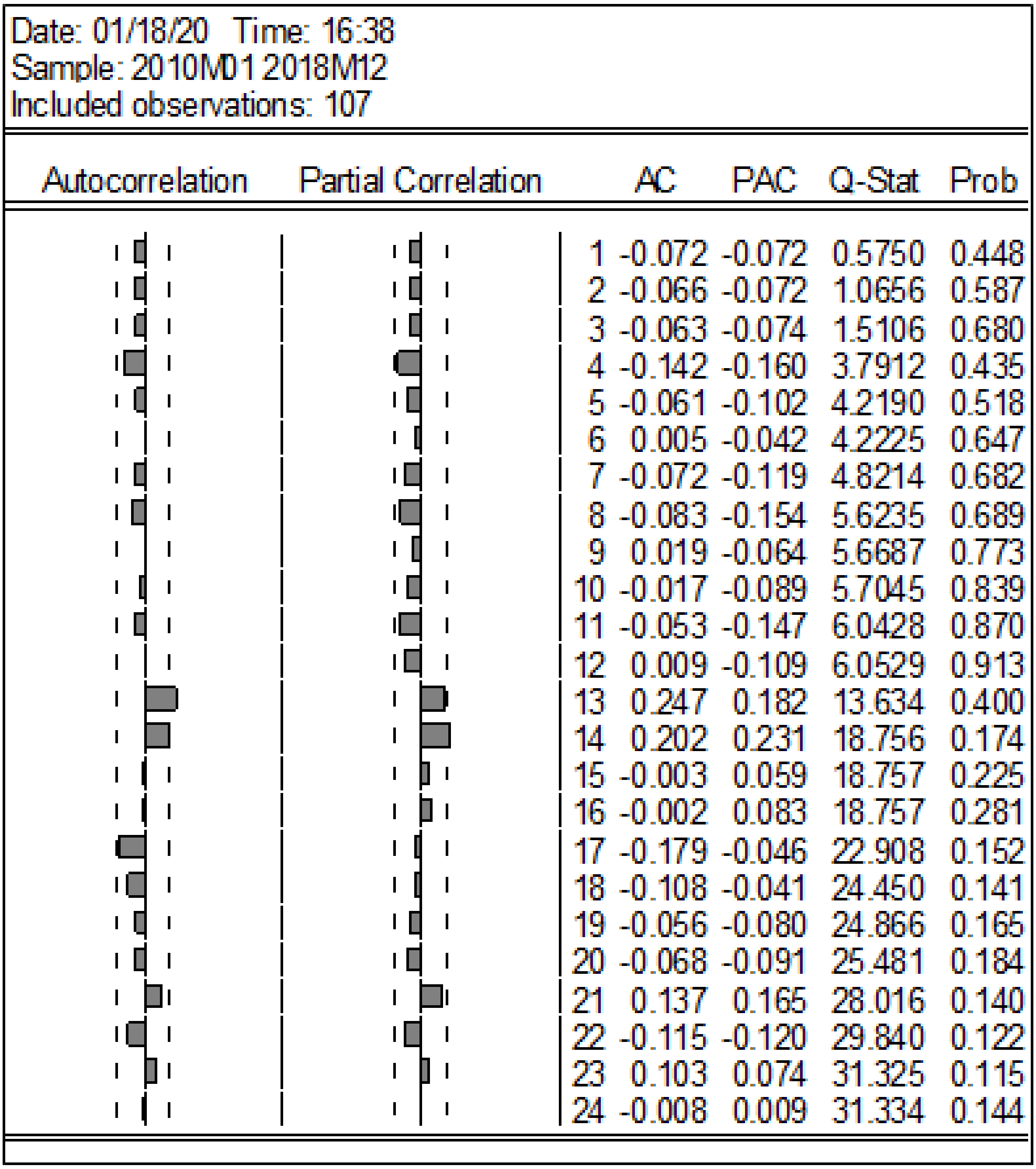


Date: 01/18/20 Time: 16:28
Sample: 2010M01 2018M12
Included observations: 94

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.026	0.026	0.0637	0.801
		2	-0.021	-0.022	0.1070	0.948
		3	-0.058	-0.057	0.4455	0.931
		4	-0.003	-0.001	0.4466	0.978
		5	-0.095	-0.098	1.3599	0.929
		6	-0.055	-0.054	1.6701	0.947
		7	0.132	0.132	3.4678	0.839
		8	-0.123	-0.149	5.0675	0.750
		9	-0.116	-0.115	6.5053	0.688
		10	0.098	0.117	7.5372	0.674
		11	0.082	0.045	8.2734	0.689
		12	0.003	0.003	8.2743	0.763
		13	0.095	0.111	9.2750	0.752
		14	0.168	0.127	12.440	0.571
		15	-0.118	-0.098	14.023	0.524
		16	-0.130	-0.078	15.975	0.455
		17	0.037	0.028	16.137	0.514
		18	0.058	0.057	16.534	0.555
		19	0.034	0.095	16.677	0.612
		20	0.043	0.030	16.904	0.659
		21	-0.017	-0.076	16.941	0.715
		22	-0.114	-0.040	18.582	0.671
		23	0.014	0.059	18.608	0.724
		24	0.054	-0.040	18.978	0.753

*Probabilities may not be valid for this equation specification.

Figure 11 Correlogram of $CRIME$


Date: 01/18/20 Time: 16:38
Sample: 2010M01 2018M12
Included observations: 107

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.072	-0.072	0.5750	0.448
		2	-0.066	-0.072	1.0656	0.587
		3	-0.063	-0.074	1.5106	0.680
		4	-0.142	-0.160	3.7912	0.435
		5	-0.061	-0.102	4.2190	0.518
		6	0.005	-0.042	4.2225	0.647
		7	-0.072	-0.119	4.8214	0.682
		8	-0.083	-0.154	5.6235	0.689
		9	0.019	-0.064	5.6687	0.773
		10	-0.017	-0.089	5.7045	0.839
		11	-0.053	-0.147	6.0428	0.870
		12	0.009	-0.109	6.0529	0.913
		13	0.247	0.182	13.634	0.400
		14	0.202	0.231	18.756	0.174
		15	-0.003	0.059	18.757	0.225
		16	-0.002	0.083	18.757	0.281
		17	-0.179	-0.046	22.908	0.152
		18	-0.108	-0.041	24.450	0.141
		19	-0.056	-0.080	24.866	0.165
		20	-0.068	-0.091	25.481	0.184
		21	0.137	0.165	28.016	0.140
		22	-0.115	-0.120	29.840	0.122
		23	0.103	0.074	31.325	0.115
		24	-0.008	0.009	31.334	0.144

Figure 12 Correlogram of ECO

Table 13 Estimations of VAR Model with std.error in ( ) & t- statistical in [ ]

Standard errors in ( ) & t-statistics in [ ]			
	DLPARTY	DECO	DCRIME
DLPARTY(-1)	-0.330953	-0.65815	-0.50325
	-0.10453	-4.45832	-3.34139
	[-3.16606]	[-0.14762]	[-0.15061]
DLPARTY(-2)	-0.260983	6.960817	6.17778
	-0.11045	-4.71087	-3.53067
	[-2.36285]	[1.47761]	[1.74975]
DLPARTY(-3)	-0.096304	6.320258	5.594572
	-0.11577	-4.93756	-3.70057
	[-0.83187]	[1.28004]	[1.51181]
DLPARTY(-4)	-0.082406	6.739085	5.682555
	-0.11885	-5.06905	-3.79911
	[-0.69336]	[1.32946]	[1.49576]
DLPARTY(-5)	-0.193303	-8.37179	-10.9195
	-0.11929	-5.0876	-3.81301
	[-1.62051]	[-1.64553]	[-2.86374]
DLPARTY(-6)	-0.078232	0.419618	-5.28768
	-0.12026	-5.12934	-3.8443
	[-0.65050]	[0.08181]	[-1.37546]
DLPARTY(-7)	0.084724	-0.23402	-1.309
	-0.11378	-4.85269	-3.63696
	[0.74464]	[-0.04822]	[-0.35992]
DECO(-1)	0.002963	-0.15992	0.007021
	-0.00276	-0.11752	-0.08808
	[1.07523]	[-1.36071]	[0.07971]
DECO(-2)	0.005069	-0.27354	0.352951
	-0.00272	-0.116	-0.08694
	[1.86363]	[-2.35820]	[4.05986]
DECO(-3)	-0.002499	-0.4019	0.107741
	-0.00299	-0.12761	-0.09564
	[-0.83521]	[-3.14943]	[1.12650]
DECO(-4)	-0.001568	-0.09074	0.178112
	-0.00308	-0.13117	-0.09831
	[-0.50999]	[-0.69176]	[1.81174]
DECO(-5)	0.000614	0.145955	0.44116

	-0.00273	-0.11624	-0.08712
	[0.22531]	[1.25565]	[5.06396]
DECO(-6)	0.001859	0.013315	0.276125
	-0.00284	-0.12132	-0.09093
	[0.65356]	[0.10975]	[3.03675]
DECO(-7)	-0.010686	0.226914	0.098075
	-0.00297	-0.12654	-0.09484
	[-3.60190]	[1.79326]	[1.03415]
DCRIME(-1)	-0.000995	0.021985	-0.47485
	-0.00372	-0.15887	-0.11907
	[-0.26714]	[0.13838]	[-3.98797]
DCRIME(-2)	0.00776	-0.13848	-0.57192
	-0.00387	-0.16491	-0.12359
	[2.00702]	[-0.83972]	[-4.62739]
DCRIME(-3)	0.000618	-0.24767	-0.41029
	-0.00373	-0.15918	-0.1193
	[0.16569]	[-1.55584]	[-3.43903]
DCRIME(-4)	0.000188	-0.36813	-0.41203
	-0.00358	-0.15272	-0.11446
	[0.05262]	[-2.41052]	[-3.59982]
DCRIME(-5)	0.000693	-0.5773	-0.27003
	-0.00364	-0.15535	-0.11643
	[0.19036]	[-3.71613]	[-2.31923]
DCRIME(-6)	0.001104	-0.42354	-0.28605
	-0.00339	-0.14447	-0.10827
	[0.32583]	[-2.93178]	[-2.64196]
DCRIME(-7)	-6.66E-07	0.013366	0.023942
	-0.00337	-0.14359	-0.10762
	[-0.00020]	[0.09308]	[0.22247]
C	0.003396	0.36313	0.552238
	-0.01221	-0.52055	-0.39014
	[0.27826]	[0.69759]	[1.41549]
R-squared	0.477819	0.637441	0.578378
Adj. R-squared	0.337232	0.539829	0.464864
Sum sq. resids	1.10857	2016.569	1132.726
S.E. equation	0.119216	5.084628	3.810791
F-statistic	3.398737	6.530346	5.095228
Log likelihood	83.21112	-292.093	-263.255

Akaike AIC	-1.224222	6.28186	5.705089
Schwarz SC	-0.651085	6.854997	6.278227
Mean dependent	0.004069	0.14	0.21
S.D. dependent	0.146438	7.49548	5.209345
Determinant resid covariance (dof adj.)			4.764111
Determinant resid covariance			2.260819
Log likelihood			-466.468
Akaike information criterion			10.64936
Schwarz criterion			12.36877
Number of coefficients			66