Algorithm of Smart Parking Planning - Final Report

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***Abstract*—We consider a linear assignment problem in future smart parking service. This problem was first solved by Hungarian Method in 1955, which is a general approach for linear assignment problems in polynomial time. To handle and supplement the supported situations of Hungarian Method, we implement bio-inspired algorithms with reference to differential evolution, particle swarm optimization, and genetic algorithm. Our bio-inspired algorithm utilizes real-time traffic data and parking spots availability, provides navigation guidance to users by user preferences and attempts to adapt more flexible environments as an approach to generalized assignment problems. With knowledge of the preference and tolerance of users, our algorithm tries to find the parking spots that optimizes prices, traveling time and distance between parking spots and destination in this paper. Our bio-inspired algorithm is evaluated with a simulated experiment in the real city data from Los Angeles and it is shown as a valuable tradeoff compared with outcome from state-of-the-art.**

***Index Terms*—smart parking, differential evolution, particle swarm optimization, genetic algorithm, assignment problem**

I. Introduction

In recent years, the concept of "Smart City" become the trend of future city development. As an indispensable part of land use in urban city planning, vehicle parking becomes a serious problem and requires a proper solution. There is an estimation that around 30% vehicles are wasting 7.8 minutes on the route to seek for a parking spot in the downtown area of our modern city [1]. According to the survey by Michael Manville and Donald Shoup [2], the parking spaces can take up to 18% of land use in the CBD of New York City, 31% in San Francisco and even 81% in Los Angeles. As the city gets more and more crowded with vehicles, a better parking system will be significantly meaningful to improve the city planning. The future parking service should provide practical guidance for drivers to shorten the parking search time in order to increase urban mobility by reducing traffic congestion. As a result, the idle time for on-street parking is also shortened and the total parking revenue will increases due to higher efficiency of parking spot's usage of drivers can find parking spots more quickly [3]. Environmental concerns are the side effect of parking problem. The vehicle cruising for parking spot in a small business area in Los Angeles can burn 47000 gallons of gasoline and produce 730 tons of carbon dioxide [4]. We desire to find a smart parking solution to improve city environment by reducing carbon generation and fuel consumption. Because the information and communication technologies associated with the "Smart City" can enhance the connectivity and efficiency of infrastructures and services, the smart parking is an essential service in the smart city [5].

In proposal, smart parking system requires supports from various sensors built in parking lots to collect information like pricing, traffic rate and space occupancy. These collected information will be passed to the data center for data processing and the smart parking planning application will provide guidance for drivers to find the suitable parking spot depending on user demands and processed data. Since smart city and smart parking integrate knowledge from different disciplines, there are a few changeable areas.

- **Basics Infrastructure:** The sensor density in the current city is not sufficient to support data collection in the smart parking. Therefore, it is important to design an efficient plan of infrastructure construction. One time-consuming and expensive example is to setup sensors spread over all on-street parking spots. City Stratford of province Ontario in Canada had announced their city plan of moving towards smart city by building a smart parking project in 2018 [6] [7]. The pilot project had a fund of $100,000, but newly built sensors in this project merely covered 78 parking spots in the downtown area.
- **Poor Compatibility:** There are some deployed smart parking applications providing detailed information of parking spaces in some cities, such as SFpark [8] in San Francisco and LA Express Park [9] in Los Angeles. However, these systems are only designed for a specific city or under a specific small scopes. It is usually hard to migrate the system to other circumstances. If the same system were to be deployed in another city, the infrastructure and applications require many comprehensive tests [3]. The migration and adjustment of the system will cost a large number of expense.
- **Lack of User Engagement:** Smart parking application is designed to help user find parking space more easily. The application provides drivers a real-time map guidance of parking availability and leads drivers to an empty space with their preferences like lower price or shorter distance. The success of the smart parking services highly depends on user participation and engagement through smart phone applications [10]. Since smart parking is currently in the developing stage, the level of user awareness of

such application is low and the service providers get low profit from the business though the smart parking has a very potential market.

Our project focuses on underlying algorithms of the smart parking planning application. We provide guidances to find the suitable parking concerning multiple attributes, such as distance, pricing and parking space occupancy.

II. Literature Review

Related literature about smart parking system or intelligent parking navigation has become popular since 1980's and there are lots of work talked about the infrastructure of a smart parking system like radio frequency identification for space recognition, reliable monitor for plate identification, stable wireless network (Wi-Fi, 5G, Bluetooth) for real-time update etc. In the previous works, there is a smart parking implementation that could manage automated check-in and check-out process in parking lot areas with an engineered RFID-reader system in [11]. With the vast growth of artificial intelligence, we can implement an integrated camera system to detect space in parking lots and to recognize license plates with some dynamically trained convolutional neural network(CNN) models [12] [13]. Those advanced computer vision systems provided a high speed recognition process and good stability. There are also related works in wireless sensor networks that showed us that we can deploy low-cost sensor nodes with periodic reports to the central database through gateways so that we can enable lots of functionalities like auto toll for timeout, warning for improper parking, online reservation, security monitoring, remote guidance, empty-space tracks etc [14] [15].

Recently, more and more literature talked about the "Internet of Things"(IoT) concept and some proposed IoT-based car parking systems [16] [17]. One proposed a complete IoT-based workflow from terminals to central and from central to clients [18]. The proposed smart parking system discussed all influential factors in the entire loop like computational power, scalability, resource allocations, storage capacity, availability, built analytical schemes to evaluate every role in the environment, and utilized modern smartphone application as potential optimal solutions. On the other hand, there is an interesting parking violation management system proposed in [19]. The proposed method is based on a location-centric IoT system. The system could detect improper parking cars in a short period. It could guide street and parking lot managers to find improper parked cars with lowest cost in order to reduce average waste time due to improper parking.

In the algorithm scope of smart parking systems, there are many factors needed to be considered including number of empty spots, expense of parking, accessibility of the parking lot, distance to destination etc. Among all of these attributes, availability of parking resources must be the most significant one. Some litureate talked about the viability to use machine learning methods to predict user's preference of parking lots based on our current infrastructure and past records. For example, some illustrated techniques to predict availability of parking space by parameterizing influential factors with multiple-level importance and training these attributes through neural networks [20] [21]. Furthermore, a Multiple Attributes Decision Making(MADM) based smart parking algorithm used only three representative decision factors (walking distance towards destination, estimated parking cost, and availability degree of vacant parking spaces) with drivers preferences options to establish a Markov-Chain based network [22]. Relative experimental results showed all preference attributes can be effectively treated as statistical factors in the network and always help users with different preferences to find their proper parking places [23].

All of the former algorithms testified that combinatorial optimization methods work well on smart parking systems because they modeled the attributing factors to quantized parameters. All of these parametrization processes are also constrained by some cost functions like tolerance of parking rate, traveling time between parking spots and destinations. If we can unify these situations we have, for instance assuming all parking spots are unified, all signal lights have the same waiting time, etc., we could simplify the problem to a linear assignment problem as lots of above algorithms did. Fortunately, we have a state-of-the-art algorithm, Hungarian method, that solves the linear assignment problem in a polynomial time [24]. This beautiful method and its derivations have shown their high efficiency and strong robustness on this set of optimization problems. However, our world is diverse and it dynamically changes itself every moment. The setup among different cities are quite different, and that's the main reason that most of our existing solutions are hard to immigrate from city to city. Human factors also influence the validity of this kind of solution. One example is that different people buy different brands and types of vehicle. A Ford Raptors usually occupy space that can hold two Mini Coopers. These kinds of conditions add more uncertainty and sometimes ruin the unified setups, because assignment problems now are no longer linear and become generalized with additional uncertainty. Generalized assignment problems are still NP-hard problems [25]. That is our motivation to construct an implementation to handle generalized situations, because none of the algorithms mentioned above can deal with the difficulty coming with the generalization. Compared with those former algorithms, our bio-inspired algorithm is a better approach to NP-hard problems mixed with differential evolution algorithm, particle swarm optimization and genetic algorithm [26]. Especially for the later two methods, the Table I and Table II show that they also have great performances in linear assignment problems. The extra amount of run time and total cost can be assumed as the compromise towards adaptability of uncertainty. In general, our approach can handle unsupported situations from former algorithms with proper and controllable sacrifices.

III. Problem Formulation and Modeling

The smart parking system receives parking queries in real time. If the smart parking system treats each query separately and processes it once it arrives, the problem can be easily solved by a greedy method. For example, the system guides each vehicle to the cheapest nearest parking lot. However, as the goal is to minimize the total cost in the whole parking system, we would like to process all queries received in a time slice together instead of processing each query immediately. In this case, the problem is turned into a combinatorial optimization problem and the greedy algorithm would usually not give us the optimal solution. In order to better address the problem, we would like to model it as an assignment problem.

The generalized assignment problem is known to be NP-hard [25]. In this project, we would like to simplify the problem as a linear assignment problem. Assume, in a time slice, there are M vehicles sending parking queries and there are N available parking lots in total. Let $V = \{v_1, ..., v_M\}$ denote the set of vehicles and $P = \{p_1, ..., p_N\}$ denote the set of available parking lots. Each vehicle has a start location, a destination and a parking duration, which can be denoted by $v_i.start$, $v_i.dest$ and $v_i.hours$. Each parking lot has a hourly rate, a time limit and a max rate, which can be denoted by $p_i.hr$, $p_i.limit$ and $p_i.max$. Let D denote the driving time matrix, as shown in (1), where d_{ij} denotes the driving time between vehicle $v_i (v_i \in V)$ and the parking space $p_j (p_j \in P)$. Let R, similar as D, denote the rate set where r_ij represents the total rate, as shown in (2), for $v_i (v_i \in V)$ to park at $p_j (p_j \in P)$. Let W denote the walking time matrix, similar as D, where w_{ij} denotes the walking time between the the parking space $p_j (p_j \in P)$ and the i$_{th}$ vehicle's destination $v_i.dest (v_i \in V)$.

$$D_{M,N} = \begin{pmatrix} d_{1,1} & d_{1,2} & \cdots & d_{1,N} \\ d_{2,1} & d_{2,2} & \cdots & d_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ d_{M,1} & d_{M,2} & \cdots & d_{M,N} \end{pmatrix} \tag{1}$$

$$r_{ij} = \begin{cases} v_i.hours * p_j.hr & \text{if } v_i.hours * p_j.hr < p_j.max \\ p_j.max & \text{otherwise} \end{cases} \tag{2}$$

Then, we would like to define a cost matrix $C_{M,N}$, as shown in (3), where $c_{i,j}$ denotes the cost of assigning v_i to p_j. We also add a penalty term $\alpha * r_{ij} * w_{ij}$ to consider the trade-off between parking rate and walk time, where α is the rate coefficient could be adjusted.

$$c_{ij} = \begin{cases} d_{ij} + w_{ij} + \alpha * r_{ij} * w_{ij} & \text{if eligible } (v_i.h <= p_j.limit) \\ \infty & \text{otherwise} \end{cases} \tag{3}$$

Then, we could have the solution matrix X where x_{ij} is defined in in (4).

$$x_{ij} = \begin{cases} 1 & \text{if } v_i \text{ is guided to } p_j \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Finally, we could define our total cost in (5).

$$cost = \sum_{i=1}^{M} \sum_{j=1}^{N} c_{ij} \times x_{ij} \tag{5}$$

Now, we could define our optimization problem as finding a solution matrix X to minimize (5) and satisfy (6) which describe the condition that each vehicle gets assigned to exactly one parking lot and each parking lot gets assigned to at most one vehicle.

$$\begin{cases} \sum_{j=1}^{N} x_{ij} = 1 \\ \\ \sum_{i=1}^{M} x_{ij} \leq 1 \end{cases} \tag{6}$$

IV. Proposed Solution

As the problem is turned into a linear assignment problem, the well-known Hungarian method could be a way to solve this problem [24]. The time complexity of this algorithm is $O(N^4)$. In a later work, the algorithm is improved to achieve a time complexity of $O(N^3)$ [27]. However, since the smart parking problem can be easily extended to a generalized assignment problem by changing parking lots to parking zones, we would still like to explore a few bio-inspired algorithms. There are a lot of strategies focusing on solving generalized assignment problems and there is no widely accepted winner. We finally decide to implement two of them, the Particle Swarm Optimization and the Genetic Algorithm. Although, in our specific case, these algorithms may not perform as well as the Hungarian method, we could still see their great potential via the comparative study later in section V.

A. Particle Swarm Optimization

Particle Swarm Optimization (PSO) is a computational method that optimizes a problem by iteratively trying to improve a candidate solution with regard to a given objective function [26]. It solves a problem by having a population of candidate solutions (particles) and moving these particles around in the search-space using a position-velocity update method. Each particle's movement is influenced by its local best known position, but is also guided toward the best known positions in the search-space, which are updated as better positions are found by other particles. This is expected to move the swarm toward the best solutions. The position update process can be defined in (7), where $v_i(t+1)$ is the velocity defined in (8). Here, $c1$ and $c2$ are the cognitive and social parameters respectively. They control the particle's behavior given two choices: (1) to follow its personal best or (2) follow the swarm's global best position, which are denoted by y and $\hat{y}$. r_{1j} and r_{2j} are random numbers. In addition, a parameter w controls the inertia of the swarm's movement.

$$x_i(t+1) = x_i(t) + v_i(t+1) \tag{7}$$

$$\begin{aligned} v_{ij}(t+1) = & w * v_{ij}(t) + c_1 r_{1j}(t)[y_{ij}(t) - x_{ij}(t)] \\ & + c_2 r_{2j}(t)[\hat{y}_j(t) - x_{ij}(t)] \end{aligned} \tag{8}$$

Another advantage of PSO is that it does not use the gradient of the problem being optimized, which means PSO does not require that the optimization problem be differentiable. This characteristic provide us with a great flexibility when developing our own objective function because it is much easier to address the constraints. A brief description of our objective function is shown in algorithm 1. As described above, because the objective function doesn't have to be differentialbe, we could take the indices of the parking lots as the decision variable instead of the whole decision matrix. In this case, we only need to address one of the constraints. And we could also enforce the constraints by simply add a very large term to the cost.

Algorithm 1: PSO Objective Function

Input : Decision vector x of length M where x_i is denoting that the v_i is assigned to p_{x_i}. Cost matrix $C_{M,N}$
Output: Total cost $cost$
Initialize a decision matrix $Y_{M,N}$ with zeros
while i *in range of* M **do**
  $Y_{i,x_i} = 1$
end
$cost = \sum y_{ij} * c_{ij}$
$penalty = 0$
for *each* j **do**
  if $\sum_{i=1}^{M} y_{ij} > 1$ **then**
    $penalty = \infty$
  end
end
return $cost + penalty$

B. Genetic Algorithm

The genetic algorithm works in a very similar way as PSO does. It also takes a population of solutions and continuously let them evolve. All GAs requires some form of recombination, as this allows the creation of new solutions that have, by virtue of their parent's success. The main difference between the PSO approach and GA is that PSO does not have genetic operators such as crossover and mutation. Particles update themselves with the internal velocity; they also have a memory important to the algorithm. Also, in PSO only the 'best' particle gives out the information to others. Additionally, GA is originally for discrete problems and PSO are for continuous problems. Therefore, GA should have a good performance in our senario.

We use a similar objective function for GA as the one mentioned in the above section. More results can be seen in the later secions of the paper.

V. Performance Evaluation

A. Dataset and Test Cases

For the analysis and evaluation of the algorithms, we use a parking dataset constructed based on open database provided by Los Angeles Department of Transportation (LADOT) [28] [29]. The LADOT database contain basic information, parking restriction and space occupancy of 33989 on-street metered parking spaces in the City of Los Angeles. To construct a compatible dataset for simple testing, we chose 298 parking spaces within six street blocks in downtown area.

To simulate the scenario when multiple drivers try to find parking lots at the same time, we randomly chose 10 pairs of start points and destination within downtown Los Angeles. The real positions are shown in Fig. 1. The blue and red markers represent start points and destinations respectively, while the orange and green markers represent occupied or vacant parking space respectively.

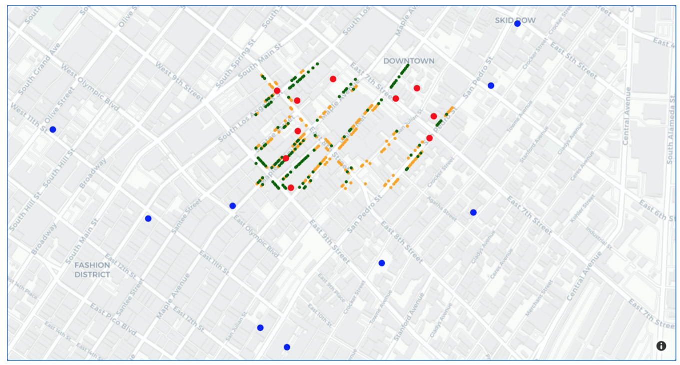


Fig. 1: Start/Destination and parking spaces for testing

B. Experiments and Results

We run experiments using five algorithms including basic greedy, Hungarian, differential evolution, PSO and genetic algorithm on our dataset to optimize the total cost among 10 test cases. Firstly, we analyze the parking space assignments and paths plotted by each algorithm. Next, we compare the total cost and run time of algorithms.

1) Routing Path Comparison: To get a clear view of the solution found by each algorithm, we chose to display 3 paths from start point to assigned parking space and destination. Fig. 2 and Fig. 6 show the paths found by each algorithm. Table I lists the parking space ID assigned to each driver by each tested algorithm.

Since our problem is defined as a linear assignment problem, it is expected to be solved in polynomial time. From Fig. 2 and Fig. 3, we can see that greedy and Hungarian algorithm agree on path 5 and path 6. Even though the driver 4 is assigned to different parking space, the difference of walking distances between the destination and the two parking spaces is not obvious. In Fig. 4 of Differential Evolution Algorithm, it is easy to see that the parking assignments are not optimal and we will later see that the total cost of differential evolution algorithm is indeed the highest among all algorithms. Fig. 5 and Fig. 6 show the results of PSO and genetic algorithm. The genetic algorithm assign driver 6 to the same parking space as greedy and Hungarian algorithm does. The three path results from PSO and genetic algorithm are reasonable under the problem formulation.

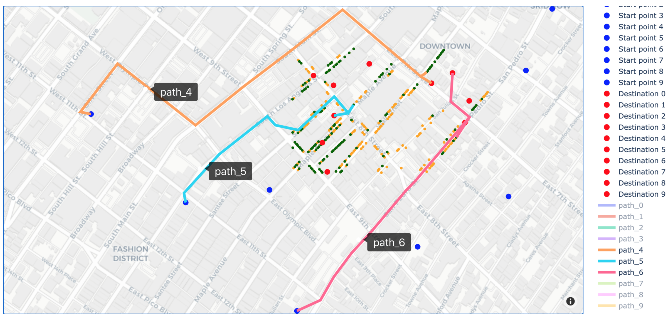


Fig. 2: Greedy Algorithm

Fig. 3: Hungarian Algorithm

Fig. 4: Differential Evolution Algorithm

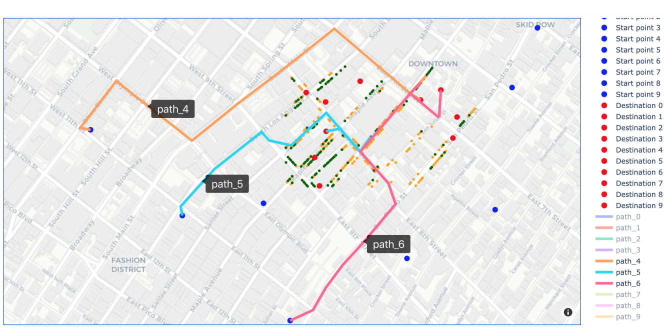


Fig. 5: Particle Swarm Optimization Algorithm

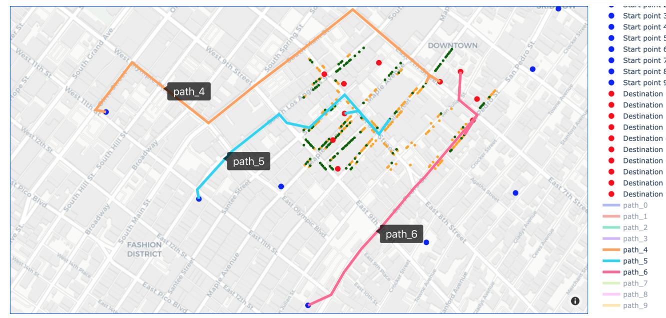


Fig. 6: Genetic Algorithm

TABLE I: The parking space ID assigned to each driver by tested algorithms

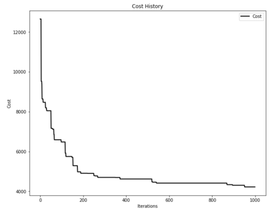
	Parking space assigned to Driver 4	**Parking space assigned to Driver 5**	**Parking space assigned to Driver 6**
Greedy	#72	#73	#97
Hungarian	#50	#73	#97
DE	#115	#117	#105
PSO	#88	#24	#106
GA	#50	#24	#97

2) Algorithm Performance Comparison: Table II shows the total cost and actual run time of five tested algorithms. Fig. 7 shows the plots of cost convergence of particle swarm optimization and genetic algorithm.

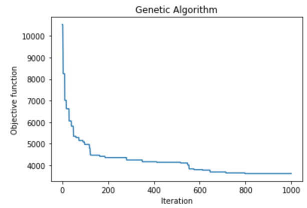
As shown in Table II, there are no much difference between the total cost and run time of greedy and Hungarian algorithm. This is because these two algorithms are relatively efficient for solving linear problem. Also, We see that the total costs of PSO and genetic algorithm are close to that of greedy and Hungarian algorithm. From previous routing path analysis, we notice that some of the results from PSO and genetic algorithm are the same as that from greedy and Hungarian algorithm. The total cost comparison result is consistent with the path comparison. However, PSO and genetic algorithm spend more time to converge to a final result. Differential evolution algorithm has the highest total cost, but it's runtime is less than PSO and genetic algorithm. From Fig. 7, we see that the cost decreases rapidly before 200 iterations in both PSO and genetic algorithm. In the first 100 iterations, genetic algorithm converges slightly faster than PSO algorithm. After 800 iterations, both costs tend to reach the final result of the optimization.

TABLE II: Total cost and runtime of tested algorithms

Algorithms	**Total Cost**	**Run Time (sec)**
Greedy	3490.92	0.000319719
Hungarian	3479.15	0.00041604
DE	8890.26	13.7536
PSO	4219.9	40.3403
GA	3619.14	59.606



(a) Cost convergence of particle swarm optimization algorithm



(b) Cost convergence of genetic algorithm

Fig. 7: Cost convergence of SI and GA

VI. Conclusions and Recommendations

We formulated the smart parking problem as a linear assignment problem to optimize the total cost of all assignments taking account of the maximum parking rate, the driving time from start point to parking space and the walking time from parking to final destination. Using real-world parking space data and simulated test cases, we evaluated five optimization algorithm, which are basic greedy, Hungarian, differential evolution, PSO and genetic algorithm. The final results show that Hungarian provides lowest total cost solution within a short runtime. Even though the bio-inspired algorithm PSO and genetic algorithm perform well on optimization, their long runtime is not feasible for solving linear assignment problem.

In this project, our real-world dataset only consists of single-space on-street parking. As a result, the smart parking problem can be modelled as a simpler problem. We can slightly refine the problem as assigning vehicles to parking lots with capacity greater than 1. The problem then turns into a more complicated problem known as generalized assignment problem. The generalized assignment problem is NP-hard so that Hungarian algorithm can not solve it within polynomial time. In this case, the bio-inspired algorithms can be used to find feasible solutions. The future smart parking system will also highly relies on the infrastructure to collect real-time data in each parking lot. The real-time data in real-world situation can improve the accuracy of the algorithm.

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