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Questão 1

① Variáveis livres: z

Variáveis ligadas: y, a, x

② $(\lambda y.\lambda a.(\lambda x.\lambda y.\ y\ x\ z)(y\ a))((\lambda x.\lambda y.\ x)(\lambda x.\lambda y.\ x))$

A expressão contém 3 β-redexes.

$\underline{(\lambda y.\lambda a.(\lambda x.\lambda y.\ y\ x\ z)(y\ a))((\lambda x.\lambda y.\ x)(\lambda x.\lambda y.\ x))}$

$\to_\beta \lambda a.(\lambda x.\lambda y.\ y\ x\ z)(((\lambda x.\lambda y.\ x)(\lambda x.\lambda y.\ x))\ a)$

$(\lambda y.\lambda a.\underline{(\lambda x.\lambda y.\ y\ x\ z)(y\ a)})((\lambda x.\lambda y.\ x)(\lambda x.\lambda y.\ x))$

$\to_\beta (\lambda y.\lambda a.\ (\lambda u.\ u\ (y\ a)\ z))((\lambda x.\lambda y.x)(\lambda x.\lambda y.\ x))$

(note-se a α-conversão de y para u)

No caso do último redex:

$(\lambda y.\lambda a.(\lambda x.\lambda y.\ y\ x\ z)(y\ a))(\underline{(\lambda x.\lambda y.\ x)(\lambda x.\lambda y.\ x)})$

$\to (\lambda y.\lambda a.(\lambda x.\lambda y.\ y\ x\ z)(y\ a))\ (\lambda v.(\lambda x.\lambda y.\ x))$

No último termo note-se a α-conversão da variável ligada y para v.

③ a) $\lambda x.\ x$
b) $(\lambda x.\ x)\ z$
c) $(\lambda x.\lambda y.\ y)((\lambda x.\ x\ x)(\lambda x.\ x\ x))$
d) $((\lambda x.\ x\ x)(\lambda x.\ x\ x))$

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Questão 2

① $F\ B\ (I\ B)\ (K\ K\ B)\ (K\ I)$

$= \underline{(F\ B)}(I\ B)\ (K\ K\ B)\ (K\ I)$

$\to_\beta \underline{((\lambda b.\ b)\ (I\ B))}(K\ K\ B)\ (K\ I)$

$\to_\beta \underline{(I\ B)}\ (K\ K\ B)\ (K\ I)$

$\to_\beta \underline{B\ (K\ K\ B)}\ (K\ I)$

$\to_\beta \underline{(\lambda y.\ \lambda z.\ (K\ K\ B)\ (y\ z))\ (K\ I)}$

$\to_\beta (\lambda z.\ (\underline{K\ K}\ B)\ ((K\ I)\ z))$

$\to_\beta (\lambda z.\ \underline{(K\ B)}\ ((K\ I)\ z))$ ✗

$\to_\beta (\lambda z.\ B\ (\underline{(K\ I)}\ z))$ ✗

$\to_\beta (\lambda z.\ B\ \underline{(I\ z)})$

$\to_\beta (\lambda z\ .\ B\ z)$

$\to_\alpha (\lambda u.\ B\ u) = (\lambda u.\ \underline{(\lambda x.\ \lambda y.\ \lambda z.\ x\ (y\ z))\ u})$

$\to_\beta B$ 0.5

B é uma forma normal

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② $B\ (F\ I) = (\lambda x.\lambda y.\lambda z.\ x\ (y\ z))\ ((\lambda a.\lambda b.\ b)\ (\lambda x.x))$

Seja EXP o λ-termo anterior. Se for tipificável, então para qualquer X tal que $EXP \rightarrow_\beta^* X$, X terá o mesmo tipo de EXP.

$B\ (F\ I)$ (ordem applicativa)

$\rightarrow_\beta\ B\ (\lambda b.\ b)$

$\rightarrow_\beta\ \lambda y.\ \lambda z.\ (\lambda b.\ b)\ (y\ z)$

$\rightarrow_\beta\ \lambda y.\ \lambda z.\ y\ z$, que se pode tipificar assim: $(\lambda y:(\alpha \rightarrow \beta).\ \lambda z:\alpha.\ y\ z) : (\alpha \rightarrow \beta) \rightarrow \alpha \rightarrow \beta$

Logo o tipo de EXP será $EXP : (\alpha \rightarrow \beta) \rightarrow \alpha \rightarrow \beta$

Com tipos nas variáveis:

$(\lambda x:\gamma.\ \lambda y:(\alpha \rightarrow \beta).\ \lambda z:\alpha.\ x\ (y\ z))\ ((\lambda a:\delta.\ \lambda b:\beta.\ b)\ (\lambda x:\pi.\ x)) : (\alpha \rightarrow \beta) \rightarrow \alpha \rightarrow \beta$ ✓

onde $\gamma \doteq \beta \rightarrow \beta$

Questão 3

① Seja $\alpha \doteq \lambda x.\ \lambda l.$ listcase l of $(\langle x, 0\rangle, \lambda h.\ \lambda t.$ if $h > 0$ then $\langle h, x\rangle$ else $\langle x, h\rangle)$

let emp $\equiv \alpha$
    par $\equiv 8$
    lis $\equiv 9 :: (5+2) :: nil$
in emp $((\lambda y.\ y * y)$ par$)$ lis

$\doteq (\lambda emp.\ \lambda par.\ \lambda lis.\ emp\ ((\lambda y.\ y*y)\ par)\ lis)\ \alpha\ 8\ (9 :: (5+2) :: nil)$

$\rightarrow_\beta^3\ \alpha\ ((\lambda y.\ y*y)\ 8)\ (9 :: (5+2) :: nil)$

$\rightarrow_\beta^2$ listcase $(9 :: (5+2) :: nil)$ of $(\langle(\lambda y.\ y*y)\ 8, 0\rangle,$
$\lambda h.\ \lambda t.$ if $h > 0$ then $\langle h, (\lambda y.\ y*y)\ 8\rangle$
else $\langle(\lambda y.\ y*y)\ 8, h\rangle$
$)$

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$\rightarrow_\beta$ $(\lambda h. \lambda t.$ if $h > 0$ then $\langle h, (\lambda y. y * y)\, 8\rangle$
else $\langle (\lambda y. y * y)\, 8, h\rangle)$
9
$(5+2) :: nil$

$\rightarrow_\beta^2$ if $9 > 0$ then $\langle 9, (\lambda y. y * y)\, 8\rangle$
else $\langle (\lambda y. y * y)\, 8, 9\rangle$

$\rightarrow_\beta$ $\langle 9, (\lambda y. y * y)\, 8\rangle$

② Seja $\pi_1 \doteq l : List\ Bool,\ f : Int \rightarrow List\ Bool \rightarrow Bool$

1. $\pi_1 \vdash \lambda n.$ listcase l of $(True, \lambda h. \lambda t.\ h \wedge (f\ n\ t)) : Int \rightarrow Bool$
1.1. $\pi_2 \vdash$ listcase l of $(True, \lambda h. \lambda t.\ h \wedge (f\ n\ t)) : Bool$
1.1.1. $\pi_2 \vdash l : List\ Bool$
1.1.2. $\pi_2 \vdash True : Bool$
1.1.3. $\pi_2 \vdash \lambda h. \lambda t.\ h \wedge (f\ n\ t) : Bool \rightarrow List\ Bool \rightarrow Bool$
1.1.3.1. $\pi_3 \vdash \lambda t.\ h \wedge (f\ n\ t) : List\ Bool \rightarrow Bool$
1.1.3.1.1. $\pi_4 \vdash h \wedge (f\ n\ t) : Bool$
1.1.3.1.1.1. $\pi_4 \vdash h : Bool$
1.1.3.1.1.2. $\pi_4 \vdash (f\ n)\ t : Bool$
1.1.3.1.1.2.1. $\pi_4 \vdash f\ n : List\ Bool \rightarrow Bool$
1.1.3.1.1.2.1.1. $\pi_4 \vdash f : Int \rightarrow List\ Bool \rightarrow Bool$
1.1.3.1.1.2.1.2. $\pi_4 \vdash n : Int$
1.1.3.1.1.2.2. $\pi_4 \vdash t : List\ Bool$

onde $\pi_2 = \pi_1,\ n : Int$
$\pi_3 = \pi_2,\ h : Bool$
$\pi_4 = \pi_3,\ t : List\ Bool$

③ a) letrec $fun \equiv \lambda l.$ listcase l of $(nil, \lambda h. \lambda t.$
listcase t of $(n :: nil, \lambda h1. \lambda t1.$
$h :: fun\ ((h + h1) :: t1)))$

in fun

$\langle exp \rangle ::= \text{empty} \mid \text{node } \langle exp \rangle\ \langle exp \rangle$
$\mid \text{treecase } \langle exp \rangle \text{ of } (\langle exp \rangle, \langle exp \rangle) \mid \ldots$

$\langle type \rangle ::= \ldots$
$\mid \text{RoseTree } \langle type \rangle$

...s de inferência de tipos

$$\frac{}{\Gamma \vdash \text{empty} : \text{RoseTree } \theta} \qquad \frac{\Gamma \vdash e_1 : \theta \quad \Gamma \vdash e_2 : \text{List } \theta}{\Gamma \vdash (\text{node } e_1\ e_2) : \text{RoseTree } \theta}$$

$$\frac{\Gamma \vdash e : \text{RoseTree } \theta \quad \Gamma \vdash e_1 : \theta' \quad \Gamma \vdash e_2 : \theta \to \text{RoseTree } \theta \to \theta'}{\Gamma \vdash \text{treecase } e \text{ of } (e_1, e_2) : \theta'}$$

...cfm $::= \ldots$
$\mid \text{empty} \mid \text{node } \langle cfm \rangle\ \langle cfm \rangle$

...s de avaliação

$$\frac{e \Rightarrow z \quad e_1 \Rightarrow z_1}{\ldots e\ e_1 \Rightarrow \text{node } z\ z_1} \qquad \frac{e \Rightarrow \text{nil} \quad e_1 \Rightarrow z}{\text{treecase } e \text{ of } (e_1, e_2) \Rightarrow z}$$

$$\frac{e \Rightarrow \text{node } z\ z_1 \quad e_2\ z\ z_1 \Rightarrow z_2}{\text{treecase } e \text{ of } (e_1, e_2) \Rightarrow z_2}$$

...tree

sumTree $\equiv \lambda t.\ \text{treecase } t \text{ of } (0, \lambda n. \lambda ts.$
$(\text{sumTreeList } ts) + n)$

sumTreeList $\equiv \lambda l.\ \text{listcase } l \text{ of } (0, \lambda t1. \lambda tts.$
$(\text{sumTree } t1) + \text{sumTreeList } tts)$

sumTree

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(4) empty $\doteq @1\langle\rangle$
node $\doteq \lambda z . \lambda z_1 . @2\langle z, z_1\rangle$

treecase e of $(e_1, e_2) \doteq$ sumcase $(\lambda\langle\rangle . e_1, \lambda\langle x, y\rangle . e_2\ x\ y)$

RoseTree $\langle type\rangle \doteq$ Unit $+ \langle type\rangle \times$ list (RoseTree $\langle type\rangle$)

Questão 3

b) letrec map $\equiv \lambda f . \lambda l .$ listcase l of (nil, $\lambda h . \lambda t . f\ h ::$ map $f\ t$)
fun $\equiv \ldots$ (*)
in fun (rec ($\lambda l .\ 1 ::$ map $(\lambda x .\ x + 4)\ l$))

(*) Ver Questão 3 a) numa folha anterior, peço desculpa pela confusão.