

## 1.5 Polynomial Functions and Complex Zeros

Name: \_\_\_\_\_

Hour: \_\_\_\_\_ Date: \_\_\_\_\_

**Directions:** For each of the following polynomials, state the total number of zeros, determine all real zeros of the polynomial and the multiplicity of each zero.

1.  $f(x) = -2x(x - 5)^3(x + 1)^2$

Total Number of Zeros: 6

Zeros (with multiplicity):

$0m^1 \quad 5m^3 \quad -1m^2$

3.  $y = 4x^3(x^2 - 2x - 8)(x^2 + x - 20)$

Total Number of Zeros: 7

Zeros (with multiplicity):

$0m^3, 4m^1, -2m^1, -5m^1, -4m^1$

2.  $g(x) = x^3(x + 4)(x - e)$

Total Number of Zeros: 5

Zeros (with multiplicity):

$\pm 4m^1, em^1, 0m^3$

4.  $p(x) = x^3 + 10x^2 + 25x$

Total Number of Zeros: 3

Zeros (with multiplicity):

$x^3 + 10x^2 + 5x$

**Directions:** Given the following complex zeros of a polynomial function, determine the conjugate that must also be a zero for the given polynomial.

5.  $x = 3 - 2i$

$3 - 2i$

6.  $x = -4 + 6i$

$-4 + 6i$

7.  $x = -3i$

$3i$

**Directions:** A polynomial has the following zeros. Determine the least possible degree of the polynomial.

8.  $x = 0, \quad x = -1$  (multiplicity 3),  $x = 5$  (multiplicity 2), and  $x = -1 + 5i$

4

2

2

**Directions:** Factor the following expressions, if possible. If the expression cannot be factored or factored further, write "Not Factorable".

9.  $(x^2 - 49)(x^2 + 5x - 14)$

 $\frac{7}{2}$ 

10.  $x^3 + 5x^2 + 6x$

11.  $(x^2 + 4)(x^2 + 9)$

$(x+7)(x-7)(x+7)(x-2)$

Not Factorable

$x(x+3)(x+2)$

Directions: The following tables give values of several polynomial functions. Determine the degree of each polynomial.

12.

$x$	$f(x)$
1	5
4	7
7	4
10	-2
13	-9
16	-15

Quadratic

13.

$x$	$g(x)$
-3	3
-1	4
1	4
3	3
5	1
7	-2

Quadratic

14.

$x$	$k(x)$
-4	12
-2	7
0	2
2	-3
4	-8
6	-13

Linear

15.

$x$	$p(x)$
0	-7
4	-5
8	-7
12	-8
16	-5
20	3

Quartic

Quartic

Quartic

16. Identify whether the following polynomials are even, odd, or neither. Show algebraic work for justification.

a)  $r(x) = -x^3 + 4x$

$$r(-x) = -(-x)^3 + 4(-x)$$

$$\in x^2 - 4x$$

Odd

b)  $p(x) = x(x-6)^2$

$$p(-x) = -x(-x-6)^2$$

neither

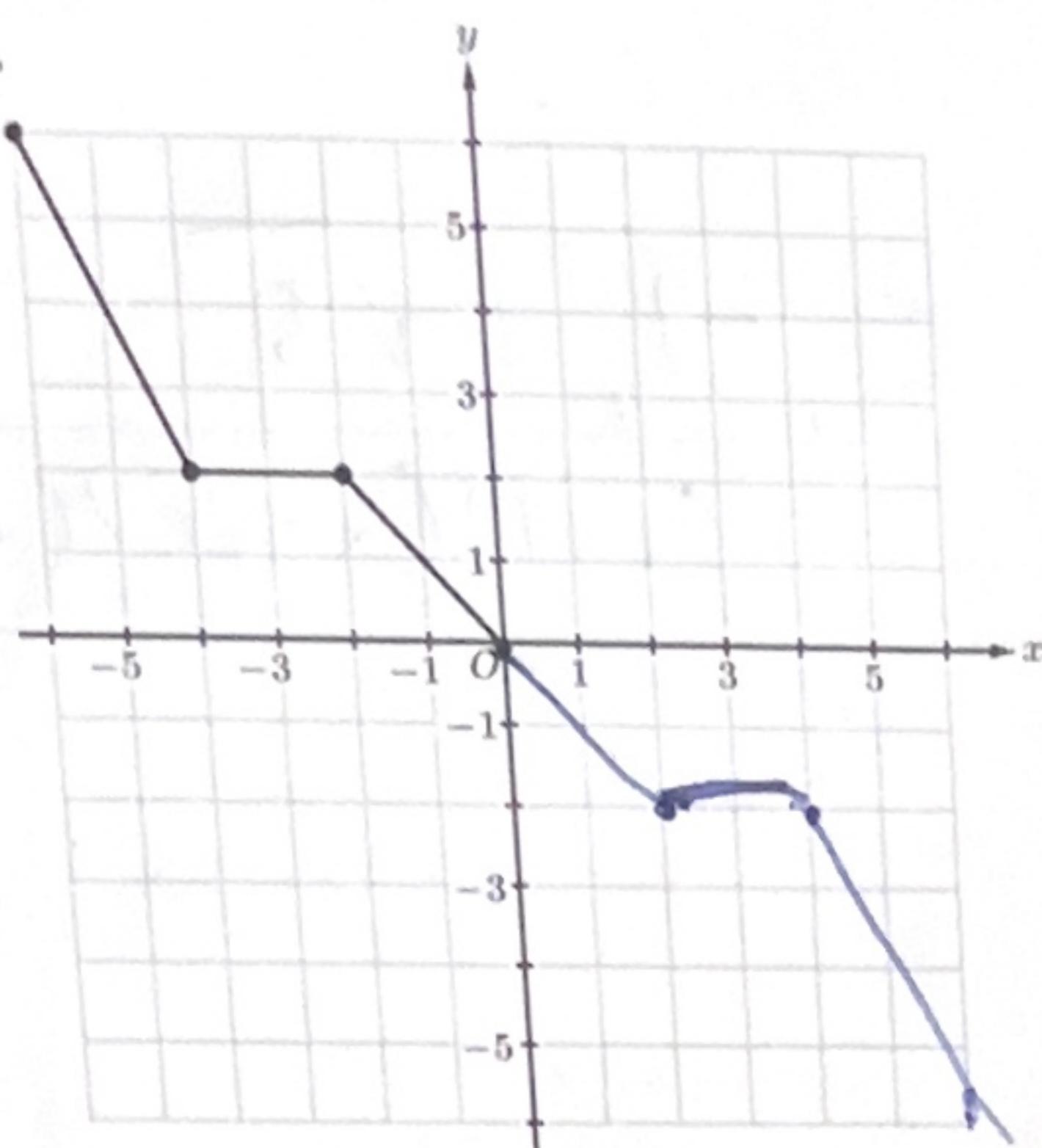
c)  $y = -x^4 + 5x^2 - 3$

$$-(-x)^4 + 5(-x)^2 - 3$$

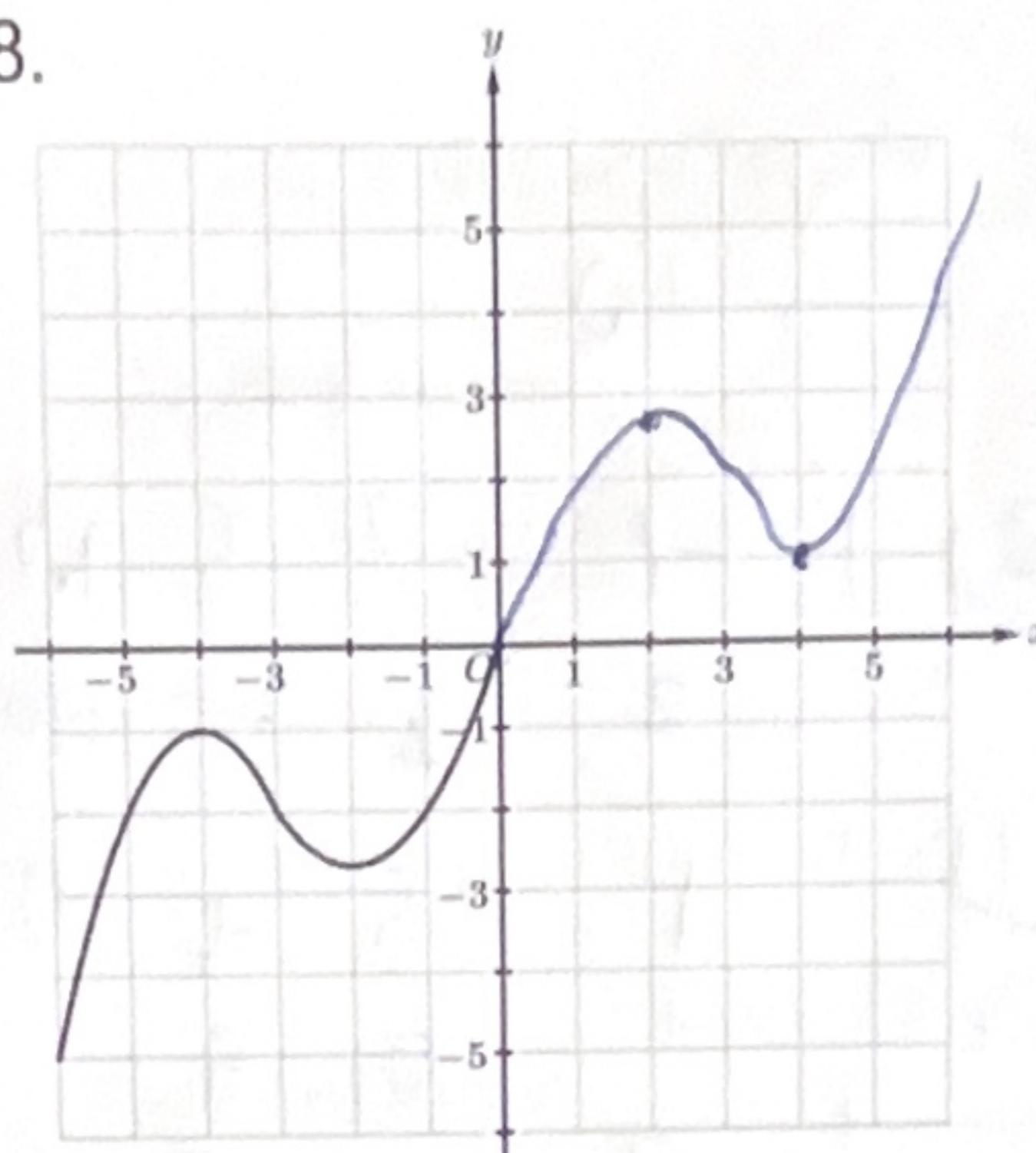
Even

**Directions:** The graphs of two odd functions are given below on the interval  $-6 \leq x \leq 0$ . Use properties of odd functions to sketch the graph of each function on the interval  $0 \leq x \leq 6$ .

17.

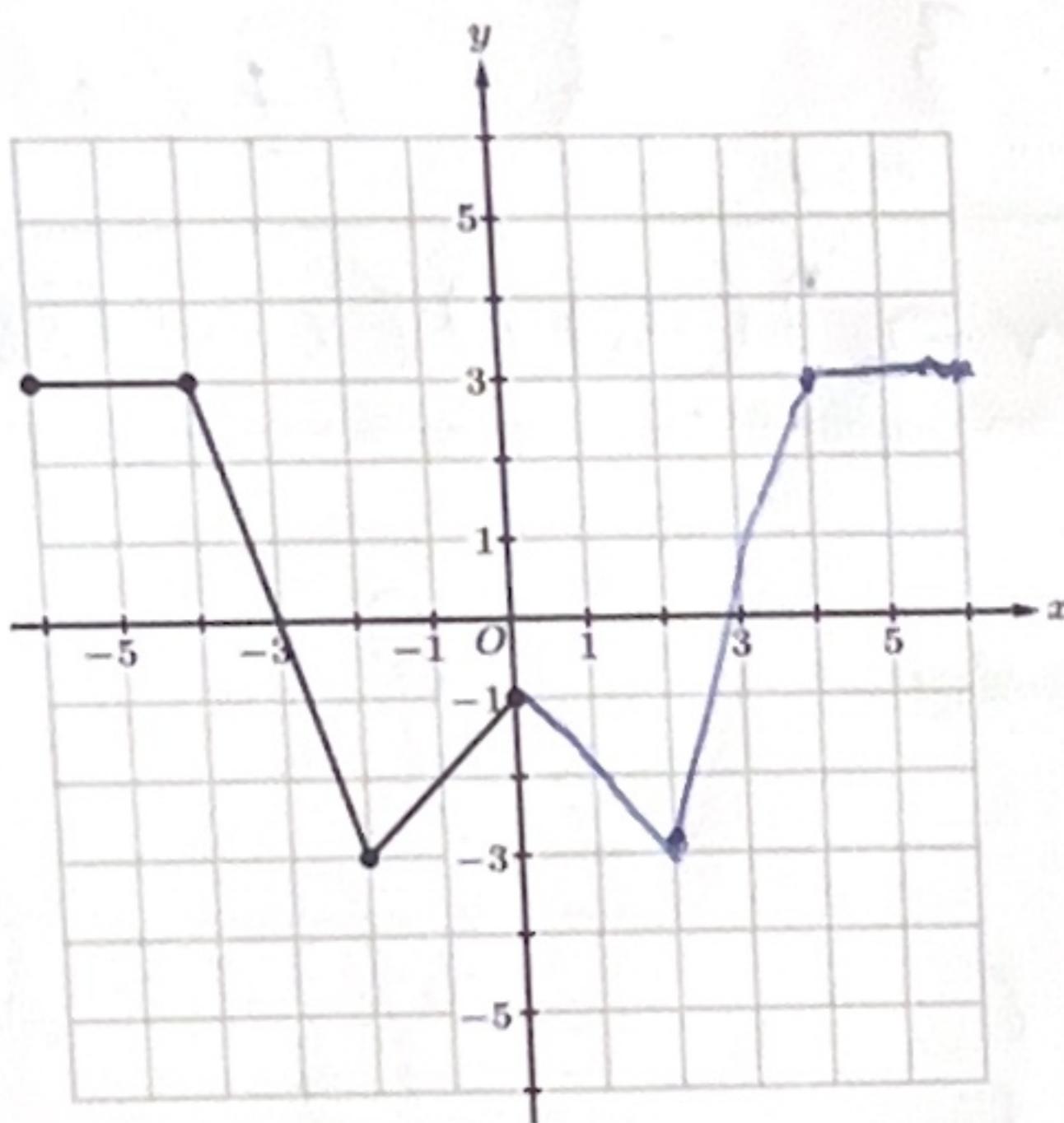


18.

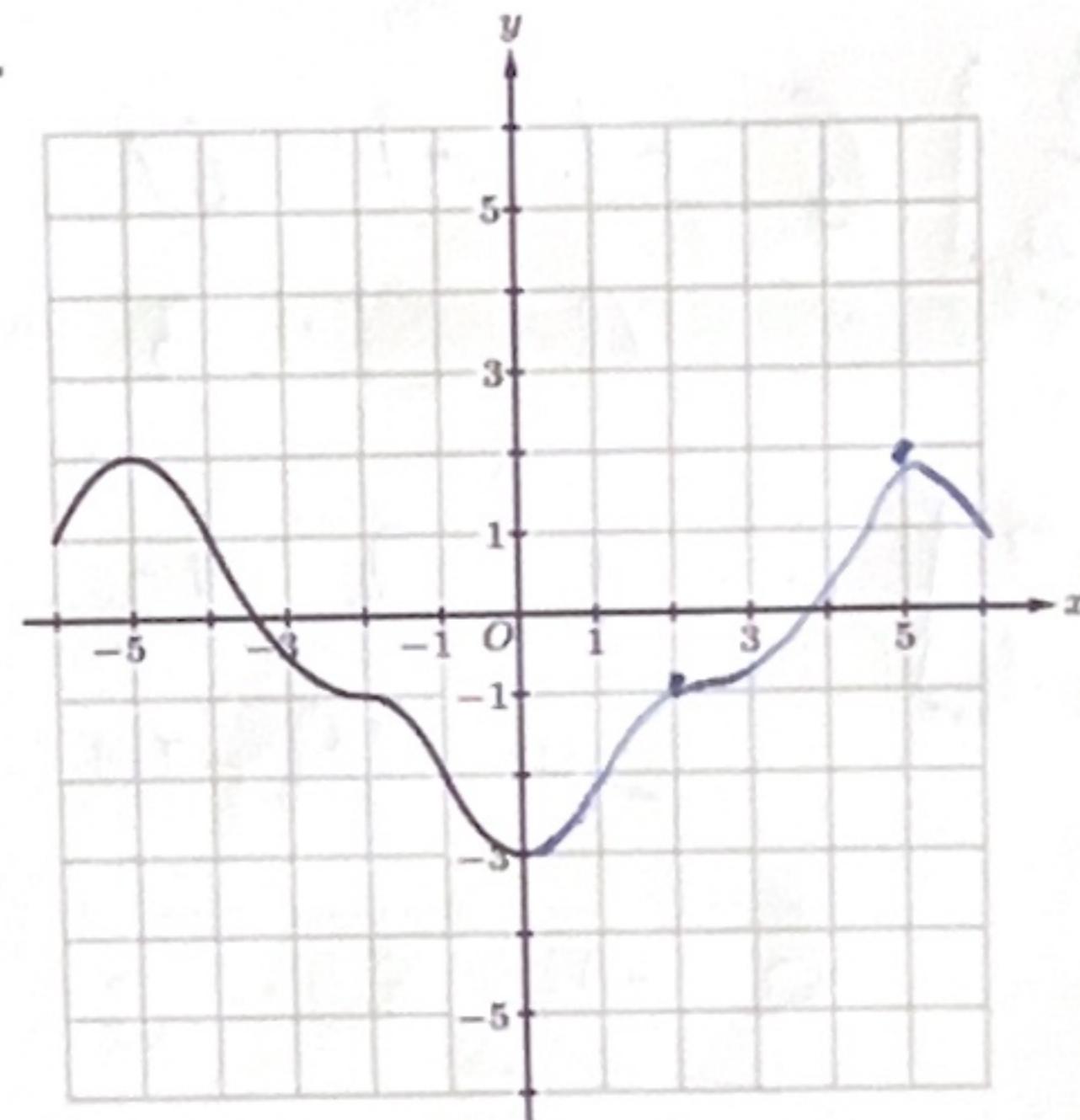


**Directions:** The graphs of two even functions are given below on the interval  $-6 \leq x \leq 0$ . Use properties of even functions to sketch the graph of each function on the interval  $0 \leq x \leq 6$ .

19.



20.



21. Given the point  $(3, -4)$  that is on  $h(x)$  find another point on  $h(x)$  given the following:  
 a)  $h(x)$  is even      b)  $h(x)$  is odd

$$(-3, -4)$$

$$(-3, 4)$$

22. Let  $g$  be an odd function that is strictly increasing. Selected values of  $g(x)$  are given in the table below. Find the values of the constants  $a, b$ , and  $c$ .

$x$	$a$	-4	-1	1	$b$	12	17
$g(x)$	-17	-11	$a+b$	$c$	11	17	23

$$a = -12$$

$$b = 4$$

$$c = -8$$

end here  
for now

Find all zeros for the polynomials. Show algebraic work and factors.

23.  $f(x) = x^4 - x^3 - 7x^2 + 5x + 10$

1 2 5 10

$$\begin{array}{r} 2 \Big) 1 \quad -1 \quad -7 \quad 5 \quad 10 \\ \quad \quad 2 \quad \quad 2 \quad -10 \quad -10 \\ \hline -1 \quad 1 \quad -5 \quad -5 \quad 0 \end{array}$$

$$\begin{array}{r} -1 \quad 0 \quad 5 \\ \quad \quad 1 \quad 0 \quad -5 \quad 0 \end{array}$$

$$x^2 - 5$$

24.  $f(x) = 6x^4 - 7x^3 - x^2 + 6x - 105$

$$\begin{array}{r} -\frac{7}{3} \quad \frac{3}{2} \Big) 6 \quad -7 \quad -1 \quad 67 \quad -105 \\ \quad \quad 9 \quad 3 \quad 3 \quad 108 \end{array}$$

4 6 30

144 - 120

$$\begin{array}{r} -\frac{7}{3} \Big) 6 \quad 2 \quad 2 \quad 10 \quad 0 \\ \quad \quad -14 \quad 28 \quad -10 \end{array}$$

$$\begin{array}{r} 12 \pm \sqrt{-576} \\ 12 \pm 24i \\ \hline 12 \end{array}$$

$$1 \pm 2i$$

$$\begin{array}{r} 6 \quad -12 \quad 30 \quad 0 \\ \hline 6x^2 - 12x + 30 \end{array}$$

$$\begin{array}{r} 2 \cdot 6 \\ 2 \cdot 6 \end{array}$$

$$\begin{array}{r} 18 \quad 2 \quad 6 \\ 2 \quad 2 \quad 3 \quad 5 \\ \hline 6 \quad 10 \\ 2 \quad 3 \end{array}$$

25.  $f(x) = x^3 + x^2 - 8x - 6$

$$\begin{array}{r} -3 \Big) 1 \quad 1 \quad -8 \quad -6 \\ \quad \quad -3 \quad 6 \quad 6 \end{array}$$

Zeros:  $-3, 1 \pm \sqrt{3}$   
Equation:  $(x+3)(x-1+\sqrt{3})(x-1-\sqrt{3})$

$$\begin{array}{r} 12 \quad 1 \quad -2 \quad -2 \quad 0 \\ 4 + 8 \end{array}$$

$$2 \pm \sqrt{ }$$

$$x^2 - 2x - 2$$

$$\frac{2 \pm 2\sqrt{3}}{2}$$

$$1 \pm 1\sqrt{3}$$

Write the polynomial equation that has the following in both factored & standard form.

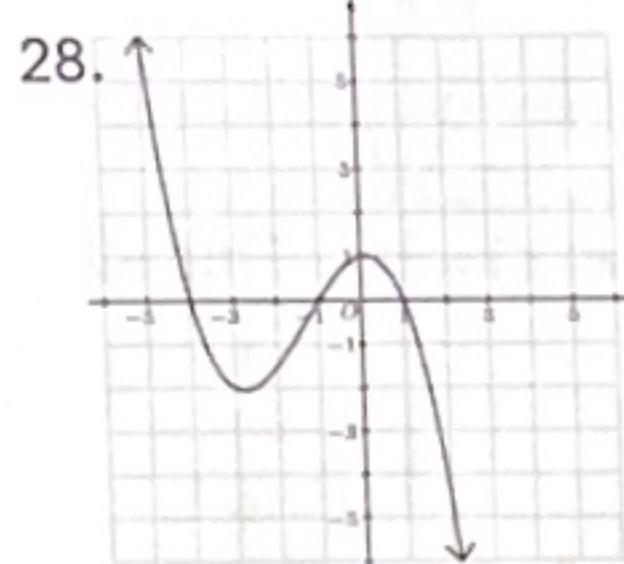
26. degree 3 with zeros 5i and 4 with y-intercept of -10

$$\begin{aligned} & \text{Graph of } y = (x^2 + 25)(x - 4) \\ & \text{y-intercept: } (0, -100) \\ & \text{zeros: } x = 4, x = -5i \\ & \text{factored form: } (x^2 + 25)(x - 4) \\ & \text{standard form: } x^3 - 4x^2 + 25x - 100 \end{aligned}$$

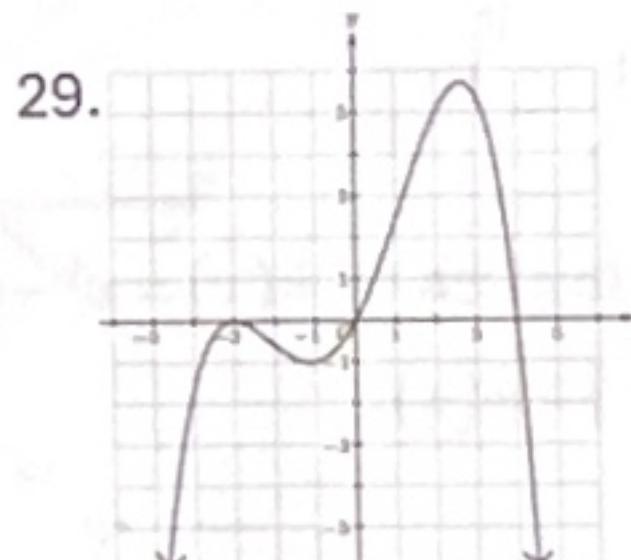
27. degree 2 with zeros  $1 - \sqrt{2}$

$$\begin{aligned} & \text{Graph of } y = (x - 1 + \sqrt{2})(x + 1 - \sqrt{2}) \\ & \text{zeros: } x = 1 - \sqrt{2}, x = -1 + \sqrt{2} \\ & \text{factored form: } (x - 1 + \sqrt{2})(x + 1 - \sqrt{2}) \\ & \text{standard form: } x^2 - 2x + 1 - 2 \end{aligned}$$

**Directions:** The graphs of several polynomial functions are shown below. Use the graphs to find all intervals of  $x$  that satisfy the given conditions.



Graph of  $f$



Graph of  $g$

28a.  $f(x) < 0$ :  $(-4, -1) \cup (1, \infty)$

28b.  $f(x) = 0$ :  $\{-4, -1, 1\}$

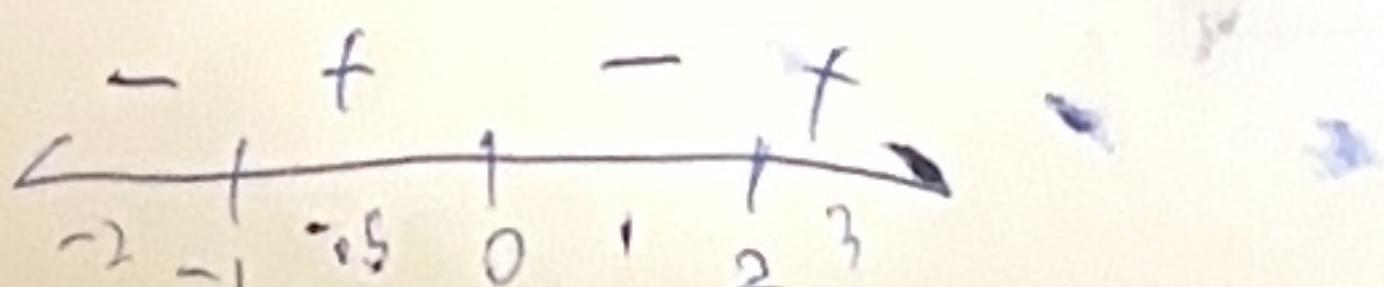
29a.  $g(x) \leq 0$ :  $(-\infty, 0] \cup [4, \infty)$

29b.  $g(x) > 0$ :  $(0, 4)$

**Directions:** Solve the Graph of  $h$  qualities. Show a sign chart and give solution in graph notation.

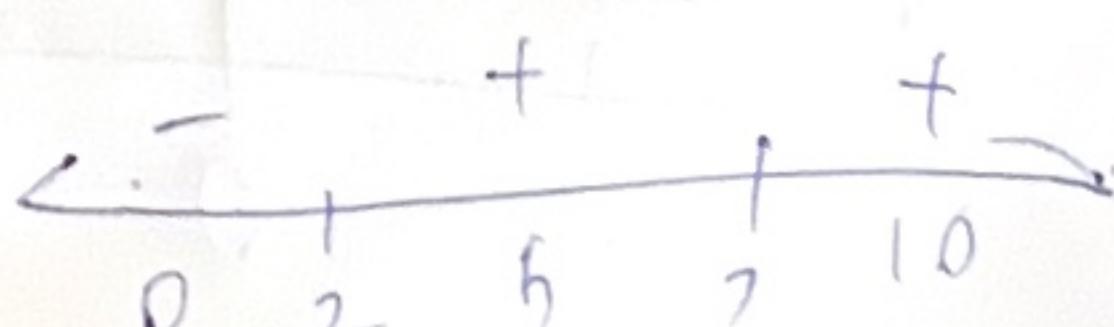
30.  $3x(x+1)(x-2) < 0$

$$\begin{array}{ccccccc} 0 & -1 & 2 & & & & \\ - & + & - & & & & \\ + & + & + & & & & \end{array} \quad (-\infty, -1) \cup (0, 2)$$



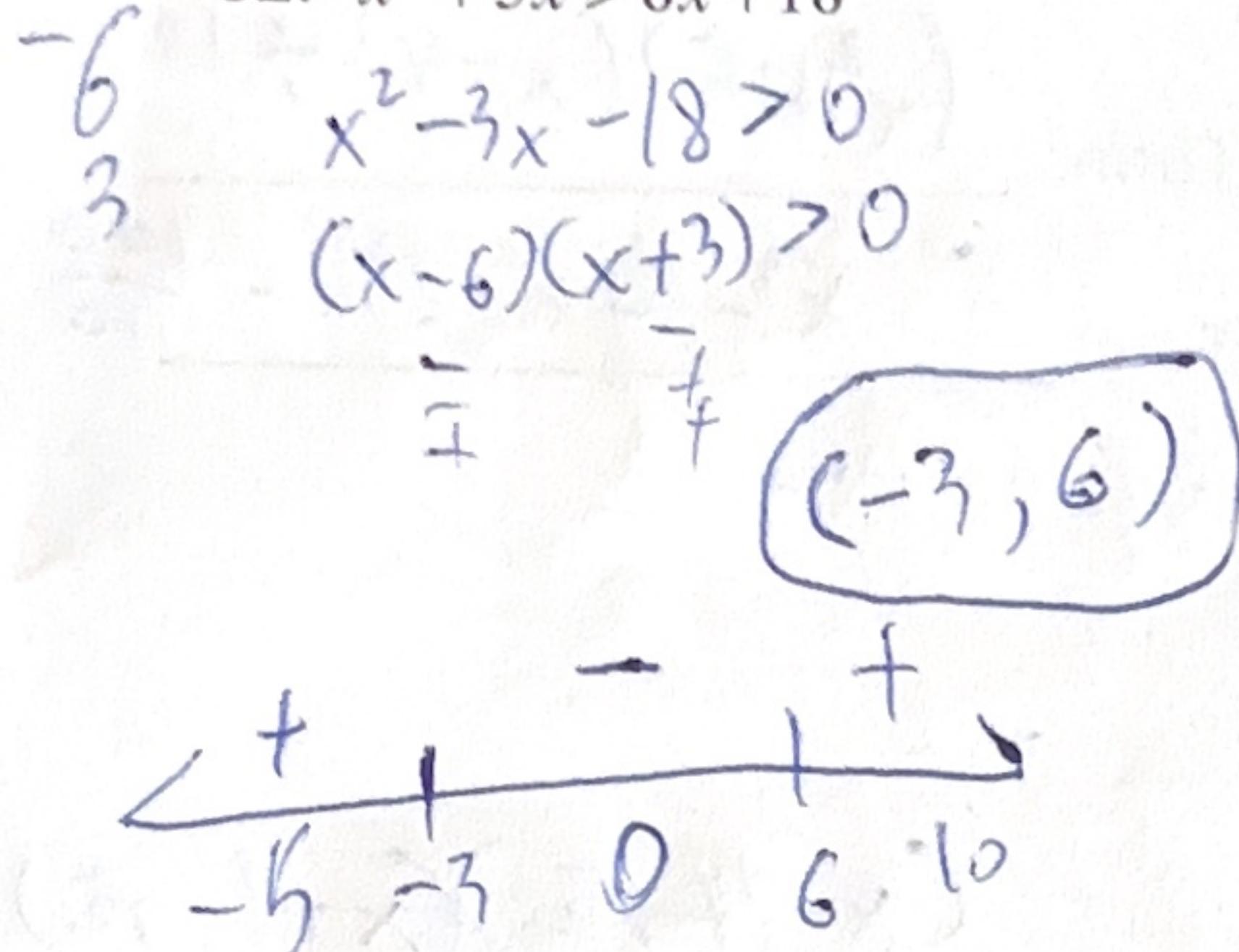
31.  $4(x-2)(x-7)^2 \geq 0$

$$\begin{array}{ccccccc} 2 & 7 & & & & & \\ - & + & & & & & \\ + & + & & & & & \end{array} \quad [2, \infty)$$



**Directions:** Solve the following inequalities. Show a sign chart and give solutions in interval notation.

32.  $x^2 + 5x > 8x + 18$



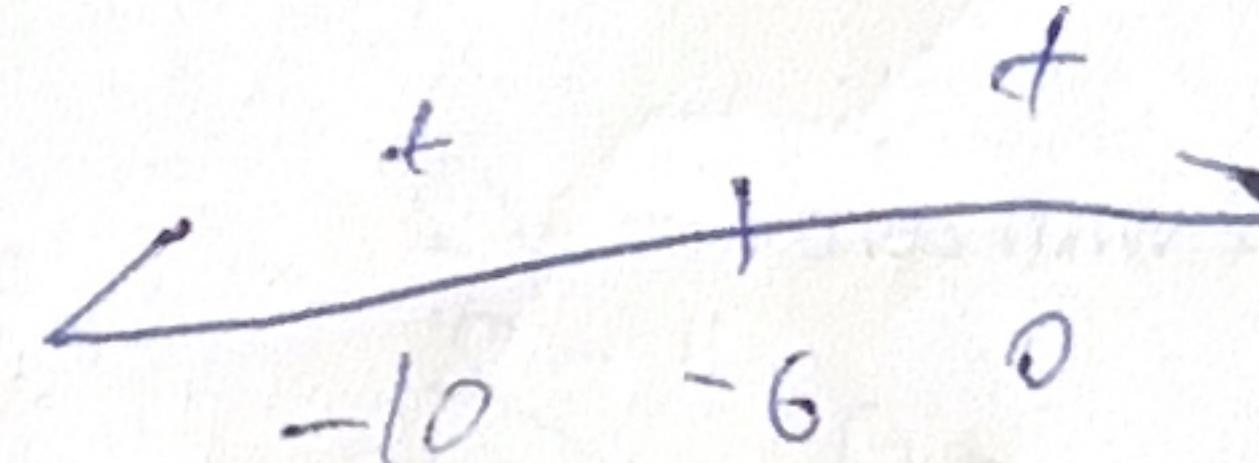
33.  $x^2 - 7x < 5x - 36$

$$x^2 - 12x + 36 < 0$$

$$(x-6)(x-6) < 0$$

$$(x-6)^2 < 0$$

*N/A*



34. The function  $f$  is given by  $f(x) = 2x^2 + 5x - 1$  and the function  $g$  is given by  $g(x) = x^2 + x + 4$ . Find all intervals where  $f(x) \geq g(x)$ .

$$2x^2 + 5x - 1 \geq x^2 + x + 4$$

$$x^2 + 4x - 5 \geq 0$$

$$(x+5)(x-1) \geq 0$$

$(-5, 1)$

35. The function  $h$  is given by  $h(x) = x^4 - 6x^2$  and the function  $k$  is given by  $k(x) = 3x^2$ . Find all intervals where  $h(x) > k(x)$ .

$$x^4 - 6x^2 > 3x^2$$

$$x^4 - 9x^2 > 0$$

$$x^2(x^2 - 9)$$

$$x^2(x+3)(x-3)$$

$$0 \quad -3 \quad 3$$

$$x^2 \quad (x+3) \quad (x-3)$$

$(-\infty, -3) \cup (3, \infty)$

