HAPC
Unit 1
1.9 & 1.10 Rational Functions: VA & Holes

Name: ____________________

Hour:_____ Date: ____________

For each of the following rational functions on # 1-6, determine any values of x where the graph has a hole or vertical asymptote. If there is a hole, also give it as a coordinate point.

1. $f(x)=\dfrac{(x-1)(x-5)}{(x-5)(x+2)}$ $\quad \dfrac{x-1}{x+2}$ $\quad \dfrac{4}{7}$

VA: $x=-2$

Hole: $(5, \frac{4}{7})$

2. $g(x)=\dfrac{(x+3)(x-1)}{(x-3)(x+1)}$

VA: $x=3$ $x=-1$

Hole: N/a

3. $h(x)=\dfrac{(x+4)(\cancel{x-6})}{(x-6)(\cancel{x-6})}$

VA: $x=6$

Hole: ~~x=~~ N/a

4. $k(x)=\dfrac{(x-8)(x+2)^2}{x(x-1)\cancel{(x+2)}}$ $\quad (x-8)(x+2)$

VA: $x=0, 1$

Hole: $(-2, 0)$

5. $r(x)=\dfrac{x^2+x-6}{x^3-4x}$ $\quad \dfrac{(x+3)\cancel{(x-2)}}{x(x+2)\cancel{(x-2)}}$ $\quad -2, -4$ $\quad 8$

VA: $x=0, 2$

Hole: $(2, \frac{5}{8})$

$\dfrac{(x+3)}{x(x+2)}$ $\quad 2, 4 \quad 8$

6. $p(x)=\dfrac{x^2-1}{x^2+1}$ $\quad \dfrac{(x+1)(x-1)}{x^2+1}$

VA = N/A

Hole = N/a

7. Graph the function without a calculator. $k(x)=\dfrac{x^2+2x-3}{x^2-1}$ $\quad \dfrac{(x+3)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}}$ $\quad \dfrac{4}{2}$

Domain: $\mathbb{R}$ $x \neq -1, 1$

Hole: $(1, 2)$

Vertical Asymptote: $x=-1$

Horizontal Asymptote: $y=1$

Slant Asymptote: N/a

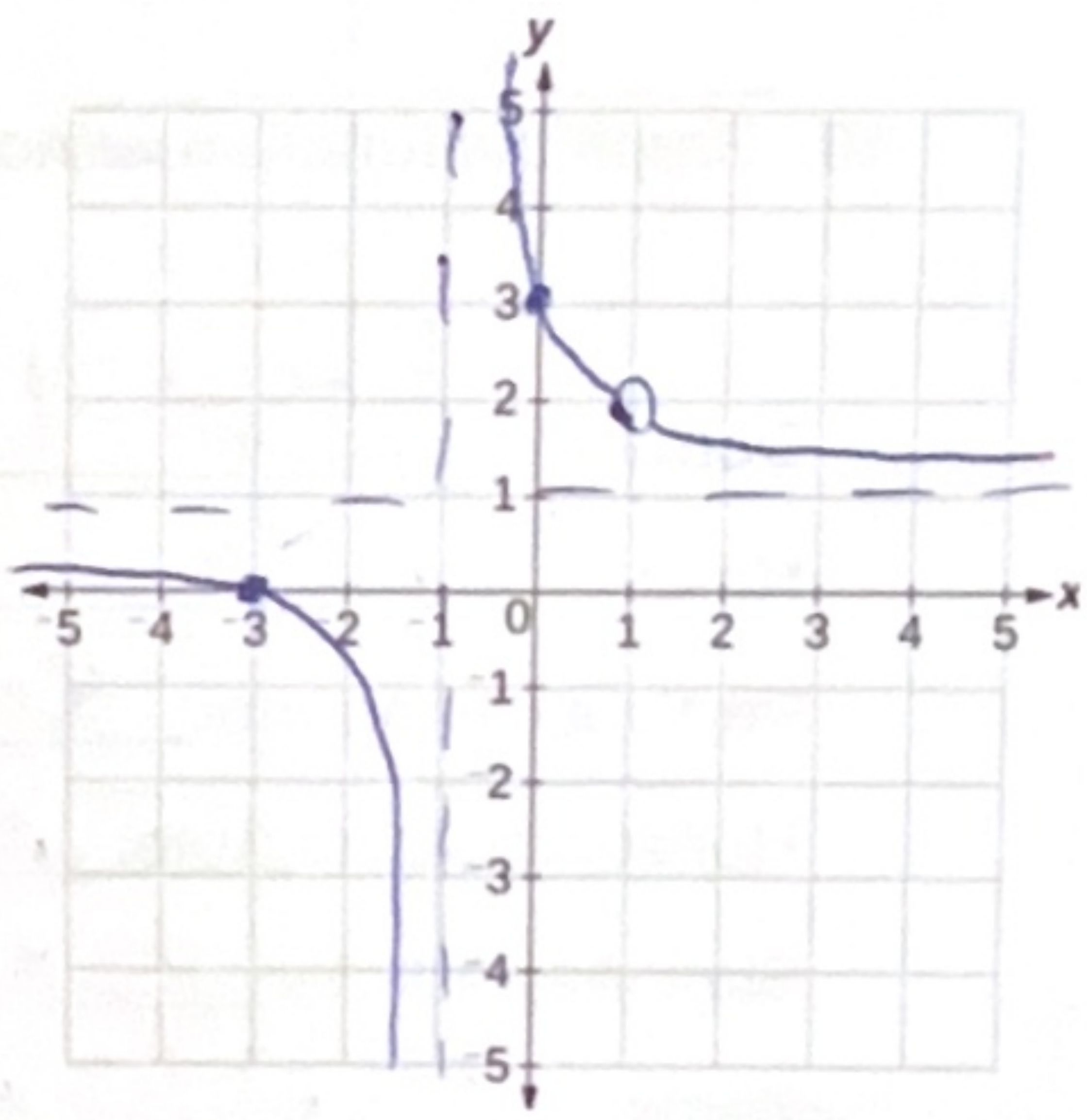
Zeros: $(-3, 0)$

y-intercept: $(0, 3)$

Increasing: N/a

Decreasing: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

Concavity $(-\infty, -1)$: down



Sign Chart: $\dfrac{x+3}{x+1}$ $\quad -1 \quad x_{-3}$

+   −   +

−4   −3   −2   −1   0

Limit Statements:

$\lim_{x\to -\infty} k(x) = 1$

$\lim_{x\to \infty} k(x) = 1$

$\lim_{x\to 1} k(x) = 2$

$\lim_{x\to -1^-} k(x) = -\infty$

$\lim_{x\to -1^+} k(x) = \infty$

9. Graph the function without a calculator. $f(x) = \frac{6x^2 + 10x + 4}{3x^2 + 6x - 9}$

$3(x^2+2x-3)$

Domain: $\mathbb{R}\ x \neq -3, 1$

Hole: N/a

Vertical Asymptote: $x = -3, 1$

Horizontal Asymptote: $y = 2$

Slant Asymptote: N/a

Zeros: $(-1, 0), (-\frac{2}{3}, 0)$

y-intercept: $y = -\frac{4}{9}$

Increasing: $(-\infty, -3) \cup (-3, x)$

Decreasing: $(x, 1) \cup (1, \infty)$

Concavity (-3, 1): down

Sign Chart:

$\frac{(3x+2)(x+1)}{(x+3)(x-1)}$

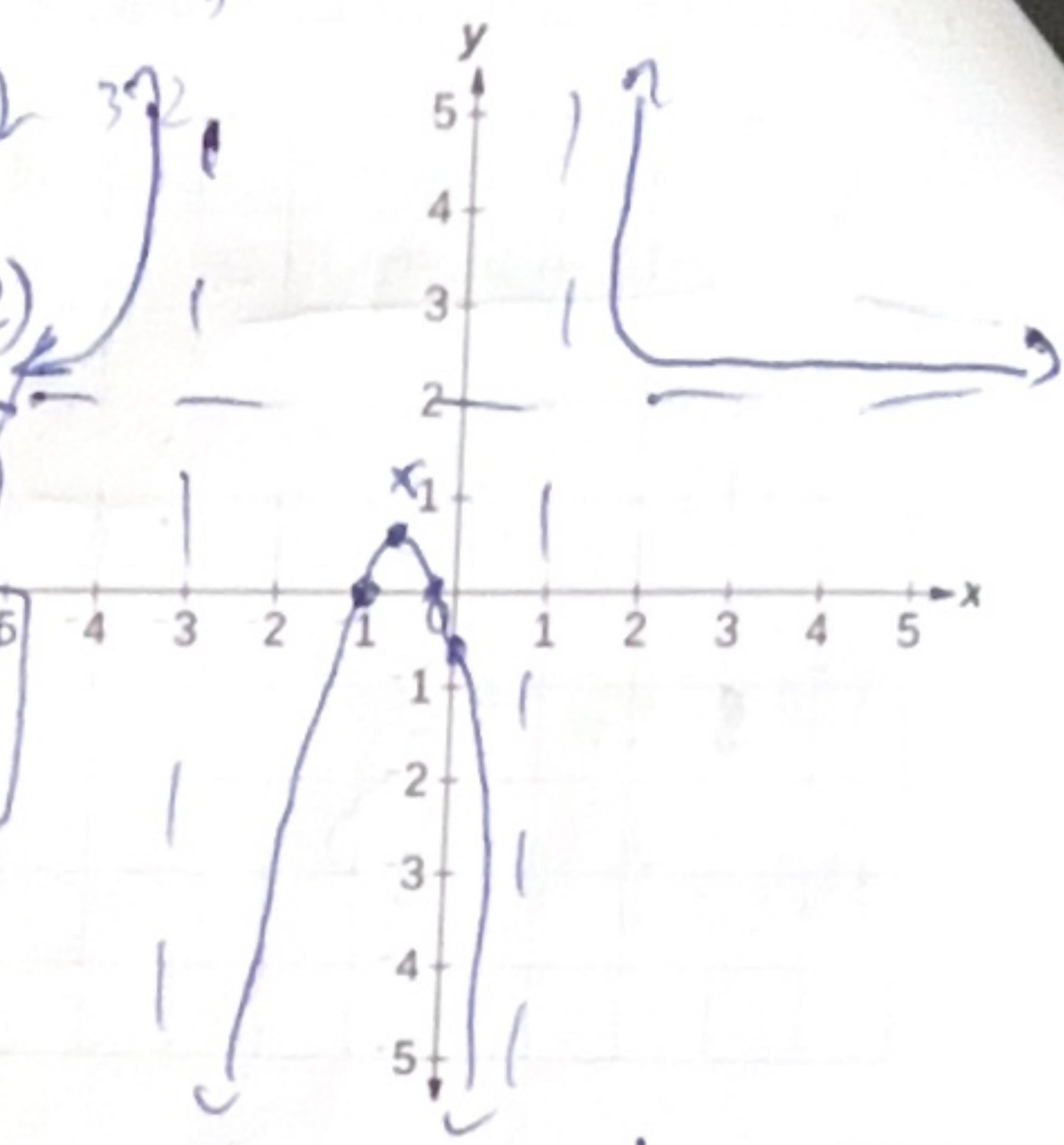
$+ \quad - \quad + \quad - \quad +$

$-4 \quad -3 \quad -2 \quad -1 \quad -\frac{5}{6} \quad -\frac{2}{3} \quad 0 \quad 1 \quad 5$

$(3x-2)(x+3)$

$\frac{2(3x^2+5x+2)}{3(x+3)(x-1)}$

$\left[\frac{2(3x+2)(x+1)}{3(x+3)(x-1)}\right]$



Limit Statements:

$\lim_{x\to\infty} f(x) = 2$

$\lim_{x\to-\infty} f(x) = 2$

$\lim_{x\to-3^-} f(x) = \infty$

$\lim_{x\to-3^+} f(x) = -\infty$

$\lim_{x\to1^-} f(x) = -\infty$

$\lim_{x\to1^+} f(x)\ \infty$

10. Graph the function without a calculator. $f(x) = \frac{x^2+2x-8}{x+1}$

$\frac{(x+4)(x-2)}{x+1}$

Domain: $\mathbb{R}\ x \neq -1$

Hole: N/a

Vertical Asymptote: $x = -1$

Horizontal Asymptote: N/a

Slant Asymptote: $y = x$

Zeros: $(-4, 0), (2, 0)$

y-intercept: $(0, -8)$

Increasing: $(-\infty, -1) \cup (-1, \infty)$

Decreasing: N/a

Concavity (-1, ∞): down

Sign Chart:

$\frac{(x+4)(x-2)}{x+1}$

$- \quad + \quad - \quad +$

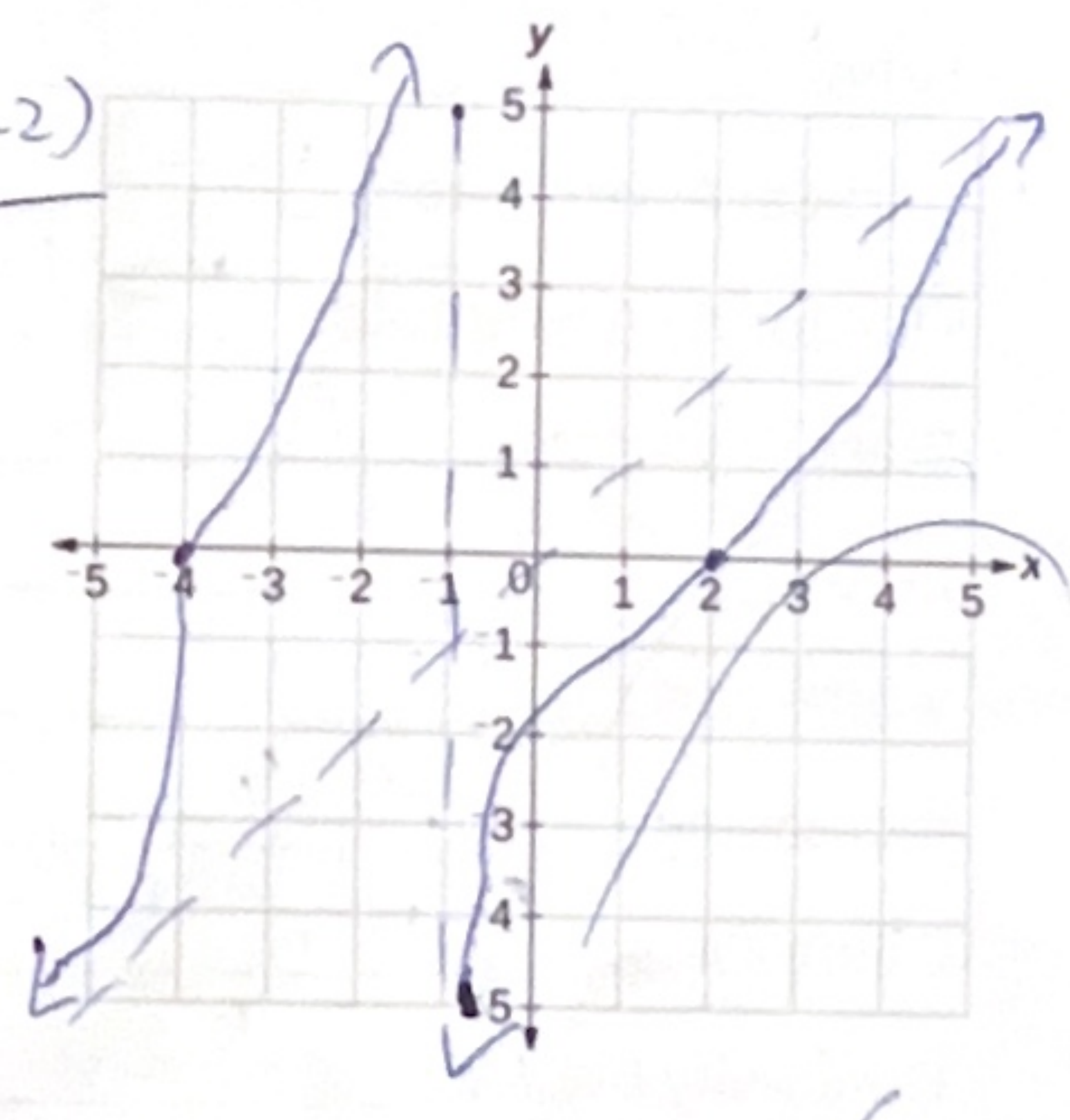
$-5 \quad -4 \quad -2 \quad -1 \quad 0 \quad 2 \quad 5$

$-1 \,|\, 1 \quad 2 \quad -8$

$\quad -2 \quad 0$

$1 \quad 0 \quad \boxed{-8}$

x



Limit Statements:

$\lim_{x\to-\infty} f(x) = -\infty$

$\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to-1^-} f(x) = \infty$

$\lim_{x\to-1^+} f(x) = -\infty$

3
-1

11. Graph the function without a calculator. $f(x) = \frac{x^2+2x-3}{x+2}$

Domain: $\mathbb{R}$, $x \neq -2$

Hole: N/a

Vertical Asymptote: $x=-2$

Horizontal Asymptote: N/a

Slant Asymptote: $y=x$

Zeros: $(-3,0)$, $(1,0)$

y-intercept: $y=-\frac{3}{2}$

Increasing: $(-\infty,-2)\cup(-2,\infty)$

Decreasing: N/a

Concavity (-∞, -2): VP

Sign Chart:

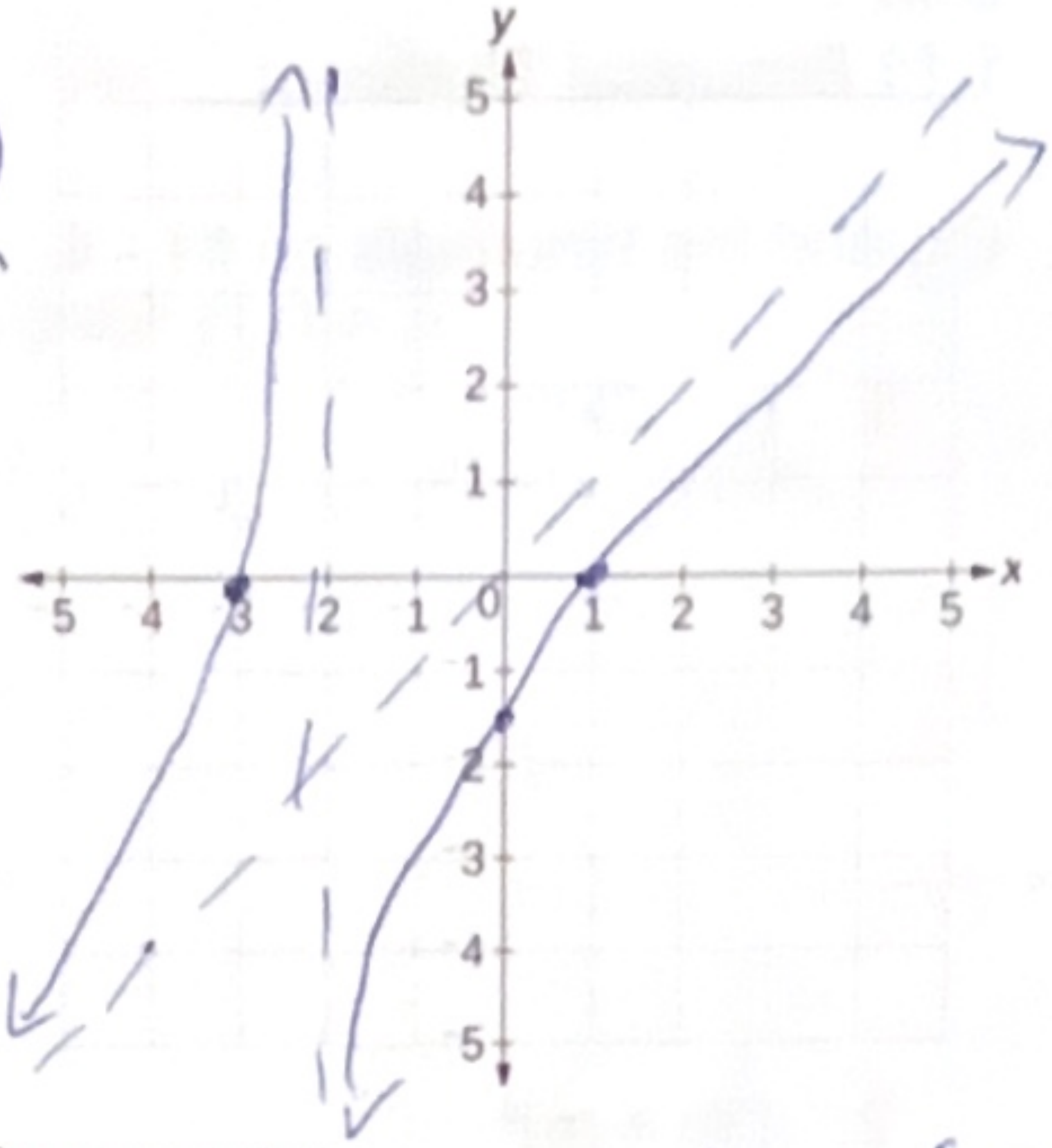
$\frac{(x+3)(x-1)}{x+2}$

$-2 \,|\, 1 \quad 2 \quad -3$

$\quad -2 \quad 0$

$1 \quad 0 \quad -3$

x



Limit Statements:

$\lim_{x\to-\infty} f(x) = -\infty$ $\lim_{x\to-2^-} f(x) = \infty$

$\lim_{x\to\infty} f(x) = \infty$ $\lim_{x\to-2^+} f(x) = -\infty$

Sign chart: $-$ $+$ $-$ $+$

$-5 \quad -3 \quad -\frac{5}{2} \quad -2 \quad 0 \quad 1 \quad 3$

$\frac{(x+3)(x-1)}{x+2}$

12. A portion of the graph of the rational function h is shown. Write an equation, in factored form, for $h(x)$.

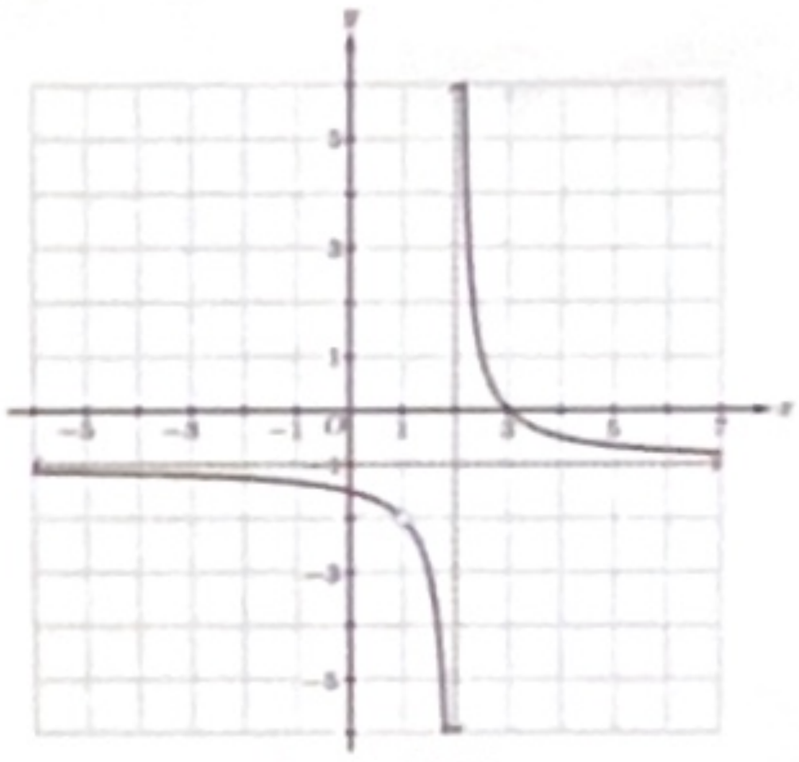
$h(x) = \left[\frac{(x-1)(x-3)}{(x-1)(x-2)}\right]$

$(3,0)$ VA: $x=2$

$\frac{(x-1)}{(x-1)}$

HA: $x=-1$

$(x+1)$



Graph of $h(x)$

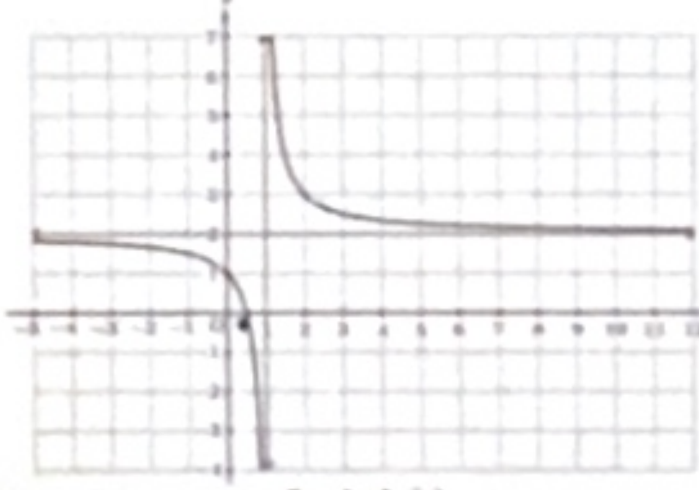
13. A portion of the graph of the rational function r is shown. Write an equation, in factored form, for $r(x)$.

$\left[r(x) = \frac{(2x-1)}{x-1}\right]$

$\left(\frac{1}{2},0\right)$ HA: 2

VA: 1

$(x-1)$



Graph of $r(x)$

14. Write an equation of a rational function that has the following properties.

a) The graph of f has a hole at $x=3$ and vertical asymptotes at $x=1$ and $x=-4$.

$f(x) = \frac{(x-3)}{(x-3)(x-1)(x+4)}$

b) The graph of g has a hole at $x=-1$, a vertical asymptote at $x=7$, and a zero at $x=-2$.

$g(x) = \frac{(x+1)(x+2)}{(x+1)(x-7)}$

c) The graph of h has a hole at $x=2$ and $x=5$, a vertical asymptote at $x=0$, and a zero at $x=1$.

$h(x) = \frac{(x-2)(x-5)(x-1)}{x(x-2)(x-5)}$