Robert Hawken 4470 #47661808 4-6pm
Hypersonics Week 4 Tutorial Questions

AERO4470 2024 Week 4 Tutorial Problems:

Task Description

- Short, open book, weekly tutorial questions to be completed individually during the tutorial session. However, students can work in groups during the tutorial and are actively encouraged to collaborate with group members in completing this learning activity.
- The assessment for each tutorial problem is pass (worth 1%) or fail (worth 0%). The total mark for the tutorial is then 1% pro-rated by the proportion of problems you passed. A pass requires that you make a reasonable attempt at the question: selecting appropriate theory and methodology from the lecture material and applying it to the problem at hand.
- Late submission will get a zero mark. Submit an electronic copy of your working via Blackboard by 1PM the day after the tutorial **(\Assessment\Tutorials\Tutorial Week X)**. Tutorials may be hand written, in which case the hardcopy should be scanned or imaged using, for example, a mobile phone and appropriate app (such as the Google Drive scan tool). Alternatively, you may take photos and insert them as images into another document type such as MS Word (see https://web.library.uq.edu.au/node/4221/3#3 under Submitting handwritten notes).
- Solutions to some or all of the tutorial problems will be presented and discussed during the course of the tutorial.

Questions

A blunt capsule is travelling at 6.2 km/s through Titan's atmosphere, which can be assumed to consist of pure nitrogen. The vehicle diameter is 4 m. The free-stream temperature and pressure are 200 K and 10.0 Pa respectively. Considering the stagnation streamline:

1. Estimate the immediate post-shock density, pressure, and temperature. What are some different approximate techniques you could use to approach this? What are the differences / discrepancies?
2. Find the immediate post-shock shock dissociation rate, i.e. $d\alpha/dt$.
3. What chemistry regime would you expect for this flow?
4. This flight point is to be simulated in a ground-based hypersonic facility, using a sub-scale model diameter of 80 mm. What test flow must be used to satisfy binary scaling?
5. For this condition, the equilibrium composition is $c_{N_2} = 0.66$, c_N=0.34. Assuming your value for $d\alpha/dt$ remains constant, estimate the post-shock distance at which this would be achieved. How accurate do you expect this estimate to be? What are some effects which might influence the real value?
6. Draw a typical stagnation streamline temperature profile for a non-equilibrium shock. How might changes in density or body size affect this?

Note that $\mathcal{M}_N = 14\,\mathrm{kg/kmol}$, $R_u = 8314\,\mathrm{J/kmol\ K}$, $R_{N_2} = 296.9\,\mathrm{J/kg\ K}$.
For nitrogen: $CT^{-n} = 2 \times 10^{17} T^{-1.5}\ \mathrm{m^3/kg/s}$ with T in kelvin, $D = 34.0\,\mathrm{MJ/kg}$, $\Theta_d = 113\,000\,\mathrm{K}$.

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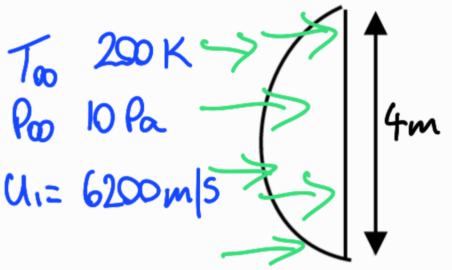
Environment Titan, 100% N_2



Assumptions
- N_2
- no recombination
- Strong shock approx.
- Ideal gas laws

$M_N = 14\,\mathrm{kg/kmol}$
$R_u = 8314\,\mathrm{J/kmol}$
$R_{N_2} = 297\,\mathrm{J/kg\cdot K}$
$CT^{-n} = 2\times10^{17}T^{-1.5}\ \mathrm{m^3/kg\cdot s}$
$D = 34\ \mathrm{MJ/kg}$
$\Theta_d = 113\,000\,\mathrm{K}$

Immediate post shock ρ_2, P_2, T_2

N_2 Ideal gas ρ_1 Density.

$$\rho_1 = \frac{P_1}{R_N\cdot T_\infty} = \frac{10\ \mathrm{Pa}}{(297\,\mathrm{J/kg\cdot K})(200\,\mathrm{K})}$$

$\rho_1 \Rightarrow 1.684\times10^{-4}\ \mathrm{m^3/kg}$

We can combine the hypersonic density limit, the Rankine-Hugoniot equations, and what is called the 'strong shock approximation' ($p_1 = 0$, $h_1 = 0$) to get a slightly simpler set of equations to solve for the immediate post-shock state.

Using Equation 62 with our $\gamma = 4/3$, we get:

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)}{(\gamma-1)} = 7 \text{ when } \gamma = 4/3 \text{ (ideal diss. gas)} \tag{64}$$

$$\boxed{\rho_2 = 7\rho_1} \tag{78}$$

$$\boxed{p_2 = \frac{6}{7}\rho_1 u_1^2} \tag{79}$$

$$\boxed{T_2 = \frac{p_2}{\rho_2 R}} \tag{80}$$

$$\boxed{u_2 = \frac{1}{7}u_1} \tag{81}$$

Density

$$\boxed{\rho_2 = 7\rho_1}$$

$\rho_2 = 7\rho_1$ Eq 78.

$\rho_2 = 7\cdot(1.684\times10^{-4}\ \mathrm{kg/m^3})$

$\rho_2 = 1.178\times10^{-3}\ \mathrm{kg/m^3}$

Pressure Eq 79 $P_2 = \frac{6}{7}\cdot\rho_1\cdot u_1^2$

$$\boxed{p_2 = \frac{6}{7}\rho_1 u_1^2}$$

$$P_2 = \frac{6}{7}\cdot(1.684\times10^{-4}\,\mathrm{kg/m^3})\cdot(6200\,\mathrm{m/s})^2$$

$P_2 = 5549$ Pascals

Post shock Temperature - No Dissociation or recombination

$$\boxed{T_2 = \frac{p_2}{\rho_2 R}}$$

Eq 80, $T_2 = \dfrac{P_2}{\rho_2 \cdot R_N}$

$T_2 = \dfrac{5549}{1.178\times10^{-3} \cdot 297}$, $T_2 \Rightarrow$ 15 859 K

Velocity post shock

$$\boxed{u_2 = \frac{1}{7}u_1}$$

$u_2 = \frac{1}{7} \cdot u_1$

$u_2 = \frac{1}{7}(6500)$, $u_2 =$ 928.6 m/s

Assuming the Rankine Hugo strong shock approximation that the dissociation has not taken place immediately post shock.

The newtonian method could be used however inaccurate as it's not based on fluid mechanics due to the pressure assumption post shock.

2. Find the immediate post-shock shock dissociation rate, i.e. $d\alpha/dt$.

Note that $\mathcal{M}_N = 14$ kg/kmol, $R_u = 8314$ J/kmol K, $R_{N_2} = 296.9$ J/kg K.
For nitrogen: $CT^{-n} = 2\times10^{17}T^{-1.5}$ m^3/kg/s with T in kelvin, $D = 34.0$ MJ/kg, $\Theta_d = 113\,000$ K.

Put ρ into formula.

$$\therefore \frac{1}{\rho}\frac{d\alpha}{dt} = \frac{u}{\rho}\frac{d\alpha}{ds} = \frac{d\alpha}{d\zeta} = CT^{-n}\rho(1-\alpha)e^{-\Theta/T} \quad (86)$$

► where $\zeta = \int_0^s \frac{\rho}{u} ds$ is known as the binary scaling parameter.

$\therefore \dfrac{d\alpha}{dt} = CT^{-n} \cdot \rho_2 (1-\alpha) e^{\left(\frac{-\theta d}{T}\right)}$

assume $\alpha = 0$ as no dissociation coefficient yet.

$\hookrightarrow$

$2\times10^{17}(15\,859\,K)^{-1.5}\ 1.178\times10^{-3}\ \text{kg/m}^3\ [1-0]e^{\left(\frac{-113000\,K}{15859\,K}\right)}$

$\dfrac{d\alpha}{dt} \Rightarrow$ 94904.5 s

3. What chemistry regime would you expect for this flow?

Use Damkohler Number

$U_\infty = 6200\,m/s$

$r = \frac{4\,m}{2\,m}$

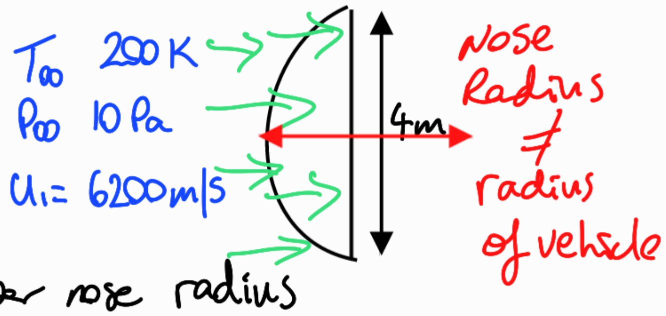
diameter to nose radius.

$$\boxed{\Omega = \frac{d\alpha/dt}{u_\infty/r}} \qquad (51)$$

- Here $\frac{d\alpha}{dt}$ is taken immediately post-shock on the stagnation streamline, and r is the nose radius. It basically compares the reaction rate to the rate at which the fluid traverses the region.
- There are no exact rules, but we can roughly say:
  - Frozen chemistry: $\Omega < 0.1$
  - Nonequilibrium chemistry: $0.1 < \Omega < 10$
  - Equilibrium chemistry: $\Omega > 10$

$$\Omega = \frac{94904.5\,s}{\left(\frac{6500\,m/s}{2\,m}\right)}$$

$$\Omega = 30.6$$



Need a proper nose radius not half diameter.

therefore with a Damkholer number over 10, this is Equilibrium chemistry.

4. This flight point is to be simulated in a ground-based hypersonic facility, using a sub-scale model diameter of 80 mm. What test flow must be used to satisfy binary scaling?

$$(\rho_\infty L)_{flight} = (\rho_\infty L)_{laboratory}$$

L = Characteristic length

ρ_∞ 4

$$= 1.684\times10^{-4}\,m^3/kg \cdot 4000 = \rho_\infty \cdot 80\,mm$$

$$= \frac{1.684\times10^{-4}\cdot 4000}{80} = \rho_\infty \Rightarrow 0.00842\,kg/m^2$$

Conservation of enthalpy pre&post shock will need to be considered

5. For this condition, the equilibrium composition is $c_{N_2} = 0.66$, c_N=0.34. Assuming your value for $d\alpha/dt$ remains constant, estimate the post-shock distance at which this would be achieved. How accurate do you expect this estimate to be? What are some effects which might influence the real value?

$C_{N_2} = 0.66$, $C_N = 0.34$ $\quad \frac{d\alpha}{dt} = 94904.5/s$

Post shock distance

$$\frac{d\alpha}{dt} = \frac{\Delta\alpha}{\Delta t} = 94904.5$$

$0.34 = \alpha = C_N$
dissociated Nitrogen.

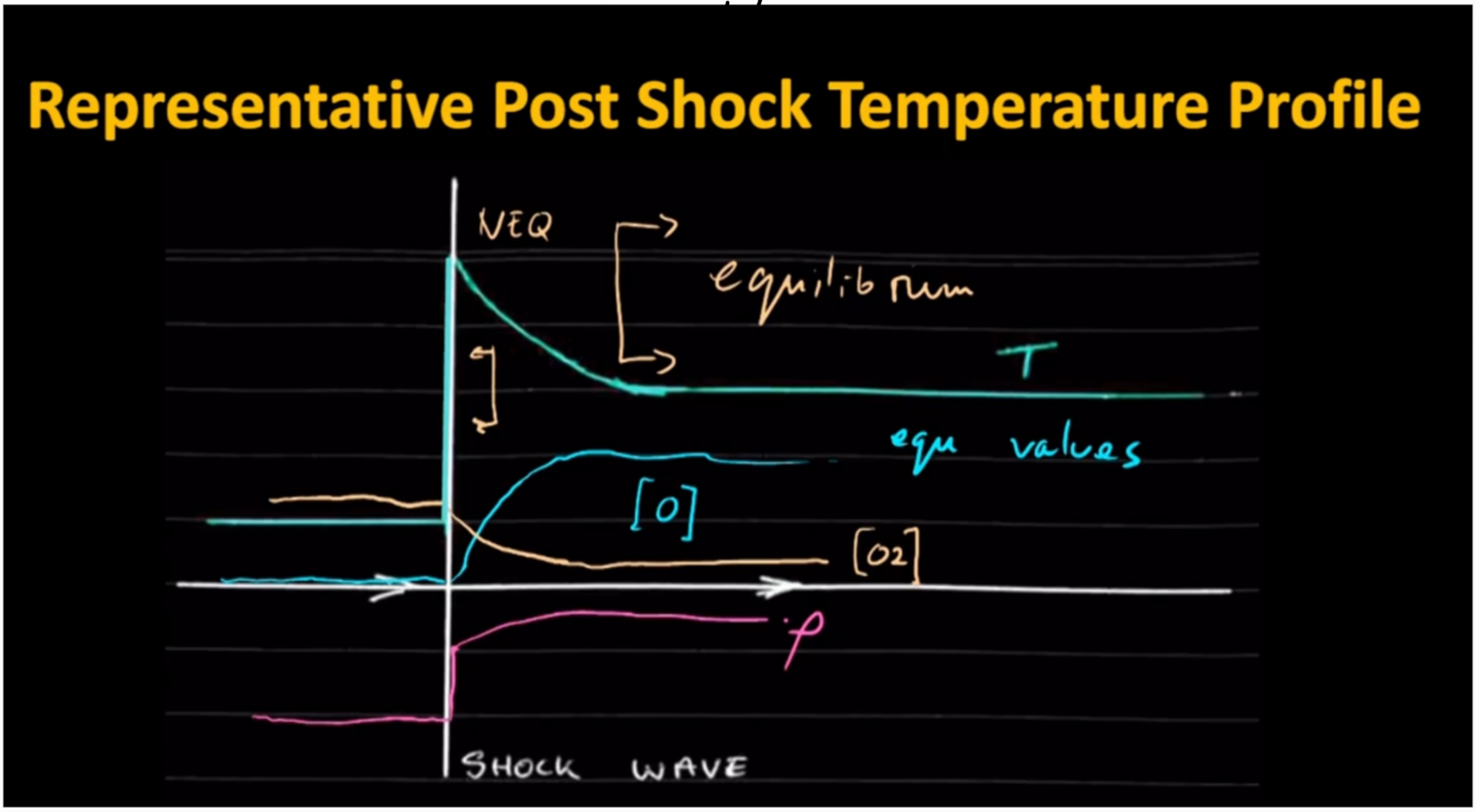
$$\frac{d\alpha}{dt} = \frac{\alpha_2 - \alpha_1}{t_2 - t_1} \therefore \frac{0.34-0}{94904.5-0}$$

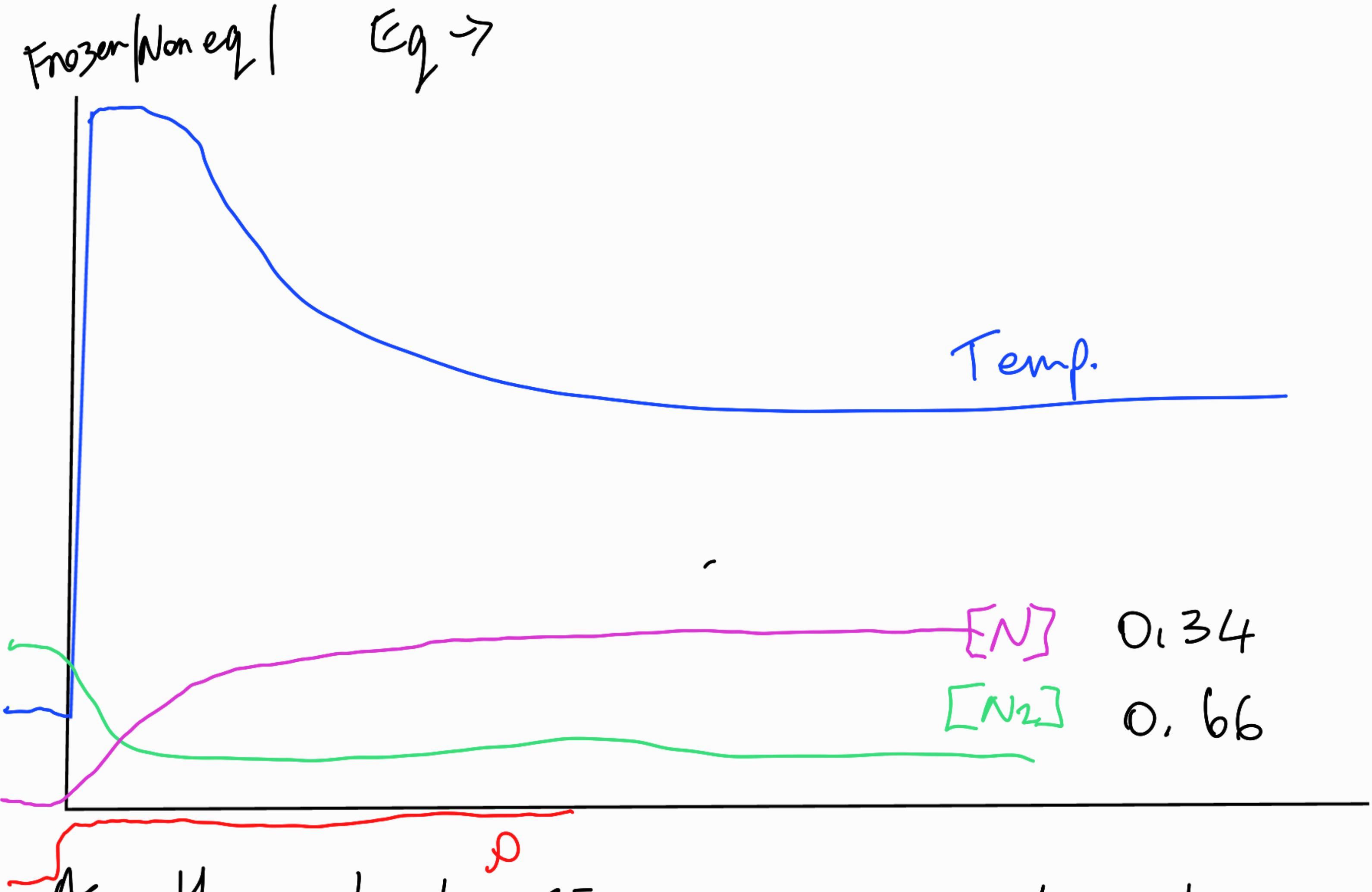
$$t = 3.583\times10^{-6}\ s$$

This value will have error due to the assumption of $d\alpha$ remaining constant. As we know dissociation has an exponential behaviour until eq is established. "Order of magnitude accuracy"

6. Draw a typical stagnation streamline temperature profile for a non-equilibrium shock. How might changes in density or body size affect this?

Richard Morgan, EDX Course.





As the vehicle size increases, so does shock stand off and an increase in density ∴ more collisions, higher temp, quicker to Eq.

End of tutorial 4