Robert Hawken 4-6pm 4470

AERO4470 2024 Week 3 Tutorial Problems:

Task Description

- Short, open book, weekly tutorial questions to be completed individually during the tutorial session. However, students can work in groups during the tutorial and are actively encouraged to collaborate with group members in completing this learning activity.
- The assessment for each tutorial problem is pass (worth 1%) or fail (worth 0%). The total mark for the tutorial is then 1% pro-rated by the proportion of problems you passed. A pass requires that you make a reasonable attempt at the question: selecting appropriate theory and methodology from the lecture material and applying it to the problem at hand.
- Late submission will get a zero mark. Submit an electronic copy of your working via Blackboard by 1PM the day after the tutorial **(\Assessment\Tutorials\Tutorial Week X)**. Tutorials may be hand written, in which case the hardcopy should be scanned or imaged using, for example, a mobile phone and appropriate app (such as the Google Drive scan tool). Alternatively, you may take photos and insert them as images into another document type such as MS Word (see https://web.library.uq.edu.au/node/4221/3#3 under Submitting handwritten notes).
- Solutions to some or all of the tutorial problems will be presented and discussed during the course of the tutorial.

Questions

1. A spacecraft is travelling at 5200 m/s through Titan's atmosphere, which can be assumed to consist of pure nitrogen. The freestream temperature and pressure are 200 K and 50 Pa respectively.
   a Calculate the post-shock composition, specific gas constant, density, and velocity. Assume that the nitrogen dissociates but does not ionise, and that the post-shock temperature and pressure are 5800 K and 21.0 kPa respectively.
   b Verify that the normal shock conservation equations for mass, momentum, and energy are satisfied. (Note that enthalpy h is mass based, so you need to use mass fractions to combine the enthalpy of the different species in the energy equation.)
   c Is the equilibrium post-shock density higher or lower than that found using a calorically perfect gas assumption? What are the competing effects? What about a vibrationally excited but non-reacting gas?
2. If we now consider a spacecraft travelling through the same conditions in Earth's atmosphere, calculate:
   a The equilibrium post-shock velocity and state variables (temperature, pressure, and density) of the post-shock gas.
   b How do the equilibrium post-shock conditions in air compare to the pure nitrogen results from above? Why might they be different?
   c Check your answer against NASA's CEA program. How well does your calculation compare?
   d Check your equilibrium post-shock temperature against the equivalent perfect gas post-shock temperature. How do they compare?

Note that $\mathcal{M}_N = 14\,\text{kg/kmol}$, $\mathcal{M}_O = 16\,\text{kg/kmol}$, $R_u = 8314\,\text{J/kmol K}$, $C_{p_{air}} = 1.007\,\text{J/kg K}$(at 200 K).

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Hypersonics Week Tutorial Questions

1. A spacecraft is travelling at 5200 m/s through Titan's atmosphere, which can be assumed to consist of pure nitrogen. The freestream temperature and pressure are 200 K and 50 Pa respectively.

   a Calculate the post-shock composition, specific gas constant, density, and velocity. Assume that the nitrogen dissociates but does not ionise, and that the post-shock temperature and pressure are 5800 K and 21.0 kPa respectively.

Note that $\mathcal{M}_N = 14$ kg/kmol, $\mathcal{M}_O = 16$ kg/kmol, $R_u = 8314$ J/kmol K, $C_{p_{air}} = 1.007$ J/kg K(at 200 K).

Initial conditions: Atm Titan Pure Nitrogen.

$V = 5200$ m/s T_∞ of N = 200K P_∞ N = 50 Pa.

Pre Shock

$T_\infty = 200K$

$P_\infty = 50Pa$

Post Shock

$T_2 = 5800K$

$P_1 = 21{,}000$ Pa

ρ_2

For Nitrogen $N_2 \rightleftharpoons 2N$ at equilibrium.

$A_1 = N_2$ $A_2 = N$

$\nu_1 = -1$ $\nu_2 = 2$

$k = P_N^2 \, P_{N_2}^{-1}$

$$\hookrightarrow \frac{P_N^2}{P_{N_2}} = e^{\frac{-\Delta G^{P=1}}{R_u \cdot T}}$$

$P_2 = P_N + P_{N_2}$ For Nitrogen @ 5800K P 228 mcbride.

$H_0^0 = 111{,}543.3 \ \frac{cal}{mol}$

$-(F_T^0 - H_0^0) = 269{,}328.1 \ \frac{cal}{mol}$

$\therefore$ Rearranging for $F_T^0 = 111{,}543.3 - 269{,}328.1$

$\hookrightarrow F_T^0 = -157784.8 \ \frac{cal}{mol}$

$\therefore G_N^{P=1} = -157784.8\,(4.184) = -659\,540.5$ J/mol

For N_2 @ 5800K from McBride.

$H_0^\circ = -2072.3 \ \frac{cal}{mol}$

$\therefore \ -(F_T^\circ - H_0^\circ) = 354\,739.5 \ \frac{cal}{mol}$

$\therefore$ as above $F_T^\circ = -354\,739.5 - 2072.3$

$= -356811.8 \ \frac{cal}{mol}$

$\therefore \ G_{N_2}^{p=1} = -356811.8(4.184) = -149\,1473.3 \ \frac{J}{mol}$

Therefore Now for each species of Nitrogen, Gibbs free energy

$$\Delta G^{p=1} = \sum_i V_i \, G_i^{p=1}$$

$$\hookrightarrow V_N \cdot G_N^{p=1} + V_{N_2} \, G_{N_2}^{p=1}$$

$$2(-659540.5) + [-1(-1491473.3)]$$
$$= 172392.4 \ \frac{J}{mol}.$$

$K_{p_1} = e^{-\Delta G^{p=1}/R_u T}$ when $R_u = 8$ pi $(8314) \cdot T = 5800K$

$$K = \frac{P_N^2}{P_{N_2}} = 0.0259$$

Therefore solve for P_N and P_{N_2}

$P_{N_2} = \frac{P_N^2}{K}$ and $P_N = P_2 - P_{N_2}$

therefore with both of these, solving

$$P_N = P_2 - \frac{P_N^2}{K}$$

$$\hookrightarrow \frac{P_N^2}{K} + P_N - P_2 = 0 \ ✓$$

When $P_2 = 21\,kpa \approx 0.207\,atm.$

$\therefore \; P_N^2 + 0.0259\,P_N - 0.00536 = 0$

$$P_N = 0.5 \cdot \left[-0.0259 \pm \sqrt{K^2 + 4(0.00536)}\right]$$

$\therefore$ $P_N > 0$, so only positive ans

$$P_N = 0.5 \cdot \left[-0.0259 + \sqrt{K^2 + 4(0.00536)}\right]$$

$= 6.18\,kpa$ after converting from atm.

$P_{N2} = 21\,kpa - 6.18\,kpa = 15.2\,kpa.$

Now Concentration Composition

$$X_{N_2} = \frac{P_{N2}}{P_2} = \frac{15.2\,\cancel{kpa}}{21\,\cancel{kpa}} = 0.72 = X_{N2}$$

$$X_N = \frac{P_N}{P_2} = \frac{6.18\,\cancel{kpa}}{21\,\cancel{kpa}} = 0.29 = X_N$$

$$M = X_N M_N + X_{N_2} M_{N2} = 0.29 \cdot \overset{MW}{14} + 0.72 \times \overset{MW}{28} = 24.22\ kg/kmol$$

Mole fraction

Assume 1kg of mixture

$$C_N = X_N \left(\frac{M_N}{M}\right)$$

$$= 0.29\left(\frac{14}{24.22}\right)$$

$= 0.17 = C_N$

$C_{N2} = 1 - C_N = 1 - 0.17 = 0.83$

8 pts

Specific Gas constants

$$R = \sum_i C_i R_i$$

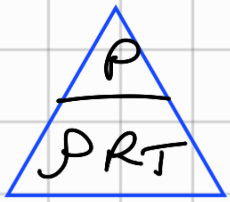
$$R_N = \frac{8314}{14} = 594\ J/kg\,K$$

$$R_{N2} = \frac{8314}{28} = 297\ J/kg \cdot K$$

∴ Overall Gas Constant

$R_T = C_N \cdot R_N + C_{N2} \cdot R_{N2}$

$R_T = (0.17 \cdot 594) + (0.83 \cdot 297) = 348\ J/kg.K$



Density from ideal gas eq.

$P = \rho R T \qquad \rho = \frac{P}{RT}$

Density post shock

$\rho_2 = \frac{21e^3\ Pa}{348 \cdot 5800\,K} = 0.0104\ kg/m^3$

R of N. = 297 J/kg.K

$\gamma = 1.4$

Now value for U_2 as $U_2 = U_1 \cdot \left(\frac{1}{\rho_2} \cdot \frac{P_1}{R \cdot T_1}\right)$

$5200\,m/s \left(\frac{1}{0.0104\ kg/m^3} \cdot \frac{50\ Pa}{297 \cdot 200\,K}\right) = U_2 = 421\,m/s$ post shock.

U_1 5200 m/s $\quad U_2 = 421\,m/s$.

END Part a

b)

b Verify that the normal shock conservation equations for mass, momentum, and energy are satisfied. (Note that enthalpy h is mass based, so you need to use mass fractions to combine the enthalpy of the different species in the energy equation.)

Sanity Check. Normal Shock conservation.

Mass from density & velocity as $\rho_1 U_1 = \rho_2 U_2$ as above.

momentum as $P_1 + \rho_1 \cdot U_1^2 = P_2 + \rho_2 \cdot U_2^2$

$P_1 + \rho_1 \cdot U_1^2$

$50\ \text{Pa} + 0.000842\ \text{kg/m}^3 \cdot 5800\ \text{m/s}$

$= 22811\ \text{Pa}$

$\rho_1 = \dfrac{50\ \text{Pa}}{297 \cdot 200\ \text{K}}$

$\rho = 0.000842\ \text{kg/m}^3$

$P_2 + \rho_2 \cdot U_2^2 = 21e^3 + 0.0104\ \text{kg/m}^3 \cdot 421\ \text{m/s}$

$= 22840$

$22811\ \text{Pa} = 22840\ \text{Pa}$

Energy w enthalpy (flow energy) as $h_i = \dfrac{(H_T^0)_i}{M_i}$

$h = \sum_i C_i h_i$

@ 200 K for Nitrogen Mcbride.

$(H_T^0)_{N_2} = -683\ \frac{\text{cal}}{\text{mol}}$

$h_{N_2} = \dfrac{-638\ \text{cal/mol}}{0.028\ \text{kg/mol}} (4.184) = -102059\ \frac{\text{J}}{\text{kg}}$

Now h @ 5800 K

$(H_T^0)_N = 141593.5\ \frac{\text{cal}}{\text{mol}}$

$(H_T^0)_{N_2} = 47307.1\ \frac{\text{cal}}{\text{mol}}$

$h_N = \dfrac{141593.5}{0.014\ \text{kg/m}^3}$

$h_N = 42.3e^6\ \text{J/kg}$

$$h_{N2} = \frac{47307.1\,(4.184)}{0.028\ \text{kg/mol}} = 7.06\,e^6\ \text{J/kg}$$

$\therefore$ ~~total~~ enthalpy @ 5800K

$$(0.17)(42.3\,\text{MJ/kg}) + (0.83)(7.06\,\text{MJ/kg}) = 13.05\,e^6\ \text{J/kg}$$

Balance Equation

$$13.05\,e^6\ \text{J/kg} + (0.5 \cdot 421^2) = \underline{13.3\,e^6}\ \text{J/kg}$$

or

$$-102\,059 + (0.5 \cdot 421^2) = \underline{13.1}\ e^6\ \text{J/kg}$$

therefore the energy equations are balanced or conserved pre or post shock.

c)

c Is the equilibrium post-shock density higher or lower than that found using a calorically perfect gas assumption? What are the competing effects? What about a vibrationally excited but non-reacting gas?

$$\frac{\rho_1}{\rho_0} = \frac{(\gamma+1)M^2}{(\gamma-1)M^2+2}$$

Nasa Normal shock pressure ratio.
Solve for Mach number as shown in Anderson 4th Ed.

$$M = \frac{u}{\sqrt{\gamma R T}}$$

Solution
For air, with $\gamma = 1.4$ and $R = 287$ joule/kg · K, the speed of sound at $T_1 = 288$ K is, from Eq. (3.20),

$$a_1 = \sqrt{\gamma R T_1} = \sqrt{(1.4)(287)(288)} = 340 \text{ m/s}$$

Hence,

$$M_1 = \frac{V_1}{a_1} = \frac{1700}{340} = 5$$

$$M = \frac{5200}{\sqrt{1.4 \cdot 297 \cdot 200}} \simeq 18.$$

$$R_N = 297\ \text{J} \cdot \text{kg/K}$$

$\therefore$

$$\frac{\rho_1}{\rho_0} = \frac{(\gamma+1)M^2}{(\gamma-1)M^2+2}$$

at $\gamma = 1.4$, $M = 18$, $\rho_1 = \frac{P_1}{R_N T_1}$

$\rho_1 = 0.000842\ \text{kg/m}^3$

$\rho_2 = 0.005\ \text{kg/m}^3$

A calorically ideal gas exhibits significantly reduced density, influenced by both the gas constant and temperature. If the gas is vibrationally excited, it retains more energy internally, which means that the temperature rises less, consequently leading to a greater density.
A non-reactive gas mixture indicates that there is no disassociation of Nitrogen, hence the gas constant remains stable. The gas constant and density would change significantly if there were further chemistry changes.

2. If we now consider a spacecraft travelling through the same conditions in Earth's atmosphere, calculate:

  a The equilibrium post-shock velocity and state variables (temperature, pressure, and density) of the post-shock gas.

A spacecraft is travelling at 5200 m/s through Titan's atmosphere, which can be assumed to consist of pure nitrogen. The freestream temperature and pressure are 200 K and 50 Pa respectively.

a) Post shock velocity U_2, T_2, P_2, ρ_2

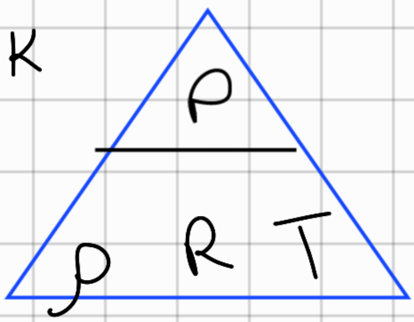
Initial conditions: EARTH

$V = 5200\,m/s$ T_∞ of $N = 200K$ P_∞ $N = 50\,Pa$

Pre Shock Post Shock

$T_\infty = 200K$
$P_\infty = 50\,Pa$

ρ_1 US Standard atmosphere at 200K
Ideal gas law



$P = \rho R T$

$\rho_1 = \frac{P_1}{R_1 T_1}$ $\rho_1 = \frac{50}{287 \cdot 200K}$

$\rho_1 = 0.00087$ kg/m³ Cp air 1.007 J/kg·K

Now we start by getting Bertin's Equation 1-10 and 1.11

conservation of Momentum and energy, in terms of post shock pressure "p"

$$PMOM = p_1 + \rho_1 \cdot U_1^2 = p_2 + \rho_2 \cdot U_2^2$$

$$HTOT = h_1 + 0.5 \cdot U_1^2 = h_2 + 0.5 \cdot U_2^2 = H_t$$

$$\Rightarrow p_2 = p_1 + \rho_1 \cdot U_1^2 - \rho_2 \cdot U_2^2 = PMOM - \rho_2 \cdot U_2^2$$

$$\Rightarrow h_2 = h_1 + 0.5 \cdot U_1^2 - 0.5 \cdot U_2^2 = HTOT - 0.5 \cdot U_2^2$$

Once we find p_2 and h_2 we can use Fig 1.17b to find T_2 and Z_2. Then we can use equation 1.20 to calculate ρ_2

$$\boxed{\rho_2 = \frac{p_2}{Z_2 \cdot R \cdot T_2}}$$

$$U_2 = \frac{\rho_1 \cdot U_1}{\rho_2}$$

then we compare values of ρ_2 calculated in successive iteration until the desired accuracy is obtained.

Since freestream air can be modelled using perfect gas relations:

$$h_1 = Cp_1 \cdot T_1 = 1.007\, kJ/kg \cdot \cancel{K} \cdot 200\, \cancel{K} = 201.4\, kJ/kg \quad 201.4e^3\, J/kg$$

$$PMOM = p_1 + \rho_1 \cdot U_1^2 = 50\,Pa + 0.00087\, kg/m^3 \cdot (5200\, m/s)^2 = 23.574\, kpa$$

$$PMOM \approx 0.233\, atm$$

$$HTOT = h_1 + 0.5 \cdot U_1^2 = 201.4e^3\, J/kg + 0.5 (5200\, m/s)^2 = 13.72e^6\, J/kg$$

For Iteration 1, to make it easy [and because U_2 is always close to 0 anyway] we start by assuming $\underline{U_2 = 0 \, m/s}$

$P_2 = P_{MOM} - \cancel{\rho_2 U_2^2}$ when $P_{MOM} \approx 0.35$ atm

as $U_2 \approx 0 \, m/s$ ∴ $\underline{P_2 = P_{MOM}}$ ≈ 0.233 atm

$h_2 = H_{TOT} - \cancel{0.5 U_2^2}$

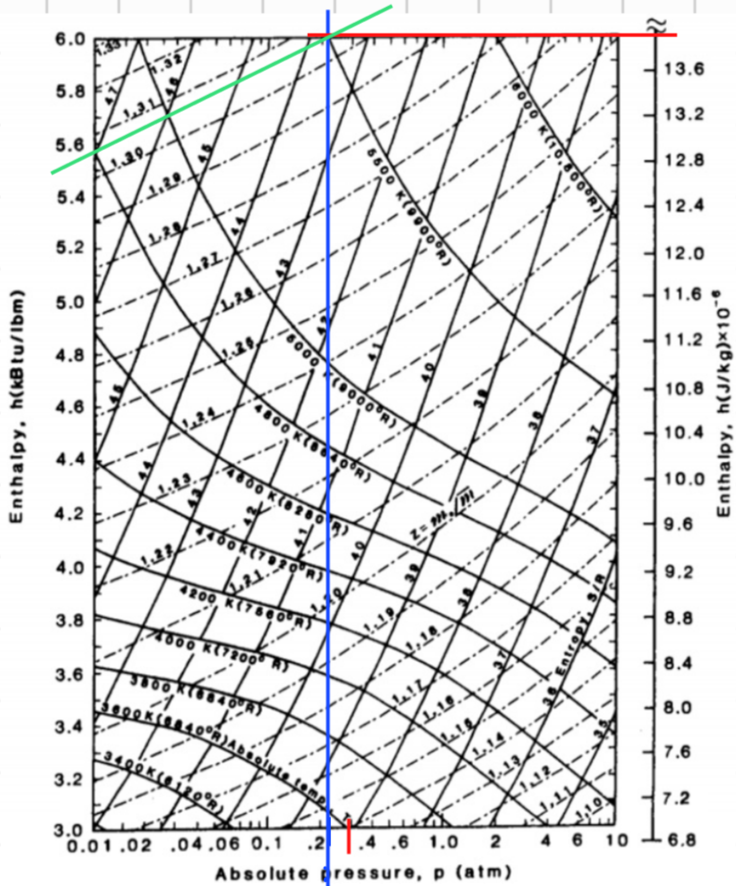
as $U_2 \approx 0 \, m/s$ ∴ $\underline{h_2 = H_{TOT}} = 13.72e^6 \, J/kg$

Using Fig 1.176: $T_2 \approx 5500K$, $z_1 \approx 1.307$

$$\rho_2 = \frac{23.574e^3 \ Pa}{1.307 \cdot 287 \, J/kg \cdot K \cdot 5500K} = 0.0114 \, kg/m^3$$

$$U_2 = \frac{\rho_1 \cdot U_1}{\rho_2} = \frac{0.00087 \, kg/m^3 \cdot 5200 \, m/s}{0.0114 \, kg/m^3}$$

$$\underline{U_2 = 383 \, m/s}$$



(b) Enthalpy from 3000 to 6000 Btu/lbm

Fig. 1.17 Continued.

For Iteration 2, assuming $U_2 = 383\,m/s$

Remember $P_2 = 23.574e^3$ Pa $\quad U_1 = 5200\,m/s$

$\rho_2 = 0.0114\,kg/m^3 \quad U_2 = 383\,m/s$

$P_2 = PMOM - \rho_2 U_2^2 \Rightarrow 23.574e^3 - 0.0114 \cdot (383^2)$ $P_2 = 21{,}901.7\,Pa$

CONVERT ~ 0.22 atm

as $U_2 \approx 383\,m/s$.

$h_2 = H_{TOT} - 0.5\,U_2^2 = 13.72e^6\,J/kg - 0.5\,(383)^2 =$

$h_2 = 13.79e^6\,J/kg$

Using Fig 1.176 : $T_2 \simeq 5400K$, $Z_1 \simeq 1.29$

$$\rho_2 = \frac{21{,}901.7\,Pa}{1.29 \cdot 287\,J/kg\,K \cdot 5400K} = 0.01096\,kg/m^3$$

$$U_2 = \frac{\rho_1 \cdot U_1}{\rho_2} = \frac{0.00087\,kg/m^3 \cdot 5200\,m/s}{0.01096\,kg/m^3}$$

$\therefore U_2$ converges to $\simeq 412\,m/s$

Stopped at 2 iterations as the convergence would be minimal

CEA results : 426 m/s