PRATHAM
TEST PREP

Comprehensive Book

on

Quantitative Aptitude

for various

UNDERGRADUATE ENTRANCE EXAMS

Written and Authored by
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Dear Student,

With the changed curriculum, we also bring you the changed book! While we are at Quantitative Aptitude, let us assure you, this is in no way as tumultuous as you think it is. In our day-to-day lives, we come across two kinds of people: those who love math and those who hate it. But, trust us, the level tested in undergraduate entrances is a piece of cake for all, if you are ready to unlearn and relearn that is.

We always equate mathematics with solving problems through formulae, theorems, etc. This in itself is the biggest fallacy as what is majorly required in solving such questions is sheer logic and common sense. Don't you agree? TRY IT! No one would bother whether you attempt an answer in 5 minutes or 5 seconds, whether you remember the formulae or you just worked by logic, whether you write down the formulae, then solve the equation and finally write "Hence, Proved" or you just simply mark the answer out of given options! Yet, mathematics is the most feared subject.

The aim of giving this book to you is that you learn by practicing the fundamentals taught to you in the class. To make the best out of it, synchronize it with your class schedule:
read the concepts before going for your class, and practice the questions after the class has been held, and you shall benefit immensely. The way we teach you math is the way you will take the shortest time possible to answer questions, at times without even solving an entire question. Your teacher would discuss the derivations of such techniques in the class. As we always say, you succeed and we succeed through you.

In name of making mathematics an easy nut to crack!

Together, let's lead the way!
Team PRATHAM

PRATHAM EDGE – I

1. Remainder of division by 9

To find the remainder when a number is divided by 9 we delete all 9's and then divide the sum of digits of the number by 9. The remainder so obtained is the answer suppose we wish to find the remainder when 6936 is divided by 9. We delete 9. Sum of remaining digits 6 + 3 + 6 = 15. When 15 is divided by 9 the remainder is 6.

2. Multiplication by 5 and powers of 5

We know,

$$5 = \frac{10}{2}, 25 = \frac{100}{4}, 125 = \frac{1000}{8}, 625 = \frac{10000}{16} etc$$

$$\left[5^x = \frac{10^x}{2^x}\right]$$

To multiply a number by 5 suffix a 0 to the number and divide by 2. To multiply the number by 25 suffix 2 zeros to the number and divide by 4. To multiply a number by 5^n suffix n zeros to the number and divide by 2^n. 52 x 5 = 520 ÷ 2 = 260, 32 x 25 = 3200 ÷ 4 = 800

48 x 125 = 48000 ÷ 8 = 6000

3. Division by 5 and power of 5

To divide a number by 5 multiply it by 2 and divide it by 10.

To divide a number by 25 multiply it by 4 and divide it by 100.

To divide a number by 5^x multiply it by 2^x and divide it by 10^x.

52 ÷ 5 = 52 x 2 ÷ 10 = 104 ÷ 10 = 10.4

36 ÷ 25 = 36 x 4 ÷ 100 = 144 ÷ 100 = 1.44

CHAPTER 1

NUMBER SYSTEM

INTRODUCTION

The chapter of number system is amongst the most important chapters in the whole of mathematics syllabus for virtually all the entrance exams inspite of the fact that it is the most elementary concept in mathematics. The fundamentals of numbers would be applied across arithmetic in different forms in algebra and also as an extended feature in fundamental principal of counting.

NUMBER SYSTEM

Number System presents the simplest mathematical structure. This system includes the real numbers, complex numbers, rational numbers, irrational numbers, fractions, integers, whole numbers and natural numbers.

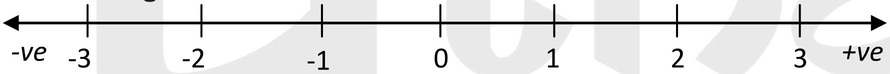
Decimal Number System

The system of number we use in general is Decimal System. There are ten digits in this system 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. These are also the face value of the digits. The place value of a digit will change according to its position in unit's place, ten's place, hundred's place and so on.
e.g. : In 2010, Face value of "2" is 2 and Place value of "2" is 1000.

Real Number

Set of all numbers that can be represented on the number line are called real numbers.
For example 6, -7, 0, 6.3777, 7, $7+\sqrt{\ddot{u}}\ \frac{3}{11}\ \ddot{u}$

A Number Line is a straight line with an arbitrary point called origin. To the right of this point are positive numbers and to the left are negative numbers.



Imaginary Numbers

In real number system square root of negative numbers does not exist.
Solution of certain equations like $x^2+3=0$ leads to the concept of imaginary numbers. Numbers which are not real.
i is the imaginary unit whose value is $\sqrt{(-1)}$ and it is written as i and known as iota.

Natural Numbers

N = {1, 2, 3, 4, 5, 6, ……} is the set of all natural numbers. These are counting numbers.

$$1+2+3+4+5+\_\_\_\_\_\_+n=\frac{n(n+1)}{2}$$

$$1^2+2^2+3^2+4^2+5^2+\_\_\_\_\_\_+n^2=\frac{n(n+1)(2n+1)}{6}$$

$$1^3+2^3+3^3+4^3+5^3+\_\_\_\_\_\_+n^3=\left[\frac{n(n+1)}{2}\right]^2$$

Can also have sum of first "n" odd and "n" even numbers together. One tends to remember all sum related formula together.

Sum of first "n" odd numbers = n^2, Sum of first "n" even numbers = $n\times(n+1)$

Whole Numbers

W = {0, 1, 2, 3, 4, 5, 6, ……..} is the set of all whole numbers.
Whole numbers are natural numbers together with zero.
It is denoted as $W = N + \{0\}$

Integers

I = {-3, -2, -1, 0, 1, 2, 3, ….} is the set of all integers. It is a set of all numbers, 0 and negative of natural numbers.

Positive Integers
These are all natural numbers, $I+ = \{1, 2, 3, 4, \ldots\ldots\}$

Negative Integers
$I- = \{-1, -2, -3, ---\}$,
0 is neither positive nor negative.

Rational Numbers
The numbers of the form p/q where p & q are integers and $q \neq 0$ are rational numbers, Every integer is a rational number.

Irrational Numbers
The numbers which when expressed in decimal form are in non-terminating and non repeating forms are called irrational number.
e.g. $\sqrt{3}$, $\sqrt{11}$,π , π is not 22/7. It is the approximate value of π

Even Numbers
These are denoted by the expression $2I$ where I is an integer.
$2 + 4 + 6 + 8 + \ldots\ldots\ldots\ldots\ldots + 2n$ terms $= n(n+1)$

Odd Numbers
These are denoted by the expression $2I + 1$ where I is an integer.
$1 + 3 + 5 + 7 + 9 + \ldots\ldots\ldots\ldots\ldots + n$ terms $= n^2$

Prime Numbers
A number which has exactly two factors, 1 and the number itself, is called a prime number.
e.g. 2, 3, 5, 7, 11, …….
The Pratham Edge: *A Prime Number >3 is **always** expressed in the form 6k±1. However it is not a test to check whether its prime or not.*

Composite Numbers
A number which has more than 2 factors is called a composite number.
e.g. 4, 6, 8, 9, 12,……..

Co-Primes
Two natural numbers a and b are said to be co-prime if their HCF is 1.
Ex. (2, 3), (4, 5), (7, 9), (8, 11) etc. are pairs of co-primes.

TEST OF DIVISIBILITY

1. ***Divisibility By 2:*** A number is divisible by 2, if its unit's digit is either 0, 2, 4, 6 or 8.
   Ex. 84932 is divisible by 2, while 65935 is not.
2. ***Divisibility By 3:*** A number is divisible by 3, if the sum of its digits is divisible by 3.
   Ex. 592482 is divisible by 3, since sum of its digits = (5 + 9 + 2 + 4 + 8 + 2) = 30, which is divisible by 3.
   But, 864329 is not divisible by 3, since sum of its digits = (8+6+4+3+2+9) = 32, which is not divisible by 3.
3. ***Divisibility By 4:*** A number is divisible by 4, if the number formed by the last two digits of the number is divisible by 4.
   Ex. 982648 is divisible by 4, since the number formed by the last two digits is 48, which is divisible by 4.
   But, 749282 is not divisible by 4, since the number formed by the last two digits is 82, which is not divisible by 4.
4. ***Divisibility By 5:*** A number is divisible by 5, if its unit's digit is either 0 or 5. Thus, 20820 and 50345 are divisible

by 5, while 30934 and 40946 are not.

5. ***Divisibility By 6:*** A number is divisible by 6, if it is divisible by both 2 and 3.
   Ex. The number 35356 is clearly divisible by 2.
   Sum of its digits = (3 + 5 + 2 + 5 + 6) = 21, which is divisible by 3.
   Thus, 35256 is divisible by 2 as well as 3. Hence, 35256 is divisible by 6.
6. ***Divisibility By 7:*** Subtract 2 times the last digit from the rest.
   Ex: 483 is divisible by 7.
   48-(3×2) = 42, which is divisible by 7.
7. ***Divisibility By 8:*** A number is divisible by 8, if the number formed by the last three digits of the given number is divisible by 8.
   Ex. 953360 is divisible by 8, since the number formed by last three digits is 360, which is divisible by 8.
   But, 529418 is not divisible by 8, since the number formed by last three digits is 418, which is not divisible by 8.
8. ***Divisibility By 9:*** A number is divisible by 9, if the sum of its digits is divisible by 9.
   Ex. 60732 is divisible by 9, since sum of digits = (6 + 0 + 7 + 3 + 2) = 18, which is divisible by 9,
   But, 68956 is not divisible by 9, since sum of digits (6 + 8 + 9 + 5 + 6) = 34, which is not divisible by 9.
9. ***Divisibility By 10:*** A number is divisible by 10, if it ends with 0.
   Ex. 96410, 10480 are divisible by 10, while 96375 is not.
10. ***Divisibility By 11:*** A number is divisible by 11, if the difference of the sum of its digits at odd places and the sum of its digits at even places, is either 0 or a number divisible by 11.
    Ex. The number 4832718 is divisible by 11, since :
    (sum of digits at odd places) – (sum of digits at even places)
    = (8 + 7 + 3 + 4) – (1 + 2 + 8) = 11, which is divisible by 11.
11. ***Divisibility By 12:*** A number is divisible by 12 if it is divisible by both 3 and 4.
12. ***Divisibility by 14*****:** Add the last two digits to twice the rest. The answer must be divisible by 14
    Ex: 364 = (3×2) + 64 = 70 is divisible by 14
13. ***Divisibility By 15:*** A number is divisible by 15 if it is divisible by both 3 and 5.
14. ***Divisibility By 16:*** A number is divisible by 16 if the number formed by its last four digits is divisible by 16.
15. ***Divisibility by 17:*** Subtract 5-times the last digit from the rest.
    Ex: 221 = 22- (1×5) = 17 is divisible by 17
16. ***Divisibility by 19:*** Add twice the last digit to the rest.
    Ex: 437 = 43 + (7×2) = 57 is divisible by 19
17. ***Divisibility by 20*****:** If the number formed by the last two digits is divisible by 20.
    Ex: 480 = (80 is divisible by 20). So, the number is divisible by 20.
18. ***Divisibility By 25:*** A number is divisible by 25 if the number formed by its last two digits are either 25, 50, 75 or 00. *and so on...*

Note : **If a number is *divisible by p* as well as q, where p and q are co-primes, then the given number is divisible by pq. If p and q are not co-primes, then the given number need not be divisible by pq, even when it is divisible by both p and q.**
Example. 36 is divisible by both 4 and 6, but it is not divisible by (4 x 6) = 24, since they are not co-prime.

DIVISION ALGORITHM

If we divide a given number by another number, then:

Dividend = (Divisor x Quotient) + Remainder

(i) $(x^n - a^n)$ is divisible by $(x - a)$ for **all** values of n.

(ii) $(x^n - a^n)$ is divisible by $(x + a)$ for all **even** values of n.

SIMPLIFICATIONS

1. **'BODMAS' Rule:** This rule depicts the correct sequence in which the operations are to be executed, so as to find out the value of a given expression.
   Here, **'B'** stands for **'Bracket** *sequencially* ***( ),{ },[ ]*** ' , **'O'** for **'of'** , **'D'** for **'Division'**, **'M'** for **'Multiplication'**, **'A'** for **'Addition'** and **'S'** for **'Subtraction'**.
   Thus, in simplifying an expression, first of all the brackets must be removed, strictly in the order (), {} and [ ].
   After removing the brackets, we must use the following operations strictly in the order:
   (i) of (ii) Division (iii) Multiplication (iv) Addition (v) Subtraction.

2. **Simplification:** When an expression contains Virnaculum, before applying the 'BODMAS' rule, we simplify the expression under the Virnaculum.

BASIC FORMULAE

1. $(a + b)^2 = a^2 + b^2 + 2ab$
2. $(a - b)^2 = a^2 + b^2 - 2ab$
3. $(a + b)^2 - (a - b)^2 = 4ab$ *(Avoid in basic formula. It can be easily derived & one doesn't need to remember it)*
4. $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$
5. $(a^2 - b^2) = (a + b)(a - b)$
6. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
7. $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$
8. $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$
9. $(a^3 + b^3 + c^3 - 3abc) = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ac)$
10. If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$
11. $(a^4 - b^4) = (a + b)(a - b)(a^2 + b^2)$
12. $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
13. $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

Recurring Decimal = If in a decimal fraction, a figure or a set of figures is repeated continuously, then such a number is called a Recurring Decimal.

eg. $0.878787..... = 0.\overline{87}$

Vulgar Fraction = A fraction in which the denominator is not 10 or powers of 10.

To Convert a Pure Recurring Decimal into Vulgar Fraction:

Rule: Write the repeated figure or figure only once in the numerator without decimal point and take as many 9's in the denominator as is the number of repeating figures.

Example 1 Express the following as vulgar fraction:

(i) $0.\overline{7}$ (ii) $0.\overline{14}$ (iii) $0.\overline{057}$

Solution: Using the above law, we have:

(i) $0.\overline{7} = 7/9$ (ii) $0.\overline{14} = 14/99$ & (iii) $0.\overline{057} = 57/999$

To convert a Mixed Recurring Decimal to Vulgar Fraction:

Rule: In the numerator take the difference between the number formed by all the digits after decimal point (taking repeated digits only once) and that formed by the digits which are not repeated. In the denominator, take the number formed by as many nines as there are repeating digits followed by as many zeros as is the number of non-repeating digits.

Example 2 Express the following as vulgar fractions:

(i) $0.1\overline{7}$ (ii) $0.12\overline{54}$ (iii) $2.63\overline{4}$

Solution: Using the above rule, we have:

(i) $0.1\overline{7} = \dfrac{17-1}{90} = \dfrac{16}{90} = \dfrac{8}{45}$

(ii) $0.12\overline{54} = \dfrac{1254-12}{9900} = \dfrac{1242}{9900} = \dfrac{69}{550}$

(iii) $2.63\overline{4} = 2 + \left(\dfrac{634-63}{900}\right) = \left(2 + \dfrac{571}{900}\right) = \dfrac{2371}{900}$

Example 3 Convert 6.3333…into a rational number.

Solution: 6.3333…= 6 + 3/9 = 6 + 1/3 = 6⅓

Example 4 Convert 24.232323… into rational number.

Solution: 24.232323…= 24 + 23/99 = $24\frac{23}{99}$

Example 5 Convert $0.\overline{71}$ into rational number.

Solution: Assume $x = 0.\overline{71}$ ____(i)

$100x = 71.71$ ____(ii)

(ii)-(i)

$\Rightarrow 100x - x = 71.\overline{71} - 0.\overline{71}$

$\Rightarrow 99x = 71$

$\Rightarrow x = 71/99$

Example 6 Convert $0.\overline{8}$ into rational number.

Solution: $x = 0.\overline{8}$ _____(i)

$10x = 8.\overline{8}$ _____(ii)

(ii) - (i)

$\Rightarrow 10x - x = 8.\overline{8} - 0.\overline{8}$

$\Rightarrow 9x = 8$ $\Rightarrow x = 8/9$

SURDS

A surd is represented by the symbol $\sqrt{d}$. It can also be defined as an irrational number.
For example, an irrational number which represents the nth root of a positive rational number a is a surd and is represented as $\sqrt[n]{a}$ or $a^{\frac{1}{n}}$
The symbol √ is called a radical, the base number is called the radicand and n is the index of the radical, also known as the order of the surd.
Therefore $\sqrt{5}, \sqrt[3]{6}, \sqrt[4]{7}$ are surds of order 2, 3 and 4 respectively.
$\sqrt[3]{6}$ is read as the third root (or the cube root) of 6 and $\sqrt[4]{7}$ is read as the fourth root of 7. If the index of the radical is not given, it is assumed to be 2.

For example: $\sqrt{5}$ is read as the square root (or the second root) of 5.

LAWS OF SURDS:

(i) $\sqrt[n]{a} = a^{\frac{1}{n}}$ **(ii)** $\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b}$ **(iii)** $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

(iv) $\left(\sqrt[n]{a}\right)^n = a$ **(v)** $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$ **(vi)** $\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$

Square Root: If $x^2 = y$, we say that the square root of y is x and we write, $\sqrt{y} = x$.
Thus, $\sqrt{4} = 2, \sqrt{9} = 3, \sqrt{196} = 14$

Cube Root: The cube root of a given number x is the number whose cube is x. We denote the cube root of x by $\sqrt[3]{x}$.

Thus, $\sqrt[3]{8} = \sqrt[3]{2 \times 2 \times 2} = 2, \sqrt[3]{343} = \sqrt[3]{7 \times 7 \times 7} = 7$ etc.

Note:

1. $\sqrt{xy} = \sqrt{x} \times \sqrt{y}$ **2.** $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}} = \frac{\sqrt{x}}{\sqrt{y}} \times \frac{\sqrt{y}}{\sqrt{y}} = \frac{\sqrt{xy}}{y}$

3. To find the value of:
   (i) $\sqrt{x \pm \sqrt{x \pm \sqrt{x \pm \cdots \ldots \ldots \ldots \infty}}}$: Suppose 'a' and 'b' are the consecutive factors of x(b > a). If there is (+) sign in the expression, the answer is b, i.e., the bigger factor and if there is (-) sign, the answer is a, i.e., the smaller factor.
   (ii) $\sqrt{x . \sqrt{x . \sqrt{x \ldots \ldots \ldots \ldots \infty}}}$: If the root goes upto ∞ in multiplication, the answer is x itself.
   (iii) $\sqrt{x\sqrt{x\sqrt{x \ldots \ldots \ldots \ldots k}}}$ times: If the root goes upto k times, the answer is $x^{\left[\frac{2^k - 1}{2^k}\right]}$
4. A square number never ends with 2, 3, 7 or 8

INDICES / EXPONENTS

1. $a^p \times a^q \times a^r = a^{(p+q+r)}$ *2.* $(a^p)^q = a^{pq}$
3. $a^0 = 1$ *4.* $a^{-p} = 1/a^p$
5. $\sqrt[q]{a^p} = a^{\frac{p}{q}}$ *6.* $(ab)^p = a^p \times b^p$
7. $\sqrt[p]{a} = a^{\frac{1}{p}}$ *8.* $a^{(p)q} = (a^p)^q$ *for example* $(2^3)^3 = 2^9 = 512$ *and* $(2^3)^2 = 2^6 = 64$

SOLVED EXAMPLES

Example 7 Find the value of $\left(\frac{27}{8}\right)^{\frac{2}{3}} + \left(\frac{81}{16}\right)^{\frac{1}{2}} + \left(\frac{243}{32}\right)^{\frac{2}{5}}$

Solution: The expression is equal to

$$\left(\frac{27}{8}\right)^{\frac{2}{3}} + \left(\frac{81}{16}\right)^{\frac{1}{2}} + \left(\frac{243}{32}\right)^{\frac{2}{5}} = \left(\frac{3}{2}\right)^2 + \left(\frac{9}{4}\right) + \left(\frac{3}{2}\right)^2$$

$$= \frac{9}{4} + \frac{9}{4} + \frac{9}{4} = \frac{27}{4}$$

Example 8 Simplify $\dfrac{\left(\dfrac{a^2b^{-4}}{a^{-1}b^2}\right)^{-7}}{\left(\dfrac{a^{-2}b^3}{a^3b^{-5}}\right)^5}$

Solution: $\dfrac{\left(\dfrac{a^2b^{-4}}{a^{-1}b^2}\right)^{-7}}{\left(\dfrac{a^{-2}b^3}{a^3b^{-5}}\right)^5} = \dfrac{\left[a^{2-(-1)}.\,b^{(-4-2)}\right]^{-7}}{\left[a^{-2-3}.\,b^{3-(-5)}\right]^5} = a^{-21+25} \times b^{42-40} = a^4b^2$

Example 9 Find the value of the expression $(216)^{\frac{2}{3}} + \dfrac{1}{(256)^{\frac{-3}{4}}} + (243)^{0.2}$

Solution: Expression could be written as $= 6^2 + 4^3 + 3^1 = 103$

LCM & HCF OF NUMBERS

Highest Common Factor: The Highest Common Factor (HCF) of two or more numbers is the greatest number that divides each of them exactly. eg. *HCF of 25 and 15 is 5.*
Least Common Multiple : Least Common Multiple (LCM) of two or more given number is the least number which is exactly divisible by each of them e.g., 30 is the LCM of 2, 3, 5, 6.

LCM x HCF = Product of two numbers

e.g. The LCM of $2^{11} \times 5^7$ and $2^7 \times 5^{11}$ is $2^{11} \times 5^{11}$ and the HCF of $2^{11} \times 5^7$ and $2^7 \times 5^{11}$ is $2^7 \times 5^7$

HCF of given fractions $= \dfrac{\text{(HCF of numerators)}}{\text{(LCM of denominators)}}$

LCM of given fractions $= \dfrac{\text{(LCM of numerators)}}{\text{(HCF of denominators)}}$

e.g. After reducing the fraction to the lowest terms.
LCM of $\dfrac{2}{7}$ and $\dfrac{37}{111}$ is $\dfrac{\text{(LCM of 2 \& 1)}}{\text{(HCF of 7 \& 3)}} = \dfrac{2}{1} = 2 \quad \therefore \left(\dfrac{37}{111} = \dfrac{1}{3}\right)$

METHOD TO FIND HCF OF THE GIVEN NUMBERS

1. By method of Prime Factorization :
Express each given number as the product of primes. Now take the product of common factors, which is HCF e.g., Find HCF of 136, 144, 168

$136 = 2 \times 2 \times 2 \times 17 = 2^3 \times 17$
$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^4 \times 3^2$

$168 = 2 \times 2 \times 2 \times 3 \times 7 = 2^3 \times 3 \times 7$ *Answer = HCF =* $2^3 = 8$

2. By division method :

Suppose two number are given.

Divide the greatest number by the lesser, divide the lesser by the remainder; divide the first remainder by the new remainder, and so on till there is no remainder. The last divisor is the HCF required.

e.g., Find the HCF of 506 and 1863.

```
506)1863(3
     1518
     345)506(1
         345
          161)345(2
              322
               23)161(7
                  161
                   0
```

Answer = 23 is the HCF of 506 and 1863.

HCF of a given set of numbers would be smaller than or equal to the smallest number in the set.

METHODS TO FIND LCM OF GIVEN NUMBER

1. By method of Prime Factorization:

Resolve each one of the given numbers into prime factors, then their LCM is the product of highest powers of all factors, that occur in these numbers.

e.g., Find LCM of 136, 144, 168

136 = 23 x 17
144 = 24 x 32
168 = 23 x 32 x 7
LCM = 24 x 32 x 17 x 23 x 7 = 17136
Find the smallest four digit multiple of 39.

Sol. First we take 1000, the smallest four digit number possible dividing 1000 by 39 we get the remainder of 25. Take the difference between the divisor 39 and the remainder 25 which is 14 and add this 14 to 1000. We get 1014, which is the smallest four digit multiple of 39.

Least Common Multiple (LCM) of two or more *numbers* is the *least number* which is divisible by each of these numbers (i.e. leaves no remainder; or remainder is zero). The same can be algebraically defined as "LCM of two or more *expressions* is the *expression of the lowest dimension* which is divisible by each of them i.e. leaves no remainder; or remainder is zero."

Highest Common Factor (HCF) is the *largest factor* of two or more given numbers. The same can be defined algebraically as "HCF of two or more algebraically expressions is *the expression of highest dimension* which divides each of them without remainder."

HCF is also GCD (Greatest Common Divisor)

Product of two numbers (a x b) = LCM x HCF

LCM is a multiple of HCF

$$LCM(p, q, r) = \frac{(p \times q \times r) \times HCF(p, q, r)}{HCF(p, q) \times HCF(q, r) \times HCF(r, p)}$$

$$HCF(p, q, r) = \frac{(p \times q \times r) \times LCM(p, q, r)}{LCM(p, q) \times LCM(q, r) \times LCM(r, p)}$$

The power of HCF in the denominator remains one less than the number of numbers.
In word problems involving "longest", "highest", "tallest", "largest", "maximum" etc, we find HCF.
In word problems involving "shortest", "smallest", "least", "minimum" etc, we find LCM.

2. By using division method, Find LCM of 18, 28, 108, 105.

2	18, 28, 108, 105
2	9, 14 54, 105
3	9, 7, 27, 105
3	3, 7, 9, 35
3	1, 7, 3, 35
5	1, 7, 1, 35
7	1, 7, 1, 7 1, 1, 1, 1

LCM = 2 x 2 x 3 x 3 x 3 x 5 x 7 = 3780
$\therefore$ LCM of a set of number would be greater than or equal to the largest number in the set.

Important Concepts of HCF and LCM

1. Greatest number that will divide a, b and c leaving remainders p, q and r respectively = **HCF (a-p, b-q, c-r)**
2. Greatest number that will divide a, b and c leaving same remainders in each case = **HCF (a-b, b-c, a-c)**
3. Least number which when divided by a, b, c leaves remainder p in each case = **LCM (a, b, c) +p**
4. Least number which when divided by a, b, c leaves remainder p, q , r respectively such that [(a-p) = (b-q) = (c-r) = z] = **LCM (a, b, c) - z**

SOLVED EXAMPLES

Example 10 If A + B = 200 & HCF of A & B is 25, A lies between 70 & 90, Then find B.

Solution: Since each number is divisible by HCF, therefore A = 75
Since, A + B = 200, Therefore B = 125.

Example 11 Find the least four digit number which when divided by 10, 12 & 30, leave 8, 10 & 28 as remainders respectively.

Solution: LCM of 10, 12 & 30 is 60.
The least number of 4 digits divisible by 60 is 1020

The common difference of 10 & 8, 12 & 10, 30 & 28 is 2.
Therefore 1020 – 2 = 1018.

FACTOR THEOREM

If $N = p_1^a \times p_2^b$ [p_1, p_2 are prime numbers]
1. Number of factors = $(a+1)(b+1)$
2. Number of prime factors = $(a+b)$
3. Number of distinct prime factors =(p_1, p_2)
4. Sum of factors = $(p_1^0 + p_1^1 + \text{---} + p_1^a)(p_2^0 + p_2^1 + \text{---} + p_2^b)$
5. Product of factors = $(N)^{f/2}$ [f = number of factors]

Example 12 **Find the number of factors of 72.**
Solution: 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72
Total factors in 72 are 12
But in case of a bigger number, this will be a tedious process. The shortcut is essentially to break the num
ber into Prime Factors.
2 | 72
2 | 36
2 | 18
3 | 9
3 | 3
$2^3 \times 3^2 \Rightarrow p_1^a \times p_2^b$
Where, p_1 and p_2 are prime numbers. Therefore, in the above case, number of factors are
(a+1) (b+1)
$72 = 2^3 \times 3^2$
$(3+1) \times (2+1) = 4 \times 3 = 12$ factors

Example 13 **Find the sum of factors of 72.**
Solution: To calculate the sum,
1 + 2 + 3 + 4 + 6 + 8 + 9 + 12 + 18 + 24 + 36 + 72 = 195
Now, the quicker way:
$72 = 2^3 \times 3^2 = (2^0 + 2^1 + 2^2 + 2^3)(3^0 + 3^1 + 3^2)$
$= (1 + 2 + 4 + 8)(1 + 3 + 9) = 15 \times 13 = 195$

Example 14 **Find the product of factors of 72.**
Solution: Where f is number of factors, product of factors would be $\mathbf{(72)^{f/2}}$
$(72)^{12/2} = 72^6$

Example 15 **Find the number of prime factors of 72.**
Solution: $72 = 2^3 \times 3^2$
$\therefore$ No. of prime factors = 3 + 2 = 5

Example 16 **Find the number of distinct prime factors of 72.**
Solution: $72 = 2^3 \times 3^2$
$\therefore$ No. of distinct Prime factors = two i.e., 2, 3

Rule of Cyclicity

A number raised to any power can be expressed as number raised to power 4k+1, 4k+2, 4k+3 and 4k. As per the rule of cyclicity, the last digit repeats in cycles of 4. This one rule helps in finding the last digit of the number.

For eg:
$2^1 = \mathbf{2}$
$2^2 = \mathbf{4}$
$2^3 = \mathbf{8}$
$2^4 = 1\mathbf{6}$
$2^5 = 2^{4k+1} = 3\underline{\mathbf{2}}$
$2^6 = 2^{4k+2} = 6\underline{\mathbf{4}}$

In case of numbers ending in 5 and 6, the last digits always remain 5 and 6 only. In case of numbers ending 4 and 9, the rule of cyclicity becomes that of alternate numbers.
For eg: what's the last digit of $(465723)^{245}$
245 can be expressed as 4k+1
Therefore, the last digit will be 3.
In case, the power was 247, then it could be expressed as 4k+3
In this case, the last digit would be $3^3 = 2\underline{\mathbf{7}}$ i.e. 7

REMAINDER THEOREM

Example 17: Find the remainder when $\frac{2^{33}}{3}$.

Solution: $\frac{2^1}{3} \rightarrow \text{Re}\ (2)$
$\frac{2^2}{3} \rightarrow \text{Re}\ (1)$
$\frac{2^3}{3} \rightarrow \text{Re}\ (2)$
$\frac{2^4}{3} \rightarrow \text{Re}\ (1)$
$\frac{2^5}{3} \rightarrow \text{Re}\ (2)$
$\frac{2^6}{3} \rightarrow \text{Re}\ (1)$

We can observe that remainder is forming a pattern i.e.., 2, 1, 2, 1, 2, 1… and so on. So Cyclicity is of 2. So to find the remainder, divide power by cyclicity. Here 33 will be divided by 2 which will give remainder as 1. So for remainder 1, answer will be 2.

Concept of Negative Remainders
Example 18: Find the remainder when $\frac{2^{32}}{17}$

Solution: We know that $2^4 = 16$
When $2^4 \div 17$ will give remainder as 16 or (-1)
Although by definition remainder cannot be negative but considering negative remainder is a very useful exam trick.

$$\text{So } \frac{2^{32}}{17}\ \frac{(2^4)^8}{17} \rightarrow \frac{(16)^8}{17} \rightarrow \frac{16 \text{ x } 16 \text{ x}16\text{...up to 8 times}}{17} \rightarrow \frac{(-1)\text{x}\ (-1)\text{x}\ (-1)\text{...up to 8 times}}{17} \rightarrow \frac{(-1)^8}{17} \rightarrow \frac{1}{17} \rightarrow \text{Re}\ (1)$$

EXPERIENCE THE PRATHAM EDGE - I

1. Which fraction is the greatest among the following: $\frac{11}{32}, \frac{15}{47}, \frac{37}{115}$?

(a) $\frac{11}{32}$ (b) $\frac{15}{47}$ (c) $\frac{37}{115}$ (d) All are same

2. Which fraction is the smallest of the following: $\frac{45}{49}, \frac{49}{53}, \frac{53}{57}$?

(a) $\frac{53}{57}$ (b) $\frac{49}{53}$ (c) $\frac{45}{49}$ (d) All are same

3. What is the remainder if 10400 is divided by 8?

(a) 0 (b) 1 (c) 2 (d) 3

4. A man gave 1/3 of his wealth to his wife, 1/2 of remaining to his first son, 1/2 of remaining to his second son & rest of Rs. 6 lakhs to his youngest son. Find out his wealth.

(a) Rs. 40 lakhs (b) Rs. 38 lakhs (c) Rs. 36 lakhs (d) Rs. 42 lakhs

5. $\frac{1}{3\times7}+\frac{1}{7\times11}+\frac{1}{11\times15}+\frac{1}{15\times19} = ?$

(a) $\frac{15}{57}$ (b) $\frac{57}{15}$ (c) $\frac{4}{57}$ (d) $\frac{57}{59}$

6. The sum of the digits of a two digit number is 8, if the digits are reversed the number is decreased by 54. Find the number.

(a) 71 (b) 17 (c) 62 (d) 53

7. $\frac{(64^3-17^3-47^3)}{64\times3\times47} = ?$

(a) 20 (b) 30 (c) 15 (d) 17

8. $\sqrt{64(x)(64)} = 32\times32, x = ?$

(a) 230 (b) 325 (c) 256 (d) 125

9. $\frac{0.004}{\sqrt{x}} = 0.0008$, x = ?

(a) 30 (b) 28 (c) 25 (d) 40

10. Find the greatest number of 4 digits which when divided by 20, 30 & 15 leaves 18, 28 & 13 as remainders respectively.

(a) 9958 (b) 9960 (c) 9985 (d) 9999

11. 1 + 2 + 3 + 4 + ----99 + 100 + 99 + 98 + ---- + 3 + 2+ 1 = ?

(a) 100000 (b) 10000 (c) 90000 (d) 9000

12. $\dfrac{(8000)^3+(0.080)^3}{(400)^3+(0.004)^3}=?$

(a) 4000 (b) 6000 (c) 8000 (d) 10000

13. $48\dfrac{1}{37}\div 37\dfrac{1}{48}=?$

(a) 47/48 (b) 48/37 (c) 38/37 (d) 48/47

14. $48\dfrac{1}{12}\div 24=\ ?$

(a) $2\dfrac{1}{288}$ (b) $5\dfrac{1}{288}$ (c) $39\dfrac{1}{288}$ (d) $3\dfrac{1}{288}$

15. Simplify: $\dfrac{1}{7}$ of $(6\times 8\times 3\times 2)+\dfrac{1}{5}\times\dfrac{7}{25}-\dfrac{1}{7}\left(\dfrac{3}{7}+\dfrac{8}{14}\right)$

(a) $40\dfrac{5}{125}$ (b) $37\dfrac{7}{125}$ (c) $39\dfrac{7}{125}$ (d) $41\dfrac{7}{125}$

16. Find the value of $16\times 4\div 2\times 5+4-6\div 9\times 4\div 2+1$.

(a) $163\frac{2}{3}$ (b) $160\frac{2}{3}$ (c) $158\frac{2}{3}$ (d) $161\frac{2}{3}$

17. $\left[2-\dfrac{1}{3}\right]\left[2-\dfrac{3}{5}\right]\left[2-\dfrac{5}{7}\right]\ldots\ldots\ldots\ldots\left[2-\dfrac{997}{999}\right]=\ ?$

(a) 1001/3 (b) 1000/7 (c) 999/5 (d) 900/7

18. Simplify $\dfrac{7+\sqrt{5}}{7-\sqrt{5}}+\dfrac{7-\sqrt{5}}{7+\sqrt{5}}$.

(a) 27/11 (b) 11/27 (c) 7/12 (d) 5/12

19. If $32^{2-n} = 64^n$, find the value of n.

(a) 8/11 (b) 9/11 (c) 10/11 (d) 1/11

20. Which among the following is greatest: $\sqrt{5},\ \sqrt[3]{11},\ \sqrt[6]{123}$?

(a) $\sqrt{5}$ (b) $\sqrt[3]{11}$ (c) $\sqrt[6]{\quad}$ (d) All are equal

21. One sixth of the number of books in a library consist of Mathematics, 3/5 of the remaining are Fiction, 8/9 of what still remains are History and the remaining books are on Science. What should be the least number of books in the library to satisfy these conditions?

(a) 52 (b) 54 (c) 56 (d) 58

22. Dividing by 4/7 and then multiplying by 8/9 is equivalent to dividing by what number?

(a) 9/8 (b) 8/9 (c) 9/14 (d) 14/9

23. Which among the following is greatest: $\sqrt{7}+\sqrt{3},\ \sqrt{5}, \sqrt{6}+2$?

(a) $\sqrt{7}+\sqrt{3}$ (b) $\sqrt{5}$ (c) $\sqrt{6}+2$ (d) All are equal

24. The product of two numbers is 1728 and their L.C.M. is 144. Find all the pairs of such numbers.
(a) (12, 144), (96, 18) (b) (12, 144), (36, 48)
(c) (36, 48), (96, 18) (d) (36, 48), (56, 30)

25. What least number must be added to 8961 to make it exactly divisible by 84?
(a) 27 (b) 28 (c) 29 (d) 30

26. A number when divided by 145 leaves the remainder 58. What is the remainder if it is divided by 29?
(a) 2 (b) 4 (c) 0 (d) 5

27. A number when divided by 5 and 7 successively, leaves the remainders 2 and 4 respectively. Find the remainder when the same number is divided by 35.
(a) 30 (b) 27 (c) 25 (d) 22

28. One pendulum ticks 57 times in 58 sec, another 608 times in 609 seconds. If they are started together, how often will they have ticked together in the first hour ?
(a) 60 (b) 57 (c) 56 (d) 50

29. Find the nearest integer to 1834 which is exactly divisible by both 12 and 16.
(a) 1830 (b) 1824 (c) 1832 (d) 1820

30. A number N when divided by 5 leaves the remainder 1, and when divided by 6 leaves the remainder 5. Find the smallest positive N.
(a) 11 (b) 12 (c) 13 (d) 14

31. Which is the largest three-digit number which when divided by 6 leaves the remainder 5, and when divided by 5 leaves the remainder 3?
(a) 999 (b) 983 (c) 987 (d) 980

32. Find the remainder when $7^{13} + 1$ is divided by 6.
(a) 2 (b) 3 (c) 4 (d) 5

33. What is the remainder when 10200 is divided by 8?
(a) 0 (b) 1 (c) 2 (d) 3

34. Find the remainder if 30^{40} is divided by 17?
(a) 0 (b) 1 (c) 2 (d) 3

35. Find the HCF of
(i) 6435, 8970, 7235
(a) 2 (b) 3 (c) 4 (d) 5
(ii) 6161, 2440, 3111
(a) 50 (b) 55 (c) 61 (d) 67

36. Find the greatest number that will divide 213, 241 and 297, leaving remainder 3 in each case.
(a) 14 (b) 16 (c) 18 (d) 20

37. Find the greatest number that will divide 364 and 532 leaving remainders 4 and 7 respectively.
(a) 13 (b) 15 (c) 17 (d) 19

38. Find the greatest number which will divide 1742, 3723 and 1843 leaving 4, 5 and 6 as remainders respectively.
(a) 11 (b) 12 (c) 13 (d) 14

39. Find the greatest number that will divide 151, 175 and 235 leaving the same remainder in each case.
(a) 11 (b) 12 (c) 13 (d) 14

40. Find the greatest number that will divide 221, 263 and 326 leaving the same remainder in each case.
(a) 19 (b) 20 (c) 21 (d) 22

41. The sum of two numbers is 180 and their HCF is 15. Find all the possible pairs of such numbers.

(a) (160, 20), (80, 100) (b) (15, 165), (75, 105)
(c) (100, 80), (90, 90) (d) (150, 30), (175, 5)

42. The sum of two numbers is 192 and their HCF is 24. Find all the possible pairs of such numbers.
(a) (24, 168), (72, 120) (b) (92, 100), (168, 24)
(c) (72, 120), (92, 100) (d) (90, 102), (60, 132)

43. The product of two numbers is 2700 and their HCF is 15. Find the all the possible pairs of such numbers.
(a) (15, 180), (45, 60) (b) (270, 10), (90, 30)
(c) (90, 30), (15, 45) (d) (270, 10), (15, 180)

44. Find the two numbers nearest to 10000 that have 169 for their HCF.
(a) 8000, 8500 (b) 9997, 9587 (c) 9971, 10140 (d) None of these

45. Find the greatest number of 4 digits and the least number of 5 digits that have 144 as their HCF.
(a) 9938, 10080 (b) 9999, 10001 (c) 9000, 11000 (d) 9936, 10080

46. 264 oranges and 693 mangoes are to be distributed among some girls so that each girl may get as many mangoes and as many oranges as another girl. Find the largest possible number of girls and the least possible number of fruits of each kind which a girl gets.
(a) (33, 8, 21) (b) (35, 10, 15) (c) (40, 15, 20) (d) (35, 8, 20)

47. Four bells toll after an interval of 8, 9, 12 and 15 seconds. When will they toll together?
(a) After 10 min. (b) After 15 min (c) After 8 min (d) After 6 min

48. Find the least number which when divided by 18, 27 and 36 will leave remainder 7 in each case.
(a) 120 (b) 115 (c) 117 (d) 118

49. Find the least number which when divided by 5, 6, 7, 8, and 9 leaves remainders 3, 4, 5, 6 and 7 respectively.
(a) 2518 (b) 2500 (c) 2080 (d) 2025

50. Find the least number of 5 digits which is exactly divisible by 5, 6, 7 and 8.
(a) 10075 (b) 10095 (c) 10080 (d) 10085

51. What is the least number which when divided by 6, 7 and 9 leaves remainder 4 in each case and is exactly divisible by 11?
(a) 1025 (b) 1010 (c) 1025 (d) 1012

52. Find the least integer closest to 67281 which when divided by 3, 8, 11 and 16 leaves remainder 1, 6, 9 and 14 respectively.
(a) 67054 (b) 67055 (c) 67523 (d) 67325

53. I have a certain number of oranges numbering between 600 and 900. If 2 oranges are taken away the remainder can be equally divided among 3, 4, 5, 6, 7 or 12 boys. Find the number of oranges I have.
(a) 845 (b) 843 (c) 842 (d) 840

54. The L.C.M. of two numbers is 1575 and their HCF is 25; one of the numbers is between 200 and 300; find the number between 200 and 300.
(a) 225 (b) 220 (c) 218 (d) 215

55. The product of two numbers is 8820 and their LCM is 1260. Find the HCF.
(a) 7 (b) 8 (c) 9 (d) 10

56. The HCF of three numbers is 12 and the LCM is 360. If two of the numbers be 24 and 36, determine the value of the third number.
(a) 20 (b) 40 (c) 60 (d) 80

57. The sum of two numbers is 126 and their L.C.M. is 180. Determine the numbers.
(a) 90, 36 (b) 100, 26 (c) 105, 21 (d) 51, 75

58. What is the remainder when 46! is divided by 47?
(a) 1 (b) 46 (c) 34 (d) 11

59. Remainder obtained in how many of the following cases would be same?
i. 10^6 divided by 7 ii. 10^{16} divided by 17 iii. 10^{18} divided by 19
(a) only (i) (b) only (ii) (c) (i) and (ii) both (d) (i), (ii) and (iii)

60. Find the remainder of $(10^{10}+10^{100}+10^{1000})/7$
   (a) 5 (b) 4 (c) 2 (d) 1
61. Which of the following numbers will have lowest total number of factors?
   (a) 48 (b) 60 (c) 30 (d) None of these
62. How many factors of 84 is the product of only two prime, distinct or same, factors?
   (a) 4 (b) 6 (c) 3 (d) 2

Directions for questions 63 and 64: Read the information below and solve the questions based on it.
$N = A^2 \times B^3 \times C^4$, where, A, B and C are prime numbers.

63. How many factors of N are perfect squares?
   (a) 6 (b) 12 (c) 8 (d) None of these
64. How many factors of N are cube of any natural number?
   (a) 2 (b) 3 (c) 4 (d) 6
65. Find the product of all the factors of 3^{16}.
   (a) 3^{33} (b) 3^{68} (c) 3^{136} (d) 3^{128}
66. What is the unit digit of $11^{(5!)} - 1$?
   (a) 1 (b) 0 (c) 2 (d) 3
67. What is the unit digit of 7^{2008}?
   (a) 2 (b) 1 (c) 0 (d) 7
68. What is the unit digit of the following expression?
   $1432^{40!\times30!\times20!} \times 2427^{4!\times5!\times10!} \times 2319^{10!\times20!\times30!} \times 1552^{12!}$?
   (a) 1 (b) 6 (c) 0 (d) 9
69. The digit in the unit place of the number represented by $(7^{95} \times \mathbf{3}^{58})$ is:
   (a) 7 (b) 0 (c) 6 (d) 4
70. What will be the unit digit of 41^{44k}, where k is positive integer?
   (a) 7 (b) 0 (c) 6 (d) 1

PRATHAM EDGE – 2

1. Multiplication by 9,99,999 etc. Let us take few examples

(a) 99 x 23 = 2277

(b) 9999 x 2831 = 28307169

Look at example (a) deduct 1 from 23, it will become 22, this will become the first part of answer. Then deduct 22 from 99, it will be 77. It will become the second part of answer. In the same way the next example (b) can be done. The same method must be applied in all multiplication of 9,99,999.......

(a)

$$\begin{array}{r} 99 \\ \times\ 23 \\ \hline 22\square\square \end{array} \qquad \begin{array}{r} 99 - 22 = 77 \\ 99 \\ \times\ 23 \\ \hline 2277 \end{array}$$

(b)

$$\begin{array}{r} 9999 \\ \times\ 2831 \\ \hline 2830\square\square\square\square \end{array} \qquad \begin{array}{r} 9999 - 2830 \\ 9999 \\ \times\ 2831 \\ \hline 28307169 \end{array}$$

2. Square of the numbers whose unit's digit is 5

Like 25^2, 35^2, 45^2

25 x 25 = 625

(i) $\begin{array}{r} 25 \\ \times\ 25 \\ \hline \square 25 \end{array}$ *multiply 5 x 5 = 25 and write it on Right hand side*

(ii) Add 1 to 2 → 1 + 2 = 3 and multiply 2 to 3 → 2 x 3 = 6. Write it on left hand side.

(iii) $\boxed{625}$

(i) $\begin{array}{r} 35 \\ \times\ 35 \\ \hline \square 25 \end{array}$ *5 x 5 = 25* (ii) 1 + 3 = 4

(iii) 4 x 3 = 12

$\boxed{1225}$

(iv)

$\begin{array}{r} 85 \\ \times\ 85 \\ \hline 7225 \end{array}$

Step 1

5 x 5 = 25

Step 2

1 + 8 = 9

9x 8 = 72

$\begin{array}{r} 95 \\ \times\ 95 \\ \hline 9025 \end{array}$

Step 1

5 x 5 = 25

Step 2

1 + 9 = 10

10 x 9 = 90

CHAPTER 2

AVERAGES

INTRODUCTION

The Concept of averages is important. Some exams test the concept of average through direct questions, and some entrance exams use the concept to set conceptual problems that ordinarily will be difficult to tackle.
For that while studying this chapter make sure to understand the faster illustrated computation in the text.

$$Average = \frac{\text{Sum of the observations}}{\text{Number of the observations}}$$

$$Average\ Increase = \frac{\text{Total Increase}}{\text{Number of the observations}}$$

$$Average\ Decrease = \frac{\text{Total Decrease}}{\text{Number of the observations}}$$

SOLVED EXAMPLES

Example 1 Average of A, B, C & D = 45, Average of B, C, D & E = 68. If A=100, Find E.

Solution: E = A + (Average of BCDE – Average of ABCD) x 4
E = 100 + (68 – 45) x 4
E = 100 + 23 x 4 = 192

Example 2 Average of A, B, C & D = p, Average of A & B = q, Average of C & D = ?

Solution: Average of CD = $\frac{(4p-2q)}{2}$ = 2p – q

Example 3 Average Salary of 24 persons decreases by Rs. 50 when one of them drawing Rs. 12200 retires and replaced by a new man. What is the salary of the newcomer.

Solution: Salary of the newcomer = Rs. (12200 - 24 x 50) = Rs. 11000

Example 4 Average of A, B, C & D = 43, Average of C, D, E & F = 53, Average of A & B = 30, Average of E & F = ?

Solution: Average of EF = $\frac{2AB+(CDEF-ABCD)\times 4}{2}$

Average of EF = $\frac{60+10\times 4}{2}$ = 50

Example 5 Average of A, B, C, D, E, F & G = $14\frac{2}{7}$, Average of A, B, C & D = 20, Average of E & F = 5, Find G.

Solution: G= $14\frac{2}{7}$ x 7 – 20 x 4 – 5 x 2 = 10

Example 6 What is the average of first 37 odd numbers?

Solution: Sum of first n odd numbers is n^2. Therefore, the average is n. Required average = 37

Example 7 What is the average of first 38 even numbers?

Solution: Sum of first n even numbers is n(n+1)
Therefore, the average is n+1. Hence, the average of first 38 even numbers is 39.

Example 8 Average of 0.1, x & 0.001 = 0.037, x = ?
Solution: $x = 0.037 \times 3 - 0.1 - 0.001 = 0.01$

Example 9 Average of 20, 30 and x = 100, Average of 30, 40, x & y = 200, y = ?
Solution: $x = 300 - (20 + 30) = 250$
$y = 800 - [30 + 40 + 250] = 480$

Example 10 Average of ABCDE = 37, Average of BCDEF = 48, A : F = 2 : 13, A = ?
Solution: F – A = (48 – 37) x 5 = 55
$\therefore$ A = 2 x 5 = 10

Example 11 Average of any 5 consecutive multiples of 28 is 56056. Find the smallest multiple out of those.
Solution: Smallest multiple of 28 = 56056 - 56
Smallest multiple of 28 = 56000

Example 12 A = 60, B = 60, C = 60, D is 60 more than average of A, B, C & D, then D = ?.
Solution: Let average of A, B, C & D be x

$\therefore D = 60 + x$

$$x = \frac{60 + 60 + 60 + (60 + x)}{4}$$

$4x = 240 + x$
$x = 80$ $\therefore D = 140$

Example 13 A = 60, B = 60, C = 60, D is 30 less than average of A, B, C & D, then D = ?
Solution: Let the average of A, B, C & D be x
$\therefore D = x - 30$

$$x = \frac{60 + 60 + 60 + (x - 30)}{4}$$

$4x = 150 + x$
$x = 50$ $\therefore D = 20$

Example 14 In Section A, 20 Students average weight is 40 kg. In Section B, 60 Students average weight is 60 kg. In Section C, 20 Students average weight is 70 kg. What will be the average weight of all the 100 students?
Solution: Average weight of all 100 students is $= \frac{(40 \times 1 + 60 \times 3 + 70 \times 1)}{5} = 58$ kg

Example 15 Average age of ABC 4 years ago was 13 years. D was born 9 years ago. What is the present average age of ABCD?
Solution: Average age of ABCD $= \frac{(17 \times 3 + 9)}{4} = 15$ years.

Example 16 Speed from A to B = 40 km/h, Speed from B to A = 60 km/h, Average speed = ?
Solution: Let Distance from A to B be x.

$$\text{Average speed} = \frac{2x}{\frac{x}{40} + \frac{x}{60}} = 48 \text{ km/h}$$

Example 17 Average of ABCD = 40, Average of BCDE = 100, E = 300, A = ?

Solution: B+C+D = 100

A = 40 x 4 -100 = 60

Example 18 Find the average of first 100 natural numbers.

Solution: Average of first 100 natural numbers = $\dfrac{\frac{n(n+1)}{2}}{n} = \dfrac{n+1}{2} = \dfrac{101}{2} = 50.5$

Example 19 What is the increase in avg, if the 16th multiple of 26 is added to any 15 consecutive multiple of 26.

Solution: Let us take first 15 multiple of 26

$26 \times 1, 26 \times 2, 26 \times 3, \ldots, 26 \times 15$

Average of first 15 multiple of 26

$= \dfrac{26\times1+26\times2+1\ldots+26\times15}{15}$

$= \dfrac{26(1+2+3+\cdots+15)}{15} = \dfrac{26\times15\times16}{15\times2} = 26 \times 8 = 208$

Now 16th multiple of 26 is added.

New average$= \dfrac{26\times1+26\times2+\cdots+26\times15+26\times16}{16}$

$\dfrac{26(1+2+3+\cdots+16)}{16} = \dfrac{26\times16\times17}{16\times2} = 13 \times 17 = 221$

$\therefore$ Increase in average $= 221 - 208 = 13$

EXPERIENCE THE PRATHAM EDGE - 2

1. Find the average of scores of the tests taken by Ram, given his scores were 92, 94, 88, 90, 90, 86, 90.
(a) 90 (b) 80 (c) 85 (d) 89

2. The monthly incomes of Ravi, his wife & their son are Rs 4000, Rs 2000 and Rs 3000 respectively. What is the average income of the family?
(a) Rs 3505 (b) Rs 3100 (c) Rs 3200 (d) Rs 3000

3. In a cricket tournament, Kapil played 7 games. His scores in the games were 71, 49, 52, 46, 48, 52 & 53. Find the average score.
(a) 54 (b) 53 (c) 51 (d) 50

4. Jeevan purchased three dozen mangoes at Rs 7 per dozen, two dozen mangoes at Rs 8.50 per dozen & one dozen mangoes at Rs 10. What is the average cost per dozen of the mangoes purchased by Jeevan?
(a) Rs 7 (b) Rs 8 (c) Rs 9 (d) Rs7.5

5. The average age of 5 boys is 19 years. A new boy joins them & their average age increases to 20 years. What is the age of new boy?
(a) 25 years (b) 24 years (c) 26 years (d) 23 years

6. 7 kg of wheat having a rate of Rs 14 per kg is mixed with 4 kg of wheat having a rate of Rs 8.50 per kg. What is the average rate per kg of the mixture?
(a) Rs 11 (b) Rs 12 (c) Rs 13 (d) Rs 14

7. A batsman scored an average of 55 runs in the first 6 tests. If the first is not counted and the seventh is counted then his average score goes up to 57. If the score in first test was 50 runs, find his score in the seventh test.
(a) 62 (b) 65 (c) 63 (d) 71

8. The average marks of three sections in tenth class were 76, 79 & 80 respectively. If the number of students in these three sections were 70, 60, 55 respectively. Find the average marks of the tenth class.
(a) 75.16 (b) 75 (c) 78.16 (d) 76

9. The sales recorded by a salesman in 6 days of a week were as follows: Rs 20,000 per day for the first 3 days. Rs 16000 on Thursday, Rs 24000 on Friday and on Saturday it was Rs 30,000. Find the average sales per day.
(a) Rs. 21667 (b) Rs. 21000 (c) Rs. 22220 (d) Rs. 21100

10. 10 years ago, the average age of a family of 4 members was 24 years. Two children having been born (with age difference of 2 yrs, the present average age of the family is the same. The present age of the youngest child is:
(a) 1 year (b) 11/2 years (c) 2 years (d) 3 years

11. The average age of a group of friends is 34 years. If five new friends with an average age of 30 years join the group, the average age of the entire group becomes 32 years. How many people were there in the group initially?
(a) 4 (b) 5 (c) 6 (d) 7

12. If 20 kg of wheat costing Rs 16 per kg is mixed with 16 kg of a second variety of wheat costing Rs 25 per kg, then what is the average cost of the resulting mixture?
(a) Rs 25 (b) Rs 24 (c) Rs 22 (d) Rs 20

13. If two kinds of grapes costing Rs 16 per kg and Rs 21 per kg are mixed in the ratio of 2 : 3 then find the cost of mixture per kg.
(a) Rs 17 (b) Rs 20 (c) Rs 18 (d) Rs 19

14. If 15 litres of 20% alcohol is mixed with 12 litres of 24% alcohol, then find the concentration of the resulting solution.
(a) 588/97 (b) 588/27 (c) 588/57 (d) 588/7

15. Find two numbers whose AM is 12.5 and GM is 10.
(a) 20, 15 (b) 18, 24 (c) 20, 5 (d) 12, 16

16. The average age of a husband and his wife was 23 years at the time of their marriage. After five years they have a one-year old child. The average age of the family now is:
(a) 19 years (b) 23 years (c) 28.5 years (d) 29.3 years

17. The average salary of all the workers in a workshop is Rs. 8000. The average salary of 7 technicians is Rs. 12000 and the average salary of the rest is Rs. 6000. The total number of workers in the workshop is:
(a) 20 (b) 21 (c) 22 (d) 23

18. 3 years ago, the average age of a family of 5 members was 17 years. A baby having been born, the average age of the family is the same today. The present age of the baby is:
(a) 1 year (b) 11/2 years (c) 2 years (d) 3 years

19. In an examination, a pupil's average marks were 63 per paper. If he had obtained 20 more marks for his Geography paper and 2 more marks for his History paper, his average per paper would have been 65. How many papers were there in the examination?
(a) 8 (b) 9 (c) 10 (d) 11

20. The average age of students of a class is 15.8 years. The average age of boys in the class is 16.4 years and that of the girls is 15.4 years. The ratio of the number of boys to the number of girls in the class is:
(a) 1 : 2 (b) 2 : 3 (c) 3 : 4 (d) 3 : 5

21. The average weight of 3 men A, B and C is 84 kg. Another man D joins the group and the average now becomes 80 kg. If another man E, whose weight is 3 kg more than that of D, replaces A, then the average weight of B, C, D and E becomes 79 kg. The weight of A is:
(a) 70 kg (b) 72 kg (c) 75 kg (d) 80 kg

22. A cricketer has a certain average for 10 innings. In the eleventh inning, he scored 108 runs, thereby increasing his average by 6 runs. His new average is:
(a) 48 runs (b) 52 runs (c) 55 runs (d) 60 runs

23. The average weight of 8 persons increases by 2.5 kg when a new person comes in place of one of them weighing 65 kg. What is the weight of the new person?
(a) 76 kg (b) 76.5 kg (c) 85 kg (d) None of these

24. Of the four numbers, the first is twice the second, the second is one-third of the third and the third is 5 times the fourth. The average of the numbers is 24.75. The largest of these numbers is:
(a) 9 (b) 25 (c) 30 (d) None of these

25. If the arithmetic mean of seventy-five numbers is calculated, it is 35. If each number is increased by 5, then mean of new numbers is:
(a) 30 (b) 40 (c) 70 (d) 90

PRATHAM EDGE – 3

LET US CONSIDER THE MULTIPLICATION OF TWO – 2 DIGIT NUMBERS

2 digit number

	x	c	d
Product			
Box Number	3	2	1

Step 1: Find b x d. If it is a one digit number write it in box 1. If it is a two digit number write its unit digit in box 1 and carry over the tens place.

Step 2: Find a x d + b x c + carried over if any. If it is a one digit number write it in box 2. If it is a two digit number then write its units digit in box 2 and carry over the rest.

Step 3: Find a x c + carried over if any. Write the reverse in box 3.

Step: 4: Read the number so obtained from left to right. This number is the required product.

$$\begin{array}{r} 43 \\ \times\, 27 \end{array}$$

Product	11	6	1
Box number	3	2	1

Step 1: 3 x 7 = 21. We write 1 in box and carry over 2.

Step 2: 4 x 7 + 3 x 2 + 2 = 36. We write 6 in box 2 and carry over 3.

Step 3: 4 x 2 + 3 = 11. We write 11 in box 3.

Step 4: The required product is 1161.

NOTES

CHAPTER 3

PERCENTAGE

INTRODUCTION

The importance of percentages is accentuated by the fact that there are a lot of questions related to the use of percentage in all the chapters of arithmetic. A closer look at the topic will evolve the result that at least 75% of the total calculations in any DI paper is formed on calculations on additions and percentages.

Percentage means $\frac{1}{100}$

- $100\% = 100 \times \frac{1}{100} = 1$
- To find $33\frac{1}{3}\%$ of a number multiply it by 1/3
- To find $66\frac{2}{3}\%$ of a number multiply it by 2/3
- To find 25% of a number multiply it by 1/4
- To find 20% of a number multiply it by 1/5
- To find $16\frac{2}{3}\%$ of a number multiply it by 1/6
- To find $14\frac{2}{7}\%$ of a number multiply it by 1/7
- To find $12\frac{1}{2}\%$ of a number multiply it by 1/8
- To find $37\frac{1}{2}\%$ of a number multiply it by 3/8
- To find $62\frac{1}{2}\%$ of a number multiply it by 5/8
- To find $11\frac{1}{9}\%$ of a number multiply it by 1/9
- To find 10% of a number multiply it by 1/10
- To find $9\frac{1}{11}\%$ of a number multiply it by 1/11
- To find $8\frac{1}{3}\%$ of a number multiply it by 1/12
- To find $7\frac{9}{13}\%$ of a number multiply it by 1/13
- To find $7\frac{1}{7}\%$ of a number multiply it by 1/14
- To find $6\frac{2}{3}\%$ of a number multiply it by 1/15
- To find $6\frac{1}{4}\%$ of a number multiply it by 1/16

Important Concepts:

1. Percentage Increase/Decrease

- Percentage increase $= \frac{\text{increase in quantity}}{\text{original quantity}} \times 100\%$
- Percentage decrease $= \frac{\text{decrease in quantity}}{\text{original quantity}} \times 100\%$
- To increase a number by x% multiply it by $\frac{100+x}{100}$
- To decrease a number by x% multiply it by $\frac{100-x}{100}$
- If A's income is r% more than B's income then B's income is $\frac{r}{100+r} \times 100\%$ less than A's income.
- If A's income is r% less than B's income then B's income is $\frac{r}{100-r} \times 100\%$ more than A's income.
- If A's income is $1/x$ more than B's income, then B's income is $1/(1+x)$ less than A's income.
- If A's income is $1/x$ less than B's income, then B's income is $1/(x-1)$ more than A's income.
- If price of a commodity increases by x% then to keep the expenses same the consumption must be decreased by $\frac{100x}{100+x}\%$.
- If price of a commodity decreases by x% then for same expense consumption must be increased by $\frac{100x}{100-x}\%$

2. Successive Percentage Increase/Decrease

- Let the successive percentages be a% and b%.
  In that case, the total percentage change will be $(a \pm b \pm \frac{(\pm a)(\pm b)}{100})\%$
- If the price of an article is successively increased by x%, y% and z%, then single equivalent increase in the price will be $\left[x + y + z + \frac{xy+yz+zx}{(100)} + \frac{xyz}{(100)^2}\right]\%$

3. Examination

- A candidate scoring x% in an examination fails by 'a' marks, while another candidate who scores y% marks gets 'b' marks more than the minimum required pass marks. Then the maximum marks for that examination are $M = \frac{100\,(a+b)}{y-x}$
- If in an examination x% of the students failed in one subject, y% failed in another subject and z% failed in both the subjects, the percentage of students who:
  (i) Failed in either of the subjects = x + y – z.
  (ii) Passed in both the subjects = 100 – (x + y – z).

4. Population

- If the original (present) population of a town is P, then the population (P_o) after n years at an annual increase of r% is given by $P_0 = P\left(1 + \frac{r}{100}\right)^n$
- If the present population is P, then the population n years ago is given by $P_0 = \frac{P}{\left(1+\frac{r}{100}\right)^n}$

5. Elections

<u>Important Terms:</u>

- List votes – People who are eligible to vote and their names are present in the list.
- Valid votes – A valid ballot paper is one which reflects the intentions of the voter unambiguously.
- Invalid Votes - A vote is defined as "spoilt" or "invalid" if the counting official is unable to clearly determine

the intention of the voter.

- Votes polled - People standing in a line waiting to vote at a polling station.

SOLVED EXAMPLES

Example 1 In a class of 60 if 36 are boys what percentage are girls?

Solution: Percentage of girls = $\frac{24}{60} \times 100\% = 40\%$

Example 2 What is 60% of 60%?

Solution: The value of 60% of 60% $= \frac{60}{100} \times \frac{60}{100} = 36\%$

Example 3 What is $6\frac{1}{4}\%$ of 64 + $13\frac{3}{4}\%$ of 64?

Solution: $6\frac{1}{4}\%$ of 64 + $13\frac{3}{4}\%$ of 64 = 20% of 64 = 12.8.

Example 4 If a number is 60 plus 95% of itself, then what is the number?

Solution: Let x be the number Then x = 60 + 95% of x

Therefore 5 % of x = 60 Hence x = 1200.

Example 5 If the price of a commodity increases by 20%, then to keep same expenditure, what is the percentage reduction in consumption?

Solution: Price ratio is 100 : 120. Consumption decreases from 120 to 100

Therefore to keep the expenditure same, % reduction in consumption would be

$\frac{20}{120} \times 100\% = 16\frac{2}{3}\%$

Example 6 A number is first increased by 30% then deceased by 20%, what is the overall change?

Solution: The overall change = 30 – 20 – $\frac{(30 \times 20)}{100}$ = +4% (4% Increase)

Example 7 After 2 successive decreases of 20% the price of a set is Rs. 12800. What is the original price?

Solution: Original Price = 12800 x $\frac{100}{80} \times \frac{100}{80}$ = Rs. 20,000.

Example 8 In a class of 60, 60% are boys. How many girls leave so that boys are 75%?

Solution: $\frac{36}{(24-x)} = \frac{75\%}{25\%} = \frac{3}{1}$, 36 = 3 (24 – x), x = 12

Example 9 Two numbers are less than a third number by 30% and 37% respectively. How much percentage is the second number less than the first?

Solution:

	:			:		
70	:	63	:	100

$\frac{7}{70}$ x 100% = 10%

Example 10 25% of A = $33\frac{1}{3}\%$ of B = $12\frac{1}{2}\%$ of C. If A = 20, A + B + C = ?

Solution: $\frac{A}{4} = \frac{B}{3} = \frac{C}{8}$

3A = 4B → 3 x 20 = 4 x B ∴ B = 15

8A = 4C → 8 x 20 = 4 x C ∴ C = 40

∴ A + B + C = 20 + 15 + 40 = 75

EXPERIENCE THE PRATHAM EDGE - 3

1. Two numbers are less than a third number by 30% and 37% respectively. How much percent is the second number of the first?
(a) 70% (b) 40% (c) 30% (d) 90%

2. The population of a village is 5000 and it increases at the rate of 2% every year. After 2 years, the population will be:
(a) 5108 (b) 5200 (c) 5202 (d) 5300

3. Out of a number of electronics items, a person purchases 60% coloured TV. 5% of these were found to be defective. The percentage of defective TVs in all is:
(a) 3% (b) 6% (c) 9% (d) 10%

4. The price of an item is increased by 20% and then decreased by 20%. The final price as compared to the original price is:
(a) 20% less (b) 20% more (c) 4% more (d) 4% less

5. In an examination, 52% of the candidates failed in English, 42% in Mathematics and 17% in both. The number of those who passed in both the subjects is:
(a) 83% (b) 23% (c) 64% (d) 55.5%

6. Two candidates fought an election. One got 65% of the votes and won by 300 votes. The total number of votes polled is:
(a) 600 (b) 800 (c) 1000 (d) 1200

7. When 40% of a number is added to 42, the result is the number itself. The number is:
(a) 82 (b) 105 (c) 70 (d) 72

8. The radius of a circle is increased by 200%. The area of circle will be increased by:
(a) 200% (b) 400% (c) 800% (d) 1600%

9. In an examination 37% students fail in English. 20% fail in Mathematics and 6% fail in both. 245 pass, total students are:
(a) 400 (b) 500 (c) 1200 (d) None of these

10. To increase a number by 40%, it must be divided by:
(a) 5/7 (b) 7/5 (c) 2/5 (d) 5/2

11. 20% of A = 60% of B. What percent of B is A?
(a) 25% (b) 400% (c) 200% (d) 300%

12. 10% of voters did not vote. 60 votes were invalid. A got 47% of votes in the list and B lost by 308 votes. The total number of voters is
(a) 6000 (b) 6200 (c) 6400 (d) 6450

13. The population of a town increases by 10% in one year and then decreases by 10% in 2nd year. On the whole there was a/an
(a) decrease of 10% (b) decrease of 1% (c) increase of 1% (d) None

EXPERIENCE THE PRATHAM EDGE - 3

14. $45 \times ? = 25\%$ of 900

(a) 16.20 (b) 500 (c) 4 (d) 5

15. 218% of 1674 = ? X 1800

(a) 4 (b) 0.5 (c) 6 (d) None of these

16. One fourth of one third of two fifth of a number is 15. What will be 40% of that number?

(a) 120 (b) 350 (c) 270 (d) 180

17. What percent of 7.2 kg is 18 gms?

(a) 25% (b) 2.5% (c) 0.25% (d) 0.025%

18. A man loses 12.5% of his money and after spending 70% of the remainder, has left Rs 210. Find the money he had at first.

(a) 600 (b) 700 (c) 800 (d) 900

19. Of the total amount received by Kiran, 20% was spent on purchases and some percentage of the remaining on transportation. If she is left with Rs 1520, the initial amount was:

(a) 2800 (b) 2000 (c) 2400 (d) Data inadequate

20. In a library, 20% books are in Hindi, 50% of the remaining are in English and the remaining 9000 are in various other languages. What is the total number of books in English?

(a) 4000 (b) 3000 (c) 2250 (d) None of these

21. A's marks in Biology are 20 less than 25% of the total marks obtained by him in Biology, Mathematics and Drawing. If his marks in Drawing be 50, what are his marks in Mathematics?

(a) 40 (b) 45 (c) 50 (d) Can't be determined

22. In an election between two candidates, one got 55% of the total valid votes. 20% of the votes were invalid. If the total number votes were 7500, the number of valid votes that the other candidates got was:

(a) 2700 (b) 2900 (c) 3000 (d) 3100

23. At an election involving two candidates, 68 votes were declared invalid. The winning candidate got 52% votes and wins by 98 votes. The total number of votes polled is:

(a) 2518 (b) 2450 (c) 2382 (d) None of these

24. Pranav spent 30% of his salary on food, 15% on clothing and 20% on other expenditure. If he saves Rs. 280 a month, what is his monthly salary?

(a) Rs. 700 (b) Rs. 800 (c) Rs. 900 (d) Rs. 1000

25. A sum of Rs. 700 was distributed among 4 people, such that B gets 20% more than A, and C gets 30% more than B. If D gets Rs. 136, how much does C get?

(a) Rs. 431 (b) Rs. 214 (c) Rs. 234 (d) Rs. 344

26. In a town there are 2500 men and 2500 women. If men's population increased by 20% and women's population decreased by 20%, now women as a percentage of men is:

(a) 60% (b) $66\frac{2}{3}\%$ (c) 80% (d) 83%

27. Al Pacino invested 40% of his money in shares, 20% of rest in property and lost 25% of the remaining in a casino. What percentage does he have now?
(a) 15% (b) 40% (c) 42% (d) 36%

Directions (for questions 28 to 30): In an election, there were only two candidates. The losing candidate received $66\frac{2}{3}$ % of the votes the winner got. If the votes polled in favour of loser were 60 less than that of winner, then

28. How many votes did the loser get?
(a) 200 (b) 150 (c) 120 (d) 100

29. How many total votes were cast?
(a) 200 (b) 300 (c) 400 (d) 500

30. What percentage of the total votes did winner get?
(a) 60 (b) 50 (c) 80 (d) 66.66

PRATHAM EDGE – 4

LET US CONSIDER THE MULTIPLICATION OF TWO – 3 DIGITS NUMBERS

		a	*b*	*c*	
	x	*d*	*e*	*f*	
Product					
Box Number	*5*	*4*	*3*	*2*	*1*

Step 1: Find c x f. If it is a one digit number write it in box 1.
If it is a two digit number write its units digits in box 1 and carry over the tens digits.

Step 2: Find b x f + c x e + carried over if any. If it is a one digit number write it in box 2.
If it is a two digit number write its units digit in box 2 and carry over its tens digits.

Step 3: Find a x f + d x c + b x e + carried over if any. If it is a one digit number write it in box 3.
If it is a two digit number, write its units digits in box 3 and carry over its tens digit.

Step 4: Find a x e + b x d + carried over if any. If it is a one digit number write it in box 4.
If it is a two digit number write its unit digit in box 4 and carry over the tens digit.

Step 5: Find a x d + carried over if any. Write the number so obtained in box 5.

Step 6: Now read the number from left to right. This number is the answer.

	2	3	5	
x	4	2	6	
10	0	1	1	0
5	4	3	2	1

Step 1: 5 x 6 = 30. Write 0 in box 1 and carry over 3.

Step 2: 3 x 6 + 2 x 5 + 3 = 31. Write 1 in box 2 and carry over 3.

Step 3: 2 x 6 + 4 x 5 + 3 x 2 + 3 = 41. Write 1 in box 3 and carry over 4.

Step 4: 2 x 2 + 4 x 3 + 4 = 20. Write 0 in box 4 and carry over 2.

Step 5: 4 x 2 + 2 = 10. Write 10 in box 5.

Step 6: The required product is 100110.

CHAPTER 4

PROFIT & LOSS

INTRODUCTION

The chapter of profit and loss is a part & parcel of every commercial activity. This chapter has a wide ranging importance in exams. The concepts of application of percentage rule, fraction to percentage change, are applicable to profit and loss.

- **Cost Price** : The price at which an article is bought is called its cost price, abbreviated as C.P.
- **Selling Price** : The price at which an article is sold is called its selling price, abbreviated as S.P.
- **Profit or Gain** = (S.P.) – (C.P.)
- **Loss** = (C.P.) – (S.P.)
- Loss or Gain is always calculated on C.P.
- **Gain%** = $\text{Gain} \times \frac{100}{\text{C.P.}}$
- **Loss%** = $\text{Loss} \times \frac{100}{\text{C.P.}}$

If Profit is P%

$$SP = \frac{100+P}{100} \times CP$$

$$CP = \frac{100}{100+P} \times SP$$

Value of Profit $= \frac{P}{100} \times CP$

If Loss is L%

$$SP = \frac{100-L}{100} \times CP$$

$$CP = \frac{100}{100-L} \times SP$$

Value of Loss $= \frac{L}{100} \times CP$

MARKED PRICE AND DISCOUNT

Marked Price (M.P): It is the price of an item which is marked on it without any discount.

Discount: It is the percentage of rebate given on the Marked Price.

Example An item is marked at Rs 1000 by the shopkeeper which actually costs Rs. 800
It is sold to the customer at a discount of 10%. Find the selling price.

Solution: Then, Here Marked Price = Rs 1000
Cost Price = Rs 800
Discount = 10% of Marked Price = 10% of 1000 = 0.1×1000 = Rs 100
Selling price = Marked Price - Discount = 1000 - 100 = Rs 900

Successive Discounts

Example Let the marked price of an article be Rs 100 and the shopkeeper provides two successive discounts of 20%. Find the selling price of the article.

Solution: M.P. = Rs 100
1st discount = 20% on M.P i.e., 20% of 100 = Rs 20
So Rate after giving first discount is Rs 80.

Again 20% discount is provided. Now, this discount will get applied on Rs 80.
2nd discount = 20% of Rs 80 = Rs 16
So final price will be Rs 80 – Rs 16 = Rs 64
Overall discount given is of Rs 100 – Rs 64 = Rs 36
In terms of percentage, it will be $\frac{36}{100}$X 100 = 36%

Alternate Method:
How to find overall discount after two successive discounts :
$\pm X \pm Y + \frac{(\pm X \times \pm Y)}{100}$, here X = first increment/decrement and Y = second increment decrement
Here, + → Increment
- → Discount
Using the given equation we can find out the overall discount for the above question.
$-20 - 20 + \frac{-20 \times -20}{100}$ = - 40 + 4 = - 36%, Here – sign indicates discount.

SOLVED EXAMPLES

Example 1 CP is 40% of SP. What is the % gain on CP?
Solution: Let the SP = Rs. 100
CP = Rs. 40
Gain = (60/40) x 100% = 150%

Example 2 The loss when SP is Rs 27.8 is equal to the gain when SP is Rs 132.2. Find CP.
Solution: CP = $\frac{\text{Rs. } 27.80 + \text{Rs. } 132.20}{2}$ = Rs. 80

Example 3 By selling 24 lemons the loss is equal to selling price of 8 lemons. Find the % Loss.
Solution: Let SP of 1 lemon = Re. 1
Loss = $\frac{8}{32} \times 100\%$ = 25%

Example 4 By selling 24 lemons the gain is equal to the selling price of 8 lemons. Find the gain %.
Solution: Let SP of 1 lemon = Re.1
Loss = $\frac{8}{16} \times 100\%$ = 50%

Example 5 A pretends to sell the goods at cost price but uses a weight of 960 g for 1 kg. Find his gain%.
Solution: Gain% = $\frac{40}{960} \times 100$ = 4.17%
(Note: we use 960 because his cost is on Rs 960 and not on Rs 1000)

Example 6 A pretends to gain 20% but also uses a weight of 800 g for 1 kg. His actual gain is.
Solution: Let CP of 1 kg = Rs. 100
therefore, SP of 800g = Rs. 120 *(Sells it at 20% gain)*
Gain = $\frac{40}{80} \times 100\%$ Therefore, Gain is 50%

Example 7 Selling price of two articles is Rs. 910 each. Gain on one is 30%. Loss on other is 30%. Find the final position.

Solution: CP of both = Rs. 910 x $\frac{100}{130}$ + Rs. 910 x $\frac{100}{70}$ = Rs. 2000

Therefore Loss = Rs. 2000 – 2 (Rs. 910) = Rs. 180

Example 8 Selling price of two articles is same. Gain on one is 20%. Loss on other is 20%. On the whole find the loss %.

Solution: Loss = 20 x $\frac{20}{100}$% = 4%, (If SP is same.)

Example 9 If SP is Rs. 48, loss is 37%. Find the SP to gain 26%.

Solution: New S.P = 126% of CP

$= \frac{126}{100} \times \left[48 \times \frac{100}{63}\right]$ = Rs. 96

Example 10 CP of 11 books = Rs. 5, SP of 10 books = Rs. 6. Find the gain percent.

Solution: Let the number of books = 110 *(LCM of 11 & 10)*

CP = Rs. 50, SP = Rs. 66

Gain = $\frac{16}{50}$ x 100 = 32%

Example 11 By selling 6 lemons for Re. 1 loss is 20 %. How many for Re. 1 must be sold to gain 20%?

Solution: SP of 6 lemons = Re. 1

New SP of 6 lemon = 120% of 1 x $\frac{100}{80}$ = Rs. 3/2

Answer = 6 ÷ (3/2) = 4

Example 12 A merchant pretends to lose 4% but uses a weight of 840 g for 1 kg. His real gain or loss percent is:

Solution: Let CP of 1 kg = Rs. 100

SP of 840 g = Rs. 96

SP of 1 kg = Rs. $\frac{96}{840}$x 1000 = Rs.$\frac{800}{7}$

Gain =$14\frac{2}{7}$ %

Example 13 A sells an article to B at a profit of 25%, B sells to C at a profit of 20%. If C pays Rs. 450 for the same article then find the CP of A.

Solution: CP of A = Rs 450 x $\frac{100}{125} \times \frac{100}{120}$ = Rs. 300

Example 14 Ramesh purchased a bicycle for Rs. 5200 and spent Rs. 800 on its repairs. He had to sell it for Rs. 5500. Find his profit or loss percent.

Solution: C.P. of the bicycle = 5200+800 = Rs. 6000

S.P. = Rs. 5500

Since, S.P.<C.P.

⇒ Loss = C.P.-S.P.=6000-5500 = Rs. 500

$\therefore$ Loss % = $\frac{\text{Loss}\times 100}{\text{C.P}} = \frac{500\times 100}{6000} = \frac{25}{3}\%$ or $8\frac{1}{3}\%$

Example 15 A sells $\frac{1}{4}$th of goods at 20% loss. What must be the percentage gain on the remaining part so that overall gain is 10%?

Solution: Let the Total CP = Rs. 400

Loss = $\frac{1}{4}$ x 400 x 20% = 20

for gaining 10%, SP = 440-80 =360

Therefore percentage gain on the second part with CP = Rs. 300 is $\frac{60}{300}$ x 100 = 20%

Example 16 Rs. 46 are received less if instead of selling the article at 2% gain, it is sold at 21% loss. Find the CP.

Solution: 23% of CP = Rs. 46
CP = Rs. 200

Example 17 What % gain is there by selling an article at a certain rate if by selling that at half rate, loss is 28%.

Solution: Let the CP = Rs 100.
Half of SP = Rs. 72
Normal SP = Rs. 144
Gain = 44%

Example 18 The gain is 44% after giving discount series of 20%, 10%. What is the percentage gain if discount is not given.

Solution: Let the CP = Rs. 100
SP = Rs. 144
Total discount = -20-10 + [(-20)(-10)]/100
Marked price = Rs. 144 x $\frac{100}{72}$
= Rs. 200 (20%, 10% = 28%)
Therefore, gain is 100% if discount is not given.

Example 19 Loss by selling an article is 20%. If the article is bought at 10% less price & sold for Rs. 38 more, the gain is 10%. Find the original price.

Solution: Initial SP = 0.8 CP
New SP = Old SP + 38
0.9 CP × 1.1 = 0.8 CP +38 $\Rightarrow$ CP = Rs. 200

Example 20 By selling 6 articles a man lost what he got for 14 articles. Find the loss percent.

Solution: Loss % = $\frac{\text{Loss}}{\text{CP}}\times 100 = \frac{14}{20}\times 100$ = $\frac{14}{20}$ x 100 = 70%

EXPERIENCE THE PRATHAM EDGE - 4

1. A man buys an article for Rs 27.50 and sells it for Rs28.60. Find his gain percent.
(a) 4% (b) 5% (c) 6% (d) 7%

2. If a radio is purchased for Rs 490 and sold for Rs 477.50. Find the loss percent
(a) 2% (b) 3% (c) 4% (d) 2.55%

3. By selling a book for Rs 115.20, a man loses 10%. At what price should he sell it to gain 5%?
(a) Rs. 150 (b) Rs 145.50 (c) Rs 134.40 (d) Rs 165.50

4. A trader lost 20% by selling a watch for Rs 1024. What percent shall he gain or lose by selling it for Rs 1472?
(a) 15% (b) 20% (c) 22% (d) 35%

5. By selling 33 metres of cloth, one gains the selling price of 11metres. Find the gain percent.
(a) 34% (b) 50% (c) 55% (d) 64%

6. A reduction of 20% in the price of sugar enables a purchaser to obtain 2.5 kg more for Rs 160. Find the original rate.
(a) Rs 12.80 (b) Rs 15.75 (c) Rs 16 (d) Rs 12

7. A grocer purchased 80 kg of rice at Rs 13.50 per kg and mixed it with 120 kg rice at Rs 16per kg. At what rate per kg should he sell the mixture to gain 16%.
(a) Rs 18 (b) Rs 18.50 (c) Rs 17.40 (d) Rs 19

8. A vendor sells 10 toffees for a rupee, gaining thereby 20%. How many did he buy for a rupee?
(a) 12 (b) 10 (c) 11 (d) 14

9. A vendor bought lemons at 6 for a rupee. How many for a rupee must he sell to gain 20%?
(a) 2 (b) 3 (c) 4 (d) 5

10. A vendor bought a number of bananas at 6 for 5 rupees and sold at 4 for 3 rupees. Find his gain percent.
(a) 10% (b) -10% (c) 5% (d) -5%

11. Find the single discount equivalent to a series discount of 20%, 10% and 5%.
(a) 35% (b) 31.6% (c) 32% (d) 36.7%

12. If a commission of 10% is given on the marked price of an article, the gain is 25%. Find the gain percent, if the commission is increased to 20%.
(a) $11\frac{1}{9}\%$ (b) $13\frac{1}{4}\%$ (c) $30\frac{1}{23}\%$ (d) $16\frac{2}{5}\%$

13. A trader marks his goods 20% above the cost price and allows a discount of 15% on it. Find his gain percent.
(a) 1% (b) 2% (c) 3% (d) 4%

14. Two-third of a consignment was sold at a profit of 5% and the remainder at loss of 2%. If the profit was Rs 400, the value of consignment was:
(a) Rs 20000 (b) Rs 15000 (c) Rs 12000 (d) Rs 10000

15. A man sells two commodities at the same price. On one he gains 20% and loses 20% on the other. He gets:
(a) 1%loss (b) 4% loss (c) 1%gain (d) no gain no loss

16. A sweet seller declares that he sells sweets at the cost price. However, he uses a weight of 450gms instead of 500gms. His profit percentage is:
(a) 10 (b) $11\frac{1}{9}$ (c) 12 (d) $12\frac{2}{9}$

17. A man loses Rs 20 by selling some toys at the rate of Rs 3 per piece and gains Rs 30 , if he sells them at Rs 3.25 per piece. The number of pieces sold by him is:
(a) 100 (b) 120 (c) 200 (d) 300

18. On selling an article for Rs 240, a trader loses 4%. In order to gain 10%, he must sell that article for:
(a) Rs 264 (b) Rs 273.20 (c) Rs 275 (d) Rs 280

19. A man bought an article and sold it at a gain of 6%. If he had bought it at 5% less and sold it for Re1 less, he would have made a profit of 10%. The CP of the article was:
(a) Rs 100 (b) Rs 150 (c) Rs 67 (d) Rs 500

20. A reduction of 25% in the price of eggs will enable one to buy 4 dozen more eggs for Rs 96. What is the original price per dozen?
(a) Rs 6 (b) Rs 8 (c) Rs 8.50 (d) Rs 9

21. Raghu bought 4 dozen oranges at Rs 12 per dozen and 2 dozen oranges at Rs 16 per dozen. He sold them all to earn 20% profit. At what price per dozen did he sell the oranges?
(a) Rs 14.40 (b) Rs 16 (c) Rs 16.80 (d) Rs 19.20

22. A single discount equivalent to a discount series of 40% and 20% is:
(a) 50% (b) 52% (c) 55% (d) 60%

23. Rajesh buys an article with 25% discount on its marked price. He makes a profit of 10% by selling it at Rs 660. the marked price is:
(a) Rs 600 (b) Rs 685 (c) Rs 700 (d) None of these

24. What price should a shopkeeper mark on an article, costing him Rs 153, to gain 20% after allowing a discount of 15%?
(a) Rs 224 (b) Rs 216 (c) Rs 184 (d) Rs 162

25. If the SP of Rs 24 results in a discount on list price, what SP would result in a 30% discount on list price?
(a) Rs 27 (b) Rs 21 (c) Rs 20 (d) Data inadequate

26. If the cost price of 12 books is same as the selling price of 16 books, the loss percentage is:
(a) 15% (b) 20% (c) 25% (d) 30%

27. A man loses the selling price of 4 apples on selling 36 apples. His loss percentage is:
(a) 12.5% (b) 11.11% (c) 10% (d) None of these

EXPERIENCE THE PRATHAM EDGE - 4

28. A man sells two cows for Rs. 4000 each, neither gaining nor losing in the deal. If he sold one cow at a gain of 25%, then the other cow is sold at a loss of:
(a) 16.66% (b) 18.22% (c) 25% (d) None of these

29. The cost price of an article is 40% of the selling price. The percentage that the selling price is of cost price is:
(a) 250% (b) 240% (c) 60% (d) 40%

30. Two horses were sold for Rs. 12000 each, one at a loss of 20% and the other at a gain of 20%. The entire transaction resulted in:
(a) no loss, no gain (b) loss of 4% (c) gain of 4% (d) gain of 8%

PRATHAM EDGE – 5

CALCULATING SQUARES OF NUMBERS

For Calculating squares, we must be aware of the basic algebraic formula.

$(a + b)^2 = a^2 + b^2 + 2ab$

$(a - b)^2 = a^2 + b^2 - 2ab$

$53^2 = (50 + 3)^2 = 2500 + 9 + 2 \times 50 \times 3$
$= 2500 + 300 + 9 = 2809$

$49^2 = (50 - 1)^2 = 2500 + 1 - 100$
$= 2401$

AT the first look it can appear a bit cumbersome, but with a regular practice, this will look much easier.

CALCULATING CUBES OF NUMBERS

Let AB be a two digit number where A is the tens digit and B is the units digit.
Let us form a row with 4 columns as follows.

Box Number →				
	4	*3*	*2*	*1*

Step 1: Find B^3. If it is a one digit number write it in box 1.
If it is a two digit or a three digit number write its unit digit in box 1 and carry over the rest.

Step 2: Find $3 \times A \times B^2$ + carried over if any. If it is a one digit number write it in box 2.
If it is a two or three digit number write its unit digit in box 2 and carry over the rest.

Step 3: Find $3 \times A^2 \times B$ + carried over if any. If it is a one digit number write it in box 3 and if it is a two digit number write its units digit in box 3 and carry over the rest.

Step 4: Find A^3 + carried over if any. Write the number obtained in box 4.

Step 5: We read the number from left to right. This is the required cube.

Eg: $(27)^3 = (AB)^3$

Step 1: $7^3 = 343$, we write '3' in box 1 and carry over **34**.

Step 2: $3 \times 2 \times 7^2 + \mathbf{34} = 3 \times 2 \times 9 + 34 = 328$, we write '8' in box 2 and carry over **32**.

Step 3: $3 \times 2^2 \times 7 + \mathbf{32} = 84 + 32 = 116$, we write '6' in box 3 and carry over **11**.

Step 4: $23 + 11 = 8 + \mathbf{11} = 19$.

Hence, the cube is '19683'

NOTES

CHAPTER 5

SIMPLE & COMPOUND INTEREST

INTRODUCTION

A clear understanding of percentage calculations is a pre-requisite to solve the problems in interest. The solved example of this chapter will help you in different dimensions. This chapter finds its varied application in solving data interpretation problems.

When a sum of money is lent by x to y, then x is called the *lender* or *creditor* and y is called the *debtor* or *borrower*.
The **money** borrowed is called ***Principal*** **or Sum.**
Extra money paid for using other's money is called ***Interest*****.**
Total money paid in the end is the sum of *Principal* & *Interest* and is known as ***Amount*****.**
Growth of money can be simple or compound.

SIMPLE INTEREST

- If the interest on certain sum for a certain period is reckoned uniformly, it is called Simple Interest (SI)
- At simple rate the simple interest is proportional to the time.
- There is no interest on interest.
- The interest is calculated on the initial sum.
- The amounts are in AP if time is same.

Formulae.
Let Principal = P, Rate = R% and time = T years. Then

$\text{S.I.} = \frac{P \times R \times T}{100}$	$P = \frac{100 \times SI}{R \times T}$	$T = \frac{100 \times SI}{P \times R}$	$R = \frac{100 \times SI}{P \times T}$

COMPOUND INTEREST

- Interest which is calculated on the initial principal as well as the interest accumulated over previous periods is called Compound interest (C.I.)
- The amount at the end of certain year becomes the principal for the next time period.
- At Compound rate there is interest on interest.
- The ratio between the amounts at compound rate is same if the time is same.
- The amounts at compound rate are in G.P. if time is same.

In other words, the amount at the end of first year (or period) will become the principal for the second year (or period); the amount at the end of second year (or period) becomes the principal for the third year (or period) and so on.

If P denotes the principal at the beginning of Period 1, then amount at the end of Period I = $P\left(1+\frac{r}{100}\right)$ = PR = Which is Principal at the beginning of Period 2.

Now, $P\left(1+\frac{r}{100}\right)^2$ = amount at the end of period 2 = PR^2 = Principal at the beginning of Period 3.
Similarly, Principal at the beginning of Period n+I = $P\left(1+\frac{r}{100}\right)^n$ = PR^n = Amount at the end of period n.

Where R = $P\left\{1+\left(\frac{r}{100}\right)\right\}$

Hence the amount after n years (periods) = PR^n = A

Interest = I = A – P = P [R^n – 1]

Some other important formulas

Let Principal = P, Rate = R% and Time = n years. Then

1. When interest is compounded Annually: In this case, Amount = $P\left(1+\frac{R}{100}\right)^n$

2. When interest is compounded Half Yearly: In this case, Amount = $P\left(1+\frac{R/2}{100}\right)^{2n}$

3. When interest is compounded Quarterly: In this case, Amount = $P\left(1+\frac{R/4}{100}\right)^{4n}$

4. When Time is Fraction of a Year: Let the time be a fraction of a year, say $3\frac{2}{3}$ years

   Then Amount = $P\left(1+\frac{R}{100}\right)^3 \times \left(1+\frac{\frac{2}{3}R}{100}\right)$

5. When interest is compounded continuously, Amount = P{e($^{rt}/_{100}$)}

6. When Rates are different for different years, say R_1, R_2, R_3, percent for 1st, 2nd and 3rd year respectively:

   In this case, Amount = $P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)$

SOLVED EXAMPLES

Example 1 Calculate the simple interest on Rs7200 for 4 years at 6.25% p.a. rate.
Solution : Simple Interest = 25% of 7200 = Rs. 1800

Example 2 Rate is 10% p.a., Time is 4 years, What is the simple Interest on a loan of Rs. 1000.
Solution: Simple Interest = 40% of Rs. 1000 = Rs 400
Example 3 A sum becomes 2.6 times in 8 years at simple rate. Find the rate.
Solution: The Rate is $\frac{1.6}{8}$ x 100 = 20%, because the rate is the interest on Rs. 100 in one year.

Example 4 A sum becomes 3 times in 8 years at simple rate. When does it become 5 times?
Solution: Let the P = Rs. 1
S I is Rs. 2 in 8 years
S I is Rs. 4 in 16 years [*When SI = Rs. 4, A = Rs. 5 hence 5 times*]
Ans = 16 years

Example 5 What sum amounts to Rs 6400 in 4 years at 3.75% p.a. simple rate.
Solution: Amount after 4 years = (100 + 4 x 3.75)% of Principal

6400 = $\frac{115}{100}$x Principal

Principal = Rs $\frac{6400}{115}$ x 100 = Rs. 5565.25

Alternate Method: Use direct formulae P = (100×A)/(100+RT)

Example 6 A sum becomes Rs. 1200 after 2 years & Rs. 1320 after 3 years at simple rate. Find the rate.

Solution: P = 1200, t = 1 year

SI = A-P $\Rightarrow$ 1320-1200 = 120

Rate = (100x120)/(1200) = 10%

Example 7 Simple interest of 2 years = Rs. 600. Find the compound interest on the same sum at 20% p.a. rate.

Solution: C. I. of 2 year = Rs. 600 + 20% of Rs. 300 [Rs. 300 is S.I. in 1 year]

Rs. 660

Example 8 What sum amounts to Rs. 9261 in 3 years at 5% p.a. compound interest.

Solution: $P\left[1+\frac{5}{100}\right]^3$ = 9261

$\therefore$ P = Rs. 8000

Example 9 A sum becomes $\frac{25}{16}$ times in 2 years at compound rate. Find the rate.

Solution : Since the sum becomes $\frac{25}{16}$ times after 2 years.

If becomes $\sqrt{\frac{25}{16}}$ = 5/4 after 1 year

Rate = $\frac{1}{4}$ = 25%

Example 10 For how many years should Rs. 600 be invested at 10% per annum in order to earn the same simple interest as earned by investing Rs. 800 at 12% per annum for 5 years?

Solution: Simple Interest = Rs. $\left(\frac{800\times12\times5}{100}\right)$ = Rs. 480

Time = $\left(\frac{100\times480}{600\times10}\right)$ = 8 years

Example 11 Vinod Kumar invested Rs. 1600 for 3 years and Rs. 1100 for 4 years at same rate of simple interest. If the total interest from these investments is Rs. 506, the rate of interest was

Solution : $\frac{1600\times3\times R}{100}+\frac{1100\times4\times R}{100}$ = 506 or 92R = 506 or R = $5\frac{1}{2}$%.

Example 12 Prabhat took a certain amount as a loan from a bank at the rate of 8% per annum simple interest and gave the same amount to Ashish as a loan at the rate of 12% per annum. If at the end of 12 years, he made a profit of Rs. 320 in the deal, what was the original amount?

Solution: Let the original amount be Rs. x, *then*

$$\frac{x\times12\times12}{100}-\frac{x\times8\times12}{100}=320 \quad\Rightarrow x=\frac{2000}{3}$$ = Rs. 666.67

Example 13 What is the amount of interest on Rs. 1000 compounded annually at rate of 10% for 3 years?

Solution: $A=1000\left(1+\frac{10}{100}\right)^3$

= Rs. 1331. Interest = Rs. 331

Example 14 Find the CI on Rs. 5000 at 8% per annum for 2 years, compounding being done annually.

Solution: P = Rs. 5000, R = 8% and n = 2 years

$5000\left(1+\frac{8}{100}\right)^2$ $\Rightarrow$ Amount = Rs. 5832

CI = Amount – Principal = Rs. (5832 – 5000) = Rs. 832

Example 15 If the CI on a certain sum of money for 3 years at 20% per annum is Rs. 728, what shall be the sum invested?

Solution: $P\left(1+\frac{20}{100}\right)^3 - P = 728$

$P\left(\frac{728}{1000}\right)$ = Rs. 728

P = Rs. 1000

EXPERIENCE THE PRATHAM EDGE - 5

1. Ramesh borrows Rs. 2000 and pays back after 3 years at 10% simple interest. The amount paid by the man, is
(a) Rs. 2400 (b) Rs. 2600 (c) Rs. 3000 (d) Rs. 2750

2. The rates of simple interest in two banks A & B are in the ratio 5:4. A person wants to deposit his total savings in two banks such that he received equal half yearly interest from both. He should deposit the savings in banks A & B in the ratio.
(a) 5:2 (b) 2:5 (c) 4:5 (d)5:4

3. A certain sum of money at simple interest amounts to Rs 901 in $1\frac{1}{2}$ years & to Rs 986 in 4 years, both at simple interest. The rate of interest per annum is
(a)2.5% (b) 3% (c) 4% (d) 5%

4. For how many years should Rs 600 be invested at simple interest of 10% pa in order to earn the same simple interest as is earned by investing Rs 800 at 12% pa for 5 years?
(a) 6 (b) 8 (c) 12 (d) 16

5. The simple interest on Rs 10 per 4 months at the rate of 3 paise per Rupee per months is
(a) Rs. 1.20 (b) Rs 12 (c) Rs. 120 (d) Rs. 1200

6. A sum of money trebles itself in 15 years 6 months. In how many years would it double itself?
(a) 6 years 3 months (b) 7 years 9 months (c) 8 years 3 months (d) 9 years 6 months

7. Out of a sum of Rs. 625 a part was lent at 5% and other at 10% simple interest. If the interest on the first part after 2 years is equal to the interest on the second part after 4 years, then the second sum (in Rs.) is
(a) 125 (b) 200 (c) 250 (d) 300

8. The SI on Rs 1820 from March 9, 1994 to May 21, 1994 at $7\frac{1}{2}$ % rate will be
(a) Rs 29 (b) Rs 28.80 (c) Rs. 27.30 (d) Rs. 22.50

9. The difference between the interests (S.I.) received from two different banks on Rs 500 for 2 years is Rs 2.50. The difference between their rates is
(a) 1% (b) 0.5% (c) 0.25% (d) 2.5%

10. In how many years will sum of money double itself at 12% simple interest per annum?
(a) 6 years 9 months (b) 7 years 6 months (c) 8 years 3 months (d) 8 years 4 months

11. If a sum of money at SI doubles in 6 years it will become 4 times in:
(a)12years (b) 14 years (c) 16 years (d) 18 years.

12. Ravi Kumar borrows Rs 300 at SI of 5% and Rs 450 at SI of 6% at the same time & on the condition that the whole loan will be repaid when the total interest amounts to Rs 126. The loan will have to be repaid after how many years?
(a) 2 (b) 3 (c) 4 (d) 5

13. A money lender finds that due to a fall in the rate of interest from 8% to 7.75 % his yearly income diminishes by Rs 60. His Capital is
(a) Rs. 26000 (b) Rs. 24000 (c) Rs. 23800 (d) Rs. 22400

14. A loan was repaid in two annual installments of Rs 121 each. If the rate of interest be 10% per annum compounded annually, the sum borrowed was
(a) Rs. 200 (b) Rs. 210 (c) Rs. 217.80 (d) Rs. 216

15. The difference between CI & SI on a sum for 2 years at 10% per annum, when the interest is compounded annually is Rs 16. If the interest were compounded half yearly, the difference in two interests would be.
(a) Rs. 24.81 (b) Rs. 31.61 (c) Rs. 32.40 (d) Rs. 26.90

16. A sum of Rs 12000 deposited at CI becomes double after 5 years. After 20 years it will become
(a) Rs. 120000 (b) Rs. 192000 (c) Rs. 124000 (d) Rs. 96000

17. A sum of money placed at CI doubles itself in 5 years. It will amount to eight times in
(a) 10 years (b) 12 years (c) 15 years (d) 20 years

18. The difference between the CI & SI earned at the end of second year on a sum of money at 10% per annum is Rs 20. The sum is
(a) Rs. 4000 (b) Rs. 2000 (c) Rs. 1500 (d) Can't be determined.

19. In how many years will a sum of Rs 800 at 10% per annum compounded semi-annually become Rs 926.10?
(a) $2\frac{1}{2}$ (b) $1\frac{1}{2}$ (c) $2\frac{1}{3}$ (d) $1\frac{1}{3}$

20. A sum of money at CI amounts to thrice itself in 3 years. In how many years will it be 9 times itself?
(a) 12 (b) 9 (c) 6 (d) 8

21. The simple interest on Rs. 500 at 6% per annum from May 3rd to July 15th in the same year is
(a) Rs. 9 (b) Rs. 6 (c) Rs. 4 (d) None of these

22. A sum of money at simple interest becomes four times in 24 years. The rate percent of interest per annum is
(a) $13\frac{}{4}\%$ (b) $12\frac{1}{2}\%$ (c) $11\frac{3}{4}\%$ (d) None of these

23. What should be the least number of years in which the simple interest on Rs 2600 at $6\frac{2}{3}\%$ will be exact number of rupees?
(a) 2 (b) 3 (c) 4 (d) 5

24. A sum of money doubles itself at CI in 15 years . In how many years will it become eight times?
(a) 45 (b) 50 (c) 55 (d) 60

25. A certain sum amounts to Rs 7300 in 2 years & to Rs 8575 in 3 years compounded annually. Find the sum
(a) Rs. 5400 (b) Rs. 5290 (c) Rs. 5600 (d) Rs. 5700

26. The amount of Rs 7500 at compound interest at 4% per annum for 2 years is.
(a) Rs. 7800 (b) Rs. 8100 (c) Rs. 8112 (d) Rs. 8082

27. If the SI on a sum of money at 5% per annum for 3years is Rs 1200, the CI on the same sum for the same period & rate is?
(a) Rs. 1260 (b) Rs. 1261 (c) Rs. 1264 (d) Rs. 1265

EXPERIENCE THE PRATHAM EDGE - 5

28. The difference between the CI & SI on a sum of money for 2 years at $12\frac{ü}{2}$ % per year is Rs 150. The sum is

    (a) Rs. 9000 (b) Rs. 9200 (c) Rs. 9500 (d) Rs. 9600

29. If the difference between CI, compounded half yearly & the SI on a sum at 10% per annum for one year is Rs 25. The sum is.

    (a) Rs. 9000 (b) Rs. 9500 (c) Rs. 10000 (d) Rs. 10500

30. If the CI on a certain sum at $16\frac{2}{3}$% for 3 years is Rs 1270, the SI on the same sum at the same rate & for the same period is

    (a) Rs. 1200 (b) Rs. 1165 (c) Rs. 1080 (d) Rs. 1220

31. A sum of money calculated at CI doubles itself in 5 years. It will amount to eight times itself in.

    (a) 10 years (b) 12 years (c) 15 years (d) 20 years

32. A sum of money at CI amounts to thrice itself in 3 years. In how many years will it be 9 times itself?

    (a) 12 (b) 9 (c) 6 (d) 8

33. Rahul borrows Rs. 20000 and pays back after 4 years at 10% simple interest. The amount paid by the man, is

    (a) Rs. 24000 (b) Rs. 28000 (c) Rs. 30000 (d) Rs. 27500

34. If the rate of simple interest is 12% p.a., the amount when Rs. 6000 is deposited for 1 year is

    (a) Rs. 7200 (b) Rs. 48513.69 (c) Rs. 50000 (d) Rs. 6720

35. In how many years will a sum of Rs. 8000 at 10% p.a., compounded semi annually become Rs. 9261?

    (a) $2\frac{1}{2}$ (b) $1\frac{1}{2}$ (c) $2\frac{1}{3}$ (d) $1\frac{1}{3}$

PRATHAM EDGE – 6

Methods of finding square root

Square root of perfect squares up to four digits can be easily found by finding their units and tens digits. Let us observe

- If units digit of a perfect square is 0, units digit of its square root is 0.
- If units digit of a perfect square is 1, units digit of its square root is 1 or 9.
- If units digit of a perfect square is 4, units digit of its square root is 2 or 8.
- If units digit of a perfect square is 5, units digit of its square root is 5.
- If units digit of a perfect square is 6, units digit of its square root is 4 or 6.
- If units digit of a perfect square is 9, units digit of its square root is 3 or 7.

Note: *Numbers ending with 2,3,7 or 8 cannot be perfect square*

Lets find the square root of 2209

Step 1: Units digit is 9, so units digit of square root is 3 or 7.

$\boxed{13}$ Or $\boxed{17}$

Step 2: Strike out the largest square that is less than 22. ($4^2 = 16 < 22$ whereas $5^2 = 25 > 22$).
So we always take the largest square that is less than the number given after striking out two digits. Hence the tens digit of square root is 4. So answer will be 43 or 47.

Step 3: By method number (5) we know $45^2 = 2025$, and as the number 2209 is bigger than 2025. Then logically and obviously the square root of 2209 is 47 not 43.

NOTES

CHAPTER 6

RATIO & PROPORTIONS

INTRODUCTION

RATIO

If the values of two quantities A and B are 4 and 6 respectively, then we say that they are in the ratio 4 : 6 (read as "four is to six"). Ratio is the relation which one quantity bears to another of the same kind, the comparison being made by considering what multiple, part or parts, one quantity is of the other. **The ratio of two quantities** "a" and "b" is represented **as a : b** and read as **"a is to b"**. Here, **"a"** is called **antecedent**, **"b"** is the **consequent**. Since the ratio expresses the number of times one quantity contains the other, it's <u>an abstract</u> quantity.

Ratio of any number of quantities **is expressed** after **removing any common factors** that all the terms of the ratio have. For example, if there are two quantities having values of 4 and 6, their ratio is 4 : 6 i.e., 2 : 3 after taking the common factor 2 between them out. Similarly, if there are three quantities 6, 8 and 18, there is a common factor between all three of them. So, dividing each of the three terms by 2, we get ratio as 3 : 4 : 9.

If two quantities whose values are A and B respectively are in the ratio a : b, and we know that some common factor k(>0) would have been removed from A and B to get the ratio a : b so we can write the original values of the two quantities (i.e., A and B) as ak and bk respectively. **For example**, if the salaries of two persons are in ratio 7 : 5, we can write their individual salaries as 7k and 5k respectively.

A ratio a : b can also be expressed as a/b. So if two items are in the ratio 2 : 3 we can say that their ratio is 2/3. If two terms are in the ratio 2, it means that they are in the ratio of 2/1, i.e., 2 : 1.

"A ratio is said to be a ratio of greater or less inequality or of equality according as antecedent is greater than, less than or equal to consequent". In other words,

- the ratio a : b where a > b is called a ratio of greater inequality *(example 3 : 2)*
- the ratio a : b where a < b is called a ratio of less inequality *(example 3 : 5)*
- the ratio a : b where a = b is called a ratio of equality *(example 1 :1)*

From this we can find that a ratio of greater inequality is diminished and a ratio of less inequality is increased by adding same quantity to both terms, i.e., in the ratio a : b, when we add the same quantity x (positive) to both the terms of the ratio, we have the following results

if a < b then $(a + x) : (b + x) > a : b$
if a > b then $(a + x) : (b + x) < a : b$
if a = b then $(a + x) : (b + x) = a : b$

This idea can also be helpful in questions on Data interpretation when we need to compare fractions to find the larger of two given fractions.
If two quantities are in the ratio a : b, then the first quantity will be a + <u>a</u> times the total of the two quantity and the second quantity will be equal to b + <u>b</u> times the total of the two quantities.

- **The compounded ratio** of ratios A : B and C : D is A x C : B x D
- **The compounded ratio** of ratios A : B, C : D and E : F is ACE : BDF
- **The duplicate ratio** of A : B is $A^2 : B^2$
- **The triplicate ratio** of A : B is $A^3 : B^3$
- **The sub duplicate ratio** of A : B is $\sqrt{A} : \sqrt{B}$
- **The sub triplicate ratio** of A : B is $\sqrt[3]{A} : \sqrt[3]{B}$

- **The inverse ratio** of A : B = B : A
  If A : B = C : D, then
- B : A = D : C. This property is called **Invertendo**.
- A : C = B : D. This property is called **Alternendo**.
- A + B : B = C + D : D. This property is called **Componendo**.
- A – B : B = C – D : D. This property is called **Dividendo**.
- A + B : A – B = C + D : C – D. This property is called **Componendo – Dividendo.**

Equal Ratios

- If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$, then each ratio is equal to $\frac{a+c+e}{b+d+f}$
- If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$, then each ratio is equal to $\frac{pa+qc+re}{pb+qd+rf}$

PROPORTION

This is represented as a: b:: c : d and is read as "a is to b (is) as c is to d".
When a, b, c and d are in proportion, then a and d are called the EXTREMES and b and c are called the MEANS.

Types of Proportion

1. **Mean Proportion –** If the given ratio is a : b :: b : c then b is the mean proportion.
   $b=\sqrt{ac}$
2. **Third Proportion –**
   Case 1- If the given ratio is a : b :: b : c then c is the third proportion $c=\frac{b^2}{a}$
   Case 2 – If the given ratio is a : b :: c : d then c is the third proportion
3. **Fourth Proportion –** If the given ratio is a : b :: c : d then d is the fourth proportion

SOLVED EXAMPLES

Example 1 The salaries of David and John are in the ratio of 5 : 9. The sum of their salaries is Rs.35000. Find their individual salaries?

Solution: Since the ratio is 5 : 9
David's salary is 5/14th of their total salary and

John's salary is 9/14th of their total salary.

David's salary = (5/14)×35000= Rs. 12500

John's salary = $\frac{9}{14}\times 35000$ = Rs. 22500

Example 2 If a : b = 3 : 5, then find (2a + 4b) : (3a + 5b) = $\frac{2a+4b}{3a+5b}$

Solution: Dividing the numerator and denominator by b, we get

$$\frac{2\frac{a}{b}+4}{3\frac{a}{b}+5}$$

Substituting the value of $\frac{a}{b}$ as $\frac{3}{5}$

$$\frac{2\times\frac{3}{5}+4}{3\times\frac{3}{5}+5}=\frac{\frac{6}{5}+4}{\frac{9}{5}+5}=\frac{26}{34}=\frac{13}{17}$$

Example 3 The runs scored by Rahul and Ramesh are in the ratio of 13 : 7. If Ramesh scored 48 runs less than Rahul, then find their individual runs.

Solution: Since the ratio of the runs of Rahul and Ramesh is 13 : 7 we can take their individual scores as 13k and 7k respectively. So the difference between their score will be 13k – 7k = 6k, which is given to be 48.

6k = 48 $\Rightarrow$ k = 8, 13k = 104 and 7k = 56

Rahul scored 104 runs and Ramesh scored 56 runs.

Example 4 What number shall be added to or subtracted from each term of the ratio 7 : 16, so that it becomes equal to 2 : 3?

Solution: Let x be the number to be added to each term of the ratio. Then we have

$$\frac{7+x}{16+x}=\frac{2}{3} \quad \Rightarrow 21 + 3x = 32 + 2x$$

x = 11. So. 11 has to be added to each term to make it 2 : 3.

Example 5 If $\frac{x}{y}=\frac{2}{5}$ then find $\frac{(2x+3y)}{(3x+7y)}$

Solution: $\frac{x}{y}=\frac{2}{5} \quad \Rightarrow x=\frac{2y}{5}$

By substituting value of x in

$$\frac{2x+3y}{3x+7y} \text{ we get } \frac{2\times\frac{2y}{5}+3y}{3\times\frac{2y}{5}+7y}=\frac{4y+15y}{6y+35y}=\frac{19}{41}$$

VARIATION

Two quantities A and B may be such that as one quantity changes in value, the other quantity also changes in value bearing certain relationship to the change in the value of the first quantity.

Direct Variation

One quantity A is said to vary directly as another quantity B if the two quantities depend upon each other in such a manner that if B is increased in a certain ratio, A is increased in the same ratio and if B is decreased in a certain ratio, A is decreased in the same ratio.

This is denoted as A α B (A varies directly as B).

If A α B then A = kB, where k is a constant, it is called a constant of proportionality.

For example, when the quantity of sugar purchased by a housewife doubles from the normal quantity, the total amount she spends on sugar also doubles, i.e., the quantity and the total amount increase (or decrease) in the same ratio.

From the above definition of direct variation, we can see that when two quantities A and B vary directly with each other, then A/B = k or the ratio of the **two quantities is a constant.** Conversely, **when the ratio of two**

quantities is a constant, we can conclude that they vary directly with each other.

If X varies directly with Y and we have two sets of values of the variables X and Y, X_1 corresponding to Y_1 and X_2 corresponding to Y_2, then, since X α Y, we can write down

$$\frac{X_1}{Y_1} = \frac{X_2}{Y_2} \Rightarrow \frac{X_1}{X_2} = \frac{Y_1}{Y_2}$$

INVERSE VARIATION

A quantity A is said to vary inversely as another quantity B if the two quantities depend upon each other in such a manner that if B is increased in a certain ratio, A is decreased in the same ratio and if B is decreased in a certain ratio, then A is increased in the same ratio.
It is the same as saying that A varies directly with 1/B. It is denoted as If A α 1/B i.e., A = k/B where k is a constant of proportionality.

For example, as the number of men doing a certain work increases, the time taken to do the work decreases and conversely, as the number of men decreases, the time taken to do the work increases.
From the definition of inverse variation, we can see that when two quantities A and B vary inversely with each other, then AB = a constant, i.e., **the product of the two quantities is a constant**. Conversely, **if the product of two quantities is a constant, we can conclude that they vary inversely with each other.**

If X varies inversely with Y and we have two sets of values of X and Y. X_1 corresponding to Y_1 and X_2 corresponding to Y_2 then since X and Y are inversely related to each other, we can write down

$$\frac{X_1}{X_2} = \frac{Y_2}{Y_1} \text{ or } X_1Y_1 = X_2Y_2$$

SOLVED EXAMPLES

Example 1 If a : b = 7 : 2, Then $\frac{4a+3b}{5a+4b} = ?$

Solution: Then expression = $\frac{4(7)+3(2)}{5(7)+4(2)} = \frac{34}{43}$

Example 2 If 4A = 5B = 7C & A + B + C = 581, C = ?

Solution:

$$A = \frac{7C}{4},\ \text{B} = \frac{7C}{5}$$

$$\frac{83C}{20} = 581 \Rightarrow \text{C} = 581 \times \frac{20}{83} = 140$$

Example 3 If 2x + 3y : 3x + 5y = 18 : 19, find x : y

Solution:

$$\frac{2\frac{x}{y}+3}{3\frac{x}{y}+5} = \frac{18}{19}$$

Cross Multiplying $38\frac{x}{y} + 57 = 54\frac{x}{y} + 90$

$$\Rightarrow -33 = 16\frac{x}{y} \quad \Rightarrow \frac{x}{y} = \frac{-33}{16}$$

Example 4 If $x : y = 2 : 3$, $3x + 2y : 2y : 2x + 5y = ?$

Solution:

$$= 3x + 2y : 2y : 2x + 5y$$

$$= 3\frac{x}{y} + 2 : 2 : 2\frac{x}{y} + 5 = 3\frac{2}{3} + 2 : 2 : 2\frac{2}{3} + 5$$

$$= 4 : 2 : \frac{19}{3} \therefore\ 12 : 6 : 19$$

Example 5 Find the fourth proportional of 3, 15, 11

Solution: $3 : 15 :: 11 : x$

$x = (15 \times 11)/3 = 55$

Example 6 Find the mean proportion of 8 & 128

Solution: $8 : x :: x : 128$

$x^2 = 8 \times 128 \Rightarrow x = 32$

Example 7. Find the third proportion of 2 & 16

Solution: $2 : 16 :: 16 : x$

$x = (16 \times 16)/2 = 128$

Example 8. Three numbers are in the ratio 3 : 2 : 5, Sum of their squares is 608. The largest of these numbers is

Solution: $9x^2 + 4x^2 + 25x^2 = 608$

$38x^2 = 608 \rightarrow x = 4$, the largest is $5x = 20$

JOINT VARIATION

If there are three quantities A, B and C such that A varies with B when C is constant and varies with C when B is constant, then A is said to vary jointly with B and C when both B and C are varying i.e., $A \propto B$ when C is constant and $A \propto C$ when B is a constant

=> $A \propto BC$

$A = kBC$ where k is the constant of proportionality.

Solved Examples

Example 1 Find the value of x, if $(3x - 2) : (2x - 1) = (4x + 8) : (7x - 2)$

Solution:

$$\frac{3x-2}{2x-1} = \frac{4x+8}{7x-2}$$

$\Rightarrow (3x - 2)(7x - 2) = (2x - 1)(4x + 8)$

$\Rightarrow 21x^2 - 20x + 4 = 8x^2 + 12x - 8$

$\Rightarrow 13x^2 - 32x + 12 = 0$

$\Rightarrow 13x^2 - 26x - 6x + 12 = 0$

$\Rightarrow 13x(x - 2) - 6(x - 2) = 0 \Rightarrow (x - 2)(13x - 6) = 0$

$\Rightarrow x = 2$ *or* $6/13$.

Example 2 If x varies inversely as $(y^2 - 6)$ and is equal to 2 when $y = 16$, find x, when $y = 6$.

Solution:

$$x \propto \frac{1}{y^2-6}$$

$$\Rightarrow x = \frac{k}{y^2-6}$$

$$at\ x=2,\ y=16$$

$$2 = \frac{k}{256-6} \quad \Rightarrow k = 500$$

$$x = \frac{500}{y^2-6} = \frac{500}{6^2-6} = \frac{50}{3}$$

Important Points to Remember

1. *The relationship between different variables is defined to frame an equation involving the variables and the constant of proportionality*
2. *One set of values of all the values of all the variables enables us to find the value of the constant of proportionality*
3. *The values of all but one variable are given and we are asked to find the value of the one variable whose is not given*
4. Note that there should be consistency of the units used for the variables, i.e., whatever be the units used to express the variables when the constant of proportionality is being calculated, the same units should be used for different variables later on also when finding the value of the variable which we are asked to find out.

Example 3 The volume of a cylinder varies jointly as its height and the area of its base. When the area of the base is 64 sq. ft. and the height is 10 ft, the volume is 480 cu.ft. What is the height of the cylinder, whose volume is 360 cu.ft. and the area of the base is 72 sq.ft.

Solution: Let V be the volume, a = area of the base and h = height.

V = mah (m is proportionality constant)

we know, a = 64 h = 10 and V = 480

480 = m (64) (10)

$\Rightarrow$ m = 0.75 V = $\frac{3ah}{4}$ Therefore, 360 = $\frac{3 \times 72 \times h}{4}$

h = $\frac{360 \times 4}{3 \times 72}$ = h = $\frac{20}{3}$ ft, Hence height of the cylinder is 6 ⅔ ft.

Example 4 $\frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{a+b}$, then $\frac{c}{b+c} = ?$

Solution: Let each expression = k

a = (b + c) k

b = (c + a) k

c = (a + b) k

Adding all three expression

(a+b+c) = 2(a+b+c)k

1=2k

k= 1/2

Therefore, a=b=c hence; $\frac{c}{b+c} = \frac{1}{2}$

Example 5 If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ then find the value of expression $\frac{a+3c-5e}{b+3d-5d}$.

Solution: Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ =k

$$\frac{bk+3dk-5fk}{b+3d-5f}=k=\frac{a}{b}$$

Example 6 33.3 % of A = 20 % of B= 20 % of C, if A + B + C = 260, A = ?

Solution:

$$\frac{A}{3}=\frac{B}{5}=\frac{C}{5}=k$$

$$3k+5k+5k=260 \quad \Rightarrow k=\frac{260}{13}=20$$

$$A=20\times 3=60$$

Example 7 $(a^2 + b^2) : (a^2 - b^2) = 17 : 8$, a : b = ?

Solution:
$8(a^2+b^2) = 17(a^2-b^2)$ [by applying componendo and dividendo]
$25b^2 = 9a^2 \quad \Rightarrow (a/b)^2 = 25/9$
a/b = 5/3

Example 8 If A : B = 3 : 4, A : C = 5 : 3, find B : C

Solution:
A : B = 3:4 _____(i)
A : C = 5:3 _____(ii)
Dividing (i) by (ii), C:B = 9:20
therfore, B:C = 20:9

Example 9 A : B = 1 : 2, A : C = 1 : 3, A : D = 1 : 4, B : C : D = ?

Solution:
Since A is same, therefore
B : C : D = 2 : 3 : 4

Example 10 A : B : C = $\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$, A + B + C = 390, A = ?

Solution:
A : B : C = 6 : 4 : 3

$$A=\frac{6}{13}\times 390=180$$

Example 11 A man lost 12.5% of his money. After spending 70% of the remainder, he has Rs. 210 left with him. How much did he have in the beginning?

Solution:
Let X be the total amount of money he had

Now 12.5% = $\frac{1}{8}$. So, money left with him, after losing = 7/8 of *x*

Money left after spending $= 30\%\ of\ \frac{7}{8}\ of\ x=\frac{30}{100}\times\frac{7}{8}\times x=210$

$$\Rightarrow \frac{3\times 7}{80}x=210 \quad \Rightarrow x = \text{Rs. } 800$$

Example 12 In an election there are only two candidates, A and B. Candidate A got 59% of votes and won by a majority of 144 votes. If there were no invalid votes, find the total number of votes.

Solution:
A got 59%, therefore B got 41%
Difference = 59 – 41 = 18
18% = 144 votes

Therefore, total votes = $\frac{144}{18}$ x 100 = 800

Example 13 Ram, Sham and Pran share profits in the ratio 12 : 1 : 5, if Pran's share is Rs. 12500, what was the profit of the firm?

Solution: Pran's share = 12500 = $\frac{5}{18}$ of total profit; Total profit = $\frac{12500 \times 18}{5}$ = Rs. 45000

Example 14 A's income is 2/3 of B's income and B's income is 75% of C's. What is C's income as a percentage of A?

Solution: B's income = $\frac{3}{2}$ of A's income

C's income = $\frac{4}{3}$ of B's income = $\frac{4}{3} \times \frac{3}{2}$ of A's income = 2 *(A's income)*

Example 15 A man has certain number of potatoes out of which 13% were rotten, 75% of the rest were sold and he was left with 261 fresh potatoes. How many potatoes did he had initially?

Solution: Let's assume he had P potatoes initially, then
Number of fresh potatoes = 100 – 13 = 87% and
Number of potatoes unsold = 25% of 87% of potatoes
$\left[\frac{87}{100} \times \frac{25}{100}\right] \times$ P= 261 or P = 1200

EXPERIENCE THE PRATHAM EDGE - 6

1. In a mixture of 60 litres, the ratio of milk and water is 2 : 1. If the ratio of milk and water is to be 1 : 2, then the amount of water (in litres) to be further added is:
(a) 20 (b) 30 (c) 40 (d) 60

2. Two numbers are in the ratio 3 : 5, If 9 be subtracted from each, then they are in the ratio 12 : 23. The second number is:
(a) 52 (b) 53 (c) 54 (d) 55

3. What is the ratio whose terms differ by 40 and the measure of which is 2/7 ?
(a) 16 : 56 (b) 14 : 56 (c) 15 : 56 (d) 16 : 72

4. A sum of Rs, 7000 is divided among A, B, C, in such a way that shares of A and B are in the ratio 2 : 3 and those of B and C are in the ratio 4 : 5. The amount received by C is:
(a) Rs. 2600 (b) Rs.2800 (c) Rs.3000 (d) Rs. 3900

5. If A's money is to B's money as 4 : 5 and B's money is to C's money as 2 : 3 and A has Rs.800, then C has:
(a) Rs. 1000 (b) Rs.1200 (c) Rs.1500 (d) Rs.2000

6. If X and Y shared Rs. 1100 in the ratio 1 : 10, how much did X get?
(a) Rs. 99 (b) Rs.100 (c) Rs.101 (d) Rs.110

7. If a : b = 3 : 4, then (6a + b) : (4a + 5b) is:
(a) 1 : 2 (b) 3 : 5 (c) 11 : 8 (d) 11 : 16

8. If A : B = 3 : 4, B : C = 5 : 6 and C : D = 11 : 9, then A : D is:
(a) 50 : 60 (b) 55 : 72 (c) 60 : 70 (d) 65 : 75

9. If A : B = 5 : 7 and B : C = 6 : 11, then A : B : C is :
(a) 55 : 77 : 66 (b) 30 : 42 : 77 (c) 35 : 49 : 42 (d) None of these

10. If A : B = 8 : 15, B : C = 5 : 8 and C : D = 4 : 5, then A : D is equal to :
(a) 2 : 7 (b) 4 : 15 (c) 8 : 15 (d) 15 : 4

11. If 2A = 3B = 4C, then A : B : C is :
(a) 2 : 3 : 4 (b) 4 : 3 : 2 (c) 6 : 4 : 3 (d) 3 : 4 : 6

12. If $^1/_3$A = $^1/_4$B and $^1/_4$B = $^1/_5$C, then A : B : C is:
(a) 4 : 3 : 5 (b) 5 : 4 : 3 (c) 3 : 4 : 5 (d) 20 : 15 : 12

13. If A = $^1/_3$B and B= $^1/_2$C, then A : B : C is:
(a) 1 : 3 : 6 (b) 3 : 1 : 2 (c) 2 : 3 : 4 (d) 3 : 2 : 6

14. If x : y = 2 : 1, then $(x^2 – y^2) : (x^2 + y^2)$ is :
(a) 3 : 5 (b) 5 : 3 (c) 1 : 3 (d) 3 : 1

15. If $(4x^2 – 3y^2) : (2x^2 + 5y^2) = 12 : 19$, then x : y is:
(a) 2 : 3 (b) 1 : 2 (c) 3 : 2 (d) 2 : 1

16. If a carton containing a dozen mirrors is dropped, which of the following cannot be the ratio of broken mirror to unbroken mirror?
(a) 2 : 1 (b) 3 : 1 (c) 3 : 2 (d) 7 : 5

17. An amount of Rs 2430 is divided among A, B & C such that if their shares be reduced by Rs 5, Rs 10, Rs 15 respectively, the remainders shall be in the ratio 3 : 4 : 5. The share of B is:
(a) Rs 605 (b) Rs 790 (c) Rs 800 (d) 810

18. X, Y, & Z share a sum of money in the ratio 7 : 8 : 16. If Y receives Rs 27 more than X, then find total money.
(a) Rs 489 (b) Rs 943 (c) Rs 837 (d) Rs 558

19. Rs 407 are to be divided among A, B, C so that shares are in the ratio of 1/4 : 1/5 : 1/6. The respective share of A, B, C, are:
(a) Rs 165, Rs 132, Rs 110 (b) Rs 165, Rs 110, Rs 132
(c) Rs 132, Rs 110, Rs 165 (d) Rs 110, Rs 132, Rs 165

20. If Rs 1066 are divided among A, B, C & D such that A : B = 3 : 4 , B : C = 5 : 6 & C : D = 7 : 5 who will get the maximum ?
(a) B (b) A (c) C (d) D

21. If the sides of a triangle are in the ratio of $\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$ & its perimeter is 104 cm. The length of longest side is ?
(a) 52 cm (b) 48 cm (c) 32 cm (d) 26 cm

22. The ratio of income of A to that of B is 5 : 4 and the expenditure of A to that of B is 3 : 2. If at the end of the year, each saves Rs 800, the income of A is :
(a) Rs 1600 (b) Rs 1800 (c) Rs 2000 (d) 2200

23. A dog takes 3 leaps of every 5 leaps of a hare. If one leap of the dog is equal to 3 leaps of the hare. The ratio of the speed of the dog to that of hare is:
(a) 8 : 5 (b) 9 : 5 (c) 8 : 7 (d) 9 : 7

24. $\frac{100}{3}$ % of a man's output is equal to 50% of a second man's daily output. If the second man turns out 1500 screws daily. Then the first man's output in terms of making screw is:
(a) 500 (b) 1000 (c) 2000 (d) 2250

25. If three numbers in the ratio 3 : 2 : 5 be such that the sum of their squares is 1862, the middle number will be:
(a) 7 (b) 14 (c) 21 (d) 35

26. A sum of Rs 1300 is divided among P, Q, R and S such that $\frac{\text{P's share}}{\text{Q's share}} = \frac{\text{Q's share}}{\text{R's share}} = \frac{\text{R's share}}{\text{S's share}} = \frac{2}{3}$
What is P's share?
(a) Rs 320 (b) Rs 240 (c) Rs 160 (d) Rs 140

27. The ratio of the number of boys & girls in a school is 3 : 2. If 20% of the boys and 25% of the girls are scholarship holders, the percentage of the school students who are not scholarship holder's is:
(a) 56 (b) 70 (c) 78 (d) 80

28. 15 L of a mixture contains 20% alcohol & the rest water. If 3L of water be mixed in it. The percentage of alcohol in the new mixture will be:
(a) 15 (b) $16\,{}^{2}/_{3}$ (c) 17 (d) $18\,{}^{1}/_{2}$

29. A & B are two alloys of gold & copper prepared by mixing metals in the ratio 7 : 2 & 7 : 11 respectively. If equal quantities of the alloys are melted to form a third alloy C, the ratio of gold & copper will be:
(a) 5 : 9 (b) 5 : 9 (c) 7 : 5 (d) 9 : 5

30. A mixture contains milk and water in the ratio 5 : 1 on adding 5 litres of water, the ratio of milk to water becomes 5 : 2. The quantity of milk in the original mixture is:
(a) 16 L (b) 25 L (c) 22.75 L (d) 32.5 L

31. Two equal glasses are respectively $\frac{1}{3}$ and $\frac{1}{4}$ full of milk . They are then filled with water & the contents mixed in a tumbler. The ratio of milk to water in tumbler is:
(a) 7 : 5 (b) 7 : 17 (c) 9 : 21 (d) 11 : 23

32. Three bottles are filled with a mixture of milk and water in the ratio 4 : 3, 5 : 4 and 7 : 6 respectively. If the contents of three bottles are poured into a single vessel, then the ratio of milk to water in the new vessel is:
(a) $\left[\frac{4}{3}+\frac{5}{4}+\frac{7}{6}:\frac{3}{4}+\frac{4}{5}+\frac{6}{7}\right]$ (b) $\left[\frac{4}{7}+\frac{5}{9}+\frac{7}{13}:\frac{3}{7}+\frac{4}{9}+\frac{6}{13}\right]$

(c) $\left[\frac{4}{3}+\frac{5}{4}+\frac{7}{6}:\frac{1}{4}+\frac{1}{5}+\frac{1}{7}\right]$ (d) $\left[\frac{1}{3}+\frac{1}{4}+\frac{1}{6}:\frac{3}{4}+\frac{4}{5}+\frac{6}{7}\right]$

33. A student's grade in a course is determined by 6 quizzes & one examination. If the examination counts thrice as much as each of the quizzes, what fraction of final grade is determined by the examination?
(a) $\frac{1}{6}$ (b) $\frac{1}{5}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

34. A student finishes the first half of an examination in two-thirds of the time it takes him to finish the second half. If the whole examination takes him an hour & half, how many minutes does he spend on the second half of the examination?
(a) 36 min (b) 54 min (c) 60 min (d) 44 min

35. A servant is paid a total of Rs 100 and a turban for a full year's service. If the servant works for only 9 month & received in return Rs 65 and the turban, what is the value of the turban?
(a) Rs 9 (b) Rs 10 (c) Rs 40 (d) Rs 50

PRATHAM EDGE – 7

METHODS OF FINDING CUBE ROOT

Cube roots of number upto six digits perfect cubes can be easily found by finding their units and tens digits. Let us observe

- Units digit of perfect cube is 1, units digit of cube root is 1.
- Units digit of perfect cube is 2, units digit of cube root is 8.
- Units digit of perfect cube is 3, units digit of cube root is 7.
- Units digit of perfect cube is 4, units digit of cube root is 4.
- Units digit of perfect cube is 5, units digit of cube root is 5.
- Units digit of perfect cube is 6, units digit of cube root is 6.
- Units digit of perfect cube is 7, units digit of cube root is 3.
- Units digit of perfect cube is 8, units digit of cube root is 2.
- Units digit of perfect cube is 9, units digit of cube root is 9.
- Units digit of perfect cube is 0, units digit of cube root is 0.

Let us find the cube root of 300763 by finding its units and tens digit.

Units digit is 3 so the units digit of cube root is 7. Let us first strike out the three digits from the right. We get the number 300.

Cube of 6 is 216 is the largest cube that is less than 300.
$6^3 = 216 < 300$, $7^3 = 343 > 300$. So we will take the largest cube less than 300, i.e. 6.

So our answer, cube root of 300763 is 67.

NOTES

CHAPTER 7

PARTNERSHIP

INTRODUCTION

Partnership is a business in which two or more persons come together with a motive to divide the profits. For this purpose we classify it into two types.

Simple Partnership
If the time period of investment of all the partners is same, even if the investment are different, then that partnership is a simple partnership. The profit in this case is divided in the ratio of the investments

Compound Partnership
If the time period of investment of any one or more partners is different even if the investment are same then that partnership is compound partnership. In this case the profit is divided in the ratio of the products of the investments and respective time period.

- Partners who share the investment also share the profits or loss if any.

- Usually all the partners take **active part** in running the business. Sometimes one partner may invest his money but not take an active part in running the business. Such a partner is called **sleeping partner**. He leaves the task of running the business to the other partner. The partner who runs the business is called the **working partner**.

Simple Partnership	*Compound Partnership*
The profit or loss is divided in the ratio of their investments.	The Profit or loss is divided in the ratio of their monthly equivalent investments.
If two partners A and B have entered a simple partnership then: Investment of A : Investment of B = Profit of A : Profit of B	If two partners A and B have entered a compound partnership then : Monthly equivalent of A : Monthly equivalent of B = Profit of A : Profit of B Monthly equivalent of A : Monthly equivalent of B = Loss of A : Loss of B
Investment of A : Investment of B = Loss of A : Loss of B	Monthly equivalent investment is the product of capital and the number of months for which it is invested.

SOLVED EXAMPLES

Example 1 A with Rs. 4000, B with Rs. 7000 and C with Rs. 12000 started a business. After a year if the combined profit of B and C is Rs. 3000 more than that of A. Find the total profit.

Solution: Ratio of investments of A : B : C = 4 : 7 : 12
Hence, (B+C) - A = (12+7-4) =15
If (B+ C) – A is Rs. 3000 then, Total profit = $\frac{23}{15} \times 3000$ = Rs. 4600

Example 2 A with Rs. 8000 and B with Rs. 11000 started a business. A gets 25% of the profit as salary being a working partner. Remaining is divided in the ratio of investments. If A gets Rs. 4300, find the total profit.

Solution: Let the total profit be x
Profit of A is $\frac{x}{4}+\frac{8}{19}\left[\frac{3x}{4}\right] = 4300 \Rightarrow \frac{43x}{76} = 4300$ Therefore, x = Rs. 7600

Example 3 A with Rs. 4000 and B with Rs. 11000 started a business. 3 months later C joined with Rs. 7000, 2 months later A leaves. After 12 months if B gets Rs. 1320 as profit. Find the total profit of A & C

Solution: Ratio of Investment of A : B : C = 4 : 11 : 7
Ratio of time periods = 5 : 12 : 9
Ratio of Profits = 20 : 132 : 63
If B gets Rs. 132 then total profit of A & C = Rs. 83
If B gets Rs. 1320 then (A + C)'s profit = Rs. 830

Example 4 A with Rs. 8000 started a business. B joined later with Rs. 12000. After 18 months they divided the profit in ratio 2 : 1, when did B join?

Solution: Ratio of profits 2 : 1
Ratio of investment 2 : 3
Ratio of time periods = $\frac{2}{2}:\frac{1}{3}$

$= 1:\frac{1}{3}$ = 3 : 1
B's investment is for $\frac{1}{3}$ x 18 months = 6 months
B joined after 1 year.

Example 5 A invests Rs. 5000 and B invests Rs. 6000. A is a working partner. A receives 1/8 of the profit as salary and rest is divided in proportion of their capitals. If A gets Rs. 460 then find the total profit.

Solution: Ratio of investment is 5 : 6
Let the total profit be x
Share of A = $\frac{x}{8}+\frac{5}{11}\times\frac{7x}{8}$ =Rs. 460
= $\frac{46x}{88}$ = 460 Therfore, x = Rs. 880

Example 6 A, B, C enter into a partnership. A advances Rs. 600 for 8 months, B advances Rs. 2800 for 4 months, C advances Rs 2000 for 5 months. They earn a profit of Rs. 585 together. Share of C is

Solution: Ratio of investment (A : B : C) = (600 : 2800 : 2000)
Ratio of time periods 8 : 4 : 5
Ratio of profits = 4800 : 11200 : 10000
= 12 : 28 : 25

Share of C = $\frac{25}{65}$x Rs. 585 = Rs. 225

Example 7 A invested Rs. 24000 for 8 months and investment of B is for 4 months. Annual ratio of profits is 3 : 4. Find the investment of B

Solution: Ratio of profit is 3 : 4
Ratio of time period is 2 : 1
Ratio of investment is $\frac{3}{2}:\frac{4}{1}=3:8$

If investment of A = Rs. 24000
Therefore investment of B = Rs. 64000

Example 8 A and B rent a pasture for 20 months. A puts in 160 oxen for 14 months. How many can B put in for the remaining 6 months, if he pays half as much as again as A.

Solution: Rent ratio of A & B = 2 : 3 (half as much again means half more)
Time ratio = 14 : 6

Oxen ratio = $\frac{1}{7}:\frac{1}{2}=2:7 \Rightarrow 160 : 560$

Example 9 Investment ratio of A, B, C is $\frac{1}{3}:\frac{1}{4}:\frac{1}{7}$ for the same time period. If A gets Rs. 560 as profit then how much does B get more than C.

Solution: Investment ratio is $\frac{1}{3}:\frac{1}{4}:\frac{1}{7}$ = 28 : 21 : 12

If A gets Rs. 28 then (B –C) = Rs. 9

If A gets Rs. 560 then (B – C) is 560 $\times\frac{9}{28}$ = Rs. 180

Example 10 A, B & C enter into a partnership for 1 year. A invests his money at the start. B invests twice the amount of A but after 6 months, C invests $3/2$ times that of B after 8 months. If total profit = Rs. 12000. What is the Share of B ?

Solution: Investment ratio A : B : C = 1 : 2 : 3
Time period of investment = 12 : 6 : 4
Profit ratio is 1 : 1 : 1

Share of B = $\frac{12000}{3}$ = Rs. 4000

Example 11 A, B and C start a business each investing Rs. 20000. After 5 months A withdrew Rs. 5000, B withdrew Rs. 4000 and C invests Rs. 6000 more. At the end of the year, a total profit of Rs. 69900 was recorded. Find the share of each.

Solution: Ratio of capitals of A, B and C
= 20000 x 5 + 15000 x 7 : 20000 x 5 + 16000 x 7 : 20000 x 5 + 26000 x 7
= 205000 : 212000 : 282000 = 205 : 212 : 282

$\therefore$ A's share = Rs.$\left(69900\times\frac{205}{699}\right)$ = Rs. 20500;

B's share = Rs. $\left(69900\times\frac{212}{699}\right)$ = Rs. 21200;

C's share = Rs.$\left(69900\times\frac{282}{699}\right)$ = Rs. 28200.

Example 12 A, B and C enter into partnership. A invests 3 times as much as B invests and B invests two-third of what C invests. At the end of the year, the profit earned is Rs. 6600. What is the share of B?

Solution: Let C's capital = Rs. x. Then, B's capital = Rs. $\frac{2}{3}x$

A's capital = Rs. $\left(3\times\frac{2}{3}x\right)$ = Rs. 2x.

$\therefore$ Ratio of their capitals = 2x : $\frac{2}{3}$ x : x = 6 : 2 : 3

Hence, B's share = Rs. $\left(6600\times\frac{2}{11}\right)$ = Rs. 1200.

Example 13 Four milkmen rented a pasture. A grazed 24 cows for 3 months; B grazed 10 cows for 5 months; C grazed 35 cows for 4 months and D grazed 21 cows for 3 months. If A's share of rent is Rs. 720, find the total rent of the field.

Solution: Ratio of shares of A, B, C, D = (24 x 3) : (10 x 5) : (35 x 4) : (21 x 3)
= 72 : 50 : 140 : 63

Let total rent be Rs. x. Then , A's share = Rs. $\frac{72x}{325}$

$$\therefore \quad \frac{72x}{325} = 720 \Rightarrow x = \frac{720 \times 325}{72} = 3250$$

Hence, total rent of the field is Rs. 3250

Example 14 A invested Rs. 76000 in a business. After few months, B joined him with Rs. 57000. At the end of the year, the total profit was divided between them in the ratio 2 : 1. After how many months did B join?

Solution: Suppose B joined after x months. Then, B's money was invested for (12 – x) months.

$$\therefore \quad \frac{76000 \times 12}{57000(12-x)} = \frac{2}{1} \Rightarrow 912000 = 114000\,(12-x)$$

$\Rightarrow$ 114 (12 – x) = 912 $\Rightarrow$ (12 – x) = 8 $\Rightarrow$ x = 4.
Hence, B joined after 4 months.

Example 15 A, B and C enter into a partnership by investing in the ratio of 3 : 2 : 4. After one year, B invests another Rs. 270000 and C, at the end of 2 year, also invests Rs. 270000. At the end of three years, profits are shared in the ratio of 3 : 4 : 5. Find the initial investment of each.

Solution: Let the initial investments of A, B and C be Rs.$3x$, Rs. $2x$ and Rs. $4x$ respectively. Then
($3x$ x 36) : [($2x$ x 12) + ($2x$+ 270000) x 24] : [($4x$ x 24) + ($4x$ + 270000) x 12]=3 : 4 : 5
$\Rightarrow$ $108x$: ($72x$ + 6480000) : ($144x$ + 3240000) = 3 : 4 : 5

$$\therefore \quad \frac{108x}{72x + 6480000} = \frac{3}{4} \Rightarrow 432x = 216x + 19440000$$

$\Rightarrow$ $216x$ = 19440000 $\Rightarrow x$ = 90000.

Hence, A's initial investment = $3x$ = Rs. 270000;
B's initial investment = $2x$ = Rs. 180000;
C's initial investment = $4x$ = Rs. 360000.

EXPERIENCE THE PRATHAM EDGE - 7

1. Alok started a business investing Rs 90000. After 3 months Shabbir joined him with a capital of Rs. 120000. If at the end of 2 years, the total profit made by them was Rs 96000, what will be the difference between their shares?
   (a) Rs. 20000 (b) Rs. 24000 (c) Rs. 8000 (d) None of these

2. A,B and C enter into a partnership. A initially invests Rs 25 lakhs and adds another Rs 10 lakhs after one year. B initially invests Rs 35 lakhs and withdraws Rs 10 lakhs after two years and C invests Rs 30 lakhs. In what ratio should the profits be divided at the end of 3 years?
   (a) 20 : 19 : 18 (b)10 : 10 : 9 (c)20 : 20 : 19 (d) 19:19:18

3. Manoj received Rs 6000 as his share out of the total profit of Rs. 9000 which he and Ramesh earned at the end of one year. If manoj invested Rs 20000 for 6 months, whereas Ramesh invested his amount for the whole year, what was the amount invested by Ramesh?
   (a) Rs. 6000 (b) Rs. 10000 (c) Rs. 4000 (d) Rs. 5000

4. Three partners A,B and C starts a business. Twice A's capital is equal to thrice B's capital and B's capital is four times C's capital. Out of the total profit of Rs 16500 at the end of the year, B's share is:
   (a) Rs. 4000 (b) Rs. 6000 (c) Rs. 7500 (d) Rs. 6600

5. A,B and C subscribe Rs 50000 for a business. A subscribes Rs 4000 more than B and B Rs 5000 more than C. Out of the total profit of Rs 35000. A receives:
   (a) Rs. 11900 (b) Rs. 8400 (c) Rs. 14700 (d) Rs. 13600

6. If 4(A' capital)=6(B's capital)=10(C's capital), then out of a profit of Rs 4650, C will receive:
   (a) Rs. 2250 (b) Rs. 1550 (c) Rs. 900 (d) Rs. 465

7. Four milkman rented a pasture. A grazed 24 cows for 3 months; B grazed 10 cows for 5 months, C grazed 35 cows for 4 months and D grazed 21 cows for 3 months. If A 's share of rents is Rs 720, the total rent of the field is:
   (a) Rs. 3000 (b) Rs. 3200 (c) Rs. 3250 (d) Rs. 3300

8. A,B,C start a business jointly. A invests 3 times as much as B invests and B invests two third of what C invests. At the end of the year, the profit earned is Rs 660. Out of it, B's share is:
   (a) Rs. 220 (b) Rs. 120 (c) Rs. 180 (d) Rs. 240

9. A and B started a business jointly. A's investment was thrice the investment of B and the period of the investment was two times the period of investment of B. If B received Rs 4000 as profit. Then their total profit is:
   (a) Rs. 16000 (b) Rs. 24000 (c) Rs. 20000 (d) Rs. 28000

10. A and B started a business with initial investments in the ratio 14:15 and their annual profit were in the ratio 7:6. If A invested the money for 10 months, for how many months did B invest his money?
    (a) 8 (b) 9 (c) 6 (d) 7

11. A,B and C enter into a partnership and their capitals are in proportion 20:15:12. A withdraws half his capital at the end of 4 months. Out of the total annual profit of Rs 847, A's share is:
    (a) Rs. 252 (b) Rs. 280 (c) Rs. 315 (d) Rs. 412

12. A, B and C entered into a partnership in the ratio $\frac{7}{2} : \frac{4}{3} : \frac{6}{5}$. After 4 months, A increases his share by 50%. If the total profit at end of one year be Rs21600, then B's share in the profit is:
(a) Rs. 2100 (b) Rs. 2400 (c) Rs. 3600 (d) Rs. 4000

13. A and B invests in a business in the ratio 3:2. If 5% of the total profit goes to charity and A's share is Rs 855, the total profit is:
(a) Rs. 1425 (b) Rs. 1500 (c) Rs. 1537.50 (d) Rs. 1576

14. A and B a sleeping partner in a business. A puts in Rs 12000 and B Rs 20000. A receives 10% of the profits for managing, the rest being divided in proportion to their capitals. Out of the total profit of Rs 9600, the money received by A is:
(a) Rs. 3240 (b) Rs. 3600 (c) Rs. 4200 (d) Rs. 4500

15. A,B and C contract a work for Rs 1100. A and B together are to do $\frac{7}{11}$ of the work. C's share is:
(a) Rs. 366.66 (b) Rs. 400 (c) Rs. 600 (d) Rs. 800

NOTES

CHAPTER 8

MIXTURES & ALLEGATIONS

INTRODUCTION

The concept of Allegation or Mixture is very important for various examinations as it can be applied to a wide variety of questions ranging from profit and loss to time and distance. It can reduce the time in solving the problems by at least 50%. As we are known to some basic concepts of ratio and proportion, now in this chapter we will learn some more interesting concepts of ratio and proportion.

Allegation: It is the rule that enables us to find the ratio in which two or more ingredients at the given price must be mixed to produce a mixture at a given price.
Mean Price: The cost price of a unit quantity of the Mixture is called the Mean Price.
Rule of Allegation: If two ingredients are mixed, then:

$$\frac{\text{Quantity of Cheaper}}{\text{Quantity of Dearer}} = \frac{(\text{CP of Dearer}) - (\text{Mean Price})}{(\text{Mean Price}) - (\text{CP of Cheaper})}$$

We represent as under :

CP of a unit quantity of cheaper (c) CP of a unit quantity of dearer (d)

Mean Price (m)

(d – m) (m – c)

$$\frac{\text{Cheaper quantity}}{\text{Dearer quantity}} = \frac{d - m}{m - c}$$

Suppose a container contains x units of liquid from which y units are taken out and replaced by water. After n operations, the quantity of

$$\textit{pure liquid} = \left[x\left(1-\frac{y}{x}\right)^n\right] \text{units}$$

SOLVED EXAMPLES

Example 1 There are two types of rice of prices Rs. 9/ kg and Rs. 11/ kg respectively. They are mixed in a certain ratio and then the average price of the mixture is Rs 9.6/kg. Find the ratio in which the two types of rice are mixed?

Solution:

Cheaper **Dearer**

9 11

9.6

1.4 0.6

(dearer – mean) **(mean – cheaper)**

Required ratio $= 1.4 : 0.6$

= 7 : 3

Example 2 In what ratio must tea at Rs 62 per kg be mixed with tea at Rs 72 per kg, so that the mixture must be worth Rs. 64.5?

Solution: Cheaper Dearer

62 72

64.5

7.5 2.5

$$\frac{Tea\ of\ I^{st}\ Kind}{Tea\ of\ II^{nd}\ Kind} = \frac{3}{1}$$

Example 3 In what ratio must rice at Rs.9.30 per kg be mixed with rice at Rs 10.80 per kg so that the mixture be worth Rs. 10 per kg ?

Solution:

9.3 10.8

10

10.8 -10 10 – 9.3
=0.8 **=0.7**

0.8 : 0.7
8 : 7
Required ratio = 8 : 7

Example 4 In what ratio must water be mixed with Milk to gain 20% by selling the mixture at cost price ? (Assume that water costs nothing).

Solution: Let CP of 1 litre Milk be Re 1 per litre, Then, SP of 1 litre of mixture = 1
Gain obtained = 20%
CP of 1 litre of mixture = $\frac{100}{120} \times 1 = \text{Re } \frac{5}{6}$

CP of 1 Ltr. of Water CP of 1 Ltr. of Milk
0 1

Mean Price
$\left(Rs.\ \frac{5}{6}\right)$

$\left(1 - \frac{5}{6}\right) = \frac{1}{6}$ $\left(\frac{5}{6} - 0\right) = \frac{5}{6}$

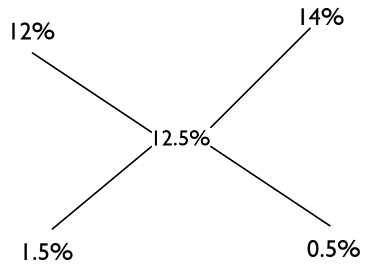
Ratio of Water and Milk = 1/6:5/6= 1 : 5

Example 5 A man lent out Rs 9600 partly at 12% and partly at 14% simple interest. His total income after 1½ year was Rs. 1800. Find the sum lent at different rates.

Solution: Total interest on Rs 9600 for 1½ year = Rs 1800

$$\text{Rate} = \frac{100 \times 1800}{9600 \times \frac{3}{2}} = 12.5\%$$

By the rule of Allegation, we have



$$\frac{\text{1st part}}{\text{2nd part}} = \frac{1.5}{0.5} = \frac{3}{1}$$

First part = Rs $(9600 \times \frac{3}{4})$ = Rs 7200 & Second part = Rs (9600 – 7200) = Rs 2400

Example 6 In a mixture of 42 litre, the ratio of milk to water is 6 : 1 respectively. Another 12 litre of water is added to the mixture. Find the ratio of milk to water in the resultant mixture.

Solution: Quantity of mixture = 42 litre

Ratio of M : W = 6 : 1

Quantity of Milk = $\frac{6}{7} \times 42 = 36$

Now 12 litre of water is added to the mixture.

Now quantity of milk in the mixture = 36 and Quantity of water in the mixture = 6 + 12 = 18.

Now ratio of milk to water is 36 : 18 or 2 : 1

Example 7 A container has 80 litres of milk. From this container 8 litres of milk was taken out and replaced by water. The process was further repeated twice. What is the amount of milk left at the end of three operations ?

Solution: Concept :

Let the container originally contain x units of liquid.

Liquid taken out in each case is y units.

The final quantity of the component from the original mixture that is not being replaced, if this operation is repeated n times is $x[1 - (y/x)]^n$ units.

Where x is the original amount of that component in the mixture.

Remember that the original amount of milk need not be equal to the volume of the container.

In this case, the original quantity of milk is 80 litres.

The total number or operations of drawing the liquid i.e., n = 3

The amount of milk left at the end of three operations

$$= 80 \times \left(1 - \frac{1}{10}\right)^3 = 80 \times \left(\frac{9}{10}\right)^3 = 80 \times \left(\frac{729}{1000}\right) = 58.32 \text{ litre}$$

Example 8 In a zoo, there are rabbits and pigeons. If heads are counted, there are 200 and if legs are counted, there are 580. How many pigeons are there ?

Solution: Average number of legs per animal = $\frac{580}{200} = \frac{29}{10} = 2.9$

Using the principle of allegations, we have $\frac{4-\left(\frac{29}{10}\right)}{\left(\frac{29}{10}\right)-2}=\frac{11}{9}$

No. of pigeons = $^{11}/_{20}$ x 200 = 110

Example 9 Two vessels A and B contains spirit and water mixed in the ratio 5 : 2 and 7 : 6 respectively. Find the ratio in which these mixtures be mixed to obtain a new mixture in vessel C containing spirit and water in the ratio 8 : 5

Solution: Let the C.P. of spirit be Re 1 per litre.

Spirit in 1 litre mixture of A = $\frac{5}{7}$ litre.

Spirit in 1 litre mix. of B = $\frac{7}{13}$ litre.

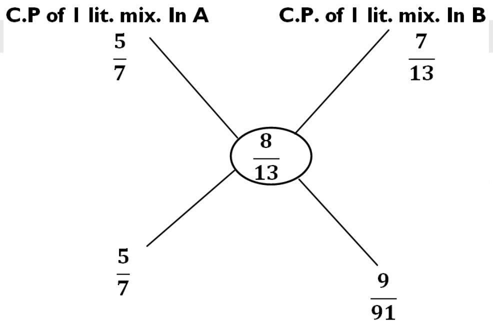
Spirit in 1 litre mix. of C = $\frac{8}{13}$ litre

C.P. of 1 litre mix, in A = Re $\frac{5}{7}$

C.P. of 1 litre mix in B = Re $\frac{7}{13}$

Mean price = Re $\frac{8}{13}$

By the rule of allegation, we have:



Required ratio = $\frac{1}{13}:\frac{9}{91}$ = 7 : 9

Example 10 A mixture of 20 kg of spirit and water contains 10% water. How much water must be added to this mixture to raise the percentage of water to 25% ?

Solution: Water in given mix. = $\frac{10}{100}$ x 20 kg = 2 kg & spirit = 18 kg

Let x kg of water be added.

Then, $\frac{x+2}{20+x}=\frac{25}{100}$ $\Rightarrow$ 4x + 8 = 20 + x or x = 4 kg

Example 11 729 ml of mixture contains milk and water in the ratio 7 : 2. How much more water is to be added to get a new mixture containing milk and water in the ratio 7 : 3 ?

Solution: Milk = $729 \times \frac{7}{9}$ ml = 567 ml

Water = (729 – 567) ml = 162 ml

Let water is to added be x ml.

$\frac{567}{162+x} = \frac{7}{3}$ or 1701 = 1134 + 7x or x = 81 ml

Example 12. A jar full of whisky contains 40% alcohol. A part of this whisky is replaced by another containing 19% alcohol and now the percentage of alcohol was found to be 26%. The quantity of whisky replaced is :

Solution:

Strength of first Jar 40% | Strength of second Jar 19%

Mean Strength 26%

7 | 14

Ratio of 1st & 2nd qualities = 7 : 14 = 1 : 2

Required quantity replace = 2/3

Example 13 In what ratio must a person mix three kinds of wheat costing him Rs. 1.2, Rs.1.44 and Rs.1.74 per kg of that the mixture may be worth Rs.1.41 per kg?

Solution:

1st 1.2 | 3rd 1.74

1.41

0.33 | 0.21

$\frac{1^{st}}{3^{rd}} = \frac{11}{7}$

1st 1.2 | 2nd 1.44

1.41

0.03 | 0.21

$\frac{1^{st}}{2^{nd}} = \frac{1}{7}$

$\therefore 1^{st} : 2^{nd} : 3^{rd}$

$= 11 : 77 : 7$

Example 14 How many kgs. of wheat costing Rs. 8 per kg must be mixed with 36 kg of rice costing Rs. 5.40 per kg so that 20% gain may be obtained by selling the mixture at Rs. 7.20 per kg?

Solution: S.P. of 1 kg mixture = Rs. 7.20, Gain = 20%

$\therefore$ C.P. of 1 kg mixture = Rs. $\left(\frac{100}{120} \times 7.20\right)$ = Rs. 6

By the rule of allegation, we have:

C.P of 1 kg wheat of 1[st] kind (800 p) | C.P of 1 kg wheat of 2[nd] kind (540 p)

600

60 | 200

Wheat of 1[st] kind: Wheat of 2[nd] kind = 60 : 200 = 3 : 10
Let x kg of wheat of 1[st] kind be mixed with 36 kg of wheat of 2[nd] kind.
Then, 3 : 10 = x : 36 or 10x = 3 x 36 or x = 10.8 kg.

Example 15 The milk and water in two vessels A and B are in the ratio 4 : 3 and 2 : 3 respectively. In what ratio, the liquids in both the vessels be mixed to obtain a new mixture in vessel C containing half milk and half water?

Solution: Let the C.P. of milk be Rs. 1 per litre.

Milk in 1 litre mixture of A = $\frac{4}{7}$ litre; Milk in 1 litre mixture of B = $\frac{2}{5}$ litre;

Milk in 1 litre mixture of C = $\frac{1}{2}$ litre.

$\therefore$ C.P. of 1 litre mixture in A = Rs.$\frac{4}{7}$; C.P. of 1 litre mixture in B = Rs.$\frac{2}{5}$.
Mean price = Rs. $\frac{1}{2}$

By the rule of allegation, we have:

C.P. of 1 litre mix. In A $\left(\frac{4}{7}\right)$ | C.P. of 1 litre mix. In B $\left(\frac{2}{5}\right)$

$\left(\frac{1}{2}\right)$

$\left(\frac{1}{10}\right)$ | $\left(\frac{1}{14}\right)$

$\therefore$ Required ratio = $\frac{1}{10}:\frac{1}{14}$ = 7 : 5

EXPERIENCE THE PRATHAM EDGE - 8

1. In what ratio must wheat at Rs. 3.20 per kg be mixed with wheat at Rs. 2.90 per kg so that the mixture be worth Rs. 3.08 per kg ?
(a) 2 : 3 (b) 3 : 2 (c) 3 : 5 (d) 5 : 3

2. In what ratio must water be mixed with milk costing Rs. 12 per litre to obtain a mixture worth of Rs. 8 per litre ?
(a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 3 : 1

3. A sum of Rs. 4000 is lent out in two parts, one at 8% simple interest and the other at 10% simple interest. If the annual interest is Rs. 352, the sum lent at 8% is:
(a) 2800 (b) 2400 (c) 3000 (d) 3200

4. A man covered a distance of 1980 km in 18 hours partly by bus at 72 kmph and partly by train at 160 kmph. The distance covered by bus is
(a) 800 km (b) 880 km (c) 736.36 km (d) 960 km

5. A mixture of 40 L of milk and water contains 10% water. How much water must be added to this mixture in order to make it a 20 % water mixture ?
(a) 4 L (b) 5 L (c) 6 L (d) 7 L

6. A man purchases two shirts for Rs. 900. He sells one at profit of 15% and other at a loss of 10% . He neither gains nor losses anything. Find the cost price of each shirt.
(a) 300, 600 (b) 350, 550 (c) 360, 540 (d) 375, 525

7. The ratio of milk and water in 66kg of adulterated milk is 5 : 1. Water is added to it to make the ratio 5 : 3. What is the quantity of water added ?
(a) 15 (b) 17 (c) 20 (d) 22

8. A mixture of 729 ml contains milk and water in the ratio 7 : 2. How much more water is added to get a new mixture containing milk and water in the ratio 7 : 3 ?
(a) 57 (b) 69 (c) 81 (d) 97

9. A dishonest milkman professes to sell his milk at cost price. He mixes it with water and thereby gains 25%. What is the percentage of water in the mixture ?
(a) 20% (b) 25% (c) 30% (d) 35%

10. A can contains a mixture of two liquids A and B in the ratio 7 : 5. When 9 L of mixture is taken out and the can is filled with B, the ratio of A and B is now 7 : 9. How many litres of liquid A was there in the can initially ?
(a) 18 L (b) 20 L (c) 21 L (d) 24 L

11. A man traveled a distance of 80 km in 7 hr, partly on foot at the rate of 8 km/hr and partly by a cycle at 16 km/hr. Find the distance traveled on foot.
(a) 30 km (b) 32 km (c) 34 km (d) 46 km

12. Two liquids are mixed in the ratio 3 : 5 and the mixture is sold at Rs. 120 with a profit of 20%. If the first liquid is costlier than the second by Rs. 2 per litre, find the costs of the two liquids.
(a) Rs. 105.25 & Rs. 95.25 per litre (b) Rs. 101.75 & Rs. 99.75 per litre

(c) Rs. 100.25 & Rs. 90.25 per litre (d) Rs. 101.25 & Rs. 99.25 per litre

13. Tea worth Rs. 126 per kg and Rs. 135 per kg are mixed with a third variety in the ratio 1 : 1 : 2. If the mixture is worth Rs. 153 per kg, the price of the third variety per kg will be:
(a) Rs. 169.50 (b) Rs. 170 (c) Rs. 175.50 (d) Rs. 180

14. A mixture contains alcohol and water in the ratio 4 : 3. If 8 L of water is added to the mixture, the ratio of alcohol and water becomes 3 : 5. Find the quantity of alcohol in the old mixture.
(a) $\frac{50}{13}$ (b) $\frac{65}{7}$ (c) $\frac{96}{11}$ (d) $\frac{33}{5}$

15. Fresh grapes contain 90% water by weight while dried grapes contain 20% water by weight. What will be the weight of dry grapes available from 20 kg of fresh grapes?
(a) 2.5 kg (b) 3 kg (c) 4.5 kg (d) 5 kg

16. A litre of water is added to 5 litres of 20% alcohol solution. Find the strength of the resulting solution.
(a) 20% (b) $15\frac{9}{5}$% (c) $16\frac{2}{3}$% (d) 25%

17. There are two mixture of wine and water, the quantity of wine being 0.25 and 0.75 of the two mixtures. If 2 litres of the first be mixed with three litres of the second, what will be the ratio of wine to water in the new mixture ?
(a) 15 : 7 (b) 12 : 13 (c) 1 : 25 (d) 11 : 9

18. Milk and water are in the ratio of 3 : 2 in a mixture of 120 litres. How much water should be added so that the ratio of milk to water is 2 : 3
(a) 20 L (b) 35 L (c) 55 L (d) 60 L

19. A cask contains 3 parts of milk and 1 part of water. How much portion of mixture must be withdrawn and water substituted in order the resulting mixture may be half and half ?
(a) $\frac{1}{5}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$

20. A dealer mixes tea costing Rs. 12 a kg with tea costing Rs. 18 a kg and sells the mixture at Rs. 18 a kg, earning a profit of 25% in what proportion does he mix the two varieties to tea ?
(a) 2 : 3 (b) 3 : 2 (c) 3 : 5 (d) 5 : 3

21. In what ratio must water be mixed with milk to gain 16.67 % on selling the mixture at cost price?
(a) 1 : 6 (b) 6 : 1 (c) 2 : 3 (d) 4 : 3

22. 8 litres are drawn from a cask full of wine and is then filled with water. This operation is performed three more times. The ratio of the quantity of wine now left in cask to that of the water is 16 : 65. How much wine did the cask hold originally?
(a) 18 litres (b) 24 litres (c) 32 litres (d) 42 litres

23. A container contains 40 litres of milk. From this container 4 litres of milk was taken out and replaced by water. This process was repeated further two times. How much milk is now contained by the container?
(a) 26.34 litres (b) 27.36 litres (c) 28 litres (d) 29.16 litres

24. How many kilograms of sugar costing Rs. 9 per kg must be mixed with 27 kg of sugar costing Rs. 7 per kg so that there may be a gain of 10% by selling the mixture at Rs. 9.24 per kg?
(a) 36 kg (b) 42 kg (c) 54 kg (d) 63 kg
25. In what ratio must a grocer mix two varieties of worth Rs. 60 a kg and Rs. 65 a kg so that by selling the mixture at Rs. 68.20 a kg he may gain 10%?
(a) 3 : 2 (b) 3 : 4 (c) 3 : 5 (d) 4 : 5

26. A merchant has 1000 kg of sugar, part of which he sells at 8% profit and the rest at 18% profit. He gains 14% on the whole. The quantity sold at 18% profit is:
(a) 400 kg (b) 560 kg (c) 600 kg (d) 640 kg

27. The cost of Type 1 rice is Rs. 15 per kg and Type 2 rice is Rs. 20 per kg. If both Type 1 and Type 2 are mixed in the ratio of 2 : 3, then the price per kg of the mixed variety of rice is:
(a) Rs. 18 (b) Rs. 18.50 (c) Rs. 19 (d) Rs. 19.50

28. A milk vendor has 2 cans of milk. The first contains 25% water and the rest milk. The second contains 50% water. How much milk should he mix from each of the containers so as to get 12 litres of milk such that the ratio of water to milk is 3 : 5?
(a) 4 litres, 8 litres (b) 6 litres, 6 litres (c) 5 litres, 7 litres (d) 7 litres, 5 litres

29. A vessel is filled with liquid, 3 parts of which are water and 5 parts syrup. How much of the mixture must be drawn off and replaced with water so that the mixture may be half water and half syrup?
(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{5}$ (d) $\frac{1}{7}$

30. A can contains a mixture of two liquids A and B in the ratio 7 : 5. When 9 litres of mixture are drawn off and the can is filled with B, the ratio of A and B becomes 7 : 9. How many litres of liquid A was contained by the can initially?
(a) 10 (b) 20 (c) 21 (d) 25

CHAPTER 9

LINEAR & QUADRATIC EQUATIONS

LINEAR EQUATIONS

Introduction

Linear Equations are the most basic kind of **algebraic function** and help to answer the questions like "Most cars won't be able to run for more than 250,000 miles, so how much longer will your car live?" There will be linear equations of **one or two unknown variables** in every problem. In general, we need as many equations as the variables as we have to solve for. Solving the equation by itself is not a difficult task. The most important part of the problem is framing the equations. In this chapter, we will deal with problems involving as many equations as the number of unknowns.

Definition

- **Equation:** An equation is a **statement of equality** of two algebraic expressions which involve one or more unknown quantities, called the **'variables'**.
  Example: $4x + 7 = 0$, $3x + 2 = 5x - 6$, $x^2 + 6 = 3x$
- **Linear equation:** An equation involving the **variable in maximum of order 1**, then it is called a linear equation.
  Example: $3x - 8 = 0$, contains power of variable as 1.
  $(x - 1)(x - 3) = 6$, contains maximum power of variable as 2 so, it is not a linear equation.

  Solution of a Linear Equation in one Variable

  A value of the variable, which when substituted for the variable in the equation makes **the two sides of the equation equal**, is called the **solution** of the variable.

  Example: Consider the equation, $2x + 3 = -7$,

  If we substitute the value -5 for x, we get

  LHS $= 2(-5) + 3 = -10 + 3 = -7 =$ RHS.

  Therefore, $x = -5$ is the solution of the equation $2x + 3 = -7$

  Other than -5, if we put any value of x, says 3, in the equation, we get

  LHS $= 2 \times 3 + 3 = 6 + 3 = 9 \neq$ RHS

  So, $x = 3$ cannot be a solution of the equation. Therefore, solving linear equation means finding the **unique value** of the variable, which satisfies the equation.

Rules for Solving an Equation

To solve an equation, following properties of equality are used.

- **Rule I:** Same quantity can be added to both side of an equation without changing the equality.
- **Rule II:** Same quantity can be subtracted from both the sides of an equation without changing the equality.
- **Rule III:** Both the sides of an equation can be multiplied by the same non-zero number without changing the equality.
- **Rule IV:** Both the sides of an equation may be divided by the same non-zero number without changing the equality.

Note: *Addition or subtraction by the same quantity on two sides of an equation is equivalent to transposition of the quantity on the other sides after changing its sign (from '+' to '-' and from '-' to '+').*

Simultaneous Linear Equations in two Variables

Two distinct linear **equations** with the **same two unknowns** will form a system of simultaneous linear equations in two variables. Then, a pair of values of variables satisfying each one of the given equations is called a solution of the system.

Example: $\left.\begin{array}{l} 8x + 5y = 9 \\ 3x + 2y = 4 \end{array}\right\}$ A system of simultaneous linear equations in variable x and y.

Algebraic Methods of solving Simultaneous Linear Equations in two Variables

The most commonly used algebraic methods of solving simultaneous linear equations in two variables are:

(i) Method of substitution

(ii) Method of elimination by equating the coefficients

(iii) Method of cross-multiplication

Example 1 Solve (1) $8x + 5y = 9$ (2) $3x + 2y = 4$

Solution: Let us eliminate 'x' from the given equations. The coefficients of x in the given equations are 8 and 3. The L.C.M. of 8 and 3 is 24. So we have to make the coefficients of x in both equations as equal to 24.

Multiplying equation (1) by 3 and equation (2) by 8, we get,

$24x + 15y = 27$ (iii)

$24x + 16y = 32$ (iv)

Subtracting (iv) from (iii), we get

$15y - 16y = 27 - 32 \Rightarrow y = 5$

Putting $y = 5$ in equation (1), we get $8x + 25 = 9 \Rightarrow x = -2$

Hence the solution is $x = -2$, $y = 5$

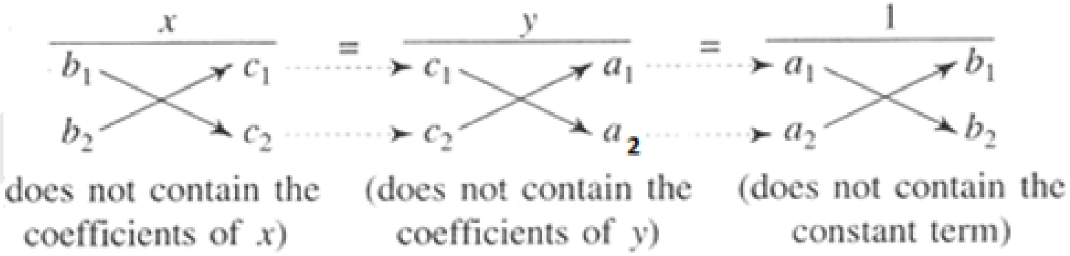
Method of Cross-Multiplication

Consider the system of simultaneous linear equations, in two variables x and y

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Now, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, i.e. $a_1 b_2 - a_2 b_1 \neq 0$, then the above system of equations has a unique solution given by the method of cross-multiplication,



Note: The arrows between two numbers indicate that they are to be multiplied. The numbers with downward arrow are multiplied first. From their product, the product of numbers with upward arrow is to be subtracted.

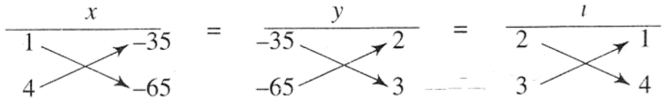
$$\frac{x}{b_1c_2-b_2c_1} = \frac{y}{c_1a_2-c_2a_1} = \frac{1}{a_1b_2-a_2b_1}$$

$$\left.\begin{array}{l} x = \frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1} \\ y = \frac{c_1a_2-c_2a_1}{a_1b_2-a_2b_1} \end{array}\right\}$$ are the required solutions of the given system of simultaneous equation

Example 2: Solve $2x + y - 35 = 0$

$3x + 4y - 65 = 0$

Solution: By using the method of cross-multiplication,



$$\frac{x}{(1\times-65)-(4\times-35)} = \frac{y}{(-35\times3)-(-65\times2)} = \frac{1}{(2\times4-3\times1)}$$

$$\frac{x}{-65+140} = \frac{y}{-105+130} = \frac{1}{8-3}$$

$$\Rightarrow \frac{x}{75} = \frac{y}{25} = \frac{1}{5} \Rightarrow x = \frac{75}{5} = 15 \text{ and } y = \frac{25}{5} = 5$$

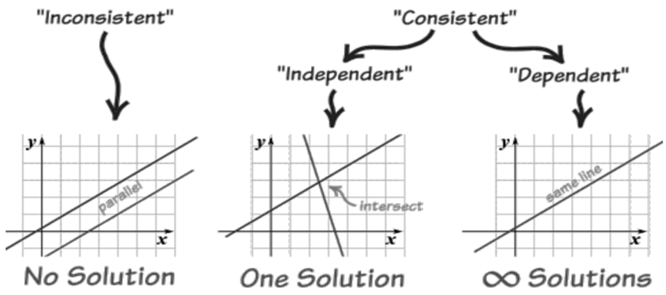
Hence the solution is $x = 15$, $y = 5$

Systems of Linear Equations

A System of Equations means when we have **two or more equations working together**. When the **number of equations** is the **same** as the number of **variables** there is likely to be a **solution**. Not guaranteed, but likely.

Solution of the linear equation

Consistent	Inconsistent
When there is one or infinitely many solutions.	When there is no solution



Consider the system of simultaneous linear equations, in two variables x and y

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Three possible cases are:

One or Unique solution	No solution	Infinitely many solutions
The lines intersect at one point. In this case, the system has a **unique solution**. $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	The lines are parallel. In this case, the system has **no solution**. i.e. $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	The lines are identical. In this case, every point on the line is a solution, and so the system has **infinitely many solutions.** i.e. $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

QUADRATIC EQUATIONS

Introduction

Quadratic functions are more than algebraic curiosities—they are widely used in science, business, and engineering. We commonly use quadratic equations in situations where **two things are multiplied together** and they both depend on the same variable. For example, when working with area, if both dimensions are written in terms of the same variable, we use a quadratic equation. Quadratic equation is a **polynomial equation of degree 2**. So, we always get **two solutions**. There are various methods of solving quadratic equations. In this chapter, we will deal with the word problems and forming quadratic equations.

Definition

Quadratic equations: A **second degree polynomial in** x **equated to zero** will be a quadratic equation. For co-efficient of x^2 should not be zero.
The most **general form** of a Quadratic equation is $ax^2 + bx + c = 0$, **where** $a \neq 0$.
The values of x that satisfy the equation are called the **ROOTS/ SOLUTION/ZEROS** of the equation. **These roots may be real or imaginary.**

The roots of a quadratic equation can be found out in three ways.

A. By factorizing the expression on the left hand side of the quadratic equation.
B. By completing the square.
C. By using the standard formula.

A. *Finding the roots by factorization:* You will understand these steps with the help of the equation $x^2 - 5x + 6 = 0$. The given equation can be written as $x^2 - 3x - 2x + 6 = 0$. The equation can be rewritten as $x(x - 3) - 2(x - 3) = 0$. If we take out $(x - 3)$ as the common factor, we can rewrite the given equation as $(x - 3)(x - 2) = 0$. **The roots of the equation are 3 and 2.**

B. *Finding the roots by completing the square:* Let's start with $x^2 + bx$ and notice that the x^2 has a coefficient of one. That is required in order to use this method. Now, to this add and subtract $(\frac{b}{2})^2$, Doing this gives the following factorable quadratic equation.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2 - \frac{b^2}{4}$$

This process is called **completing the square**

C. *Finding the roots by using the formula:*

Use this formula: $[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $D = b^2 - 4ac$ is called discriminant]

- **Sum and Product of roots of a Quadratic Equation ax^2+bx+c :**
  Sum of the roots are $= -b/a$
  Product of the roots are $= c/a$
- **Nature of the Roots:**

$b^2 - 4ac \geq 0$	Real roots
When $b^2 - 4ac = 0$	The roots are rational and equal
When $b^2 - 4ac > 0$ and a perfect square	The roots are rational and unequal
When $b^2 - 4ac > 0$ but not a perfect square	The roots are irrational (and unequal)

- **Signs of the Roots:**

Sign of product of the roots	**Sign of sum of the roots**	**Sign of the roots**
+ ve	+ ve	Both the roots are positive
+ ve	- ve	Both the roots are negative
- ve	+ ve	One roots is positive and the other negative; the numerically larger root is positive.
- ve	- ve	One root is positive and the other negative; the numerically larger root is negative.

- **Constructing a Quadratic Equation:** If the roots of the quadratic equation are given as α and β, the equation can be written as

$(x - \alpha)(x - \beta) = 0$ i.e., $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

Or

$x^2 - Sx + P = 0$ (S= sum of roots, P= product of roots)

- **Maximum or Minimum value of the Quadratic Expression:** As x varies from $-\infty$ to $+\infty$, the quadratic expression $ax^2 + bx + c$:
  (i) Has a **minimum value** whenever $a > 0$. The minimum value of the quadratic expression is $(4ac - b^2)/4a$ and it occurs at $x = -b/2a$.
  (ii) Has a **maximum value** whenever $a < 0$. The maximum value of the quadratic expression is $(4ac - b^2)/4a$ and it occurs at $x = -b/2a$

EXPERIENCE THE PRATHAM EDGE - 9

1. Solve: $9x + 11y = 53$; $11x + 9y = 47$
(a) $x = 2, y = 3$ (b) $x = 3, y = 2$ (c) $x = 4, y = 1$ (d) $x = 1, y = 4$

2. Three years ago, Anurag's age was thrice that of Bharagav. Two years hence. Anurag's age will be twice that of Bhargav. What is the present age of Anurag (in years)?
(a) 16 (b) 18 (c) 20 (d) 22

3. If a number is divided into two unequal parts, then the difference of the squares of the two unequal parts is 48 times the difference of the two unequal parts. What is the number?
(a) 96 (b) 72 (c) 120 (d) 48

4. Three sharpeners and four erasers cost Rs. 25. Four sharpeners and three erasers cost Rs. 24. What are the respective costs of each sharpener and each eraser?
(a) Rs. 4, Rs. 3 (b) Rs. 3, Rs. 4 (c) Rs. 3, Rs. 3 (d) Rs. 4, Rs. 4

5. One samosa and two puffs cost Rs. 14. Three samosas and one puff cost Rs. 17. What is the cost of 5 samosas and 5 puffs (in Rs.)?
(a) 40 (b) 45 (c) 50 (d) 55

6. How many values of x and y satisfy the equations $2x + 6y = 12$ and $3x + 9y = 18$?
(a) 0 (b) 1 (c) ∞ (d) 2

7. A two-digit number is such that the sum of its digits is five times the difference of its digits. If the number exceeds the number formed by interchanging the digits by 18, then find the number.
(a) 96 (b) 64 (c) 32 (d) 42

8. Five years ago, a man was five times as old as his son. Two years hence, the man will be three times as old as his son. What is the present age of the man (in years)?
(a) 50 years (b) 35 years (c) 42 years (d) 40 years

9. Venkat takes 2 hours more than Vatsa to cover a distance of 600 km. If instead Venkat doubles his speed he would reach the destination 4 hours before Vatsa. Find Vatsa's speed.
(a) 100km/hr (b) 50km/hr (c) 60km/hr (d)120km/hr

10. The sum of the ages of two friends Avish and Lakhan 14 years ago was one-third of the sum of their ages today. If the ratio of the present ages of Avish and Lakhan is 4 : 3, then what is the present age of Lakhan?
(a) 32 years (b) 18 years (c) 24 years (d) 21 years

11. Seven burgers and eight pizzas together, cost Rs. 780 while twelve burgers and five pizzas cost Rs. 945. Find the cost of each pizza.
(a) Rs. 60 (b) Rs. 40 (c) Rs. 45 (d) Rs. 50

12. Varun's present age is thrice that of Tarun's age three years ago. Nine years hence, Varun would be thrice as old as Taran today. Find the sum of their present ages.
(a) 26 years (b) 7 years (c) 11 years (d) Cannot be determined

13. The ratio of number of chocolates with Seoni and Varsha is 7 : 9. If Varsha has 14 chocolates more than Seoni, then find the total number of chocolates with them.
(a) 48 (b) 80 (c) 96 (d) 112

14. Durgesh has only Rs. 2 and Rs. 5 coins with him. If he has in all 57 coins worth Rs. 150 with him. How many Rs.2 coins does he have?
(a) 12 (b) 20 (c) 40 (d) 45

15. Find the smaller of the two numbers such that their sum is 250 and the difference of their squares is 9000.
(a) 109 (b) 141 (c) 107 (d) 100

16. Find the value of k if the equations $3x + 4y = 24$, and $15x + 20y = 8k$ are consistent.
(a) 5 (b) 120 (c) 30 (d) 15

17. A fraction becomes 2/3 if its numerator is increased by 1 and the denominator by 2. It becomes 3/4 if its numerator is increased by 4 and the denominator by 5. Find the fraction.
(a) 4/5 (b) 3/5 (c) 5/7 (d) 2/7

18. The cost of three pens, five pencils and two books is Rs. 68. The cost of six pens, seven pencils and four books is Rs. 121. The cost of nine pens, fifteen pencils and six books is Rs. 204. Find the cost of each book.
(a) Rs. 7 (b) Rs. 11 (c) Rs. 5 (d) Cannot be determined

19. Manish was asked to find 5/6 times a number and he instead multiplied it by 6/5. As a result, he got an answer which was more than the correct answer by 649. What was the number?
(a) 1870 (b) 1770 (c) 1860 (d) 1760

20. In an examination 3/5 th of the students that appeared failed by 10 marks and l/5th of the students got 10 marks above the pass mark. Each of the remaining students got 20 marks above the pass mark. Their average mark is 62 marks. The pass mark is
(a) 64 (b) 66 (c) 62 (d) Cannot be determined

21. Rohan has a certain amount of money and Sohan has a greater amount. If Sohan gives Rs. 60 to Rohan, then the amounts with Rohan and Sohan are the same. If Rohan gives Rs. 30 to Sohan, the amount with Sohan is thrice the amount with Rohan. What is the total amount with Rohan and Sohan together?
(a) Rs. 540 (b) Rs. 480 (c) Rs. 420 (d) Rs. 360

22. Dheeraj has twice as many sisters as he has brothers. If Deepa, Dheeraj's sister has the same number of brothers as she has sisters, then Deepa has how many brothers?
(a) 2 (b) 3 (c) 4 (d) 6

23. A man had enough money to purchase 16 apples or 10 mangoes. If the man buys 4 apples, and five mangoes and is left with Rs. 20, which is one-fourth of the money, he initially had, then what is the difference in the prices of an apple and a mango?
(a) Rs. 2 (b) Rs. 3 (c) Rs. 4 (d) Rs. 6

24. Nathasha was asked to calculate 4/7 of the number. She calculated 4/17 of the number and got an answer, which was 840 less than the correct answer. What is the number to be multiplied?
(a) 2600 (b) 2599 (c) 2499 (d) 2737

25. P, Q and R are successive even positive integers in the ascending order. Four times R is 4 more than five times P. What is the value of Q?
(a) 12 (b) 14 (c) 16 (d) 10

Directions for Questions 26 to 28: Find the roots of the following equations.

26. $15x^2 - 7x - 36 = 0$
(a) $\frac{5}{9}, -\frac{4}{3}$ (b) $\frac{9}{5}, -\frac{4}{3}$ (c) $\frac{9}{5}, -\frac{3}{4}$ (d) none of these

27. $6x^2 + 40 = 31x$
(a) $\frac{3}{8}, \frac{2}{5}$ (b) $\frac{3}{8}, \frac{3}{2}$ (c) $0, \frac{8}{3}$ (d) $\frac{8}{3}, \frac{5}{2}$

28. $2\left(x^2 + \frac{1}{x^2}\right) - 3\left(x + \frac{1}{x}\right) - 1 = 0$
(a) $2, \frac{1}{2}$ (b) -2, 4 (c) $2, \frac{3}{2}$ (d) none of these

29. Determine k such that the quadratic equation $x^2 + 7(3 + 2k) - 2x(1 + 3k) = 0$ has equal roots:
(a) 2, 7 (b) 7, 5 (c) $2, \frac{-10}{9}$ (d) none of these

30. Discriminant of the equation $-3x^2 + 2x - 8 = 0$ is :
(a) -92 (b) -29 (c) 39 (d) 49

31. The nature of the roots of the equation $x^2 - 5x + 7 = 0$ is :
(a) no real roots (b) 1 real root (c) can't be determined (d) none of these

32. The roots of $a^2x^2 + abx = b^2$, $a \neq 0$ are :
(a) equal (b) non-real (c) unequal (d) none of these

33. The equation $x^2 - px + q = 0$, $p, q \in R$ has no real roots if:
(a) $p^2 > 4q$ (b) $p^2 < 4q$ (c) $p^2 = 4q$ (d) none of these

34. Determine the value of k for which the quadratic equation $4x^2 - 3kx + 1 = 0$ has equal roots :
(a) $\pm\left(\frac{2}{3}\right)$ (b) $\pm\left(\frac{4}{3}\right)$ (c) ± 4 (d) ± 6

35. Find the value of k such that the equation $x^2 - (k + 6)x + 2(2k - 1) = 0$ has sum of the roots equal to half of their product:
(a) 3 (b) 4 (c) 7 (d) 10

36. Find the value of k so that the sum of the roots of the quadratic equation is equal to the product of the roots : $(k + 1)x^2 + 2kx + 4 = 0$
(a) -2 (b) -4 (c) 6 (d) 8

37. If -4 is a root of the quadratic equation $x^2 - px - 4 = 0$ and the quadratic equation $x^2 - px + k = 0$ has equal roots, find the value of k.
(a) 9/4 (b) 1 (c) 2.5 (d) 3

38. Find the value of p for which the quadratic equation $x^2 + p(4x + p - 1) + 2 = 0$ has equal roots :
(a) $-1, \frac{2}{3},$ (b) 3, 5 (c) $1, -\frac{4}{3}$ (d) $\frac{3}{4},$

39. If α and β are the roots of the equation $3x^2 + 7x + 3 = 0$ Find the value of $\alpha\beta$:
(a) 1 (b) 0 (c) 2 (d) 3

40. If α and β are the roots of the equation $x^2 - 3x + 2 = 0$. Find the quadratic equation whose roots are $-\alpha$ and $-\beta$:
(a) $x^2 - 3x + 2 = 0$ (b) $x^2 + 3x + 2 = 0$ (c) $x^2 + 3x - 2 = 0$ (d) none of these

41. Find the quadratic equation whose roots are $\sqrt{3}$ and $2\sqrt{3}$:
(a) $x^2 + 3\sqrt{3}x - 6 = 0$ (b) $x^2 - 3\sqrt{3}x + 6 = 0$ (c) $x^2 + 3\sqrt{3}x + 5 = 0$ (d) none of these

42. If α and β are the roots of equation $6x^2 + x - 2 = 0$, find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$
(a) $-\frac{25}{12}$ (b) $-\frac{12}{75}$ (c) $-\frac{4}{9}$ (d) none of these

43. If a and c are such that the quadratic equation $ax^2 - 5x + c = 0$ has 10 as the sum of the roots and also as the product of the roots, find a and c respectively.
(a) $\frac{1}{2}, 6$ (b) $\frac{1}{2}, 5$ (c) $\frac{1}{2}, 8$ (d) none of these

44. If p and q ($\neq 0$) are the roots of the equation $x^2 + px + q = 0$, then the least value of $x^2 + px + q$ ($x \in R$) is:
(a) $-\frac{1}{4}$ (b) $\frac{1}{4}$ (c) $\frac{-9}{4}$ (d) $\frac{9}{4}$

45. Equation $ax^2 + 2x + 1$ has one double root if:
(a) a = 0 (b) a = -1 (c) a = 1 (d) a = 2

46. The value of $\sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \cdots \infty}}}}$ is:
(a) 2 (b) 3 (c) 4 (d) 5

47. The value of p for which the difference between the roots of the equation $x^2 + px + 8 = 0$ is 2 are:
(a) ± 2 (b) ± 4 (c) ± 6 (d) ± 8

48. If $x = 7 + 4\sqrt{3}$ and $xy = 1$, then the value of $\frac{1}{x^2} + \frac{1}{y^2}$ is:
(a) 64 (b) 128 (c) 184 (d) 194

CHAPTER 10

TIME, SPEED & DISTANCE, RACES & GAMES

INTRODUCTION

Your ability to solve the problems on relative motion, circular motion, and problems based on trains, boats, clock, races etc. will depend only on the depth of your understanding

1. Speed = Distance/Time
2. Distance = Speed × Time
3. Time = Distance/ Speed
4. x km/hr = (x×5/18) m/sec
5. y m/sec = (y× 18/5) km/hr
6. If a certain distance is covered at x km/hr and the same distance is covered at y km/hr, then the average speed during the whole journey = $\frac{2xy}{(x+y)}$ km/hr
7. If a person changes his speed in the ratio m: n. then the ratio of time taken becomes n : m
8. For the same distance speed varies inversely as time and in the same time distance varies directly as speed
9. Average Speed = $\frac{\text{Total distance traveled}}{\text{Total time taken}}$

   Please note that the average speed of a moving body is NOT EQUAL to the average of the speed

10. If a body covered one part of the journey at speed p and the remaining part of the journey at speed q and the distances of the two parts of the journey are in the ratio m : n, then the average speed for the entire journey is $\frac{(m+n)pq}{mq+np}$

Conversions

- *To convert cm to metre divide by 100.*
- *To convert km to metre multiply by 1000.*
- *To convert hours to min. multiply by 60.*
- *To convert hours to sec multiply by 3600.*
- *To convert minutes to hours divide by 60.*
- *To convert seconds to hours divide by 3600.*

Important Results

- If a body covers a distance d_1 with speed V_1 in time t_1 and then it covers a distance d_2 with speed V_2 in time t_2 then, distance covered; $D = d_1 + d_2$, time taken; $T = t_1 + t_2$, average speed = $\frac{D}{T}$

- If a body moves a distance d_1 with speed V_1 in time t_1, then it moves a distance d_2 with speed V_2 in time t_2 and then a distance d_3 with speed V_3 in time t_3, then, distance covered; $D = d_1 + d_2 + d_3$, time taken; $T = t_1 + t_2 + t_3$, average speed = $\frac{D}{T}$

- P and Q are two points on a straight road. Mr. A starts from P and travels to Q in time t_1. Also Mr. B starts from Q and travels to P along the same straight road in time t_2.

$$\text{Now, } \frac{Speed\ of\ A}{Speed\ of\ B} = \frac{Time\ taken\ \text{by B}}{Time\ taken\ \text{by A}}$$

SOLVED EXAMPLES

Example 1 Time taken at 48 km/h is 24 minutes. What is the time if speed decreases by 25%?

Solution: Speed ratio = 4: 3
Time ratio = 3: 4 = 24: 32
Time ratio = 32 minutes

Example 2 Walking at 48% less speed, time taken is 75 minutes. Find the time taken at usual speed.

Solution: Speed ratio = 100: 52 → 25:13
Time ratio = 13: 25 (speed and time vary inversely)
= 39: 75
Time = 39 minutes

Example 3 Speed from A to B = 10 km/h and from B to A is 40 km/h. What is the average speed?

Solution: Average Speed = $2 \times 10 \times \frac{40}{50} \Rightarrow 16$ km/hr.
(If the distance is same then average speed is the harmonic mean)

Example 4 Express 17.5 m/s into km/h

Solution: $17.5 \times 18/5$ km/h = 63 km/h (1 m/s = 18/5 km/h)

Example 5 Mohan goes to school at 10 km/h and reaches the school 5 minutes late. Next day he goes to school at 12 km/h and reaches the school 10 minutes earlier than the scheduled time. Find the distance of his school from the house.

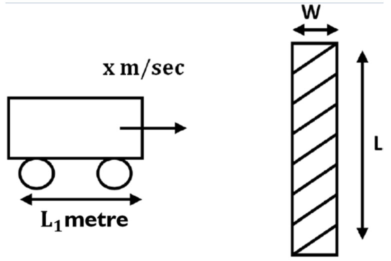
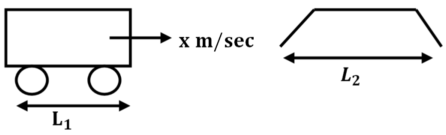
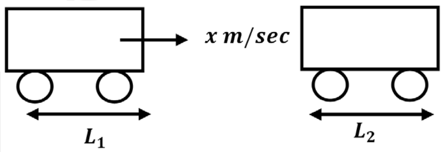
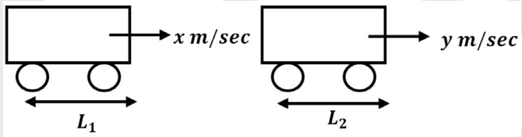
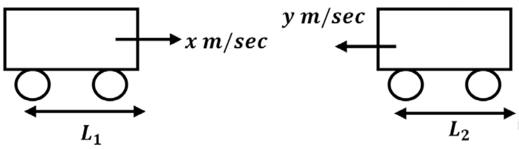
Solution: Let x be the distance.
Time= 5 minutes + 10 minutes ⇒15 minutes
⇨1/4 hours.
Therefore, $x/10 - x/12 = ¼$
Distance = x = 15 km

RELATIVE SPEED

- The speed of one (moving) body in relation to another moving body is called the relative speed of these two bodies , i.e., it is the speed of one moving body as observed, from the second moving body.
- If two bodies are moving in the same direction, the relative speed is equal to the difference of the speed of the two bodies.
- If two bodies are moving in opposite direction, the relative speed is equal to the sum of the speeds of the two bodies.

TRAINS

When we are dealing with trains, the length of the train will be distance covered when the train is passing a stationary point, if the train is crossing a platform (or a bridge), the distance covered by the train is equal to the length of the train plus the length of the platform (or bridge). If two trains pass each other (traveling in the same direction or in opposite directions), the total distance covered is equal to the sum of the lengths of the two trains.

<table>
<tr><td>Case-1: Train crossing a pole
$T=\frac{L_1}{x}$ sec</td><td></td></tr>
<tr><td>Case 2: Train crossing a bridge/platform
$T=\frac{L_1+L_2}{x}$ sec</td><td></td></tr>
<tr><td>Case 3: Train crossing another train
(a) Train crossing stationary train
$T=\frac{L_1+L_2}{x}$ sec</td><td></td></tr>
<tr><td>(b) Train crossing another moving train.
(i) In the same direction
$T=\frac{L_1+L_2}{x-y}$ sec</td><td></td></tr>
<tr><td>(ii) In opposite direction
$T=\frac{L_1+L_2}{x+y}$ sec</td><td></td></tr>
</table>

SOLVED EXAMPLES

Example 1 Two trains of length 216 m and 264 m are running towards each other on parallel lines. One at 64 km/h, other at 80 km/h. What is the time to clear each other?

Solution: Total Distance to cover = 216 + 264 $\Rightarrow$ 480 m

Relative Speed = 64 + 80 $\Rightarrow$ 144 km/hr

$$\Rightarrow 144\times\frac{5}{18} \Rightarrow 40 \text{ m/sec}$$

Time taken to clear each other = $\frac{\text{Total Distance}}{\text{Total Speed}}$ = 480/40 $\Rightarrow$ 12 sec.

Example 2 A 240m long train travelling at 36 km/h clears a 600 m long platform in how much time.

Solution: Speed of Train = 36 × 5/18 m/s = 10m/sec.

Time = Distance/Speed $\Rightarrow$ 840m/10m/s = 84sec.

Example 3 A train running at 25 km/h takes 18 seconds to pass a platform. Next it takes 13 ½ seconds to pass a man walking at 5 km/h in the same direction. Find the length of the train.

Solution: Relative Speed = 25 – 5 ⇒ 20 km/hr

$= 20 \times \frac{5}{18} \quad = \frac{100}{18}$ m/sec

Length of the train = Speed of the train × Time

$= \frac{100}{18} \times \frac{27}{2} = 75$ m

Example 4 A train at 148 km/h overtakes a motorcyclist at 58 km/h in the same direction in 10 sec. What is the length of the train?

Solution: Distance = Speed × time

= (148-58) × 5/18 × 10 m ⇒ 250m

Example 5 Express a speed of 54 km/hr in meters/second

Solution: $54 \times \frac{5}{18} = 15$m/sec.

Example 6 A car can cover 350 km in 4 hours. If the speed is decreased by 12½ kmph, how much time does the car take to cover a distance of 450 km?

Solution: Speed $= \frac{\text{Distance}}{\text{Speed}}$ = 350/4 = 87.5 kmph

Now this is reduced by 12.5 kmph. Hence, speed is 75 kmph.

At this speed time taken = 450/75 = 6 hours.

Example 7 A person covers a certain distance at a certain speed. If he increases his speed by 25% then he takes 12 minutes less to cover the same distance. Find the time taken by him initially to cover the distance at the original speed.

Solution: When the speed increases by 25% the new speed is 125% of the original speed; it is $\frac{5}{4}$ times original speed, Since speed and time are inversely related, if speed is $\frac{5}{4}$ times the original speed, then the time will be $\frac{4}{5}$ times the original times.

This means that the new time is $1 - \frac{4}{5} = \frac{1}{5}$ part less than the original time.

But we know the new times is less by 12 minutes, so, the original time

= 5 x 12 = 60 minutes = 1hour.

Example 8 A car covers a certain distance going at a speed of 80 kmph and return to the starting point at a speed of 40 kmph. Find the average speed for the whole journey

Solution: We know the average speed is 2pq/p+q where p and q are the speeds in the two directions, for equal distances.

Average speed $= \frac{2pq}{p+q} = \frac{2 \times 40 \times 80}{40+80} = 53.3$ kmph

Example 9 What is the time taken by a train running at 72 km/hr to cross a man standing on a platform, the length of the train being 180 m?

Solution: Speed of train = 72 kmph = $72 \times \frac{5}{18}$ = 20 m/s

Distance = length of the train = 180 m

∴ Time $= \frac{180}{20}$ = 9 sec

Example 10 How long will a train 100m long and traveling at a speed of 45 kmph, take to cross a platform of length 250 m?

Solution: Distance = length of the train + length of the platform = 100 + 250 = 350 m

Speed of the train = 45 kmph

$= 45 \times \frac{5}{18} = 12.5$ m/sec. Therefore, Time $= \frac{350}{12.5} = 28$ sec.

Example 11 Find the length of bridge, which a train 120 m long traveling at 54 kmph can cross in 30 seconds.

Solution: Speed of the train = 54 kmph $= 54 \times \frac{5}{18} = 15$ m/sec

Distance covered in 30 seconds = 15 × 30 = 450 m

Length of bridge = Distance covered – Length of the train = 450 – 120 = 330 m.

Example 13 A worker reaches his work place 15 minutes late by walking at 4 km/hr from his house. The next day he increases his speed by 2 kmph and reaches on time. Find the distance from his house to his workplace.

Solution: Let the distance be x.

Then, time taken on the 1st day $= \frac{x}{4}$

Time taken 2nd day $= \frac{x}{6}$

We are given

$\frac{x}{4} - \frac{x}{6} = \frac{15}{60} \Rightarrow x = 15 \times \frac{12}{60} = 3$ km

In general, if a person traveling between two points reaches p hours late traveling at a speed of u kmph and reaches q hours early traveling at v kmph, the distance between the two points is given by $\frac{vu}{v-u} \times (p+q)$

Example 14 A person leaves his house and traveling at 4 kmph reaches his office 10 minutes late. Had he traveled at 7 kmph he would have been 20 minutes early. Find the distance from his house to the office.

Solution: As per the rule above,

$$\frac{4 \times 7}{7-4} \times \frac{20+10}{60} = \frac{28}{3} \times \frac{30}{60} = \frac{14}{3} km$$

Example 15 Find the time taken by a train 50 m long running at a speed of 63 kmph to cross another train of length 100 m running at a speed of 45 kmph in the same direction.

Solution: Total distance covered = sum of length of the two trains = 100 + 50 = 150 m

Relative speed of the two trains = 63 – 45 = 18 kmph

(since the trains are running in the same direction the relative speed will be the difference in the speeds) $18 \times \frac{5}{18} = 5$ m/s

Time = 150/5 = 30 seconds

Example 16 A train crosses two persons, cycling in the same direction as the train in 12 and 18 seconds respectively. If the speeds of the two cyclists are 9 and 18 kmph respectively, find the length and the speed of the train.

Solution: Relative speed of over taking first cyclist

= (s – 9) kmph (s kmph being speed of train.)

Time took to over take the first cyclist = 12 seconds

Hence length of train = $12 \times (s - 9) \times \frac{5}{18}$ ------------------------------ (i)

Similarly, considering the case of overtaking the second cyclist, length of train

$= 18\times(s - 18)\times\frac{5}{18}$ ------------------------------ (ii)

Equating (i) and (ii), $12\times(s - 9)\times\frac{5}{18} = 18\times(s - 18)\times\frac{5}{18}$

$\Rightarrow 2s - 18 = 3s - 54 \Rightarrow s = 36$ kmph

Length $= 12\times(s - 9)\times\frac{5}{18} = 12\times27\times\frac{5}{18} = 90$ meters.

Example 17 Two trains running at 45 kmph and 54 kmph cross each other in 12 seconds. When they run in the same direction, a person in the faster train observes that he overtake the other train in 32 seconds. Find the lengths of the two trains.

Solution: Let p, q be the lengths of the slow and faster trains respectively. When trains are traveling in the opposite direction, relative speed

= 45 + 54 = 99 kmph = 27.5 m/s

Distance covered = sum of length of 2 trains = p + q

Then we have p + q = 12 x 27.5 $\Rightarrow$ p + q = 330m -------------------(i)

When trains are traveling in the same direction, since we are given the time noted by a person in the faster train as 32 seconds the distance covered is equal to the length of the slower train, distance covered = p

Relative speed = 54 – 45 = 9 = 2.5 m/sec

p = 2.5 x 32 = 80 m ----------------(ii)

From (i) and (ii) we get p = 80 m and q = 250 m

Example 18 Two trains of length 150 m and 250 m run on parallel lines. When they run in the same direction it will take 20 seconds to cross each other and when they run in opposite direction it will take 5 seconds. Find the speeds of the two trains.

Solution: Let the speeds of the two trains be p & q m/sec.

Total distance covered = sum of length of two trains = 150 + 250 = 400 m

When they run in the same direction, relative speed (p – q) is given by,

$p - q = \frac{400}{20} = 20$ ---------------(i)

When they are running to opposite directions, relative speed p + q is given by.

$p + q = \frac{400}{5} = 80$ -------------- (ii)

Solving (i) and (ii), we get

p = 50 m/s and q = 30 m/s

$\therefore$ Speeds of two trains are 180 kmph and 108 kmph.

BOATS AND STREAMS

Problems related to boats and streams are different in the computation of relative speed from those of train/cars.

- When a boat is moving in the same direction as the stream or water current, the boat is said to be moving **WITH THE STREAM OR CURRENT.**
- When a boat is moving in a direction opposite to that of the stream or water current, it is said to be moving **AGAINST THE STREAM OR CURRENT.**
- If the boat is moving with a certain speed in water that is not moving, the speed of the boat is then be called

the **SPEED OF THE BOAT IN STILL WATER.**

- When the boat is moving upstream, the speed of the water opposes (and hence reduces) the speed of the boat.
- When the boat is moving downstream, the speed of the water add (and thus adds to) the speed of the boat. Thus have

Speed of the boat against stream	*= Speed of the boat in still water – Speed of the stream*
Speed of the boat with the stream	*= Speed of the boat in still water + Speed of the stream*

These two speeds, the speed of the boat against the stream and the speed of the boat with the stream, are **RELATIVE SPEED**

If u is the speed of the boat down the stream and v is the speed of the boat up the stream, then we have the following two relationships,

Speed of the boat in still water $= \frac{u+v}{2}$

Speed of the water current $= \frac{u-v}{2}$

NOTE : In problems, instead of a boat, it may be a swimmer but the approach is exactly the same, instead of boats/swimmers in water. It could also be a cyclist cycling against or along the wind. The approach to solve problems still remains the same.

Example 19 A boat travels 30km upstream in 6 hours and 20 km downstream in 5 hours. Find the speed of the boat in still water and the speed of water current.

Solution: Upstream speed = 30/6= 5 kmph
Downstream speed = 20/5 = 4 kmph
Speed in still water = (5 + 4)/2 = 4.5 kmph
Speed of the water current = (5-4)/2= 0.5 kmph

Example 20 A man can row 8 km in one hour in still water. If the speed of the water current is 2 km/hr and it takes 3 hours for him to go to a new place and return, find distance from the starting point to the new place.

Solution: Let the distance be x km
Upstream speed = 8 – 2 = 6 kmph
Downstream speed = 8 + 2 = 10 kmph

Total time = time taken upstream + time taken down stream = $\frac{x}{6}+\frac{x}{10}$ kmph

$\frac{x}{6}+\frac{x}{10}=\frac{16x}{60}=3 \Rightarrow x=11.25$ km

Example 21 A man takes twice as long to row a distance against the stream as to row the same distance in favour of the stream. The ratio of the speed of the boat (in still water) and the stream is:
(a) 2 : 1 (b) 3 : 1 (c) 3 : 2 (d) 4 : 3

Solution: (b): Let man's rate upstream be x kmph.
Then, his rate downstream = 2x kmph.

∴ (Speed in still water): (Speed of stream) = $\left(\frac{2x+x}{2}\right):\left(\frac{2x-x}{2}\right)=\frac{3x}{2}:\frac{x}{2}=3:1$

Example 22 A boat can travel 1½ times the distance down the stream than up the stream in the same time. If the speed of the current is 3 kmph, find the speed of the boat in still water.

Solution: If the distance covered down the stream is $1\frac{1}{2}$ times that covered up the stream, the speed down the stream will also be $1\frac{1}{2}$ times the speed up the stream.
Let the speeds of the boat in still water be u.
We get $(u + 3) / (u - 3) = 3/2 \Rightarrow u = 15$ kmph

Example 23 A man can row $2/7^{th}$ of a kilometer upstream in 25 minutes and return in 10 minutes. Find the speed of the man in still water.

Solution: Upstream speed $= \dfrac{\frac{2}{7}}{\frac{25}{60}} = \dfrac{24}{35}$ kmph

Downstream speed $= \dfrac{\frac{2}{7}}{\frac{10}{60}} = \dfrac{12}{7}$ kmph

Speed in still water $= \dfrac{\frac{24}{35} + \frac{12}{7}}{2} = \dfrac{84}{35} \times \dfrac{1}{2} = 1.2$ km/hr

Example 24 A man sails 12 km/h in still water. It takes him twice as long to row up as to row downstream. Find the rate of stream?

Solution: Let × be the speed of stream
Thus, $(12 + x) = 2(12 - x)$
Therefore Speed of the Stream $= x = 4$ km/h.

RACES AND CIRCULAR TRACKS

Introduction
Problems on **races** depend on the fundamental thing that the **speeds of runners/ competitors** remain the **same** throughout the **course of a race**, though practically the same is not possible. The **length** of the course of a race is **normally predetermined** and from the starting point to the winning point, the speed of the individual contes tants remain the same.

Definition
- **Race** - A race is a contest of **speed** in running, riding, driving, sailing, rowing etc over a particular distance.
- **Race Course** - Race course is the **ground** or path on which contests are conducted.
- **Starting Point** - Starting Point is the point from which a **race starts**.
- **Winning Point (or Goal)** - Winning Point (or Goal) is the point where a **race finishes**.
- **Dead-heat Race** - A race is said to be a dead-heat race if all the persons contesting the race reach the **winning point (goal)** exactly at the **same time**.
- **Winner** - Winner is the person who **first** reaches the **winning point**.
- **Games:** A game of 100, means that the person among the contestants who scores 100m first is the winner. If A scores 100 points while B scores only 80 points, then we say that A can give B 20 points.

Some of the general statements and their mathematical interpretations:

Let A and B be two contestants in a race.

Statements	**Mathematical interpretations**
A beats B by t seconds	A finishes the race t seconds before B finishes.

A gives B a start of t seconds	A starts t seconds after B starts from the same starting point.
A gives B a start of x meters	While A starts from the starting point, B starts x meters ahead from the same starting point at the same time. To cover a race of 100 meters in this case, A will have to cover 100 meters while B will have to cover only (100 - x) meters.
Game of 100	A game in which the participant who scores 100 points first wins.
In a game of 100, A can give B 20 points	While A scores 100 points, B scores only 100-20=80 points.

When two or more persons are running around a circular track (starting at the same point and at the same time), then we will be interested in two main issues:

- When they will meet for the first time and
- When they will meet for the first time at the starting point

To solve the problems on circular tracks, you should keep the following points in mind.

When two persons are running around a circular track in OPPOSITE directions

- The relative speed equal to the sum of the speeds of the two individuals and
- From one meeting point to the next meeting point, the two of them TOGETHER cover a relative distance equal to the length of the track.

When two persons are running a circular track in the SAME direction.

- The relative speed is equal to the difference of the speeds of the two individuals and
- From one meeting point to the next meeting point, the faster person covers one COMPLETE ROUND more than the slower person.

We can now calculate the time taken by the persons to meet for the time ever or for the first time at the starting point in various cases

When two people are running around a circular track

Let the two people A and B with respective speeds of a and b (a > b) be running around a circular track (of length L) starting at the same point at the same time. Then,

	When the two persons are running in the SAME direction	When the two persons are running in OPPOSITE directions
Time taken to meet for the FIRST TIME EVER	L/(a-b)	L/(a+b)
TIME taken to meet for the first time at the STARTING POINT	LCM of L/a, L/b	LCM of L/a, L/b

Please note that when we have to find out the time taken by the persons to meet for the first time at the starting point, what we have to do is to find out the time taken by each of them to complete one full round and then take the LCM of these two timings (L/a and L/b are the timings taken by the two of them respectively to complete on full round).

When three people are running around a circular track

Let the three people A, B and C with respective speeds of a, b and c (a > b > c) be running around a circular track (of length L) starting at the same point at the same time. In this case we consider the three persons running in the same direction as the general case.

Example 25 In a race of 1000 m. A beats B by 50 m or 5 seconds.
Find (i) B's speed (ii) A's speed (iii) Time taken by A to complete the race.

Solution: Since A beats B by 50 m, it means by the time A reaches the winning point, B is 50 m away and as A beats B by 5 seconds, it means B takes 5 seconds to reach the winning point. This means B covers 50 m in 5 seconds i.e., B's speed is 50/5 = 10 m/s. Since A wins by 50 m, in the times A covers 1000 m, B covers 950 m at 10 m/s, B can cover 950 m in 950/10.
i.e., 95 seconds i.e., 1 min 35 seconds.
A completes the race in 1 min 35 seconds
A's speed is $\frac{1000}{95} = 10\frac{10}{19}$ m/s

Example 26 Rakesh runs $1\frac{1}{3}$ times as fast as Mukesh. In a race, if Rakesh gives a lead of 60 m to Mukesh. Find the distance from the starting point where both of them will meet.

Solution: Since Rakesh runs $1\frac{1}{3}$ times as fast as Mukesh, in the time Mukesh runs 3 meters, for every 4 meters he runs.
Since he has given a lead of 60m, he will gain this distance by covering $4 \times 60 = 240$m
Hence they will meet at a point 240 m from the starting point.

Example 27 In a 1500 m race, Tarun beats Manoj by 150 m and in the same race Manoj beats Rahul by 75 m. By what distance does Tarun beat Rahul

Solution: Let us write the data given as below

$$\frac{Tarun}{1500} = \frac{Manoj}{1350} \quad \& \quad \frac{Manoj}{1500} = \frac{Rahul}{1425}$$

This will be equal to $1350 \times \frac{1425}{1500} = 1282.5$

But Manoj running 1350 m is the same as Tarun running 1500m. Hence, Tarun beats Rahul by 217.5 m.

Example 28 In a 500 m race, the ratio of speeds of two runners P and Q is 3 : 5. P has a start of 200 m who wins the race and what is the distance between P and Q at the finish of the race?

Solution: Since the ratio of speeds of P and Q is 3 : 5, in the time P runs 300 m, Q runs 500 m. Since P has a start of 200 m, at the time Q start at the starting point, P has already covered 200 m and he has another 300 m to cover. In the time P covers this 300 m. Q can cover 500 m, thus reaching the finish point exactly at the same time as P
Both P and Q reach the finishing point at the same time.

Example 29 In a circular race of 1200 m, A and B start from the same point and at the same time with speeds of 27 km/hr and 45 km/hr. Find when will they meet again for the first time on the track when they are running
(i) In the same direction,
(ii) In the opposite directions.

Solution: Length of the track, L = 1200 m
Speed of A = $27 \times \frac{5}{18}$ = 7.5 m/s

Speed of B = $45 \times \frac{5}{18}$ = 12.5 m/s

(i) <u>Same direction:</u> Time = $\frac{L}{\text{Relative speed}} = \frac{1200}{(12.5-7.5)}$ = 240 seconds

(ii) <u>Opposite direction</u>: Time = $\frac{L}{\text{Relative speed}} = \frac{1200}{(12.5+7.5)}$ = 60 seconds

Example 30 In a circular race of 1200 m length A and B start with speeds of 18 km/hr and 27 km/hr respectively starting at the same time from the same point. When will they meet for the first time at the starting point when running.

(i) In the same direction (ii) In opposite direction

Solution: L = 1200 m.

Speed of A = $18 \times \frac{5}{18} = 5$ m/sec

Speed of B = $27 \times 18 = 7.5$ m/sec

Time taken by A to complete one round = $\frac{1200}{5} = 240$ s

Time taken by B to complete one round = $\frac{1200}{7.5} = 160$ s

(i) <u>Same direction</u>

They will meet at the starting point at a time which is the LCM of the timings taken by each of them to complete one full round. i.e., the LCM of 160 s and 240 s which is 480 s.

(ii) <u>Opposite direction</u>

They will meet at the starting point at a time which is the LCM of the timing taken by each of them to complete one full round, i.e., the LCM of 160 s and 240 s which is 480 s.

(Please note that the time taken by them to meet for the first time at the starting point does not change in the two cases i.e., it does not depend on whether the two persons are running in the same direction or in opposite directions).

Example 31 a, b and c with respective speeds of 9, 18, 27 km/hr, run around a circular track 1200 m long. If they started at the same time from the same point and run in the same direction, when will they meet for the first time?

Solution: L = 1200 m

Speed of a = $9 \times \frac{5}{18} = 2.5$ m/sec

Speed of b = $18 \times \frac{5}{18} = 5$ m/sec

Speed of c = $27 \times \frac{5}{18} = 7.5$ m/sec

They will meet for the first time at a time which is the LCM of $\frac{L}{(a-b)}$ & $\frac{L}{(b-c)}$

$$\frac{L}{(b-a)} = \frac{1200}{(5-2.5)} = 480 \text{ s}$$

$$\frac{L}{(c-b)} = \frac{1200}{(7.5-5)} = 480 \text{ s}$$

$\therefore$ They will meet for the first time after 480 seconds i.e., 8 minutes after they start.

EXPERIENCE THE PRATHAM EDGE - 10

1. Ravi travels in forest for 100 km at 50 kmph, next 50 km at 25 kmph and then 90 km at 45 kmph , his average speed for the whole journey (in kmph) is
   (a) 35 (b) 45 (c) 55 (d) 40

2. Kumar walks at the rate of 5km/h for 6 hours and at 4 km/h for 12 hours. The average speed of the man (in km/hr) is:
   (a) 4.33 (b) 4.66 (c) 4.5 (d) 5

3. Tinu goes to the school with the speed of 3 kmph and returns with the speed 2 kmph. If he takes 5 hour in all, then the distance (in kms) between the village and the school is
   (a) 6 (b) 7 (c) 8 (d) 9

4. Mukesh walks from his house at 5 kmph and reaches his school 10 minutes late. If his speed had been 6 kmph he would have reached 15 minutes early. The distance of his school from this house is
   (a) 2.5 km (b) 12.5 km (c) 5.5km (d) 3.6 km

5. A man can row 18kmph in still water. It takes him thrice as long to row up as to down the river. Find the rate of the stream.
   (a) 6 km/hr (b) 9 km/hr (c) 12 km/hr (d) Data Inadequate

6. Two trains of lengths 120 m and 80 m are running in the same direction with velocities of 40 kmph and 50 kmph respectively. The time taken by them to cross each other, is:
   (a) 60 sec (b) 72 sec (c) 75 sec (d) 80 sec

7. Two trains whose lengths are 180 m and 220 m respectively are running in directions opposite to one another with respective speed of 40 km/hr and 50 km/hr. Time taken by them in crossing one another will be:
   (a) 16 sec (b) 17 sec (c) 18 sec (d) 22 sec

8. Raman standing on a railway platform observes that a train going in one direction taken 4 seconds to pass him. Another train of same length going in the opposite direction takes 5 seconds to pass him. The time taken (in seconds) by the two trains to cross each other will be:
   (a) $\frac{31}{9}$ (b) $\frac{40}{9}$ (c) $\frac{49}{9}$ (d) $\frac{50}{9}$

9. A motor boat takes 2 hours to travel a distance of 9 km down the current and it takes 6 hours to travel the same distance against the current. The speed of the boat in still water and that of the current (in km/hr) respectively are:
   (a) 3, 1.5 (b) 3, 2 (c) 3.5, 2.5 (d) 3, 1

10. Sunil travels first 50 km at 25 kmph, next 40 km at 20 kmph and then 90 kmph. His average speed for the whole journey (in kmph) is:
    (a) 25 (b) 20 (c) 18 (d) Data inadequate

11. A covers 1/4th of his journey at 20 km/h and 1/3rd of the rest at 25 km/h and half of the rest at 30 km/h and rest at the speed of 40 km/h. What is the average speed of A?
    (a) $13\frac{78}{89}$ km/h (b) 12 km/h (c) $26\frac{86}{89}$ km/h (d) 28 km/h

12. Ravi goes uphill with an average speed 24 kmph and comes down with an average speed of 36 kmph. The distance travelled in both the cases being the same, the average speed for the entire journey is:
    (a) 30 kmph (b) 24 kmph (c) 28.8 kmph (d) 32.6 kmph

13. A car covers four successive 3 km stretches at speeds of 10 kmph, 20 kmph, 30 kmph and 60 kmph respectively. Its average speed over this distance is :
(a) 10 kmph (b) 30 kmph (c) 20 kmph (d) 25 kmph

14. A person X starts from Lucknow and another persons Y starts from Kanpur to meet each other. Speed of X is 25 km/h, while speed of Y is 35 km/h. If the distance between Lucknow and Kanpur be 120 km and both X and Y start their journey at the same time, when will they meet?
(a) 1 h later (b) 2 h later (c) 1/2 h later (d) 3 h later

15. Excluding stoppages, the speed of a bus is 54 kmph and including stoppages, it is 45 kmph . For how many minutes does the bus stop per hour ?
(a) 9 (b) 10 (c) 12 (d) 20

16. Two men starting from the same place walk at the rate of 4 kmph and 4.5 kmph respectively. What time will they take to be 8.5 km apart, if they walk in the same direction?
(a) 1 hour (b) 4 hrs 15 min (c) 17 hrs (d) 2 hrs 55 min

17. Two cyclists starts from the same place in opposite directions. One goes towards north at 18 kmph and the other goes towards south at 20 kmph. What time will they take to be 47.5 km apart ?
(a) 2 hrs 15 mins (b) 1hr 15 mins (c) 2 hrs 23 mins (d) 23 hrs 15 mins

18. A man covers a certain distance on scooter. Had he moved 3 kmph faster, he would have taken 40 min. less. If he had moved 2 kmph slower, he would have taken 40 min. more. The distance (in km) is:
(a) 20 (b) 35 (c) 37.5 (d) 40

19. If a train runs 40 kmph, it reaches destination late by 11 minutes but if it runs at 50 kmph it is late by 5 minutes only. The correct time for the train to complete its journey is:
(a) 13 min. (b) 15 min (c) 19 min (d) 21 min.

20. Two buses travel to a place at 45 kmph and 60 kmph respectively. If the second bus takes 5 ½ hours less than the first for the journey, the length of the journey is :
(a) 900 km (b) 945 km (c) 990 km (d) 1350 km

21. Walking $5/6$ of his usual speed, a man is 10 minutes to late. The usual time taken by him to cover that distance is :
(a) 1 hour (b) 50 min (c) 12 min (d) 8 min 20 sec.

22. A car travels a distance of 840 km at a uniform speed. If the speed of the car is 10 kmph more, then it takes 2 hours less to cover the same distance. The original speed of the car (in kmph) was :
(a) 45 (b) 50 (c) 60 (d) 70

23. Distance between two towns P and Q is 240 km. A motor cycle rider starts from P towards Q at 8 p.m. at a speed of 40 kmph. At the same time another motor cycle rider starts from Q towards P at 50 kmph. At what time will they meet?
(a) 9.45 p.m. (b) 10.30 p.m. (c) 10.40 p.m. (d) 11 p.m.

24. A train M leaves Meerut at 5 a.m. and reaches Delhi at 9 a.m. Another train N leaves Delhi at 7 a.m. and reaches Meerut at 10.30 a.m. At what time do the two trains cross one another?
(a) 8.26 a.m. (b) 8 a.m. (c) 7.36 a.m. (d) 7.56 a.m.

25. Two trains start at the same time from Aligarh and Delhi and proceed towards each other at 36 kmph and 42 kmph respectively. When they meet, it is found that one train has traveled 48 km more the other. The distance between the two stations (in km) is :
    (a) 624 (b) 636 (c) 544 (d) 460

26. A man travels 35 km partly at 4 kmph and partly at 5 kmph. If he covers the former distance at 5 kmph and the latter distance at 4 kmph, he could cover 2 km at 4 kmph more in the same time. The time taken to cover the whole distance at the original rate (in hours) is:
    (a) 4.5 (b) 8.1 (c) 7.9 (d) 7.5

27. A is twice as fast as B and B is thrice as fast as C. C covered the journey in 42 minutes then it will be covered by A in :
    (a) 14 min (b) 7 min (c) 28 min (d) 34 min

28. A certain distance is covered at a certain speed. If half of this distance is covered in double the time, the ratio of the two speeds is :
    (a) 4 : 1 (b) 1 : 4 (c) 2 : 1 (d) 1 : 2

29. The ratio between the rates of walking of A and B is 2 : 3 and therefore A takes 10 min. more than the time taken by B to reach the destination. If A had walked at double the speed, he would have covered the distance in :
    (a) 15 min (b) 20 min (c) 25 min (d) 20 min

30. A thief steals a car at 1.30 p.m. and drives it at 40 kmph. The theft is discovered at 2 p.m. and the owner sets off in another car at 50 kmph. He will overtake the thief at :
    (a) 3.30 p.m. (b) 4 p.m. (c) 4.30 p.m. (d) 6 p.m.

31. Rohit has to cover a distance of 6 km in 45 min. If he covers one half of the distance in (2/3) of the time, what should be his speed to cover the remaining distance in the remaining time?
    (a) 12 kmph (b) 16 kmph (c) 3 kmph (d) 8 kmph

32. Two boys start together to walk to a certain distance, one at 3.75 km an hour and another at 3 km an hour. The former arrives half an hour before the latter. The distance (in km) is :
    (a) 9.5 (b) 8 (c) 7.5 (d) 6

33. The ratio between the rates of walking of A and B is 3 : 4. If the time taken by B to cover a certain distance is 48 minutes, the time taken (in minutes) by A to cover that distance is :
    (a) 36 (b) 64 (c) $21\frac{5}{7}$ (d) $27\frac{3}{7}$

34. A man walking at the rate of 3 kmph crosses a square field diagonally in 2 minutes. The area of the field (in areas) is :
    (a) 2500 m^2 (b) 3000 m^2 (c) 5000 m^2 (d) 6000 m^2

35. A man riding a cycle at 12 km/hr can reach a village in $4\frac{1}{2}$ hours. If he is delayed by $1\frac{1}{2}$ hours at the start, then in order to reach his destination in time, he should ride with a speed of :
    (a) 15 km/hr (b) 16 km/hr (c) 18 km/hr (d) 20 km/hr

36. A man takes 5 hours 45 min in walking to a place and riding back. He would gain 2 hours by riding both ways. The time he would take to walk both ways is:

(a) 3 hours 45 min (b) 7 hours 30 min (c) 7 hours 45 min (d) 11 hours 45 min.

37. Renu rides at the rate of 10 km per hour but stops for 10 minutes to take rest at the end of every 15 km. How many hours will she take to cover 100 km ?
(a) 10 (b) 10.6 (c) 12 (d) 11

38. Assume that the distance that a car runs on one litre of petrol varies inversely as the square of the speed at which it is driven. It gives a run of 25 km per litre at a speed of 30 kmph. At what speed should it be driven to get a run of 36 km per litre?
(a) $20\frac{5}{6}$ km/hr (b) 25 km/hr (c) 35 km/hr (d) 43.2 km/hr

39. The radius of a wheel is 1.75 m and it makes 3 revolutions per second. The speed of the wheel (in km/hr) is :
(a) 121 (b) 118.8 (c) 163.5 (d) 165.8

40. A train 280 m long, running with a speed of 63 kmph will pass an electric pole in:
(a) 160 sec (b) 16 sec (c) 18 sec (d) 15 sec

41. A train 360 m long is running at a speed of 54 kmph. Time taken by the train to cross a tunnel 360 m long, is :
(a) 34 sec (b) 50 sec (c) 36 sec (d) 48 sec

42. A train 700 m long is running at the speed of 72 km/hr. If it crosses a tunnel in 1 minute, then the length of the tunnel (in meters) is :
(a) 700 (b) 600 (c) 550 (d) 500

43. If a 200 m long train crosses a platform of the same length as that of the train in 20 seconds, then the speed of the train is :
(a) 50 km/hr (b) 54 km/hr (c) 72 km/hr (d) 90 km/hr

44. A train 270 m long is moving at a speed of 25 kmph. It will cross a man coming from the opposite direction at a speed of 2 kmph in :
(a) 28 sec (b) 24 sec (c) 32 sec (d) 36 sec

45. Two trains whose length are 180 m and 220 m respectively are running in direction opposite to one another with respective speeds of 40 km/hr. Time taken by them in crossing one another will be :
(a) 16 sec (b) 17 sec (c) 18 sec (d) 22 sec

46. A train of length 150 m takes 10 seconds to pass over another train 100 m long coming from the opposite direction. If the speed of the first train be 30 kmph, the speed of the second train is :
(a) 54 km/hr (b) 60 km/hr (c) 72 km/hr (d) 36 km/hr

47. A train speeds pass a pole in 15 seconds and a platform 100 m long in 25 seconds. Its length in meters is :
(a) 50 (b) 150 (c) 200 (d) 375

48. Driving at 40% faster speed than normal, Rajesh saves 25 minutes. Find out the time he normally takes.
(a) 78.5 min (b) 87.5 min (c) 68.5 min (d) 98.5 min

49. In a 1000 meters race, A beats B by 100 meters and C by 200 meters. If A takes 50 seconds to finish the race, what is the ratio of the times taken by A, B and C to finish the race?

(a) 36 : 40 : 45 (b) 15 : 30 : 45 (c) 60 : 65 : 70 (d) 20 : 40 : 60

50. A walks at 14 km/hr instead of 10 km/hr, he would have walked 20 km more. The actual distance travelled by him is:
(a) 50 km (b) 56 km (c) 70 km (d) 80 km

51. In a race A runs at 10 km/hr and B at 13 km/hr. A has a start of 100 metres and also A sets off 3 minutes before B. How soon will B overtake A?
(a) 10 minutes (b) 11 minutes (c) 12 minutes (d) None of these

52. In a game of 100 points A can give B 20 points and C 40 points. How many points B can give C?
(a) 25 points (b) 24 points (c) 10 points (d) None of these

53. In a game of 90 points A can give B 15 points and C 30 points. How many points can B give C in a game of 100 points?
(a) 15 points (b) 28 points (c) 10 points (d) 20 points

54. In a game A can give B 25 points in 75 and C 18 points in 90. How many points can C give B in a game of 120?
(a) 15 points (b) 18 points (c) 72 points (d) None of these

55. A and B run a 5 km race on a round course of 400 m. If their speeds be in the ratio 5:4, how often does the winner pass the other?
(a) 2 times (b) 6 times (c) 3 times (d) None of these

56. A and B walk around a circle of circumference 1800 m at the speed of 150 m/minutes and 60 m/minute respectively. If both start simultaneously from the same point and walk in the same direction, when will they be together again for the first time.
(a) 20 minutes (b) 18 minutes (c) 16 minutes (d) None of these

57. A, B and C walk around a circle of circumference 600 metres at 150, 130 and 100 metres/minute respectively. All three start simultaneously from the same point and walk in the same direction. When will the three be together again for the first time?
(a) 40 min (b) 20 min (c) 60 min (d) None of these

58. In a race of 200 metres, B can give a start of 10 metres to A, and C can give a start of 20 metres to B. The start that C can give to A, in the same race, is
(a) 30 metre (b) 25 metre (c) 29 metre (d) 27 metre

59. A runs twice as fast as B and B runs thrice as fast as C. The distance covered by C in 72 minutes, will be covered by A in:
(a) 18 minutes (b) 24 minutes (c) 16 minutes (d) 12 minutes

60. In a kilometre race, A beats B by 30 seconds and B beats C by 15 seconds. If A beats C by 180 metres, the time taken by A to run 5 kilometre is
(a) 1025 seconds (b) 1005 seconds (c) 1200 seconds (d) 1210 seconds

NOTES

CHAPTER 11

TIME & WORK

INTRODUCTION

1. A person P_1 can finish a job alone in x days and P_2 can finish the same job alone in y days. Then both of them working together can finish the work in $\frac{xy}{(x+y)}$ days.

   Rejesh and Ajay can complete a job in 25 days and 30 days respectively. Then, both of them working together can complete the job in $= \frac{25\times 30}{20+30}\ days = 15\ days$

2. Three persons P_1,P_2 and P_3 complete a work in x, y and z days when working alone. When they work together they can complete the work in: $\frac{xyz}{xy+yz+zx}days$.

3. There are three persons P_1,P_2 and P_3 pairwise complete a work i.e. P_1 and P_2 working together can complete in x days similarly P_2 and P_3 in y days and P_3 and P_1 in z days.
   When they work together they can finish the job in $\left(\frac{2xyz}{xy+yz+zx}\right)days$

   Suppose three person P_1, P_2 and P_3 pairwise complete a work in 24 days. 20 days and 30 days respectively. Then when they work together they can complete the work in
   $\frac{2\times 24\times 20\times 30}{24\times 20+20\times 30+30\times 24}$ days i. e., 16 days

4. A can complete a piece of work in x days working alone, when A and B work together can complete the work in y days.
   $\therefore$ B alone can finish the work alone in $\frac{xy}{x-y}$ days.

5. A and B can finish a piece of work in T days. When A working alone takes x days more than A and B, and B working alone takes y days more than A and B together.
   $\Rightarrow$ T = $\sqrt{xy}$

SOLVED EXAMPLES

Example 1 4 men can do a work in 12 days. If there were 2 men less, in how many days can they complete one-fourth of the original work?

Solution: Total man-days required to complete the work = 48

$\frac{1}{4}^{th}$ work requires 12 man-days

2 men to complete 12 man-days require = $\frac{12}{2}$ = 6 days

Example 2 A can do a piece of work in 10 days and B can do it in 15 days. In how many days will they complete the work together?

Solution: A's 1 day work = $\frac{1}{10}$ and B's 1 day work = $\frac{1}{15}$

(A + B)'s 1 day work = $\frac{1}{10}+\frac{1}{15}=\frac{5}{30}=\frac{1}{6}$

They together complete it in 6 days.

Example 3 A and B can separately do a piece of work in 20 days and 30 days respectively. They work together for some days and then B stops. If A completes the rest of the work in 10 days. Then find the number of days B worked.

Solution: (A + B)'s 1 day work = $\frac{1}{20}+\frac{1}{30}=\frac{1}{12}$

A and B can complete it in 12 days.

A's 10 days work $= \frac{1}{20} \times 10 = \frac{1}{2}$

Remaining work done by A and B = $1/2$;
$1/12$ work done by A and B = 1 day;
$1/2$ work done by A and B = 6 days.
Hence, B worked for 6 days.

Example 4 Two pipes can fill a cistern in 3 hours and 4 hours respectively, and a drain pipe can empty it in 6 hours. If the cistern is empty and all the pipes are opened simultaneously. In how many hours will the cistern be full?

Solution: If all the three pipes are opened simultaneously the part of the tank filled in one hour is

$$\frac{1}{3}+\frac{1}{4}-\frac{1}{6}=\frac{4+3-2}{12}=\frac{5}{12}$$

i.e. $\frac{12}{5} = 2\frac{2}{5}$ hours = 2 hours 24 minutes

Example 5 Three men and six boys can complete a work in six days. Four men and five boys can complete the same work in five days. The work done by eleven boys is equal to the work of how many men?

Solution: (3m+ 6b)'s 1 day work = $\frac{1}{6}$
(4m+5b)'s 1 day work = $\frac{1}{5}$
(18m+36b)'s 1 day work = (20m+25b)'s 1 day work
2 men's work = 11 boys work.

Example 6 6 men earn as much as 8 women, 2 women earn as much as 3 boys, 4 boys earn as much as 5 girls. If a girl earns Rs. 500 per day, then find the earnings of a man per day.

Solution: Let 1 man's earnings per day be Rs. R.
6 men = 8 women, 4 boys = 5 girls, 1 girl = Rs. 500
$\therefore$ 1 boy = $(5/4)$ x Rs. 500 = Rs. 625
$\therefore$ 1 woman = Rs. 625 x $(3/2)$ = Rs. $\frac{1875}{2}$
$\therefore$ Earnings of 1 man/day = $\frac{1875}{2}\times\frac{8}{6}$ = Rs. 1250

Example 7 A takes 16 days to finish a job alone, while B takes 8 days to finish the same job. What is the ratio of their efficiency and who is less efficient.

Solution: Since A takes more time than B to finish the same job hence A is less efficient or
Efficiency of A = $\frac{100}{16}$ = 6.25% and,
Efficiency of B = $\frac{100}{8}$ = 12.5%
Ratio of efficiency of A & B = $\frac{1}{16}:\frac{1}{8}$ = 1 : 2
Hence, B is twice efficient of A.

Example 8 P is thrice as efficient as Q and is therefore able to finish a piece of work in 60 days less than Q. Find the time in which P and Q can complete the work individually.

Solution: Efficiency of P : Q = 3 : 1
Required number of days of P : Q = 1 : 3
i.e. if P requires x days then Q requires 3x days but 3x – x = 60
2x = 60
x = 30 and 3x = 90

Thus P can finish the work in 30 days and Q can finish the work in 90 days.

Example 9 A tub can be filled in 20 minutes but there is a leakage in it which can empty the full tub in 60 minutes. In how many minutes it can be filled?

Solution: In 1 minute part of tub that gets filled (when both are functional) = $\frac{1}{20}-\frac{1}{60}=\frac{2}{60}=\frac{1}{30}$

Hence, total time required to fill it = 30 minutes

Example 10 20 men working 8 hours a day can completely build a wall of length 200 metres, breadth 10 metres and height 20 metres in 10 days. How many days will 25 men working 12 hours a day require to build a wall of length 400 m, breadth 10 m and height of 15 m.

Solution: Total man hours required to construct volume equivalent to

Man hours = 20x8x10=1600 man hours

Volume build = 200 x 10 x 20 = 40000 m^3

Total work to be done = 400 x 10 x 15 m^3 = 60000 m^3

Man hours required to complete 60000 m^3

$1600\times\frac{60,000}{40,000}=1600\times\frac{3}{2}$ = 2400 man hours

25 men working 12 hours a day will do 300 men hours in a day

Total days required = $\frac{2400}{300}$ = 8. Hence, days required is 8.

Example 11 A and B together can complete a piece of work in 4 days. If A alone can complete the same work in 12 days, in how many days can B alone complete that work?

Solution: (A + B)'s 1 day's work = $\frac{1}{4}$, A's 1 day's work = $\frac{1}{12}$

$\therefore$ B's 1 day's work = $\left(\frac{1}{4}-\frac{1}{12}\right)=\frac{1}{6}$

Hence, B alone can complete the work in 6 days.

Example 12 45 men can complete a work in 16 days. Six days after they started working, 30 more men joined them. How many days will they now take to complete the remaining work?

Solution: Total man days = 45x16 = 720 days

Man days completed = 6x45 = 270 days

Man days remaining = 720 – 270 = 450

Days to complete work = $\frac{450}{75}$ = 6

Example 13 A can do a certain job in 12 days. B is 60% more efficient than A. How many days does B alone take to do the same job?

Solution: Ratio of times taken by A and B = 160 : 100 = 8 : 5

Suppose B alone takes x days to do the job.

Then, 8 : 5 : : 12 : x → 8x = 5 x 12 → x = $7\frac{1}{2}$ days.

Example 14 Two pipes A and B can fill a tank in 24 min and 32 min respectively. If both the pipes are opened simultaneously, after how much time B should be closed so that the tank is full in 18 minutes?

Solution: Let B be closed after x minutes. Then,

Part filled by (A + B) in x min. + part filled by A in (18 – x) min. = 1

$$\therefore \quad x\left(\frac{1}{24}+\frac{1}{32}\right)+(18-x)\times\frac{1}{24}=1$$

$$\Rightarrow \frac{x}{32}+\frac{18}{24}=1 \quad \Rightarrow x = 8$$

Example 15 A cistern has two taps which fill it in 12 minutes and 15 minutes respectively. There is also a waste pipe in the cistern. When all the three are opened, the empty cistern is full in 20 minutes. How long will the waste pipe take to empty the full cistern?

Solution: Work done by the waste pipe in 1 minute

$$= \frac{1}{20} - \left(\frac{1}{12} + \frac{1}{15}\right) = -\frac{1}{10}$$ [- ve sign means emptying]

$\therefore$ Waste pipe will empty the full cistern in 10 minutes.

Example 16 If two pipes function simultaneously, the reservoir will be filled in 12 hours. One pipe fills the reservoir 10 hours faster than the other. How many hours does it take the second pipe to fill the reservoir?

Solution: Let the reservoir be filled by first pipe in x hours.
Then, second pipe will fill it in (x + 10) hours.

$$\therefore \quad \frac{1}{x} + \frac{1}{(x+10)} = \frac{1}{12} \quad \Rightarrow \frac{x+10+x}{x(x+10)} = \frac{1}{12}$$

$\leftrightarrow$ $x^2 - 14x - 120 = 0$
$\leftrightarrow$ $(x - 20)(x + 6) = 0$
$\leftrightarrow$ $x = 20$

So, the second pipe will take (20 + 10) hrs i.e. 30 hrs to fill the reservoir.

Example 17 An electric pump can fill a tank in 3 hours. Because of a leak in the tank, it took $3\frac{1}{2}$ hours to fill the tank. If the tank is full, how much time will the leak take to empty it?

Solution: Work done by the leak in 1 hour $= \left[\frac{1}{3} - \frac{1}{\left(\frac{7}{2}\right)}\right] = \frac{1}{3} - \frac{2}{7} = \frac{1}{21}$

$\therefore$ The leak will empty the tank in 21 hours.

Example 18 A can do a piece of work in 7 days of 9 hours each and B can do it in 6 days of 7 hours each. How long will they take to do it, working together $8\frac{2}{5}$ hours a day?

Solution: A can complete the work in (7 x 9) = 63 hours.
B can complete the work in (6 x 7) = 42 hours.

$\therefore$ A's 1 hour's work $= \frac{1}{63}$ and B's 1 hour's work $= \frac{1}{42}$.

(A + B)'s 1 hour's work $= \left(\frac{1}{63} + \frac{1}{42}\right) = \frac{5}{126}$.

$\therefore$ Both will finish the work in $\frac{126}{5}$ hrs.

Number of days of $8\frac{2}{5}$ hrs each $= \left(\frac{126}{5} \times \frac{5}{42}\right)$ = 3 days.

Example 19 Two pipes A and B can fill a tank in 36 min. and 45 min. respectively. A water pipe C can empty the tank in 30 min. First A and B are opened for 7 minutes, then C is also opened. In how much time, the tank is full?

Solution: Part filled in 7 min. = $7\left(\frac{1}{36}+\frac{1}{45}\right)=\frac{7}{20}$

Remaining part = $\left(1-\frac{7}{20}\right)=\frac{13}{20}$

Net part filled in 1 min. when A, B and C are opened = $\left(\frac{1}{36}+\frac{1}{45}-\frac{1}{30}\right)=\frac{1}{60}$

Now, $\frac{1}{60}$ part is filled in 1 min.

$\frac{13}{20}$ part is filled in $\left(60\times\frac{13}{20}\right)$ = 39 min.

$\therefore$ Total time = 39 min + 7 min = 46 mins

Example 20 Two pipes A and B can fill a tank in 36 hours and 45 hours respectively. If both the pipes are opened simultaneously, how much time will be taken to fill the tank?

Solution: Part filled by A in 1 hour $=\frac{1}{36}$; Part filled by B in 1 hour = $\frac{1}{45}$.

Part filled by (A + B) in 1 hour = $\left(\frac{1}{36}+\frac{1}{45}\right)=\frac{9}{180}=\frac{1}{20}$.

Hence, both the pipes together will fill the tank in 20 hours.

EXPERIENCE THE PRATHAM EDGE - 11

1. 2 boys and 3 girls can do a piece of work in 10 days while 3 boys and 2 girls can do the same work in 8 days. In how many days, 5 boys and 4 girls will finish the work
(a) 4 (b) 5 (c) 6 (d) 7

2. A certain number of men complete a piece of work in 80 days which could be finished in 8 days less if no. of men increase by 8. How many were originally present.
(a) 64 (b) 60 (c) 66 (d) 72

3. 2 men and 7 women can do a piece of work in 14 days. 3 men and 8 women can do the same work in 11 days. Then 8 men and 6 women can do 2 times the work in
(a) 14 days (b) 21 days (c) 28 days (d) 24 days

4. Taps A, B, C are connected to a water tank and the rate of flow of water is 42 lts/hr, 56 lts/hr and 48 lts/hr respectively. Tap A and B fill the tank while tap C empties the tank. If all the three taps are opened simultaneously, the tank gets completely filled in 20 hours. What is the capacity of the tank?
(a) 980 lts (b) 940 lts (c) 960 lts (d) 1000 lts

5. A cistern is filled in 8 hours, but takes four hours longer due to a leak at its bottom. If the cistern is full, in what time will the leak empty it?
(a) 4 hours (b) 20 hours (c) 24 hours (d) 32 hours

6. Anoop can do a job in 20 days, Brajesh in 30 days and Christopher in 60 days. If Anoop is helped on every 3rd day by Brajesh and Christopher, then in how many days, the job is finished.
(a) 15 (b) 18 (c) 16 (d) 17

7. A tap can fill a cistern in 27min and another can fill it in 33 min. If both are opened simultaneously, find the time when the cisten will be full.
(a) 14.85 min (b) 15.6 min (c) 15.2 min (d) 14.76 min

8. Vandana and Shilpa can do a job in 15 and 10 days respectively. They began the work together but Shilpa leaves after somedays and Vandana finished the remaining job in 5 days. After how many days did Shilpa leave?
(a) 5 (b) 4 (c) 3 (d) 6

9. If 15 men can do a piece of work in 20 days. How many men are required to do the work in 30 days.
(a) 10 (b) 8 (c) 6 (d) 12

10. A and B can do a piece of work in 10 days, B and C in12 days, C and A in 15 days. How long will A take to seperately do the work.
(a) 24 days (b) 20 days (c) 18 days (d) 30 days

11. A is 30% more efficient than B. How much time will they work together to complete a job which A alone could have done in 23 days?
(a) 11 days (b) 13 days (c) $20\frac{3}{17}$ (d) None of these

EXPERIENCE THE PRATHAM EDGE - 11

12. A machine P can print one lakh books in 8 hours, machine Q can print the same number of books in 10 hours while machine R can print them in 12 hours. Machine P started at 9 am and closed at 11 am. and the remaining two machines complete the work of printing 1 lakh books. Approximately at what time will the work be finished?
(a) 2:30 a.m. (b) 3 p.m. (c) 2 p.m. (d) 1 p.m.

13. A and B together can do a piece of work in 30 days. A having worked for 16 days, B finishes the remaining work alone in total 44 days. In how many days shall B finish the whole work alone?
(a) 30 days (b) 40 days (c) 60 days (d) 70 days

14. A and B can complete a work in 15 days and 10 days respectively. They started doing the work together but after 2 days B had to leave and A alone completed the remaining work. The whole work was completed in:
(a) 8 days (b) 10 days (c) 12 days (d) 15 days

15. Three men, four women and six children can complete a work in seven days. A woman does double the work a man does and a child does half the work a man does. How many women alone can complete this work in 7 days?
(a) 7 (b) 8 (c) 12 (d) Cannot be determined

16. 10 men and 15 women together can complete a work in 6 days. It takes 100 days for one man alone to complete the same work. How many days will be required for one woman alone to complete the same work?
(a) 90 (b) 125 (c) 145 (d) None of these

17. A can finish a work in 24 days, B in 9 days and C in 12 days. B and C start the work but are forced to leave after 3 days. The remaining work was done by A in:
(a) 5 days (b) 6 days (c) 10 days (d) 10.5 days

18. Two pipes A and B together can fill a cistern in 4 hours. Had they been opened separately, then B would have taken 6 hours more than A to fill the cistern. How much time will be taken by A to fill the cistern separately?
(a) 1 hr (b) 2 hr (c) 6 hr (d) 8 hr

19. A large tanker can be filled by two pipes A and B in 60 minutes and 40 minutes respectively. How many minutes will it take to fill the tanker from empty state if B is used for half the time and A and B fill it together for the other half?
(a) 15 min. (b) 20 min. (c) 27.5 min. (d) 30 min.

20. An electric pump can fill a tank in 3 hours. Because of a leak in the tank, it took $3\frac{1}{2}$ hours to fill the tank. If the tank is full, how much time will the leak take to empty it?
(a) 21 hours (b) 20 hours (c) 22 hours (d) 23 hours

CHAPTER 12

CLOCKS & CALENDARS

CLOCK

Clock is a device which shows time. A clock has **two hands**, the smaller one is called the **hour hand or short hand** while the larger one is called the **minute hand or long hand.** Irrespective of the shape of the dial of the clock, the **tips of their hands** – hour as well minute describe a **circular path. Clock** based problems are one of the frequently asked questions in most of the competitive exam.

Some Important Points:

- The **hour hand** and **minute hand** of a clock move in a relation to each other continuously and at any given point of time, they make an **angle between 0^0 and 180^0** with each other
- When we say angle between the hands, we normally refer to the **acute/obtuse angles** (upto 180^0) between the hands and **not the reflex angle** ($>180^0$)
- The Face or dial of a watch is a circle whose circumference is divided into **60 equal parts**, called **minute spaces**.
- All angles are measured in the **clockwise direction** starting from the vertical line 12 o' clock.

Note: *Minutes hand covers 360^0 in 1 hr i.e. in 60 mins. Hence, Minute Hand covers 6^0 per minute. Hour hand covers 360^0 in 12 hrs. Hence Hours hand covers 30^0 per hour or Hour hand covers $1/2^0$ per minute*

The following additional points also should be remembered. In a period of 12 hours, the hands make an angle of

- **0^0** with each other (i.e. they coincide with each other), **11 times.**
- **180^0** with each other (i.e. they lie on the same straight line), **11 times.**
- **90^0** or any other angle with each other, **22 times.**

Concept of too fast and too slow:

- **Too fast** - When the clock **indicates time more** than the correct time, it is said to be running too fast by the difference between the correct time and the time indicated by the clock.
  Example - the clock indicated 10:20 am when the correct time is 10:05 am, it is said to be 15 minutes fast.
- **Too slow** - When the **time indicated** by the clock is **less** than the correct time, it is said to be too slow.
  Example - the clock indicated 9:30 am when the correct time is 9:35 am, it is said to be slow by 5 minutes.

CALENDARS

Calendar is an important part of quantitative aptitude. If someone asks what day it was on 10th may 1575, then we may call him crazy for asking such silly questions. But it is not a difficult task. In this chapter we will concentrate our discussion on finding the answer i.e. on **what day of the week does a particular date falls**. The clue to the **process of finding** it lies in calculating the number of **odd days** which is very different from the odd num bers.

Definition

- **Odd days:** In a given period, the number of **days more than** the **complete week** are called odd days. Odd day is the remainder obtained by dividing the total number of days by seven.
  Example: 52 days $\div$ 7 = 3 odd days
- **Leap year:** leap year has 366 days **(52 complete weeks+2 days)**. The two extra days are odd days.
  Facts: i. Every year divisible by 4 is a leap year, if it is not a century.
  ii. Every 4th century is a leap year and no other century is a leap year.
  Example: i. Each of the year 1948, 2004, 1676 etc. is a leap year.
  ii. Each of the years 400, 800, 1200, 1600, 2000 etc. is a leap year.
  iii. None of the years 2001, 2002, 2003, 2005, 1800, 2100 is a leap year.

- **Ordinary years:** The year which is not a leap year is called an ordinary year. An ordinary year has 365 days **(52 complete weeks+1 day)**.

Counting of odd days:

i. **Ordinary year** = 365 days = (52 weeks + 1 days). ∴ 1 ordinary year has **1 odd day**.

ii. **1 leap year** = 366 days = (52 weeks + 2 days). ∴ 1 leap year has **2 odd days**.

iii. **100 years** = 76 ordinary years + 24 leap years. ∴ Number of **odd days in 100 years** = **5**

Number of **odd days in 200 years =** (5 × 2) = 3 odd days

Number of **odd days in 300 years =** (5 × 3) = 1 odd days

Number of **odd days in 400 years =** (5 × 4 × 1) = 0 odd days.

Similarly, each one of 800 years, 1200 years, 1600 years, 200 years etc. has 0 odd days.

Days of the week related to odd days:

No. of days	0	1	2	3	4	5	6
Days	Sun.	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.

Note:
- *Last day of a century cannot be either Tuesday, Thursday or Saturday.*
- *The first day of a century must either be Monday, Tuesday, Thursday or Saturday.*

Symmetricity of Calendars (Repetition of the calendar)

(a) **For a leap Year:** Let us see, for example, the case of 2004.

Year	2004	2005	2006	2007	2008
Odd days	2	1	1	1	2

Since, the number of odd days are 7 , so days of the year 2004 and 2009 will be same from 1st January to 28 th February . (Because 2009 is not a leap year)

To know which year will have the same calendar as the given leap year, add 28 to given leap year i.e., 2004 +28 =2032 will have the same calendar like 2004 for the whole year

(b) **For any (leap year + 1) year:** Let us take an example , 2005

Year	2005	2006	2007	2008	2009	2010
Odd days	1	1	1	2	1	1

Since the number of odd days are 7, so calendars of 2005 and 2011 will be same for whole year.

Any (leap year +1) , the same calendar will happen after 6 years .

2005 + 6 =2011

(c) **For any (leap year + 2) year:** Let us take an example , 2006

Year	2006	2007	2008	2009	2010	2011
Odd days	1	1	2	1	1	1

Since the number of odd days are 7, so calendars of 2006 and 2012 will be same till 28th February.

Any (leap year + 2), the same calendar will happen after 6 years .

2006 + 6 =2012

(d) **For any (leap year + 3) year:** Let us take an example, 2007

Year	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Odd days	1	2	1	1	1	2	1	1	1	2	1

Since the number of odd days are 14 , so calendars of 2007 and 2018 will be same for whole year.

Any (leap year +3) , the same calendar will happen after 11 years .

2007 + 11 =2018

EXPERIENCE THE PRATHAM EDGE - 12

1. What is the angle covered by the minute hand in 22 minutes?
(a) 66° (b) 110° (c) 121° (d) 132°
2. By how many degrees does an hour hand move in one quarter of an hour?
(a) 5° (b) 7.5° (c) 10° (d) 12.5°
3. By how many degrees will the minute hand move, in the same time, in which the hour hand moves 6°?
(a) 54° (b) 84° (c) 72° (d) 60°
4. What is the angle between the hands of the clock, when it shows 40 minutes past 6?
(a) 40° (b) 70° (c) 80° (d) 90°
5. When the clock shows 3 hours 14 minutes, what is the angle between the hands of the clock?
(a) 10° (b) 12° (c) 13° (d) 14°
6. What is the angle between the two hands of a clock when the time is 25 minutes past 7 O'clock?
(a) 62½° (b) 66½° (c) 72½° (d) 69½°
7. When the clock shows 20 minutes past 11 O'clock, what is the angle between the two hands of the clock?
(a) 110° (b) 120° (c) 130° (d) 140°
8. At what time between 9 and 10 O'clock, will both the two hands of the clock coincide?
(a) $43\frac{3}{11}$ minutes past 9 O'clock (b) $54\frac{6}{11}$ minutes past 9 O'clock
(c) $49\frac{1}{11}$ minutes past 9 O'clock (d) $49\frac{6}{11}$ minutes past 9 O'clock
9. At what time between 4 and 5 O'clock are the hands of a clock in the opposite directions?
(a) $52\frac{3}{11}$ minutes past 4 O'clock (b) $54\frac{6}{11}$ minutes past 4 O'clock
(c) $51\frac{7}{11}$ minutes past 4 O'clock (d) $53\frac{9}{11}$ minutes past 4 O'clock
10. The angle between the hands of a clock is 20° and the hour hand is in between 2 and 3. What is the time shown by the clock?
(a) 2 hours $7\frac{3}{11}$ minutes (b) 2 hours $14\frac{6}{11}$ minutes
(c) 2 hours $15\frac{6}{11}$ minutes (d) Both (a) and (b)
11. Which of the following can be the time shown by the clock, when the hour hand is in between 4 and 5 and the angle between the two hands of the clock is 60°?
(a) $16\frac{4}{11}$ min past 4 (b) $18\frac{9}{11}$ min past 4 (c) $32\frac{8}{11}$ min past 4 (d) $36\frac{5}{11}$ min past 4
12. How many times, the hands of a clock will be at 30° with each other in a day?
(a) 36 (b) 40 (c) 44 (d) 48
13. How many times, the minute hand of a clock overlaps with the hour hand from 9:00 a.m. to 4:00 p.m. in a day?
(a) 5 (b) 6 (c) 7 (d) 8
14. A watch which gains uniformly was observed to be 1 minute slow at 8:00 a.m. on a day. At 6:00 p.m. on the same day it was 1 minute fast. At what time did the watch show the correct time?
(a) 12:00 noon (b) 1:00 p.m. (c) 2:00 p.m. (d) 3:00 p.m.
15. A watch, which gains uniformly was observed to be 6 minutes slow at 9:00 a.m. on a-Tuesday and 3 minutes fast at 12:00, noon on the subsequent Wednesday. When did the watch show the correct time?
(a) 9:00 p.m. on Tuesday (b) 12:00 a.m. on Wednesday
(c) 3:00 a.m. on Wednesday (d) 6:00 a.m. on Wednesday
16. A watch showed 10 minutes past 6 O'clock on Thursday morning when the correct time was 6 O'clock. It loses

uniformly and was observed to be 15 minutes slow at 8 O'clock on Saturday morning. When did the watch show the correct time?

(a) 1 O'clock on Friday afternoon (b) 12 O'clock noon on Friday
(c) 4 O' clock on Friday evening (d) 2 O'clock on Friday morning

17. The minute hand of a clock overtakes the hour hand at intervals of 60 minutes of correct time. How much time does the clock gain or lose in one hour of correct time?

(a) Gains $5\frac{5}{11}$ minutes (b) Loses $5\frac{5}{11}$ minutes
(c) Gains $5\frac{1}{11}$ minutes (d) Loses $5\frac{1}{11}$ minutes

18. The minute hand of a clock overtakes the hour hand after every 70 minutes of correct time. How much time does the clock lose or gain in a day of normal time?

(a) $93\frac{39}{77}$ minutes (b) $91\frac{31}{77}$ minutes
(c) $92\frac{24}{77}$ minutes (d) $94\frac{56}{77}$ minutes

Directions for Questions 19 and 20: Find the time between 2 and 3 O'clock at which the minute hand and the hour hand

19. Make an angle of 60° with each other:

(a) $21\frac{9}{11}$ min past 2 (b) $22\frac{9}{11}$ min past 2 (c) $21\frac{9}{11}$ min past 2 (d) $21\frac{9}{11}$ min past 7

20. Minute hand and the hour hand overlap:

(a) $10\frac{10}{11}$ past 2 (b) $10\frac{9}{11}$ past 2 (c) $12\frac{10}{11}$ past 2 (d) $10\frac{7}{11}$ past 2

21. If the time in a clock is 10 hours 40 minutes, then what time does its mirror image show?

(a) 1 hour 25 minutes (b) 1 hour 15 minutes (c) 1 hour 10 minutes (d) 1 hour 20 minutes

22. The reflection of a wall clock in a mirror shows the time as 3 hours 40 minutes. What is the actual time?

(a) 8 hours 20 minutes (b) 8 hours 15 minutes (c) 8 hours 45 minutes (d) 8 hours 35 minutes

23. If the seconds hand moves by 240°, then by how many degrees does the minute hand move in the same time?

(a) 1° (b) 2° (c) 3° (d) 4°

24. When the time is 10:30, if the minute hand points towards south, the hour hand will point towards

(a) North-east (b) North-west (c) South-east (d) South-west

25. What is the angle between the minute hand and the hour hand of a clock at 3 hour 40 minutes?

(a) 20^0 (b) 130^0 (c) 90^0 (d)70^0

26. If 01.01.2012 was Sunday, what was the day of the week on 31.12.2012?

(a) Tuesday (b) Wednesday (c) Sunday (d) Monday

27. If 01.01.2008 was Tuesday, what was the day of the week on 01.01.2009?

(a) Thursday (b) Wednesday (c) Tuesday (d) Monday

28. 01.01.1982 was Friday. What was the day of the week on 09.01.1983?

(a) Friday (b) Sunday (c) Saturday (d) Tuesday

29. If today is Monday, what will be the day of the week after 161 days?

(a) Monday (b) Tuesday (c) Sunday (d) Wednesday

30. If today is Wednesday, what will be the day of the week after 72 days?

(a) Saturday (b) Friday (c) Sunday (d) None of these

EXPERIENCE THE PRATHAM EDGE - 12

31. If December 3rd is Monday, what will be the day of the week of January 1st of the succeeding year?
(a) Monday (b) Wednesday (c) Tuesday (d) None of these

32. August 15th of 2013 was Thursday. What will be the day of the week on August 15th 2014?
(a) Tuesday (b) Wednesday (c) Thursday (d) None of these

33. If 27th March 2003 was Thursday, what was the day of the week on 27th March 2004?
(a) Saturday (b) Friday (c) Thursday (d) None of these

34. If 9th March 2013 was Saturday, what was the day of the week on 9th March 2012?
(a) Monday (b) Saturday (c) Thursday (d) Friday

35. If 1st August 2012 was Wednesday, what was the day of the week on 1st August 2010?
(a) Sunday (b) Monday (c) Saturday (d) None of these

36. If January 1st is a Saturday, how many Fridays are there in that month?
(a) 1 (b) 5 (c) 4 (d) 3

37. If October 25th is Friday, how many Tuesdays are there in the month?
(a) 5 (b) 4 (c) 3 (d) None of these

38. If January 1st 2012 was Sunday, how many Wednesdays are there in the month?
(a) 6 (b) 5 (c) 3 (d) 4

39. If the independence day of a year falls on Friday, what will be the day of the week of Gandhi Jayanthi of that year?
(a) Thursday (b) Wednesday (c) Friday (d) Cannot be determined

40. If the Republic Day of the year 2008 was on Saturday, what was the day of the week of the Independence Day of the year 2009?
(a) Sunday (b) Monday (c) Saturday (d) None of these

41. 27th September 2006 was Wednesday. What was the day of the week on 12th July 2007?
(a) Wednesday (b) Friday (c) Saturday (d) None of these

42. If in a year June 15th was Sunday, what was the day of the week on May 15th of that year?
(a) Tuesday (b) Thursday (c) Monday (d) Saturday

43. If in a year December 25th was on Saturday, what was the day on September 23rd of that year?
(a) Tuesday (b) Friday (c) Sunday (d) None of these

44. If 15th May 1992 was Friday, what was the day on 7th December 1993?
(a) Saturday (b) Sunday (c) Friday (d) Tuesday

45. Which year will have the same calendar as that of 2005?
(a) 2006 (b) 2007 (c) 2008 (d) 2011

46. 26th April 1994 was Tuesday, what was the day on 22nd September 2011?
(a) Tuesday (b) Thursday (c) Monday (d) Cannot be determined

47. What was the day of the week on 15th of August 1947?
(a) Wednesday (b) Monday (c) Saturday (d) Friday

48. What was the day of the week on 23rd March 2007?
(a) Wednesday (b) Friday (c) Sunday (d) None of these

49. What will be the day of the week on May 11th 2039?
(a) Monday (b) Wednesday (c) Thursday (d) None of these

50. When was the calendar of 2002 used again?
(a) 2011 (b) 2012 (c) 2013 (d) 2010

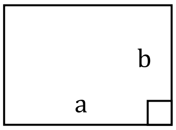
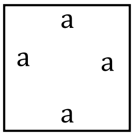
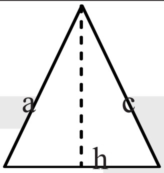
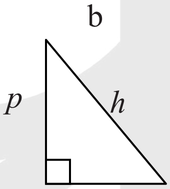
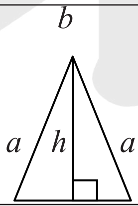
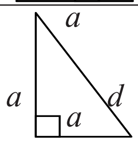
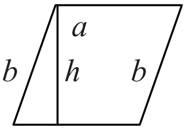
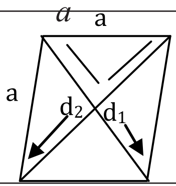
CHAPTER 13

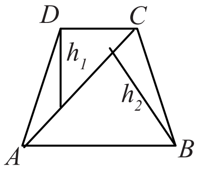
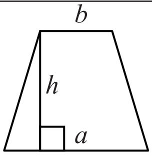
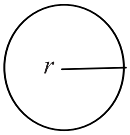
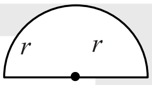
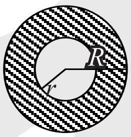
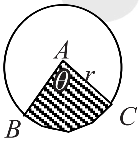
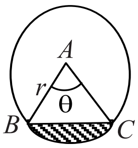
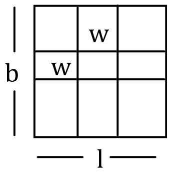
MENSURATION

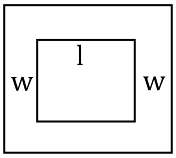
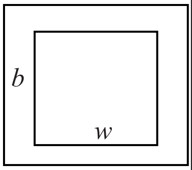
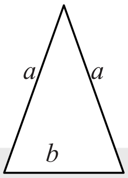
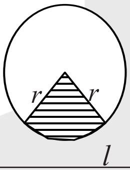
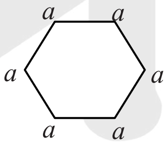
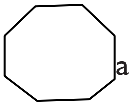
INTRODUCTION

AREA

Mensuration
Part-I : *Plane Figures*

S.No.	Name	Figure	Perimeter	Area	Nomenclature
1.	Rectangle		2(a + b)	ab	*a = Length* *b = Breadth*
2.	Square		4a	a^2	*a = Side*
3.	Triangle		a + b + c = 2s	$=\frac{1}{2}\times b\times h$, & $\sqrt{S(s-a)(s-b)(s-c)}$	*b is the base and h is the altitude a, b, c are three sides of Δ s is the semiperimeter*
4.	Right triangle		b + h + p	$\frac{1}{2}bp$	*(hypotenuse)* $h=\sqrt{p^2+b^2}$
5.	Equilateral triangle		3a	$\frac{1}{2}ha$, $\frac{\sqrt{3}}{4}a^2$	*a = side* *h = Altitude* $=\frac{\sqrt{3}}{2}a$
6.	Isosceles right triangle		2a + d	$\frac{1}{2}a^2$	*d(hypotenuse)=a√2, a = Each of equal sides.*
7.	Parallelogram		2(a + b)	ah	*a = Side* *b = Side adjacent to a* *h = Distance between the parallel sides a & b*
8.	Rhombus		4a	$\frac{1}{2}d_1\ d_2$	*a = Side of rhombus d_1, d_2 are the two diagonals.*

S.No.	Name	Figure	Perimeter	Area	Nomenclature
9.	Quadrilateral		Sum of its four sides	$\frac{1}{2}$(AC) (h_1 + h_2)	*AC is one of its diagonals and h_1, h_2 are the altitudes on AC from D, B respectively.*
10.	Trapezium		Sum of its four sides	$\frac{1}{2}$h (a+b)	*a, b are parallel sides and h is the perpendicular distance between parallel sides.*
11.	Circle		$2\pi r$	πr^2	*r = Radius of the circle* *π = 22/7 or 3.1416 (approx.)*
12.	Semicircle		$\pi r + 2r$	$\frac{1}{2}\pi r^2$	*r = Radius of the circle*
13.	Ring (shaded region)		--	$\pi(R^2 - r^2)$	*R= Outer radius* *r= inner radius*
14.	Sector of a circle		s + 2r Where $s = \frac{\theta}{360^0}.2\pi r$	$\frac{\theta}{360^o}\pi r^2$	*θ = Central angle of the sector* *r= Radius of the arc*
15.	Segment of a circle		$\frac{\theta}{360^0}2\pi r - 2r\sin\frac{\theta}{2}$	Area of segment ABC (Minor segment) $\frac{\theta}{360^o}\pi r^2 - \frac{1}{2}sin\theta r^2$	*r = Radius* *θ = angle of the related sector AOB.*
16.	Pathways running across the middle of a rectangle		2 (b + l)	A = w(l+b-w)	*l = Length* *b = Breadth* *w = Width of the path*

S.No.	Name	Figure	Perimeter	Area	Nomenclature
17.	Pathways outside		4 (b + l+2w)	A=2w(l+b+2w)	
18.	Pathways inside		4 (b + l-2w)	A= 2w(l+b-2w)	
19.	Isosceles		2a + b	$\frac{b}{4}\sqrt{4a^2-b^2}$	
20.	Sector of a Circle		$2r + r\theta$	$\frac{r^2\theta}{2}$	l = length of arc r = radius θ = Angle of the related sector in radian
21.	Hexagon Regular		6a	$\frac{3\sqrt{3}}{2}a^2$	
22.	Octagon Regular		8a	$2(1+\sqrt{2})a^2$	

SOLVED EXAMPLES

Example 1 If the sides of a square are increased by 10%, then find the percentage change in the area of the square

Solution: % change in area = $2x + \frac{x^2}{100}$

% change = $2(10) + \frac{10^2}{100} = 21\%$

Example 2 If the sides of a square are reduced by 25% then find the percentage change in the area of the square

Solution: % change in area = $2x - \frac{x^2}{100}$

% change = $2(25) - \frac{x^2}{100}$ = 43.75%

Example 3 What is the ratio of areas of an inscribed circle to a circumscribing circle in a square?

Solution: Let the radius of inscribed circle be x

Then side of square = 2x

Diameter of circumcircle = Diagonal of Square

$2R = 2\sqrt{2}x$ or $R = \sqrt{2}x$

Ratio is $x^2 : 2x^2 \Rightarrow 1:2$

Example 4 What is the ratio of areas of an inscribed circle to a circumscribing circle in an equilateral triangle?

Solution: If the sides of triangle is 'a' units.

Then $r = \frac{a}{2\sqrt{3}}$ and $R = \frac{a}{\sqrt{3}}$

$$\frac{\text{Area of in circle}}{\text{Area of circum circle}} = \frac{\pi\left[\frac{a}{2\sqrt{3}}\right]^2}{\pi\left[\frac{a}{\sqrt{3}}\right]^2} = 1 : 4$$

Example 5 Find the ratio of area of the equilateral triangle drawn with the side of a square as its base and the area of equilateral triangle described on the diagonal of the square

Solution: Let side of square be a units.

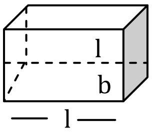
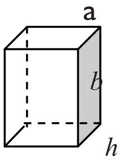
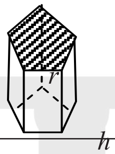
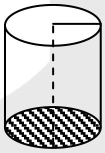
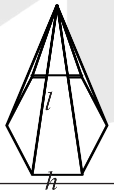
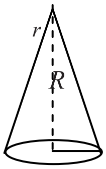
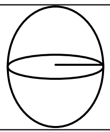
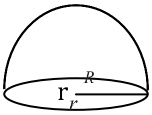
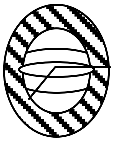
Then Area of equilateral triangle with side as 'a' = $\frac{\sqrt{3}}{4}a^2$

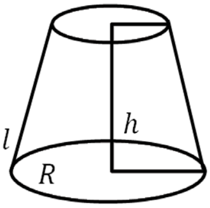
and Area of eq. triangle with side as diagonal of square = $\frac{\sqrt{3}}{4}(\sqrt{2}\,a)^2$

Required ratio = $\frac{\sqrt{3}}{4}a^2 : \frac{\sqrt{3}}{4}2a^2 = 1 : 2$

VOLUME

Mensuration
Part-II : *Three Dimentional Figures*

S.No.	Name	Figure	Lateral/curved surface area	Total Surface Area	Volume	Nomenclature
1.	Cuboid		2h (l + b)	2(lb + bh + lh)	*lbh*	l = Length b=Breadth h=Height
2.	Cube		$4a^2$	$6a^2$	a^3	a = Edge
3.	Right prism		(Perimeter of base) x Height	2(Area of base) + Lateral Surface area	Area of base x Height	--
4.	Right circular cylinder		$2\pi rh$	$2\pi r(r + h)$	πr^2h	r=Radius of base h=Height of the cylinder
5.	Right pyramid		$\frac{1}{2}$x (Perimeter of the base) x (slant height)	Area of the base + Lateral surface area	$\frac{1}{3}$ x (Area of the base) x Height	
6.	Right circular cone		πrl	$\pi r(l + r)$	$\frac{1}{3}\pi r^2 h$	h = Height r = Radius l = Slant height
7.	Sphere		--	$4\pi r^2$	$\frac{4}{3}\pi r^3$	r = Radius
8.	Hemisphere		$2\pi r^2$	$3\pi^2$	$\frac{2}{3}\pi r^3$	r = Radius
9.	Spherical shell		--	$4\pi(R^2 + r^2)$	$\frac{4}{3}\pi(R^3 - r^3)$	R = Outer radius r = inner radius

S.No.	Name	Figure	Lateral/curved surface area	Total Surface Area	Volume	Nomenclature
10.	Frustum of a cone		$\pi(R + r)\, l$	$\pi R^2 + \pi r^2 + \pi l(R+r)$	Volume = $\frac{1}{3}\pi h[R^2 + r^2 + Rr]$	$l = \sqrt{(R-r)^2 + h^2}$

Example 7 If the edges of a cube are doubled then find the percentage change in its volume

Solution: Let the edge of cube be 'a' units, Then edge after doubling will be = 2a units

Ratio of volumes = $a^3 : (2a)^3 = 1 : 8$

Example 8 Two cubes of equal size are joined face together then find the percentage loss in the total surface area

Solution: Let us assume that edge of cube is 1 unit.

Then S. area of both the cubes = 6 + 6 = 12 sq. units

And S. area of cuboid = $2\,(1 \times 2 + 2 \times 1 + 1 \times 1)$ = 10 sq. units.

% loss in surface area = $\frac{2}{12} \times 100 = 16\frac{2}{3}\%$

Example 9 Six cubes each with 10 cm edge are joined end to end. Find the surface area of the resulting cuboid?

Solution: Length of cuboid = $10 \times 6 = 60$ cm

S. Area = $2\,(60 \times 10 + 10 \times 10 + 60 \times 10) = 2600\ cm^2$

Example 10 The external dimensions of a closed box are 42cm, 30 cm and 20 cm. If the box is made of thickness 1cm. Find the volume of the wood used.

Solution: Volume or wood used = volume of bigger cuboids – volume of air cavity.

Volume $= 42 \times 30 \times 20 - 40 \times 28 \times 18 = 5040\ cm^3$

Example 11 A Cone, Sphere and a Cylinder have same base radii and same heights.

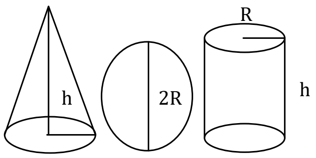
Then find the ratio of their volumes

Solution: Since heights are same h = 2 R

Ratio of volumes = $\frac{1}{3}\pi R^2 h : \frac{4}{3}\pi R^3 : \pi R^2 \times h$

$= \frac{1}{3}\pi R^2 \times 2R : \frac{4}{3}\pi R^3 : \pi R^2 \times 2R$

or the ratio of volumes = 1 : 2 : 3



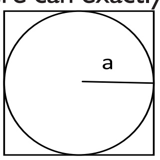
Example 12 Find the ratio of volume of a cube to a sphere if the sphere can exactly fit into the cube

Solution: Let the radius of sphere be 'a' unit.

Then side of cube = 2a

Ratio of volume = volume of cube : volume of sphere.

$= (2a)^3 : \frac{4}{3}\pi a^3 \qquad = 6 : \pi$



Example 13 The surface areas of three adjacent sides of a cuboid are x, y, z, sq. units respectively. Find the volume of the cuboid

Solution: For the cuboid

Let length × height = x , Length × breadth = y Breadth × height = z

The (length × breadth × height)^2 = xyz or volume = $\sqrt{xyz}$ cubic units

Example 14 How many cubes of surface area 24 dm can be made from a meter cube?

Solution: $6a^2 = 24 \Rightarrow a = 2$ dm = 20 cm [1 dm = 10 cm]

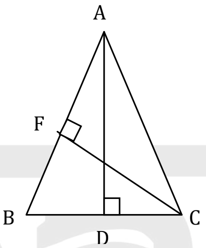
$$\text{Number of cubes} = \frac{\text{Volume of meter cube}}{\text{Volume of each smaller cube}}$$

$$n = \frac{100\times100\times100}{20\times20\times20} = 125$$

Example 15 In the figure given below AB = 6.4 cm CF = 2.6 cm, AD = 3.2 cm. Find the length of BC

Solution: AB = 6.4 cm (given)

Altitude on AB i.e., CF = 2.6 cm



$\therefore$ Area = $\frac{1}{2}$ x b x h = $\frac{1}{2}$ x 6.4 x 2.6

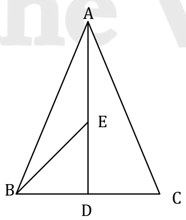
= 8.32 sq. cm

If we take BC as base, then corresponding altitude is AD $\therefore$ 8.32 = $\frac{1}{2}$ x BC x 3.2

$\Rightarrow$ BC = 5.2 cm

Example 16 In the figure given below, ABC is a triangle in which D is the mid point of side BC and E is the midpoint of AD. What is the ratio of areas of ΔBED and ΔABD?

Solution: Since BE is the median on AD and the median divides the triangle into two halves.



ΔBED = ΔAEB = $\frac{1}{2}$(ΔABD)

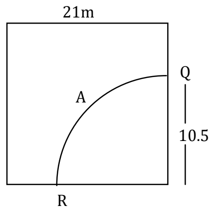
$\therefore$ Area of the ΔBED = $\frac{1}{2}$(ΔABD)

$\therefore$ Area of ΔABD = $\frac{1}{2}$: 1 = 1 : 2

Example 17 A goat is tied to one corner of a square plot of side 21 m with a rope 10.5 m long. Find the area that the goat can graze and also the area it cannot graze.

Solution: Area of square = 21 x 21 = 441 sq cm

The area that the goat can graze is sector ARQ.



The radius of the sector is 10.5 m, which is the length of the rope and angle of the sector is 90°

Area of ARQ = $\frac{90}{360} \times \frac{22}{7}$ x 10.5 x 10.5

= 86.625 sq.m

Area that the goat cannot graze = 441 – 86.625

= 354.375 sq.m

Example 18 A circular path runs all around and outside a circular garden of radius 42 m. If the difference between the outer circumference of the path and the circumference of the garden is 88 m, find the width of the path.

Solution: Circumference of inner circle

$2 \times \frac{22}{7} \times 21 = 132$ m

Circumference of the outer circle = 132 + 88 = 220 m

$2\pi R = 220$, where R is the outer radius of the path

$R = (220\times7)/(2\times22) = 35$m

width of the path= R-r =35-21=14m.

Example 19 A circular garden of radius 15 m is surrounded by a circular path of width 7 m. If the path is to be covered with tiles at a rate of Rs.10 per sq. m then find the total cost of the work.

Solution: Area of the ring (circular path) = $\pi(R^2 - r^2)$

$= \pi(22^2 - 15^2) = \pi(37)\ 7 = 814$ sq.m

$\therefore$ Total cost at Rs.10 per sq.m= 814 x 10 = Rs 8140

Example 20 The area of a parallelogram is 72 sq.cm and its height is 8 cm. Find its base.

Solution: The area of parallelogram = base x height

$\therefore$ height = $\frac{72}{8}$ = 9 cm

Example 21 A rectangle has twice the area of a square. The length of the rectangle is 8 cm greater than the side of the square and the breadth is equal to the side of the square. Find the perimeter of the square.

Solution: Let the side of the square be a.

$\Rightarrow$ area of the square = a^2

$\therefore$ area of the rectangle = $2a^2$

$\therefore (a + 8)a = 2a^2 \Rightarrow a^2 + 8a = 2a^2 \Rightarrow a = 8$

$\therefore$ perimeter of the square = 4 x 8 = 32 cm

Example 22 In a square of side 6 cm, find the length of the diagonal.

Solution: Length of the diagonal = $\sqrt{2}a$, where

a is side of the square

$\therefore d = 6\sqrt{2} = 6\sqrt{2}$ cm

Example 23 The cross section of a canal is trapezium, 5 m width at the top and 2 m, at the bottom, the depth is 3 m. Find the quantity of earth dug out in digging 100 m. length of the canal.

Solution: Area of cross section of the canal

= Volume of trapezium = ½ [5 + 2] x 3 = 10.5 sq.m

Volume of earth dug out

= area of cross section x length

= 10.5 x 100 = 1050 cu.m

Example 24 A wall of measurement 18m x 1m x 2m is to be constructed with bricks of dimensions 12 cm x 12 cm x 10 cm. Find the number of bricks required to constructs the wall.

Solution: Volume of the wall = 1800 x 100 x 200 cc
Volume of brick = 12 x 12 x 10 cc

Number of brick = $\frac{1800 \times 100 \times 200}{12 \times 12 \times 10}$ = 25,000

Example 25 A swimming pool 200 m long and 40 m wide is 1 m deep at the shallow end 5 m at the deep end. Find the volume of water contained in the pool.

Solution: Area of cross section perpendicular to width
= ½ (200) (1 + 5) = 600 sq.m
Volume = area of cross section x width
= 600 x 40 = 24000 cubic meters.

Example 26 The metallic solid cylinder of 8 cm diameter and 6 cm height is melted and made into 72 solid spheres of equal size. What is the diameter of each sphere?

Solution: Volume of cylinder = $\pi r^2 h$

= $\frac{22}{7}$ x 4^2 x 6 = $\frac{2112}{7}$ cc

Volume of each sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times r^3 = \frac{88r^3}{21}$

Volume of 72 spheres = $\frac{72 \times 88}{21} r^3$

Equating these two volumes, we have

$\frac{72 \times 88}{21} r^3 = \frac{2112}{7}$ => $r^3 = 1$

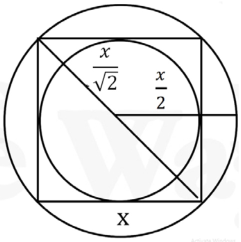
∴ The diameter of each sphere is 2 cm.

Example 27 Find the ratio of the areas of the incircle and circumcircle of a square.

Solution: Let the side of the square be x. Then, its diagonal = $\sqrt{2}x$.

Radius of incircle = $\frac{x}{2}$ and

Radius of circumcircle = $\frac{\sqrt{2}x}{2} = \frac{x}{\sqrt{2}}$



∴ Required ratio = $\left(\frac{\pi x^2}{4} : \frac{\pi x^2}{2}\right) = \frac{1}{4} : \frac{1}{2}$ = 1 : 2

Example 28 If the radius of a circle is decreased by 50%, find the percentage decrease in its area.

Solution: Let original radius = R.

New radius = $\frac{50}{100} R = \frac{R}{2}$

Original area = πR^2 and New area = $\pi\left(\frac{R}{2}\right)^2 = \frac{\pi R^2}{4}$

∴ Decrease in area = $\left(\frac{3\pi R^2}{4} \text{x} \frac{1}{\pi R^2} \times 100\right)\%$ = 75%.

Example 29 The difference between two parallel sides of a trapezium is 4 cm. The perpendicular distance between them is 19 cm. If the area of the trapezium is 475 cm^2, find the lengths of the parallel sides.

Solution: Let the two parallel sides of the trapezium be a cm and b cm.

Then, a – b = 4 .... (i)

And, $\frac{1}{2}$x (a + b) x 19 = 475 ⇒ (a + b) = $\left(\frac{475\times 2}{19}\right)$ ⇒ a + b = 50 .... (ii)

Solving *(i)* and *(ii)*, we get : a = 27, b = 23.

So, the two parallel sides are 27 cm and 23 cm.

Example 30 In measuring the sides of a rectangle, one side is taken 5% in excess, and the other 4% in deficit. Find the error percent in the area calculated from these measurements.

Solution: Side taken in excess = 5%

Another side taken in deficit = 4%

Error = 5-4-$\frac{4\times 5}{100}$ = 1-0.2 = 0.8%

Example 31 The diameter of the driving wheel of a bus is 140 cm. How many revolutions per minute must the wheel make in a minute in order to keep a speed of 66 kmph?

Solution: Distance to be covered in 1 min. = $\left(\frac{66\times 1000}{60}\right)$ m = 1100 m

Circumference of the wheel = $\left(2\times\frac{22}{7}\times 0.70\right)$ m = 4.4 m

∴ Number of revolutions per min. =$\left(\frac{1100}{4.4}\right)$ = 250

Example 32 The radii of the bases of a cylinder and a cone are in the ratio of 3 : 4 and their heights are in the ratio 2 : 3. Find the ratio of their volumes.

Solution: Let the radii of the cylinder and the cone be 3r and 4r and their heights be 2h and 3h respectively.

$$\therefore \frac{Volume\ of\ cylinder}{Volume\ of\ cone}=\frac{\pi\times(3r)^2\times 2h}{\frac{1}{3}\pi\times(4r)^2\times 3h}=\frac{9}{8}=9:8$$

Example 33 Find the slant height, volume, curved surface area and the whole surface area of a cone of radius 21 cm and height 28 cm.

Solution: Here, r = 21 cm and h = 28 cm.

∴ Slant height, $l=\sqrt{r^2+h^2}=\sqrt{(21)^2+(28)^2}=\sqrt{1225}=35\ cm$

Volume = $\frac{1}{3}\pi r^2h$ = $\left(\frac{1}{3}\times\frac{22}{7}\times 21\times 21\times 28\right)$ cm³ = 12936 cm³.

Curved surface area = πrl = $\left(\frac{22}{7}\times 21\times 35\right)$ cm² = 2310 cm².

Total surface area = $\pi rl+\pi r^2$ = $\left(2310+\frac{22}{7}\times 21\times 21\right)$ cm² = 3696 cm².

Example 34 A cone and a sphere have equal radii and equal volumes. Find the ratio of the diameter of the sphere to the height of the cone.

Solution: Let radius of each be R and height of the cone be H.

Then, $\frac{4}{3}\pi R^3 = \frac{1}{3}\pi R^2 H$ or $\frac{R}{H} = \frac{1}{4}$ or $\frac{2R}{H} = \frac{2}{4} = \frac{1}{2}$.

$\therefore$ Required ratio = 1 : 2

Example 35 A hemispherical bowl of internal radius 9 cm contains a liquid. This liquid is to be filled into cylindrical shaped small bottles of diameter 3 cm and height 4 cm. How many bottles will be needed to empty the bowl?

Solution: Volume of bowl $= \left(\frac{2}{3}\pi \times 9 \times 9 \times 9\right)$ cm³ = 486π cm³.

Volume of 1 bottle $= \left(\pi x \frac{3}{2} \times \frac{3}{2} \times 4\right)$ cm³ = 9π cm³.

Number of bottles $= \left(\frac{486\pi}{9\pi}\right) = 54$.

Example 36 The heights of two right circular cones are in the ratio 1 : 2 and the perimeters of their bases are in the ratio 3 : 4. Find the ratio of their volumes.

Solution: Let the radii of their bases be r and R and their heights be h and 2h respectively.

Then, $\frac{2\pi r}{2\pi R} = \frac{3}{4} \Rightarrow \frac{r}{R} = \frac{3}{4} \Rightarrow R = \frac{4}{3}r.$

$$\therefore \text{Ratio of volumes} = \frac{\frac{1}{3}\pi r^2 h}{\frac{1}{3}\pi\left(\frac{4}{3}r\right)^2(2h)} = \frac{9}{32} = 9:32$$

EXPERIENCE THE PRATHAM EDGE - 13

1. What is the sum of the squares of the sides of triangle ABC if the sum of the squares of its medians is 36 sq.cm?
(a) 40 sq.cm (b) 48 sq.cm (c) 52 sq.cm (d) 55 sq.cm

2. The perimeter of a square is equal to that of a regular hexagon. Find the ratio of their areas.
(a) $2 : 3\sqrt{3}$ (b) $3\sqrt{3} : 2$ (c) $2 : \sqrt{3}$ (d) $\sqrt{3} : 2$

3. The perimeter of the sector of a circle of radius 21 cm is 64 cm. Find the area of the sector.
(a) 225 sq.cm (b) 231 sq.cm (c) 248 sq.cm (d) 257 sq.cm

4. If each side of an equilateral triangle is increased by 4 cm, the area increases by $16\sqrt{3}$ sq.cm. Find the altitude of the new triangle.
(a) $2\sqrt{3}$ cm (b) $3\sqrt{3}$ cm (c) $4\sqrt{3}$ cm (d) $5\sqrt{3}$ cm

5. Find the two perpendicular sides of a right-angled triangle whose hypotenuse is 65 cm and the perimeter 144 cm.
(a) 16 cm, 63 cm (b) 29 cm, 60 cm (c) 25 cm, 45 cm (d) None of these

6. Three cubes of metals whose edges are in the ratio 3 : 4 : 5 are melted and one cube is formed. If the diagonal of the cube is $12\sqrt{\ }$ cm, then find the edge of the smallest cube.
(a) 6 cm (b) 9 cm (c) 12 cm (d) None of these

7. A cube and a sphere have the same surface area. Find the ratio of the volume of the sphere to that of the cube.
(a) $\sqrt{2} : \sqrt{3\pi}$ (b) $\sqrt{6} : \sqrt{\pi}$ (c) $\pi : 2\sqrt{6}$ (d) $\sqrt{\pi} : \sqrt{2}$

8. The area of a trapezium is 105 sq.cm. One of the parallel sides is 8 cm longer than the other. Find the longer of the two parallel sides, if distance between them is 7 cm.
(a) 7 cm (b) 19 cm (c) 11 cm (d) 15 cm

9. A cow is put outside a fenced rectangular plot 40 m x 14 m and is tethered to one corner by a rope 21 m long. Find the total area that it can graze.
(a) 996 sq.m (b) 1022 sq.m (c) 1078 sq.m (d) 1124 sq.m

10. The radius of a cylindrical box is 12 cm and its height is 5 cm. By increasing the height or radius by x cm, the increase in volume is the same. What is the value of x?
(a) 4 (b) $\frac{24}{5}$ (c) $\frac{7}{6}$ (d) $\frac{12}{5}$

11. The radius of a cylinder and a cone are equal. Their heights are also equal. The curved surface area of the cylinder to that of the cone is 8 : 5. Find the ratio of the radius to the height.
(a) 3 : 4 (b) 4 : 3 (c) 2 : 3 (d) 3 : 2

12. The girth of a cylindrical tree is 440 cm and its height is 1.5 m. Wood sells at Rs. 1500 per cu. Ft. If there was a wastage of 10% then what was the total realization? (1 inch = 2.5 cm. 1 ft = 12 inch)
(a) Rs.62,500 (b) Rs.87,750 (c) Rs.1,15,500 (d) None of these

13. An ink pen with a cylindrical barrel of diameter 1 cm and height 7 cm can write 2200 words. How many

words can be written using a 200 ml bottle?
(a) 65,000 (b) 80,000 (c) 1,20,000 (d) 1,55,000

14. The parallel sides of a trapezium of area 220 sq.cm. are 20 cm and 24 cm. Find the length of the non-parallel side if they are equal in length.
(a) $2\sqrt{26}$ cm (b) $3\sqrt{32}$ cm (c) 5 cm (d) $4\sqrt{7}$ cm

15. If the cost of painting $1m^2$ is Rs. 50, then what will be the maximum amount saved in painting the room in the most economical way, if the sum of length, breadth and height is 21 m, and all sides are integers? [floor is not painted]
(a) Rs. 9100 (b) Rs. 9200 (c) Rs. 9300 (d) Rs. 9400

16. The length of a rectangular plot is 60% more than its breadth. If the difference between the length and the breadth of that rectangle is 24 cm, what is the area of that rectangle?
(a) 2400 sq. cm (b) 2480 sq. cm (c) 2560 sq. cm (d) Data inadequate

17. The ratio between the length and the breadth of a rectangular park is 3 : 2. If a man cycling along the boundary of the park at the speed of 12 km/hr completes one round in 8 minutes, then the area of the park (in sq. m) is:
(a) 15360 (b) 153600 (c) 30720 (d) 37200

18. The area of a rectangle is 460 square metres. If the length is 15% more than the breadth, what is the breadth of the rectangular field?
(a) 15 metres (b) 26 metres (c) 34.5 metres (d) None of these

19. The sides of a rectangular field are in the ratio 3 : 4. If the area of the field is 7500 sq. m, the cost of fencing the field @ 25 paise per metre is:
(a) Rs. 55.50 (b) Rs. 67.50 (c) Rs. 86.50 (d) Rs. 87.50

20. A rectangular paper, when folded into two congruent parts had perimeter of 34 cm for each part folded along one set of sides and the same is 38 cm when folded along the other set of sides. What is the area of the paper?
(a) $140\ cm^2$ (b) $240\ cm^2$ (c) $560\ cm^2$ (d) None of these

21. If the length and breadth of a rectangular plot be increased by 50% and 20% respectively, then how many times will its area be increased?
(a) $1\frac{1}{3}$ (b) 2 (c) $4\frac{1}{5}$ (d) None of these

22. The length of a rectangle is halved, while its breadth is tripled. What is the percentage change in area?
(a) 25% increase (b) 50% increase (c) 50% decrease (d) 75% decrease

23. The length of a rectangle is decreased by r%, and the breadth is increased by (r + 5)%. Find r, if the area of the rectangle is unaltered.
(a) 5 (b) 10 (c) 15 (d) 20

24. A rectangular park 60 m long and 40 m wide has two concrete cross roads running in the middle of the park and rest of the park has been used as a lawn. If the area of the lawn is 2109 sq. m, then what is the width of the road?
(a) 2.91 m (b) 3 m (c) 97 m (d) None of these

EXPERIENCE THE PRATHAM EDGE - 13

25. If the length of a rectangular plot increases by 30% while its breadth remains same, what will be the ratio of the areas of new and old figures?
(a) 1 : 3 (b) 3 : 1 (c) 10 : 13 (d) None of these

26. An error of 2% in excess is made while measuring the side of a square. The percentage of error in the calculated area of the square is:
(a) 2% (b) 2.02% (c) 4% (d) 4.04%

27. If the area of a square increases by 69%, then the side of the square increases by:
(a) 13% (b) 30% (c) 39% (d) 69%

28. The side of a triangle are in the ratio of $\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$. If the perimeter is 52 cm, then the length of the smallest side is:
(a) 9 cm (b) 10 cm (c) 11 cm (d) 12 cm

29. If the area of a square with side *a* is equal to the area of a triangle with base *a*, then the altitude of the triangle is:
(a) $\frac{a}{2}$ (b) a (c) 2a (d) 4a

30. If the side of an equilateral triangle is decreased by 20%, its area is decreased by:
(a) 36% (b) 40% (c) 60% (d) 64%

31. A man runs round a circular field of radius 50 m at the speed of 12 km/hr. What is the time taken by the man to take twenty rounds of the field?
(a) 30 min. (b) 32 min. (c) 34 min. (d) None of these

32. The wheel of a motorcycle, 70 cm is diameter, makes 40 revolutions in every 10 seconds. What is the speed of the motorcycle in km/hr?
(a) 22.32 (b) 27.68 (c) 31.68 (d) 36.24

33. If the radius of a circle is increased by 75%, then its circumference will increase by:
(a) 25% (b) 50% (c) 75% (d) 100%

34. If the radius of a circle is diminished by 10%, then its area is diminished by:
(a) 10% (b) 19% (c) 20% (d) 36%

35. A hemisphere and a cone have equal bases. If their heights are also equal, then the ratio of their curved surfaces will be:
(a) 1 : 2 (b) 2 : 1 (c) 1 : √2 (d) √2: 1

36. A hemispherical bowl is filled to the brim with a beverage. The contents of the bowl are transferred into a cylindrical vessel whose radius is 50% more than its height. If the diameter is same for both the bowl and the cylinder, the volume of the beverage in the cylindrical vessel is:
(a) $66\frac{2}{3}$% (b) $78\frac{1}{2}$%
(c) 100% (d) More than 100%(i.e. some liquid will be left in the bowl)

37. A solid piece of iron of dimensions 49 x 33 x 24 cm is molded into a sphere. The radius of the sphere is:

(a) 21 cm (b) 28 cm (c) 35 cm (d) None of these

38. The volume of greatest sphere that can be cut off from a cylindrical log of wood of base radius 1 cm and height 5 cm is:
(a) $\frac{4}{3}\pi$ (b) $\frac{10}{3}\pi$ (c) 5π (d) $\frac{20}{3}\pi$

39. How many cubes of 10 cm edge can be put in a cubical box of 1 m edge?
(a) 10 (b) 100 (c) 1000 (d) 10000

40. If the radius of the base and the height of a right circular cone are doubled, then its volume becomes:
(a) 2 times (b) 3 times (c) 4 times (d) 8 times

41. Water flows through a cylindrical pipe of internal diameter 7 cm at 2 m per second. If the pipe is always half full, then what is the volume of water (in litres) discharged in 10 minutes?
(a) 2310 (b) 3850 (c) 4620 (d) 9240

42. A cylindrical tube open at both ends is made of metal. The internal diameter of the tube is 11.2 cm and its length is 21 cm. The metal everywhere is 0.4 cm thick. The volume of the metal is:
(a) 280.52 cm^3 (b) 306.24 cm^3 (c) 310 cm^3 (d) 316 cm^3

43. A cylindrical tank of diameter 35 cm is full of water. If 11 litres of water is drawn off, the water level in the tank will drop by:
(a) $10\frac{1}{2}$ cm (b) $11\frac{3}{7}$ cm (c) $12\frac{6}{7}$ cm (d) 14 cm

44. The radius of the base and height of a cone are 3 cm and 5 cm respectively whereas the radius of the base and height of a cylinder are 2 cm and 4 cm respectively. The ratio of the volume of cone to that of the cylinder is:
(a) 1 : 3 (b) 15 : 8 (c) 15 : 16 (d) 45 : 16

45. The length of an edge of a hollow cube open at one face is $\sqrt{3}$ meters. What is the length of the largest pole that it can accommodate?
(a) $\sqrt{3}$meters (b) 3 meters (c) $3\sqrt{3}$ (d) None of these

46. What is the volume of a cube (in cubic cm) whose diagonal measures $4\sqrt{3}$cm?
(a) 8 (b) 16 (c) 27 (d) 64

47. The volumes of two cubes are in the ratio 8 : 27. The ratio of their surface areas is:
(a) 2 : 3 (b) 4 : 9 (c) 12 : 9 (d) None of these

48. A cube of edge 5 cm is cut into cubes each of edge 1 cm. The ratio of the volume of one of the small cubes to that of the large cube is equal to:
(a) 1 : 5 (b) 1 : 25 (c) 1 : 125 (d) 1 : 625

49. The perimeter of one face of a cube is 20 cm. Its volume must be:
(a) 125 cm^3 (b) 400 cm^3 (c) 1000 cm^3 (d) 8000 cm^3

50. A rectangular sheet of iron foil is 44 cm long and 20 cm wide. A cylinder is made out of it by rolling the foil once along the width. Find the volume of the cylinder.
(a) 3080 cm^3 (b) 3077.8 cm^3 (c) 3078 cm^3 (d) 3077 cm^3

CHAPTER 14

GEOMETRY

INTRODUCTION

ANGLE AND LINES

An angle of 90° is a right angle; an angle less than 90° is acute angle; an angle between 90° and 180° is an obtuse angle; and angle between 180° and 360° is a reflex angle.
The sum of all angles on one side of a straight line AB at a point O by any number of lines joining the line AB at O is 180°. Below, the sum of the angles u, v, x, y and z is equal to 180°.
When any number of straight lines join at a point, the sum of all the angles around that point is 360°. In below figure the sum of the angle u, v, x, y and z is equal to 360°

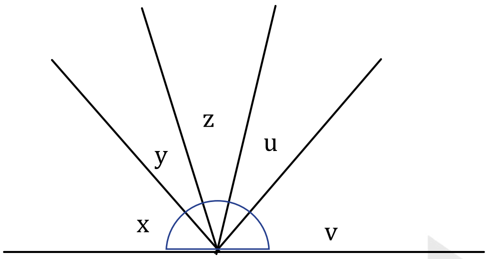


fig.01

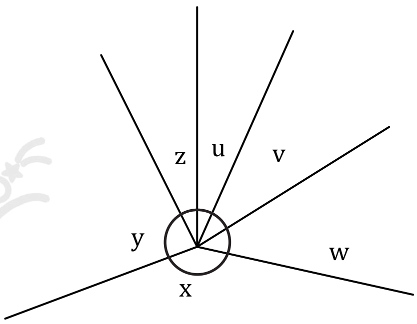


fig.02

Two angles whose sum is 90° are said to be complementary angles and two angles whose sum is 180° are said to be supplementary angles.
When two straight lines intersect, vertically. Opposite angles are equal. ∠AOB and ∠COD are vertically opposite angles and ∠BOC and ∠AOD are vertically opposite angles. So, we have ∠AOB = ∠COD and ∠BOC = ∠AOD.

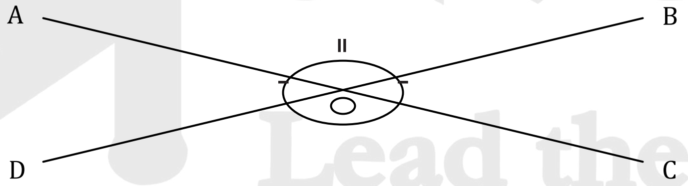


fig.03

Two lines which make an angle of 90° with each other are said to be PERPENDICULAR to each other.
If a line l_1 passes through the mid-point of another line l_2, then the line l_1 is said to be the BISECTOR of the line l_2, i.e., the line l_2 is divided into two equal parts.
Any point on the perpendicular bisector of a line is EQUIDISTANT from both ends of the line.

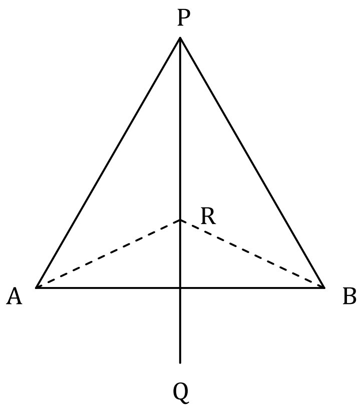


fig.04

Line PQ is the perpendicular bisector of line AB. A point P on the perpendicular bisector of AB will be equidistant from A and B, i.e., PA = PB.
Similarly, for any point R on the perpendicular bisector PQ, RA = RB

PARALLEL LINES

When a straight line cut two or more parallel lines, then the cutting line is called the TRANSVERSAL. When a straight line XY cuts two parallel lines PQ and RS, the following are the relationships between various angles that are formed. [M and N are the points of intersection of XY with PQ and RS respectively]

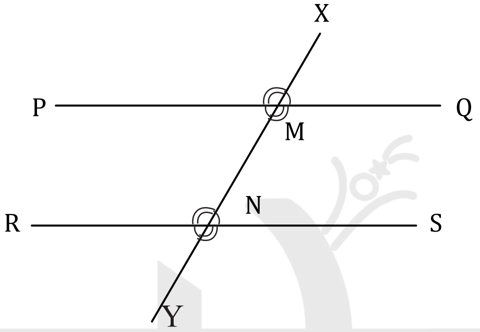


fig.05

(a) *Alternate angles are equal i.e.*
$\angle PMN = \angle MNS$ and $\angle QMN = \angle MNR$

(b) *Corresponding angles are equal, i.e.*
$\angle XMQ = \angle MNS$; $\angle QMN = \angle SNY$;
$\angle XMP = \angle MNR$; $\angle PMN = \angle RNY$

(c) *Sum of interior angles on the same side of the cutting line is equal to 180°, i.e.*
$\angle QMN + \angle MNS = 180^\circ$ and $\angle PMN + \angle MNR = 180^\circ$

(d) *Sum of exterior angles on the same side of the transversal is equal to 180°, i.e.*
$\angle XMQ + \angle SNY = 180^\circ$; and $\angle XMP + \angle RNY = 180^\circ$

If three or more parallel lines make intercepts on a transversal in a certain proportion, then they make intercepts in the same proportion on any other transversal as well. The lines AB, CD and EF are parallel and the transversal XY cuts them at the points P, Q and R. If we now take a second transversal, UV, cutting the three parallel lines at the points J, K and L, then we have $\frac{PQ}{PR} = \frac{JK}{JL}$

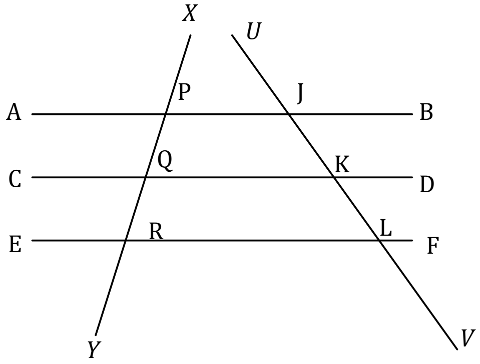
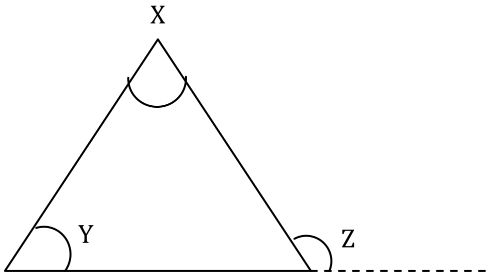


fig.06

If three or more parallel lines make equal intercepts on one transversal, they make equal intercepts on any other transversal as well.

TRIANGLES



Sum of the three angles of a triangle is 180°

z = x+y

fig.07

The exterior angle of the triangle (at each vertex) is equal to the sum of the two opposite interior angles. Exterior angle is the angle formed at any vertex, by one side and the extended portion of the second side at that vertex.

A line perpendicular to a side and passing through the midpoint of the side is said to be the perpendicular bisector of the side. It is not necessary that the perpendicular bisector of a side should pass through the opposite vertex in a triangle in general.

In triangle, the internal bisector of an angle bisects the opposite side in the ratio of the other two sides. In triangle ABC, if AD is the angular bisector of angle A, then $\frac{BD}{DC}=\frac{AB}{AC}$ This is called the Angular Bisector Theorem.

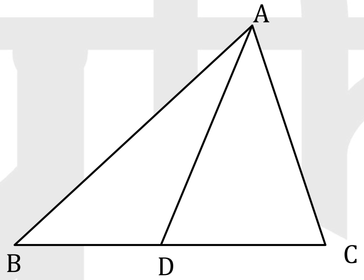


fig.08

In ΔABC, if AD is the median from A to side BC (meeting BC at its mid point D), then $2(AD^2+BD^2)=AB^2+AC^2$. This is called the **Apollonius Theorem**. This will be helpful in calculating the lengths of the three medians given the lengths of the three sides of the triangle (refer to Fig. 08).

GEOMETRIC CENTRES OF A TRIANGLE CIRCUMCENTRE

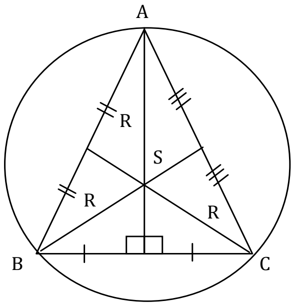


fig.09

The three **perpendicular bisector** of a triangle meet at a point called Circumcentre of the triangle and it is represented by S. The circumcentre of a triangle is equidistant from its vertices & the distance of the circumcentre from each of the three vertices is called circumradius (represented by R) of the triangle. The circle drawn with the circumcentre as centre and circumradius as radius is called the cirumcircle of the triangle and passes through all three vertices of the triangle. (refer to Fig. 09).

INCENTRE

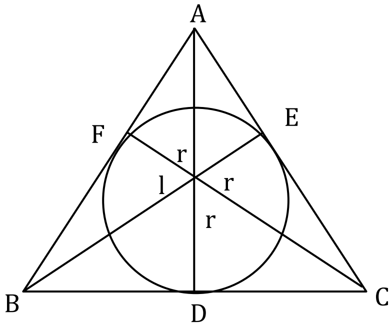


fig.10

The **internal bisectors** of the three angles of a triangle meet at a point called incentre of the triangle and it is represented by I. Incentre is equidistant from the three sides of the triangle i.e., the perpendiculars' drawn from the incentre to the three sides are equal in length and this length is called the inradius (represented by r) of the triangle. The circle drawn with incentre as centre and inradius as radius is called the incircle of the triangle and it touches all three sides on the in side.

$\angle BIC = 90^0 + \frac{1}{2}\angle A$, where I is the incentre.

EXCENTRE

If the internal bisector of one angle and the external bisector of the other two angles are drawn, they meet at a point called Excentre. There will be totally three excentres for the triangle – one corresponding to the internal bisector of each angle.

ORTHOCENTRE

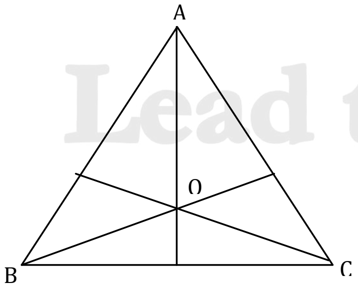


fig.11

The three altitudes meet at a point called Orthocentre and it is represented by O.

CENTROID

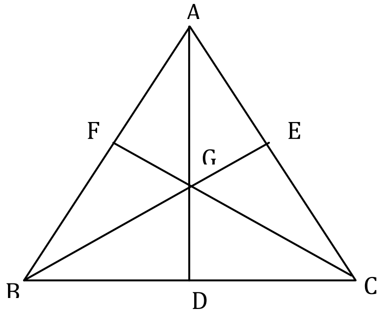


fig.12

The three medians of a triangle meet at a point called the Centroid and it is represented by G.

SIMILARITY OF TRIANGLES

Two triangles are said to be similar if the three angles of one triangle are equal to the three angles of the second triangle. Similar triangles are alike in shape only. The corresponding angles of two similar triangles are equal but the corresponding sides are **proportional.**

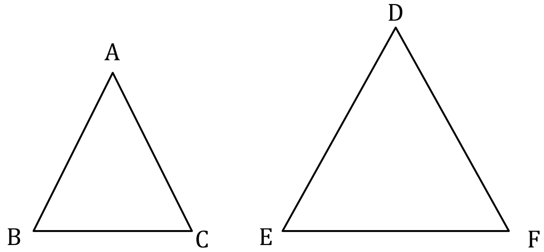


fig.13

For example, in Fig.13, if ΔABC is similar to ΔDEF where $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$, then we have ratio of the corresponding sides equal as given below.

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$

By "corresponding sides", we mean that if we take a side opposition to a particular angle in one triangle, we should consider the side opposite to the equal angle in the second triangle in this case. Since AB is the side opposite to $\angle C$ in ΔABC, we have taken DE which is the side opposite to $\angle F$ in ΔDEF since $\angle C = \angle F$.

Two triangles are similar if.

- the three angles of one are respectively equal to the three angles of the second triangle.
- two sides of one triangle are proportional to two sides of the other and the included angles are equal.
- three sides of one triangle are proportional to the three sides of second triangle.

In two similar triangles.
(a) Ratio of sides = Ratio of heights (altitudes) = Ratio of the lengths of the medians = Ratio of the lengths of the angular bisectors = Ratio of in radii = Ratio of circum radii
(b) Ratio of areas = Ratio of squares of corresponding sides

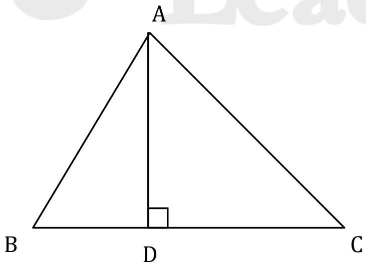


fig.14

In a right angled triangle, the altitude drawn from the vertex where there is a right angle to the hypotenuse divides the given triangle into two similar triangles, each of which is in turn similar to the original triangle. In triangle ABC in Fig. 14, ABC is a right angled triangle where $\angle A$ is a right angle. AD is the perpendicular drawn to the hypotenuse BC. The triangles ABD, ADC and ABC are similar because of the equal angles given below.

In triangle ABC, $\angle A = 90^\circ$. If $\angle B = \theta$, then $\angle C = 90^\circ - \theta$. In triangle ABD, $\angle ADB = 90^\circ$. We already know that $\angle B = \theta$; hence $\angle BAD = 90^\circ - \theta$
In triangle ADC, $\angle ADC = 90^\circ$. We already know that $\angle C = \theta$; hence $\angle CAD = 90^\circ - \theta$.

$$\Delta ABC \sim \Delta ADC$$

We can write down the relationship between the sides in these three triangles. One important relationship that emerges out of this exercise is $AD^2 = BD.DC$

CONGRUENCY OF TRIANGLES

Two triangles that are identical in all respects are said to be congruent.
In two congruent triangles.

- The corresponding sides (i.e., sides opposite to equal angles) are equal.
- The corresponding angles (angles opposite to equal sides) are equal.
- The areas of the two triangles will be equal.

Two triangles will be congruent if at least one of the following conditions is satisfied:

- Three sides of one triangle are respectively equal to the three sides of the second triangle (normally referred to as the S-S-S rule, i.e., the side-side-side congruency).
- Two sides and the included angle of one triangle are respectively equal to two sides and the included angle of the second triangle (normally referred to as the S-A-S rule, i.e., side-angle-side congruency).
- Two angles and included side of a triangle are respectively equal to two angles and the corresponding side of the second triangle (normally referred to as the A-S-A rule, i.e., angle-side-angle congruency).
- Two right-angled triangles are congruent if the hypotenuse and one side of one triangle are respectively equal to hypotenuse and one side of the second right-angled triangle.

Some more useful points about triangles

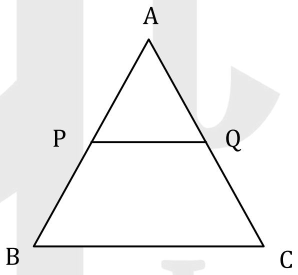


fig.15

A line drawn parallel to one side of a triangle divides the other two sides in the same proportion. For example, PQ is drawn parallel to BC in ΔABC. This will divide the other two sides AB and AC in the ratio, i.e., $\frac{AP}{PB} = \frac{AQ}{QC}$

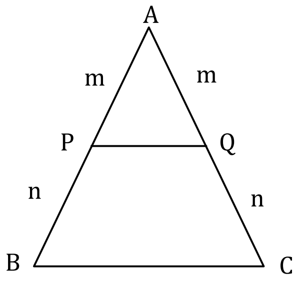


fig.16

Conversely, a line joining two points (each) dividing two sides of a triangle in the same ratio is parallel to the third side. The ratio, the length of this line segment bears to that of the third side is the same as that in which it cuts each of the first two sides. P divides AB in the ratio m : n and Q divides AC in the ratio m : n. Now, the line joining P and Q will be parallel to the third side BC and the length of PQ will be equal to $\frac{m}{m+n}$ times the length of BC.
On the basis of the above, we can say that a line drawn through one point on one side of the triangle and parallel to a second side will cut the third side at a point which will divide the third side in the same ratio as the first point divides the first side.
The line joining the midpoints of two sides of a triangle is parallel to the third side and it is half the third side. Two triangles having the same base and their third vertex on the same line parallel to the base have their areas equal.

CIRCLES

A circle is a curve drawn such that any point on the curve is equidistant from a fixed point. The fixed point is called the centre of the circle and the distance from the centre to any point on the circle is called the radius of the circle.

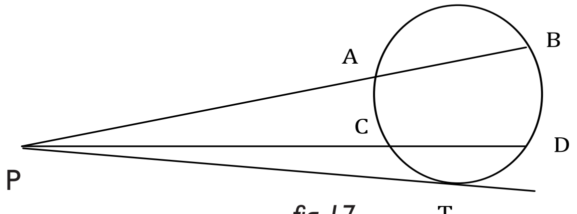


fig.17

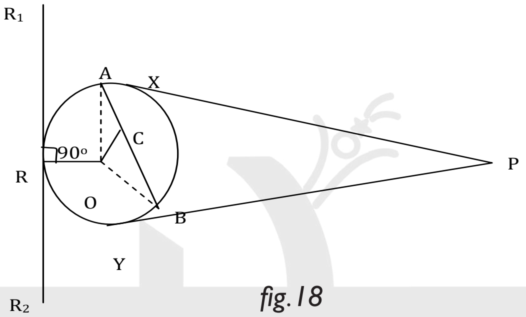


fig.18

Diameter is a straight line passing through the centre of the circle and joining two points on the circle A circle is symmetric about any diameter.
A chord is a point joining two points on the circumference of a circle. Diameter is the largest chord in a circle. *(fig. 18, line AB)*
A secant is a line intersecting a circle in two distinct points and extending outside the circle also.
If PAB and PCD are two secants. then PA.PB = PC.PD as shown in *fig. 17.*
A line that touches the circle at only one point is a tangent to the circle (R_1R_2 is a tangent touching the circle at the point R. *(in fig 18.)*
Two tangents can be drawn to the circle from any point outside the circle and these two tangents are equal in length (P is the external point and the two tangents PX and PY are equal) *as shown in fig. 18.*
A perpendicular drawn from the centre of the circle to a chord bisects the chord (OC, the perpendicular from O to the chord AB bisects AB) and conversely, the perpendicular bisector of a chord passes through the centre of the circle.
Two chords that are equal in length will be equidistant from the centre, and conversely two chords which are equidistant from the centre of the circle will be of the same length.
One and only one circle passes through any three given non-collinear points.

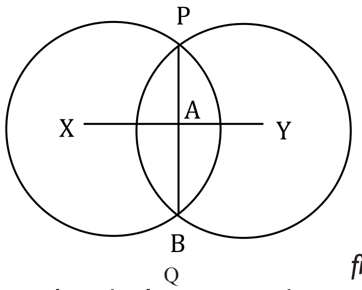


fig.19

When there are two intersecting circles, the line joining the centres of the two circles will perpendicularly bisect the line joining the points of intersection. The two circles with centres X and Y respectively intersect at the two points P and Q. The line XY (the line joining the centres) bisects PQ (the line joining the two points of intersection).
XA =AY, PA = AQ & $\angle PAY = \angle PAX = \angle XAQ = \angle YAQ = 90^0$ *as shown in fig. 19*

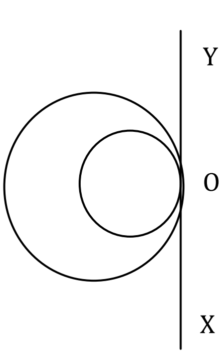


fig.20

Two circles are said to touch each other if a common tangent can be drawn touching both the circles at the same point. This is called the point of contact of the two circles. The two circles may touch each other internally or externally. When two circles touch each other, then the point of contact and the centres of the two circles are collinear, i.e., the point of contact lies on the line joining the centres of the two circles.

If two circles touch internally, the distance between the centres of the two circles is equal to the difference in the radii of the circles. (*as shown in fig. 20)*

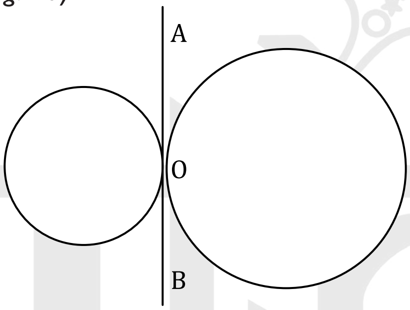


fig.21

When two circles touch each other externally, then the distance between the centres of the two circles is equal to the sum of the radii of the two circles. (*as shown in fig. 21)*

A tangent drawn common to two circles is called a common tangent. In general, for two circles, there can be anywhere from zero to four common tangents drawn depending on the position of the two circles.

If the common tangent cuts the line joining the centres not between the two circles but on one side of the circles, such a common tangent is called a direct common tangent. A common tangent that cuts the line joining the centres in between the two circles is called transverse common tangent.

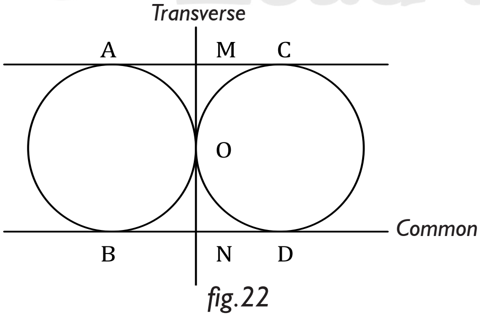


fig.22

If two circles are such that one lies completely inside the other (without touching each other), then there will not be any common tangent to these circles.

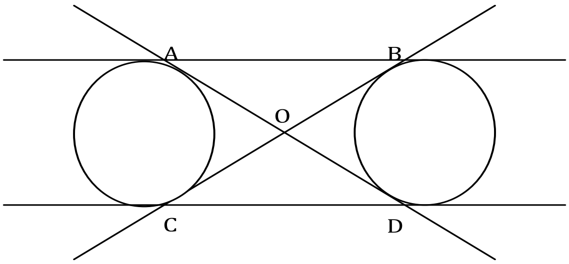


fig.23

Two circles touching each other internally (i.e., still one circle lies inside the other), then there is only one common tangent possible and it is drawn at the point of contact of the two circles.

Two intersecting circles have two common tangents. Both of these are direct common tangents and the two intersecting circles do not have a transverse common tangent.

Two circles touching each other externally have three common tangents. Out of these, two are direct common tangents and one is a transverse common tangent. The transverse common tangent is at the point of contact.
(as shown in fig. 22)

Two circles are said to be concentric if they have the same centre. It is obvious, here the circle with smaller radius lies completely within the circle with bigger radius.

ARCS AND SECTORS

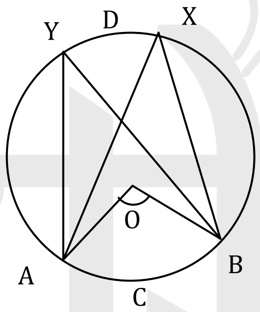


fig.24

An arc is a segment of a circle. ACB is called minor arc and ADB is called major arc. In general, if we talk of an arc AB, we refer to the minor arc. AOB is called the angle formed by the arc (at the centre of the circle) *as shown in fig 24*

The angle subtended by an arc at the centre is double the angle subtended by the arc in the remaining part of the circle.
$\angle AOB = 2\ \angle AXB$.
Angles in the same segment are equal. $\angle AXB = \angle AYB$.

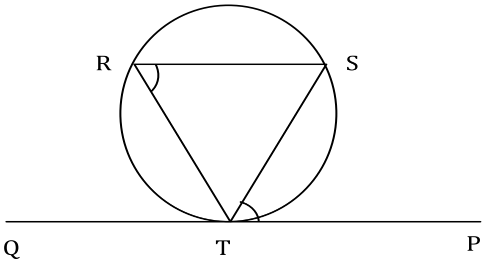


fig.25

The angle between a tangent and a chord through the point of contact of the tangent is equal of the angle made by the chord in the alternate segment (i.e., segment of the circle on the side other than the side of location of the angle between the tangent and the chord). This is normally referred as to the "alternate segment theorem." *In Fig. 25,* PQ is a tangent to the circle at the point T and TS is a chord drawn at the point of contact. Considering $\angle PTS$ which is the angle between the tangent and the chord, the angle TRS is the angle in the "alternate segment". So $\angle PTS = \angle TRS$.

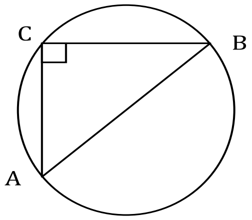


fig.26

The angle in a semicircle (or the angle the diameter subtends in a semicircle) is a right angle. The converse of the above is also true and is very useful in a number of cases – in a right angled triangle, a semi-circle can be drawn passing through the third vertex with the hypotenuse as the diameter.

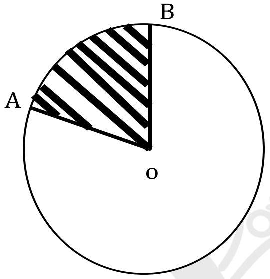
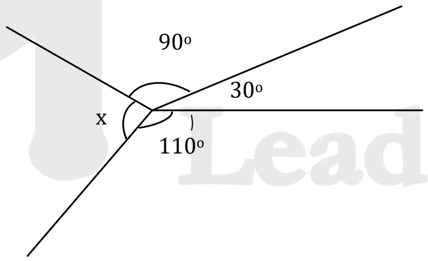


fig.27

The area formed by an arc and the two radii at the two end points of the arc is called sector.
$\angle$AOB is a sector. *(as shown in fig. 27)*
As we have already seen in quadrilaterals, the opposite angles of a cyclic quadrilateral are supplementary and that the external angle of a cyclic quadrilateral is equal to the interior opposite angle.

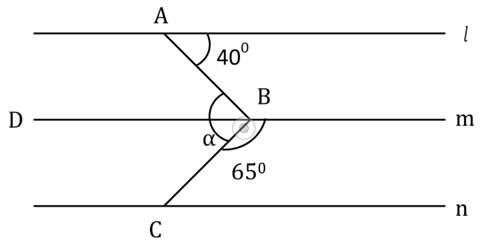
SOLVED EXAMPLES

Example 1 Calculate the value of x in the figure given below



Solution: The angles at a point add up to 360° Hence, x = 360-[90 + 30 + 110] $\Rightarrow$ x = 130°

Example 2 Calculate the measures of angle α in the figure below *l*, m & n are parallel to each other where α = angle ABC and D is a point on m.



Solution: $\alpha = \angle ABD + \angle CBD$
$\alpha = 40 + 115 = 155°$

Example 3 How many degrees are there in an angle which is equal to $\frac{1}{5}^{th}$ of its supplement?

Solution: Let the angle be x

Then its supplement is (180° – x)

$x = \frac{1}{5}(180 - x)$ [given] $\Rightarrow \frac{6x}{5} = 36$ $\Rightarrow x = 30°$

Example 4 How many degrees are there in an angle, which equals $1/3^{rd}$ of its complement?

Solution: Let angle be x.

Then its complement is (90° – x)

$x = \frac{1}{3}(90 - x)$ [given] $\Rightarrow \frac{4x}{3} = 30$ $\Rightarrow x = 22.5°$

Example 5 The base of right-angled triangle is 5 cm and its hypotenuse is 13 cm. Find its area.

Solution: Given base = 5 cm and hypotenuse = 13 cm

Applying Pythagoras theorem, we can find the third side as $\sqrt{13^2 - 5^2}$ = 12 cm

Area of the triangle = $\frac{1}{2}\text{bh} = \frac{1}{2} \times 5 \times 12$ = 30 sq.cm

Example 6 Triangle ABC and DEF are similar, If ∠C = ∠E, ∠B = ∠D, AB = 4.2 cm, DE = 2.1 cm BC = 6.3 cm and EF = 2.8 cm then, find DF and AC.

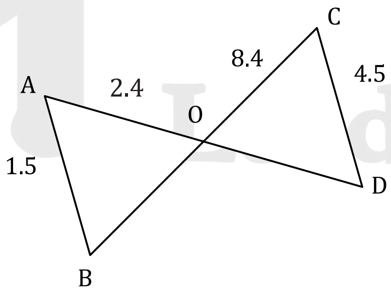
Solution: Given that ABC and DEF are similar, ∠C= ∠E, ∠B = ∠D so we have

since, ∠ C = ∠ E, ∠ B = ∠ D and Δ ABC = Δ FDE

$\frac{BC}{DE} = \frac{AB}{DF} = \frac{AC}{EF}$ $\Rightarrow$ $\frac{6.3}{2.1} = \frac{4.2}{DF} = \frac{AC}{2.8}$

DF = 1.4, AC = 8.4

Example 7 In the figure given below triangles ABO and DCO are similar. Find the length of BO & OD.

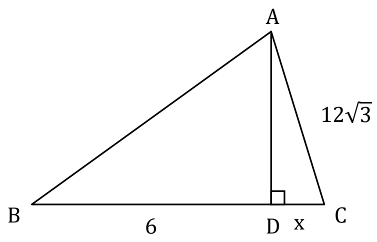


Solution: Given that ABO and DCO are similar,

$\frac{\text{CD}}{\text{AB}} = \frac{\text{OC}}{\text{OB}} = \frac{\text{OD}}{\text{OA}}$ $\frac{4.5}{1.5} = \frac{8.4}{\text{BO}} = \frac{\text{OD}}{2.4}$

BO = 2.8 cm and OD = 7.2 cm

Example 8 In the figure given below ∠A = 90°, AD is perpendicular to BC. Find x.



Solution: In a right angled triangle if a perpendicular is drawn from the right angled vertex, it will divide the triangle into two similar triangle and each in turn is similar to the original ΔABC

Triangle BAD, CAD and ABC are similar

$$\frac{12\sqrt{3}}{x+6} = \frac{x}{12\sqrt{3}} \Rightarrow 432 = x^2 + 6x$$

$x^2 + 6x - 432 = 0 \Rightarrow x = 18$ m

Example 9 A and B leave a point at the same time. A travels South at a speed of 16km/hr and B towards East at a speed of 12 km/hr. What is the distance between A and B after 4 hours?

Solution: Let the starting point be O. After 4 hours, A will be at X, which will be 4 x 16 = 64 km from O, B will be 4 x 12 = 48 km from O in direction perpendicular to OX.

Using Pythagoras theorem, we can find the third side XY as $\sqrt{64^2 + 48^2} = 80$km.

Hence A and B will be 80 km from each other.

Example 10 The wheel of a motorcar makes 1000 revolutions in moving 660m. Find the diameter of the wheel.

Solution: Distance covered = number of revolutions x circumference of the wheel

$\Rightarrow$ 660 = 1000 x πd, where d is the diameter of the wheel.

$\therefore$ d $= \frac{660 \times 7}{1000 \times 22}$ = 0.21 m $\therefore$ d =21cm.

Example 11 A copper wire is bent in the shape of a square, enclosing an area of 342.25 sq.cm. If the same wire is bent in the form of a circle, find the radius of the circle.

Solution: Area of square = 342.25 sq.cm

$\therefore$ side = $\sqrt{342.25}$ = 18.5 cm

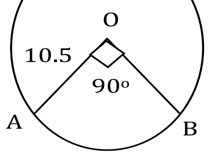
Perimeter = 4 x 18.5 cm = 74 cm

This will be the circumference of the wheel

$2\pi r = 74$

r = $\frac{74 \times 7}{2 \times 22}$ $\therefore$ r = 11.77 cm

Example 12 In the figure given, find the length of the major arc AB



Solution: $\angle AOB = 90°$

Angle of the major sector = 360 – 90 = 270°

$\therefore$ Length of the major arc

$$\frac{270}{360} \times \frac{2 \times 22}{7} \times 10.5 = 49.5 \text{ cm}$$

Example 13 If the circumference of one circle is ½ times that of the other, how many times the area of the smaller one is the larger one?

Solution: Let the radius of smaller circle = r

$\Rightarrow$ Radius of bigger circle =2r

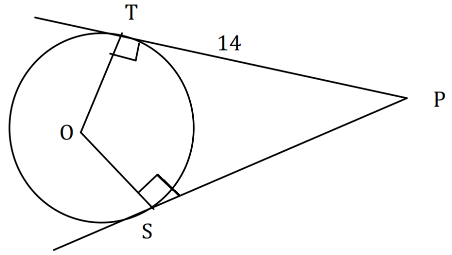
Area of smaller circle = πr^2

Area of bigger circle = $\pi(2r)^2 = 4\pi r^2$

$\therefore$ Ratio of area of bigger circle to that of smaller circle $= \frac{4\pi r^2}{\pi r^2} \Rightarrow \frac{4}{1}$

Hence, area of the smaller one = $\frac{1}{4}th$ of the larger one.

Example 14 In the figure given below, find the length of PO, given PT = 14 cm and OT = 10.5 cm



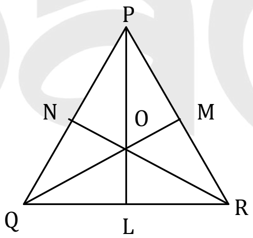
Solution: Since ΔPTO is a right angled triangle with
$\angle T = 90°$, $PO^2 = PT^2 + OT^2 = (14)^2 + (10.5)^2$
$\Rightarrow PO = 17.5$ cm

Example 15 If a + b + c = 2s, then the value of $(s - a)^2 + (s - b)^2 + (s - c)^2$ will be equal to:

Solution: $(s - a)^2 + (s - b)^2 + (s - c)^2 = s^2 + a^2 - 2as + s^2 + b^2 - 2bs + s^2 + c^2 - 2cs$
$= 3s^2 + a^2 + b^2 + c^2 - 2s(a + b + c) = 3s^2 + a^2 + b^2 + c^2 + - 2s \times 2s$
$= a^2 + b^2 + c^2 - s^2 = \frac{3}{4}(a^2 + b^2 + c^2) - \frac{1}{2}(ab + bc + ca)$

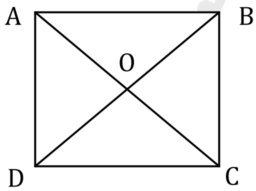
Example 16 In ΔPQR the medians QM and RN intersect at O. PO meets QR in L. If OL is 2.5 cm, then PL is equal to:

Solution: Medians get divided in the ratio 2 : 1
Here PO = 2OL
$\therefore$ PO = 2 x 2.5 cm = 5 cm
$\therefore$ PL = PO + OL = (5 + 2.5) cm = 7.5 cm



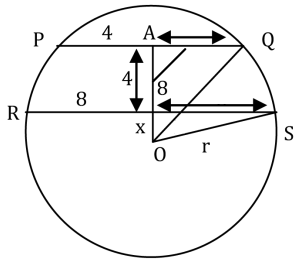
Example 17 If ABCD is a quadrilateral such that its diagonals AC and BD intersect at O to form four Δs equal in area. Then ABCD must be a:

Solution: Whenever intersection of diagonals divides
The quadrilateral into 4 equal Δ's, it has to be a parallelogram.



Example 18 PQ and RS are two parallel chords of a circle with centre O such that PQ = 8 cm and RS = 16 cm. If the chords are on the same side of the centre and the distance between them is 4 cm, then the radius of the circle is:

Solution: In ΔSBO, $r^2 = x^2 + 8^2$ .... (1)
In ΔQAO, $r^2 = (4 + x)^2 + 4^2$ .... (2)
From (1) and (2), $x^2 + 64 = 16 + x^2 + 8x + 16$
Or $8x = 32$ or $x = 4$
$\therefore$ from (1), $r = \sqrt{16+64} = \sqrt{80} = 4\sqrt{5}$ cm.



Example 19 AB = 2r is the diameter of a circle. If a chord CD intersects AB at right angle at a point L in the ratio 1 : 2, then CD is equal to:

Solution: AL : LB = 1 : 2 and AB = 2r

$\therefore$ x + 2x = 2r or x = (2/3)r

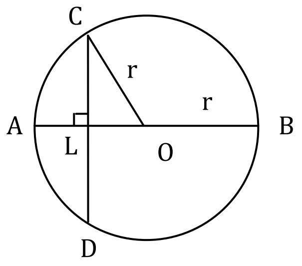
→ LO = AO – AL = r - (2/3)r = (1/3)r

In Δ CLO, $OC^2 = CL^2 + LO^2$

Or $CL^2 = OC^2 - LO^2 = r^2 - (1/9)r^2$

Or CL = $\sqrt{\frac{8}{9}r^2}$ $\frac{2\sqrt{2}}{3}r$

$\therefore$ CD = 2CL = $\frac{4\sqrt{2}}{3}r$



Example 20 Two circles touch internally at point P and a chord AB of the circle of larger radius intersects the other circle in C and D. Which of the following holds good?

a) ∠ CPA =∠DPB b) ∠CPB = ∠DPB

c) ∠CPD = ∠DPB d) None of these

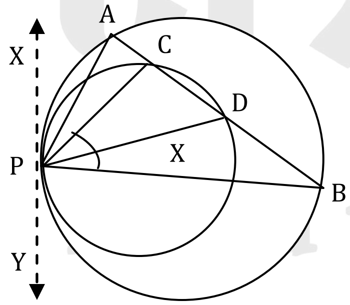
Solution: In bigger circle, by alt. segment theorem ∠ APX = ∠ABP ….. (1)

In smaller circle, by alt. segment theorem ∠CPX = ∠PDC

Or ∠APX + ∠CPA = ∠PDC or ∠ABP + ∠CPA = ∠PDC

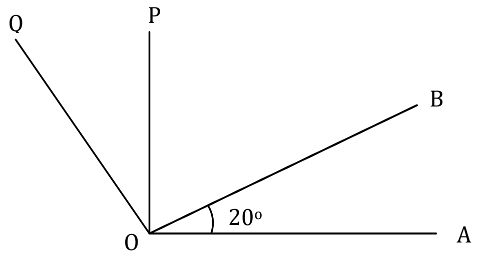
Or ∠ABP + ∠CPA = ∠ABP + ∠DPB [by ext. theorem in ΔPDB]

Or ∠CPA = ∠DPB, Hence answer is a).



EXPERIENCE THE PRATHAM EDGE - 14

1. In the following figure OP ⊥ OA and OQ ⊥ OB. Find ∠POQ if ∠AOB = 20°

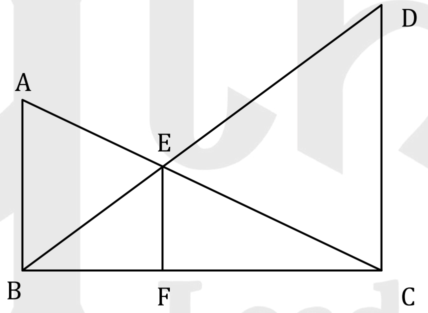


(a) 20° (b) 30° (c) 40° (d) None of these

2. If each angle of a regular polygon of n sides is 144°. Find the value of n.
(a) 8 (b) 10 (c) 12 (d) None of these

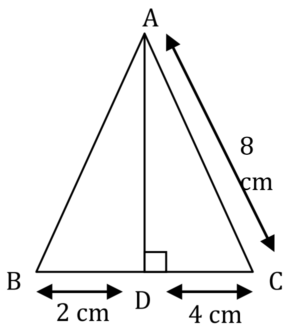
3. How far from the centre of a circle of diameter 130 cm is a chord 32 cm long?
(a) 48 cm (b) 63 cm (c) 66 cm (d) None of these

4. In the given figure, AB = 30 cm, CD = 45 cm and BC = 15 cm, find EF.



(a) 12 cm (b) 18 cm (c) 20 cm (d) 24 cm

5. Find the length of AB from the given figure.

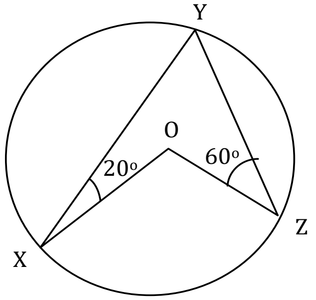


(a) 7 cm (b) $\sqrt{50}$ cm (c) $\sqrt{52}$ cm (d) None of these

6. In triangle ABC, AD ⊥ BC, D divides BC in the ratio 1 : 3 internally. Find BC, if AB = 9 cm and AC = 21 cm.
(a) $12\sqrt{5}$ cm (b) $15\sqrt{5}$ cm (c) $15\sqrt{5}$ cm (d) $18\sqrt{5}$ cm

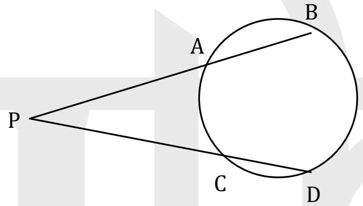
7. Find the distance between the tops of two poles 7 m and 4 m high if they are 4 m apart.
(a) 5 cm (b) 8 m (c) 9.767 m (d) None of these

8. Find $\angle XOZ$ from the given figure, (O is the centre of the circle)



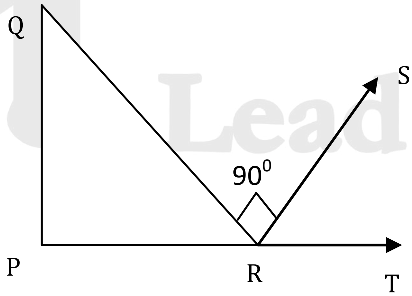
(a) 80° (b) 100° (c) 140° (d) 160°

9. In the given figure, PA = 4 cm; AB = 6 cm and PD = 8 cm; find CD.



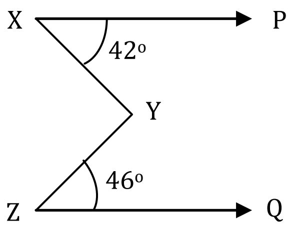
(a) 1 cm (b) 2 cm (c) 3 cm (d) 4 cm

10. In the following figure, $\angle QRP = 2\angle QPR = 2\angle SRT$. If $\angle QRS = 90°$ find $\angle QRP$.



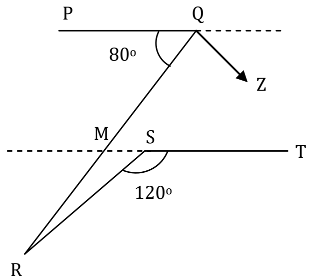
(a) 20° (b) 30° (c) 40° (d) 60°

11. Find $\angle XYZ$ from the given figure, if XP || ZQ.



(a) 42° (b) 46° (c) 66° (d) 88°

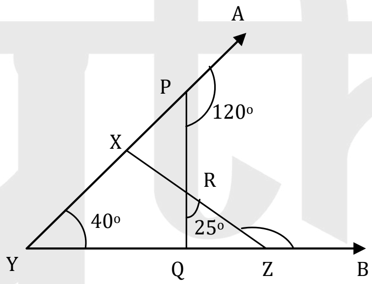
12. From the following figure, find ∠RQZ if ∠RQZ = 2∠QRS and PQ || ST.



(a) 20° (b) 30° (c) 40° (d) 60°

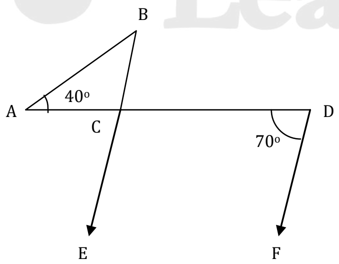
13. In ΔABC, ∠ABC =80°, ∠ACB = 40°, AP is the bisector of ∠BAC and AQ ⊥ BC, and ∠PAQ.
(a) 10° (b) 15° (c) 20° (d) 35°

14. From the following figure find ∠BZX.



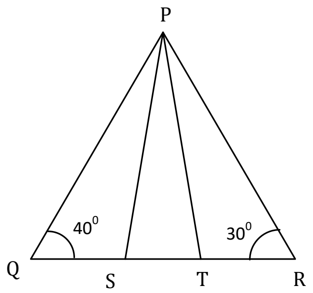
(a) 80° (b) 90° (c) 110° (d) 125°

15. From the given figure, find ∠ABC, if BE || DF.



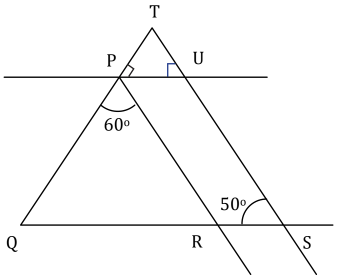
(a) 30° (b) 40° (c) 35° (d) None of these

16. In the given figure, PS = QS and PT = TR. Find ∠SPT.



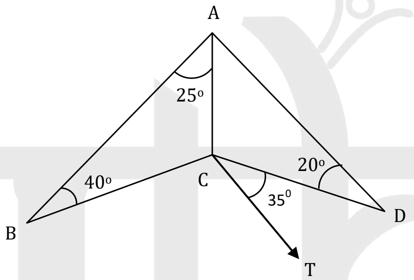
(a) 25° (b) 35° (c) 40° (d) 45°

17. In the given figure, PR || TS and PU || RS. Find ∠TPU.



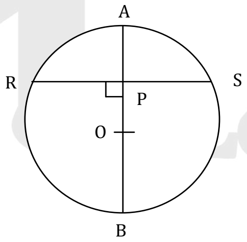
(a) 60° (b) 70° (c) 80° (d) 100°

18. From the given figure find ∠BCT, if AC = CD.



(a) 50° (b) 70° (c) 80° (d) None of these

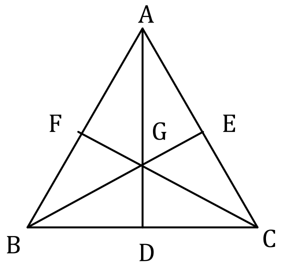
19. In the given figure, RS = 6 cm and radius of circle is 5 cm. Find PB.



(a) 9 cm (b) 8 cm (c) 7.5 cm (d) 10 cm

20. Three lines intersect at a point generating six angles. If one of these angles is 90°, then the maximum number of other distinct angles is:
(a) 4 (b) 3 (c) 2 (d) 1

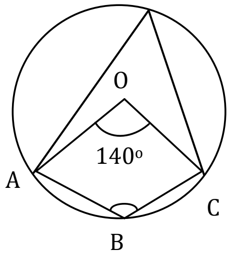
21. In ΔABC medians BE and CF intersect at G. If the straight line AGD meets BC at D in such a way that GD = 1.5 cm, then the length of AG is:



(a) 2.5 cm (b) 3 cm (c) 4 cm (d) 4.5 cm

22. If PL, QM and RN are the altitudes of ΔPQR whose orthocentre is O, then P is the orthocentre of:
(a) Δ PQO (b) ΔPQL (c) ΔQLO (d) ΔQRO

23. In the following figure, it is given that O is the centre of the circle and $\angle AOC = 140°$. find $\angle ABC$



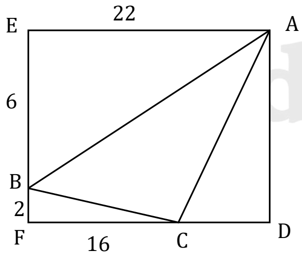
(a) 110° (b) 120° (c) 115° (d) 130°

24. Find the length of a chord that is at a distance of 12 cm from the centre of a circle of radius 13 cm:
(a) 9 cm (b) 10 cm (c) 12 cm (d) 5 cm

25. If one of the interior angles of a regular polygon is found to be equal to 9/8 times of one of the interior angles of a regular hexagon, then the number of side of the polygon is:
(a) 4 (b) 5 (c) 7 (d) 8

26. In ΔABC, P and Q are the mid-points of AB and AC, PQ is produced to R such that PQ = QR, then PRCB is:
(a) rectangle (b) square (c) rhombus (d) parallelogram

27. In the given figure, EADF is a rectangle and ABC is a triangle whose vertices lie on the sides of EADF. AE = 22, BE = 6, CF = 16 and BF = 2. Find the length of the line joining the mid points of the sides AB and BC.



(a) $4\sqrt{2}$ (b) 5 (c) 3.5 (d) None of these

28. Consider the following statements:
1. Every equilateral triangle is necessarily an isosceles triangle.
2. Every right angled triangle is necessarily an isosceles triangle.
3. A triangle in which one of the medians is perpendicular to the side it meets, is necessarily an isosceles triangle.
The correct statement are:
(a) 1 and 2 (b) 2 and 3 (c) 1 and 3 (d) 1, 2 and 3

29. If AD, BE, CF are the altitudes of ΔABC whose orthocentre is H, then C is the orthocentre of:
(a) ΔABH (b) ΔABD (c) ΔBDH (d) ΔBEA

30. The exterior angle of a regular polygon is one -third of its interior angle. How many sides does the polygon have?
(a) 6 (b) 7 (c) 8 (d) 9

CHAPTER 15

TRIGONOMETRY

INTRODUCTION

Trigonometry literally means **study of measuring triangles**. The basic task of trigonometry is the solution of triangles means **finding unknown quantities of triangles** from given values. This chapter deals with all the basic identities and property of trigonometry.

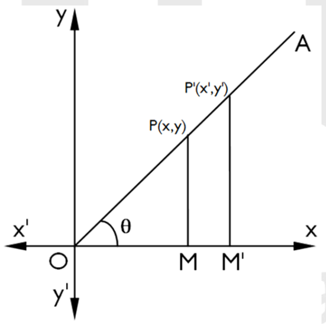
Definition

- **Degree measure** - The **measure** of an angle is determined by the **amount of rotation** from the initial side to the terminal side. One way to measure an angle is in terms of **degrees**. A measure of **one degree** (1°) is equivalent to a **rotation of** $\frac{1}{360}$ of a **complete revolution**.

- **Radian measure –** The **radian measure** of an angle drawn in standard position in the plane is equal to the **length of arc** on the **unit circle** subtended by that angle. One radian (1^c) is the measure of an **angle** subtended at the **center of a circle by an arc of length equal to the radius of the circle**.

Conversion between degrees and radians:
- To convert from degrees to radians, multiply the angle by $\frac{\pi}{180}$.
- To convert from radians to degrees, multiply the angle by $\frac{180}{\pi}$.

TRIGONOMETRIC RATIOS OR FUNCTIONS



In the right angled triangle OMP, we have Base = OM = x, Perpendicular = PM= y and Hypotenuse =OP= r

We define the following trigonometric ratios also known as trigonometric functions.

Sine $\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{y}{r}$ and is written as **sin** θ;

Cosine $\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{x}{r}$ and is written as **cos** θ;

Tangent $\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{y}{x}$ and is written as **tan** θ;

Cosecant $\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{r}{y}$ and is written as **cosec** θ;

Secant $\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{r}{x}$ and is written as **sec** θ;

Cotangent $\theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{x}{y}$ and is written as **cot** θ.

Special Angle and their Trigonometric Values:

Angle / Trigonometric Function	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined	0	not defined	0
cosec	not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	not defined	-1	not defined
Sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	not defined	-1	not defined	1
cot	not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	not defined	0	not defined

Fundamental Trigonometric Identities:

(i) $\sin\theta = \frac{1}{\operatorname{cosec}\theta}$ or $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

(ii) $\cos\theta = \frac{1}{\sec\theta}$ or $\sec\theta = \frac{1}{\cos\theta}$

(iii) $\cot\theta = \frac{1}{\tan\theta}$ or $\tan\theta = \frac{1}{\cot\theta}$

(iv) $\tan\theta = \frac{\sin\theta}{\cos\theta}$

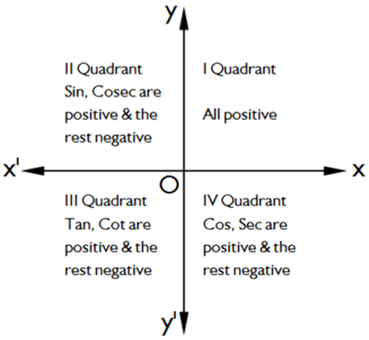
(v) $\cot\theta = \frac{\cos\theta}{\sin\theta}$

(vi) $\sin^2\theta + \cos^2\theta = 1$

(vii) $1 + \tan^2\theta = \sec^2\theta$

(viii) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

Signs of the Trigonometric Ratios or Functions



ALGORITHM

Step I: See whether the given angle α is positive or negative if it is negative, make it positive by using the following:
sin (-θ) = - sin θ, cos (-θ) = cos θ, tan (-θ) = - tan θ etc.

Step II: Express the positive angle α obtained in step I in the form $\alpha = \mathbf{90^\circ} \times n \pm \theta$, where θ is an acute angle.

Step III: Determine the **quadrant** in which the terminal side of the angle α lies.

Step IV: Determine the **sign** of the given **trigonometrical function** in the quadrant obtained in step III.

Step V: If **n** in step II is an **odd integer**, then $\sin\alpha = \pm\cos\theta$, $\cos\alpha = \pm\sin\theta$, $\tan\alpha = \pm\cot\theta$, $\sec\alpha = \pm\operatorname{cosec}\theta$ and $\operatorname{cosec}\theta = \pm\sec\theta$. The sign on RHS will be the sign obtained in step IV.
If **n** in step II is an **even integer**, then $\sin\alpha = \pm\sin\theta$, $\cos\alpha = \pm\cos\theta$, $\tan\alpha = \pm\tan\theta$, $\sec\alpha = \pm\sec\theta$ and

cosec α = $\pm$ cosec θ. The sign on RHS is the sign obtained in step IV.

TRIGONOMETRIC FORMULAS

- **Sum and difference formulas :**
  i. $\cos(A - B) = \cos A \cos B + \sin A \sin B$
  ii. $\cos(A + B) = \cos A \cos B - \sin A \sin B$
  iii. $\sin(A - B) = \sin A \cos B - \cos A \sin B$
  iv. $\sin(A + B) = \sin A \cos B + \cos A \sin B$
  v. $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$
  vi. $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$
- **Product to sum formulas :**
  i. $\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$
  ii. $\cos A \cos B = \frac{1}{2}[\cos(A + B) + \cos(A - B)]$
  iii. $\sin A \cos B = \frac{1}{2}[\sin(A + B) + \sin(A - B)]$
  iv. $\cos A \sin B = \frac{1}{2}[\sin(A + B) - \sin(A - B)]$
- **Sum to product formulas :**
  i. $\sin A + \sin B = 2\sin\frac{(A+B)}{2}\cos\frac{(A-B)}{2}$
  ii. $\sin A - \sin B = 2\cos\frac{(A+B)}{2}\sin\frac{(A-B)}{2}$
  iii. $\cos A + \cos B = 2\cos\frac{(A+B)}{2}\cos\frac{(A-B)}{2}$
  iv. $\cos A - \cos B = -2\sin\frac{(A+B)}{2}\sin\frac{(A-B)}{2}$

Periodicity Identities – radians
- $\text{Sin}(2\pi + x) = \text{Sin}\,x$
- $\text{Tan}(\pi + x) = \text{Tan}\,x$
- $\text{Cos}(2\pi + x) = \text{Cos}\,x$
- $\text{Cot}(\pi + x) = \text{Cot}\,x$

Periodicity Identities – degrees
- $\text{Sin}(x + 360^\circ) = \text{Sin}\,x$
- $\text{Tan}(x + 180^\circ) = \text{Tan}\,x$
- $\text{Cos}(x + 360^\circ) = \text{Cos}\,x$
- $\text{Cot}(x + 180^\circ) = \text{Cot}\,x$

Double Angle Identities
- $\text{Sin}(2x) = 2\sin(x)\cos(x)$
- $\text{Cos}(2x) = 2\cos^2(x) - 1$
- $\text{Cos}(2x) = \cos^2(x) - \sin^2(x)$
- $\text{Cos}(2x) = 1 - 2\sin^2(x)$
- $\text{Tan}(2x) = \frac{2\tan x}{1 - \tan^2 x}$

Half Angle Identities
- $Sin\frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{2}}$
- $Cos\frac{x}{2} = \pm\frac{\sqrt{1 + \cos x}}{2}$
- $tan\frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{1 + \cos x}}$

General Solution:

1. $\text{Sin}\,\alpha = \text{Sin}\,\beta \Rightarrow \alpha = k + (-1)^k\beta$
2. $\text{Cos}\,\alpha = \text{Cos}\,\beta \Rightarrow \alpha = \beta + 2k$ (or) $\alpha = -\beta + 2k$
3. $\text{Tan}\,\alpha = \text{Tan}\,\beta \Rightarrow \alpha = \beta + k$

Example 1: Solve for x in the following equations 2 Sin x – 1 = 0

Solution: $2 \sin x - 1 = 0 \Rightarrow \sin x = \frac{1}{2}$

$\sin x = \sin \frac{\pi}{6} \Rightarrow x = \frac{\pi}{6} + k(2\pi)$ (or)

$x = \frac{5\pi}{6} + k(2\pi)$, where K is an integer.

Example 2: $\cos x = \frac{\sqrt{2}}{2}$

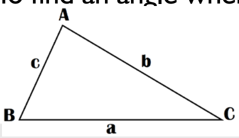
Solution: $\cos x = \cos \frac{\pi}{4} \Rightarrow x = \frac{\pi}{4} + k(2\pi)$ **or** $x = -\frac{\pi}{4} + k(2\pi)$

Law of Sine and Cosine

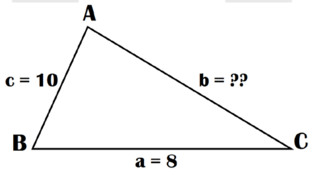
- $\cos C = \frac{a^2+b^2-c^2}{2ab}$
- $\cos A = \frac{b^2+c^2-a^2}{2bc}$
- $\cos B = \frac{c^2+a^2-b^2}{2ac}$
- $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

The law of cosines is used:

1. To find the third side of a triangle given two sides and the included angle.
2. To find an angle when all 3 sides are given.



Example 1: In triangle ABC, side a= 8 cm, c = 10 cm, and the angle at B is 60. Find side b, angle A and angle C.



Solution: 1. Side b:

$b^2 = a^2 + c^2 - 2ac \cos \angle B$

$b = \sqrt{a^2 + c^2 - 2ac \cos \angle B}$

$b = \sqrt{8^2 + 10^2 - 2.8.10 \cos(60^o)}$

$b = \sqrt{84} = 2\sqrt{21}$

2. Angle A:

$a^2 = b^2 + c^2 - 2bc \cos\angle A$

$8^2 = 10^2 + (\sqrt{84})^2 - 2.10.\sqrt{84} \cos\angle A$

$\cos\angle A = \frac{120}{20\sqrt{84}}$

$\cos\angle A = \frac{3}{\sqrt{21}}$

$\angle A = \cos^{-1}\left(\frac{3}{\sqrt{21}}\right) \approx 49.1^\circ$

3. Angle C:

$\angle A + \angle B + \angle C = 180^\circ$

$49.1^\circ + 60^\circ + \angle C = 180^\circ$

$\angle C = 70.9^\circ$

Directions for Questions 1 to 7: Find the value of each of the following:

1. $\frac{\sec^2\theta - 1}{\tan^2\theta} = ?$

(a) 1 (b) 2 (c) 3 (d) 4

2. $\sin^4\theta + \sin^2\theta\cos^2\theta$

(a) 0 (b) 1 (c) 2 (d) $\sin^2\theta$

3. $\frac{\sin\theta \text{ cosec }\theta \tan\theta \cot\theta}{\sin^2\theta + \cos^2\theta}$

(a) 0 (b) 1 (c) -1 (d) 2

4. $\sin^2 A \cot^2 A + \cos^2 A \tan^2 A$

(a) -1 (b) 0 (c) 3 (d) 1

5. $\tan\theta + \cot\theta$

(a) 1 (b) $\tan\theta$ (c) $\text{cosec }\theta \cot\theta$ (d) $\sec\theta \text{ cosec }\theta$

6. $\frac{3 - 4\sin^2\theta}{\cos^2\theta} + \tan^2\theta$

(a) 1 (b) 2 (c) 3 (d) None of these

7. $\frac{\cot A + \tan B}{\cot B + \tan A}$

(a) $\tan A \cot B$ (b) $\cot A \tan B$ (c) 1 (d) None of these

8. If $\sin\theta = \frac{21}{29}$, find the value of $\sec\theta + \tan\theta$, if θ lies between 0 and $\pi/2$:

(a) 1 (b) $\pi/2$ (c) 5/2 (d) None of these

9. If A is in the fourth quadrant and $\cos A = \frac{5}{13}$ find the value of $\frac{13\sin A + 5\sec A}{5\tan A + 6\text{ cosec }A}$:

(a) -2/37 (b) -3/27 (c) 2/37 (d) Can't be determined

10. Find the value of $\frac{4}{3}\cot^2 30^0 + 3\sin^2 60^0 - 2\text{ cosec}^2 60^0 - 3/4\tan^2 30°$:

(a) 10/3 (b) 11/3 (c) 4 (d) None of these

11. If $\tan\theta = \frac{a}{b}$, find the value of $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$

(a) $\frac{a^2 - b^2}{a^2 + b^2}$ (b) $\frac{b^2 - a^2}{b^2 + a^2}$ (c) $\frac{a^2 + b^2}{a^2 - b^2}$ (d) None of these

12. If A + B = 45°, find the value of tan A + tan B + tan A tan B:

(a) -1 (b) $\frac{1}{2}$ (c) $\sqrt{3}$ (d) 1

13. If $\cos A = \frac{3}{5}$ and $\sin B = \frac{5}{13}$, find the value of $\frac{\tan A + \tan B}{1 - \tan A \tan B}$:

(a) $\frac{63}{16}$ (b) $\frac{36}{16}$ (c) $\frac{61}{36}$ (d) None of these

14. Find the value of Cos28° Cos32° - Sin28° Sin32°
(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) Can't be determined

15. The value of $\cos^4\theta - \sin^4\theta$ is:
(a) Cos2 θ (b) Sin2 θ (c) tan2θ (d) None of these

16. Find the value of tan 75°
(a) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ (b) $\frac{\sqrt{3}}{2+\sqrt{2}}$ (c) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (d) $\frac{2+\sqrt{2}}{\sqrt{3}}$

17. Find the value of $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} + \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$
(a) 2 Sec θ (b) Sec θ (c) 2 Cosec θ (d) None of these

18. If $\sin\theta =$ 4/5, find the value of sin 2θ :
(a) 24/25 (b) 16/25 (c) 9/20 (d) None of these

19. $\frac{\sin 2A}{1+\cos 2A}$ is:
(a) tan 2A (b) cos 2A (c) tan A (d) None of these

20. Find the value of cot 3θ :
(a) $\frac{\cot^3\theta - 3\cot\theta}{3\cot^2\theta}$ (b) $\frac{\cot^3\theta - 3\cot\theta}{3\cot^2\theta - 1}$ (c) $\frac{3\cot\theta - \cot^3\theta}{3\cot^2\theta - 1}$ (d) None of these

NOTES

CHAPTER 16

HEIGHTS & DISTANCES

INTRODUCTION

IMPORTANT FACTS AND FORMULAE

1. We already know that:
   In a rt. angled ΔOAB, where $\angle BOA = \theta$,

   (i) $\sin\theta = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}} = \dfrac{AB}{OB} = \dfrac{P}{H}$;

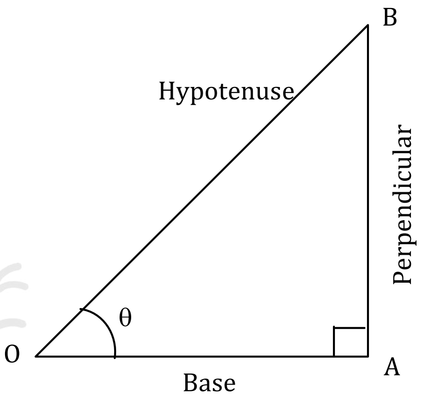
   (ii) $\cos\theta = \dfrac{\text{Base}}{\text{Hypotenuse}} = \dfrac{OA}{OB} = \dfrac{B}{H}$;

   (iii) $\tan\theta = \dfrac{\text{Perpendicular}}{\text{Base}} = \dfrac{AB}{OA} = \dfrac{P}{B}$;

   (iv) $\text{cosec}\,\theta = \dfrac{1}{\sin\theta} = \dfrac{OB}{AB}$;

   (v) $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{OB}{OA}$;

   (vi) $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{OA}{AB}$.



2. Trigonometrical Identities:

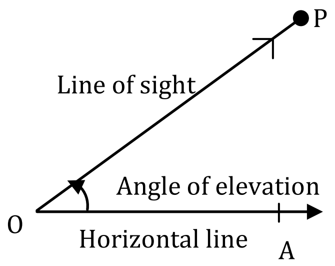
(i) $\sin^2\theta + \cos^2\theta = 1$ (ii) $1 + \tan^2\theta = \sec^2\theta$ (iii) $1 + \cot^2\theta = \text{cosec}^2\theta$

3. Values of T-ratios:

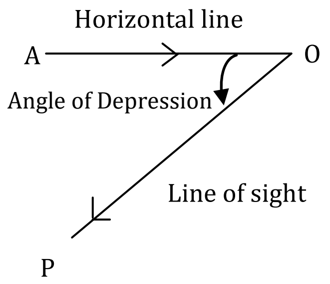
θ	0°	$\frac{\pi}{6}, 30^\circ$	$\frac{\pi}{4}, 45^\circ$	$\frac{\pi}{3}, 60^\circ$	$\frac{\pi}{2}, 90^\circ$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined

4. **Angle of Elevation:** Suppose a man from a point O looks up at an object P, placed above the level of his eye. Then, the angle which the line of sight makes with the horizontal through O, is called the angle of elevation of P as seen from O:

   $\therefore$ Angle of elevation of P from O = $\angle AOP$



5. Angle of Depression: Suppose a man from a point O looks down at an object P, placed below the level of his eye, then the angle which the line of sight makes with the horizontal through O, is called the angle of depression of P as seen from O.



SOLVED EXAMPLES

Example 1 If the height of a pole is $2\sqrt{3}$ meters and the length of its shadow is 2 metres, find the angle of elevation of the sun.

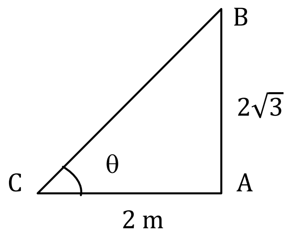
Solution: Let AB be the pole and AC be its shadow.

Let angle of elevation, $\angle ACB = \theta$.

Then, $AB = 2\sqrt{3}$ m, AC = 2 m.

$\tan\theta = \frac{AB}{AC} = \frac{2\sqrt{3}}{2} = \sqrt{3} \Rightarrow \theta = 60^0$

So, the angle of elevation is 60°.

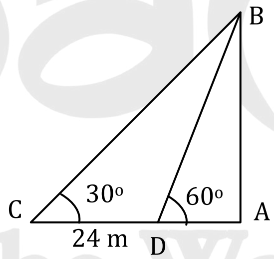


Example 2 The angle of elevation of the top of a tower at a point on the ground is 30°. On walking 24 m towards the tower, the angle of elevation becomes 60°. Find the height of the tower.

Solution: Let AB be the tower and C and D be the points of observation. Then,

$\frac{AB}{AD} = \tan 60° = \sqrt{3} \Rightarrow AD = \frac{AB}{\sqrt{3}} = \frac{h}{\sqrt{3}}$

$\frac{AB}{AC} = \tan 30^0 = \frac{1}{\sqrt{3}} \Rightarrow AC = AB\sqrt{3} = h\sqrt{3}$



$CD = (AC - AD) = \left(h\sqrt{3} - \frac{h}{\sqrt{3}}\right)$

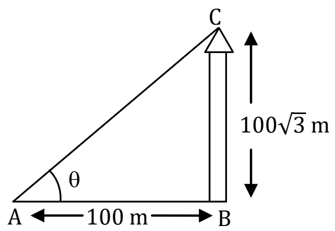
$\therefore \left(h\sqrt{3} - \frac{h}{\sqrt{3}}\right) = 24 \Rightarrow h = 12\sqrt{3} = (12 \times 1.73) = 20.76$ m

Example 3 Find the angle of elevation of the top of a tower of height $100\sqrt{3}$ m from a point at a distance of 100 m from the foot of the tower of an horizontal plane.

Solution: Let the height of the tower $BC = 100\sqrt{3}$ m and AB = 100 m

$\therefore$ In ΔABC $\tan\theta = \frac{BC}{AC} \Rightarrow \tan\theta = \frac{100\sqrt{3}}{100} = \sqrt{3}$

$\tan\theta = \sqrt{3} \Rightarrow \theta = 60°$.



Example 4 A tree breaks due to the storm, and the broken part bends so that the top of the tree touches the ground making an angle of 30° with the ground. The distance from the foot of the tree to the point, where the top touches the ground is 10 metres. Find the height of the tree.

Solution: Let AB be a tree

The broken part AC touches the ground at D, making an angle of 30° with the ground

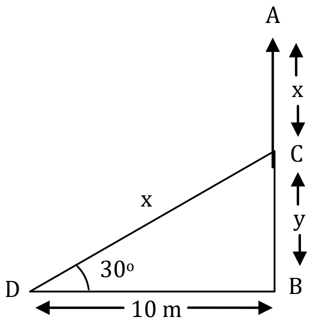
Let AC = x $\therefore$ CD = x

In right angles ΔBCD $\tan 30° = \frac{BC}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{y}{10} \quad \Rightarrow y = \frac{10}{\sqrt{3}}$

Also $\cos 30^\circ = \frac{BD}{DC} \quad \Rightarrow \frac{\sqrt{3}}{2} = \frac{10}{x} \quad \Rightarrow x = \frac{20}{\sqrt{3}}$ m

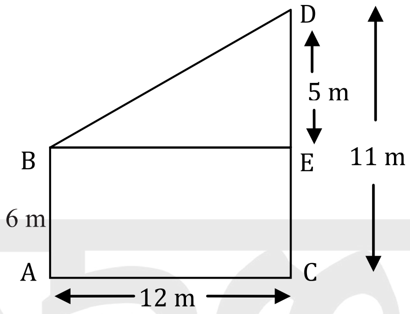
Height of the tree = AB = x + y = $\frac{20}{\sqrt{3}} + \frac{10}{\sqrt{3}} = 10\sqrt{3}$ m.



Example 5 Two poles of height 6 m and 11 m stand vertically on the ground. If the distance between their feet is 12 m. Find the distance between their tops

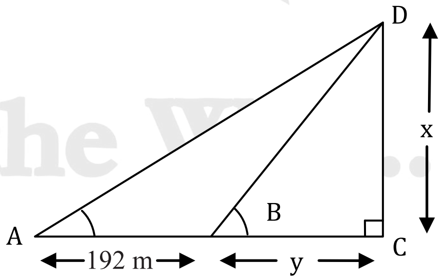
Solution: Let AB and CD represent the poles and AC is the distance between their feet.

Let BE ⊥ CD
∴ BE = AC = 12 m
DE = CD – EC = 11 – 6 = 5 m
In right angle Δ BED
$BD^2 = BE^2 + DE^2$
$= 12^2 + 5^2 = 144 + 25 = 169$
∴ BD = $\sqrt{169}$= 13 m
Distance between the tops of the poles = 13 m.



Example 6 At a point on level ground, the angle of elevation of a vertical tower is found to be such that its tangent is 5/12 on walking 192 meters towards the tower, the tangent of the angle is found to be 3/4. Find the height of the tower.

Solution: Suppose height of the tower CD = x m
Let A and B be the points of observations.
And distance BC = y
$\tan A = \frac{5}{12}$, $\tan B = \frac{3}{4}$



now in right angle Δ BCD

$\tan B = \frac{CD}{BC} \rightarrow \frac{x}{y} = \frac{3}{4}$ ….. (i)

again in right angle ΔACD

$\tan A = \frac{CD}{AC} \Rightarrow \frac{CD}{AB+BC} = \tan A$

$\rightarrow \frac{x}{192+y} = \frac{5}{12}$ ….. (ii)

Dividing (i) and (ii) we get,

$\Rightarrow \frac{x}{y} \times \frac{192+y}{x} = \frac{3}{4} \times \frac{12}{5}$

$\Rightarrow \frac{192+y}{y} = \frac{9}{5} \rightarrow 9y = 5(192 + y)$

$\rightarrow 9y - 5y = 960 \qquad \rightarrow 4y = 960 \qquad \therefore y = 240$ m.

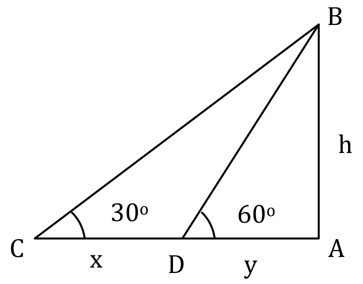
Putting the value of y in (i) we get

$\frac{x}{240} = \frac{3}{4} \rightarrow 4x = 720 \rightarrow x = 180$ m

Example 7 A man on the top of a tower, standing on the seashore finds that a boat coming towards him takes 10 minutes for the angle of depression to change from 30° to 60°. Find the time taken by the boat to reach the shore from this position.

Solution: Let AB be the tower and C and D be the two positions of the boat.

Let AB = h, CD = x and AD = y.



$\frac{h}{y} = \tan 60° = \sqrt{3}$ → $y = \frac{h}{\sqrt{3}}$

$\frac{h}{x+y} = \tan 30° = \frac{1}{\sqrt{3}}$ → $x + y = \sqrt{3}h.$

$\therefore x = (x + y) - y = \left(\sqrt{3}h - \frac{h}{\sqrt{3}}\right) = \frac{2h}{\sqrt{3}}$

Now $\frac{2h}{\sqrt{3}}$ is covered in 10 min.

$\therefore \frac{h}{\sqrt{3}}$ will be covered in $\left(10 \times \frac{\sqrt{3}}{2h} \times \frac{h}{\sqrt{3}}\right) = 5$ min.

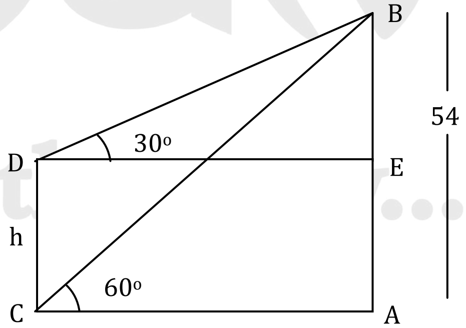
Example 8 There are two temples, one on each bank of a river, just opposite to each other. One temple is 54 m high. From the top of this temple, the angles of depression of the top and the foot of the other temple are 30° and 60° respectively. Find the width of the river and the height of the other temple.

Solution: Let AB and CD be the two temples and AC be the river.

Then, AB = 54 m.

Let AC = x metres and CD = h metres.

$\angle ACB = 60°$, $\angle EDB = 30°$.



$\frac{AB}{AC} = \tan 60° = \sqrt{3}$

$\Rightarrow AC = \frac{AB}{\sqrt{3}} = \frac{54}{\sqrt{3}} = \left(\frac{54}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}\right) = 18\sqrt{3}\ m$

DE = AC = $18\sqrt{3}$ m.

$\frac{BE}{DE} = \tan 30^0 = \frac{1}{\sqrt{3}}$

→ BE = $\left(18\sqrt{3} \times \frac{1}{\sqrt{3}}\right)$ = 18 m.

$\therefore$ CD = AE = AB – BE = (54 – 18) m = 36 m.

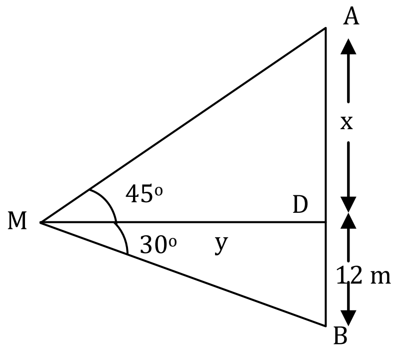
So, width of the river = AC = $18\sqrt{3}$ m = (18 x 1.73) m = 31.14 m.

Height of the other temple = CD = 36 m.

Example 9 A man on the deck of a ship is 12 m above water level. He observes that the angle of elevation of the top of a cliff is 45°, and the angle of depression of the base is 30°. Calculate the distance of the cliff from the ship and the height of the cliff.

Solution: Let the cliff be AB and point of observation is M and deck of ship BD is 12 m.

Let AD = x, MD = y



In right angled ΔMDA

$\frac{x}{y} = \tan 45°$ → $x = y$ [$\tan 45° = 1$]

In right angled ΔBDM

$\tan 30^0 = \frac{BD}{MD}$

→ $\frac{12}{y} = \tan 30°$

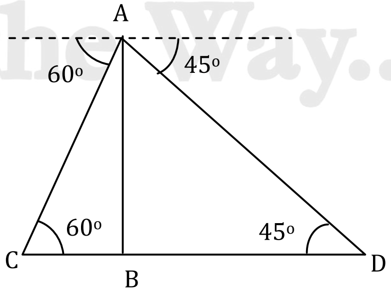
→ $\frac{12}{y} = \frac{1}{\sqrt{3}}$ → $y = 12 \times \sqrt{3} = 12 \times 1.732 = 20.784$ m

Height of cliff = $12 + 12\sqrt{3} = 32.784$ m

Example 10 Two ships are sailing in the sea on the either side of the light house, the angles of depression of two ships as observed from the top of the light-house are 60° and 45° respectively. If distance between 2 ships is $200\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right)$. Find height of the tower?

Solution: AB = height of the light house AB = h m

C, D = be the position of two ships.



Such that CD = $200\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right)$

In right angled Δ ABC, $\frac{AB}{BC} = \tan 60°$

$\frac{h}{CB} = \sqrt{3}$ → $CB = h/\sqrt{3}$

In right angled Δ ABD, $\frac{AB}{BD} = \tan 45°$ → $\frac{h}{BD} = 1$ → $h = BD$

Now CD = CB + BD

$CD = \frac{h}{\sqrt{3}} + h$ → $200\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right) = h\left(\frac{1}{\sqrt{3}}+1\right)$

→ $h = \frac{200\left(\sqrt{3}+1\right)}{\left(\sqrt{3}+1\right)} = 200$

Hence the height of the light house = 200 m.

EXPERIENCE THE PRATHAM EDGE - 16

1. From a point P on a level ground, the angle of elevation of the top of a tower is 30°. If the tower is 100 m high, the distance of point P from the foot of the tower is:
   (a) 149 m (b) 156 m (c) 173 m (d) 200 m

2. A man is watching from the top of a tower a boat speeding away from the tower. The boat makes an angle of depression of 45° with the man's eye when at a distance of 60 metres from the tower. After 5 seconds, the angle of depression becomes 30°. What is the approximate speed of the boat, assuming that it is running in still water?
   (a) 32 kmph (b) 36 kmph (c) 38 kmph (d) 40 kmph

3. The top of a 15 metre high tower makes an angle of depression of 60° with the bottom of an electric pole and angle of depression of 30° with the top of the pole. What is the height of the electric pole?
   (a) 5 metres (b) 8 metres (c) 10 metres (d) 12 metres

4. A man on the top of a vertical tower observes a car moving at a uniform speed coming directly towards it. If it takes 12 minutes for the angle of depression to change from 30° to 45°, how soon after this will the car reach the observation tower?
   (a) 14 min. 35 sec. (b) 15 min. 49 sec. (c) 16 min. 23 sec. (d) 18 min. 5sec.

5. A man standing at a point P is watching the top of a tower, which makes an angle of elevation of 30° with the man's eye. The man walks some distance towards the tower to watch its top and the angle of elevation becomes 60°. What is the distance between the base of the tower and the point P?
   (a) $4\sqrt{3}$ units (b) 8 units (c) 12 units (d) Data inadequate

6. The angle of elevation of the sun, when the length of the shadow of a tree is $\sqrt{3}$ times the height of the tree, is:
   (a) 30° (b) 45° (c) 60° (d) 90°

7. The angle of depression from the top of a light-house of two boats are 45° and 30° towards the east. If the two boats are 60 m apart, then the height of the light-house is:
   (a) $60/\sqrt{3}$ (b) $30/(\sqrt{3}+1)$ (c) $30/(\sqrt{3}-1)$ (d) $60/(\sqrt{3}-1)$

8. Two men on either side of a cliff 100 m high observe the angle of elevation on the top of the cliff to be 30° and 45° respectively. The distance between the two men is:
   (a) $100\sqrt{3}$ m (b) 200 m (c) $100(\sqrt{3}+1)$ (d) $100(\sqrt{3}-1)$

9. The angle of elevation of the top of an incomplete tower at a point 40 m from its base is 45°. If the elevation of the completed tower at the same point is 60°, then height through which the tower has been raised is:
   (a) $40\sqrt{3}$m (b) $40(\sqrt{3}+1)$ m (c) $40(1-\frac{1}{\sqrt{3}})$ (d) $40(\sqrt{3}-1)$

10. The upper part of a tree broken by the wind in two parts, makes an angle of 60° with the ground. If the top of the tree touches the ground at a distance of $2\sqrt{3}$ m from the foot of the tree, then the height of the tree is equal to:
    (a) 14 m (b) $(6+2\sqrt{3})$ m (c) $(6-4\sqrt{3})$ m (d) $(6+4\sqrt{3})$ m

11. From the top of a 60 m high tower, the angle of depression of the top and bottom of a building are observed to be 30° and 60° respectively. The height of the building is:
(a) $60\sqrt{3}$ m (b) $40\sqrt{3}$m (c) 40 m (d) 20 m

12. The angles of elevation of a cloud from a point h m above the surface of a lake is 30° and the angle of depression of its reflection is 60°. Then the height of the cloud above the surface of the lake is:
(a) $h\sqrt{3}$ (b) h/3 (c) $\sqrt{2}$h (d) 2 h

13. The angle of elevation of a jet fighter from a point C on the ground is 60°. After five seconds of flight, the angle of elevation changes to 45°. If the jet is flying at a height of 3000 m, then the speed of the jet, in m/s, is:
(a) $1000(3 - \sqrt{3})$ (b) $200(3 - \sqrt{3})$ (c) $1000\sqrt{3}$ (d) 600

14. Two posts are k m apart. If from the middle point of the line joining their feet, an observer finds the angles of elevations of their tops to be 60° and 30° respectively, then the ratio of heights of the posts is:
(a) $\sqrt{3}$ (b) $\frac{k}{\sqrt{3}}$ (c) $k\sqrt{3}$ (d) 3

15. Two observers are stationed due east of a tower at a distance of 20 m from each other. If the elevations of the tower observed by them are 30° and 45°, respectively, then the height of the tower is:
(a) 10 m (b) 17.32 m (c) $10(\sqrt{3} + 1)$ m (d) 30 m

16. The angle of elevation of the top of a tree of height 18 m is 30° when measured from a point P in the plane of its base. The distance of the base of the tree from P is:
(a) 6 m (b) $6\sqrt{3}$ m (c) 18 m (d) $18\sqrt{3}$ m

17. A person aims at a bird on top of a 5 m high pole with an elevation of 30°. If a bullet is fired, it will travel k m before reaching the bird. The value of k (in m) is:
(a) $10/\sqrt{3}$ (b) 10 (c) $5\sqrt{3}$ (d) $10\sqrt{3}$

18. A light house is 60 m high, its base being at the sea level. If the angle of depression of a boat in the sea from the top of the light house is 15°, then the distance of the boat from the foot of the light house is equal to:
(a) $\frac{60(1-\sqrt{3})}{(\sqrt{3}-1)}$ m (b) $\frac{60(\sqrt{3}+1)}{(\sqrt{3}-1)}$ m (c) $\frac{60(1+\sqrt{3})}{(\sqrt{3}+2)}$ m (d) $\frac{60(1-\sqrt{3})}{(\sqrt{3}+2)}$ m

19. The angles of elevation of the top of a tower from two points at distances m and n metres are complementry. If the two points and the base of the tower are on the same straight line, then the height of the tower is
(a) $\sqrt{mn}$ (b) mn (c) m/n (d) None of these

20. A man is standing on the 8 m long shadow of a 6 m long pole. If the length of the shadow is 2.4m, then the height of the man is
(a) 1.4 m (b) 1.6 m (c) 1.8 m (d) 2.0 m

CHAPTER 17

PROGRESSIONS

INTRODUCTION

A succession of numbers, a_1, a_2, a_3, _ _ _ are formed according to a certain definite rule is called a sequence or progression. In this chapter we will study the following progressions:
- Arithmetic Progression
- Geometric Progression
- Harmonic Progression

Arithmetic Progression (A.P):
The progression of the form a, a + d, a + 2d, a + 3d… is known as A.P. with first term = a and common difference = d.
In an A.P. a, a + d, a + 2d, a + 3d… We have:

(i) n th term, $T_n = a + (n - 1)d$. where, $n > 0$

(ii) Sum of n terms, $s_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a + c)$, where c is the last term

(iii) If a, b, c are in A.P., then b is called arithmetic mean (A.M.) between a and c.
In this case, $b = \frac{a+c}{2}$

Geometric Progression (G.P):
The Progression of the form a, ar, ar^2, ar^3… is known as G.P. with first term = a and common ratio = r.

(i) nth term, $T_n = ar^{n-1}$

(ii) Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$ when r < 1 and when r > 1, $S_n = \frac{a(r^n-1)}{r-1}$

(iii) If a, b, c are in G.P., then b is the geometric mean (G.M.) between a and c. In this case, $b^2 = ac$

Harmonic Progression (H.P.) :
If $\frac{1}{a_1}, \frac{1}{a_2} \cdots\cdots\cdots \frac{1}{a_n}$ is in A.P. then the Progression a_1, a_2, a_3, …. is called a H.P.

(i) If a, b, c are in H.P., then b is the harmonic mean between a and c. $\left(\frac{2}{b} = \frac{1}{a} + \frac{1}{c}\right)$
In this case, $b = \frac{2ac}{a+c}$

Relation between A.M., G.M. and H.M.

Let there are two numbers 'a' and 'b', a, b > 0 then, AM = $\frac{a+b}{2}$ GM = $\sqrt{ab}$ HM = $\frac{2ab}{a+b}$

$\therefore$ AM × HM = $\frac{a+b}{2} \times \frac{2ab}{a+b} = ab = \left(\sqrt{ab}\right)^2 = (GM)^2$

Note that these means are in G.P.
Hence AM, GM, HM follows the rules of G.P. i.e. G.M. = $\sqrt{A.M. \times H.M.}$
Now, let us see the difference between AM and GM

AM – GM = $\frac{a+b}{2} - \sqrt{ab} = \frac{(\sqrt{a})^2 + (\sqrt{b})^2 - 2\sqrt{a}\sqrt{b}}{2} = \frac{(\sqrt{a} - \sqrt{b})^2}{2} > 0$ i.e. AM ≥ GM ….(a)

Similarly, G.M. – H.M. = $\sqrt{ab} - \frac{2ab}{a+b} = \frac{\sqrt{ab}(\sqrt{a}-\sqrt{b})^2}{a+b} > 0$ So, GM ≥ HM ….(b)

Combining both results (a) and (b), we get AM ≥ GM ≥ HM

SOLVED EXAMPLES

Example 1 Find the sum of all odd numbers lying between 10 and 100.

Solution: The required numbers will be 11, 13, 15, 17, 19… 99.

Let the number of terms be n. Then, $99 = 11 + (n - 1) \times 2$ or $n = 45$.

Required sum $= \frac{n}{2}(a + c) = \frac{45}{2} \times (11 + 99) = 2475$

Example 2 Find 10^{th} term of the G.P. 7, 14, 28, …

Solution: In this G.P. we have $a = 7$ and $r = 2$

10^{th} term $= ar^{(10-1)} = ar^9 = 7 \times 2^9 = (7 \times 512) = 3584$.

Example 3 A boy saves Rs. 1 on first day of the month, Rs.2. on 2^{nd} day of the month, Rs 4 on third day, Rs.8 on 4^{th} day. Rs. 16 on 5^{th} day and so on. How much does he save in 12 days?

Solution: His savings are 1, 2, 4, 8, 16… up to 12 terms.

This is a G.P. with $a = 1$ and $r = 2$.

Savings = Sum of above G.P. up to 12 terms $= \frac{a(r^n - 1)}{r - 1} = \frac{1 \times (2^{12} - 1)}{2 - 1} = 4095$

Example 4 Find the 14^{th} term of an A.P., whose first term is 3 and common difference is 2.

Solution: n^{th} term $= a + (n - 1)d$, where a is the 1^{st} term, d is the common difference.

$\therefore$ 14^{th} term $= 3 + (14 - 1)2 = 29$

Example 5 Find the number of terms in an A.P., if the first term is 2 and the last term is 41. Given, the common difference is 3.

Solution: Last term in an A.P $= a + (n - 1)d = 2 + (n - 1)3$

$\rightarrow 3n - 1 = 41$

$3n = 42$ $\therefore n = 14$

So. There are 14 terms in an A.P.

Example 6 Find the first term and the common difference of an A.P. if the 3^{rd} term is 6 and the 17^{th} term is 34.

Solution: If a is the first term and the common difference d, then we have

$a + 2d = 6$ ….. (1)

$a + 16d = 34$ ….(2)

(2) – (1) gives

$14d = 28$ $\therefore d = 2$

Substituting this value of d in (1), we get $a = 2$

$\therefore$ $a = 2$ and $d = 2$

First term = 2; Common difference = 2

Example 7 Find the 1^{st} term, common difference and number of terms of an A.P. given its 4^{th} term is 12, 16^{th} term is 20 and last term is 26.

Solution: Let 1^{st} term = a

Common difference = d

Number of terms = n

$a + 3d = 12$ …. (1)

$a + 15d = 20$ …. (2)

$a + (n - 1)d = 26$ ….(3)

Solving (1) and (2) we get $d = 2/3$ and $a = 10$ Substituting these values in (3)

We get $n = 25$.

Example 8 Find the 7^{th} term of the G.P., whose 1^{st} term is 6 and common ratio is $\frac{2}{3}$.

Solution: n^{th} term $= a \cdot r^{n-1}$; 7^{th} term $= 6\left(\frac{2}{3}\right)^6$

$$= \frac{6\times 64}{729} = \frac{384}{729} = \frac{128}{243}$$

Example 9 Find the sum to 5 terms of a G.P. whose 1^{st} term is 16 and the common ratio $\frac{1}{2}$.

Solution: Sum to 5 terms $= \frac{a(1-r^n)}{(1-r)}$

$$= \frac{16\left\{1-\left(\frac{1}{2}\right)^5\right\}}{\left(1-\frac{1}{2}\right)} = \frac{16\left(1-\frac{1}{32}\right)}{\left(1-\frac{1}{2}\right)} = 16\times\frac{31}{32}\times\frac{2}{1} = 31$$

Example 10 Find the 3 numbers in G.P. whose sum is 26 and product is 216.

Solution: Let the 3 numbers be $\frac{a}{r}$, a and ar.

Given that

$\frac{a}{r} \times a \times ar = 216$; → $a^3 = 216$; $a = 6$

$\frac{a}{r} + a + ar = 26$ → $6r^2 + 6r + 6 = 26r$

→ $6r^2 - 20r + 6 = 0$

→ $6r^2 - 18r - 2r + 6 = 0$

→ $6r(r-3) - 2(r-3) = 0$

∴ $r = 1/3$ (or) $r = 3$

Hence the three numbers are 2, 6 and 18.

Note : Even if the other value of r is taken, values of numbers will be same.

Example 11 How many terms are there in an A.P.?

20, 25, 30, ….. 100

Solution: Let n = No. of terms

$t_n = 100$, $a = 20$, $d = 5$

∴ $t_n = a+(n-1)d$

$100 = 20+(n-1)5$ → $80 = (n-1)5$

→ $n - 1 = 16$ → $n = 17$

Example 12 Find $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\ldots$ to n terms.

Solution:

Here given sequence is a G.P. and $a = 1$, $r = \frac{1}{2}$, $n = n$

$$\rightarrow S_n = \frac{a(1-r^n)}{1-r} = \frac{1.\left[1-\left(\frac{1}{2}\right)^n\right]}{1-\frac{1}{2}} = 2\left[1-\left(\frac{1}{2}\right)^n\right]$$

EXPERIENCE THE PRATHAM EDGE - 17

1. How many numbers are there between 103 and 750 which are divisible by 6?
(a) 125 (b) 108 (c) 107 (d) 113

2. The sum of all 2 digit numbers is:
(a) 4750 (b) 4905 (c) 3776 (d) 4893

3. If (k + 1), 3k and (4k + 2) be any three consecutive terms of an A.P. then the value of k is :
(a) 3 (b) 0 (c) 1 (d) 2

4. The 5^{th} and 13^{th} terms of an A.P. are 5 and -13 respectively. The first term of the A.P. is :
(a) 1 (b) 14 (c) -15 (d) 2

5. The sum of the series 5 + 13 + 21 + …. + 181 is:
(a) 2139 (b) 2475 (c) 472 (d) 2219

6. The sum of all odd numbers between 100 and 200 is:
(a) 6200 (b) 6500 (c) 7500 (d) 3750

7. Arithmetic mean between 14 and 18 is:
(a) 16 (b) 15 (c) 17 (d) 32

8. How many terms of the A.P. 3, 6, 9, 12, 15… must be taken to make the sum 108?
(a) 6 (b) 7 (c) 8 (d) 36

9. The 8^{th} term of the G.P. 2, 6, 18, 54… is:
(a) 2187 (b) 4374 (c) 1458 (d) 13122

10. If n^{th} term of G.P. is 2^n, then its 6^{th} term is :
(a) 2^6 (b) 2^5 (c) $2^6/6$ (d) $(2^6 \times 6)$

11. If a, b, c are in G.P. and $a^x = b^y = c^z$, then x, y, z are in :
(a) A.P. (b) G.P. (c) H.P. (d) None

12. If a, b, c are in G.P., then log a, log b, log c are in :
(a) A.P. (b) G.P. (c) H.P. (d) None

13. If $\frac{1}{3}$, y_1, y_2, 9 are in G.P., the value of y_2 is :
(a) 3 (b) 6 (c) 1 (d) Cannot be determined

14. Relation between A.M. and G.M. is:
(a) A.M. $\leq$ G.M. (b) A.M. $\geq$ G.M. (c) A.M. $= \frac{3}{4}$ GM (d) None of these

15. If a, G, b is in G.P. then:
(a) $G = ab$ (b) $G^2 = ab$ (c) $G = \frac{1}{3}ab$ (d) $G = \frac{1}{3}(a+b)$

16. The 6^{th} and 8^{th} terms of an A.P. are 12 and 22 respectively its 2^{nd} term is:
(a) 8 (b) -8 (c) 6 (d) -3

17. The 3^{rd} and 5^{th} terms of G.P. are 12 and 48. Its second term is:
(a) ± 6 (b) 4 (c) 8 (d) 1/2

18. The AM of 2 numbers is 34 and their GM is 16. One of the numbers will be
(a) 4 (b) 16 (c) 18 (d) 12

19. Find the sum of all numbers between 100 and 500 divisible by 6.
(a) 67 (b) 19,600 (c) 30,000 (d) 20,100

20. Find four numbers in Arithmetic Progression such that their sum is 40 and the product of the first and the last is 64.
(a) 2, 6, 10, 16 (b) 8, 12, 16, 20, (c) 4, 8, 12, 16 (d) 1, 5, 9, 13

21. If sum to n terms of a sequence be $n^2 + 2n$ then that sequence will be
(a) A. P. (b) G. P. (c) H. P. (d) None of these

22. If x, y, z be the 4^{th}, 10^{th} and 16^{th} terms of a G. P. then x, y, z will be in
(a) A. P. (b) G. P. (c) H. P. (d) None of these

23. $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+.....$ to n terms
(a) $2^{-n} - 1$ (b) $2^{-n} - n - 1$ (c) $2^{-n} + n - 1$ (d) $1 - 2^{-n}$

24. If $S = 1 + a + a^2 + a^3 + a^4 + ...$ to infinity then a = ?
(a) $\frac{S}{S-1}$ (b) $\frac{S}{1-S}$ (c) $\frac{S}{1+S}$ (d) $\frac{S-1}{S}$

25. $4^3 + 5^3 + 6^3 + ... + 10^3 = ?$
(a) 2989 (b) 1918 (c) 1819 (d) 2829

CHAPTER 18

PERMUTATIONS & COMBINATIONS

INTRODUCTION

Permutations and combinations deals with the arrangement and selection of a set of given items. The notations used for permutations is ^nP_r and that used for combinations is ^nC_r.

1. Permutations of n different things taken 'r' at a time is denoted by ^nP_r and is given by $^nP_r = \frac{n!}{(n-r)!}$

2. The total number of permutations of n dissimilar things taken r at a time with repetitions = n^r

3. (a) Number of circular permutations of n different things taken all at a time = (n – 1)!
   (b) Number of circular permutations of n different things taken r at time = $\frac{^nP_r}{r}$

   (c) Number of circular permutations of n different things when clockwise and anticlockwise directions are same = $\frac{(n-1)!}{2}$

4. The number of permutations when things are not all different : if there are n things, p of them of one kind, q of another kind, r of still another kind and so on, then the total number of permutations of all the things is given by $\frac{n!}{(p!q!r!......)}$

5. Number of combinations of n dissimilar things taken 'r' at a time is denoted ^nC_r and is given by
   $^nC_r = \frac{n!}{[(n-r)!r!]}$

6. $^nC_r = {}^nC_{n-r}$

7. n! = n(n-1)(n-2)(n-3) 3.2.1

8. $^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n = 2^n$

9. $^nP_r = r! \times {}^nC_r$

10. $^nC_x = {}^nC_y$, then either x = y or x + y = n.

SOLVED EXAMPLES

Example 1 Find the total number of ways in which the letters of the word HEXAGON can be rearranged?

Solution: HEXAGON has 7 different letters total no. of arrangements = 7! - 1 = 5039 ways

Example 2 Find the total number of ways in which the letters of the word ARRANGE can be arranged?

Solution: ARRANGE has 7 letters with 2 A's and 2 R's

Total no. of arrangements = $\frac{7!}{2! \times 2!}$ = 1260 ways

Example 3 Find the total number of ways in which the letters of the word HEXAGON can be arranged when the vowels are always together?

Solution: HEXAGON has 3 vowels (E, A, O) and 4 consonants.
4 + 1 Group = 5 things have to be arranged.
But 3 vowels can be arranged in between them.
Total no. of arrangements = 5! × 3! = 720 ways

Example 4 Find the total number of ways in which the letters of the word HEXAGON can be arranged so that vowels are never together?

Solution: Total no. of arrangements = 5040 -720 = 4320

Example 5 How many different numbers can be formed by using any four digits out of 1, 2, 3, 4, 5, 6, 7, 8, 9 without repetition ?

Solution: There are 9 different digits from which 4 are to be taken
Number of arrangements = $^9P_4 = 3024$

Example 6 How many three digit even numbers can be formed using the digits 0, 1, 2, 3, 4, 5 without repetition?

Solution: If we fix 0 at unit place
then three digit even numbers without repetion = 4 x 5= 20
Similarly, if we fix 2 at unit place = 4 x 4 = 16
If we fix 4 at unit place = 4 x 4 = 16
$\therefore$ Total number formed = 20 + 16 + 16 = 52

Example 7 With 3 red, 2 yellow, and 2 green flags, how many different signals can be made?

Solution: Total no. of flag = 3 + 2 + 2 = 7
Total no. of signals = $\frac{7!}{3!\times 2!\times 2!} = 210$

Example 8 Find the total number of ways in which 8 people can sit on a round table with two particular people always sitting together?

Solution: With 2 people sitting together we have – 6 people + 1 Group.
Number of arrangements = $(7 - 1)! \times 2!$
Since, In circle, we subtract by 1 = $(6)! \times 2!$ = 1440 ways

Example 9 In an examination paper there are 6 questions. In how many ways can an examinee choose 4 questions

Solution: Number of choices = $^6C_4 = \frac{6!}{4!\times 2!} = 15$

Example 10 In a party there were 66 hand shakes when each person shook hands with the other person only once. Find how many people attended the party?

Solution: If 'n' people attended
No. of hand shakes = $^nC_2 = 66$
or $\frac{n(n-1)}{2} = 66$ $\quad$ n = 12

Example 11 Find the number of diagonals in a decagon?

Solution: Number of diagonals = (Total lines – sides)
Total number of lines from 10 points = $^{10}C_2$
Number of Diagonals = $^{10}C_2 - 10 = 35$

Example 12 If P(n, r) = 720 and C (n, r) = 120, find r

Solution: $C(n, r) = \frac{P(n,r)}{r!}$
$120 \times r! = 720 \quad r! = 6 = 3!$ or $r = 3$

Example 13 Evaluate: $\frac{30!}{28!}$

Solution: We have, $\frac{30!}{28!} = \frac{30\times 29\times (28!)}{28!} = (30\times 29) = 870$

EXPERIENCE THE PRATHAM EDGE - 18

1. How many signals can be made with 5 different flags by hosting any number of them in a vertical line?
(a) 300 (b) 325 (c) 400 (d) 625

2. How many numbers of 6 digits can be formed from the digits 4, 5, 6, 7, 8, 9, no digits being repeated?
(a) 200 (b) 400 (c) 500 (d) 720

3. How many three digits are there, with no digit repeated?
(a) 648 (b) 720 (c) 764 (d) 986

4. In a group of 6 boys and 4 girls, four children are to be selected. In how many different ways can they be selected such that at least one boy should be there?
(a) 159 (b) 194 (c) 205 (d) 209

5. In how many ways can the letters of the word PENCIL, be arranged so that N is always next to E?
(a) 100 (b) 120 (c) 140 (d) 160

6. There are three blue balls, four red balls and five green balls. In how many ways can they be arranged in a row?
(a) 27720 (b) 2770 (c) 26720 (d) 26620

7. How many 4 digit even numbers are there such that of 5 is one of the digits and 7 is the following digit.
(a) 180 (b) 145 (c) 185 (d) None of these

8. In how many ways can the letter of the word ARRANGE be permuted?
(a) 1260 (b) 2432 (c) 3625 (d) 4865

9. In how many ways can 6 boys and 5 girls be arranged for a group photograph, if the girls are to sit on chairs in a row and the boys are to stand in a row behind them?
(a) 86400 (b) 72000 (c) 120000 (d) 764000

10. If all the permutations of the word AGAIN are arranged for a dictionary, what is the 50^{th} word?
(a) NAAGI (b) NAAIG (c) NAIAG (d) NAIGA

11. How many numbers can be formed with the digits 1, 2, 3, 4, 3, 2, 1 so-that odd digits always occupy the odd places.
(a) 16 (b) 18 (c) 20 (d) 24

12. Find the number of ways in which 5 different beads can be arranged to form a necklace.
(a) 60 (b) 90 (c) 24 (d) 12

13. A family of 4 brothers and 3 sisters is to be arranged for a photograph in one row. In how many ways can they be seated if all the sisters sit together.
(a) 120 (b) 600 (c) 720 (d) 5040

14. In how many ways can a student choose 5 courses out of 9 courses, if 2 courses are compulsory?
(a) 30 (b) 35 (c) 45 (d) 60

15. A man has 5 friends. In how many ways can he invite one or more of his friends to dinner?
(a) 30 (b) 31 (c) 35 (d) 60

16. A boy has 3 library tickets and 8 books of his interest in the Library. Of these 8, he does not want to borrow Mathematics Part II unless Mathematics Part I is also borrowed. In how many ways can he choose the three books to be borrowed?

(a) 38 (b) 40 (c) 41 (d) 42

17. From a class of 25 students, 10 are to be chosen for an excursion. There are 3 students who decide that either all of them will join or none of them will join. In how many ways can they be chosen?
(a) 817190 (b) 817000 (c) 819710 (d) 819770

18. A regular polygon has 23 vertices and consequently 23 sides. How many additional lines need to be drawn so that every pair of vertices will be connected?
(a) 210 (b) 230 (c) 240 (d) 255

19. From a class of 14 boys and 10 girls, ten students are to be chosen for a competition, at least including 4 boys and 4 girls. The 2 girls who won the prize last year should be included. In how many ways can the selection be made?
(a) 266266 (b) 196196 (c) 182182 (d) 154154

20. A man has 6 friends. In how many ways can he invite one or more to a party?
(a) 31 (b) 63 (c) 127 (d) 100

21. How many numbers greater than 2000 can be formed with the digits 1, 2, 3, 4, 5, without repetition ?
(a) 210 (b) 216 (c) 256 (d) 280

22. In a cultural festival, 6 programmes are to be staged, 3 on a day for 2 days. In how many ways could the programmes be arranged ?
(a) 320 (b) 360 (c) 675 (d) 720

23. How many numbers lying between 3000 and 4000 are made with the digits 3, 4, 5, 6, 7 and 8 without repetition are divisible by 5 ?
(a) 5! (b) 4! (c) 12 (d) 6

24. Five persons A, B, C, D and E occupy seats in a row such that A and B sit next to each other. In how many possible ways can these 5 people sit?
(a) 24 (b) 48 (c) 72 (d) 96

25. In a tournament 7 teams are participating. Each team plays with every other participating team once and the winner is decided by the total points accumulated by the end of all these matches. Find the total number of matches in the tournament.
(a) 71 (b) 70 (c) 20 (d) 21

26. In how many different ways can the letters of the word 'LEADING' be arranged in such a way that the vowels always come together?
(a) 360 (b) 480 (c) 720 (d) 5040

27. In how many different ways can the letters of the word 'JUDGE' be arranged in such a way that the vowels always come together?
(a) 48 (b) 120 (c) 124 (d) 160

28. A box contains 2 white balls, 3 black balls and 4 red balls. In how many ways can 3 balls be drawn from the box, if at least one black ball is to be included in the draw?
(a) 32 (b) 48 (c) 64 (d) 96

29. From a group of 7 men and 6 women, five persons are to be selected to form a committee so that at least 3 men are there on the committee. In how many ways can it be done?
(a) 564 (b) 645 (c) 735 (d) 756

30. How many parallelograms will be formed if 7 parallel horizontal lines intersect 6 parallel vertical lines ?
(a) 42 (b) 294 (c) 315 (d) None of these

NOTES

CHAPTER 19

PROBABILITY

INTRODUCTION

Probability means the chance or occurrence of an event.

P (Occurrence of an event) = $\frac{\text{No.of favorable outcomes}}{\text{Total no.of outcomes}}$

For an event A is sure to occur, P(A) = 1

For an event A is sure not to occur, P(A) = 0

For any event, $0 \leq P(A) \leq 1$

Sample Space: It is defined in the context of a random experiment and denotes the set representing all the possible outcomes of the random experiment. [e.g. Sample space when a coin is tossed is (Head, Tail). Sample space when a dice is thrown is (1, 2, 3, 4, 5, 6).

Mutually Exclusive Events: A set of events is mutually exclusive when the occurrence of any one of them means that the other events cannot occur. (If head appears on a coin, tail will not appear and vice versa.)

Equally Likely Events: If two events have the same probability or chance of occurrence they are called equally likely events. (In a throw of a dice, the chance of 1 showing on the dice is equal to 2 is equal to 3 is equal to 4 is equal to 5 is equal to 6 appearing on the dice.)

Exhaustive Set of Events : A set of events that includes all the possibilities of the sample space is said to be an exhaustive set of events. (e.g. in a throw of a dice the number is less than three or more than or equal to three.)

Independent Events: An event is described as such if the occurrence of an event has no effect on the probability of the occurrence of another event. (If the first child of a couple is a boy, there is no effect on the chances of the second child being a boy.)

Expectation: The expectation of an individual is defined as
Probability of winning × Reward of winning.

1. Additional Law of Probability:
For two events A and B
P (A or B) = P (A) + P (B) – P (A and B)
If the events are mutually exclusive
P (A and B) = 0

2. Multiplication Law of Probability:
If the events A and B are independent
P (A and B) = P (A) × P (B)

3. Complement of an event:
$P(A^{\circ}) = 1 - P(A)$

4. A conditional probability is the probability of one event, given that another event has occurred
P(A/B)=P(A and B)/P(B) [the conditional probability of A given that B has occurred]
P(B/A)=P(A and B)/P(A) [the conditional probability of B given that A has occurred]

SOLVED EXAMPLES

Example 1 Find the probability that a leap year chosen at random will have 53 Sundays.

Solution: A leap year has 366 days = 52 weeks + 2 days

2 days can be (Sun-Sat, Sun-Mon, Mon-Tue, Tue-Wed, Wed-Thurs, Thurs-Fri, Fri-Sat)

P (53 Sundays) = $\frac{2}{7}$

Example 2 3 coins are tossed together. Find the probability of getting

i) Exactly one head ii) At least one head iii) At most 1 head

Solution: Total number of cases when three coins are tossed together = 2^3 = 8. Now we will solve each of the above question one by one.

i) The cases of exactly one head are {(HTT)(TTH)(THT)}

$\therefore$ Probability of getting exactly one head = 3/8

ii) Probability of getting atleast one head = 1- probability of getting no head = 1- (1/8) = 7/8

iii) The cases of at most one head {(TTT)(HTT)(TTH)(THT)}

$\therefore$ Probability of getting at most one head = 4/8 = 1/2

Example 3 Three unbiased coins are tossed. What is the probability of getting at least 2 heads?

Solution: P (at least 2 heads) = P (2 heads) + P (3 heads)

2 heads = {(H, H, T), (H, T, H), (T, H, H)}

3 heads = {(H, H, T)}

P (at least 2 heads) = $\frac{3}{8}+\frac{1}{8}=\frac{1}{2}$

Example 4 An urn contains 9 red, 7 white and 4 black balls. If two balls are drawn at random. Find the probability that both the balls are red?

Solution: Total no of balls = 9 + 7 + 4 = 20

no. of ways of selecting 2 balls = ${}^{20}C_2$

no. of ways of selecting 2 balls from 9 red balls = ${}^{9}C_2$

P (Both red) = $\frac{{}^{9}C_2}{{}^{20}C_2} - = \frac{18}{95}$

Example 5 The letters of the word LUCKNOW are arranged among themselves. Find the Probability of always having NOW in such words.

Solution: Required Probability = $\frac{\textit{No. of } \text{words having NOW}}{\textit{Total } \text{No. of words}} = \frac{5!}{7!} = \frac{1}{42}$

Example 6 Probability that A and B can solve the problem are 0.2 & 0.3 respectively. If both of them try what is the Probability that the problem will be solved?

Solution: P (problem solved) = 1 – P (problem not solved)

P (solved) = 1 – (1 – 0.2) (1 – 0.3) = 1 – (0.8 × 0.7)

P (solved) = 0.44

Example 7 A speaks truth in 80% cases and B speaks truth in 75% cases. Find the probability in how many cases will they contradict each other?

Solution: P (Contradict) = P (A lies) P (B tells truth) + P (A tells truth) P (B lies)

P (Contradict) = $\frac{1}{5}\times\frac{3}{4}+\frac{4}{5}\times\frac{1}{4}$ = 7/20

Example 8 A man holds 20 out of 500 tickets of a lottery. If the reward for the winning ticket is Rs. 100. Find the expectation of the man?

Solution: Expectation = P (win) × Reward

$= \frac{20}{500} \times 100$ = Rs. 4

Example 9 The probability that A will solve the problem is 1/5. What is the probability that he solves at least one problem out of ten problems?

Solution: The non-event is defined as:

He solves no problem i.e. he doesn't solve the first problem and the doesn't solve the tenth problem.

Probability of non-event = $\left[\frac{4}{5}\right]^{10}$

Hence, probability of the event is $1 - \left[\frac{4}{5}\right]^{10}$

Example 10 A bag contains 6 white and 4 black balls. Two balls are drawn at random. Find the probability that they are of the same colour.

Solution: Let S be the sample space. Then,

n (S) = Number of ways of drawing 2 balls out of (6 + 4) = $^{10}C_2 = \frac{(10 \times 9)}{(2 \times 1)}$ = 45

Let E = Event of getting both balls of the same colour. Then,

n (E) = Number of ways of drawig (2 balls out of 6) or (2 balls out of 4)

$= (^6C_2 + {^4C_2}) = \frac{6 \times 5}{2 \times 1} + \frac{4 \times 3}{2 \times 1}$ = (15 + 6) = 21

$\therefore$ P (E) = $\frac{n(E)}{n(S)} = \frac{21}{45} = \frac{7}{15}$

EXPERIENCE THE PRATHAM EDGE - 19

1. Find the chance of drawing 2 red balls in succession from a bag containing 5 red and 7 blue balls, if the balls are not being replaced.

   (a) $\frac{5}{33}$ (b) $\frac{21}{64}$ (c) $\frac{7}{22}$ (d) $\frac{21}{61}$

2. From a pack of 52 cards, two cards are drawn at random. Find the chance that one is a knave & the other a queen.

   (a) $\frac{8}{663}$ (b) $\frac{1}{663}$ (c) $\frac{14}{663}$ (d) $\frac{1}{12}$

3. A bag contain 3 green and 7 white balls. Two balls are drawn from the bag in succession without replacement. What is the probability that both are white?

   (a) $\frac{1}{7}$ (b) $\frac{5}{9}$ (c) $\frac{7}{9}$ (d) $\frac{7}{15}$

4. 100 students appeared for two examinations. 60 passed the first, 50 passed the second and 30 passed both. Find the probability that a student selected at random has failed in both the examination

   (a) $\frac{1}{5}$ (b) $\frac{1}{7}$ (c) $\frac{5}{7}$ (d) $\frac{5}{6}$

5. A bag contains four black and five red balls. If three balls from the bag are chosen at random, what is the chance that they are all black?

   (a) $\frac{1}{21}$ (b) $\frac{1}{20}$ (c) $\frac{2}{23}$ (d) $\frac{1}{9}$

6. If a number of two digits is formed with the digits 2, 3, 5, 7, 9 without repetition of digits, what is the probability that the number formed is 35?

   (a) $\frac{1}{10}$ (b) $\frac{1}{20}$ (c) $\frac{2}{11}$ (d) $\frac{1}{11}$

7. $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{2}$, $P(A \cap B) = \frac{1}{3}$, then find $P(A^C \cap B^C)$

   (a) $\frac{5}{12}$ (b) $\frac{1}{2}$ (c) $\frac{3}{12}$ (d) $\frac{9}{12}$

8. A and B are two candidates seeking admission to the IIMs. The probability that A is selected is 0.5 and the probability that both A and B are selected is at most 0.3. Is it possible that the probability of B getting selected is 0.9 ?

   (a) No (b) Yes (c) Either (a) or (b) (d) Can't say

9. The odds against an event is 3 : 5 and the odds in favour of another independent event is 7 : 5. Find the probability that at least one of the two event will occur.

   (a) 52/96 (b) 69/96 (c) 81/96 (d)13/96

10. If 8 coins are tossed, what is the chance that only one will turn up Head?

    (a) $\frac{1}{16}$ (b) $\frac{3}{35}$ (c) $\frac{3}{32}$ (d) $\frac{1}{32}$

11. Six boys and six girls sit in a row randomly. Find the probability that all the six girls sit together.

    (a) $\frac{3}{22}$ (b) $\frac{1}{132}$ (c) $\frac{7}{1584}$ (d) $\frac{1}{66}$

12. From a group of 7 men and 4 women a committee of 6 persons is formed. What is the probability that the committee will consist of exactly 2 women?

    (a) $\frac{5}{11}$ (b) $\frac{3}{11}$ (c) $\frac{7}{11}$ (d) $\frac{2}{11}$

EXPERIENCE THE PRATHAM EDGE - 19

13. A bag contains 5 red, 4 green and 3 black balls. If three balls are drawn out of it at random, find the probability of drawing exactly 2 red balls.
(a) $\frac{7}{22}$ (b) $\frac{10}{33}$ (c) $\frac{7}{12}$ (d) $\frac{7}{11}$

14. Three students appear at an examination of Mathematics. The probability of their success are $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ respectively. Find the probability of success of at least two.
(a) $\frac{1}{6}$ (b) $\frac{2}{5}$ (c) $\frac{3}{4}$ (d) $\frac{3}{5}$

15. In a race where 12 horses are running, the chance that horse A will win is 1/6, that B will win is 1/10 and C will win is 1/8. Find the chance that one of them will win.
(a) $\frac{47}{120}$ (b) $\frac{1}{480}$ (c) $\frac{1}{160}$ (d) $\frac{1}{240}$

16. In a class, there are 15 boys and 10 girls. Three students are selected at random. The probability that 1 girl and 2 boys are selected, is:
(a) $\frac{21}{46}$ (b) $\frac{21}{117}$ (c) $\frac{21}{50}$ (d) $\frac{21}{25}$

17. Two cards are drawn from a pack of well shuffled deck of cards. Find the probability of getting a queen of club or a king of heart.
(a) $\frac{1}{13}$ (b) $\frac{1}{26}$ (c) $\frac{1}{52}$ (d) $\frac{87}{1326}$

18. A bag contains 7 red, 5 blue, 4 white and 4 black balls. Find the probability that a ball drawn at random is red or white?
(a) $\frac{11}{20}$ (b) $\frac{9}{20}$ (c) $\frac{4}{20}$ (d) $\frac{13}{20}$

19. A coin is tossed three times. What is the chance of getting head and tail alternatively?
(a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $\frac{1}{8}$ (d) $\frac{3}{8}$

20. If seven coins are tossed, find the probability of obtaining no head?
(a) $\frac{1}{128}$ (b) $\frac{1}{64}$ (c) $\frac{1}{32}$ (d) $\frac{1}{16}$

21. What is the probability of getting a sum 9 from two throws of a dice?
(a) $\frac{1}{6}$ (b) $\frac{1}{8}$ (c) $\frac{1}{9}$ (d) $\frac{1}{12}$

22. From a pack of 52 cards, two cards are drawn together at random. What is the probability of both the cards being kings?
(a) $\frac{1}{15}$ (b) $\frac{25}{57}$ (c) $\frac{35}{256}$ (d) $\frac{1}{221}$

23. A bag contains 4 white, 5 red and 6 blue balls. Three balls are drawn at random from the bag. The probability that all of them are red, is:
(a) $\frac{1}{22}$ (b) $\frac{3}{22}$ (c) $\frac{2}{91}$ (d) $\frac{2}{77}$

24. A speaks truth in 75% cases and B in 80% of the cases. What is the maximum percentage of cases are they likely to contradict each other, whole narrating the same incident?
(a) 5% (b) 15% (c) 35% (d) 40%

25. A bag contains 80 envelops of which 30 are airmails and the rest are ordinary. Out of the 80 envelops in the bag, 48 are stamped and the rest are unstamped. There are 20 unstamped ordinary envelops in the bag. If one envelope is chosen at random from the bag then the probabilities that this is an unstamped airmail envelop is :
(a) 5% (b) 15% (c) 35% (d) 40%

CHAPTER 20

DATA INTERPRETATION

INTRODUCTION

TABULATION

This section comprises of questions in which certain data as production over a period of a few years: imports, exports, income of workers of a factory students applying for and qualifying a certain field of study etc. are given in the form of a table. The candidate is required to understand the given information and thereafter answer the given question on the basis of comparative analysis of the data.

Thus, here the data collected by the investigator are arranged in a systematic form in a table called the tabular form. In order to avoid some heads again and again, tables are made consisting of horizontal lines called ***rows*** and vertical lines called ***columns*** with distinctive heads, known as ***captions***. Units of measurements are given with the captions.

SOLVED EXAMPLES

1. The following table gives the sales of batteries manufactured by a company over the years. Study the table and answer the questions that follow:

NUMBER OF DIFFERENT TYPES OF BATTERIES SOLD BY A COMPANY OVER THE YEAR (NUMBERS IN THOUSANDS)

	TYPES OF BATTERIES					
Years	4AH	7AH	32AH	35AH	55AH	Total
1992	75	144	114	102	108	543
1993	90	126	102	84	126	528
1994	96	114	75	105	135	525
1995	105	90	150	90	75	510
1996	90	75	135	75	90	465
1997	105	60	165	45	120	495
1998	115	85	160	100	145	605

1. The total sales of all the seven years is the maximum for which battery?
(a) 4AH (b) 7AH (c) 32AH (d) 35AH (e) 55AH

2. What is the difference in the number of 35AH batteries sold in 1993 and 1997?
(a) 24000 (b) 28000 (c) 35000 (d) 39000 (e) 42000

3. The percentage of 4AH batteries sold to the total number of batteries sold was maximum in the year:
(a) 1994 (b) 1995 (c) 1996 (d) 1997 (e) 1998

4. In the case of which battery there was a continuous decrease in sales from 1992 to 1997?
(a) 4AH (b) 7AH (c) 32AH (d) 35AH (e) 55AH

5. What was the approximate percentage increase in the sales of 55AH batteries in 1998 compared to that in 1992?
(a) 28% (b) 31% (c) 33% (d) 34% (e) 37%

Solutions

1. (c) : The total sales (in thousands) of all the seven years for various batteries are:
For 4AH = 75 + 90 + 96 + 105 + 90 + 105 + 115 = 676
For 7AH = 144 + 126 + 114 + 90 + 75 + 60 + 85 = 694

For 32AH = 114 + 102 + 75 + 150 + 135 + 165 + 160 = 901
For 35AH = 102 + 84 + 105 + 90 + 75 + 45 + 100 = 601
For 55AH = 108 + 126 + 135 + 75 + 90 + 120 + 145 = 799.
Clearly, sales are maximum in case of 32AH batteries.

2. (d) : Required difference = [(84 – 45) × 1000] = 39000.
3. (d) : The percentages of sales of 4AH batteries to the total sales in different years are :

For 1992 = $\left(\frac{75}{543}\times100\right)$ % = 13.81%; **For 1993** = $\left(\frac{90}{528}\times100\right)$ % = 17.05%;

For 1994 = $\left(\frac{96}{525}\times100\right)$% = 18.29%; **For 1995** = $\left(\frac{105}{510}\times100\right)$% = 20.59%;

For 1996 = $\left(\frac{90}{465}\times 100\right)$%% = 19.35%; **For 1997** = $\left(\frac{105}{495}\times100\right)$% = 21.21%

For 1998 = $\left(\frac{115}{605}\times100\right)$ % = 19.01%.

Clearly, the percentage is maximum in 1997.

4. (b) : From the table it is clear that the sales of 7AH batteries have been decreasing continuously from 1992 to 1997.
5. (d) : Required Percentage = $\left[\frac{(145-108)}{108}\times100\right]$% = 34.26% = 34%.

2. Study the following table carefully and answer these questions:

NUMBER OF CANDIDATES APPEARED AND QUALIFIED IN A COMPETITIVE EXAMINATION FROM DIFFERENT STATES OVER THE YEARS

Year / State	1997		1998		1999		2000		2001	
	App.	Qual.	App.	Qual.	App.	Qual.	App.	Qual.	App.	Qual.
M	5200	720	8500	980	7400	850	6800	775	9500	1125
N	7500	840	9200	1050	8450	920	9200	980	8800	1020
P	6400	780	8800	1020	7800	890	8750	1010	9750	1250
Q	8100	950	9500	1240	8700	980	9700	1200	8950	995
R	7800	870	7600	940	9800	1350	7600	945	7990	885

1. Combining the states P and Q together in 1998, what is the percentage of the candidates qualified to that of the candidates appeared?
(a) 10.87% (b) 11.49% (c) 12.35% (d) 12.54% (e) 13.05%

2. The percentage of the total number of qualified candidates to the total number of appeared candidates among all the five states in 1999 is:
(a) 11.49% (b) 11.84% (c) 12.21% (d) 12.57% (e) 12.73%

3. What is the percentage of candidates qualified from State N for all the years together, over the candidates appeared from State N during all the years together?
(a) 12.36% (b) 12.16% (c) 11.47% (d) 11.15% (e) None of these

4. What is the average of candidates who appeared from State Q during the given years?
   (a) 8700 (b) 8760 (c) 8810 (d) 8920 (e) 8990

5. In which of the given years the number of candidates appeared from State P has maximum percentage of qualified candidates?
   (a) 1997 (b) 1998 (c) 1998 (d) 2000 (e) 2001

6. Total number of candidates qualified from all the states together in 1997 is approximately what percentage of the total number of candidates qualified from all the states together in 1998?
   (a) 72% (b) 77% (c) 80% (d) 83% (e) 86%

Solutions

1. (c) : Required Percentage $= \left[\frac{(1020+1240)}{(8800+9500)}\times 100\right]\% = \left(\frac{2260}{18300}\times 100\right)\% = 12.35\%.$

2. (b) : Required Percentage $= \left[\frac{(850+920+890+980+1350)}{(7400+8450+7800+8700+9800)}\times 100\right]\%$

   $= \left(\frac{4990}{42150}\times 100\right)\% = 11.84\%.$

3. (d) : Required Percentage $= \left[\frac{(840+1050+920+980+1020)}{(7500+9200+8450+9200+8800)}\times 100\right]\%$

   $= \left(\frac{4810}{43150}\times 100\right)\% = 11.15\%.$

4. (e) Required Average $= \frac{8100+9500+8700+9700+8950}{5} = \frac{44950}{5} = 8990$

5. (e) : The percentages of candidates qualified to candidates appeared from State P during different years are:

   For 1997 $= \left(\frac{780}{6400}\times 100\right)\% = 12.19\%;$

   For 1998 $= \left(\frac{1020}{8800}\times 100\right)\% = 11.59\%;$

   For 1999 $= \left(\frac{890}{7800}\times 100\right)\% = 11.41\%;$

   For 2000 $= \left(\frac{1010}{8750}\times 100\right)\% = 11.54\%;$

   For 2001 $= \left(\frac{1250}{9750}\times 100\right)\% = 12.82\%;$

   $\therefore$ Maximum percentage is for the year 2001.

6. (c) : Required Percentage $= \left[\frac{(720+840+780+950+870)}{(980+1050+1020+1240+940)}\times 100\right]\% = \left(\frac{4160}{5230}\times 100\right)\% \approx 80\%$

3. The following table gives the percentage of marks obtained by seven students in six different subjects in an examination. Study the table and answer the questions based on it. The numbers in the brackets gives the maximum marks in each subject.

Subjects (Max. Marks) Students	Maths (150)	Chemistry (130)	Physics (120)	Geography (100)	History (60)	Computer Science (40)
Ayush	90	50	90	60	70	80
Aman	100	80	80	40	80	70
Sajal	90	60	70	70	90	70
Rohit	80	65	80	80	60	60
Muskan	80	65	85	95	50	90
Tanvi	70	75	65	85	40	60
Tarun	65	35	50	77	80	80

1. What was the aggregate of marks obtained by Sajal in all the six subjects?
   (a) 409 (b) 419 (c) 429 (d) 439 (e) 449

2. What is the overall percentage of Tarun?
   (a) 52.5% (b) 55% (c) 60% (d) 63% (e) 64.5%

3. What are the average marks obtained by all the seven students in Physics? (rounded off to two digits after decimal)
   (a) 77.26 (b) 89.14 (c) 91.37 (d) 96.11 (e) 103.21

4. The number of students who obtained 60% and above marks in all the subjects is:
   (a) 1 (b) 2 (c) 3 (d) None (e) None of these

5. In which subject is the overall percentage the best?
   (a) History (b) Maths (c) Physics (d) Chemistry (e) Geography

Solution

1. (e) : Aggregate marks obtained by Sajal
   = [(90% of 150) + (60% of 130) + (70% of 120) + (70% of 100) + (90% of 60) + (70% of 40)]
   = 135 + 78 + 84 + 70 + 54 + 28 = 449.

2. (c) : Aggregate marks obtained by Tarun
   = [(65% of 150) + (35% of 130) + (50% of 120) + (77% of 100) + (80% of 60) + (80% of 40)
   = 97.5 + 45.5 + 60 + 77 + 48 + 32 = 360.

   Total maximum marks (of all the six subjects)
   = (150 + 130 + 120 + 100 + 60 + 40) = 600.
   Overall Percentage of Tarun = $\left(\frac{360}{600}\times 100\right)$% = 60%.

3. (b) : Average marks obtained in Physics by all the seven students
   = $\frac{1}{7}\times$ [(90% of 120) + (80% of 120) + (70% of 120) + (80% of 120) + (85% of 120) + (65% of 120)+(50% of 120)]
   = $\frac{1}{7}\times$ [(90 + 80 + 70 + 80 + 65 + 50)% of 120] = $\frac{1}{7}\times$ [520% of 120] = $\frac{624}{7}$ = 89.14.

4. (b) : From the table it is clear that Sajal and Rohit have 60% or more marks in each of the six subjects.
5. (b) : We shall find the overall percentage (for all the seven students) with respect to each subject. The overall percentage for any subject is equal to the average of percentages obtained by all the seven students since the maximum marks for any subject is the same for all the students.
Therefore, overall percentage for:

$$\text{(i) Maths} = \left[\frac{1}{7}(90+100+90+80+80+70+65)\right]\% = \left[\frac{1}{7}(575)\right]\% = 82.14\%$$

$$\text{(ii) Chemistry} = \left[\frac{1}{7}(50+80+60+65+65+75+35)\right]\% = \left[\frac{1}{7}(430)\right]\% = 61.43\%.$$

$$\text{(iii) Physics} = \left[\frac{1}{7}(90+80+70+80+85+65+50)\right]\% = \left[\frac{1}{7}(520)\right]\% = 74.29\%.$$

$$\text{(iv) Geography} = \left[\frac{1}{7}(60+40+70+80+95+85+77)\right]\% = \left[\frac{1}{7}(507)\right]\% = 72.43\%.$$

$$\text{(v) History} = \left[\frac{1}{7}(70+80+90+60+50+40+80\right]\% = \left[\frac{1}{7}(470)\right]\% = 67.14\%.$$

$$\text{(vi) Computer Science} = \left[\frac{1}{7}(80+70+70+60+90+60+80)\right]\% = \left[\frac{1}{7}(510)\right]\% = 72.86\%.$$

Clearly, the percentage is highest for Maths.

4. Study the following table carefully and answer the questions given below:

CLASSIFICATION OF 100 STUDENTS BASED ON THE MARKS OBTAINED BY THEM IN PHYSICS AND CHEMISTRY IN AN EXAMINATION

Marks out of 50 *Subject*	*40 and above*	*30 and above*	*20 and above*	*10 and above*	*0 and above*
Physics	9	32	80	92	100
Chemistry	4	21	66	81	100
(Aggregate) Avg.	7	27	73	87	100

1. The number of students scoring less than 40% marks in aggregate is :
(a) 13 (b) 19 (c) 20 (d) 27 (e) 34
2. If at least 60% marks in Physics are required for pursuing higher studies in Physics, how many students will be eligible to pursue higher studies in Physics?
(a) 27 (b) 32 (c) 34 (d) 41 (e) 68
3. What is the difference between the number of students passed with 30 as cut-off marks in Chemistry and those passed with 30 as cut-off marks in aggregate?
(a) 3 (b) 4 (c) 5 (d) 6 (e) 7
4. The percentage of the number of students getting at least 60% marks in Chemistry over those getting at least 40% marks in aggregate, is approximately:
(a) 21% (b) 27% (c) 29% (d) 31% (e) 34%
5. If it is known that at least 23 students were eligible for a Symposium on Chemistry, the minimum qualifying marks in Chemistry for eligibility to Symposium would lie in the range:
(a) 40-50 (b) 30-40 (c) 20-30 (d) Below 20 (e) Cannot be determined

Solutions

1. (d): We have 40% of 50 = $\left(\frac{40}{100}\times 50\right)$ = 20.

$\therefore$ Required number = Number of students scoring less than 20 marks in aggregate
= 100 – number of students scoring 20 and above marks in aggregate = 100 – 73 = 27.

2. (b) : We have 60% of 50 = $\left(\frac{60}{100}\times 50\right)$ = 30.

   $\therefore$ Required number = Number of students scoring 30 and above marks in Physics = 32.

3. (d) : Required difference = (Number of students scoring 30 and above marks in Chemistry) – (Number of students scoring 30 and above marks in aggregate) = 27 – 21 = 6.

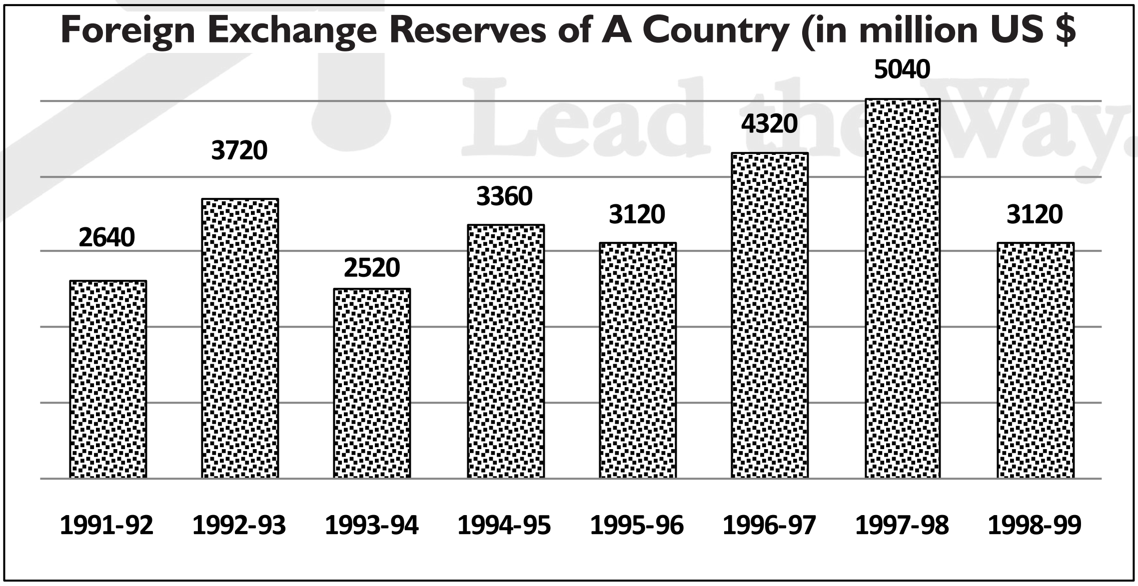
4. (c) : Number of students getting at least 60% marks in Chemistry
   = Number of students getting 30 and above marks in Chemistry = 21.
   Number of students getting at least 40% marks in aggregate
   = Number of students getting 20 and above marks in aggregate = 73.
   $\therefore$ Required Percentage = $\left(\frac{21}{73}\times 100\right)$% = 28.77% = 29%.

5. (c) : Since 66 students get 20 and above marks in Chemistry and out of these 21 students get 30 and above marks, therefore to select top 35 students in Chemistry, the qualifying marks should lie in the range 20-30.

BAR GRAPHS

This section comprises of questions in which the data collected in a particular discipline are represented in the form of vertical or horizontal bars drawn by selecting a particular scale. One of the parameters is plotted on the horizontal axis and the other on the vertical axis. The candidate is required to understand the given information and thereafter answer the given questions on the basis of data analysis.

I. **The bar graph given below shows the foreign exchange reserves of a country (in million US $) from 1991-92 to 1998-99. Answer the questions based on this graph.**



1. The foreign exchange reserves in 1997-98 was how many times that in 1994-95?
   (a) 0.7 (b) 1.2 (c) 1.4 (d) 1.5 (e) 1.8

2. What was the percentage increase in the foreign exchange reserves in 1997-98 over 1993-94?
   (a) 100 (b) 150 (c) 200 (d) 620 (e) 2520

3. For which year, the percent increase exchange reserves over the previous year, is the highest?
(a) 1992-93 (b) 1993-94 (c) 1994-95 (d) 1996-97 (e) 1997-98

4. The foreign exchange reserves in 1996-97 were approximately what percent of the average foreign exchange reserves over the period under review?
(a) 95% (b) 110% (c) 115% (d) 125% (e) 140%

5. The ratio of the number of years, in which the foreign exchange reserves are above the average reserves, to those in which the reserves are below the average reserves, is:
(a) 2 : 6 (b) 3 : 4 (c) 3 : 5 (d) 4 : 4 (e) 5 : 3

Solutions

1. (d) : Required ratio = $\frac{5040}{3360}$ = 1.5.

2. (a) : Foreign exchange reserves in 1997-98 = 5040 million US $.
Foreign exchange reserves in 1993-94 = 2520 million US $.
∴ Increase = (5040 - 2520) = 2520 million US $.
∴ Percentage increase = $\frac{2520}{2520}$ x100% = 100%.

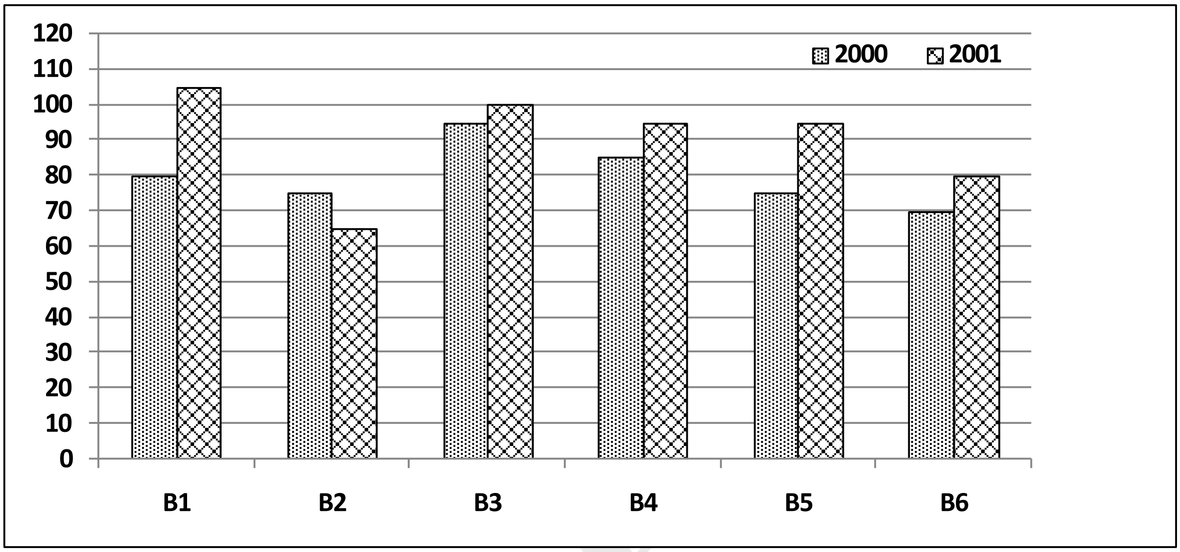
3. (a) : There is an increase in foreign exchange reserves during the years 1992-93, 1994-95, 1996-97 and 1997-98 as compared to previous year (as shown by bar-graph). The percentage increase in reserves during these years compared to previous year are:
(i) For 1992-93 = $\left[\frac{(3720-2640)}{2640}\times 100\right]$% = 40.91%
(ii) For 1994-95 = $\left[\frac{(3360-2520)}{2640}\times 100\right]$% = 33.33%
(iii) For 1996-97 = $\left[\frac{(4320-3120)}{3120}\times 100\right]$% = 38.46%
(iv) For 1997-98 = $\left[\frac{(5040-4320)}{4320}\times 100\right]$% = 16.67%
Clearly, the percentage increase over previous year is highest for 1992-93.

4. (d) : Average foreign exchange reserves over the given period
= $\left[\frac{1}{8}(2640+3720+2520+3360+3120+4320+5040+3120)\right]$ million US $ = 3480 million US $.
Foreign exchange reserves in 1996-97 = 4320 million US $.
∴ Required Percentage = $\left(\frac{4320}{3480}\times 100\right)$% = 124.14% = 125%.

5. (c) : Average foreign exchange reserves over the given period = 3480 million US $. The country had reserves above 3480 million US $ during the years 1992-93, 1996-97 and 1997-98 i.e., for 3 years and below 3480 million US $ during the years 1991-92, 1993-94, 1994-95, 1995-96 and 1998-99 i.e., for 5 years.
Hence, required ratio = 3 : 5.

II. **The bar-graph provided below gives the sales of books (in thousand numbers) from six branches of a publishing company during two consecutive years 2000 and 2001. Answer the questions based on this bar-graph.**

Sales of Books (in thousand numbers) from Six Branches-
B1, B2, B3, B4, B5, and B6 of a Publishing Company in 2000 and 2001



1. Total sales of branches B1, B3 and B5 together for both the years (in thousand numbers) is:
   (a) 250 (b) 310 (c) 436 (d) 550 (e) 585

2. Total sales of branch B6 for both the years is what percent of the sales of branch B3 for both the years?
   (a) 68.54% (b) 71.11% (c) 76.92% (d) 75.55% (e) 77.26%

3. What is the average sale of the branches (in thousand numbers) for the year 2000?
   (a) 73 (b) 80 (c) 83 (d) 88 (e) 96

4. What is the ratio of the total sales of branch B2 for both years to the total sales of branch B4 for both years?
   (a) 2 : 3 (b) 3 : 5 (c) 4 : 5 (d) 5 : 7 (e) 7 : 9

5. What percent of the average sales of branches B1, B2 and B3 in 2001 is the average sales of branches B1, B3 and B6 in 2000?
   (a) 75% (b) 77.5% (c) 82.5% (d) 85% (e) 90.7%

Solutions

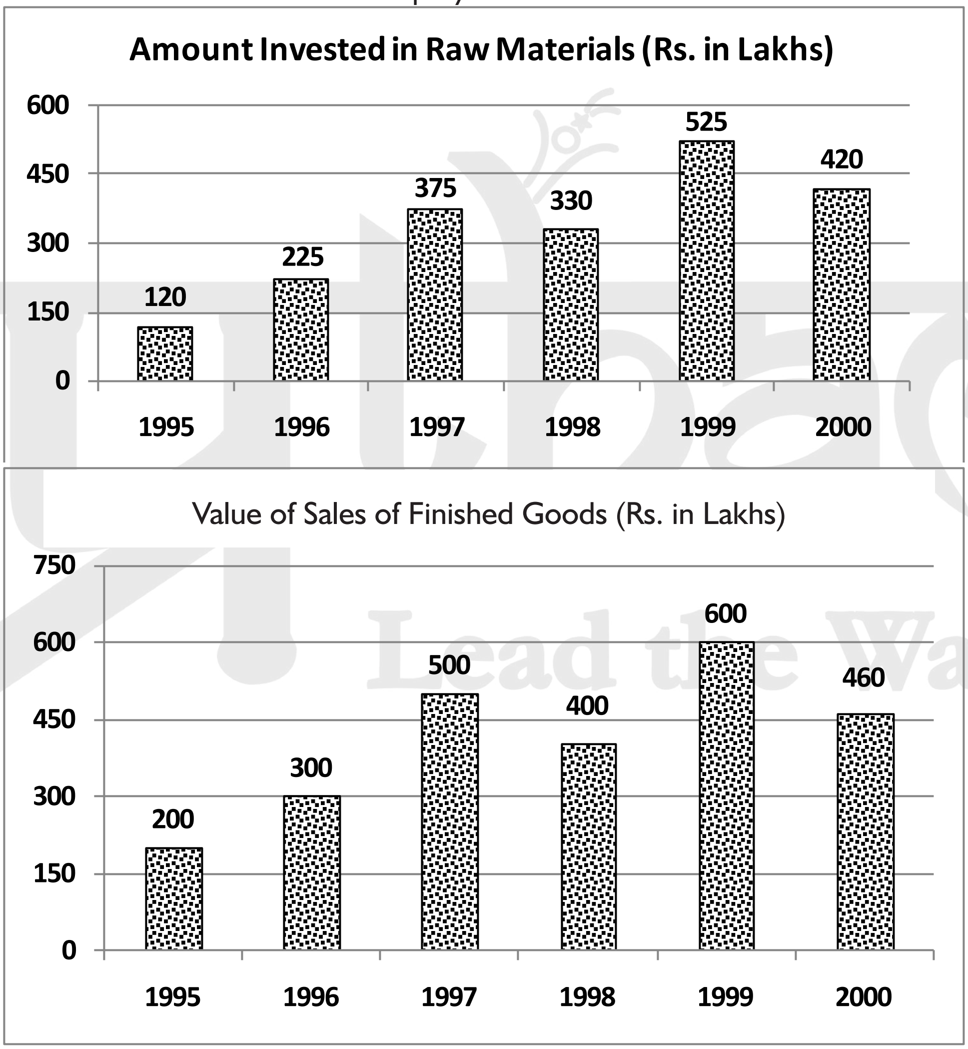
1. (d) : Total sales of branches B1, B3 and B5 for both the years (in thousand numbers)
   = (80 + 105) + (95 + 100) + (75 + 95) = 550.
2. (c) : Required Percentage = $\left[\frac{(70+80)}{(95+100)}\times 100\right]\% = \left(\frac{150}{195}\times 100\right)\% = 76.92\%$
3. (b) : Average sales of all the six branches (in thousand numbers) for the year
   2000 = 1/6× [80 + 75 + 95 + 85 + 75 + 70] = 80.
4. (e) : Required ratio = $\frac{(75+65)}{(85+95)} = \frac{140}{180} = \frac{7}{9}$
5. (e) : Average sales (in thousand numbers) of branches B1 B3 and B6 in 2000
   $= \frac{1}{3}\times (80 + 95 + 70) = \left(\frac{245}{3}\right)$

   Average sales (in thousand numbers) of branches B1, B2 and B3 in 2001
   $= \frac{1}{3}\times (105 + 65 + 100) = \frac{270}{3}$

$$\therefore \text{ Required Percentage } = \left[\frac{\left(\frac{245}{3}\right)}{\left(\frac{270}{3}\right)}\times 100\right]\% = \left(\frac{245}{270}\times 100\right)\% = 90.7\%$$

III. **Out of the two bar graphs provided below, one shows the amounts (in Lakh Rs.) invested by a Company in purchasing raw materials over the years and the others shows the values (in Lakh Rs.) of finished goods sold by the Company over the years. Study the two bar graphs and answer the questions based on them.**

Amount Invested in Raw Materials and the Value of Sales of Finished Goods for A Company over the Years



1. In which year, there has been a maximum percentage increase in the amount invested in Raw Materials as compared to the previous years?
(a) 1996 (b) 1997 (c) 1998 (d) 1999 (e) 2000

2. In which year, the percentage change (compared to the previous year) in the investment on Raw Materials is the same as that in the value of sales of finished goods?
(a) 1996 (b) 1997 (c) 1998 (d) 1999 (e) 2000

3. What was the difference between the average amount invested in Raw Materials during the given period and the average value of sales of finished goods during this period?
(a) 77.5 lakhs (b) 80.8 lakhs (c) 72.7 lakhs (d) 92.6 lakhs (e) None

4. The value of sales of finished goods in 1999 was approximately what percent of the amount invested in Raw Materials in the years 1997, 1998 and 1999?
(a) 33% (b) 37% (c) 45% (d) 49% (e) 53%

5. The maximum difference between the amount invested in Raw Materials and the value of sales of finished goods was during the year:
(a) 1995 (b) 1996 (c) 1997 (d) 1998 (e) 1999

Solutions

1. (a) : The percentage increase in the amount invested in raw-materials as compared to the previous year, for different year are:

For 1996 = $\left[\frac{(225-120)}{120}\times 100\right]\% = 87.5\%$

For 1997 = $\left[\frac{(375-225)}{225}\times 100\right]\% = 66.67\%$

For 1998 there is a decrease.

For 1999 = $\left[\frac{(525-330)}{330}\times 100\right]\% = 59.09\%$

For 2000 there is a decrease.

$\therefore$ There is a maximum percentage increase in 1996.

2. (b) : The percentage change in the amount invested in raw-materials all in the value of sales of finished goods for different years are:

Year	Percentage change in Amount invested in raw-material	Percentage change in value of sales of finished goods
1996	$\left[\frac{(225-120)}{120}\times 100\right]\% = 87.5\%$	$\left[\frac{(300-200)}{200}\times 100\right]\% = 50\%$
1997	$\left[\frac{(375-225)}{225}\times 100\right]\% = 66.67\%$	$\left[\frac{(500-300)}{300}\times 100\right]\% = 66.67\%$
1998	$\left[\frac{(330-375)}{375}\times 100\right]\% = -12\%$	$\left[\frac{(400-500)}{500}\times 100\right]\% = -20\%$
1999	$\left[\frac{(525-330)}{330}\times 100\right]\% = 59.09\%$	$\left[\frac{(600-400)}{400}\times 100\right]\% = 50\%$

2000	$\left[\frac{(420-525)}{525}\times 100\right]$ % = -20%	$\left[\frac{(460-600)}{600}\times 100\right]$% = -23.33%

Thus, the percentage difference is same during the year 1997.

3. (a) : Required difference

$$= \text{Rs. } \left[\frac{1}{6}(200+300+500+400+600+460)-\frac{1}{6}(120+225+375+330+525+420)\right]\text{lakhs}$$

$$= \text{Rs. } \left[\left(\frac{2460}{6}\right)-\left(\frac{1995}{6}\right)\right] \text{ lakhs} = \text{Rs. } (410 - 332.5) \text{ lakhs} = \text{Rs. } 77.5 \text{ lakhs.}$$

4. (d) : Required percentage = $\left[\frac{600}{(375+330+525)}\times 100\right]\%$ = 48.78% =49%.

5. (c) : The differences between the amount invested in raw material and the value of sales of finished goods for various years are:

   For 1995 = Rs. (200 – 120) lakhs = Rs. 80 lakhs.

   For 1996 = Rs. (300 – 225) lakhs = Rs. 75 lakhs.

   For 1997 = Rs. (500 – 375) lakhs = Rs. 125 lakhs.

   For 1998 = Rs. (400 – 330) lakhs = Rs. 70 lakhs.

   For 1999 = Rs. (600 – 525) lakhs = Rs. 75 lakhs.

   For 2000 = Rs. (460 – 420) lakhs = Rs. 40 lakhs.

   Clearly, maximum difference was during 1997.

PIE-CHARTS

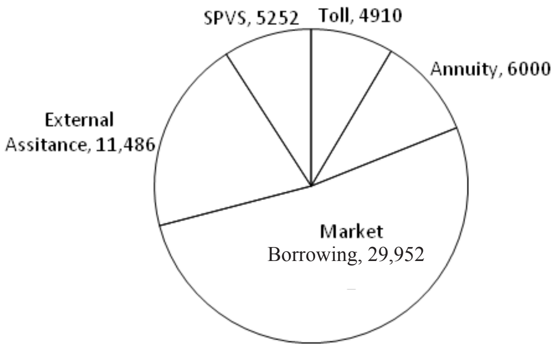
The pie-chart or a pie-graph is a method of representing a given numerical data in the form of sectors of a circle. The sectors of the circle are constructed in such a way that the area of each sector is proportional to the corresponding value of the component of the data.
From geometry, we know that the area of the sector of a circle is proportional to the central angle.
So, the central angle of each sector must be proportional to the corresponding value of the component.
Since the sum of all the central angles is 360°, we have

$$\text{Central angle of the component} = \left(\frac{Value\ of\ \text{the component}}{Total\ Value}\times 360\right)^0$$

SOLVED EXAMPLES

The procedure of solving problems based on pie-charts will be clear from the following solved examples.

I. **The following pie-chart shows the sources of funds to be collected by the National Highways Authority of India (NHAI) for its Phase II projects. Study the pie-chart and answer the question that follow.**



Total funds to be arranged for Projects (Phase II) = Rs. 57,600 crores.

1. Near about 20% of the funds are to be arranged through:
   (a) SPVS (b) External Assistance (c) Annuity (d) Market Borrowing

2. The central angle corresponding to Market Borrowing is:
   (a) 52° (b) 137.8% (c) 187.2° (d) 192.4°

3. The approximate ratio of the funds to be arranged through Toll and that through Market Borrowing is:
   (a) 2 : 9 (b) 1 : 6 (c) 3 : 11 (d) 2 : 5

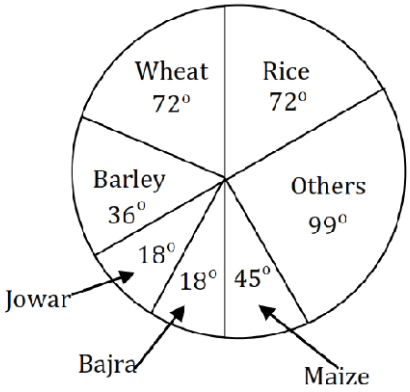
4. If NHAI could receive a total of Rs. 9695 crores as External Assistance, by what percent (approximately) should it increase the Market Borrowings to arrange for the shortage of funds?
   (a) 4.5% (b) 7.5% (c) 6% (d) 8%

5. If the toll is to be collected through an outsourced agency by allowing a maximum 10% commission, how much amount should be permitted to be collected by the outsourced agency, so that the project is supported with Rs. 4910 crores?
   (a) Rs. 6213 crores (b) Rs. 5827 crores (c) Rs. 5401 crores (d) Rs. 5216 crores

Solutions

1. (b) : 20% of total funds to be arranged = Rs. (20% of 57600) crores
   = Rs. 11520 crores = Rs. 11486
   Rs. 11486 crores is the amount of funds to be arranged through External Asistance.
2. (c) : Central angle corresponding to Market Borrowing = $\frac{29952}{57600} \times 360^0$ = 187.2°
3. (b) : Required ratio = $\frac{4910}{29952} = \frac{1}{6.1} = \frac{1}{6}$
4. (c) : Shortage of funds arranged through External Assistance
   = Rs. (11486 – 9695) croces = Rs. 1791 croces.
   ∴ Increase required in Market Borrowings = Rs. 1791 crores.
   Percentage increase required = $\left(\frac{1791}{29952} \times 100\right)$% = 5.98% = 6%.
5. (c) : Amount permitted = (Funds required from Toll for projects of Phase II) + (10% of these funds)
   = Rs. 4910 crores + Rs. (10% of 4910) crores
   = Rs. (4910 + 491) crores = Rs. 5401 croces

II. **The pie-chart provided below gives the distribution of land (in a village) under various food crops. Study the pie-chart carefully and answer the questions that follow.**



1. Which combination of three crops contribute to 50% of the total area under the food crops?
   (a) Wheat, Barley and Jowar (b) Rice, Wheat and Jowar
   (c) Rice, Wheat and Barley (d) Bajra, Maize and Rice

2. If the total area under jowar was 1.5 million acres, then what was the area (in-million acres) under rice?
   (a) 6 (b) 7.5 (c) 9. (d) 4.5

3. If the production of wheat is 6 times that of barley, then what is the ratio between the yield per acre of barley and wheat?
   (a) 3 : 2 (b) 3 : 1 (c) 12 : 1 (d) 2 : 3
4. If the yield per acre of rice was 50% more than that of barley, then the production of barley is what percent of that of rice?
   (a) 30% (b) $33\frac{1}{3}$ % (c) 35% (d) 36%

5. If the total area goes up by 5%, and the area under wheat production goes up by 12% then what will be the angle for wheat in the new pie-chart?
   (a) 62.4° (b) 76.8° (c) 80.6° (d) 84.2°

Solutions

1. (c) : The total of the central angles corresponding to the three crops which cover 50% of the total area, should be 180°. Now, the total of the central angles for the given combinations are:

   (i) Wheat, Barley and Jowar $= (72° + 36° + 18°) = 126°$

   (ii) Rice, Wheat and Jowar $= (72° + 72° + 18°) = 162°$

   (iii) Rice, Wheat and Barley $= (72° + 72° + 36°) = 180°$

   (iv) Bajra, Maize and Rice $= (18° + 45° + 72°) = 135°$

   Clearly, (iii) is the required combination
2. (a) : The area under any of the food crops is proportional to the central angle corresponding to that crop.
   Let, the area under rice production be x million acres.

Then, 18 : 72 = 1.5 : x $\Rightarrow$ x = $\left(\frac{72\times1.5}{18}\right)$ = 6.

Thus, the area under rice production = 6 million acres.

3. (b) $\frac{\text{Land under barley}}{\text{Land under wheat}}=\frac{36}{72}=\frac{1}{2}$ $\qquad$ $\frac{\text{Production of barley}}{\text{Production of wheat}}=\frac{1}{6}$

$\therefore$ Required Ratio $=\frac{1/2}{1/6}$ = 3:1

4. (b) : Let Z acres of land be put under barley production.

Thus, $\frac{\text{Area under rice production}}{\text{Area under barley production}}=\frac{72°}{36°}=2$

$\therefore$ Area under Rice production = 2 × Area under barley production = (2Z) acres.
Now, if p tones be the yield per acre of barley then, yield per acre of rice
= (p + 50% of p) tones = $\left(\frac{3}{2}p\right)$tonnes.

$\therefore$ Total production of rice = (yield per acre) × (area under production)
= $\left(\frac{3}{2}p\right)$ × 2Z = (3pZ) tones.

And, Total production of barley, = (pZ) tones.
$\therefore$ Percentage production of barley to that of rice = $\left(\frac{pZ}{3pZ}100\right)$% = $33\frac{1}{3}$%

5. (b) : Initially, let t acres be the total area under consideration.

Then, area under wheat production initially was = $\left(\frac{72}{360}t\right)$acres = $\left(\frac{t}{5}\right)$acres.

Now, if the total are under consideration be increased by 5%, then the new value of the total are = $\left(\frac{105}{100}t\right)$ acres.

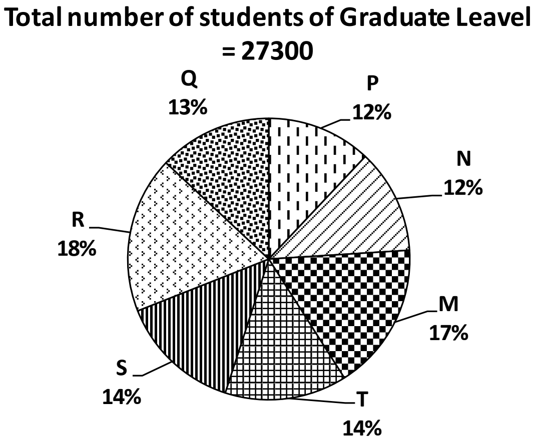
Also, if the area under wheat production be increased by 12%, then the new value of the area under wheat = $\left[\frac{t}{5}+\left(12\%of\ \frac{t}{5}\right)\right]$acres = $\left(\frac{112t}{500}\right)$ acres.

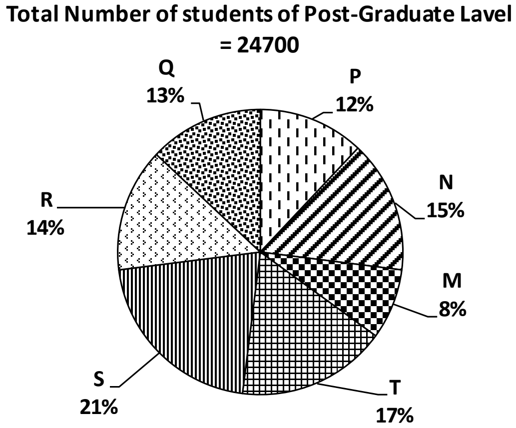
$\therefore$ Central angle corresponding to wheat in the new pie-chart

$$=\left[\frac{\text{Area under wheat(new)}}{\text{Total area(new)}}360\right]^{\circ}=\left[\frac{\left(\frac{112t}{100}\right)}{\left(\frac{105t}{100}\right)}360\right]^{\circ} = 76.8°.$$

III. **The following pie-charts show the distribution of students of graduate and post-graduate levels in seven different institutes – M, N, P, Q, R, S and T in a town.**

DISTRIBUTION OF STUDENTS AT GRADUATE AND POST-GRADUATE LEVELS IN SEVEN INSTITUTES – M, N, P, Q, R, S AND T





1. How many students of institutes M and S are studying at graduate level?
(a) 7516 (b) 8463 (c) 9127 (d) 9404
2. Total number of students studying at post-graduate level from institutes N and P is:
(a) 5601 (b) 5944 (c) 6669 (d) 7004
3. What is the total number of graduate and post-graduate level students in institute R?
(a) 8320 (b) 7916 (c) 9116 (d) 8372
4. What is the ratio between the number of students studying at post-graduate and graduate levels respectively from institute S?
(a) 14 : 19 (b) 19 : 21 (c) 17 : 21 (d) 19 : 14
5. What is the ratio between the number of students studying at post-graduate level from institute S and the number of students studying at graduate level from institute Q?
(a) 13 : 19 (b) 21 : 13 (c) 13 : 8 (d) 19 : 13

Solutions

1. (b) : Students of institute M at graduate level = 17% of 27300 = 4641.
Students of institute S at graduate level = 14% of 27300 = 3822.
$\therefore$ Total number of students at graduate level in institutes M and S = 4641 + 3822 = 8463.

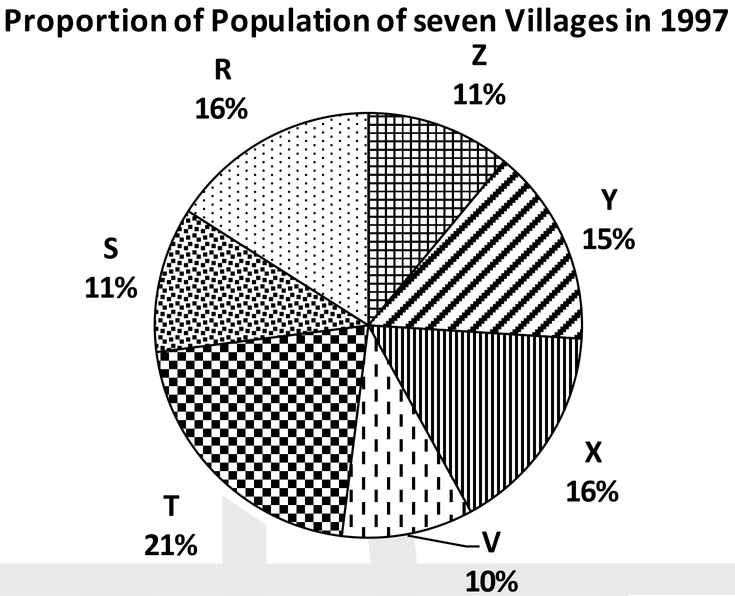
2. (c) : Required number = (15% of 24700) + (12% of 24700) = 3705 + 2964 = 6669.

3. (d) : Required number = (18% of 27300) + (14% of 24700) = 4914 + 3458 = 8372.

4. (d) : Required ratio = $\dfrac{(21\% \text{ of } 24700)}{(14\% \text{ of } 27300)} = \dfrac{21 \times 24700}{14 \times 27300} = \dfrac{19}{14}$

5. (d) : Required ratio = $\dfrac{(21\% \text{ of } 24700)}{(13\% \text{ of } 27300)} = \dfrac{21 \times 24700}{13 \times 27300} = \dfrac{19}{13}$

IV. **Study the following pie-chart and the table and answer the questions based on them.**



Village	% Population Below Poverty Line
X	38
Y	52
Z	42
R	51
S	49
T	46
V	58

1. Find the population of village S if the population of village X below poverty line in 1997 is 12160.
(a) 18500 (b) 20500 (c) 22000 (d) 26000

2. The ratio of population of village T below poverty line to that of village Z below poverty line in 1997 is:
(a) 11 : 23 (b) 13 : 11 (c) 23 : 11 (d) 11 : 13

3. If the population of village R in 1997 is 32000, then what will be the population of village Y below poverty line in that year?
(a) 14100 (b) 15600 (c) 16500 (d) 17000

4. If in 1998, the population of villagers Y and V increase by 10% each and the percentage of population below poverty line remains unchanged for all the villages, then find the population of village V below poverty line in 1998, given that the population of village Y in 1997 was 30000.
(a) 11250 (b) 12760 (c) 13140 (d) 13780

5. If in 1999, the population of village R increases by 10% while that of village Z reduces by 5% compared to that in 1997 and the percentage of population below poverty line remains unchanged for all the villages, then find the approximate ratio of population of village R below poverty line to the ratio of population of village Z below poverty line for the year 1999.
(a) 2 : 1 (b) 3 : 2 (c) 4 : 3 (d) 5 : 4

Solutions

1. (c) : Let the population of village X be x.

Then, 38% of x = 12160 $\Rightarrow x = \frac{12160 \times 100}{38} = 32000.$

Now, if s be the population of village S, then

16 : 11 = 32000 : S $\Rightarrow s = \frac{11 \times 32000}{16} = 22000.$

2. (c) : Let N be the total population of all the seven villages.
Then, population of village T below poverty line = 46% of (21% of N)
And population of village, Z below poverty line = 42% of (11% of N)

$\therefore$ Required ratio $= \frac{46\% \text{ of}(21\% \text{ of N})}{42\% \text{ of}(11\% \text{ of N})} = \frac{46 \times 21}{42 \times 11} = \frac{23}{11}$

3. (b) : Population of village R = 32000 (given).
Let the population of village Y be y.

Then, 16 : 15 = 32000 : y $\Rightarrow y = \frac{15 \times 32000}{16} = 30000.$

$\therefore$ Population of village Y below poverty line = 52% of 30000 = 15600.

4. (b) : Population of village Y in 1997 = 30000 (given).
Let the population of village V in 1997 be v.

Then, 15 : 10 = 30000 : v → $v = \frac{30000 \times 10}{15} = 20000.$

Now, population of village V in 1998 = 20000 + (10% of 20000) = 22000.
$\therefore$ Population of village V below poverty line in 1998 = 58% of 22000 = 12760.

5. (a) : Let the total population of all the seven villages in 1997 be N.

Then, population of village R in 1997 = 16% of N = $\frac{16}{100}$ N

And population of village Z in 1997 = 11% of N = (11/100) N.

$\therefore$ Population of village R in 1999 = $\left\{\frac{16}{100}\text{N} + \left(10\% \text{ of } \frac{16}{100}\text{N}\right)\right\} = \frac{1760}{10000}N$

And population of village Z in 1999 = $\left\{\frac{11}{100}\text{N} + \left(5\% \text{ of } \frac{11}{100}\text{N}\right)\right\} = \frac{1045}{10000}N$

Now, population of village R below poverty line for 1999 = 51% of $\left(\frac{1760}{10000}N\right)$

And population of village Z below poverty line for 1999 = 42% of $\left(\frac{1045}{10000}\text{N}\right)$

$\therefore$ Required ratio = $\frac{51\%\text{of}\left(\frac{1760}{10000}\text{N}\right)}{42\%\text{of}\left(\frac{1045}{10000}\text{N}\right)} = \frac{51 \times 1760}{42 \_ 1045} = \frac{2}{1}$

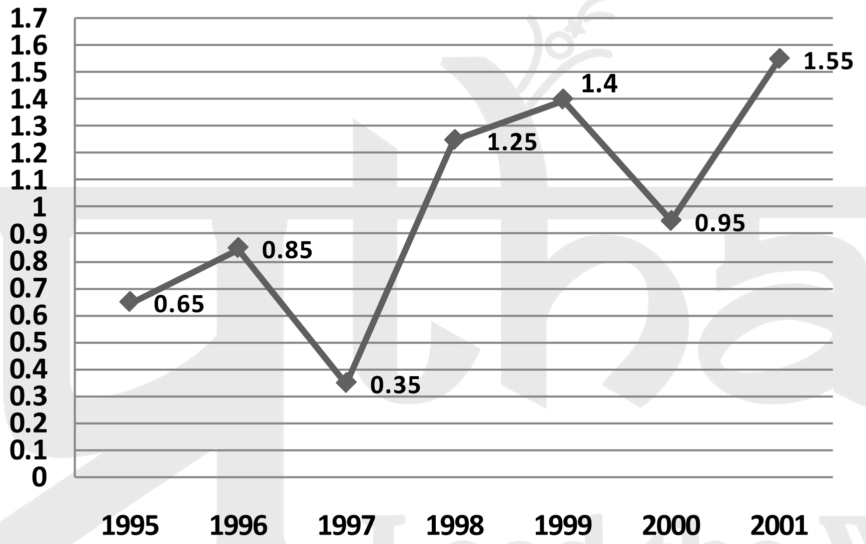
LINE-GRAPHS

This section comprises of questions in which the data collected in a particular discipline are represented by specific pattern joined together by straight lines. The points are plotted on a two-dimensional plane taking one parameter on the horizontal axis and the other on the vertical axis. The candidate is required to analyze the given information and thereafter answer the given questions on the basis of the analysis of data.

Solved Examples

I. **The following line-graph gives the ratio of the amounts of imports by a company to the amount of exports from that Company over the period from 1995 to 2001. The questions given are based on this graph.**

Ratio of Value of Imports to Exports by a Company over the Years.



1. In how many of the given years were the exports more than the imports?
   (a) 1 (b) 2 (c) 3 (d) 4 (e) None of these

2. The imports were minimum proportionate to the exports of the Company in the year:
   (a) 1995 (b) 1996 (c) 1997 (d) 2000 (e) 2001

3. If the imports of the Company in 1996 was Rs. 272 crores, the exports from the Company in 1996 was:
   (a) Rs. 370 crores (b) Rs. 320 crores (c) Rs. 357 crores
   (d) Rs. 420 crores (e) None of these

4. What was the percentage increase in imports from 1997 to 1998?
   (a) 72 (b) 56 (c) 28 (d) None of these (e) Data inadequate

5. If the imports in 1998 was Rs. 250 crores and the total exports in the years 1998 and 1999 together was Rs. 500 crores, then the imports in 1999 was:
   (a) Rs. 250 crores (b) Rs. 300 crores (c) Rs. 357 crores
   (d) Rs. 420 crores (e) None of these

Solutions

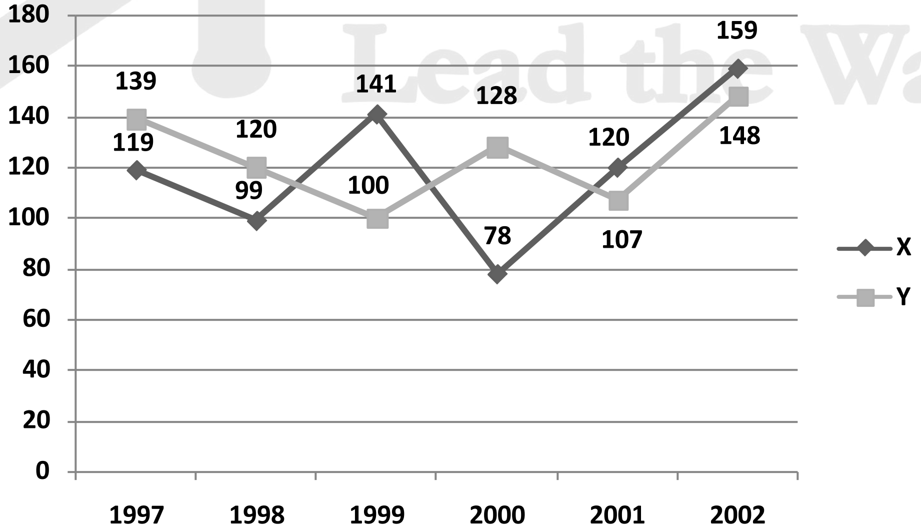
1. (d) : The exports are more than the imports implies that the ratio of value of imports to exports is less than 1.
   Now, this ratio is less than 1 in the years 1995, 1996, 1997 and 2000.
   Thus, there are four such years.
2. (c) : The imports are minimum proportionate to the exports implies that the ratio of the value of imports has the minimum value.
   Now, this ratio has a minimum value of 0.35 in 1997, i.e., the imports are minimum proportionate to the exports in 1997.
3. (b) : Ratio of imports to exports in the year 1996 = 0.85.
   Let the exports in 1996 = Rs. x crores.
   Then, $\frac{272}{x} = 0.85 \Rightarrow x = \frac{272}{0.85} = 320.$
   $\therefore$ Exports in 1996 = Rs. 320 crores.
4. (e) : The graph gives only the ratio of imports for different years. To find the percentage increase in imports from 1997 to 1998, we require more details such as the value of imports or exports during these years. Hence, the data is inadequate to answer this question.

5. (d) : The ratio of imports for the years 1998 and 1999 are 1.25 and 1.40 respectively.
   Let the exports in the year 1998 = Rs. x crores.
   Then, the exports in the year 1999 = Rs. (500 – x) crores.
   $\therefore 1.25 = \frac{250}{x} \Rightarrow x = \frac{250}{1.25} = 200$ [Using ratio for 1998]

   Thus, the exports in the year 1999 = Rs. (500 – 200) crores = Rs. 300 crores.
   Let the imports in the year 1999 = Rs. y crores.
   Then, $1.40 = \frac{y}{300} \Rightarrow y = (300 \times 1.40) = 420.$
   $\therefore$ Imports in the year 1999 = Rs. 420 crores.

II. **Study the following line-graph and answer the questions based on it.**

Number of Vehicles Manufactured by Two Companies over the Years (In thousand)



1. What is the difference between the total productions of the two Companies in the given years?
   (a) 19000 (b) 22000 (c) 26000 (d) 28000 (e) 29000
2. What is the difference between the numbers of vehicles manufactured by Company Y in 2000 and 2001?
   (a) 50000 (b) 42000 (c) 33000 (d) 21000 (e) 13000

3. What is the average number of vehicles manufactured by Company X over the given period ? (rounded off to the nearest integer)
   (a) 119333 (b) 113666 (c) 112778 (d) 111223 (e) None of these

4. In which of the following years, the difference between the productions of Companies X and Y was the maximum among the given years?
   (a) 1997 (b) 1998 (c) 1999 (d) 2000 (e) 2001

5. The production of Company Y in 2000 was approximately what percent of the production of Company X in the same year?
   (a) 173% (b) 164% (c) 132% (d) 97% (e) 61%

Solutions

From the line-graph it is clear that the productions of Company X in the years 1997, 1998, 1999, 2000, 2001, and 2002 are 119000, 99000, 141000, 78000, 120000 and 159000 respectively and those of Company Y are 139000, 120000, 100000, 128000, 107000 and 148000 respectively.

1. (c) : Total production of Company X from 1997 to 2002
   = 119000 + 99000 + 141000 + 78000 + 120000 + 159000 = 716000.

   And total production of Company Y from 1997 to 2002
   = 139000 + 120000 + 100000 + 128000 + 107000 + 148000 = 742000.

   Difference = 742000 – 716000 = 26000.

2. (d) : Required difference = 128000 – 107000 = 21000.

3. (a) : Average number of vehicles manufactured by Company X
   $= \frac{1}{6} \times (119000 + 99000 + 141000 + 78000 + 120000 + 159000) = 119333.$

4. (d) : The difference between the productions of Companies X and Y in various years are:

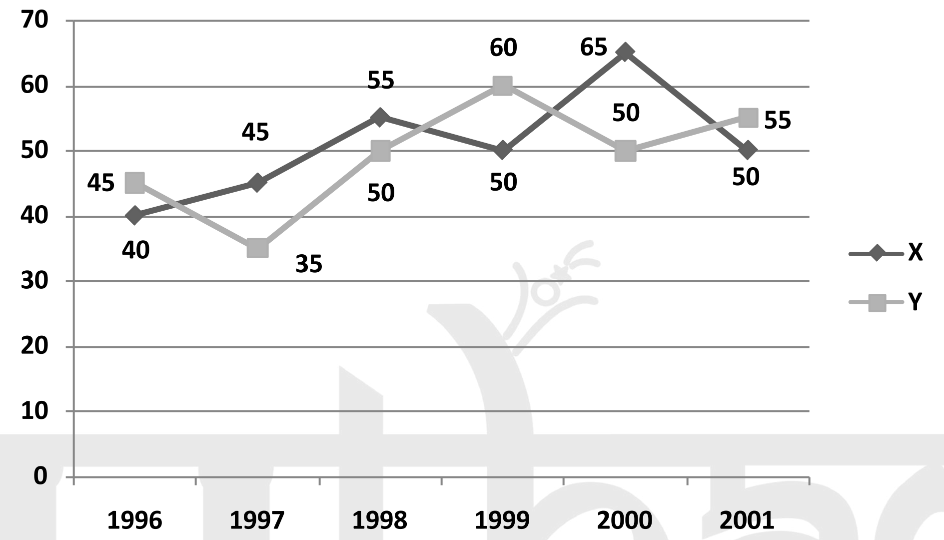
   For 1997 = (139000 – 119000) = 20000;
   For 1998 = (120000 – 99000) = 21000;
   For 1999 = (141000 – 100000) = 41000;
   For 2000 = (128000 – 78000) = 50000;
   For 2001 = (120000 – 107000) = 13000;
   For 2002 = (159000 – 148000) = 110000.
   Clearly, maximum difference was in 2000.

5. (b) Required percentage = $\left(\frac{128000}{78000} \times 100\right)\%$ = 164%.

III. **The following line-graph gives the percent profit earned by two Companies X and Y during the period 1996 – 2001. Study the line-graph and answer the questions that are based on it.**

Percentage Profit Earned by Two Companies X and Y over the Given Years

$$\% \text{ Profit / Loss} = \frac{\text{Income - Expenditure}}{\text{Expenditure}} \times 100$$



1. If the expenditure of Company Y in 1997 was Rs. 220 crores, what was its income in 1997?
(a) Rs. 312 crores (b) Rs. 297 crores (c) Rs. 283 crores
(d) Rs. 275 crores (e) Rs. 261 crores

2. If the incomes of the two Companies were equal in 1999, then what was the ratio of expenditure of Company X to that of Company Y in 1999?
(a) 6 : 5 (b) 5 : 6 (c) 11 : 6
(d) 16 : 15 (e) 15 : 16

3. The incomes of the Companies X and Y in 2000 were in the ratio of 3 : 4 respectively. What was the respective ratio of their expenditure in 2000?
(a) 7 : 22 (b) 14 : 19 (c) 15 : 22
(d) 27 : 35 (e) 33 : 40

4. If the expenditures of Companies X and Y in 1996 were equal and the total income of the two Companies in 1996 was Rs. 342 crores, what is the total profit of the two Companies together in 1996? (Profit = Income – Expenditure)
(a) Rs. 240 crores (b) Rs. 171 crores (c) Rs. 120 crores
(d) Rs. 102 crores (e) Rs. 255 crores

5. The expenditure of Company X in the year 1998 was Rs. 200 crores and the income of Company X in 1998 was the same as its expenditure in 2001. The income of Company X in 2001 was:
(a) Rs. 465 crores (b) Rs. 385 crores (c) Rs. 335 crores
(d) Rs. 295 crores (e) Rs. 255 crores

Solutions

1. (b) : Profit percent of Company Y in 1997 = 35.

   Let the income of Company Y in 1997 be Rs. x crores.

   Then, 35 = $\frac{x-220}{220} \times 100 \Rightarrow x = 297$.

   $\therefore$ Income of Company Y in 1997 = Rs. 297 crores.

2. (d) : Let the incomes of each of the two Companies X and Y in 1999 be Rs. x. And let the expenditure of Companies X and Y in 1999 be E_1 and E_2 respectively.
   Then, for Company X we have:
   $$50 = \frac{x-E_1}{E_1} \times 100 \Rightarrow \frac{50}{100} = \frac{x}{E_1} - 1 \Rightarrow x = \frac{150}{100} E_1 \quad \text{(i)}$$
   Also, for Company Y we have:
   $$60 = \frac{x-E_2}{E_2} \times 100 \Rightarrow \frac{60}{100} = \frac{x}{E_2} - 1 \Rightarrow x = \frac{160}{100} E_2 \quad \text{(ii)}$$
   From (i) and (ii) we get:
   $$\frac{150}{100}E_1 = \frac{160}{100}E_2 \Rightarrow \frac{E_1}{E_2} = \frac{160}{150} = \frac{16}{15}$$ (Required ratio)

3. (c) : Let the incomes in 2000 of Companies X and Y be 3x and 4x respectively. And let the expenditures in 2000 of Companies X and Y be E_1 and E_2 respectively.
   Then, for Company X we have:
   $$65 = \frac{3x-E_1}{E_1} \times 100 \Rightarrow \frac{65}{100} = \frac{3x}{E_1} - 1 \Rightarrow E_1 = 3x \times \left(\frac{100}{165}\right) ....(i)$$
   For Company Y we have:
   $$50 = \frac{4x-E_2}{E_2} \times 100 \Rightarrow \frac{50}{100} = \frac{4x}{E_2} - 1 \Rightarrow E_2 = 4x \times \left(\frac{100}{150}\right) \quad ...\text{(ii)}$$
   From (i) and (ii), we get:
   $$\frac{E_1}{E_2} = \frac{3x\left(\frac{100}{165}\right)}{4x\left(\frac{100}{150}\right)} = \frac{3150}{4165} = \frac{15}{22}$$ (Required ratio).

4. (d) : Let the expenditures of each of the Companies X and Y in 1996 be Rs. x crores. And let the income X in 1996 be Rs. z crores so that the income of Company Y in 1996 = Rs. (342 – z) crores.
   Then, for Company X we have:
   $$40 = \frac{z-x}{x} \times 100 \Rightarrow \frac{40}{100} = \frac{z}{x} - 1 \Rightarrow x = \frac{100z}{140} \quad ...\text{(i)}$$
   Also, for Company Y we have:
   $$45 = \frac{(342-z)-x}{x} \times 100 \Rightarrow \frac{45}{100} = \frac{(342-z)}{x} - 1 \Rightarrow x = \frac{(342-z)100}{145} \quad ...\text{(ii)}$$

From (i) and (ii), we get:

$\frac{100z}{140} = \frac{(342 - z)100}{145} \Rightarrow$ z = 168.

Substituting z = 168 in (i), we get : x = 120.

$\therefore$ Total expenditure of Companies X and Y in 1996 = 2x = Rs. 240 crores.

Total income of Companies X and Y in 1996 = Rs. 342 crores.

$\therefore$ Total profit = Rs. (342 – 240) crores = Rs. 102 crores.

5. (a) : Let the income of Company X in 1998 be Rs. x crores.

Then, 55 = $\frac{x - 100}{200} \times 100 \Rightarrow x = 310$.

$\therefore$ Expenditure of Company X in 2001

= Income of Company X in 1998 = Rs. 310 crores.

Let the income of Company X in 2001 be Rs. z crores.

Then, 50 = $\frac{z - 310}{310} \times 100 \Rightarrow z = 465$.

$\therefore$ Income of Company X in 2001 = Rs. 465 crores.

EXPLANATORY ANSWERS

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EXPERIENCE THE PRATHAM EDGE 1: NUMBER SYSTEM

1. (a): $\frac{11}{32} > \frac{1}{3}, \frac{15}{47} < \frac{1}{3} \;\&\; \frac{37}{115} < \frac{1}{3}$

Alternate Method:

Since what we can say by seeing 11/32, 15/47, 37/15

11/32 will be equal to 1/3 if 11/33

15/47 will be equal to 1/3 if 15/45

37/115 will be equal to 1/3 if 37/111

So, 11/32 > 1/3 Since 32 < 33

(11/32 > 11/33 =1/3)

15/47 < 1/3, Since 47 > 45

15/47 < 15/45 = 1/3

37/115 <1/3, Since 115 > 111

37/115 < 37/111 = 1/3

37/115 < 1/3

Hence 11/32 will be the greatest

2. (c): $\frac{45}{49}, \frac{49}{53}, \frac{53}{57}.$

When the difference of numerator and denominator of all the numbers are same, then the fraction with greater numerator is greater and with smallest numerator is smallest.

Therefore, 45/49 is the answer.

3. (a): Last 3 digits = 400 which is divisible by 8

4. (c): Let wealth be equal to w

Given to wife = w/3

Given to 1st son $= \frac{1}{2}\left(w - \frac{w}{3}\right) = \frac{w}{3}$

Given to 2nd son$= \frac{1}{2}\left(w - \frac{w}{3} - \frac{w}{3}\right) = \frac{w}{6}$

Youngest son = 6 lac = $w - \frac{w}{3} - \frac{w}{3} - \frac{w}{6} = w/6$

Therefore, w = 36 lakhs

Alternate Method:

By looking at the question, we can see remaining wealth is 6 lakhs and then by looking at the options only 36 and 42 are multiple of 6.

Then, check both the numbers and verify the answer.

5. (c): $\frac{1}{3\times7} + \frac{1}{7\times11} + \frac{1}{11\times15} + \frac{1}{15\times19}$

$\frac{1}{4}\left(\frac{1}{3} - \frac{1}{7}\right) + \frac{1}{4}\left(\frac{1}{7} - \frac{1}{11}\right) + \frac{1}{4}\left(\frac{1}{11} - \frac{1}{15}\right)$

$+ \frac{1}{4}\left(\frac{1}{15} - \frac{1}{19}\right)$

$= \frac{1}{4}\left(\frac{1}{3} - \frac{1}{19}\right) = \frac{4}{57}$

6. (a): x + y = 8

(10x + y) – (10y + x) = 54

9(x – y) = 54 ⇒ x – y = 6

Solving this we get, x= 7, y = 1

Alternate Method:

Work through the options

7. (d): $\frac{\left(64^3 - 17^3 - 47^3\right)}{3\times47\times64} = \frac{3\times47\times17\times64}{3\times47\times64} = 17$

Since if a + b + c = 0, then $a^3 + b^3 + c^3 = 3abc$

8. (c): $\sqrt{64\times x\times64} = 32 \times 32$

$\Rightarrow (64 \times 64) \times x = (32)^2 \times (32)^2$

$\Rightarrow x = 16 \times 16 = 256$

9. (c): $\frac{0.004}{\sqrt{x}} = 0.0008,\ \sqrt{x} = \frac{1}{0.2} = 5 \quad \therefore x = 25$

10. (a): LCM of 20, 30 & 15 is 60 highest multiple of 60 less than 10,000 = 9960

Remainder ⇒ 18, 28, 13 for 20, 30, 15

20 – 18 = 2, 30 – 28 = 2, 15 – 13 = 2

Hence, number = 9960 – 2 = 9958

11. (b): 1 + ……+ 100 + 99 + …….. + 1

2(1 + 2 + 3 + ….+ 99) + 1(100)

$2\text{x}\frac{99(99+1)}{2} + 100 = 99 \text{ x } 100 + 100 = 10,000$

12. (c): $\frac{(8000)^3 + (0.080)^3}{(400)^3 + (0.004)^3} = 20^3\left[\frac{400^3 + (0.004)^3}{400^3 + 0.004^3}\right] = 8000$

13. (b): $48\frac{1}{37} \div 37\frac{1}{48}$

$= \frac{48\times37+1}{37} \times \frac{48}{48\times37+1} = \frac{48}{37}$

ANSWERS

14. (a): $48\frac{1}{12}\times\frac{1}{24}=\left(48+\frac{1}{12}\right)\frac{1}{24}$

$= 2+\frac{1}{12\times 24} \;=\; 2\frac{1}{288}$

15. (d): 1/7 of (6 x 8 x 3 x 2)$+\frac{1}{5}\times\frac{7}{25}-\frac{1}{7}\left(\frac{3}{7}+\frac{8}{14}\right)$

$=\frac{1}{7}\times 288+\frac{7}{125}-\frac{1}{7}$

$=\frac{1}{7}\times 287+\frac{7}{125}=41\frac{7}{125}$

16. (a): 16 x 4 ÷ 2 x 5 + 4 – 6 ÷ 9 x 4 ÷ 2 + 1

= 16x 4/2 x 5 + 4 $-\frac{6}{9}\times\frac{4}{2}$ + 1

= 160 + 4 $-\frac{12}{9}$ + 1 = 165 – 1 – 1/3 = $163^{2}/_{3}$

17. (a): $\left[2-\frac{1}{3}\right]\left[2-\frac{3}{5}\right]\left[2-\frac{5}{7}\right]\ldots\ldots\ldots\left[2-\frac{997}{999}\right]$

$\frac{5}{3}\times\frac{7}{5}\times\frac{9}{7}\times\ldots\ldots\frac{1001}{999}=\frac{1001}{3}$

18. (a): $\frac{7+\sqrt{5}}{7-\sqrt{5}}+\frac{7-\sqrt{5}}{7+\sqrt{5}}=\frac{\left(7+\sqrt{5}\right)^2+\left(7-\sqrt{5}\right)^2}{7^2-\left(\sqrt{5}\right)^2}$

$=\frac{2\times\left(7^2+\left(\sqrt{5}\right)^2\right)}{49-5}=\frac{2\times(49+5)}{44}=\frac{27}{11}$

19. (c): $32^{(2-n)}=64^{n}\Rightarrow 2^{5(2-n)}=2^{6n}$

5(2-n) = 6n ⇒ 10-5n = 6n

11n = 10 ⇒ n = 10/11

20. (a): $5^{1/2}=125^{1/6};11^{1/3}=121^{1/6};123^{1/6}$

21. (b): Books of Math = B/6

Books of Fiction = (B – B/6) 3/5 = B/2

Books are History = (B – B/2 – B/6)8/9 = 8B/27

Books of Science = B – B/6 – B/2 – 8B/27

= (54 - 9 - 27 - 16)/54 B = B/27

Since there is more than 1 book of science hence minimum books

⇒ 2 = B/27 Therefore, B = 54

22. (c): $x\div 4/7\times 8/9=7x/4\times 8/9=14x/9=x\div 9/14$

23. (c): $\sqrt{7}+\sqrt{3}\approx 2.7+1.7=4.4;\ \sqrt{5}\approx 2.4$

$\sqrt{6}+2\approx 2.5+2=4.5$

24. (b): The LCM of (a) and (c) is not 144 therefore option (a) and (c) are wrong.

Also product of 56 and 30 is not 1728

Therefore option (d) is also wrong.

Hence option (b) is the correct answer.

25. (a): 8961 divided by 84 leaves a remainder 57

Therefore, 27 should be added.

26. (c): 145 is divisible by 29; *(145x +58 is the no.)*

Now divide remainder 58 by 29

27. (d): Number is $5(7x+4)+2$

28. (c): Pendulum ticks in 58/57, 609/608

LCM of 58/57, 609/608 = 1218/19

= (LCM of 609 & 58) ÷ (HCF of 608 & 57)

= $64\ ^{2}/_{19}$ seconds

⇒ In 1 hr, (3600×19)/1218 = 56.17 times

29. (b): LCM of 12 & 16 = 48

Nearest no. to 1834 is 1824

30. (a): This question can be best solved by checking options.

Now when 11 is divided by 5 it leaves remainder 1 and when divided by 6 it leaves remainder 5.

Hence option (a) is the correct answer.

31. (b): Only option (b) leaves remainder of 3 when divided by 5.

32. (a): $\frac{(6+1)^{13}+1}{6}$, remainder will be 1 + 1 = 2

33. (a): 200 is divisible by 8

34. (b): $30^{40}\div 17=\frac{(34-4)^{40}}{17}=\frac{\left({}^{40}C_{39}\times 34\times 4^{39}\times 4^{40}\right)}{17}$

$=\frac{4^{40}}{17}=\frac{(4^4)^{10}}{17}$

Therefore, remainder = 1

35. (i) (d) $\begin{array}{r|l} 5 & 6435,\ 8970,\ 7235 \\ \hline & 1287,\ 1794,\ 1447 \end{array}$

(ii) (c) $\begin{array}{r|l} 61 & 6161,\ 2440,\ 3111 \\ \hline & 101,\ 40,\ 51 \end{array}$

36. (a): HCF of = (213 – 3), (241 – 3), (297 – 3)

= (210, 238, 294) = 14

37. (b): HCF of = (364 – 4) & (532 – 7)

= (360, 525) = 15

38. (a): HCF of = (1742 – 4), (3723 – 5), (1843 – 6)

i.e. 1738, 3718, 1837 all are divisible by 11

ANSWERS

39. (b): HCF of (175 – 151), (235 – 175), (235 – 151)
HCF of 24, 60, 84.
HCF = 12
40. (c): HCF of 263 – 221, 326 – 263
$\Rightarrow$ 42, 63 $\therefore$ 21
41. (b): (15, 165), (75, 105)
15 + 165 = 180
HCF = 15
Similarly 75 + 105 =180
HCF = 15 $\Rightarrow$ Hence, (15, 165), (75, 105)
42. (a): (24, 168)(72, 120)
24 + 168 = 192, HCF = 24
72 + 120 = 192, HCF = 24
Hence (24, 168) (72, 120)
43. (a): (15, 180), (45, 60)
Product 15 × 180 = 2700
45 × 60 = 2700, HCF = 15
44. (c): 169 × 60 =10140
HCF of (9971, 10140) = 169
45. (d): 144 ×75 = 10080
HCF of (9936, 10080) = 144
46. (a): HCF of (264, 693) = 33
$\therefore$ (33, 8, 21)
Alternate Method:
Since 264 is divided by 33 only & not by 35, 40, 35
$\therefore$ equal fruits can be given to only 33 girls
Hence (33, 8, 21)
47. (d): LCM of 8, 9, 12 & 15
$\Rightarrow$ 8 × 9 × 5 = 360 seconds
$\Rightarrow$ 6 min
48. (b): LCM of 18, 27, 36
$\Rightarrow$ 18 × 3 × 2 = 108
Remainder of 7 then no is 108 + 7 = 115
49. (a): LCM of 5, 6, 7, 8, 9
$\Rightarrow$ 5 × 6 × 7 × 4 × 3 = 2520
Remainder of 3, 4, 5, 6, 7
Diff = 2, 2, 2, 2, 2
No. is = 2520 – 2 = 2518
50. (c): Directly from the options
Since 10080 is divisible by 6 and all other number are not divisible by 6. (last digit is 5 of every number)
Alternate Method
LCM of 5, 6, 7, 8 is 5 × 6 × 7 × 4 = 840
Last 5 digit no. divisible by 840 is 10080
51. (d): Directly check which no. is divisible by 11 and leaves remainder '0'
Answer is 1012
Alternate Method:
LCM of 6, 7, 9 is 6 × 7 × 3 = 126
No. is of the form 126 x + 4
No. is divisible by 11,
Hence 5x + 4 should be divisible for x = 8
$\therefore$ No. is 1012
52. (a): LCM of 3, 8, 11, 16 = 3 x 16 x 11 = 528
No. close to 67218 which is divisible by 528
is 67584 & 67056; leaves remainder (No. -2) is
67582, 67054
53. (c): LCM of 3, 4, 5, 6, 7 & 12
= 3 × 4 × 5 × 7 = 420
Now between 600 & 900, we get 840
Adding 2, we get 842
54. (a): Since HCF = 25
Directly see from the options which no. is divisible by 25 and only 225 is divisible by 25.
Hence 225 is the answer.
55. (a): Product = HCF × LCM
HCF = 8820 / 1260 = 7
56. (c): HCF = 12, LCM = 360 = 12 × 2 × 3 × 5
No.s 24, 36 LCM of these 2 = 72
3rd no $\Rightarrow$ 12 × 5 = 60
57. (a): Sum = 126
LCM = 180
Check from the options whose LCM is 180.
58. (b) (46! + 1) is divisible by 47. (Since 47 is a prime number)
Hence, when 46! is divided by 47, then remainder obtained = -1 = 46.
59. (d) $\frac{10^6}{7} = \frac{3^6}{7} = \frac{9^3}{7} = \frac{2^3}{7} = \frac{8}{7}$

Hence remainder obtained is 1. Similarly, remainder obtained in each of the given cases = 1.
60. (a) Remainder obtained when $10^{10}/7$ = Remainder obtained when $3^{10}/7 = 9^5/7 = 2^5/7 = 32/7$. Hence, remainder obtained is 4.
Similarly, remainder obtained when $10^{100}/7$ = remainder obtained when $3^{100}/7$ = remainder obtained when 3^{96} x $3^4/7 = 4$.
Similarly, when 10^{1000} is divide by 7, remainder = 4.
Hence, net remainder = (4+4+4)/7 = 5

ANSWERS

61. (c) $48 = 2^4 \times 3$ Number of factors = 5×2 = 10
$60 = 2^2 \times 3 \times 5$ Number of factors = 3×2×2 = 12
30 = 3×2×5 Number of factors = 2×2×2 = 8
Hence 30 has minimum number of factors.

62. (a) There are only four factors of 84 which are the product of only two primes (same or distinct) = 4, 6, 14, 21.

63. (b) $N = A^2 \times B^3 \times C^4$, where, A, B and C are prime numbers.
Perfect Square will be obtained for A^0 and A^2, B° and B^2, C° and C^2 and C^4.
Number of ways powers of A is used = 2
Number of ways powers of B is used = 2
Number of ways powers of C is used = 3
Hence, number of perfect square factors = 2x2x3 = 12

64. (c) $N = A^2 \times B^3 \times C^4$, where A, B and C are prime numbers.
Cubes will be obtained for A^0, B° and B^3, C° and C^3
Number of ways powers of A is used = 1
Number of ways powers of B is used = 2 Number of ways powers of C is used = 2. Hence, number of perfect square factors = 1 x 2 x 2 = 4

65. (c) Number of factors = 17
Product of factors = $N^{f/2} = (3^{16})^{17/2} = 3^{136}$

66. (b) Unit digit of $11^{(5!)} = 1$. Hence, unit digit of $11^{(5!)} - 1 = 0$

67. (b) $7^{2008} = (7^4)^{502}$
Last digit of $7^4 = 1$

68. (b) $1432^{40! \times 30! \times 20!}$ unit digit = 6
$2427^{4! \times 5! \times 10!}$ unit digit = 1
$2319^{10! \times 20! \times 30!}$ unit digit = 1
$1552^{12!}$ unit digit = 6

69. (a) 7^{95} unit digit of $7^3 = 3$
3^{58} unit digit of $3^2 = 9$
Unit digit = 3×9 = 27

70. (d) $41^{44k} = (41^4)^{11k} = 41^{4n}$
Hence unit digit is 1.

EXPERIENCE THE PRATHAM EDGE 2: AVERAGES

1. (a): $\dfrac{92+94+88+90+90+86+90}{7}$
Subtract 90 from each
$\Rightarrow \dfrac{2+4-2+0+0-4+0}{7} = 0$
Hence, 90 is the average score.

2. (d): $\dfrac{4000+2000+3000}{3} = 3000$

3. (b): $\dfrac{71+49+52+46+48+52+53}{7} = \dfrac{371}{7} = 53$

4. (b): $\dfrac{3\times7+2\times8.5+10}{3+2+1} = \dfrac{48}{6} = 8$

5. (a): Combined age of 5 boys = 5 × 19 = 95
Combined age of 6 boys = 6 × 20 = 120
Age of new boy = 120 – 95 = 25
$\Rightarrow$ Inc in age = (20 – 19) =1
Age of new boy = old avg ± (increase/decrease)
= 19 + 6 = 25

6. (b): $\dfrac{7\times14+4\times8.5}{11} = \dfrac{98+34}{11} = 12$

7. (a): Run scored in first 6 tests = 6 × 55 = 330
Run scored in 2nd to 7th test = 6 x 57 = 342
Run scored in 7th test – Run scored in 1st test = Run scored in 2nd to 7th test – Run scored in 1st to 6th
R – 50 = 342 – 330; $\therefore$ R = 62

Alternate Method:

$$\frac{1st+2nd+3rd+4th+5th+6th}{6} = 55$$

$$\frac{2nd+3rd+4th+5th+6th+7th}{6} = 57$$

7th = 57 × 6 - 55 × 6 + 50 = 62

8. (c): Avg. Marks =
$$\frac{76\times70+79\times60+80\times55}{70+60+55} = 78.16$$

9. (a): Avg. Sales =

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$$\frac{60000+16000+24000+30000}{6}=\frac{130000}{6}=21667$$

10. (d): Combined age 10 yr ago = 24 × 4 = 96
Combined age presently = 24 × 6 = 144
Combined age of children
= 144 - (96 + 10 + 10 + 10 +10) = 8
Let age of younger child be x
Then 8 = $x + x + 2 = 2x + 2$;
$\therefore x = 3$

11. (b): Old combined age = $34x$
New combined age
= $(5 + x) \times 32 = 34x + (5 \times 30)$
$\Rightarrow 32x + 160 = 34x + 150$
$\therefore$ x = 5

12. (d): Avg = (20 × 16 + 16 × 25)/(20 + 16)
= 720/ 36 = 20

13. (d): Avg = (16 × 2 + 21 × 3)/(2+3) = 95/5 = 19

14. (b): (15 × 20 + 12 × 24)/(15 + 12) = (588)/27

15. (c): a + b = 25; ab = 100
a + 100/a = 25
$\Rightarrow a^2 + 100 - 25a = 0 \Rightarrow a^2 - 25a + 100 = 0$
$\Rightarrow a^2 - 20a - 5a + 100 = 0$
$\Rightarrow a(a-20) - 5(a-20) = 0$
$\Rightarrow (a-5)(a-20) = 0$
Therefore, a = 5 or a = 20
If a = 5; b = 20
If a = 20; b = 5

16. (a): New avg. age = $\frac{(23+23)+(5+5)+1}{3}=\frac{57}{3}=19$

17. (b): $8000=\frac{7\times12000+x\times6000}{x+7}$
$8x + 56 = 84 + 6x$
$\Rightarrow x = 14$.
Hence Total number of workers are 7+14 = 21

18. (c): Combined age of family 3 years ago = 5×17 = 85
Combined age of family now = 6 × 17 = 102
Baby's age,
$\Rightarrow$ 102 = 85 + 5 x 3 + B $\therefore$ B = 2

19. (d): Old avg. = 63
New avg. = 65
Let n be number of papers
$65n = 63n + 20 + 2 \Rightarrow n = 11$

20. (b): Let n be ratio of boys to girls
$15.8=\frac{16.4n+15.4}{n+1}$ => $.6n = .4$ => $n = 2/3$
Alternate Method: B:G = x:y
[(16.4)(x)+(15.4)(y)]/(x+y) = 15.8(x+y)
x:y = 2:3

21. (c): A + B + C = 84 × 3 = 252
A + B + C+ D = 80 × 4 = 320
D = 68
E = D + 3 = 71
B + C + D + E = 79 × 4 = 316
B + C + D = 245
A = 320 – 245 = 75

22. (a): Let the old Average = x
$\frac{10x+108}{11}=x+6\Rightarrow x=42$
$\therefore$ New average = 42 + 6 = 48

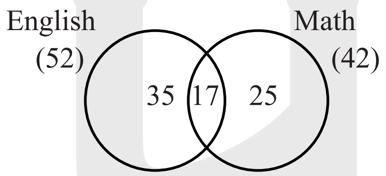
23. (c): Old combined weight = $8x$
New combined weight
= $8(x + 2.5) = 8x – 65$ + new person
$\therefore$ New person = 85

24. (d): First no = x
Second no = $x/2$
Third no = $3x/2$
Fourth no = $3x/10$
Avg. = $\frac{10\quad 5+15\quad 3}{10\times4}x \quad \frac{33}{40}x$
$\Rightarrow x=\frac{990}{33}=30$
Largest number is 45

25. (b): Mean of new
= mean of old + avg. increment
= 35 + 5 = 40

ANSWERS

EXPERIENCE THE PRATHAM EDGE 3: PERCENTAGE

1. (d): 3rd no = x, 1st no = $(1 - 0.30)x = 0.7x$
   2nd no = $(1 - 0.37)x = 0.63x$
   $= \dfrac{2nd}{1st} = \dfrac{0.63x}{0.7x} = 0.9 = 90\%$
2. (c): $5000\,(1.02)^2$ = new population = 5202
3. (a): % Defective TV
   $= \dfrac{\text{no of TV} \times \text{defective}}{\text{Total}} = 60\% \times 5\% = 3\%$
4. (d): Let initial price be 100, Increasing price by 20%, the new price = 120
   Decreasing price by 20% then new price
   $= 120 \times 80\% = 96 \Rightarrow 4\%$ less
5. (b):


   Failed in at least one = 25 + 35 + 17 = 77%
   Passed in both = 100 – 77 = 23%
6. (c): Winning margin = 300 = Won – Loss
   = 65% - 35% = 30%
   Hence total votes = 300/(30%) = 1000
7. (c): Let number be n
   $0.4n + 42 = n \Rightarrow n = 70$
8. (c): Old radius = r; Old area = πr^2
   New radius = r(1 + 2) = 3r
   New area = $\pi\,(3r)^2 = 9\pi r^2$
   Increase in area% $= \dfrac{9\pi r^2 - \pi r^2}{\pi r^2} = 800\%$
9. (b): Students failing in at least 1
   = 37% + 20% - 6% = 51%
   Students who passed both = 245 $=(100-51)x$
   = 49%
   Hence total students = 245/49% = 500
10. (a): Increase a no. by 40% = 140%
    Let no be x then $\frac{140}{100}x = \frac{7}{5}x = \frac{x}{5}$
    It must be divided by 5/7
11. (d): 20% A = 60% B $\Rightarrow$ A = $\frac{60\%}{20\%} \times$ B
    A = 3B = 300% B
12. (b): Valid votes = $(100 - 10)\%\ x - 60 = 0.9x - 60$
    Votes for A = $0.47x$
    Votes for B = $0.47x - 308$
    Hence $0.47x + 0.47x - 308 = 0.9x - 60$
    $\Rightarrow 0.04x = 248 \quad \therefore x = 6200$
13. (b): Let initial population be 100
    Population after 1st yr = (100 + 10) = 110
    Population after 2nd year = (110-11) = 99
    Therefore, total decrease of 1%
14. (d): $45 \times ? = 25\%$ of $900 \Rightarrow ? = \dfrac{0.25 \times 900}{45} = 5$
15. (d): 218% of 1674 = ? × 1800
    $? = \dfrac{2.18 \times 1674}{1800} = 2.02$
16. (d): $\dfrac{1}{4}\left(\dfrac{1}{3}\left(\dfrac{2}{5}n\right)\right) = 15$
    $\Rightarrow n = 15 \times 5 \times 3 \times 2 = 450 \quad \Rightarrow 0.4n = 0.4 \times 450 = 180$
17. (c): $\dfrac{18}{7200} \times 100 = \dfrac{1}{4} = 0.25\%$
18. (c): (100 – 12.5) (100 – 70)% money = 210
    Money = $210 \times \dfrac{1}{0.3} \times \dfrac{8}{7} = 800$
19. (d): Data is inadequate. We need transportation expenses as % of remaining or total amount
20. (d): Books in Hindi = 20% B
    Books in English = (100 – 20)% (100– 50)%
    = 0.8×0.5 B = 0.4 B
    Remn. books = B – 0.2B – 0.4B= 0.4B= 9000
    Books in English = 0.4B = 9000
21. (d): Marks in drawing = 50
    Marks in Biology = B = 25% (B + M + D) – 20
    $\Rightarrow 0.75B = 0.25M - 7.5$
    Therefore, Cannot be determined
22. (a): Valid votes = 80% × 7500 = 6000
    Other candidate votes = 45% × 6000 = 2700
23. (s): $\Rightarrow 4\%\,(x - 68) = 98$
    $\Rightarrow x - 68 = 2450$
    $\Rightarrow x = 2518.$

ANSWERS

24. (b): Saving =100 – 30% - 15% - 20%
= 35% = 280
Salary = 280 / 35% = 800

25. (c): B= 1.2A and C = 1.3B
700 = A + B + C+ D =

$$\frac{C}{1.2\times1.3}+\frac{C}{1.3}+C+136$$

$$=C\left(\frac{1.56+1.2+1}{1.56}\right)+136 \therefore C=\frac{564\times1.56}{3.76}=234$$

26. (b): $\frac{\text{No of women}}{\text{No of Men}}=\frac{2500\times(100-20)}{2500\times(100+20)}=\frac{80}{120}$

$\% = \frac{80}{120}\times100 = 66\frac{2}{3}\%$

27. (d): Money Left
= (100 – 40)% (100 – 20)% (100 – 25)%
= 60% × 80% × 75% = 36%

For Q. 28-30 $\frac{loser}{winner}=66\frac{2}{3}\%=\frac{2}{3}$

Winner-Loser =60 = W-$\frac{2W}{3}=\frac{W}{3}$

W = 180 ; L = 120 $\therefore$ Total = 120 + 180 = 300

28. (c)
29. (b)
30. (a): 180/300 = 60%

EXPERIENCE THE PRATHAM EDGE 4: PROFIT & LOSS

1. (a): Gain = $\frac{28.6-27.50}{27.50}=\frac{1.1}{27.5}=\frac{1}{25}=4\%$

2. (d): % Loss =$\frac{490-477.5}{490}=\frac{12.5}{490}=0.025=2.5\%$

3. (c): SP of book = 115.20; Loss = 10%

CP = $\frac{100}{90}\times115.20=128$

Gain = 5%; SP = $\frac{105}{100}\times128=134.4$

4. (a): Loss = 20%; SP = 1024

CP = $\frac{100}{100-L}\times1024=1280$ SP =1472

Gain = $\frac{1472-1280}{1280}\times100=15\%$

5. (b): SP of 33m = CP of 33m + P of 33m
= SP of 11m + CP of 33m
SP of 22m = CP of 33 m
P% = 11/22 ×100 = 50%

6. (c): Let Price = P Quantity = Q
PQ = (0.8P) (Q + 2.5) = 160
$\Rightarrow$ P = 16

7. (c): $\frac{80\times13.5+120\times16}{80+120}=\frac{1080+1920}{200}=15$

To gain 16% $\Rightarrow$ 15×1.16 = 17.4

8. (a): CP of 10 toffees = $\frac{1\times100}{120}=\frac{5}{6}$

No of toffees in Rs 5/6 = 10

No of toffees in Re 1 = $\frac{60}{5}=12$

9. (d): SP of 6 lemons = $\frac{120}{100}\times1=\frac{6}{5}$

No of lemons in Rs. 6/5 = 6
No of lemons in Rs 1 = 5

10. (b):CP of 1 Banana = 5/6; SP of 1 Banana = 3/4

Gain % $=\left(\frac{3}{4}-\frac{5}{6}\right)\times\frac{100}{5/6}$ = -10%

11. (b): (1 – 0.2) (1 – 0.1) (1 – 0.05)
= 0.8×0.9×0.95 = 0.684 = (1 – 0.316)

12. (a): Let the Marked price be Rs 100
Then SP = Rs (90/100)×100 = Rs 90
Gain = 25%
CP = (100/125)×90 = Rs 72
New Commission = Rs 20, New SP = Rs 80

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New Profit = (8/72)×100 = Rs $11\frac{1}{9}$ %

13. (b): SP = (1 + 0.2) (1 – 0.15)CP = 1.2×0.85CP =1.02CP
14. (b): P = 2/3(5%) + 1/3(-2%) = 8/3% CP = 400
    CP = 400×300/8 = 15,000
15. (b): SP = $1.2CP_1 = 0.8\,CP_2$

$$CP_1 + CP_2 = \frac{SP}{1.2} + \frac{SP}{0.8} = SP\left(\frac{1.2+0.8}{1.2\times0.8}\right) = \frac{2}{0.96}SP$$

$\Rightarrow$ Since 2 items 2SP = 0.96 $(CP_1 + CP_2)$

= (1- 0.04) $(CP_1 + CP_2)$

Alternate Method:

Using direct formula:

$$\text{Loss }\% = \left[\frac{\text{Common Loss \& Gain\%}}{10}\right]^2$$

= $(20/10)^2$ = 4% loss

16. (b): Profit% $= \frac{500-450}{450}\times100 = \frac{50\times100}{450} = 11\frac{1}{9}\%$
17. (c): Let price = P Quantity = Q
    PQ – 3Q = 20, PQ – 3.25Q = -30
    $\therefore$ Q = 200
18. (c): (1.00 – 0.04)CP = 240 $\Rightarrow$ CP = $\frac{240}{0.96}$
    1.1CP = 1.1×240/0.96 = 275
19. (c): Initial SP = 1.06 CP
    New SP = Old SP – 1 = (0.95)CP×(1.1) = 1.045CP
    1.06CP – 1 = 1.045CP
    $\Rightarrow$ 0.015CP = 1 => CP = 66.66
20. (b): Let Price be P & Quantity be Q
    PQ = 0.75P (Q + 4) = 96........... (i)
    $\Rightarrow$ 96 = 3P + 72 $\Rightarrow$ P = 8 Q = 12
21. (b): CP = (4×12 + 2×16)/ (4 + 2)= 80/6
    SP = 1.2CP = 1.2 x 80/6 = 16
22. (b): SP = (1 – 0.4) (1- 0.2)CP = 0.6×0.8CP = 0.48
    CP = (1 – 0.52) CP
23. (d): SP = 660 = 1.1CP => CP = 600
    CP = (1 - 0.25)MP = 0.75MP

$$\Rightarrow MP = \frac{CP}{0.75} = \frac{600}{0.75} = 800$$

24. (b): 0.85 MP = SP = 1.2CP

$$\Rightarrow MP = \left(\frac{1.2}{0.85}\right)CP = \frac{1.2}{0.85}\times 153 = 216$$

25. (d): We need discount on the current price
26. (c): CP of 12 books = SP of 16 books
    = CP of 16 books + Profit of 16 books
    CP of 4 books = Loss of 16 books
    CP of 1 book = loss of 4 books
    Loss = 1/4 CP = 25% CP

Alternate Method:

Let the CP of 1 book = Re 1

CP of 16 books = Rs 16

SP of 16 Books = Rs 12

Loss % = (4/16)×10 = 25%

27. (c): Let SP of 1 apple = x; SP of 36 apples = 36x
    Loss = 4x $\therefore$ CP = 40x
    Therefore, Loss %age = (4/40)×100 = 10%
28. (a): 4000 = 1.25 CP_1
    CP_1 = 3200, Profit = 800
    CP_2 = 4000 + 800 = 4800

    Loss = $\frac{800}{4800}$ = 16.66%
29. (a): CP= 40% of SP = 0.4 SP
    SP = 2.5 CP = 250% of CP
30. (b): 1.2 CP = 12000 => CP =10000
    P = 2000
    0.8 CP = 12000 => CP = 15000
    L = 3000
    Loss% = $\frac{1000\times100}{10000+15000} = \frac{1\times100}{25} = 4\%$

Alternate Method: Using direct formula:

$$\text{Loss }\% = \left[\frac{\text{Common Loss \& Gain\%}}{10}\right]^2 = 4\%\text{ Loss}$$

ANSWERS

EXPERIENCE THE PRATHAM EDGE 5: SIMPLE & COMPOUND INTEREST

1. (b): SI = $\frac{10 \times 3 \times 2000}{100}$ = 600

   A = 2000 + 600 = 2600

2. (c): $\frac{SI_1}{SI_2} = \frac{P_1R_1T_1}{P_2R_2T_2}$ = $T_1 = T_2$, $SI_1 = SI_2$ & $R_1/R_2 = 5/4$

   $\frac{P_1}{P_2} = \left(\frac{SI_1}{SI_2}\right)\left(\frac{R_2}{R_1}\right)\left(\frac{T_2}{T_1}\right) = \frac{4}{5}$

3. (c): $901 = \frac{3/2PR}{100} + P;\quad 85 = \frac{2.5PR}{100}$

   $\Rightarrow$ PR = 3400

   P = 986 - 4PR/100 = 986 – 136 = 850

   PR = 3400 $\Rightarrow$ R = 4

4. (b): $\frac{600 \times 10 \times T}{100} = \frac{800 \times 12 \times 5}{100}$ $\Rightarrow$ T = 8 years

5. (a): SI = (10 ×4 ×3)/100 = 1.2

6. (b): 3P = P + SI = P + (P ×R ×15.5)/100

   $\Rightarrow$ 2 = (31/ 200) R $\Rightarrow$R=400/31

   2P = P + SI $\Rightarrow$ P = (P×R×T)/100

   $\frac{P \times 400}{31 \times 100} \times T$ $\Rightarrow$ T = $\frac{31}{4} = 7\frac{3}{4}$

7. (a): $SI_1 = SI_2 \Rightarrow \frac{(625-p)5 \times 2}{100} \Rightarrow \frac{p \times 10 \times 4}{100}$

   $\Rightarrow$ 625 – p = 4p $\Rightarrow$ p = 125

8. (c): (may 21 – march 9) 1994 = 2 month 12 days

   = $2^{12}/_{30}$ = 2.4 month

   SI = $\frac{1820 \times \frac{2.4}{12} \times 7.5}{100} = 27.3$

9. (c): $SI_1 - SI_2 = 2.5$

   $= \frac{P_1R_1T_1}{100} - \frac{P_2R_2T_2}{100} = \frac{500R_1 2}{100} - \frac{500R_2 2}{100}$

   2.5 = 10 (R_1 - R_2)

   => R_1 - R_2 = 0.25

10. (d): SI = P = $\frac{PRT}{100}$ $\Rightarrow$ T = $\frac{100}{R} = \frac{100}{12}$ =8 yrs 4 month

11. (d): SI = P is 6years

    SI = 3P is 6 × 3 yrs = 18 yrs

12. (b): 126 = $\frac{300 \times 5 \times T}{100} + \frac{450 \times 6 \times T}{100}$ = 42T $\therefore$ T = 3

13. (b): 60 = $\frac{P(8-7.75) \times 1}{100} = \frac{.25P}{100}$ $\Rightarrow$ P = 24000

14. (b): Amount paid at the end of 2nd year = 121

    Principal paid at End of 2nd year = P_1

    121 = $\frac{P_1 \times 10 \times 1}{100}$ + P_1= 110

    Amount remaining at end of 1st year =

    110+121= 231

    Principal due at start of 1st year

    $\Rightarrow$ 231 = $\frac{P_0 \times 10 \times 1}{100}$ + P_0 $\Rightarrow$ P_0 = 210

15. (a): CI-SI = 16, A-P-SI= 16

    $P\left[\left(1+\frac{10}{100}\right)^2 - 1 - \frac{20}{100}\right] = 16 \Rightarrow P = 1600$

    New CI = $P(1+\frac{r}{200})^{2t} - P$

    = 1600[(1+ .05)^4 -1] = 344.81

    New SI = Old SI = $\frac{1600 \times 10 \times 2}{100}$ = 320

    Difference = 24.81

16. (b): A = P(1+ $\frac{r}{100}$)^t

    2 = A/P = (1 + $\frac{r}{100}$)^5 $\Rightarrow$ $\frac{r}{100}$ = $2^{1/5}$-1

    A = P (1+$\frac{r}{100}$)^{20} = 12000 (1 + $2^{1/5}$- 1)^{20}

    = 1200 × (2)^4 = 192000

17. (c): 2 = A/P = (1 + $\frac{r}{100}$)^5 $\Rightarrow$ $\frac{r}{100}$ = $2^{\frac{1}{5}}$– 1

    A/P = 8 = (1 + $\frac{r}{100}$)^t

    $\Rightarrow$ $2^3 = 2^{t/5}$ $\Rightarrow$ t = 15

18. (b): $\frac{PR^2}{100^2}$ = 20

    $\frac{P(10)^2}{100^2} = 20 \Rightarrow P = 2000$

19. (b): $926.10 = 800(1+\frac{.10}{2})^{2t}$ ==> $(\frac{21}{20})^3 = \left(\frac{21}{20}\right)^{2t}$

    T = 3/2 yrs.

20. (c): 3P = P (1 + r)^3; R = (3)$^{1/3}$ – 1

    A = 9P = P(1 + $3^{1/3}$ – 1)^t

    $\Rightarrow$ 9 = $3^{t/3}$ $\Rightarrow$ t = 6

ANSWERS

21. (b): Time from May 3rd to July 15th

=28 days of May + 30 days of June and 15 days of July

=73 days = 1/5 years

$$\therefore\ I = \frac{P \times R \times T}{100} = \frac{500 \times 6 \times \frac{1}{5}}{100} = Rs.6$$

22. (b): We have, n=4 and T = 24 years

$\therefore$ Rate of interest

$$= \frac{100(n-1)}{T} = \frac{100(4-1)}{24} = 12\frac{1}{2}\%$$

23. (b): $SI = \dfrac{2600 \times \frac{20}{3} \times t}{100} = \dfrac{5200}{3}t$

$\therefore$ T has to be a multiple of 3

24. (a): $A = 2P = P\left(1 + \frac{r}{100}\right)^{15} \quad \Rightarrow \frac{r}{100} = 2^{1/15} - 1$

$A = 8P = P(1+2^{15} - 1)^{t} \quad \Rightarrow 8 = 2^{3} = 2^{t/15}$

$\therefore$ T = 45

25. (b): $A = P\left(1+\frac{r}{100}\right)^{t}$; $\Rightarrow$ $7300 = P\left(1 + \frac{r}{100}\right)^{2}$

$\Rightarrow 8575 = P\left(1 + \frac{r}{100}\right)^{3} \quad \Rightarrow \quad \frac{8575}{7300} = \frac{343}{292} = 1 + \frac{r}{100}$

$$P = 7300\left(\frac{1}{1+\frac{r}{100}}\right)^{2} = 7300 \times \frac{292}{343} \times \frac{7300}{8575} = 5290$$

26. (c): $A = 7500\,(1+.04)^{2} = 7500 \times 1.0816 = 8112$

27. (b): $SI = 1200 = \frac{P \times 5 \times 3}{100} \Rightarrow P = 8000$

$A = 8000(1.05)^{3} = \frac{9.261}{8.00}(8000) = 9261$

CI = 9261 – 8000 = 1261

28. (d): SI & CI for 1st yr

$\Rightarrow 150 = \dfrac{SI \times 12\frac{1}{2} \times 1}{100}$; SI = 600

$\Rightarrow 600 = \dfrac{P \times 12\frac{1}{2} \times 1}{100} \quad \Rightarrow P = 9600$

29. (c): Interest for half year = 5%

$$\Rightarrow 25\left(\frac{100}{5}\right) \times \left(\frac{100}{5}\right) = 10{,}000$$

30. (c): $CI = A - P = P\left(1 + \frac{r}{100}\right)^{t} - P$

$$P\left[\left(1+\frac{r}{100}\right)^{t} - 1\right] = 1270 \quad \Rightarrow P\left[\left(1+\frac{1}{6}\right)^{3} - 1\right] = 1270$$

$$P = \frac{1270 \times 216}{127} \quad \Rightarrow P = 2160$$

31. (c): Money doubles itself in 5 years
Money becomes 4 times itself in (5 + 5) years
Money becomes 8 times itself in (5+5+5) =15 yrs

32. (c): Money becomes thrice itself in 3years
Money becomes (3 × 3) times in (3 + 3) yrs

33. (b): A=P+SI= 20000 + 20000 × 4 ×10/100=28000

34. (d): $A = P + SI = 6000 + \dfrac{6000 \times 12 \times 1}{100} = 6720$

35. (b): $A = P\left(1+\frac{r}{100}\right)^{t} = P\left(1+\frac{r}{200}\right)^{2t} = \textit{for half annual}$

$$\left(1+\frac{5}{100}\right)^{2t} = \left(\frac{21}{20}\right)^{2t} = \frac{A}{P} = \frac{9261}{8000} = \left(\frac{21}{20}\right)^{3} \Rightarrow t = \frac{3}{2}$$

ANSWERS

EXPERIENCE THE PRATHAM EDGE 6: RATIO & PROPORTIONS

1. (d): $\frac{Milk}{Water} = \frac{2}{1} = \frac{40}{20}$

   $New \ \frac{milk}{water} = \frac{1}{2} = \frac{40}{80} = \frac{40}{20+60}$

2. (d): $\frac{3x-9}{5x-9} = \frac{12}{23} \Rightarrow \ 69x - 207 = 60x - 108$

   $\Rightarrow 9x = 99 \quad \therefore x = 11$

   Second number is $5x$ i.e. 55

3. (a): 2/7 diff =(7 – 2) = 5

   Difference = 40 $\Rightarrow$ 40/5 = 8

   Multiplying numerator & denominator by 8

   $\Rightarrow$ 16/56

Alternate Method:

Work with options, answer is option (a) as this is the only option where diff. of terms are 40

4. (c): A : B = 2 : 3 B : C = 4 : 5

   A : B : C = 8 : 12 : 15

   $7000 = A + B + C = \frac{8}{15}C + \frac{12}{15}C + C = 35C/15$

   $\Rightarrow 7000 = 7/3\ C \quad \therefore C = 3000$

5. (c): A : B = 4 : 5 B : C = 2 : 3

   A = 800

   B = 5/4 A $\Rightarrow$ 5/4 × 800 = 1000

   C = 3/2 B $\Rightarrow$ 3/2 × 1000 = 1500

6. (b): $x : y$ = 1 : 10 = 100 : 1000

7. (d): a : b = 3 : 4

   $$\frac{6a+b}{4a+5b} = \frac{6\left(\frac{a}{b}\right)+1}{4\left(\frac{a}{b}\right)+5} = \frac{6\left(\frac{3}{4}\right)+1}{4\left(\frac{3}{4}\right)+5} = \frac{\frac{11}{2}}{8} = \frac{11}{16}$$

8. (b): A/C = (A/B)× (B/C) = 3/4 × 5/6 = 15/24

   A/D = (A/C)× (C/D) = 15/24 × 11/9 = 55/72

   A:D = 55:72

9. (b): A : B = 5 : 7 B : C = 6 : 11

   A : B : C = 5× 6 : 7 × 6 : 11 × 7 = 30 : 42 : 77

10. (b): A : B = 8 : 15 B : C = 5 : 8

    C : D = 4 : 5

    $A : C = A : B \times B : C = \frac{8}{15} \times \frac{5}{8} = \frac{1}{3}$

    $A : D = \frac{A}{C} \times \frac{C}{D} = \frac{1}{3} \times \frac{4}{5} = \frac{4}{15}$

11. (c): 2A = 3B = 4C =k

    $A : B : C = \frac{k}{2} : \frac{k}{3} : \frac{k}{4} = 6 : 4 : 3$

12. (c): $\frac{A}{3} = \frac{B}{4} \ \& \ \frac{B}{4} = \frac{C}{5}$

    $\frac{A}{B} = \frac{3}{4} \ \& \ \frac{B}{C} = \frac{4}{5} \ \Rightarrow$ A: B : C = 3: 4 : 5

13. (a): $\frac{A}{B} = \frac{1}{3} \ \frac{B}{C} = \frac{1}{2} = \frac{3}{6} \quad \Rightarrow$ A : B : C = 1 : 3 : 6

14. (a): x : y = 2 : 1

    $$\frac{x^2 - y^2}{x^2 + y^2} = \frac{\frac{x^2}{y^2} - 1}{\frac{x^2}{y^2} + 1} = \frac{4-1}{4+1} = \frac{3}{5}$$

15. (c):

    $$\frac{12}{19} = \frac{4x^2 - 3y^2}{2x^2 + 5y^2} = \frac{4\frac{x^2}{y^2} - 3}{2\frac{x^2}{y^2} + 5} \Rightarrow \frac{24x^2}{y^2} + 60 = 76\frac{x^2}{y^2} - 57$$

    $$\Rightarrow \frac{x^2}{y^2} = \frac{117}{52} = \frac{9}{4} \Rightarrow \frac{x}{y} = \frac{3}{2}$$

16. (c): Sum of both ratios should be less than 12 & should be a factor of 12

    (a) 2+1 = 3

    (b) 3 + 1 = 4

    (c) 3 + 2 = 5

    (d) 7 + 5 = 12

17. (d): (A – 5) : (B – 10) : (C – 15) = 3 : 4 : 5

    = 600 : 800 : 1000

    A + B + C = 605 + 810 + 1015 = 2430

    B = 810

18. (c): x : y : z = 7 : 8 : 16

    y – x = 27 diff of ratio = 8 – 7 = 1

    Hence, multiply by 27

    x : y : z = 7 : 8 : 16 = 189 : 216 : 432

    x + y + z = 189 + 216 + 432 = 837

19. (a): A : B : C = 1/4 : 1/5:1/6 = 30 : 24 : 20

    $A + B + C = 407 = A + \frac{24}{30}A + \frac{20}{30}A = \frac{74}{30}A$

    A = 165 , B = 132 , C = 110

ANSWERS

20. (c): A : B = 3 : 4 , B : C = 5 : 6, C : D = 7 : 5
A : B : C = 3 × 5 : 4 × 5 : 6 × 4 = 15 : 20 : 24
A : B : C : D = 15 × 7 : 20 × 7 : 24 × 7 : 24 × 5
= 105 : 140 : 168 : 120, Maximum is C

21. (b): a : b : c = 1/2 : 1/3:1/4 = 6 : 4 : 3
104 = a + b + c = a+ 4/6 a + 3/6 a = 13/6 a
a = 48 b = 32 c = 24

22. (c): The income of A and B is 5x and 4x and expenditure of A and B is 3y and 2y
5x - 3y = 800
4x - 2y = 800
Solving these equations give x = 400
Hence income of A is 5x = 2000

23. (b): D = 1 leap of dog = 3 leaps of hare
T = 3 leaps of dog = 5 leaps of hare

$$\frac{speed\ of\ dog}{speed\ of\ hare} = \frac{Distance_{dog} / Time_{dog}}{Distance_{Hare} / Time_{Hare}}$$

$$= \frac{\frac{D}{t/3}}{\frac{D}{3t/5}} = \frac{3}{\frac{5}{3}} = \frac{9}{5}$$

24. (d): (100/3)% A = 50% B

$$= \frac{A}{B} = \frac{50\%}{\frac{100}{3}\%} = \frac{3}{2}$$

B = 1500 A = 3/2×B = 2250

25. (b): 3 : 2 : 5 = a : b : c
$1862 = a^2 + b^2 + c^2$

$$\left(\frac{3}{2}b\right)^2 + b^2 + \left(\frac{5}{2}b\right)^2 = b^2\left(\frac{9}{4}+1+\frac{25}{4}\right)$$

$$b^2 = \frac{4\times 1862}{38} \Rightarrow b = 14$$

26. (c): $\frac{P}{Q}=\frac{Q}{R}=\frac{R}{S}=\frac{2}{3};\ \ Q = \frac{3}{2}P;\ R = \frac{3}{2}Q = \frac{9}{4}P$

$$S = \frac{3}{2}R = \frac{9}{4}Q = \frac{27}{8}P$$

P + Q + R + S = 1300

$$1300 = P + \frac{3}{2}P + \frac{9}{4}P + \frac{27}{8}P = \frac{P}{8}(8+12+18+27) = \frac{65}{8}P$$

P = 160

27. (c) :

$$\frac{b}{g} = \frac{3}{2} \quad \frac{scholarship}{Total} = \frac{.2b+.25g}{b+g} = \frac{.2\frac{b}{g}+.25}{\frac{b}{g}+1}$$

$$\frac{Non\ scholarship}{total} = 1 - \frac{scholarship}{Total} = 1 - \frac{.2\frac{3}{2}+.25}{\frac{3}{2}+1}$$

$$= 1 - \frac{1.1}{5} = .78$$

28. (b): Alcohol = 15 ×.2 = 3L water = 15 – 3 = 12 L

$$\text{New ratio} = \frac{3}{12+3+3} = \frac{3}{18} = \frac{1}{6} = 16\frac{2}{3}\%$$

29. (c): $A \Rightarrow \frac{q}{c} = \frac{7}{2}\ B \Rightarrow \frac{q}{c} = \frac{7}{11}$
Mixing 1q of A & 1q of B

$$= \frac{\left(\frac{7}{9}q + \frac{7}{18}q\right)\text{gold}}{\left(\frac{2}{9}q + \frac{11}{18}q\right)\text{copper}} = \frac{21\ \text{gold}}{15\ \text{copper}} = 7:5$$

30. (b): New Ratio = $\frac{M}{W} = \frac{\frac{5}{6}n}{\frac{n}{6}+5} = \frac{5}{2}\ \therefore\ n = 30$

$$\therefore \text{Milk} = \frac{5n}{6} = \frac{5}{6}\times 30 = 25$$

31. (b): Total milk $= (\frac{1}{3}+\frac{1}{4})$glass $= \frac{7}{12}$glass

Total water $= 2 - \frac{7}{12} = \frac{17}{12}$ Ratio= 7:17

32. (b): $\frac{milk}{water} = \left[\frac{4}{7}+\frac{5}{9}+\frac{7}{13} : \frac{3}{7}+\frac{4}{9}+\frac{6}{13}\right]$

33. (c): 3q = exam
Final = 6 quiz + 1 exam = 9 quiz
= 3 exam, Examination = 1/3 Final

34. (b): Time for 1st part = 2/3 time for 2nd part
Total time =
$1^1/_2$ hr = Ist part + IInd part= (2/3+1) IInd part

$\Rightarrow$ IInd part $= \frac{3}{2}\times\frac{3}{5} = \frac{9}{10}$ hr = 54min

35. (c): 12 month = 100 + turban
9 month = 3/4(100 + turban) = 65 + turban
75 + turban (3/4)= 65 + turban⇒ turban = 40

ANSWERS

EXPERIENCE THE PRATHAM EDGE 7: PARTNERSHIP

1. (d): $\frac{Profit\ Alok}{profit\ Shabbir}$

$= \frac{(time \times money)\,alok}{(time \times money)\,Shabbir} = \frac{90,000 \times 24}{120000 \times 21} = \frac{6}{7}$

Total profit = 96,000 = Profit Alok + Profit Shabbir

Alok's profit = 6/13×96000

Shabbir's profit = 7/13×96,000

Difference = 96000/13 = 7385

2. (d): Profit (A:B:C)

= (25×1+35×2) : (35×2+25×1) : (30 × 3)

= 95 : 95 : 90 = 19 : 19 : 18

3. (d): $\frac{Profit\ Mahesh}{profit\ Ramesh} = \frac{6000}{3000} = \frac{20,000 \times 6}{x \times 12}$

$\Rightarrow x = \frac{3000}{6000}(20,000) \times \frac{6}{12} = 5000$

4. (b): 2A = 3B, B = 4C

$\frac{Profit\ of\ B}{total\ Profit} = \frac{B}{\frac{3}{2}B + B + \frac{B}{4}} = \frac{4}{11}$

B's profit = (4/11) 16500 = 6000

5. (c): A – B = 4000, B – C = 5000

50,000 = A + B + C = B + 4000 + B + B – 5000

B = 17, 000; A = 21, 000; C = 12, 000

$\frac{A\ profit}{total\ profit} = \frac{21000}{50000} = 0.42$

A's profit = 0.42×35000 = 14700

6. (c): $\frac{C}{A+B+C} = \frac{C}{\frac{10}{4}C + \frac{10}{6}C + C} = \frac{12}{62} = \frac{6}{31}$

$C = \frac{6}{31} \times 4650 = 900$

7. (c): $\frac{A\ rent}{Total\ rent}$

$= \frac{24 \times 3}{24 \times 3 + 10 \times 5 + 35 \times 4 + 21 \times 3} = \frac{72}{325}$

$Total\ rent = \frac{325}{72} \times 720 = 3250$

8. (b): A = 3B, B = 2/3 C

$\frac{B\ profit}{Total} = \frac{B}{A+B+C} = \frac{B}{3B + B + \frac{3}{2}B} = \frac{2}{11}$

B profit = 2/11 × 660 = 120

9. (d): Investment A = 3 investment B

Time A = 2 time B

$\frac{Profit\ B}{Total} = \frac{Time\ \text{B inv B}}{invest\ A\ time\ A + \text{invest B time B}}$

$= \frac{1}{(2 \times 3)+1} = \frac{1}{7}$

Total profit = 7 × B profit = 7 × 4000 = 28000

10. (a): $\frac{Profit\ A}{Profit\ B} = \frac{7}{6} = \frac{14 \times 10}{15x}$

$\Rightarrow x = \frac{14}{15} \times 10 \times \frac{6}{7} = 8$

11. (b): PA:PB:PC = 10×8 + 20×4 : 15×12 : 12×12

= 160 : 180 : 144 = 40 : 45 : 36

$\text{A's Share} = \frac{40}{40+45+36} \times 847 = \frac{40}{121} \times 847 = 280$

12. (d): $\frac{B's\ share}{total\ profit} = \frac{\frac{4}{3} \times 12}{\frac{7}{2} \times 4 + \frac{7}{2} \times \frac{3}{2} \times 8 + \frac{4}{3} \times 12 + \frac{6}{5} \times 12}$

$= \frac{16}{14+42+16+14.4} = \frac{16}{86.4}$

$B = \frac{16}{86.4} \times 21600 = 4000$

13. (b): Total Profit = A + B + 5% of total P

$\text{Total Profit} = \frac{1}{.95} \times (A + B) = \frac{1}{.95}(A + \frac{2}{3}A)$

$= \frac{1}{.95} \times \frac{5}{3} \times 855 = 1500$

14. (c): Extra amount received by

A = 10% of 9600 = 960

Investment of A : Investment of B = 3 : 5

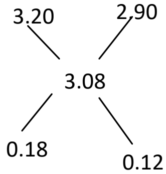
Amount invested by A $(9600 - 960)\frac{3}{8} + 960 = 4200$

15. (b): C's share [1-(A+B)] × profit =

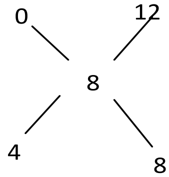
$(1 - \frac{7}{11}) \times 1100 = 400$

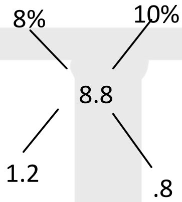
ANSWERS

EXPERIENCE THE PRATHAM EDGE 8: MIXTURES & ALLEGATIONS

1. (b): 

$\frac{.18}{.12} = \frac{3}{2}$

2. (a): $\frac{4}{8} = \frac{1}{2}$ 

3. (b): 

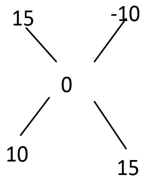
$\frac{352}{4000} = 8.8\%$

$\frac{1.2}{1.2+.8} = \frac{12}{20}$

Money invested at 8% $= \frac{12}{20} \times 4000 = 2400$

4. (c): 72x + 160y = 1980 ---- (i)
x + y = 18 ---- (ii)
from (i) and (ii)
x = 900/88
$\therefore$ 72x = 736.36 km

5. (b): Milk in mixture = 0.9 × 40 = 36L
$\frac{milk\ in\ mixture}{Total} = .8 = \frac{36L}{total\ mix}$
Total mixture = 45 lts.
Water = 45× (20/100) = 9 lts
Therefore, 5 lts need to be added.

6. (c): 1.15x + .9 (900 – x) = 900
$\Rightarrow 0.25x = 90$
$\Rightarrow$ x = 360 

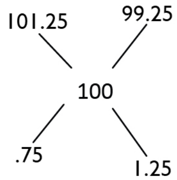
7. (d):
Milk $= \frac{5}{5+1} \times 66 = 55L$
New quantity $= \frac{milk}{proportions\ of\ milk} = \frac{55}{\frac{5}{8}} = 88L$
Water added = 88 – 66 = 22L

8. (c): Milk = 7/9 × 729 = 567
New quantity $= \frac{milk}{proportion\ of\ milk} = \frac{567}{\frac{7}{10}} = 810$
Water added = 810 – 729 = 81

9. (a): For gaining 25%
100 ml of milk sold as 125 ml of mixture
% of water $= \frac{125-100}{125} = \frac{25}{125} = 20\%$

10. (c): $\frac{A}{A+B} = \frac{7}{16} = \frac{\frac{7}{12}(x-9)}{x}$
$\Rightarrow 12x = 16(x-9) \qquad x = 36$
$A = \frac{7}{12} \times 36 = 21$

11. (b): 8x + 16(7 - x) = 8
=> x = 4
$\therefore$ 8x = 32

12. (d): 

3:5

13. (c): $153 = \frac{126+135+2x}{1+1+2} \Rightarrow 612 = 261 + 2x$
$x = 351/2 = 175.50$

14. (c): $\frac{alcohol}{water} = \frac{3}{5} = \frac{4x}{3x+8}$
$\Rightarrow 9x + 24 = 20x \quad \therefore x = 24/11$
Alcohol = $4x$ = 96/11

15. (a): 20 kg of fresh grapes have pulp =20×0.1= 2 kg
Dry grapes having 2 kg of pulp = 2/(.8) = 2.5

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16. (c): $\frac{alcohol}{mixture}=\frac{5\times .2}{5+1}=\frac{1}{6}=16\frac{2}{3}\%$

17. (d): $\frac{wine}{water}=\frac{2\times 0.25+3\times 0.75}{2(1-.25)+3(1-.75)}=\frac{2.75}{2.25}=\frac{11}{9}$

18. (d): $\frac{milk}{water}=\frac{2}{3}=\frac{\frac{3}{3+2}120}{\frac{2}{2+3}120+x}=\frac{72}{48+x}$

$\Rightarrow 96+2x=216 \qquad \Rightarrow x=60$

19. (c): $\frac{milk}{water}=\frac{1}{1}=\frac{\frac{3}{3+1}(1-x)}{\frac{1}{3+1}(1-x)+x}=\frac{3-3x}{1+3x}$

$1+3x = 3-3x \qquad \Rightarrow x = 1/3$

20. (b): CP = $\frac{18}{1+.25}=\frac{18}{1.25}=14.4$

12   18

14.4

3.6   2.4

$\frac{3.6}{2.4}=\frac{3}{2}$

21. (a): Let CP of 1 litre milk be Re 1
SP of 1 litre of mixture = Re 1, Gain = 50/3%
$\therefore$ CP of 1 litre of mixture = 100×3/350×1 = Re 6/7
By the rule of alligation, we have

CP of 1 ltr of water   CP of 1 ltr of milk

0   Re 1

Mean price
Re 6/7

1/7   6/7

$\therefore$ Ratio of water and milk = 1/7:6/7=1:6

22. (b):

$\frac{wine}{mixture}=\frac{16}{16+65}=\frac{16}{81}=\left(1-\frac{8}{x}\right)^{3+1}=\left(\frac{x-8}{x}\right)^{4}$

$\frac{2}{3}=\frac{x-8}{x} \quad \Rightarrow 2x = 3x - 24 \quad \Rightarrow x = 24$

23. (d): milk = $40\left(1-\frac{4}{40}\right)^{3}=40\left(\frac{9}{10}\right)^{3}$

$= 40 \times .729 = 29.16$

24. (d): CP = $\frac{9.24}{1.1}=8.4 \quad \Rightarrow 8.4=\frac{9x+27}{x+27}$

$\Rightarrow 8.4x + 8.4\times 27 = 9x + 27$
$\Rightarrow .6x = 1.4\times 27 \quad \therefore x = 63$

25. (a): CP = 68.2/1.1 = 62

60   65

62   (3 : 2)

3   2

26. (c): Quantity at 18%

8%   18%

14%

4%   6%

$=\frac{6}{4+6}\times 1000=600$

27. (a): $\frac{15\times 2+20\times 3}{2+3}=\frac{90}{5}=18$

28. (b):

$\frac{1}{4}$ $\frac{1}{2}$

$\frac{3}{8}$

$\frac{1}{8}$ $\frac{1}{8}$

$\frac{1}{8}:\frac{1}{8}=1:1$
6 lts : 6 lts

29. (c): $\frac{Syrup}{mix}=\frac{1}{2}=\frac{\frac{5}{8}(1-x)}{1}$

$\Rightarrow 1 = \frac{5}{4}(1-x) \Rightarrow x = \frac{1}{5}$

30. (c): $\frac{A}{A+B}=\frac{7}{7+9}=\frac{7}{16}=\frac{\frac{7}{12}(x-9)}{x}$

$3x = 4(x-9); \quad x = 36; A = 21; B = 15$

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EXPERIENCE THE PRATHAM EDGE 9: LINEAR & QUADRATIC EQUATIONS

1.
(d) $9x + 11y = 53$ ………….. (1)
$11x + 9y = 47$ ………….. (2)
$(1) + (2) \Rightarrow (9x + 11x) + (11y + 9y) = 53 + 47$
$\Rightarrow \quad 20(x + y) = 100 \Rightarrow x + y = 5$ ………….. (3)
$(1) - (2) \Rightarrow (9x - 11x) + (11y - 9y) = 53 - 47$
$\Rightarrow \quad 2(y - x) = 6 \Rightarrow y - x = 3$ ………….. (4)
$(3) + (4) \Rightarrow (x + y) + (y - x) = 5 + 3$
$\Rightarrow 2y = 8 \Rightarrow y = 4$
From (3), $x + 4 = 5 \Rightarrow x = 1$. $\therefore x = 1$ and $y = 4$

2. (b) Let the present age of Bhargav be 'b'.
Let the present age of Anurag be 'a'.
From the data,
$(a - 3) = 3(b - 3) \Rightarrow 3b - a = 6 \rightarrow (1)$
$(a + 2) = 2(b + 2) \Rightarrow a - 2b = 2 \rightarrow (2)$,
By solving, $b = 8$
Substituting $b = 8$ in (1) $\Rightarrow a = 18$
$\Rightarrow$ The present age of Anurag is18 years.

3. (d) Let the number be divided into x and y.
From the data, $x^2 - y^2 = 48(x - y)$
$(x + y)(x - y) = 48(x - y) \Rightarrow x + y = 48$
$\therefore$ The number $= x + y = 48$

4. (b) Let the cost of each sharpener and eraser be S and E respectively.
From the data,
$3S + 4E = 25 \rightarrow (1)$
$4S + 3E = 24 \rightarrow (2)$
$(1) + (2) \Rightarrow 7S + 7E = 49$
$\Rightarrow 7(S + E) = 49 \Rightarrow S + E = 7 \rightarrow (3)$
$(1) - 3 \times (3) \Rightarrow$
$3S + 4E = 25$
$\underline{3S + 3E = 21}$
$ E = 4$

Substituting $E = 4$, in (3) we get
$S + 4 = 7 \Rightarrow S = \text{Rs. } 3$
$\therefore$ The cost of each sharpener is Rs. 3 and that of each eraser is Rs. 4

5. (b) Let the cost of each Samosa and Puff be Rs. S and Rs. P respectively. From the data,
$S + 2P = 14 \rightarrow (1)$
$3S + P = 17 \rightarrow (2)$
$2x\,(1) + (2) \Rightarrow 2S + 4P = 28$

$\underline{3S + P = 17}$
$5S + 5P = 45$

$\therefore$The cost of 5 Samosa and 5 Puffs is Rs. 45

6. (c) $2x + 6y = 12 \rightarrow (1)$
$3x + 9y = 18 \rightarrow (2)$
$3 \times (1) \Rightarrow 6x + 18y = 36 \rightarrow (3)$
$2 \times (2) \Rightarrow 6x + 18y = 36 \rightarrow (4)$
Equations (3) and (4) are same
$\therefore$ Infinite solutions are possible

7. (b) Let, the number be $10x + y$
As the numbers formed by interchanging the digits is less than the original, $x > y$.
$x + y = 5(x - y) \Rightarrow 4x = 6y$
or $2x = 3y \rightarrow (1)$
also, $(10x + y) - (10y + x) = 18$
$\Rightarrow 9(x - y) = 18 \Rightarrow x - y = 2 \rightarrow (2)$
$(1) - 2 \times (2) \Rightarrow 2x - 3y = 0$
$\underline{2x - 2y = 4}$
$ -y = -4$

$\Rightarrow y = 4$
$2x = 3y \Rightarrow 2x = 3(4) \Rightarrow x = 6$

8. (d) Let the present age of the father be x years and that of the son be y years. From the given conditions, we have, $x - 5 = 5(y - 5) \Rightarrow x - 5y = -20$ …(1)
$x + 2 = 3\,(y + 2) \Rightarrow x - 3y = 4$ ... (2)
(1) — (2)
$\Rightarrow x - 5y = -20$
$\underline{x - 3y = 4}$
$ -2y = -24$

$y = 12$ years and $x = 40$
Present age of man is 40 years

9.
(c) Let, Venkat's speed be x km/h and that of Vatsa be y km/h.
Given, $\frac{600}{x} - \frac{600}{y} = 2$
$\Rightarrow \frac{1}{x} - \frac{1}{y} = \frac{1}{300}$ ………….. (1)
and $\Rightarrow \frac{600}{y} - \frac{600}{2x} = 4$
$\Rightarrow \quad \frac{1}{y} - \frac{1}{2x} = \frac{2}{300}$ ………….. (2)
$(1) + (2) \Rightarrow \frac{1}{x} - \frac{1}{2x} = \frac{1}{300} + \frac{2}{300} \Rightarrow x = 50$
From $\frac{1}{50} - \frac{1}{y} = \frac{1}{300} \Rightarrow \frac{1}{y} = \frac{5}{300} = \frac{1}{60}$
$\therefore y = 60$
$\therefore$ Vasta's speed is 60 km/h.

10. (b) Let, the sum of the present ages of Avish and Lakhan be 'S'.
Sum of their ages, 14 years ago $= S - 28$
$\therefore S - 28 = \frac{1}{3}(S), \Rightarrow \frac{2}{3}S = 28 \Rightarrow$ 'S' = 42 years
As the ratio of present ages of Avish and Lakhan is

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$= 4 : 3$

The present age of Lakhan $= \frac{3}{7} \times 42 =$ 18 years.

11. (c) Let the cost of a burger be Rs. p, and that of a pizza be Rs. q.

$7p + 8q = 780$ …………….. (1)

$12p + 5q = 945$ …………….. (2)

$12 \times (1) - 7 \times (2)$

$84p + 96q = 9360$

$\underline{84p + 35q = 6615}$

$61q = 2745$

$\therefore q = 45$

$\therefore$ The cost of each pizzas is Rs. 45.

12. (d) Let Varun's present age be x and that of Tarun be y. Then, $x = 3(y - 3) \Rightarrow x - 3y + 9 = 0$ …… (1)

Also, $x + 9 = 3y \Rightarrow x - 3y + 9 = 0$ …… (2)

Since, the two equations are the same, the sum of their present ages cannot be uniquely determined.

13. (d) Let the number of chocolates with Seoni and Varsha be 7x and 9x respectively.

Given, $9x - 7x = 14 \Rightarrow x = 7$

Total number of chocolates $= 7x + 9x = 16x = 16(7) = 112$

$\therefore$ Total number of chocolates with them is 112.

14. (d) Let the number of Rs. 2 coins be x and Rs. 5 coins be y.

$x + y = 57$ …………….. (1)

$2x + 5y = 150$ …………….. (2)

$5x\,(1) - (2)$

$\Rightarrow 5x + 5y = 285$

$\underline{2x + 5y = 150}$

$\underline{3x = 135}$

$x = 45$

$\therefore$ The number of Rs. 2 coins with Durgesh is 45.

15. (c) Let the two numbers be x and y

$x + y = 250$ --- (I)

$x^2 - y^2 = 9000 \Rightarrow (x + y)(x - y) = 9000$

$x + y = 250$

$x - y = 36$ ---(2)

From (1) and (2)

$2y = 214 \Rightarrow y = 107$ and y is the smaller number.

16. (d) $3x + 4y = 24$

Multiply both the sides with 5. Then,

$15x + 20y = 120$ …………….. (1)

$15x + 20y = 8k$ …………….. (2)

To have (1) and (2) as consistent,

$\Rightarrow 120 = 8k,$ $\therefore k = 15$

17. (c) Let, the numerator be x and the denominator be y Now,

$\frac{x+1}{y+2} = \frac{2}{3} \Rightarrow 3x - 2y = 1$ …………….. (1)

$\frac{x+4}{y+5} = \frac{3}{4} \Rightarrow 4x - 3y = -1$ …………….. (2)

$3 \times (1) - 2 \times (2)$

$\Rightarrow 9x - 6y = 3$

$8x - 6y = -2$

$x = 5$

From (1), $3(5) - 2(y) = 1$

$\Rightarrow y = 7 \Rightarrow$ The fraction is 5/7.

18. (d) Let the cost of a pen be Rs. x, that of a pencil be Rs. y, and that of a book be Rs. z.

Given, $3x + 5y + 2z = 68$ …………….. (1)

$6x + 7y + 4z = 121$ …………….. (2)

$9x + 15y + 6z = 204$ …………….. (3)

$(1) + (2) - (3) \Rightarrow$

$3x + 5y + 2z = 68$

$6x + 7y + 4z = 121$

$9x + 15y + 6z = 204$

$0x - 3y + 0z = -15$

$\Rightarrow y = 5$

Substituting y = 5 in (1), (2), (3)

$3x + 5(5) + 2z = 68 \Rightarrow 3x + 2z = 43$

$6x + 7(5) + 4z = 121 \Rightarrow 6x + 4z = 86$

$\Rightarrow 3x + 2z = 43$

$9x + 15(5) + 6z = 204$

$\Rightarrow 9x + 6z = 129 \Rightarrow 3x + 2z = 43$ So, the value of z cannot be determined.

19. (b) Let the number be x.

$\Rightarrow \frac{6}{5}x - \frac{5}{6}x = 649 \Rightarrow \frac{11x}{30} = 649 \Rightarrow x = 59 \times 30 \Rightarrow x = 1770$

$\therefore$ The number is 1770

20. (c) Let the pass mark be p and the total number of students be t.

As the total number of students is t, 3t/5 students scored (p – 10) marks, t/5 students scored (p + 10) marks and the remaining t/5 students scored (p + 20) marks.

$\therefore$ The average $= \frac{\frac{3t}{5}(p-10) + \frac{t}{5}(p+10) + \frac{t}{5}(p+20)}{t} = 62$

$\Rightarrow \frac{tp}{t} = 62 \Rightarrow p = 62$

21. (d) Let the amounts with Rohan and Sohan be R and S respectively.

Given, $S - 60 = R + 60 \Rightarrow S - R = 120$……….. (1)

and $(R - 30)3 = S + 30 \Rightarrow 3R - S = 120$……….. (2)

Solving (1) and (2) gives, R = Rs. 120 and S = Rs. 240

$\therefore$ Rohan and Sohan together have Rs. 360.

22. (b) Let the number of sons and daughters that Dheeraj's parents have b and s respectively.
Dheeraj has b – 1 brothers.
Given $2(b-1) = s$ ……….. (1)
Also Deepa has s – 1 sisters.
Given, $s - 1 = b$ ……….. (2)
From (1) and (2), $2(b-1) = b + 1 \Rightarrow b = 3$
$\therefore$ Deepa has 3 brothers.

23. (b) After purchasing 4 apples and 5 mangoes, the man will be left with ¼ of what he initially had, which is Rs. 20. He had Rs. 80 to start with.
With Rs. 80, the man can purchase 16 apples. Each apple costs 80/16= Rs. 5
With Rs. 80, if the man can purchase 10 apples. Each mango costs 80/10 = Rs. 8.
$\therefore$ The difference in the prices of an apple and a mango is Rs.3

24. (c) Let the number to be multiplied be x.
$\frac{4}{7}x - \frac{4}{17}x = 840 \Rightarrow 4x\left[\frac{1}{7} - \frac{1}{17}\right] = 840$
$x\left[\frac{17-7}{119}\right] = 210 \Rightarrow 10x = 210 \times 119$
$\therefore x = 2499$

25. (b) P, Q and R are successive even positive integers in the ascending order.
$\Rightarrow R - Q = 2$ and $Q - P = 2$
$\Rightarrow R - P = 4$
$4R = 5P + 4$
$\Rightarrow 4R - 5P = 4 \Rightarrow 4R - 4P - p = 4$
But $R - P = 4$
$\therefore 4(R - P) - P = 4$
$\Rightarrow 4(4) - P = 4 \Rightarrow P = 12$
$\therefore Q = P + 2 = 12 + 2 = 14$

26. (b) $\because x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
$\therefore \quad x = \frac{7 \pm \sqrt{49-4\times15\times(-36)}}{2\times15}$
$\Rightarrow x = \frac{7\pm47}{30}$
$\therefore \quad x = \frac{54}{30}$ or $x = \frac{-40}{30}$
$x = \frac{9}{5}$ or $x = \frac{-4}{3}$

27. (d) $6x^2 - 31x + 40 = 0$
$\Rightarrow \left(x - \frac{8}{3}\right)\left(x - \frac{5}{2}\right) = 0$
$\Rightarrow x = \frac{8}{3}, \frac{5}{2}$

28.
(a) $2\left(x^2 + \frac{1}{x^2}\right) - 3\left(x + \frac{1}{x}\right) - 1 = 0$
$\Rightarrow 2\left[x^2 + \frac{1}{x^2} + 2 - 2\right] - 3\left(x + \frac{1}{x}\right) - 1 = 0$
$\Rightarrow 2\left[\left(x + \frac{1}{x}\right)^2 - 2\right] - 3\left(x + \frac{1}{x}\right) - 1 = 0$
$\Rightarrow 2\left(x + \frac{1}{x}\right)^2 - 3\left(x + \frac{1}{x}\right) - 5 = 0$
$\Rightarrow 2t^2 - 3t - 5 = 0$ (substituting $x + \frac{1}{x} = t$)
Now solve it and you will get
$t = -1$ and $t = \frac{5}{2}$
Now if $t = -1$, then $x + \frac{1}{x} = -1$.
$\Rightarrow x^2 + 1 + x = 0$
$\Rightarrow x^2 + x + 1 = 0$
$\therefore \quad x = \frac{-1\pm\sqrt{1-4}}{2} = \frac{-1\pm\sqrt{-3}}{2} \rightarrow$ not possible
and if $t = \frac{5}{2}$ then $x + \frac{1}{x} = \frac{5}{2}$
$\Rightarrow 2x^2 - 5x + 2 = 0 \Rightarrow x = \frac{1}{2}, 2$

29. (c) For equal roots D = 0
i.e., $b^2 - 4ac = 0$
$\Rightarrow [-2(1 + 3k)]^2 - 4 \times 1 \times 7 \times (3 + 2k) = 0$
$\Rightarrow 36k^2 - 32k - 80 = 0$
$\Rightarrow (k - 2)\left(k + \frac{10}{9}\right) = 0$
$\Rightarrow k = 2, -10/9$

30. (a) $D = b^2 - 4ac$
$= 4 - 4 \times (-3) \times (-8) = -92$

31. (a) $D = b^2 - 4ac = 25 - 4 \times 1 \times 7 = -3$
Since D < 0, therefore roots are not real, i.e., roots will be imaginary.

32. (c) $a^2x^2 + abx - b^2 = 0$
$D = b^2 - 4ac = (ab)^2 - 4 \times a^2 \times (-b^2)$
$= (ab)^2 + 4a^2b^2$
$= (ab)^2[1 + 4] = (ab)^2(5)$
$\therefore$ The roots are real and unequal

33. (b) For non-real roots D < 0
$\therefore \quad b^2 - 4ac < 0$
$\therefore \quad (-p)^2 - 4 \times 1 \times q < 0$
$\Rightarrow \quad p^2 - 4q < 0$
$\Rightarrow \quad p^2 < 4q$

34. (b) $4x^2 - 3kx + 1 = 0$
$D = b^2 - 4ac = 0$
$\therefore \quad 9k^2 - 4 \times 1 \times 4 = 0$
$\Rightarrow k^2 = \frac{16}{9} \Rightarrow k = \pm\frac{4}{3}$

35. (c) Let α, β be the roots of the equation, then

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$\alpha + \beta = \frac{1}{2}\alpha\beta$

$-\frac{b}{a} = \frac{1}{2}.\frac{c}{a}$

$\Rightarrow$ -b $= \frac{c}{2}$

$\Rightarrow (k + 6) = \frac{2(2k-1)}{2}$

$\Rightarrow k = 7$

36. (a) $\alpha + \beta = \alpha\beta$

$\Rightarrow \frac{-b}{a} = \frac{c}{a} \Rightarrow -b = c$

$\therefore$ $-2k = 4 \Rightarrow k = -2$

37. (a) Since -4 is a root of $x^2 - px - 4 = 0$

$\therefore (-4)^2 - p(-4) - 4 = 0 \Rightarrow p = -3$

$\therefore$ The equation becomes $x^2 + 3x + k = 0$

Since both the roots of this equation are equal.

$\therefore$ Sum of the roots $(\alpha + \beta) = 2\alpha = -3 \Rightarrow \alpha = -3/2$

$\therefore$ Product of the roots $(\alpha \cdot \beta) = \alpha^2 = (-3/2)^2 = 9/4 = k$

$\therefore k = 9/4$

38. (a) For equal roots, D =0

i.e., $b^2 - 4ac = 0$

But now $x^2 + p(4x + p - 1) + 2 = 0$

$\Rightarrow x^2 + 4px + p^2 - p + 2 = 0$

$\Rightarrow x^2 + 4px + (p^2 - p + 2) = 0$

$\Rightarrow (4p)^2 - 4 \times 1 \times (p^2 - p + 2) = 0$

$\Rightarrow 16p^2 - 4p^2 + 4p - 8 = 0$

$\Rightarrow 12p^2 + 4p - 8 = 0$

$\Rightarrow 12p^2 + 12p - 8p - 8 = 0$

$\Rightarrow 12p(p + 1) - 8(p + 1) = 0$

$\Rightarrow (p + 1)(12p - 8) = 0 \Rightarrow p = -1$ or $p = \frac{8}{12} = \frac{2}{3}$

39. (a) $\alpha\beta = \frac{c}{a} = \frac{3}{3} = 1$

40. (b) $x^2 - 3x + 2 = 0$

$\Rightarrow x^2 - x - 2x + 2 = 0$

$\Rightarrow (x - 1)(x - 2) = 0$

$\Rightarrow \alpha = 1$ and $\beta = 2$

$\therefore$ $-\alpha = -1$ and $-\beta = -2$

$\therefore$ The required equation is $[x - (-\alpha)][x - (-\beta)] = 0$

$\Rightarrow (x + 1)(x + 2) = 0 \Rightarrow x^2 + 3x + 2 = 0$

41. (b) Method I. If α and β be the roots of a quadratic equation, then the equation will be $(x - \alpha)(x - \beta) = 0$

$\therefore$ $(x - \sqrt{3})(x - 2\sqrt{3}) = 0$

$\Rightarrow x^2 - 3\sqrt{3}x + 6 = 0$

Method II $x^2 - (\alpha + \beta)x + (\alpha \cdot \beta) = 0$

$\Rightarrow x^2 - (\sqrt{3} + 2\sqrt{3})x + (\sqrt{3} \cdot 2\sqrt{3}) = 0$

$\Rightarrow x^2 - 3\sqrt{3}x + 6 = 0$

42. (a) $\alpha + \beta = \frac{-b}{a} = \frac{-1}{6}$

$\alpha\beta = \frac{c}{a} = -\frac{2}{6} = -\frac{1}{3}$

$\therefore \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$

$= \frac{(-1/6)^2-2(-1/3)}{-1/3} = \frac{\frac{1}{36}+\frac{2}{3}}{-\frac{1}{3}} = \frac{\frac{25}{36}}{-\frac{1}{3}} = -\frac{25}{12}$

43. (b) $\alpha + \beta = 10$ and $\alpha \cdot \beta = 10$

$\Rightarrow -\frac{b}{a} = 10$ and $\frac{c}{a} = 10$

$\Rightarrow \frac{5}{a} = 10 \Rightarrow a = \frac{1}{2} \therefore \frac{c}{1/2} = 10 \Rightarrow c = 5$

$\therefore$ $a = \frac{1}{2}$ and $c = 5$

44. (c) $y_{min.} = \frac{-b^2+4ac}{4a}$ at $x = -\frac{b}{2a}$

$\therefore$ $y_{min.} = \frac{-p^2+4\times1\times q}{4\times1} = \frac{4q-p^2}{4}$

Now, since p and q are the roots of the equation $x^2 + px + q = 0$.

$\therefore$ $p + q = -p \Rightarrow q = -2p$

and $pq = q \Rightarrow p = 1$ $(\because q \pm 0)$

$\therefore$ $q = -2$

$\therefore$ $y_{min.} = \frac{4q-p^2}{4} = \frac{4\times(-2)-(1)^2}{4} = -\frac{9}{4}$.

45. (c) One double root means a single root appears two times i.e., two roots are equal.

$\therefore$ $D = 0 \Rightarrow (2)^2 - 4a \cdot 1 = 0$

$\Rightarrow a = 1$.

46. (b) Let $x = \sqrt{6 + \sqrt{6 + \ldots\infty}} \Rightarrow x^2 = x + 6 \Rightarrow x^2 - x - 6 = 0 \Rightarrow x = 3, -2$, but $x > 0$.

47. (c) Let α, β be the roots of the equation $x^2 + px + 8 = 0$. Then $\alpha + \beta = -p$ and $\alpha\beta = 8$

But $\alpha - \beta = 2 \Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = (2)^2$.

$\Rightarrow p^2 - 32 = 4 \Rightarrow p = \pm 6$.

48. (d) $x = 7 + 4\sqrt{3}$

$y = \frac{1}{7+4\sqrt{3}} = 7 - 4\sqrt{3}$ $(xy = 1)$

$\frac{1}{x^2} + \frac{1}{y^2} = \frac{x^2+y^2}{(xy)^2}$

$= \frac{(7+4\sqrt{3})^2 + (7-4\sqrt{3})^2}{[(7+4\sqrt{3})(7-4\sqrt{3})]^2}$

$= 2(49 + 48) = 194$

ANSWERS

EXPERIENCE THE PRATHAM EDGE 10: TIME, SPEED & DISTANCE, RACES & GAMES

1. (d): Avg. Speed = $\dfrac{100+50+90}{\dfrac{100}{50}+\dfrac{50}{25}+\dfrac{90}{45}} = \dfrac{240}{6} = 40$

2. (a): Avg. speed = $\dfrac{30+48}{6+12} = \dfrac{78}{18} = 4.33$

3. (a): Let the time taken while going be t,
   D = 3t = 2 (5– t)
   $\Rightarrow$ t = 2 $\Rightarrow$ D = 3t = 6

4. (b): $D = 5\left(t + \dfrac{10}{60}\right)$ & $D = 6\left(t - \dfrac{15}{60}\right)$
   $\dfrac{D}{5} - \dfrac{1}{6} = \dfrac{D}{6} + \dfrac{1}{4} \Rightarrow \dfrac{D}{30} = \dfrac{5}{12}$ $\therefore$ D = 12.5

5. (b): $3t = \dfrac{D}{18-V}$ & $t = \dfrac{D}{18+v}$
   Dividing, we get
   $\Rightarrow$ 3(18 –v) = 18 + v $\therefore$ v = 9

6. (b): D = 120 + 80 = 200
   V = 50 – 40 = 10 Km/hr = 10× 5/18m/sec
   $T = \dfrac{200}{50/18} = 72$ sec

7. (a): D = 180 + 220 = 400m
   V = 50 + 40 = 90 Km/hr = 90 ×5/18
   = 25m/hr
   $\Rightarrow$ T = 400/25 = 16 sec

8. (b): $v_1 = \dfrac{D}{4}$ & $v_2 = \dfrac{D}{5}$
   $t = \dfrac{2D}{v_1+v_2} = \dfrac{2D}{\dfrac{D}{4}+\dfrac{D}{5}} = \dfrac{2}{\dfrac{9}{20}} = \dfrac{40}{9}$

9. (a): v – u = 9/6 = 1.5 & v + u = 9/2 = 4.5
   $\therefore$ v = 3; u = 1.5

10. (d): We need distance for last part also

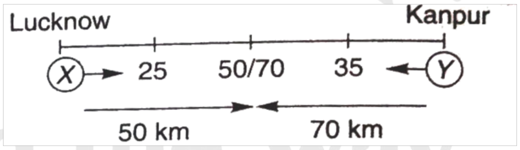
11. (c): Suppose the total distance equals to 4 times the LCM of the speeds.
   $\therefore$ Total distance = 2400 km
   $\therefore$ Total time = $\dfrac{600}{20} + \dfrac{600}{25} + \dfrac{600}{30} + \dfrac{600}{40}$
   = 30 + 24 + 20 + 15 = 89 h
   $\therefore$ Average speed = $\dfrac{2400}{89} = 26\dfrac{86}{89}$ km/h

12. (c): Avg Speed $= \dfrac{D+D}{\dfrac{D}{24}+\dfrac{D}{36}} = \dfrac{2}{\dfrac{5}{72}} = \dfrac{144}{5} = 28.8$

13. (c): Avg speed $= \dfrac{3+3+3+3}{\dfrac{3}{10}+\dfrac{3}{20}+\dfrac{3}{30}+\dfrac{3}{60}} = \dfrac{12}{3\left(\dfrac{6+3+2+1}{60}\right)} = 20$

14. (b): Effective speed = 25 + 35 =60 km/h
   Total distance to be covered = 120 km
   $\therefore$ Time taken = $\dfrac{120}{60}$ = 2h

HINT: Since in each hour X and Y together covers 60 km.



15. (b): With stoppages bus travels = 45 Km/hr
   Bus will take = 45/54 = 5/6 hr = 50 min
   $\therefore$ Stoppages = 60 min – 50min= 10min

16. (c): Difference in speed = 4.5 – 4 = 0.5 Km/hr
   Difference in distance = 8.5 Km
   Time Taken = 8.5/(.5) = 17 hr

17. (b): v = 18 + 20 = 38
   $T = \dfrac{47.5}{38} = 1\dfrac{1}{4} =$ 1hr 15 min

18. (d): $D = vt = (v+3)\left(t - \dfrac{40}{60}\right) = (v-2)\left(t + \dfrac{40}{60}\right)$
   $\Rightarrow vt = vt + 3t - \dfrac{2}{3}v - 2 \Rightarrow 3t - \dfrac{2}{3}v = 2$
   $\Rightarrow vt = vt - 2t + \dfrac{2}{3}v - \dfrac{4}{3} \Rightarrow -2t + \dfrac{2}{3}v = \dfrac{4}{3}$
   $\therefore t = \dfrac{10}{3}$, v = 12 and d = 40

19. (c): $40\left(t + \dfrac{11}{60}\right) = 50\left(t + \dfrac{5}{60}\right)$
   $\Rightarrow 10t = \dfrac{190}{60} \Rightarrow t =$ 19 min

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20. (c):T = $5\frac{1}{2}$

$\frac{D}{45} - \frac{D}{60} = \frac{11}{2}$

D = 990

21. (b): $D = vt = \frac{5v}{6}\left(t + \frac{10}{60}\right) \Rightarrow t = 50\,\text{min}.$

22. (c): 840 = vt = (v + 10)(t – 2) = vt + 10t – 2v - 20

$\Rightarrow$ 10t = 20 + 2v

$\Rightarrow 840 = vt = \frac{v(20+2v)}{10} = \frac{v^2+10v}{5}$

$\Rightarrow v^2 + 10v - 4200 = 0 \quad \therefore\ v = 60 \text{ or } -70$

23. (c): $t = \frac{D}{v_1+v_2} = \frac{240}{40+50} = \frac{8}{3} = 2\frac{2}{3}$ = 2hr 40 min

Time = 8pm + 2.4 = 10.40 pm

24. (d): M travels distance in 9 – 5 = 4 hrs

N travels distance in 10.30 – 7= 3.5hr

M would be 1/2 way at 7 am when N starts.

Time taken to meet = $\frac{\frac{D}{2}}{\frac{D}{4}+\frac{D}{\frac{7}{2}}} = \frac{\frac{1}{2}}{\frac{15}{28}} = \frac{14}{15}$ == 56 min

25. (a): Time taken to meet

$\frac{\text{more difference}}{\text{diff in speed}} = \frac{48}{42-36} = \frac{48}{6} = 8\text{hr}$

D = (42 + 36) × 8 = 624

26. (b): $t = \frac{d}{4} + \frac{35-d}{5} = \frac{d}{5} + \frac{35-d+2}{4}$

$\Rightarrow \frac{35-2d}{5} = \frac{37-2d}{4}$

$\Rightarrow 140 - 8d = 185 - 10d \Rightarrow d = 22.5$

$t = \frac{22.5}{4} + \frac{12.5}{5} = 5.6 + 2.5 = 8.1$

27. (b): C covers the journey in 42 min

B covers it in 42/3 = 14 min

A covers it in = 14/2 = 7 min

28. (a):

$\frac{v_1}{v_2} = \frac{\frac{D}{t}}{\frac{\left(\frac{D}{2}\right)}{2t}} = \frac{4}{1}$

29. (a):

$v\alpha\frac{1}{t} \quad \Rightarrow \frac{V_a}{V_b} = \frac{T_b}{T_a}$

$\Rightarrow \frac{2}{3} = \frac{t_a - 10}{t_a}; \quad t_a = 30 \Rightarrow \frac{t_a}{2} = 15$

30. (b): Distance covered by thief at 2 pm

= 40×1/2 =20km

Overtakes in = 20/(50-40) = 2hrs

Time = 2pm + 2 hr = 4 pm

31. (a): Distance left = D/2 = 6/2 = 3

Time left = 1/3 × 45 min = 15min

Speed = $\frac{3}{15/60} = 12$

32. (c): Distance covered by 2nd boy in half hr

= 3×1/2 = 1.5km

Time taken by 1st boy to create this

=1.5/(3.75-3) = 2 hrs

Total difference = 2 × 3.75 = 7.5

33. (b): $v\alpha\frac{1}{t} = \frac{v_a}{v_b} = \frac{t_b}{t_a} \quad \Rightarrow \frac{3}{4} = \frac{48}{ta} \Rightarrow t_a = 64$

34. (c): length of diagonal = 3× 5/18×2 × 60 = 100

$a^2 + a^2 = (\text{diagonal})^2 = (100)^2$

$a^2 = 10000/2 = 5000$

35. (c): D = 12 × 41/2 = 54hr

New speed = $\frac{54}{4\frac{1}{2} - 1\frac{1}{2}} = \frac{54}{3}$ = 18 km/hr

36. (c): let speed of walking = w

Let speed of riding back = r

$5\frac{45}{60} = \frac{D}{w} + \frac{D}{r} = \frac{D}{r} + \frac{D}{r} + 2 \quad \Rightarrow \frac{2d}{r} = 3\frac{3}{4}$

$\Rightarrow \frac{2D}{W} = \frac{23}{2} - \frac{2D}{r} = \frac{23}{2} - \frac{15}{4} = \frac{31}{4} = 7\frac{3}{4}$

37. (d): For covering 15 km = $\frac{15}{10} + \frac{10}{60}$ = 1.67

For covering 90 km;

Renu will take = 1.67 × 6 = 10.02 = 10hr(app)

For next 10 km = 10/10 = 1 hr

Total time = 10 + 1 = 11

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38. (b): $d \alpha \frac{1}{v^2} \quad \Rightarrow 25\alpha \frac{1}{(30)^2}$............(i)

$\Rightarrow 25\alpha \frac{1}{(v)^2}$............(ii)

Dividing 2 by 1

$= \left(\frac{30}{v}\right)^2 = \frac{36}{25} \Rightarrow \frac{30}{v} = \frac{6}{5} \quad \Rightarrow v = 30 \times \frac{5}{6} = 25$

39. (b): In 1 second wheel travels = $(2\pi r) \times 3$
Speed= $33\times 18/5 = 118.8$ km/hr

40. (b): speed = 63 Km/hr = $63\times5/18 = 17.5$m/sec
t = 280/17.5 = 16 sec

41. (d): $t = \frac{360+360}{54\times\frac{5}{18}} = \frac{720}{15}$ =720/15 = 48 sec

42. (d): t = 1 min = 60sec $\quad = \frac{700+d}{72\times\frac{5}{18}} = \frac{720+d}{20}$
D = 1200 – 700 = 500

43. (c): $v = \frac{200+200}{20}$ = 20m/sec = $20\times18/5$ = 72km/hr

44. (d): $t = \frac{270m}{25+2Km/hr} = \frac{270}{27Km/hr} = \frac{270}{27\times\frac{5}{18}} = 36$ sec

45. (c): $t = \frac{180+220}{(40+40)\frac{5}{18}} = \frac{400\times18}{80\times5} = 18$sec

46. (b):10 =(150+100)/v ⇒ v =25m/s=25×18/5=90km/hr
Speed of 2nd train = 90 – 30 = 60 km/hr

47. (b): d = 15 v
⇒(100 + d) = 25v $\quad$ ⇒ 100 = 25v – 15v
⇒ v = 10m/s $\quad$ ∴ d = 15v = 150

48. (b): vt = 1.4v (t - 25/60)

$$.4t = 1.4\times\frac{25}{60} = \frac{35}{60} \Rightarrow t = \frac{35}{60}\times\frac{1}{4} = 87.5\,\text{min}$$

49. (a): A travels 1000m = B travels 900m
= C travels 800m
$50 : \frac{500}{9} : \frac{125}{2} = 36 : 40 : 45$

50. (a): Let the actual distance travelled be x km.
Then,
$\frac{x}{10} = \frac{x+20}{14} \Leftrightarrow 14x = 10x + 200$
4x =200 ⇒ x= 50 km

51. (c) Distance covered by A in 3 minutes
$= 10\times\frac{3}{60} = \frac{1}{2}$ km = 500 m.
Hence, A has a start of total (100 m + 500 m) = 600 m. B's speed -A's speed = (13-10) km/hr = 3km/hr
$= \frac{3000\text{ m}}{60\text{ min}} = 50$ m/min
B gains 50 m over A in 1 minute
∴ B will cover 600 m lead in = $\frac{600}{50}$=12 minutes

52. (a) A: B : C = 100 : (100-20) : (100-40)= 100:80:60
$B: C = \frac{80}{60} = \frac{100}{C} \Rightarrow C = 75$
∴ B can give C (100-75) = 25 points
Note: We have assumed here that even the game between B and C is of 100 points.

53. (d) A:B:C=90:75:60
$B: C = \frac{75}{60} = \frac{100}{C} \Rightarrow C = 80$

54. (d) When A scores 75 points, B scores 50 points.
When A scores 90 points, C scores (90-18) = 72 points,
∴ When A scores 75 points, C scores
$= \frac{72}{90}\times 75 = 60$ points
A:B:C=75:50:60
$C:B = \frac{60}{50} = \frac{120}{100}$
So, in a game of 120, C can give B (120-100) = 20 points

55. (a) When A makes 5 rounds, B makes 4 rounds.
In order to pass each other, the difference in number of rounds made by each must be one. Here, A passes B each time, when A makes 5 rounds.
Distance covered by A in 5 rounds = $\frac{5\times400}{1000}$=2 km
In covering 2 km A passes B 1 time.
∴ In covering 5 km, A passes B
$= \frac{5}{2} = 2\frac{1}{2}$ times or 2 times

56. (a) A walks faster than B. They will be together again for the first time when A gains one complete round over B.
Circumference of the circle = 1800 metres.
∴ A gains (150-60)=90 m in 1 minute.
∴ A gains 1800 m in = $\frac{1800}{90}$ = 20 minutes
So, A and B will be together again for the first time after 20 minutes.

57. (c) A gains (150-130)=20 m over B in 1 minute.

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$\therefore$ A gains 600 m over B in = $\frac{600}{20}$ = 30 minutes
B gains (130-100)=30 m over C in 1 minute.
$\therefore$ B gains 600 m over C in $\frac{600}{30}$ = 20 minutes.
A and B will be together after 30 minutes while B and C will be together after 20 minutes.
LCM of 30 and 20=60
Hence, A, B and C will be together again for the first time after 60 minutes

58. (c) According to the question.
$\because$ When B runs 200 metres, A runs 190 metres
$\therefore$ When B runs 180 metres, A runs = $\frac{190}{200} \times 180$= 171 metres
When C runs 200m, B runs 180 metres.
Hence, C will give a start to A by = 200-171=29 metres

59. (d) Ratio of the speed of A, B and C = 6:3:1
Ratio of the times taken = $\frac{1}{6}:\frac{1}{3}:\frac{1}{1} = 1:2:6$
$\therefore$ Time taken by A = $\frac{72}{6}$=12 minutes

60. (a) A beats B by 30 seconds and B beats C by 15 seconds.
Clearly A beats C by 45 seconds.
Also, A beats C by 180 metres.
Hence, C covers 180 metres in 45 seconds.
$\therefore$ Speed of C = $\frac{180}{45}$ = 4 m/sec
$\therefore$ Time taken by C to cover 1000m = 1000/4=250 sec
$\therefore$ Time taken by A to Cover 1000m = 205 sec
So, time taken by A to Cover 5000m = 205 x 5 = 1025 sec

EXPERIENCE THE PRATHAM EDGE 11: TIME & WORK

1. (b): $\frac{2}{b}+\frac{3}{g}=\frac{1}{10}$; $\frac{3}{b}+\frac{2}{g}=\frac{1}{8}$
$Adding\ \frac{1}{b}+\frac{1}{g}=\frac{9}{200}$, $Subtracting\ \frac{1}{g}-\frac{1}{b}=\frac{1}{40}$
$\Rightarrow\frac{1}{b}=\frac{7}{200}\Rightarrow\frac{1}{g}=\frac{2}{200}$ $\Rightarrow\frac{5}{b}+\frac{4}{g}=\frac{38}{200}\sim\frac{1}{5}$

2. (d): m α 1/80; $(m+8)\,\alpha\,\frac{1}{80-8}$
Dividing the two equations, $\frac{m}{m+8}=\frac{72}{80}$ ⇒m =72

3. (a): $\left(\frac{2}{m}+\frac{7}{w}=\frac{1}{14}\right)\times 3$
$\left(\frac{3}{m}+\frac{8}{w}=\frac{1}{11}\right)\times 2$
$\frac{21-16}{w}=\frac{3}{14}-\frac{2}{11}=\frac{5}{154}\Rightarrow w=154, m=77$
$Hence, \frac{8}{77}+\frac{6}{154}=\frac{22}{154}=\frac{1}{7}$

4. (d): 1 hour work=42 + 56 - 48=50
Capacity of Tank = 50 × 20=1000 litres

5. (c): $\frac{1}{8}-\frac{1}{x}=\frac{1}{12}\Rightarrow\frac{1}{x}=\frac{1}{8}-\frac{1}{12}=\frac{3-2}{24}=\frac{1}{24}$

6. (a): Brajesh + Christopher $=\frac{1}{30}+\frac{1}{60}=\frac{1}{20}$
3 day work $=\frac{3}{20}+\frac{1}{20}=\frac{1}{5}$
So, work would be finished in 3 × 5 = 15 days

7. (a): full = $\frac{1}{27}+\frac{1}{33}=\frac{11+9}{297}=\frac{20}{297}$
⇒ 14.85 min

8. (b): Work Vandana did alone $=\frac{5}{15}=\frac{1}{3}$
Combined work $=\frac{2}{3}=\frac{n}{15}+\frac{n}{10}=\frac{n}{6}$
Therefore, $n = 4$

9. (a): $\frac{15}{m}=\frac{1}{20}$ ⇒ m = 300 days
Men required = 300/30=10 men

10. (a): $\frac{1}{A}+\frac{1}{B}=\frac{1}{10}$...........$(i)$; $\frac{1}{B}+\frac{1}{C}=\frac{1}{12}$..........$(ii)$
$\frac{1}{C}+\frac{1}{A}=\frac{1}{15}$...........$(iii)$
Adding all
$2\left(\frac{1}{A}+\frac{1}{B}+\frac{1}{C}\right)=\frac{1}{10}+\frac{1}{12}+\frac{1}{15}=\frac{6+5+4}{60}=\frac{1}{4}$.....(4)
Subtracting (2) from (4)
$\frac{1}{A}=\frac{1}{8}-\frac{1}{h}=\frac{1}{24}$ ⇒ A = 24 days

11. (b): Let B = x unit/day
A = 1.3 x unit/day
total work = 1.3x * 23

number of days in which work will be completed if A and B work together = 1.3x*23/2.3x = 13 days

12. (b): P prints 1 lac books in 8 hrs
Q prints 1 lac books in 10 hrs
R prints 1 lac books in12 hrs
P's work = 2/8 = 25%
Remaining = 1-1/4 =3/4
Q & R will complete it in

$$\frac{3}{4} \div \left(\frac{1}{10}+\frac{1}{12}\right) = \frac{3}{4} \div \frac{11}{60} = \frac{45}{11} = 4hr.5\min.$$

Time = 11 am = 4 hr 5m ~ 3 pm

13. (c): $\frac{1}{A}+\frac{1}{B}=\frac{1}{30}; \quad \frac{16}{A}+\frac{44}{B}=1$

Solving the 2 equations we get, B = 60 days

14. (c): $\frac{x}{15}+\frac{2}{10}=1 \quad \Rightarrow$ x = 12 days

15. (a): $\frac{3}{M}+\frac{4}{W}+\frac{6}{C}=\frac{1}{7}; \qquad W=\frac{M}{2}=\frac{C}{4}$

$$\frac{1}{7}=\frac{3}{2W}+\frac{4}{W}+\frac{6}{4W}=\frac{14}{2W}=\frac{7}{W}$$

7 women will do the work in 7 days

16. (d): $\frac{10}{M}+\frac{15}{W}=\frac{1}{6}$

$$\frac{1}{W}=\frac{1}{15}\left(\frac{1}{6}-\frac{1}{10}\right)=\frac{1}{15}\times\frac{1}{15}=\frac{1}{225}$$

17. (c): B & C work for 3 days,

$$\frac{3}{9}+\frac{3}{12}=\frac{7}{12}$$

$$\text{Remaining work} = 1-\frac{7}{12}=\frac{5}{12}$$

A completes 5/12 work in 5/12 ×24=10 days

18. (c): $\frac{1}{4}=\frac{1}{A}+\frac{1}{B}$ and B = A + 6

$$\Rightarrow \frac{1}{4}=\frac{1}{A}+\frac{1}{A+6}$$

$\Rightarrow$ $A^2 + 6A = 8A + 24$
$\Rightarrow A^2 - 2A - 24 = 0$
A=6 or – 4

19. (d):

$$1=\frac{t}{40}+t\left(\frac{1}{40}+\frac{1}{60}\right) \quad \Rightarrow \frac{3t+3t+2t}{120}=\frac{t}{15}$$

$\Rightarrow$ t = 15 min & 2t = 30 min

20. (a): Work done by the leak in 1 hr =

$$\left[\frac{1}{3}-\frac{1}{\left(\frac{7}{2}\right)}\right]=\left(\frac{1}{3}-\frac{2}{7}\right)=\frac{1}{21}$$ The leak will empty the tank in 21 hours.

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EXPERIENCE THE PRATHAM EDGE 12: CLOCKS & CALENDARS

1. (d) Angle Covered by minute hand in 60 minutes = 360°
Angle Covered in 1 minute = $\frac{360}{60}$=6°

Angle Covered in 22 minutes = 6 × 22 = 132°
2. (b) Angle Covered by hour hand in 12 hours = 360°
Angle Covered in 1 hour = 30°
Angle Covered in 1 minute = $\frac{30}{60} = \frac{1^\circ}{2}$
Angle Covered in 15 minutes = 7.5° [quarter of a hour=15 minutes]
3. (c) Hour hand Covers $\frac{1^\circ}{2}$ in = 1 minute
Hour hand Covers 6° in = 12 minutes
Angle Covered by minute hand in 12 minutes = 6 × 12 = 72°
4. (a) Angle between the hands can be calculated by $\theta = \left|\frac{11}{2}m - 30h\right|$, where m is minutes and h is hour

40 minutes past 6 ⇒ h = 6, m = 40
$\theta = \left|\frac{11}{2}(40) - 30(6)\right| = 40°$
5. (c) 3 hours 14 minutes
h = 3, m = 14
Angle between the hands $\theta = \left|\frac{11}{2}(14) - 30(3)\right|$
= |-13°|=13°
6. (c) 25 minutes past 7 o'clock.
h = 7 and m = 25
Angle between the hands $\theta = \left|\frac{11}{2}(25) - 30(7)\right| = 72\frac{1}{2}^\circ$
7. (d) 20 minutes past 11 o'clock
Angle between the two hands $\theta = \left|\frac{11}{2}(m) - 30(h)\right|$
$= \left|\frac{11}{2}(20) - 30(11)\right| = 220°$
This is reflex angle. In case of clocks, we consider angles less than 180°
So angle between the two hands = 360° - 220° = 140°
8. (c) Hands will coincide when angle between them (θ) is zero.
So θ = 0°, h = 9
$\Rightarrow \theta = \left|\frac{11}{2}(m) - 30(h)\right|$
$\Rightarrow 0° = \frac{11}{2}(m) - 270 \Rightarrow m = \frac{270\times2}{11} = 49\frac{1}{11}$
Hands will coincide at 491/11 minutes past 9 o'clock.
9. (b) Hands will be in opposite direction when θ = 180°
h=4
$\Rightarrow \theta = \left|\frac{11}{2}(m) - 30(h)\right|$
$\Rightarrow 180 = \left|\frac{11}{2}(m) - 120\right|$
$\Rightarrow \frac{300\times2}{11} = m \Rightarrow m = 54\frac{6}{11}$
Hands will be in opposite direction at 546/11 minutes past 4 o'clock
10. (d) θ =20° h=2
$\theta = \left|\frac{11}{2}(m) - 30(h)\right|$
⇒ we have two cases
Case 1 ⇒ $20° = \frac{11}{2}(m) - 30(2)$
⇒ m = $\frac{80\times2}{11} = 14\frac{6}{11}$ minutes
Case 2 ⇒ $20° = -\left(\frac{11}{2}(m) - 60\right)$
⇒ $\frac{40\times2}{11} = m \Rightarrow m = 7\frac{3}{11}$ minutes
Hence time will be 2 hours $14\frac{6}{11}$ minutes
or 2 hours $7\frac{3}{11}$ minutes
11. θ =60° h=4
$$\theta = \frac{11}{2}(m) - 30(h)$$
$$\Rightarrow 60° = \frac{11}{2}(m) - 120$$
$$\Rightarrow \frac{180\times2}{11} = m \Rightarrow m = 32\frac{8}{11} \text{ minutes}$$
Time shown by the clock will be $32\frac{8}{11}$ minutes past 4
12. (c) Any angle except 0° and 180° is made 22 times in a period of 12 hours.
So in a day i.e. 24 hours 30° angle is made 44 times.
13. (b) Minute hand overlaps with the hour hand time in each hour except between 12 and 1
So from 9:00 am to 4:00 pm, they will coincide 6 times.
14. (b) Time interval between 8:00 am and 6:00 pm is 10 hours
∴ Total time gained by watch in 10 hours = 2 minutes
Watch would indicate Correct time if it gains 1 minute because initially it was slow by 1 minute
Watch gains 2 minutes in 10 hours
∴Watch will gain 1 minute in 5 hours

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Therefore, 5 hours after 8:00 am is 1:00 pm.

15. (c) Time interval from 9:00 am on Tuesday to 12 noon on Subsequent Wednesday will be 27 hours.
Total time gained in 27 hours = 9 minutes
Correct time will be shown by the watch if it gains 6 minutes because earlier, it was slow by 6 minutes
$\Rightarrow$ 9 minutes are gained in 27 hours
$\Rightarrow$ 6 minutes will be gained in 18 hours.
Correct the will be shown by the watch 18 hours after 9:00 am i.e. 3:00 am on Wednesday

16. (d) Time interval from 6 o'clock on Thursday morning to 8 o'clock on Saturday morning will be = $(24\times2)+2=50$ hours
Total time lost by watch in 50 hours = 25 minutes
Watch will show correct times if it loses 10 minutes because earlier, it was fast by 10 minutes
$\Rightarrow$ The watch lost 25 minutes in 50 hours
$\Rightarrow$ The watch will lose 10 minutes in 20 hours
Therefore 20 hours after 6 am on Thursday will be 2 o'clock on Friday morning

17. (a) In a Correct clock, the minute hand gains 55 minutes spaces over the hour hand in 60 minutes. To overtake, the minute hand must gain 60 minute spaces over the hour hand.
Since 55 minute spaces are gained in 60 minutes
$\Rightarrow$ 60 minute spaces are gained in
$\frac{60}{55}\times 60 = 65\frac{5}{11}$ minutes

But it is given minute hand overtakes the hour hand at interval of 60 minutes
So in 60 minutes clocks gains = $\left(65\frac{5}{11}-60\right)$ minutes

18. (a) 55 minute space are gained by minute hand in 60 minutes
$\Rightarrow$ 60 minute spaces are gained in
$\frac{60}{55}\times 60 = 65\frac{5}{11}$ minutes

It is given that minute hand in interval of 70 minutes
In 70 minutes clock loses
$\left(70-65\frac{5}{11}\right)$ minutes = $\frac{50}{11}$ minutes

In 1 day (24×60 minutes) clock loses
= $\frac{50}{11\times70}$ $24\times60 = 93\frac{39}{77}$ minutes

19. (a) In the formula $\theta = \left|\frac{11}{2}m-30h\right|$
$\theta = 60^0$ and h = 2
$\therefore 60 = \frac{11}{2}m-30\times2$
$\frac{11}{2}m = 120$
$m = \frac{240}{11} = 21\frac{9}{11}\min\, past\ 2$
Or
$60 = 30\times 2 - \frac{11}{2}m$
$\therefore \frac{11}{2}m = 0$
m = 0

Therefore, the angle between the hour hand and the minute hand is 60° at 2 O'clock and at $21\frac{9}{11}$ minutes past 2 O'clock.

20. (a) When the two hand overlap, the angle between them is 0°
$\theta = \left|\frac{11}{2}m-30h\right|$
$\therefore \theta = 0^0$ and h = 2
$\frac{11}{2}\ m = 30\times2$
$m = \frac{120}{11} = 10\frac{10}{11}\min\, past\ 2.$

21. (d) If the given time is between 1 and 11. For mirror time Subtract it from 11.60
Hence minor time is 11.60-10.40 $\Rightarrow$1 hour 20 minutes

22. (a) Mirror time is 11.60-3.40=8 hour 20 minutes

23. (d) Seconds hand move 360° in 1 minute
Seconds hand move 240° in $\frac{1}{360}\times240 = \frac{2}{3}$ minutes

In 60 minute, minute hands covers = 360°
In $\frac{2}{3}$ minute it Covers = $\frac{360}{60}\times\frac{2}{3} = 4°$

24. (b) When time is 10:30, hour hand will point towards north west.

25. (b) The angle between the hand can be calculated by $\theta = \left|\frac{11}{2}m-30h\right|$,where m is the minutes and h is the hours. Here,
m = 40 and h = 3
$\theta = \left|\frac{11}{2}\times40-30\ \times3\right| = 130^0$

26. (d) In an ordinary year, January 1st and December 31st will be the same day of the week. In a leap year, January 1st and December 30th will be the same day of the week. 2012 is a leap year. So 30.12.2012 is

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Sunday and 31.12.2012 is Monday

27. (a) 2008 was a leap year. So 31.12.2008 is Wednesday and 01.01.2009 is Thursday
28. (b) 1982 is an ordinary year. So 31.12.1982 was Friday i.e.01.01.1983 was Saturday. Now 8th January will again be Saturday (1+7) i.e. 09.01.1983 will be Sunday.
29. (b) The required answer = Monday +161 days = Monday+23 x 7 days=Monday (in 161 days there are no odd days since 161 is a multiple of 7). After 161 Days, it will be Tuesday.
30. (a) The required answer = Wednesday + 72 days = Wednesday + 7 x 10 + 2 = Wednesday + 2 = Friday. After 72 Days, it will be Saturday.
31. (c) December 3rd is Monday. How many remaining days are there in December ⇒ (31-3) i.e. 28 days i.e. 0 odd days. So what is January 1st. There will be one odd day (of the 1st January) and the answer will be Monday +1 = Tuesday.
32. (d) When we get such a problem, see whether February 29th of a leap year is included in the given time period
since the number of odd days is dependent on the same. Here 2014 is not a leap year and the number of odd days from August 15 th 2013 to August 15 th 2014 is 1 and hence the required answer is Thursday +1 = Friday.
33. (a) 2004 is leap year and so in the time period 27.03.2003 -27.03.2004, 29th February of 2004 is included. So the number of odd days during the time period = 2 and the required answer is Thursday + 2 = Saturday.
34. (d) Though 2012 was a leap year, in the given time period i.e. 9th March 2013 - 9th March 2012, February 29th is not included. So there is only one odd day during the time period. But here calculation is backwards i.e. the required answer is Saturday-1 = Friday.
35. (a) 1st August 2012 - 1st August 2010.2012 is a leap year and 29th February of 2012 is included in the time period. So the number of odd days - 2 + 1 = 3 odd days and so the required answer is Wednesday - 3 = Sunday.
36. (c) January 1st is Saturday and hence 8th is a Saturday i.e. 7th is a Friday. So the first Friday of the month is 7th recurring on 14th, 21st and 28th. So there are 4 Fridays during the month.
37. (a) 25th is Friday and since Tuesday=Friday +4,29th of the month will be Tuesday and preceding on 29 - 7 i.e. 22nd, 22 - 7 = 15th, 15 - 7 = 8th and 8 - 7 = 1st. So there are 5 Fridays during the month.
38. (d) January 1st 2012 = Sunday i.e. the first Wednesday of the month is on 1 + 3=4th and succeeding on4 + 7 =11th, 18th, 25th and 25 + 7= 32 is not possible and the answer is 4.
39. (a) Since the time period is not crisscrossing the month of February, the concept of ordinary year and leap year is not relevant. Independence day is on 15th of August and Gandhi Jayanthi is on 2nd October. The remaining number of days in August =31 - 15 = 16 days=2 odd days. Number of days in September = 30 = 2. Number of days in October = 2. So the total number of odd days = 2 + 2 + 2 - 6. So the answer is Friday + 6 = Thursday .
40. (c) 26th January 2008 was Saturday. 2008 was a leap year. So upto 26th January 2009, the number of odd days = 2.
In January 2009, the remaining number of days=31 - 26 = 5. February2009 = 28 days = 0 odd day. March2009=31 days=3 odd days. April 2009=30 days =2 odd days. May2009=3 odd days. June 2009 = 2 odd days. July2009 = 3 odd days. August 2009 = 15 days = 1 odd day. So the total number of odd days = 2 + 5 + 0 + 3 + 2 + 3 + 2 + 3 + 1 = 0. Therefore the Independence Day of the year 2009 was a Saturday.
41. (d) In September 2006, the remaining days = 30 -27 = 3 days. Oct.06 =3., Nov.06 = 2., Dec.06 = 03.
Now 2007 is an ordinary year .In an ordinary year from January 1st to June 30th (i.e. the half year) there are 6 odd days.
In a leap year from January 1st to June 30th there are 0 odd days. Therefore, Jan. 2007 - Jun.2007 = 6. 12th July 2007 =12 = 5. So the total number of odd days is 3 + 3 + 2 + 3 + 6 + 5 = 1. Therefore the answer is Wednesday + 1 = Thursday
42. (b) Remaining days in June =15-1 = 14 (calculation is backward) = 0 odd days. In May, how many

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days are there. In May there are (31 - 15) +1 days (Here 15th May is also inclusive since 15th June, the reference day is already excluded. The concept is very simple. Out of the two dates, the date for which day has been mentioned will be excluded and the other included). So number of odd days = 17 = 3. So answer is Sunday - 3 = Thursday.

43. (d) December 25th = Saturday. Remaining days in December=24 = 3 odd days. November = 2 odd days. October = 3. September = 30-23+ 1 = 8 = 1 odd day. Total odd days = 3 + 2 + 3 + 1= 2. Therefore the required answer is Saturday - 2 = Thursday.
44. (d) 15th May 1992 -15th May 1993 = 1 odd day. Remaining days in May 1993 = 31 - 15 = 16 = 2 odd days. June 1993 = 2., July = 3., Aug. = 3., Sep. = 2., Oct. = 3. Nov.= 2 ., Dec. 1993 = 7 = 0. Therefore the total number of odd days = 1 +2 + 2 + 3 + 3 + 2 + 3 + 2 = 4. Therefore the answer is Friday+4 = Tuesday. Though 1992 is a leap year in the time period, 29th February is not included.
45. (d) year = 2005 + 2006 + 2007 + 2008 + 2009 + 2010 odd days = 1 + 1 + 1 + 2 + 1 + 1
 Total odd day = 0, Hence 2011 will have the same calendar as that of 2005.
46. (b) The number of odd days from 26th April 1994 to 26th April 2010 is as below.

1995	1996	1997	1998	1999	2000	2001	2002
1	2	1	1	1	2	1	1

2003	2004	2005	2006	2007	2008	2009	2010
1	2	1	1	1	2	1	1

From 26th April 2010 to 26th April 2011 there is only 1 odd day (not included in the table to avoid confusion). Total number of odd days upto 26th April 2011 =0 (Repeat: Never add 1+2+1+1+1+2+ 1 + 1 + 1 + 2+1 + 1 + 1 +2+1 +1 +1. Just see the clusters that add up to 7 and discard them and whatever remaining only need to be counted).
Remaining odd days in April 2011 =4. May 2011 =3., June 2011 =2., July 2011 =3 August 2011 = 3 22nd September2011 =22= 1.
So the total number of odd days=0 + 4 + 3 + 2 + 3 + 3 + 1 = 2 odd days. Therefore the answer is Tuesday+2=Thursday.

47. (d) 1600 - 0 odd days. 300 years -1 odd day.
 In 46 years there are 11 leap years ($\frac{46}{4} \times 11$) and 35 ordinary years.
 Therefore the number of odd days= 11x2+35x1 =4x2+0x1 = 8=1.
 1947 is an ordinary year and from 1st January to 30th June 1947 there are 6 odd days. July 1947 - 3 odd days. August 1947 = 15 = 1 odd day.
 Therefore the total number of odd days = 1 +1 + 6 + 3+1=5. Therefore 15th August 1947 was Sunday+5=Friday.
48. (b) In 2000 years there are 0 odd days. In 6 years there is one leap year and 5 ordinary years. So the number of odd days in this 6 years = 1 x 2+5 x 1= 7=0 odd days. 2007 is an ordinary year. January 2007 - 3 odd days. February 2007 - 0 odd days. March 23rd 2007 = 23 = 2 odd days. So the total number of odd days = 0 + 0 + 3 + 0 + 2=5days. Therefore 23rd March 2007 is Sunday + 5=Friday.
49. (b) In 3 8 years there are 9 leap years and 29 ordinary years. So the number of odd days=2x2+1 x 1=5 o dd days.
 Odd days in 2039: Jan. = 3., Feb =0., March =3., April =1, May=4. So the total number of odd days = 5 + 3 + 0 + 3 + 2+4 = 3 odd days. Therefore May 11th 2039=Sunday + 3 = Wednesday.
50. (c) The answer is 2002 +11 = 2013 because from 2002 to 2012 there are 0 odd days

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EXPERIENCE THE PRATHAM EDGE 13: MENSURATION

1. (b): Sum of square of sides
$= 4/3 \times$ (sum of square of Median)
$\Rightarrow \frac{4}{3} \times 36 = 48$

2. (d): $4a = 6h; \Rightarrow \frac{a}{h} = \frac{3}{2}$
$$\frac{Area\ of\ Square}{Area\ of\ Hexagon} = \frac{a^2}{\frac{3\sqrt{3}h^2}{2}} = \frac{2}{3\sqrt{3}} \times \frac{9}{4} = \frac{\sqrt{3}}{2}$$

3. (b): Perimeter of sector
$= 2r + \theta r = 2 \times 21 + 21\theta = 64$
$\Rightarrow \theta = 22/21$
Therefore, Area $= \frac{\theta}{2} r^2 = \frac{22}{2 \times 21} \times 21 \times 21 = 231$

4. (d): new area – old area
$16\sqrt{3} = \frac{\sqrt{3}}{4}(a+4)^2 - \frac{\sqrt{3}}{4}a^2$
$a^2 + 8a - a^2 + 16 = (64\sqrt{3})/\sqrt{3} = 64$
$\Rightarrow a = 8 - 2 = 6$
Altitude of new triangle $= \frac{\sqrt{3}}{2}(a+4) = 5\sqrt{3}$

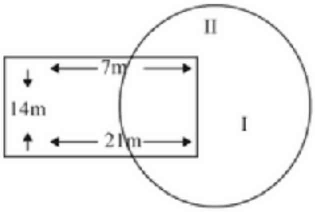
5. (a): $a + b + c = 144$
$\Rightarrow a + b = 144 - 65 = 79$
$(65)^2 = a^2 + b^2 = a^2 + (79 - a)^2$
$= 2a^2 + (79)^2 - 158a$
$a = 63$ and $a = 16$
PSS: (16, 63 and 65) is a Pythagorean triplet

6. (a): Total volume of cube
$= (3x)^3 + (4x)^3 + (5x)^3 = 216x^3$
Side of new cube $= (12\sqrt{3})/\sqrt{3} = 12$
Volume $= (12)^3 = 216x^3 \quad \Rightarrow x = 2$
$\therefore$ Side of smallest cube is $3x$ i.e. 6

7. (b): $6a^2 = 4\pi r^2$ (a is side of the cube)
$\frac{r}{a} = (\frac{6}{4\pi})^{1/2}$
Ratio of volume $= \frac{4}{3}\pi \times \frac{3}{2\pi} \times \frac{\sqrt{3}}{\sqrt{2\pi}} = \sqrt{\frac{6}{\pi}}$

8. (b): Area of trapezium $= 105 = \frac{1}{2}(x+x+8) \times 7$
$\Rightarrow 2x + 8 = 30$ Hence, $x = 11$
$\therefore$ Longer side $= 11+8 = 19$

9. (c):
The cow can graze the shaded areas numbered I and II.



Area of the region I $= \frac{22}{7} \times 21 \times 21 \times \frac{270}{360}$
$= 1039\frac{1}{2}\ m^2$
Area of the region II $= \frac{22}{7} \times 7 \times 7 \times \frac{90}{360} = 38\frac{1}{2}$
Therefore, Total area that the cow can graze
$= 1039\frac{1}{2} + 38\frac{1}{2} = 1078\ m^2$

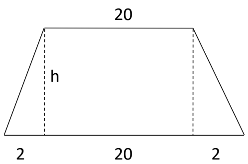
10. (b): $\pi r^2(h+x) = \pi(r+x)^2 h$
$\Rightarrow 144\,(5+x) = (12+x)^2 \times 5$
$\Rightarrow 720 + 144x = (144 + 24x + x^2) \times 5$
$\therefore x = 24/5$

11. (a): $\frac{CSA\ of\ cylinder}{CSA\ of\ cone} = \frac{8}{5} = \frac{2\pi rh}{\pi r\left(r^2+h^2\right)^{1/2}}$
$64\,(r^2+h^2) = 100\,h^2$
$r/h = 6/8 = 3/4$

12. (c): $2\pi r = 440$; r=70 cm, h=1.5m
R=70 cm = 70/ 30 feet; h = 150/30 feet
Volume $= \pi r^2 h = \frac{22}{7} \times (\frac{7}{3})^2 \times 5 = \frac{770}{9}$
Area realized $= 0.9 \times 770/9 = 77$
Price $= 77 \times 1{,}500 = 1{,}15{,}500$

13. (b): Volume $= \frac{22}{7} \times (\frac{1}{2})^2 \times 7$
$\Rightarrow 11/2\ cm^3$, can write 2200 words
Bottle = 200 ml = 200 cm^3
Words that can be written $= \frac{200}{11/2} \times 2200 = 80{,}000$

14. (a):



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220 = ½ (20+24) ×h

$\Rightarrow$ h = 10

Non parallel side =

$[10^2 + 2^2]^{1/2} = \sqrt{104} = 2\sqrt{26}$

15. (c): Area to be painted = $2(lh + bh) + lb$

$l + b + h = 21;$

Area maximum when $l=b=h=7$

Area = $5l^2$=245

Max cost = 50 ×245 = 12250

For minimum, l=19, b=h=1

Area = 2(19+1) + 19 = 59

Minimum cost = 50 × 59 = 2950

$\therefore$ Difference = 12250 – 2950 = 9300

16. (c): L = 1.6B; L – B=24;

B=24/0.6 = 40; L = 64

$\therefore$ Area = 64 × 40 = 2560

17. (b): Distance covered

$$= 2(L+B) = 12 \times \frac{5}{18} \times 8 \times 60 = 1600$$

$$\frac{l}{b} = \frac{3}{2} \Rightarrow l = \frac{3}{2}b$$

$$b = \frac{2}{5} \times 800 = 320;\ l = 480$$

Therefore, $l \times b$ = 153600

18. (d): l = 1.15b

Area = (1.15 b)b = 460

$\Rightarrow$ b = 20

19. (d): L=3/4 B

Area = 7500 = LB = ¾ B^2

$\Rightarrow$ B=100, L = 75

Cost of fencing = 0.25 × 2 (L+B)

= 0.5 × 175 = 87.5

20. (a):

$2\left(l + \frac{b}{2}\right) = 34$ .... (i)

$2\left(b + \frac{l}{2}\right) = 38$ .... (ii)

From (i) and (ii)

$b = 14, l = 10$

Area = $l \times b$ = 10 x 14 = 140 cm^2

21. (d): $\frac{new\ area}{old\ area} \Rightarrow = \frac{(1.5l)(1.2b)}{lb} = 1.8:1$

22. (b): % change = $\frac{new\ area - old\ area}{old\ area}$

$$\frac{\frac{l}{2} \times 3b - lb}{lb} = \frac{3}{2} - 1 = \frac{1}{2} = 50\%$$

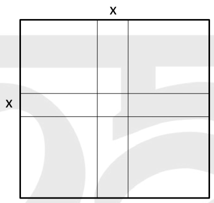
23. (d): $lb = l\left(\frac{100-r}{100}\right).b\left(\frac{100+r+5}{100}\right)$

$\Rightarrow 10000 = -r^2 - 5r + 10500$

$\Rightarrow r^2 + 5r - 500 = 0;$

$\therefore$ r = - 25 *or* 20

24. (b):



Area of lawn =

$2109 = (60 - x)(40 - x) = x^2 + 2400 - 100x$

$\therefore x = 3$

25. (d): $\frac{new\ length}{old\ length} = \frac{1.3area}{area} = \frac{13}{10}$

26. (d): error in area = $(1.02)^2a^2 - a^2$ = 4.04%

27. (b): New area = 1.69 $a^2 = (1.3a)^2$

Side of square increases by 30%

28. (d): a:b:c = 1/2:1/3:1/4 = 6:4:3

Perimeter = $6x + 4x + 3x = 52$

$\Rightarrow x = 4$

$\therefore$ Smallest side = 3×4 = 12

29. (c): a^2= ½ a × h $\Rightarrow$ h=2a

30. (a): area = $(\sqrt{3}/4)a^2$

New area = $(\sqrt{3}/4)\ 0.8a^2$

$=0.64\ (\sqrt{3}/4)a^2$

% change = - 36%

31. (b): 32 min

$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{20 \times 2 \times \frac{22}{7} \times 50}{12 \times \frac{5}{18}} = \frac{1320}{7}\text{sec} = \frac{22}{7}\text{min}$$

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32. (c): Speed = distance/ time

$$= \frac{2\times\frac{22}{7}\times\frac{70}{2}\times 40}{10} = 880$$

$\Rightarrow 8.8\times18/5 = 31.68$ km/hr

33. (c): Area and Circumference has direct proportion relation.
Since, Area increases by 75%, Therefore circumference increases by 75%

34. (b): New Area = $\pi(0.9r)^2 = 0.81\pi r^2$
$\Rightarrow (1 - 0.19)\pi r^2$

35. (d): Curved area of hemisphere/ curved area of cone =

$$\frac{2\pi r^2}{\pi r\sqrt{r^2+h^2}} = \frac{2r}{\sqrt{r^2+h^2}} = \frac{\sqrt{2}}{1}$$

36. (c): Volume of hemisphere = $(2/3)\pi r^3$
Volume of cylinder = $\pi r^2 h = \pi r^2 (2/3) r$
$= 2/3\ \pi r^3$
Hence, Volume is equal

37. (a): $4/3\ \pi r^3 = 49\times33\times24$
$r = 21$

38. (a): r=1cm; h=1 cm
$V = 4/3\ \pi r^3 = (4/3)\pi$

39. (c): $a^3 = (100)^3 = n\,(10)^3$
n = 1000

40. (d): $\frac{\text{new Vol.}}{\text{old Vol.}} = \frac{\pi(2r)^2(2h)}{\pi r^2 h} = 8$

41. (c):

Note: 1m = 100cm, 1 litre = $1000cm^3$
Volume of water flown in one second = $\pi r^2 h$
$= \frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times 200 = 7700cm^3$
∴ Volume of water flow in 10 minutes i.e 600 seconds
$= 7700\times 600cm^3$
$= \left(\frac{4620000}{1000}\right) litre$
= 4620 litre

42. (b): $V = \pi(d^2 - d_2^2)H_2$
$= \pi\times 21(6^2 - (5.6)^2)$
$= \pi\times 21(36-31.36)$
$= \pi\times 21\,(4.64)$
= 306.24

43. (b): $V = 11l = 11000cm^3 = \frac{22}{7}\times\frac{35}{2}\times\frac{35}{2}\times h$

$$h = \frac{11000\times 2}{11\times5\times35} = 11\frac{3}{7}$$

44. (c): V of cone/ V of cylinder =

$$\frac{\frac{1}{3}\times\pi\times3^2\times5}{\pi\times2^2\times4} = \frac{15}{16}$$

45. (b): Diagonal will be longest pole
length of diagonal = $\sqrt{3}\times s$
$= \sqrt{3}\times\sqrt{3} = 3$

46. (d): side = diagonal/$\sqrt{3}$ = $(4\sqrt{3})/\sqrt{3} = 4$
$\Rightarrow$ Volume = $4^3 = 64$

47. (b): $\frac{A^3}{a^3} = \frac{8}{27}$

Surface area $= \frac{6A^2}{6a^2} = \frac{4}{9}$

48. (c): Small cube/ Large cube = $\frac{(1cm)^3}{(5cm)^3} = \frac{1}{125}$

49. (a): 4a = 20cm
a = 5cm
$V = 5^3 = 125\ cm^3$

50. (a): The cylinder obtained from the foil has perimeter 44 cm and height 20 cm
$\Rightarrow 2\pi R = 44 \Rightarrow R = 22/\pi$ cm = 7 cm
$\Rightarrow$ Volume of the cylinder = $\pi r^2 h$
$= \pi\times7^2\times20 = 3080\ cm^3$

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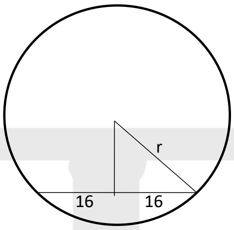
EXPERIENCE THE PRATHAM EDGE 14: GEOMETRY

1. (a): $\angle POQ = 90 - \angle POB$
$= 90 - (90 - \angle BOA) = 20^0$

2. (b): $\left(\dfrac{n-2}{n}\right)\pi = 144$
$\Rightarrow (n-2)180 = 144n$
$n = 360/36 = 10$

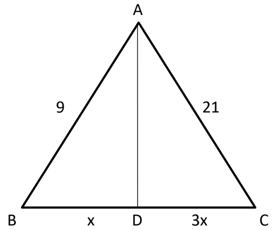
3. (b): r = 65, chord = 32
Distance from centre
$= [(65)^2 - (16)^2]^{1/2} = 63$ cm



4. (b): In ΔABC and ΔCEF,
$\dfrac{CF}{BC} = \dfrac{EF}{AB} = \dfrac{CF}{EF} = \dfrac{15}{30} = \dfrac{1}{2}$.......(i)
In ΔBCD and ΔBEF,
$\dfrac{EF}{CD} = \dfrac{BF}{BC} \Rightarrow \dfrac{BF}{EF} = \dfrac{BC}{CD} = \dfrac{15}{45} = \dfrac{1}{3}$.....(ii)
Adding (i) & (ii)
$\dfrac{(CF+BF)}{EF} = \dfrac{BC}{EF} = \dfrac{15}{EF} = \dfrac{1}{3} + \dfrac{1}{2} = \dfrac{5}{6}$
EF = 18

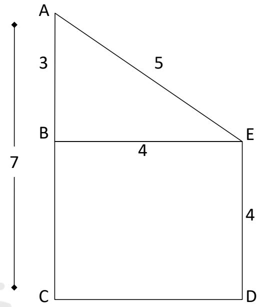
5. (c): $AD = \sqrt{8^2 - 4^2} = \sqrt{48}$
$AB^2 = AD^2 + BD^2 = 48 + 4 = 52$
$AB = \sqrt{52}$

6. (a):



$\Rightarrow AD^2 = 21^2 - (3x)^2 = 9^2 - x^2$
$\Rightarrow 8x^2 = 21^2 - 9^2 = 441 - 81 = 360$
$\Rightarrow x^2 = 45; \quad \therefore x = 3\sqrt{5}$
$BC = 4x = 4 \times 3\sqrt{5} = 12\sqrt{5}$

7. (a):



BC = DE = 4
AB = 7 – 4 = 3
$AE = \sqrt{AB^2 + BE^2} = 5$

8. (d): $\angle OYZ = \angle OZY = 60^0$
$\angle OXY = \angle OYX = 20^0$
$\angle XYZ = \angle OYX + \angle OYZ = 60^0 + 20^0 = 80^0$
$\angle XOZ = 2\angle XYZ = 160^0$

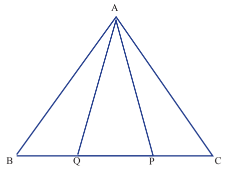
9. (c): PA. PB = PC. PD
$\Rightarrow 4(4+6) = 8 \times PC$
$\Rightarrow PC = 5 = PD - CD = 8 - CD$
$\therefore CD = 3$ cm

10. (d): $90^0 + 3\angle SRT = 180^0$
$\Rightarrow \angle SRT = 30^0$
Hence $\angle QRP = 60^0$

11. (d): $\angle XYZ = \angle PXY + \angle QZY = 42^0 + 46^0 = 88^0$

12. (c): $\angle PQM = \angle QMS = 80^0$
$\angle RMS = 180^0 - \angle QMS = 100^0$
$\angle RSM = 180^0 - \angle TSR = 60^0$
$\angle SRM = 180^0 - \angle RMS - \angle RSM$
$= 180^0 - 100^0 - 60^0 = 20^0$
$\angle RQZ = 2\angle QRS = 40^0$

13. (c): $\angle BAC = 180^0 - 40^0 - 80^0 = 60^0$
$\angle QAC = 180^0 - (90^0 + 40^0) = 50^0$
$\angle CAP = 1/2(60)^0 = 30^0$
$\angle PAQ = \angle CAQ - \angle CAP = 20^0$



14. (d): $\angle PQY = \angle APQ - \angle XYZ$
$= 120^0 - 40^0 = 80^0$
$\angle RQZ = 180^0 - \angle PQY = 100^0$

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$\angle RZB = \angle ZRQ + \angle RQZ = 100^0 + 25^0 = 125^0$

15. (a): $\angle D = \angle ACE = 70^0$
$\angle B = \angle ECA - \angle A = 70^0 - 40^0 = 30^0$

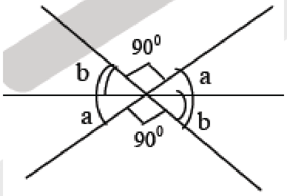
16. (c): $\angle RPQ = 180^0 - \angle Q - \angle R = 110^0$
$\angle QPS = \angle Q = 40^0$ as QS = PS
$\angle TPR = \angle R = 30^0$ as RT = TP
$\angle SPT = \angle QPR - \angle QPS - \angle TPR$
$= 110^0 - 40^0 - 30^0 = 40^0$

17. (b): $\angle UPR = \angle USR = 50^0$
$\angle TPU = 180^0 - \angle UPR - \angle QPR = 70^0$

18. (b): $\angle CAD = \angle CDA = 20^0$ or AC = CD
$\angle ACB = 180^0 - \angle CAB - \angle ABC$
$= 180^0 - 40^0 - 25^0 = 115^0$
$\angle ACD = 180^0 - \angle CAD - \angle ADC$
$= 180^0 - 20^0 - 20^0 = 140^0$
$\angle BCT = 360^0 - \angle ACB - \angle ACD - \angle DCT$
$= 360^0 - 115^0 - 140^0 - 35^0 = 70^0$

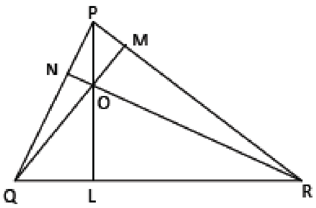
19. (a): $PO^2 + PR^2 = OR^2$
$OP = \sqrt{(25-9)} = 4$
PB = OP + OB = 4 + 5 = 9

20. (c): As we can see from figure
Maximum number of other distinct $\angle$ are 2



21. (b): AD is a median &
1/3=GD/AD; AD = 3GD = 4.5
$\therefore$ AG = AD – GD = 4.5 – 1.5 = 3

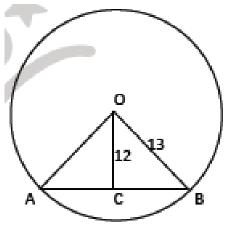
22. (d): Orthocentre is the point of intersection of altitudes



Therefore, P is the orthocentre for ΔQOR

23. (a): Construction draw line from A & C and intersect at a point on circle.
Then by using cyclic quadrilateral property
$\angle ABC = 110^0$

24. (b):



$AC = \sqrt{13^2 - 12^2} = 5$
Length of the chord = 5×2=10

25. (d): Interior $\angle$ of hexagon =

$$\left(\frac{n-2}{n}\right) \times 180 = \frac{4}{6} \times 180^0 = 120^0$$

Polygon = 9/8 ×120^0 = 135^0

$$\left(\frac{n-2}{n}\right) \times 180^0 = 135^0 \Rightarrow 4n - 8 = 3n \therefore n = 8$$

26. (d):PB || RC and PB = RC
PR || BC and BC = PR

27. (b):*Midpoint Theorem*
$AC^2 = AD^2 + CD^2$, AC = 10
therefore, Through Mid-point theorem, the length of the line = 5

28. (c): (i) is true as 2 sides need to be equal in isosceles triangle
(ii) is not true (iii) is true

29. (a): ΔABH

30. (c): Each exterior angle of a regular polygon of

n sides = $\frac{360^0}{n}$

Each interior angle of a regular polygon of

n sides $= (2n-4) \times \frac{90^0}{n}$

According to the Question, we get

$$\frac{360^0}{n} = \frac{(2n-4)}{3n} \times 90^0$$

$12 = 2n - 4 \Rightarrow 2n = 16 \Rightarrow n = 8$
Hence, the given polygon has 8 sides.

Alternative Method :
Exterior angle+interior angle = 180^0
$\Rightarrow$ Exterior angle+ 3 exterior angles = 180^0
$\Rightarrow$ 4 Exterior angle = 45^0
$\therefore$ Number of sides of the regular polygon
$= \frac{360}{45} = 8$

ANSWERS

EXPERIENCE THE PRATHAM EDGE 15: TRIGONOMETRY

1. (a) $\frac{\sec^2\theta-1}{\tan^2\theta}=\frac{\tan^2\theta}{\tan^2\theta}=1\ [1+\tan^2\theta=\sec^2\theta]$

2. (d) $\sin^4\theta+\sin^2\theta\cos^2\theta$
$=\sin^2\theta(\sin^2\theta+\cos^2\theta)\ [\sin^2\theta+\cos^2\theta=1]=\sin^2\theta$

3. (b) $\frac{\sin\theta\ \text{Cosec}\theta\ \tan\theta\ \cot\theta}{\sin^2\theta+\cos^2\theta}=\sin\theta\left(\frac{1}{\cos\theta}\right)\tan\theta\left(\frac{1}{\tan\theta}\right)=1$

4. (d) $\sin^2A\cot^2A+\cos^2A\tan^2A$
$=\sin^2A\,\frac{\cos^2A}{\sin^2A}+\cos^2A\frac{\sin^2A}{\cos^2A}\ \left[\cot A=\frac{\cos A}{\sin A},\tan A=\frac{\sin A}{\cos A}\right]$
$=\cos^2A+\sin^2A=1$

5. (d) $\tan\theta+\cot\theta=\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$
$=\frac{1}{\sin\theta\cos\theta}=\sec\theta\ \text{Cosec}\theta$

6. (c) $\frac{3-4\sin^2\theta}{\cos^2\theta}+\tan^2\theta$
$=\frac{3-4\sin^2\theta}{\cos^2\theta}+\frac{\sin^2\theta}{\cos^2\theta}$
$=\frac{3-4\sin^2\theta+\sin^2\theta}{\cos^2\theta}=\frac{3-3\sin^2\theta}{\cos^2\theta}=\frac{3(1-\sin^2\theta)}{\cos^2\theta}=3$

7. (b) $\frac{\cot A+\tan B}{\cot B+\tan A}=\frac{\frac{\cos A}{\sin A}+\frac{\sin B}{\cos B}}{\frac{\cos B}{\sin B}+\frac{\sin A}{\cos A}}$
$=\frac{\frac{\cos A\cos B+\sin A\sin B}{\sin A\cos B}}{\frac{\cos A\cos B+\sin A\sin B}{\sin B\cos A}}$
$=\frac{\sin B\cos A}{\sin A\cos B}=\tan B\cot A$

8. (c) $\sin\theta=\frac{21}{29}$ $\cos\theta=\sqrt{1-\left(\frac{21}{29}\right)^2}=\sqrt{1-\frac{441}{841}}=\frac{20}{29}$
$\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{21}{20}$
$\sec\theta+\tan\theta=\frac{29}{20}+\frac{21}{20}=\frac{50}{20}=\frac{5}{2}$

9. (a) Since A is in fourth quadrant. Sin A and tan A is negative
$\cos A=\frac{5}{13}$
$\sin A=-\sqrt{1-\frac{25}{169}}=-\sqrt{\frac{144}{169}}=\frac{-12}{13}$, $\tan A=\frac{-12}{5}$
$\Rightarrow\frac{13\sin A+5\sec A}{5\tan A+6\,\text{Cosec}A}=\frac{13\left(\frac{-12}{13}\right)+5\left(\frac{13}{5}\right)}{5\left(\frac{-12}{5}\right)+6\left(\frac{-13}{12}\right)}$
$=\frac{-12+13}{-12-\frac{13}{2}}=\frac{-2}{37}$

10. (a) $\frac{4}{3}\cot^2 30°+3\sin^2 60°-2\,\text{Cosec}^2 60°-\frac{3}{4}\tan^2 30°$
$=\frac{4}{3}(\sqrt{3})^2+3\left(\frac{\sqrt{3}}{2}\right)^2-2\left(\frac{2}{\sqrt{3}}\right)^2-\frac{3}{4}\left(\frac{1}{\sqrt{3}}\right)^2$
$=4+\frac{9}{4}-\frac{8}{3}-\frac{1}{4}=\frac{40}{12}=\frac{10}{3}$

11. (a) $\tan\theta=\frac{a}{b}$
$\frac{a\sin\theta-b\cos\theta}{a\sin\theta+b\cos\theta}$
Dividing numerator and denominator by Cos θ.
$\Rightarrow\frac{a\tan\theta-b}{a\tan\theta+b}=\frac{a\left(\frac{a}{b}\right)-b}{a\left(\frac{a}{b}\right)+b}=\frac{a^2-b^2}{a^2+b^2}$

12. (d) $A+B=45°$
$\Rightarrow\tan(A+B)=\tan 45°\Rightarrow\frac{\tan A+\tan B}{1-\tan A\tan B}=1$
$\Rightarrow\tan A+\tan B=1-\tan A\tan B$
$\Rightarrow\tan A+\tan B+\tan A\tan B=1$

13. (a) $\cos A=\frac{3}{5}$ $\quad\sin B=\frac{5}{13}$
$\sin A=\sqrt{1-\frac{9}{25}}=\frac{4}{5}$ $\quad\cos B=\sqrt{1-\frac{25}{169}}=\frac{12}{13}$
$\tan A=\frac{4}{3}$ $\quad\tan B=\frac{5}{12}$
$\frac{\tan A+\tan B}{1-\tan A\tan B}=\frac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}\times\frac{5}{12}}=\frac{63}{16}$

14. (b) $\cos 28°\cos 32°-\sin 28°\sin 32°$
$=\cos(28°+32°)=\cos 60°=\frac{1}{2}$

15. (a) $\cos^4\theta-\sin^4\theta=(\cos^2\theta-\sin^2\theta)(\cos^2\theta+\sin^2\theta)$
$=\cos 2\theta$

16. (a) $\tan 75=\tan(45+30)$
$=\frac{\tan 45+\tan 30}{1-\tan 45\tan 30}=\frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}=\frac{\sqrt{3}+1}{\sqrt{3}-1}$

17. (c) $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}+\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$
$=\sqrt{\frac{1+\cos\theta}{1-\cos\theta}\times\frac{1+\cos\theta}{1+\cos\theta}}\times\sqrt{\frac{1-\cos\theta}{1+\cos\theta}\times\frac{1-\cos\theta}{1-\cos\theta}}$
$=\frac{1+\cos\theta}{\sin\theta}+\frac{1-\cos\theta}{\sin\theta}=\frac{2}{\sin\theta}=2\,\text{Cosec}\theta$

18. (a) $\sin\theta=\frac{4}{5}$, $\cos\theta=\sqrt{1-\frac{16}{25}}=\frac{3}{5}$
$\sin 2\theta=2\sin\theta\cos\theta=2\times\frac{4}{5}\times\frac{3}{5}=\frac{24}{25}$

19. (c) $\frac{\sin 2A}{1+\cos 2A}=\frac{2\sin A\cos A}{2\cos^2 A}\ [\cos 2A=2\cos^2A-1]$
$=\frac{\sin A}{\cos A}=\tan A$

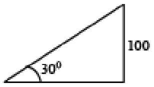
20. (b) $\tan 3\theta=\dfrac{3\tan\theta-\tan^3\theta}{\frac{1}{3}-3\tan^2\theta}$

$\Rightarrow\frac{1}{\cos 3\theta}=\dfrac{\frac{3}{\cot\theta}-\frac{1}{\cot^3\theta}}{1-\frac{3}{\cot^2\theta}}\Rightarrow\cot 3\theta=\frac{\cot^3\theta-3\cot\theta}{3\cot^2\theta-1}$

ANSWERS

EXPERIENCE THE PRATHAM EDGE 16: HEIGHTS & DISTANCES

1. (c): $\tan 30^0 = \frac{P}{B} = \frac{100}{B}$



$$B = \frac{100}{\tan 30^0} = \frac{100}{1/\sqrt{3}} = 100\sqrt{3} = 173\text{m}$$

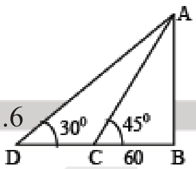
2. (a): $\tan 45^0 = AB/60 \quad \Rightarrow AB = 60$

$$\tan 30^0 = \frac{AB}{60 + CD} \Rightarrow 60 + CD = \frac{60}{1/\sqrt{3}}$$

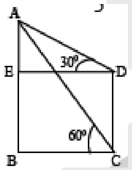
$CD = 60\,(\sqrt{3} - 1)$

Speed =

$$\frac{60\left(\sqrt{3}-1\right)}{5} \times \frac{18}{5} = 31.6$$



3. (c):

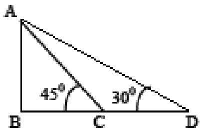


$$\tan 60^0 = \frac{15}{BC} \Rightarrow BC = \frac{15}{\sqrt{3}} = 5\sqrt{3}$$

$$\tan 30^0 = \frac{AE}{BC} \Rightarrow AE = 5\sqrt{3} \times \frac{1}{\sqrt{3}} = 5$$

$DC = AB - EA = 15 - 5 = 10$

4. (c):



$\tan 45^0 = AB/BC \quad \Rightarrow AB = BC$

$$\tan 30^0 = \frac{1}{\sqrt{3}} = \frac{AB}{BD} \Rightarrow AB = \frac{BD}{\sqrt{3}}$$

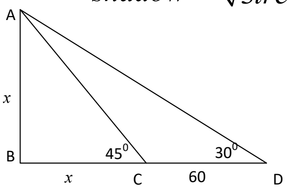
$CD = 12\text{min} = BD - BC = BC\,(\sqrt{3} - 1)$

$$BC = \frac{12}{\left(\sqrt{3}-1\right)} = 16 \text{ min } 23 \text{ sec}$$

5. (d): We need at least 1 distance once to know any reference distance

6. (a): $\tan\theta = \frac{tree}{shadow} = \frac{tree}{\sqrt{3}tree} = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^o$

7. (d):

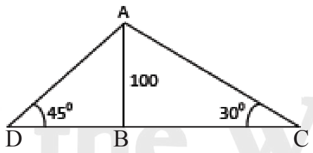


$$\tan 30^0 = \frac{x}{x+60} = \frac{1}{\sqrt{3}} \quad \Rightarrow x = \frac{60}{(\sqrt{3}-1)}$$

8. (c): $\tan 45^0 = AB/BD \quad \Rightarrow AB = BD = 100$

$$\tan 30^0 = \frac{AB}{BC} \Rightarrow BC = \frac{AB}{1/\sqrt{3}} = 100\sqrt{3}$$

$BD + BC = 100\,(\sqrt{3}+1)$

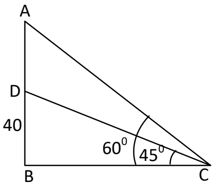


9. (d): $\tan 45^0 = BD/BC \quad \Rightarrow BD = BC = 40\text{m}$

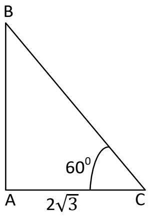
$\tan 60^0 = \sqrt{3} = AB/BC$;

$AB = \sqrt{3}\,BC = 40\sqrt{3}$

$= 40\sqrt{3} - 40 = 40\,(\sqrt{3} - 1)$



10. (d):



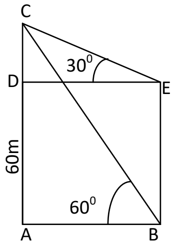
$AB = AC \tan 60^0 = 2\sqrt{3} \times \sqrt{3} = 6$

$$BC = \frac{AB}{\sin 60^0} = \frac{6}{\sqrt{3}/2} = 4\sqrt{3}$$

$AB + BC = 6 + 4\sqrt{3}$

ANSWERS

11. (c):



$$AB = DE = \frac{AC}{\tan 60^0} = \frac{60}{\sqrt{3}} = 20\sqrt{3}$$

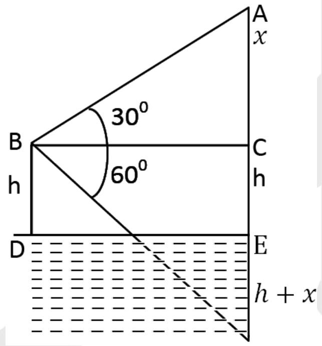
$$CD = DE \tan 30^0 = 20\sqrt{3} \times \frac{1}{\sqrt{3}} = 20$$

$$BE = AC - CD = 60 - 20 = 40$$

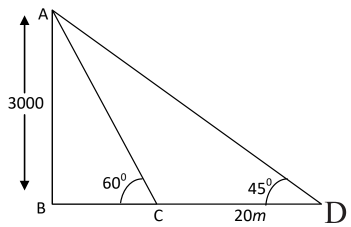
12. (d): $\tan 30 = \dfrac{x}{BC}$ $\quad \tan 60 = \dfrac{2h + x}{\sqrt{3}x}$

$BC = x\sqrt{3}$ $\quad \Rightarrow h = x$

$Height\ of\ cloud\ above\ sealand = h + x = 2h.$



13. (b):



$$\tan 60^0 = \frac{3000}{BC} \Rightarrow BC = \frac{3000}{\sqrt{3}}$$

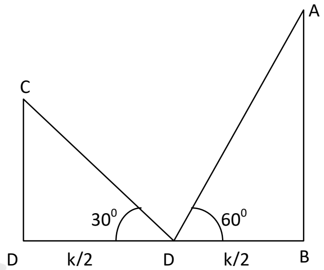
$$CD = 3000\left(1 - \frac{1}{\sqrt{3}}\right) = 1000\sqrt{3}(\sqrt{3} - 1)$$

$$Speed = \frac{CD}{5} = \frac{1000(3 - \sqrt{3})}{5} = 200\ (3 - \sqrt{3})$$

14. (d): $AB = k/2 \tan 60^0 = \sqrt{3}/2\ k$

$CD = k/2 \tan 30^0 = k/(2\sqrt{3})$

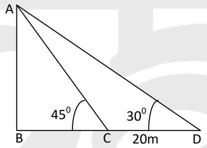
$$Ratio = \frac{BA}{CD} = \frac{\frac{\sqrt{3}\ k}{2}}{\frac{k}{2\sqrt{3}}} = 3$$



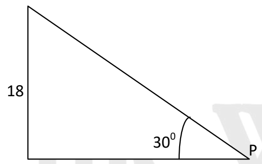
15. (c): AB = BC

BD = BC + CD = AB + 20 = AB

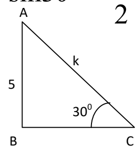
$$AB = \frac{20}{\sqrt{3} - 1} = 10\ (\sqrt{3}+1)$$



16. (d): Base = $18/\tan 30^0 = 18\sqrt{3}$



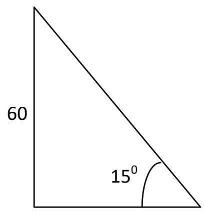
17. (b): $\sin 30^0 = \dfrac{1}{2} = \dfrac{5}{AC}$;



$\therefore$ AC = 10

18. (b): $\tan 15^0 = 60/$ Base

$$\Rightarrow \text{Base} = \frac{60}{\tan 15^0} = \frac{60(\sqrt{3} + 1)}{(\sqrt{3} - 1)}$$

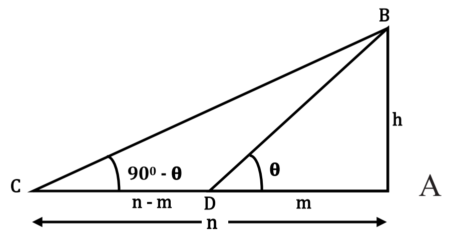


19. (a): From ΔABC,

$h/m = \tan\theta$ and

$h/n = \tan(90^0 - \theta) = \cot\theta$

ANSWERS



$\therefore \frac{h}{m} \times \frac{h}{n} = \tan\theta \times \cot\theta = 1 \Rightarrow h = \sqrt{mn}$

20. (c): Let h be the height of the man.

$\therefore \frac{6}{8} = \frac{h}{2.4} \Rightarrow h = \frac{3}{4}(2.4) = 1.8$ m

EXPERIENCE THE PRATHAM EDGE 17: PROGRESSIONS

1. (c): lowest no. = 108/6 = 18th term
   Highest no. = 744/6 = 124th term
   Total no. = 124 – 18 + 1 = 107
2. (b): 10 + 11 + 12 ………..+ 99 = 90/2 (99 + 10)
   = 45 × 109 = 4905
3. (a): 4k + 2 – 3k = 3k – (k + 1)
   k + 2 = 2k – 1 $\Rightarrow$ k = 3
4. (b): a + 4d = 5 $\Rightarrow$ d = - 9/4
   a + 12d = - 13 $\Rightarrow$ a = 14
5. (a): 5 + 13 + 21 + ……. + 181

   $(n-1) = \frac{181-5}{13-5} = \frac{176}{8} = 22 \Rightarrow n = 23$

   Sum = 23/2 (5 + 181) = 93× 23 = 2139
6. (c): 101 + 103 + ……. 199

   $n - 1 = \frac{199-101}{2} = 49 \Rightarrow n = 50$

   Sum = 50/2 (101 + 199) = 7500
7. (a): mean = $\frac{14+18}{2} = 16$
8. (c): $108 = \frac{n}{2}[2\times3 + (n-1)3]$

   $\Rightarrow 216 = (n + n^2)3$
   $\Rightarrow n^2 + n - 72 = 0$; $n = -9, n = 8$
9. (b): 8th term = $2 \times 3^7 = 4374$
10. (a): $T_6 = 2^6$
11. (c): $b^2 = ac$; $a^x = b^y = c^z$
    2 log b = log a + log c;
    x log a = y log b = z log c = k
    log a = k/x; log b = k/y; log c = k/z

    $\frac{2k}{y} = \frac{k}{x} + \frac{k}{z}$

    $\therefore \frac{2}{y} = \frac{1}{x} + \frac{1}{z}$
12. (a): $b^2 = ac$
    2 log b = log a + log c
13. (a): a = 1/3 $ar^3 = 9$
    $r^3 = 27$ $\Rightarrow$ r = 3

    $y_2 = ar^2 = \frac{1}{3}\times 3^2 = 3$
14. (b): AM $\geq$ GM
15. (b): $G^2 = ab$
16. (b): 12 = a + 5d $\Rightarrow$ d = 5
    22 = a + 7d a = - 13
    T_2 = a + d = - 13 + 5 = - 8
17. (a): $T_3 = ar^2 = 12$
    $T_5 = ar^4 = 48$ $\Rightarrow$ a= 3, r = ±2
    T_2 = ar = 3 × 2 = ±6
18. (a): a + b = 2 × 34 = 68
    ab = $(16)^2$ = 256 $\Rightarrow$ a= 64; b = 4
19. (d): 1st term = 102; last term = 498
    n – 1 = (498 – 102) / 6 = 396/6 = 66
    $\therefore$ n = 67
    Sum = 67/2 (102 + 498) = 67 × 300 = 20100
20. (c): Let the terms be
    a – 3d, a – d, a + d, a + 3d
    Sum = 4a = 40 $\Rightarrow$ a = 10
    (a – 3d) (a + 3d) = 64 = $a^2 - 9d^2$ $\Rightarrow$ d = 2
    $\therefore$ The terms are 4, 8, 12, and 16

ANSWERS

21. (a): $n^2 + 2n$ is 2nd degree polynomial which is formula in sum of AP

22. (b): $x = ar^3$
$y = ar^9$
$z = ar^{15}$
$y^2 = a^2r^{18} = (ar^3)(ar^{15}) = xz$

23. (c):
$$\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+\ldots\ldots\ldots n \text{ terms}$$
$$= (1-\frac{1}{2}) + (1-\frac{1}{4}) + (1-\frac{1}{8}) + \ldots n \text{ terms}$$
$$= (1 + 1 + 1 \ldots\ldots + n \text{ term})$$
$$- (\frac{1}{2}+\frac{1}{4}+ \ldots\ldots n \text{ terms})$$
$$= n - \frac{\frac{1}{2}\left[1-\left(\frac{1}{2}\right)^n\right]}{1-(\frac{1}{2})} = n-1+\left(\frac{1}{2}\right)^n$$
$$= 2^{-n} + n - 1$$

24. (d): $S = 1 + a + a^2 + a^2 \ldots\ldots\ldots\ldots\ldots\infty$
$$\frac{1}{1-a} \Rightarrow 1-a = \frac{1}{S} \Rightarrow a = \frac{S-1}{S}$$

25. (a): $4^3 + 5^3 + 6^3 + \ldots\ldots 10^3$
$1^3 + 2^3 + 3^3 + 4^3 \ldots\ldots + 10^3 - (1^3 + 2^3 + 3^3)$
$$= \left(\frac{10(10+1)}{2}\right)^2 - \left(\frac{3(3+1)}{2}\right)^2$$
$= 3025 - 36 = 2989$

EXPERIENCE THE PRATHAM EDGE 18: PERMUTATIONS & COMBINATIONS

1. (b): Positions = 5, Objects = 5, Any no. of lines
5 flags = 5! = 120
4 flags = $^5C_4 . 4!$ = 120
3 flags = $^5C_3 . 3!$ = 60
2 flags = $^5C_2 . 2!$ = 20
1 flags = $^5C_1 . 1!$ = 5
Therefore,
Total signals = 120+120+60+20+5 = 325

2. (d): positions = 6 objects = 6
Combination = $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

3. (a): 1st digit = 1, 2, 3 …. 9 = 9
2nd digit = 0, 1, 2, 3 …9 = 10–1= 9 (no repeats)
3rd digit = 0, 2 … 9 = 10 – 2 = 8 (no repeats)
Combination = $9 \times 9 \times 8 = 648$

4. (d): 1b & 3g; 2b & 2g; 3b & 1g; 4b
$^6C_1 \times {}^4C_3 + {}^6C_2 \times {}^4C_2 + {}^6C_3 \times {}^4C_1 + {}^6C_4$
= 24 + 90 + 80 + 15 = 209

5. (b): P<EN>CIL
Objects $\Rightarrow$ 5 combination = 5! = 120

6. (a): $\frac{(3+4+5)!}{3!4!5!} = \frac{12\times11\times10\times9\times8\times7\times6}{3\times2\times4\times3\times2}$
= 27, 720

7. (d): last digit (2, 4, 6, 8, 0)
Digit can be of form 57_ _
$\Rightarrow$ (0-9) & (2, 4, 6, 8, 0) $= 10 \times 5 = 50$
Digits can be of form _ 57_
$\Rightarrow$ (1-9) & (0, 2, 4, 6, 8) $= 9 \times 5 = 45$
Total = 95

8. (a): ARRANGE letters=7 $\Rightarrow$2 repeats
$$P = \frac{7!}{2!2!} = \frac{5040}{4} = 1260$$

9. (a): $6! \times 5! = 720 \times 120 = 86400$

10.(b):

AA <GIN> 3 X 2 = 6 4 X 3 X 2 = 24 6 + 24 = 30	GA <AIN> 3 X 2 = 6 (4 X 3 X 2)/2 =12 6 + 12 = 18	N A A G I 49^{th} N A A I G 50^{th}

11.(b): $\frac{4!}{2!2!} \times \frac{3!}{2!} = 2\times3\times3 = 18$

12.(d): $\frac{4!}{2} = 12$

13.(c): $5! \times 3! = 720$
4 brothers & 1 sister group
Group can be arranged in 3! ways

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14.(b): 2 are compulsory, rest P = $^7C_3 = \frac{7\times6\times5}{3\times2} = 35$

15.(b): 5 friends can be invited or not; hence 2 choices
$= 2^5 - 1 = 31$

16.(c): Doesn't borrow both math book
$= {}^6C_3 = \frac{6\times5\times4}{3\times2} = 20$
Does borrow both math book = $^6C_1 = 6$
Borrows only math I and not Math II
$= {}^6C_2 = (6\times5)/2 = 15$
Total = 26 + 15 = 41

17.(a): If all join = $^{22}C_7 = \frac{22\times21\times20\times19\times18\times17\times16}{7\times6\times5\times4\times3\times2}$
= 170544
None join = $^{22}C_{10}$ = 646646
Total = 817190

18.(b): Total no. of lines between 23 vertices
= (23×22)/2 = 253
Out of these, 23 are sides of the polygon
∴ Total difference = 253 – 23 = 230

19.(a): 2 girls are fixed; remaining 14 boys(b) & 8 girls(g)
Choices are 6b, 2g = $^{14}C_6 \times {}^8C_2$ = 84084
5b, 3g = $^{14}C_5 \times {}^8C_3$ = 112112
4b, 4g = $^{14}C_4 \times {}^8C_4$ = 70070
Total = 266266

20.(b): $2^6 - 1 = 63$

21.(b): 4 digit numbers, 1st digit can't be 1;
4 × 4 × 3 × 2 = 96
5 digits = 5! = 120
Total = 216

22.(d): choosing 3 prog for 1st day = $^6C_3$ = 20
Arranging for day 1 = 3 × 2 × 1 = 6
Arranging for day 2 = 3 × 2 = 6
Total = 20 × 6 ×6 = 720

23.(c): first digit = 3 last digit = 5
No of combinations = 4 × 3 = 12

24.(b): A & B From a group; rest can be arranged in 4!
Group can be arranged in 2! ways.
Total = 4! × 2! = 48

25.(d): each team would play with 6 teams
Total matches = (7×6)/2 = 21

26.(c): Vowels = EAI = 3,
this can be arranged in 6 ways
Consonant + group of vowels = 5! = 120
Total = 120 × 6 = 720 ways

27.(a): Vowels, 2 = 2!
Consonant + group of vowels
= 4! = 24
Total = 24 × 2 = 48

28.(c): Combination 1b, 1r, 1w = 4 × 3 × 2 = 24
1b, 2r = 3 × (4×3)/2 = 18
1b, 2w = 3 × 1 = 3
2b, 1r = 3 × 4 = 12
2b, 1w = 3 × 2 = 6
3b = 1
Total = 64

29.(d): combinations 3m & 2w = $^7C_3 \times {}^6C_2$
= 35 ×15 = 525
4m & 1w = $^7C_4 \times {}^6C_1$ = 35 × 6 = 210
5m = $^7C_5$ = 21
=756

30. (c): $^7C_2 \times {}^6C_2$ = 315

EXPERIENCE THE PRATHAM EDGE 19: PROBABILITY

1. (a): Total balls = 5+7 = 12
Probability of drawing 2 balls = $^{12}C_2$ = 66
Probability of drawing 2 red balls = $^5C_2$ = 10
Required probability = 10/66 = 5/33

2. (a): $\frac{knave\ \&\ queen}{^{52}C_2} = \frac{^4C_1 \times {}^4C_1}{^{52}C_2}$
$\frac{4\times4\times2}{52\times51} = \frac{8}{663}$

3. (d): Required probability = $^7C_2 / {}^{10}C_2$ = 7/15

4. (a): $\frac{failed\ both}{total} = \frac{100-(60+50-30)}{100}$
$\Rightarrow \frac{20}{100} = \frac{1}{5}$

5. (a): Required probability = $^4C_3 / {}^9C_3$ = 1/21

6. (b): $\frac{1}{5\times4} = \frac{1}{20}$

7. (b): P(A) = 1/3, P(B) = 1/2,
P(A ∩ B) = 1/3

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$P(A^c \cap B^c) = 1 - P(A) - P(B) + P(A \cap B)$

$$= 1 - \frac{1}{3} - \frac{1}{2} + \frac{1}{3} = \frac{1}{2}$$

8. (a): $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Max $P(A \cup B) = 1$
Max $P(B) = P(A \cup B) - P(A) + P(A \cap B)$
$= 1 - 0.5 + 0.3 = 0.8$
Hence, $P(B) = 0.9$ is not possible

9. (c): In favour P(A) = 5/8 is favour
P(B) = 7/12
At least 2 will happen
= P(A') P(B) + P(B') P(A) + P(A) P(B)

$$= \frac{3}{8} \times \frac{7}{12} + \frac{5}{8} \times \frac{5}{12} + \frac{5}{8} \times \frac{7}{12} = \frac{81}{96}$$

10.(d): Probability if 1 head and 7 tails

$$= {}^{8}C_{1}\left(\frac{1}{2}\right)^{8} = 8\left(\frac{1}{2}\right)^{8} = \frac{1}{32}$$

11.(b): $\dfrac{6\ girls\ together}{all\ combinations} = \dfrac{6!7!}{12!} = \dfrac{1}{132}$

12.(a): $\dfrac{exactly\ 2\ women}{all\ combinations} = \dfrac{{}^{4}C_{2} \times {}^{7}C_{4}}{{}^{11}C_{6}} = \dfrac{5}{11}$

13. (a):

$$\frac{2\ red\ balls}{all\ combinations} = \frac{2red(1green + 1blue)}{{}^{12}C_{3}}$$

$$= \frac{{}^{5}C_{2}[4+3]}{220} = \frac{70}{220} = \frac{7}{22}$$

14.(a): At least 2 =

$$\frac{1}{3} \times \frac{1}{4} \times \frac{4}{5} + \frac{1}{3} \times \frac{3}{4} \times \frac{1}{5} + \frac{2}{3} \times \frac{1}{4} \times \frac{1}{5} + \frac{1}{3} \times \frac{1}{4} \times \frac{1}{5}$$

$$= \frac{4+3+2+1}{60} = \frac{1}{6}$$

15.(a): one of them will win =

$$\frac{1}{6} + \frac{1}{10} + \frac{1}{8} = \frac{20+12+15}{120} = \frac{47}{120}$$

16.(a): $\dfrac{1g\ \&\ 2b}{3chosen} = \dfrac{{}^{10}C_{1} \times {}^{15}C_{2}}{{}^{25}C_{3}}$

$$= \left(\frac{10 \times 15 \times 14}{2}\right)\left(\frac{3 \times 2}{25 \times 24 \times 23}\right) = \frac{21}{46}$$

17.(b): Probability of getting a queen of club or a king of heart = $\dfrac{2}{52} = \dfrac{1}{26}$

18.(a): $\dfrac{red\ or\ white}{all\ combination} = \dfrac{7+4}{{}^{20}C_{1}} = \dfrac{11}{20}$

19.(a): $\dfrac{THT + HTH}{all\ combination} = \dfrac{2}{2^{3}} = \dfrac{1}{4}$

20.(a): $\dfrac{No\ heads}{(all\ combinations)} \Rightarrow \dfrac{1}{2^{7}} = \dfrac{1}{128}$

21.(c): $\dfrac{(3,6),(4,5),(5,4)(6,3)}{6 \times 6} = \dfrac{4}{36} = \dfrac{1}{9}$

22.(d): $\dfrac{{}^{4}C_{2}}{{}^{52}C_{2}} = \dfrac{4 \times 3}{52 \times 51} = \dfrac{1}{221}$

23.(c): $\dfrac{all\ red}{all\ combinations} = \dfrac{{}^{5}C_{3}}{{}^{15}C_{3}}$

$$= \left(\frac{5 \times 4}{2}\right) \times \left(\frac{3 \times 2}{15 \times 14 \times 13}\right) = \frac{2}{91}$$

24.(c): P(A) = Probability of A speaking truth = 0.75
P(B) = Probability of B speaking truch = 0.80
Probability of A & B contradicting
P(A)P(B')+P(A')P(B)
= 3/4 * 1/5 + 1/4 * 4/5 = 7/20
Required % = 7/20 * 100
= 35%

25.(b): There are 20 unstamped ordinary envelopes
⇒There are 30 stamped ordinary envelopes
⇒There are 18 stamped airmail envelopes.
So the required probability
= (30-18)/80 = 12/80
⇒(12/18) × 100 = 15%