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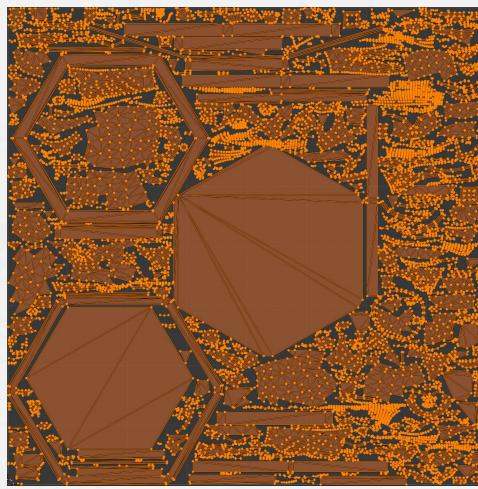
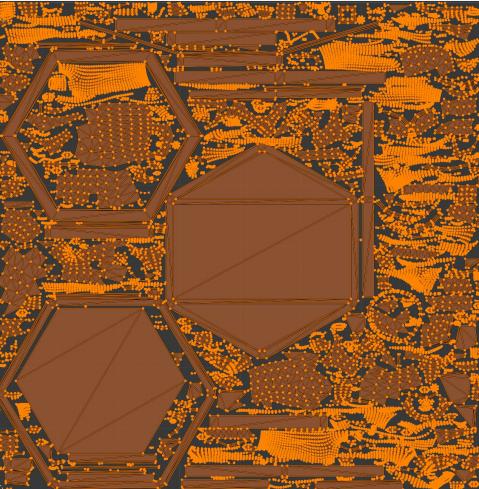
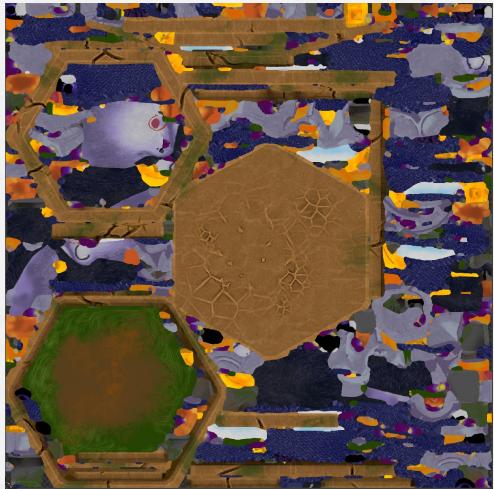
04 不保纹理的模型减面

05 总结



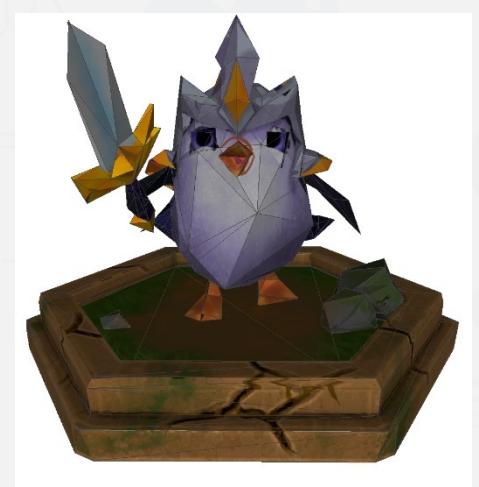
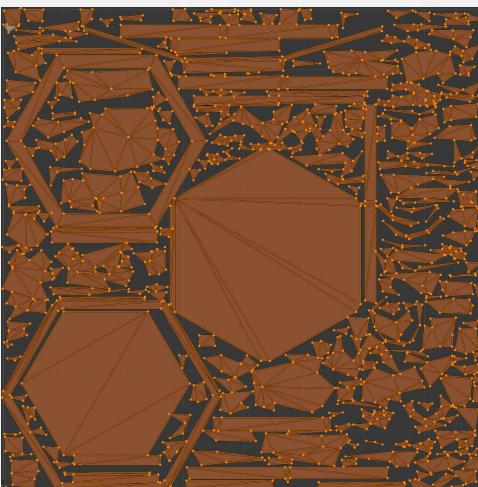
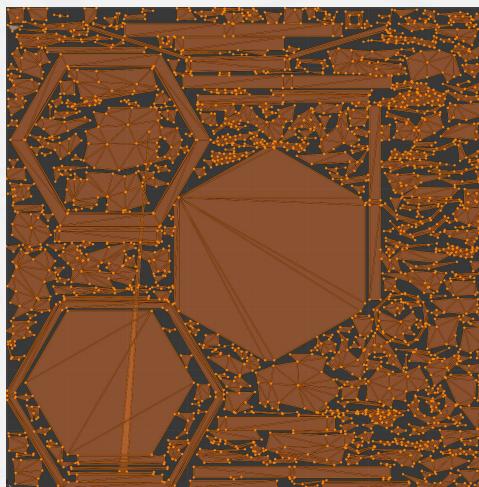
保纹理的模型减面

保纹理的减面算法



面数 : 18163

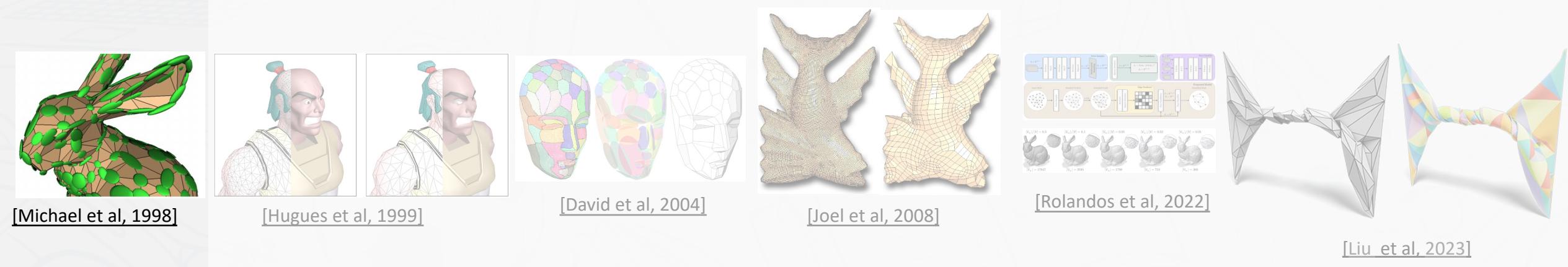
9081



1815

908

保纹理的减面算法



- [Michael et al, 1998] M. Garland and P. S. Heckbert, "Simplifying surfaces with color and texture using quadric error metrics," Proceedings Visualization '98, 1998.
- [Hugues et al, 1999] Hugues Hoppe, "New quadric metric for simplifying meshes with appearance attributes," In Proceedings of the conference on Visualization '99: celebrating ten years (VIS '99), 1999.
- [David et al, 2004] David Cohen-Steiner, Pierre Alliez, and Mathieu Desbrun, "Variational shape approximation," ACM Trans. Graph. (August 2004), 2004.
- [Rolandos et al, 2022] R. A. Potamias, S. Ploumpis and S. Zafeiriou, "Neural Mesh Simplification," 2022 IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR), 2022.
- [Liu et al, 2023] Derek Liu, Mark Gillespie, Benjamin Chislett, Nicholas Sharp, Alec Jacobson, and Keenan Crane, "Surface Simplification using Intrinsic Error Metrics", ACM Trans. Graph. 2023.

问题描述

- 输入
 - 2D/3D 网格，UV 网格，纹理材质
 - 减面数/比例
- 输出
 - 新的2D/3D 网格和UV 网格
- 要求
 - 保拓扑
 - 无自相交
 - 保距离
 - ...

简化框架



- 简化操作单元误差计算
- 构建操作单元的优先队列
- 当队列不为空并且未满足停止条件，执行
 - 取出队列的第一个操作单元
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 - 如果没有通过，重复以上步骤
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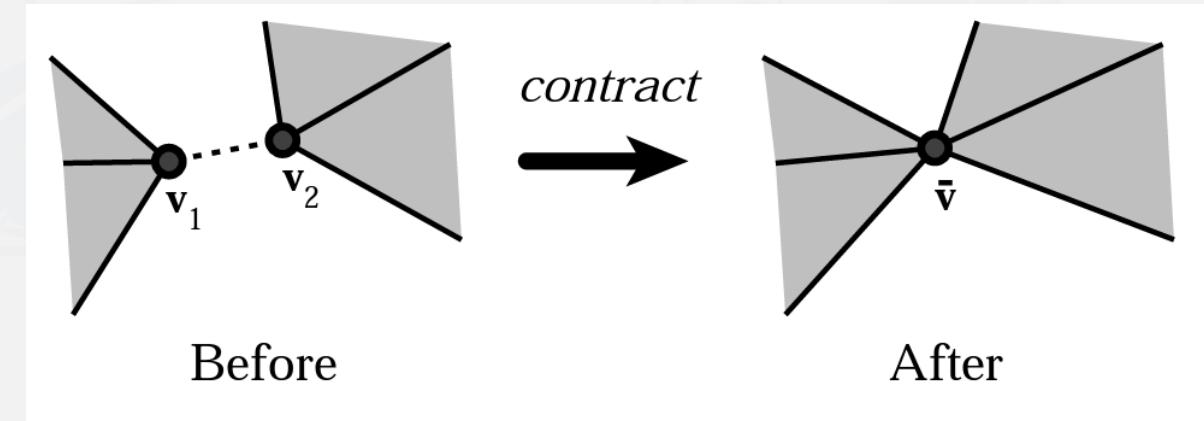
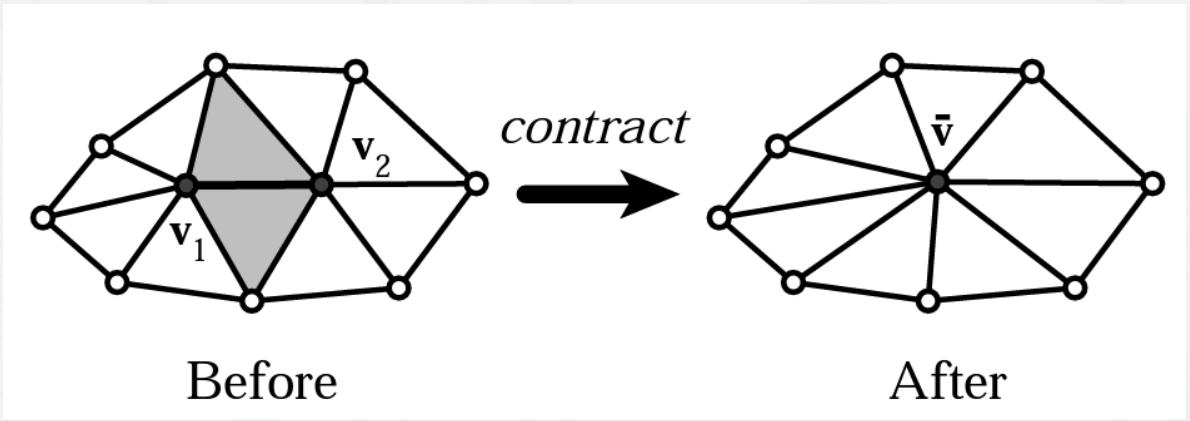
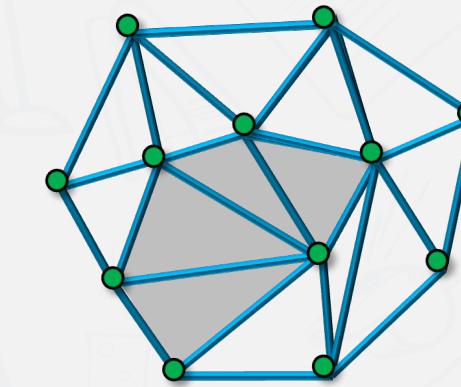
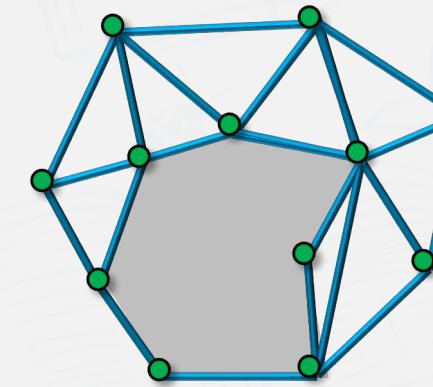
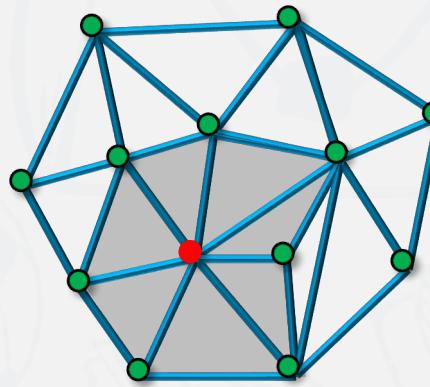
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简化操作单元



移除点



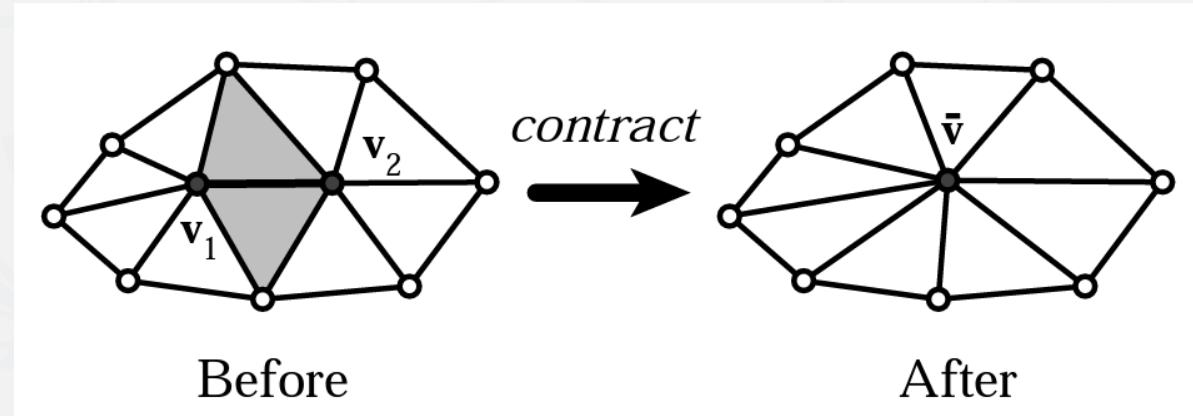
移除边

移除邻近点

简化框架的核心



- 简化操作单元的排序
- 新的顶点的位置



移除边

简化框架

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误差计算

- 几何误差
 - Hausdorff 距离
 - Quadratic Error Metric
- 视觉误差

误差计算

- 几何误差
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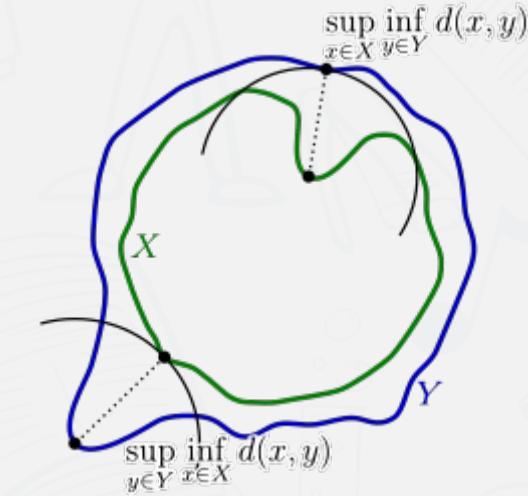
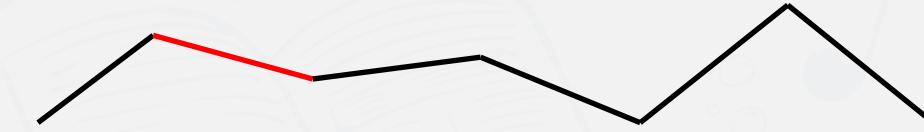


Image : Wikipedia

采样法计算实现：<https://github.com/cnr-isti-vclab/vcglib/tree/main/apps/metro>

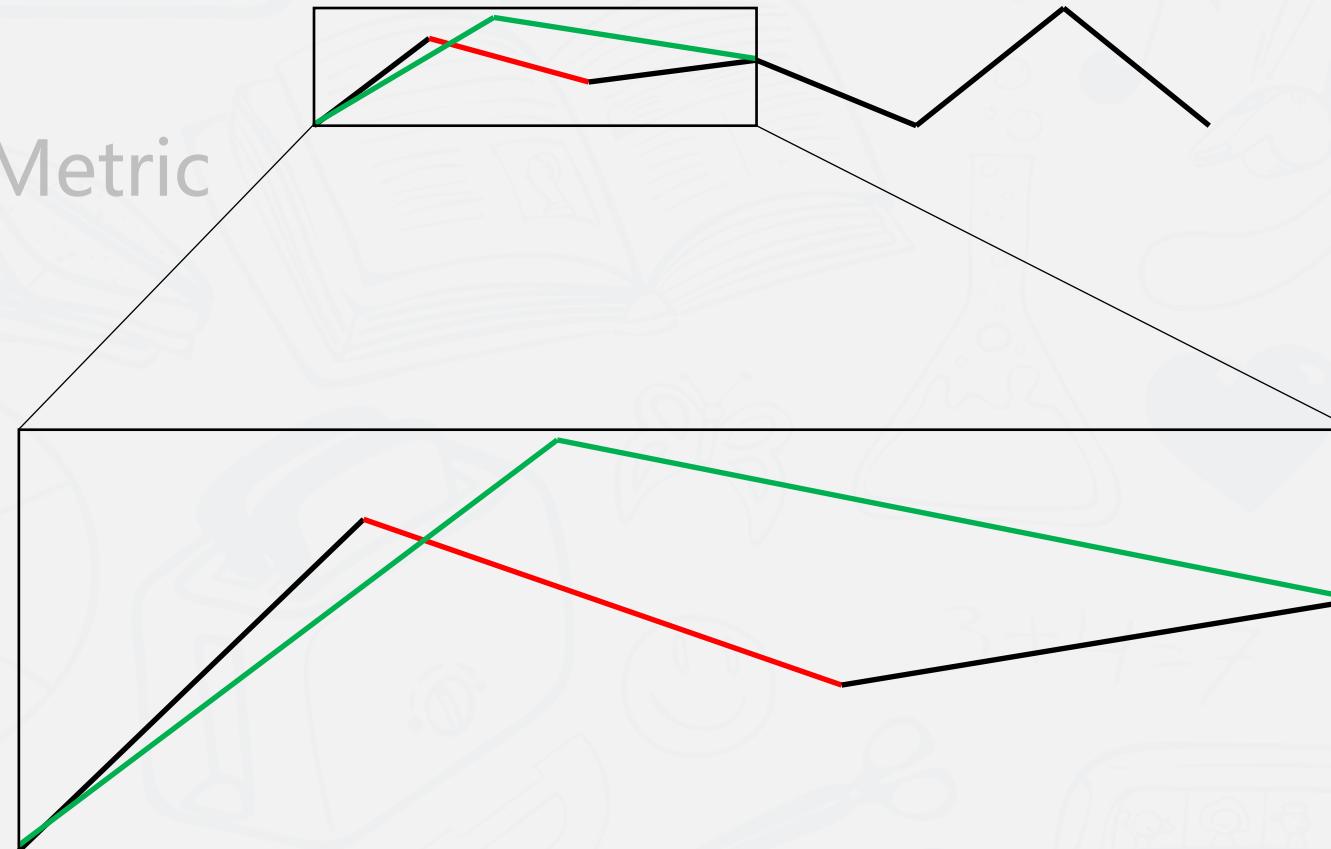
误差计算

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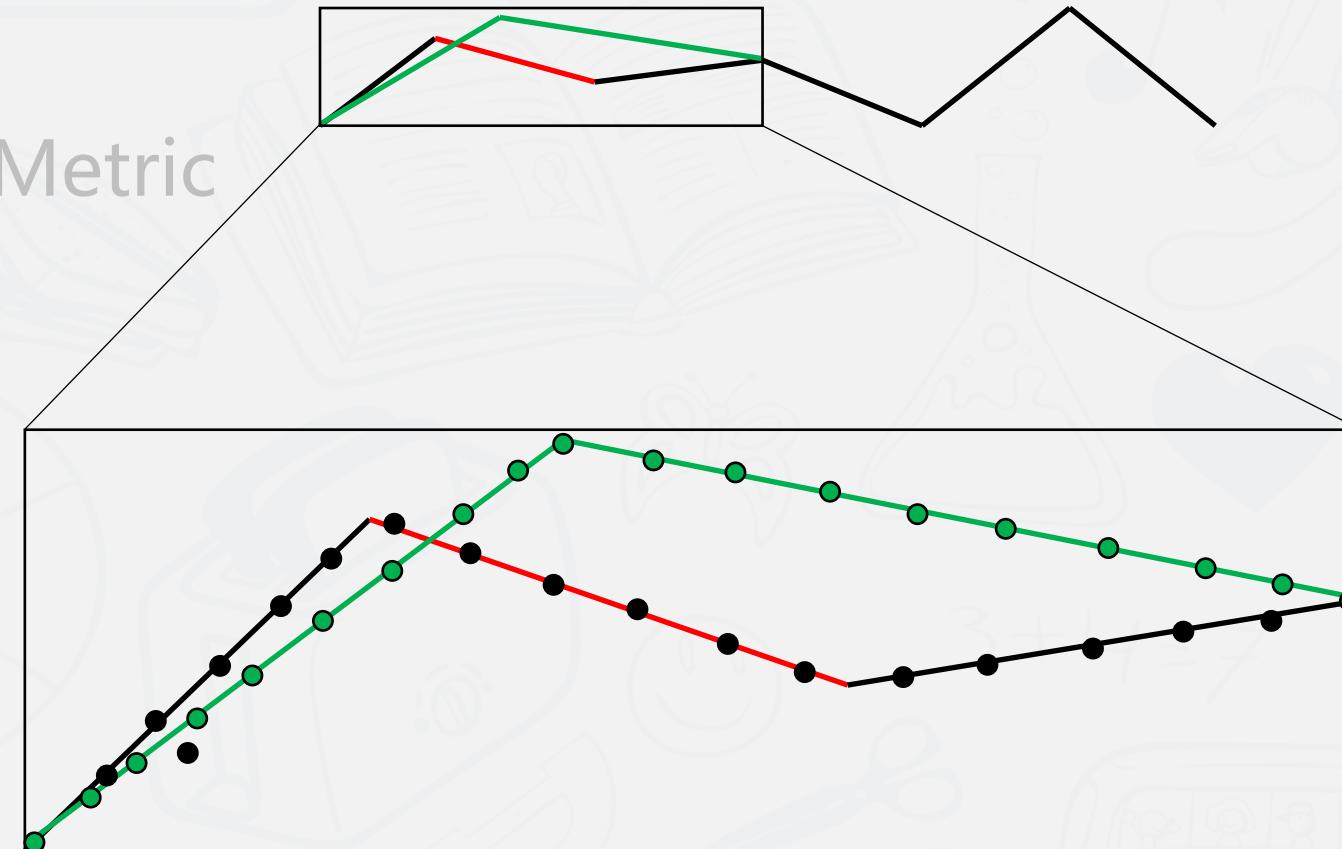
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 - Hausdorff 距离
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误差计算

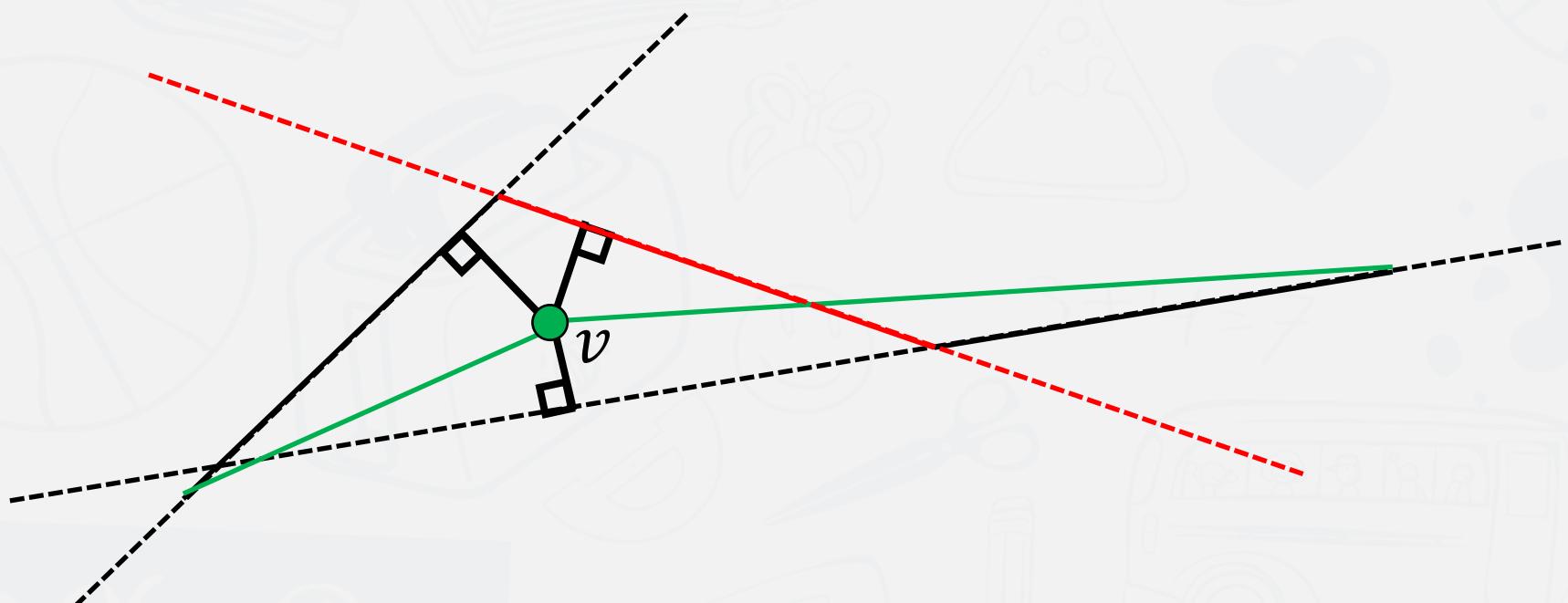
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 - Hausdorff 距离
 - Quadratic Error Metric
- 视觉误差



误差计算

- 几何误差
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$$\Delta = \sum_p Dist(v, p)^2$$



误差计算

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- 视觉误差

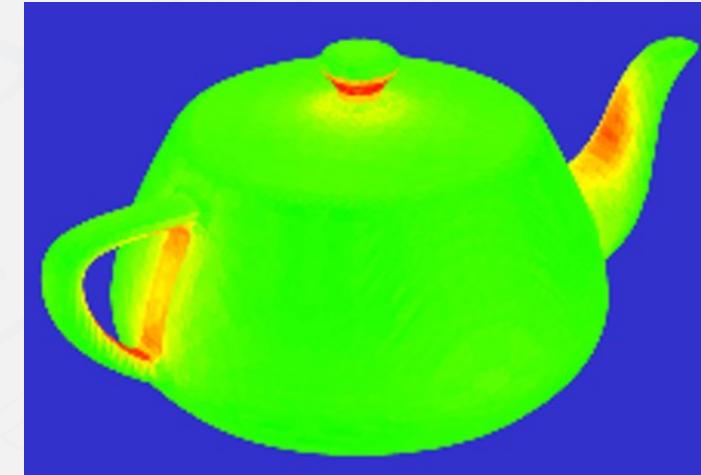
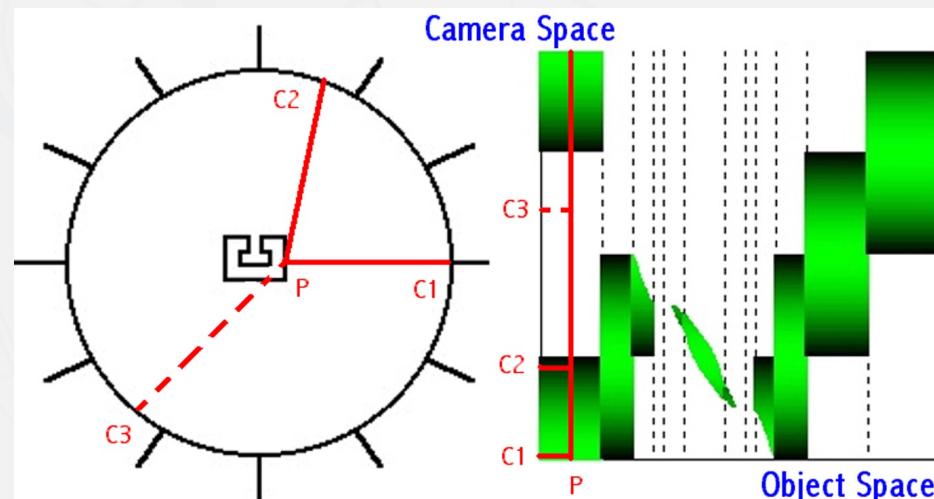


Image : Eugene Zhang and Greg Turk. 2002. Visibility-guided simplification. IEEE Visualization '02 (VIS '02).

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Quadric Error Metric (QEM)



平面公式 $ax + by + cz + d = 0,$ $a^2 + b^2 + c^2 = 1,$

平面系数 $\mathbf{p} = [a, b, c, d],$

点的坐标 $\mathbf{v} = [x, y, z, 1],$

点在平面上 $\mathbf{v}\mathbf{p}^T = 0$

Quadric Error Metric (QEM)



平面公式 $ax + by + cz + d = 0,$ $a^2 + b^2 + c^2 = 1,$

平面系数 $\mathbf{p} = [a, b, c, d],$

点的坐标 $\mathbf{v} = [x, y, z, 1],$

点在平面上 $\mathbf{v}\mathbf{p}^T = 0$

QEM $\Delta = \sum_{\mathbf{p}} (\mathbf{v}\mathbf{p}^T)^2 = \sum_{\mathbf{p}} \mathbf{v}\mathbf{p}^T \mathbf{p}\mathbf{v}^T$

$$= \mathbf{v} \left(\sum_{\mathbf{p}} \mathbf{p}^T \mathbf{p} \right) \mathbf{v}^T = \mathbf{v} \mathbf{Q} \mathbf{v}^T$$

Quadric Error Metric (QEM)



$$Q = \begin{bmatrix} a^2 & ab & ac & ad \\ ab & b^2 & bc & bd \\ ac & bc & c^2 & cd \\ ad & bd & cd & d^2 \end{bmatrix} = \begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{12} & q_{22} & q_{23} & q_{24} \\ q_{13} & q_{23} & q_{33} & q_{34} \\ q_{14} & q_{24} & q_{34} & q_{44} \end{bmatrix}$$

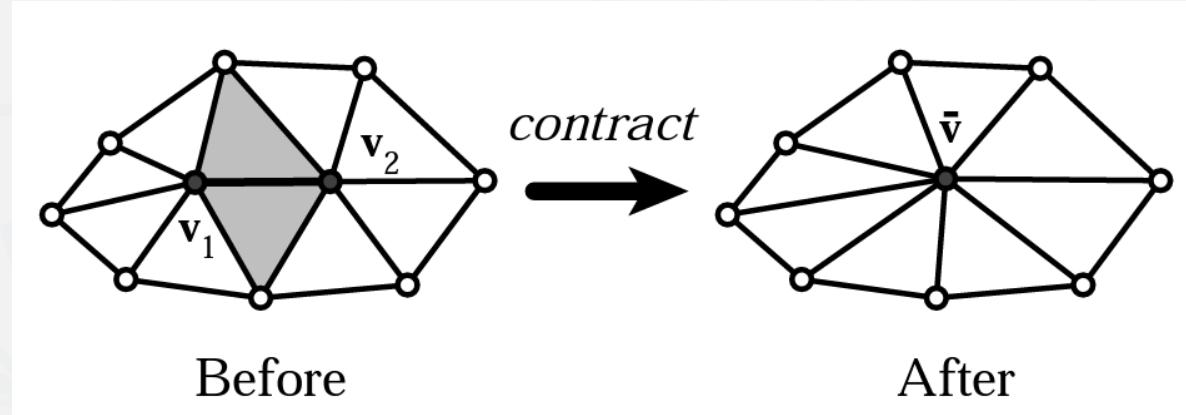
$$\begin{aligned}\Delta &= v Q v^T \\ &= q_{11}x^2 + 2q_{12}xy + 2q_{13}xz + 2q_{14}x + q_{22}y^2 \\ &\quad + 2q_{23}yz + 2q_{24}y + q_{33}z^2 + 2q_{34}z + q_{44}\end{aligned}$$

用QEM进行移除边操作



$$\bar{Q} = Q_1 + Q_2$$

$$\Delta = \bar{v} \bar{Q} \bar{v}^T$$



移除边

$$\min \bar{v} \bar{Q} \bar{v}^T : \quad \frac{\partial \Delta}{\partial x} = \frac{\partial \Delta}{\partial y} = \frac{\partial \Delta}{\partial z} = 0$$

用QEM进行移除边操作



$$\begin{aligned}\Delta(v) &= v Q v^T \\&= q_{11}x^2 + 2q_{12}xy + 2q_{13}xz + 2q_{14}x + q_{22}y^2 + 2q_{23}yz \\&\quad + 2q_{24}y + q_{33}z^2 + 2q_{34}z + q_{44}\end{aligned}$$

$$\frac{\partial \Delta}{\partial x} = \frac{\partial \Delta}{\partial y} = \frac{\partial \Delta}{\partial z} = 0$$

$$2q_{11}x + 2q_{12}y + 2q_{13}z + 2q_{14} = 0$$

$$2q_{12}x + 2q_{22}y + 2q_{23}z + 2q_{24} = 0$$

$$2q_{13}x + 2q_{23}y + 2q_{33}z + 2q_{34} = 0$$

用QEM进行移除边操作



$$2q_{11}x + 2q_{12}y + 2q_{13}z + 2q_{14} = 0$$

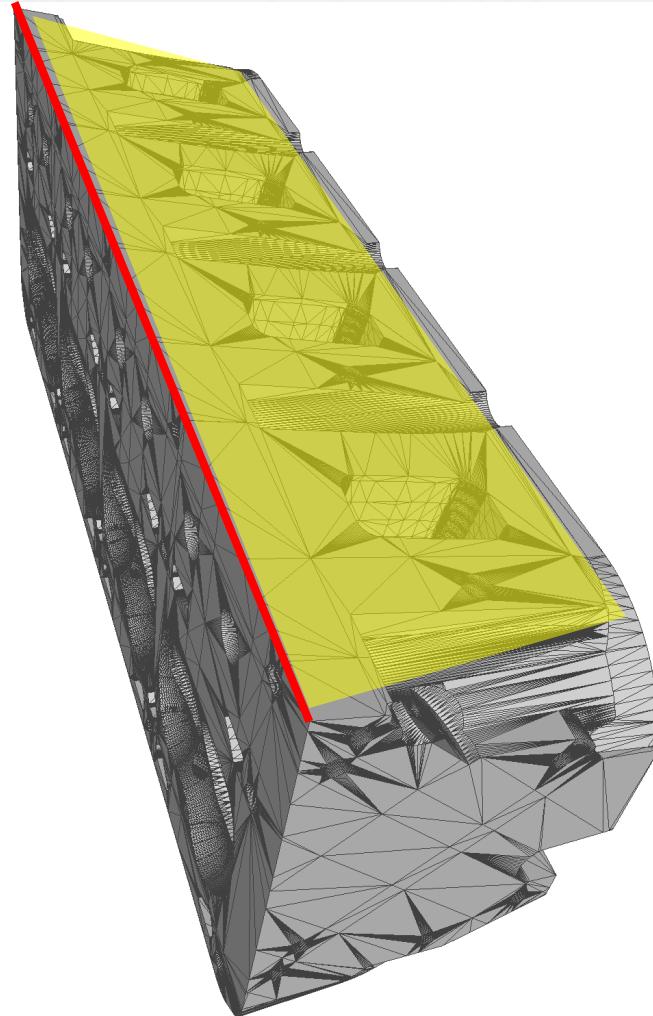
$$2q_{12}x + 2q_{22}y + 2q_{23}z + 2q_{24} = 0$$

$$2q_{13}x + 2q_{23}y + 2q_{33}z + 2q_{34} = 0$$

$$\begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{12} & q_{22} & q_{23} & q_{24} \\ q_{13} & q_{23} & q_{33} & q_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Quadric Error Metric (QEM)

- 三种解空间
 - 点
 - 矩阵可逆
 - 线
 - 面



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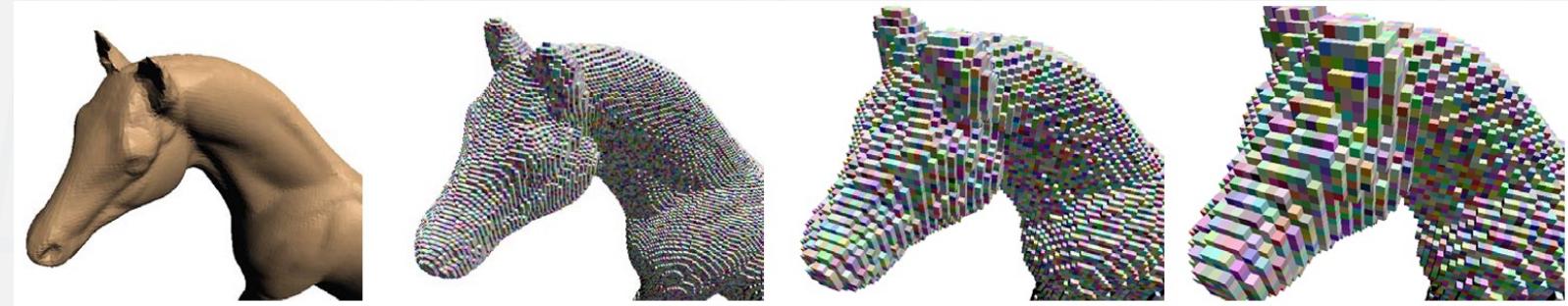
有效性检查



- 距离
 - Hausdorff 距离
- 拓扑
 - Link Condition
- 法向
 - 新表面法向与旧表面法向的点乘
- 穿模
 - 自相交检查
- 数值鲁棒性
 - Bounding-sphere

有效性检查

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Steve Zelinka and Michael Garland. 2002. Permission grids: practical, error-bounded simplification. ACM TOG.

有效性检查



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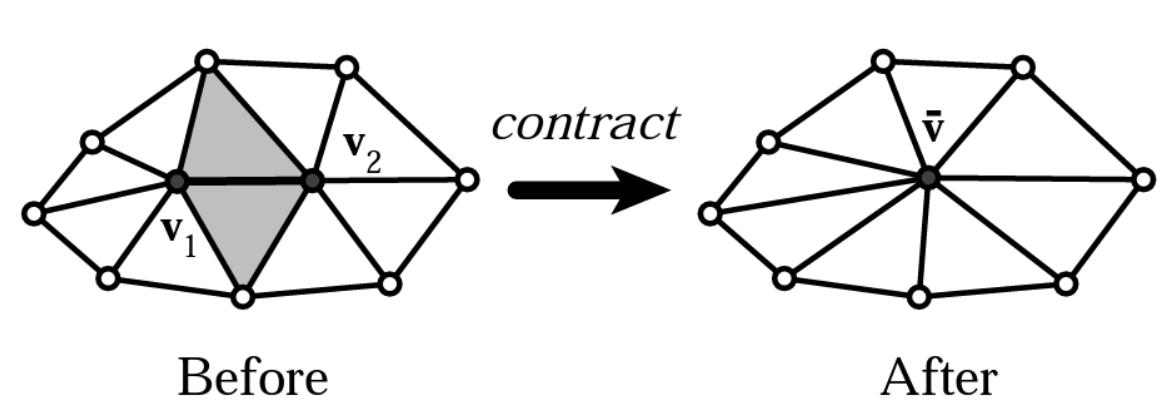
欧拉 — 庞克来公式

$$v - e + f = 2s - 2g - h$$

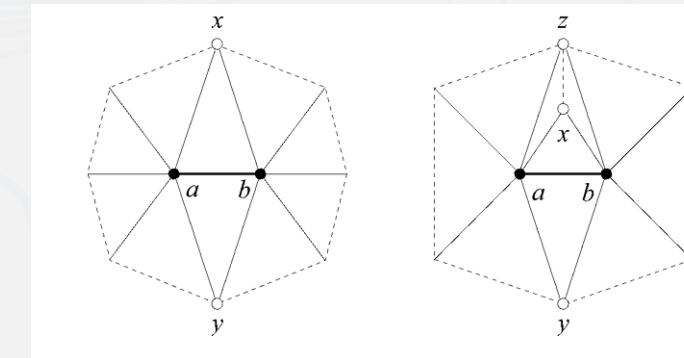
顶点数 边数 面数 连通数 亏格 洞数

有效性检查

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$$\text{link}(v_1) \cap \text{link}(v_2) = \text{link}(v_1v_2)$$

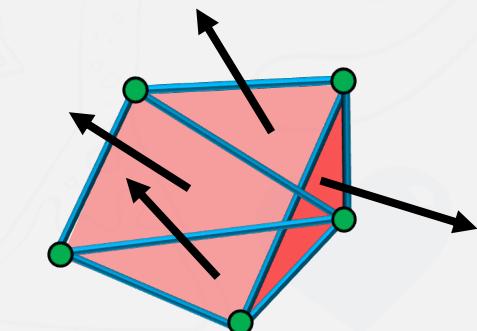
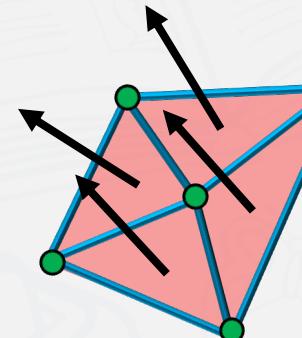


$$\text{link}(a) \cap \text{link}(b) \neq \text{link}(ab)$$

有效性检查



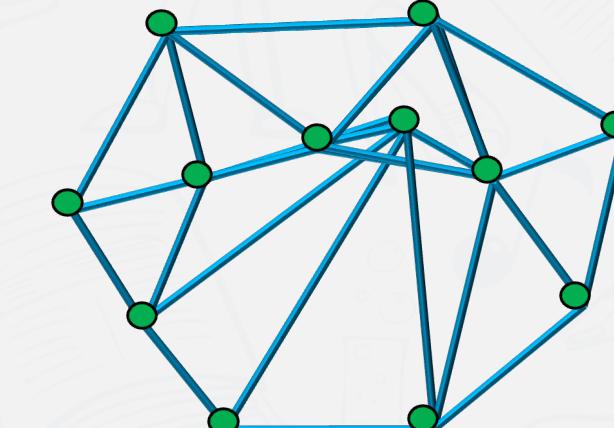
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有效性检查



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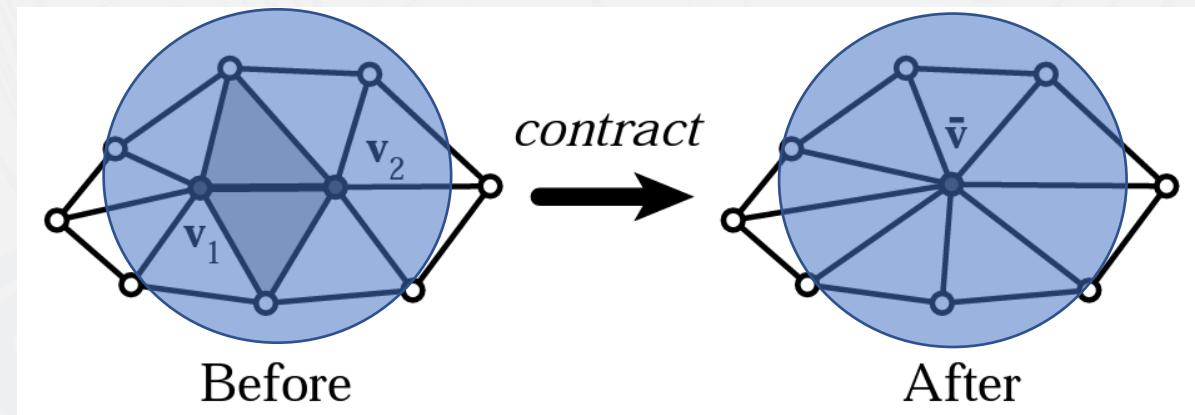
邻近相交



非邻近相交

有效性检查

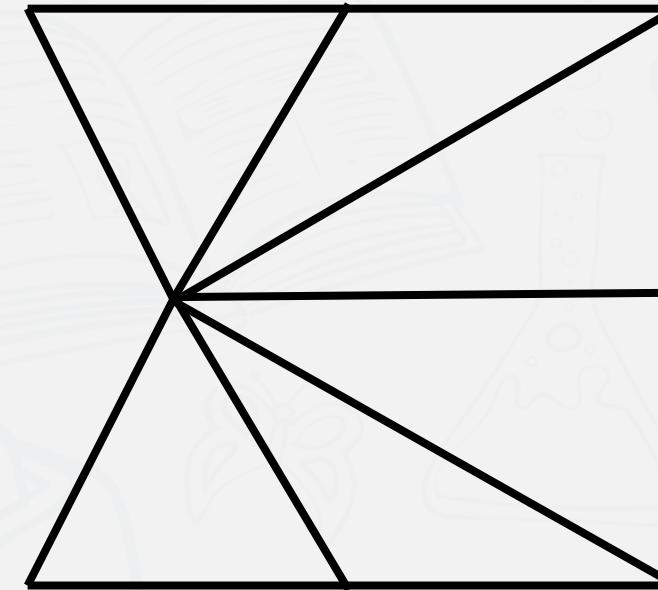
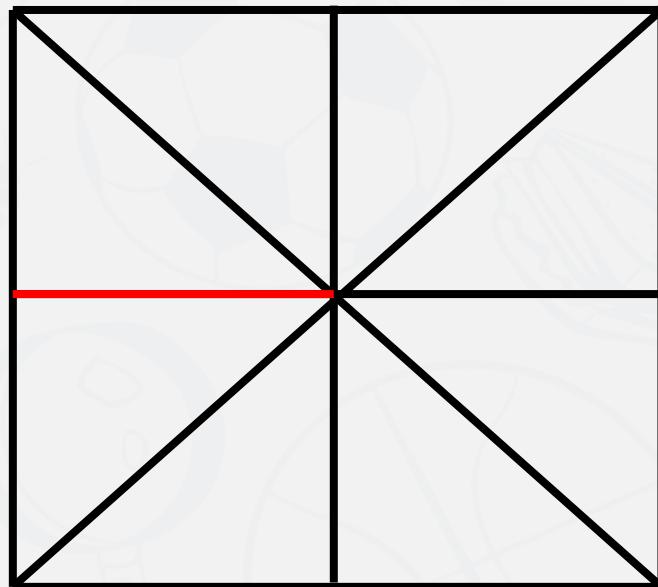
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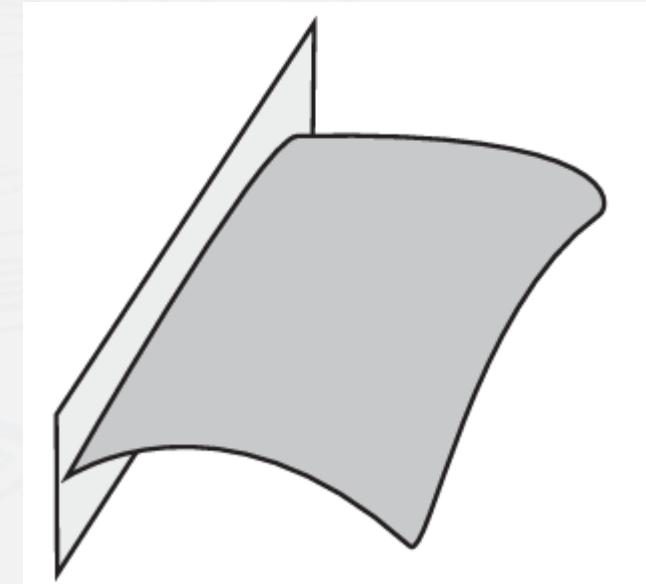
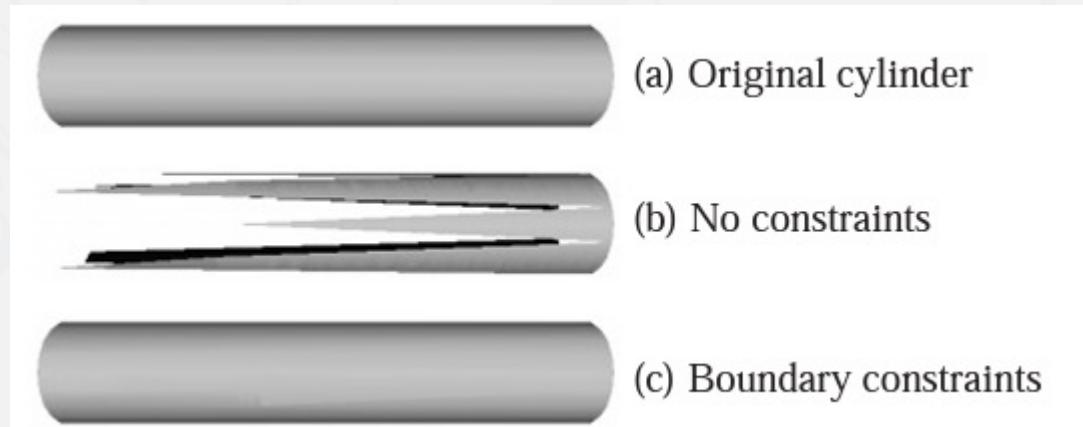
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保持模型边界

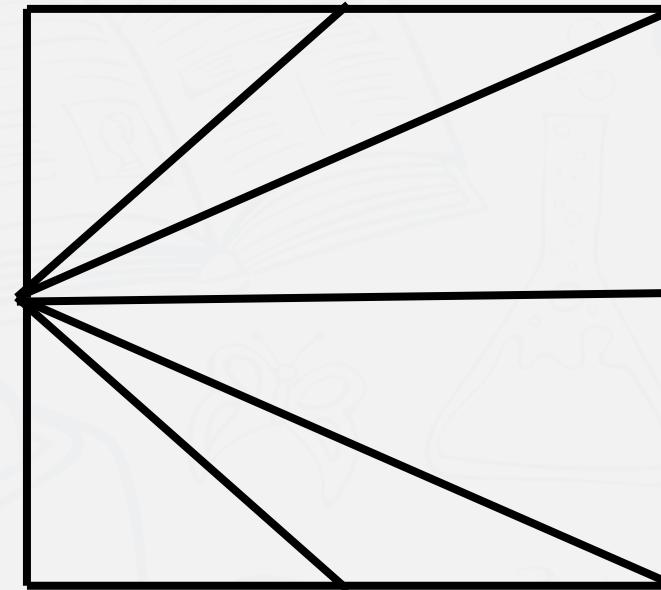
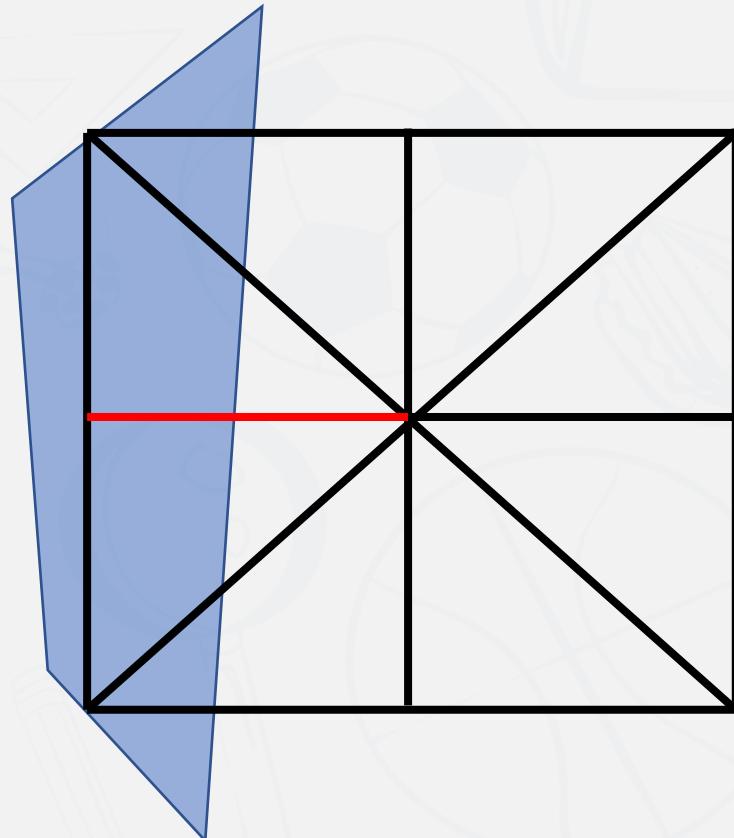


保持模型边界

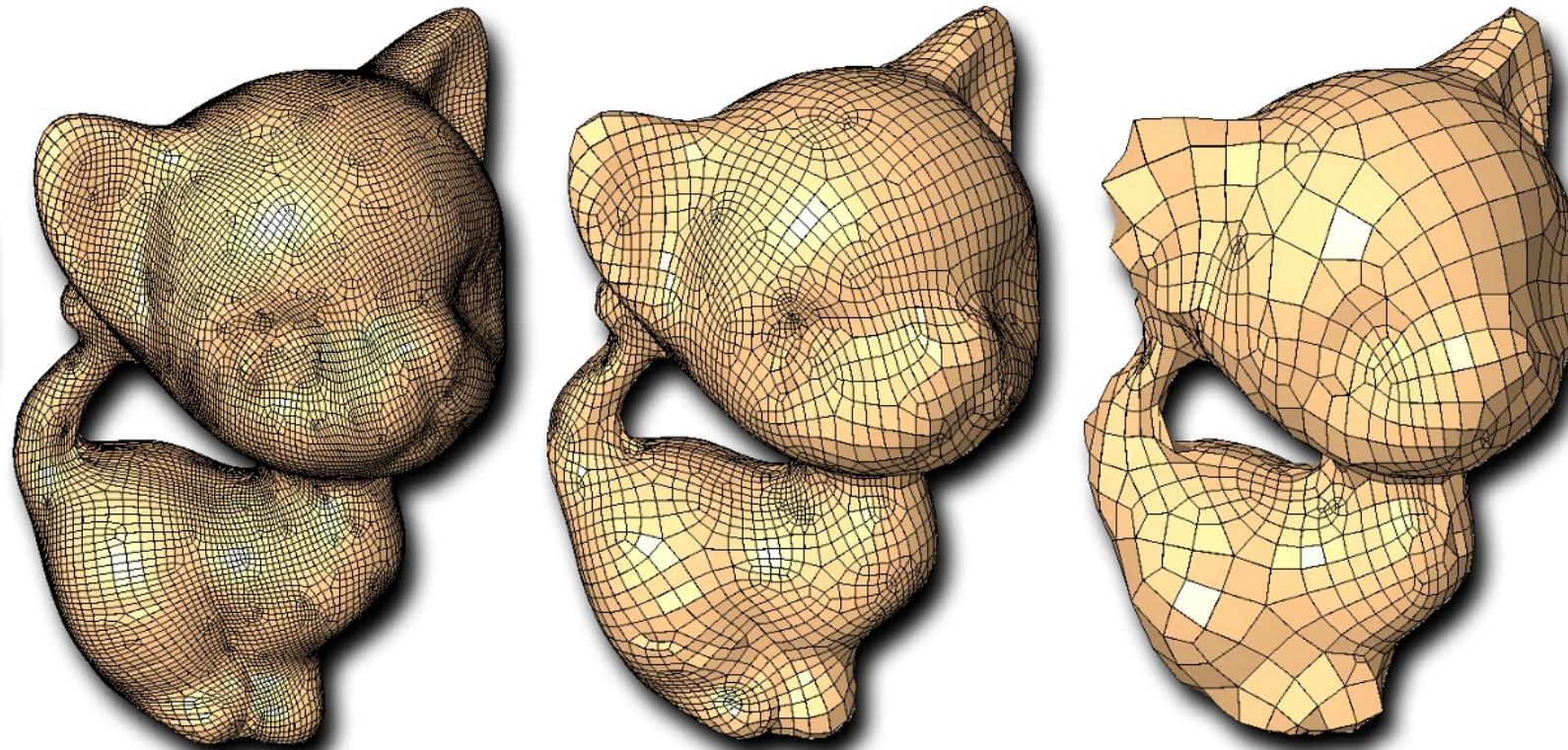


M. Garland and P. S. Heckbert, "Simplifying surfaces with color and texture using quadric error metrics," Proceedings Visualization '98.

保持模型边界

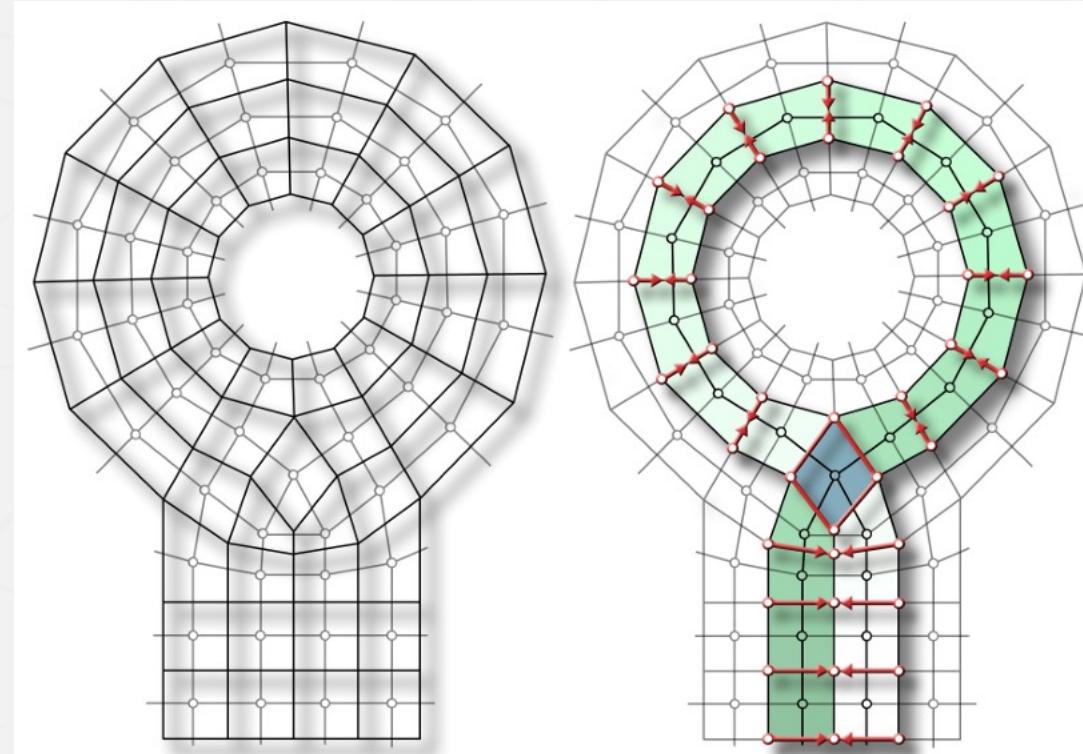


四边形网格的简化



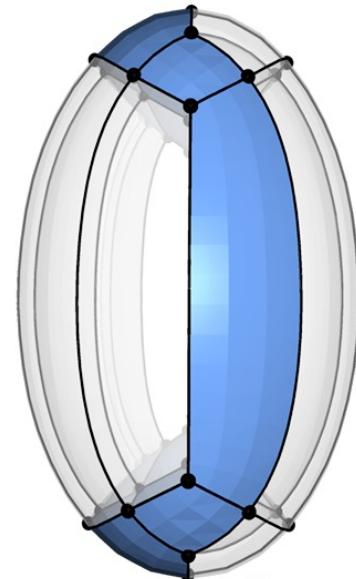
Joel Daniels, Cláudio T. Silva, Jason Shepherd, and Elaine Cohen. 2008. Quadrilateral mesh simplification. In ACM SIGGRAPH Asia 2008 papers (SIGGRAPH Asia '08)

四边形网格的简化



Joel Daniels, Cláudio T. Silva, Jason Shepherd, and Elaine Cohen. 2008. Quadrilateral mesh simplification. In ACM SIGGRAPH Asia 2008 papers (SIGGRAPH Asia '08)

六面体网格的简化



Base complex sheet

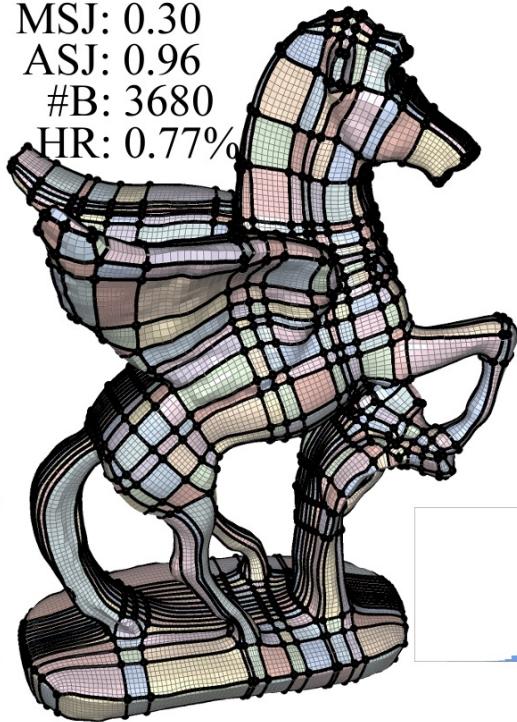


Base complex chord



Xifeng Gao, Daniele Panozzo, Wenping Wang, Zhigang Deng, Guoning Chen, Robust Structure Simplification for Hex Re-meshing, ACM Transactions on Graphics (SIGGRAPH ASIA, 2017)

六面体网格的简化



Xifeng Gao, Daniele Panozzo, Wenping Wang, Zhigang Deng, Guoning Chen, Robust Structure Simplification for Hex Re-meshing, ACM Transactions on Graphics (SIGGRAPH ASIA, 2017)

其它属性

- 顶点属性
 - 位置 (x, y, z)
 - 颜色 (r, g, b)
 - 法向 (m, n, o)
 - UV (u, v)
 - 其它



M. Garland and P. S. Heckbert, "Simplifying surfaces with color and texture using quadric error metrics," Proceedings Visualization '98.