中国科学技术大学
University of Science and Technology of China

GAMES 102在线课程

几何建模与处理基础

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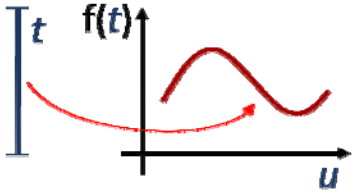
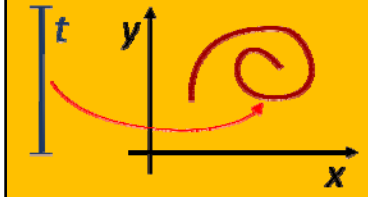
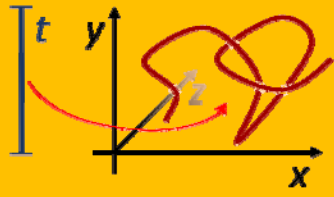
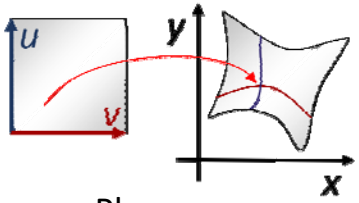
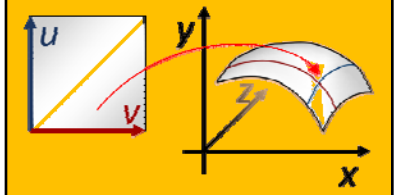
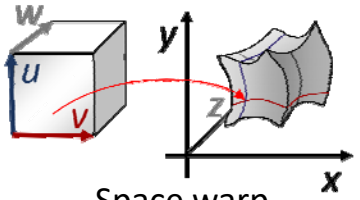
GAMES 102在线课程：几何建模与处理基础

离散曲线处理

回顾： R^2和R^3中的曲线/曲面

	Output: 1D	Output: 2D	Output: 3D
Input: 1D	 Function graph	 Plane curve	 Space curve
Input: 2D		 Plane warp	 Surface
Input: 3D			 Space warp

映射的维数

$$f: X \to Y$$

$$f: R^1 \to R^1$$
$$y = f(x)$$

一元函数

$$f: R^m \to R^1$$
$$y = f(x_1, x_2, \cdots, x_m)$$

多元函数

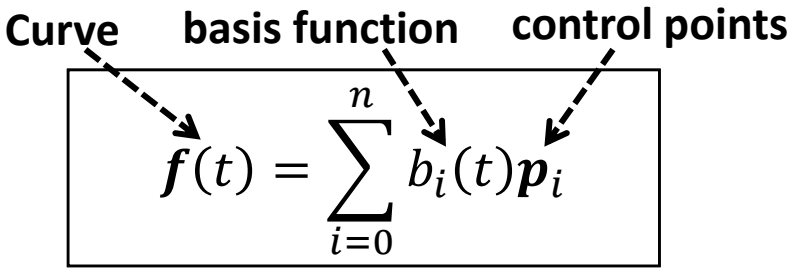
$$f: R^1 \to R^n$$
$$y_1 = f_1(x)$$
$$y_2 = f_2(x)$$
$$\vdots$$
$$y_n = f_n(x)$$

高维（单参数）曲线

$$f: R^m \to R^n$$
$$y_1 = f_1(x_1, x_2, \cdots, x_m)$$
$$y_2 = f_2(x_1, x_2, \cdots, x_m)$$
$$\vdots$$
$$y_n = f_n(x_1, x_2, \cdots, x_m)$$

高维曲面($m < n$)/降维映射($m > n$)

Curve Modeling in R^2 （建模/造型）



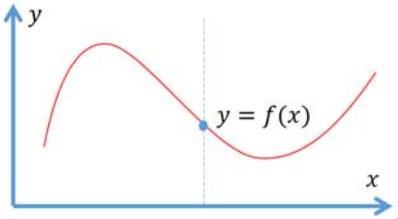
- Fitting (Reconstruction) for reverse engineering (interpolation, approximation, aggression…)
  - 从代数观点：需要函数空间表达能力足够
  - 输入：采样点$\{\boldsymbol{s}_j, j = 0{\sim}m\}$及基函数$\{b_i(t), i = 0{\sim}n\}$
  - 输出：拟合函数的系数$\{\boldsymbol{p}_i, i = 0{\sim}n\}$
- Design for interactive modeling
  - 从几何观点：具有好性质的基函数使得交互设计更直观
  - 输入：交互输入（或者反求）控制顶点$\{\boldsymbol{p}_i, i = 0{\sim}n\}$
  - 输出：曲线$\boldsymbol{f}(t)$

曲线（形状）的不同表达方法

优劣比较?

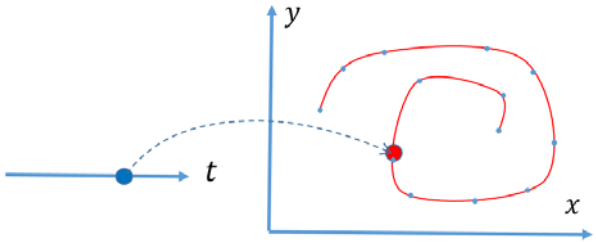
- 显式函数曲线

$$f: R^1 \to R^1$$
$$y = f(x)$$



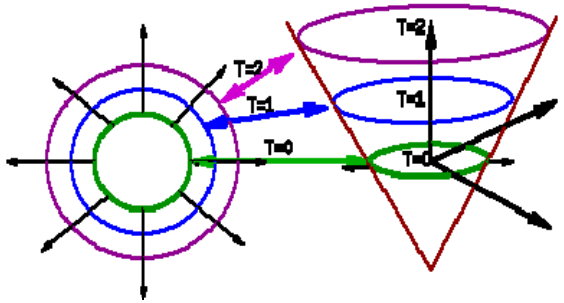
- 参数曲线

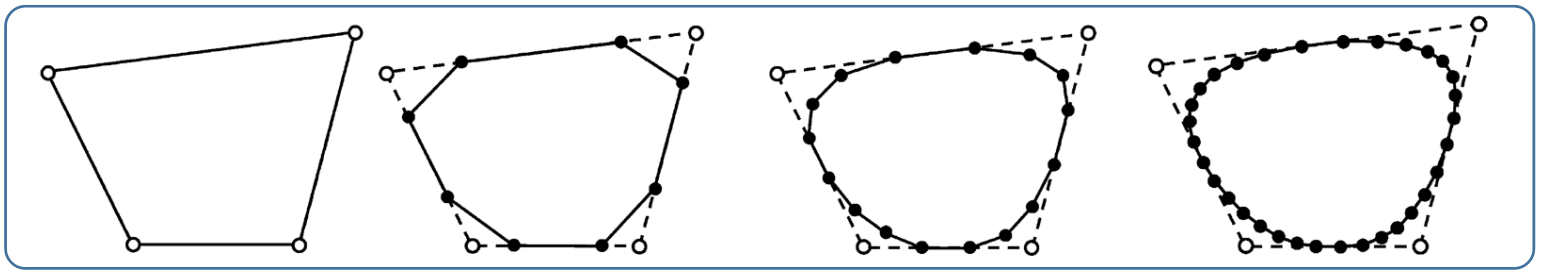
$$\boldsymbol{p}: R^1 \to R^2$$
$$x = x(t)$$
$$y = y(t)$$



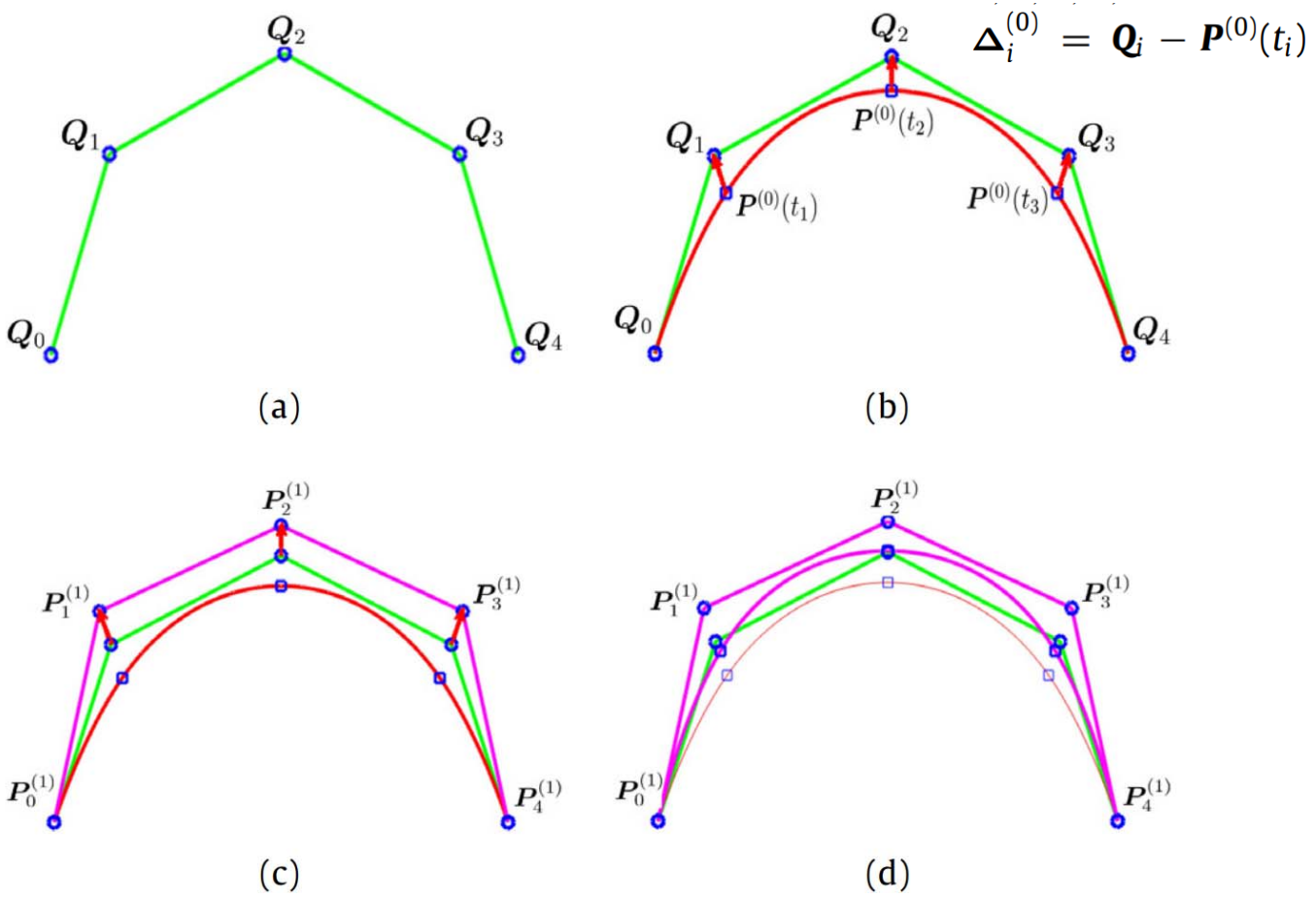
- 隐式曲线
  - Level set (水平集)
- 细分曲线

$$f(x, y) = 0$$





几何迭代法（渐进迭代逼近）
(progressive-iterative approximation, PIA)



齐东旭、de Boor、蔺宏伟

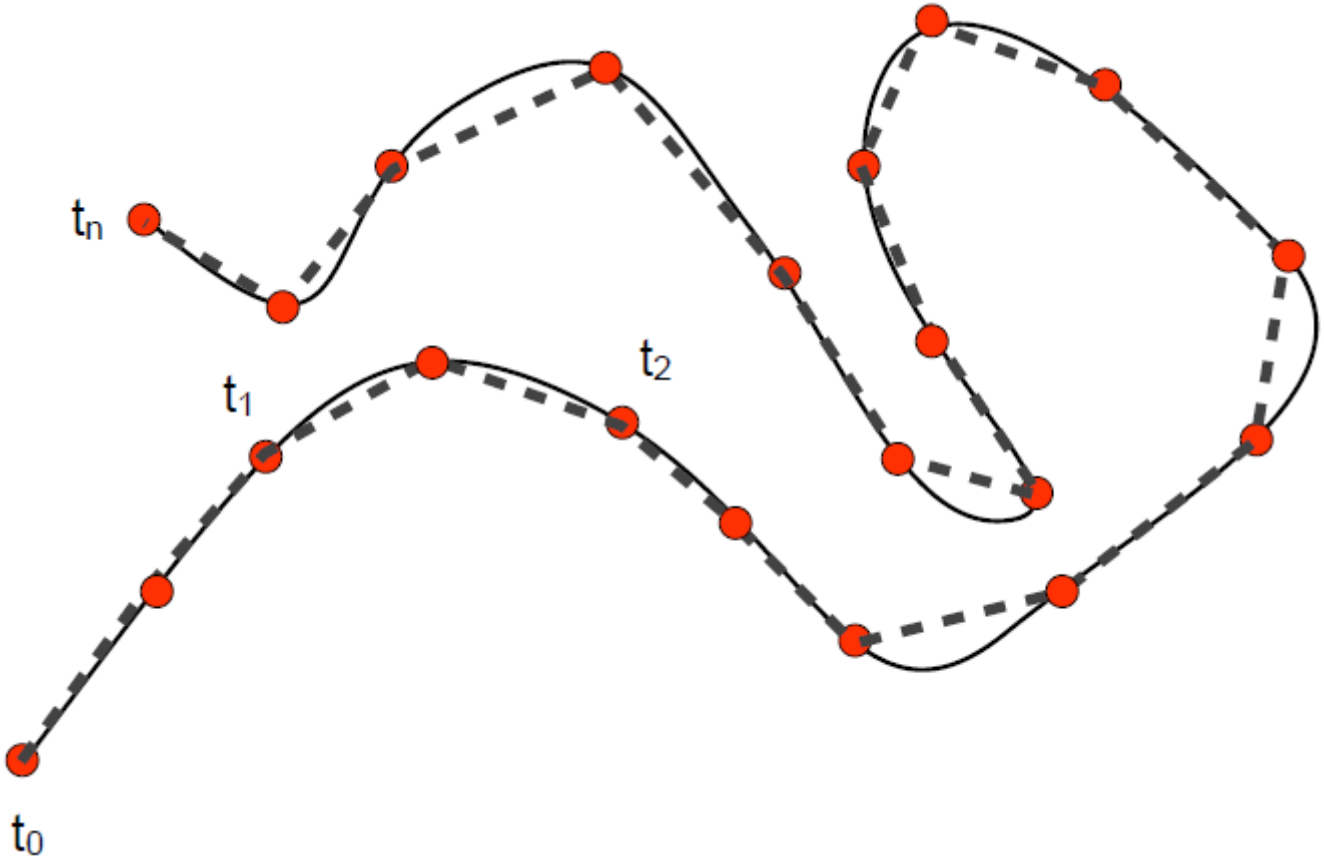
曲线的离散

从连续到离散

- 对象的表达
  - 在数学上，连续表达与计算
  - 在计算机中，离散表达与计算
- 数值方法：数值微分、数值积分、数值优化
  - 数值分析：离散计算对精确计算的近似程度
  - Fourier分析/变换：离散Fourier分析/变换
  - 卷积（滤波）
- 在计算机科学（计算机图形学）中，采样无处不在
  - 计算机只能表达离散的数值
  - 例子：int型的数据（量化）

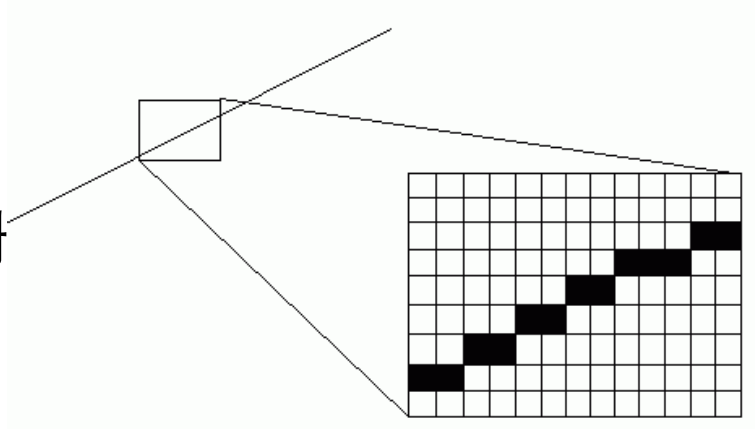
曲线的离散化

- 将连续性表达转化为多边形表达（分段线性）

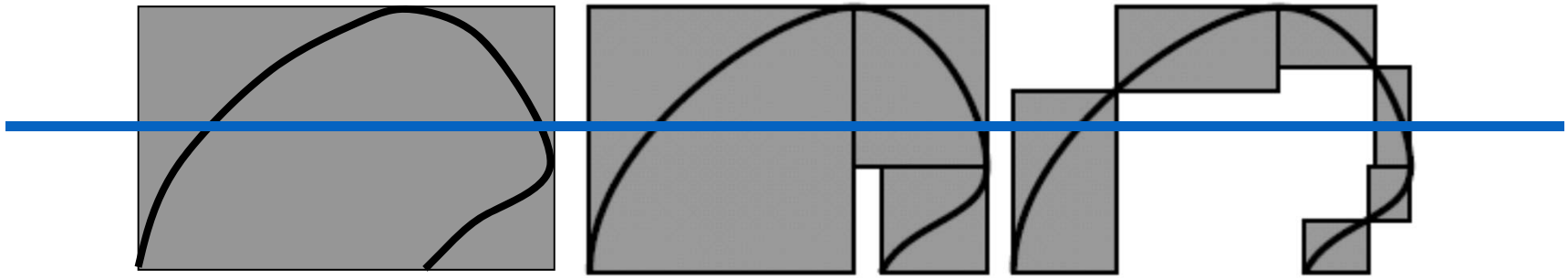


为何要离散化?

- 渲染的必要性
  - 算法和硬件：线段/圆的光栅



- 计算的必要性
  - 直线求交、多项式求根

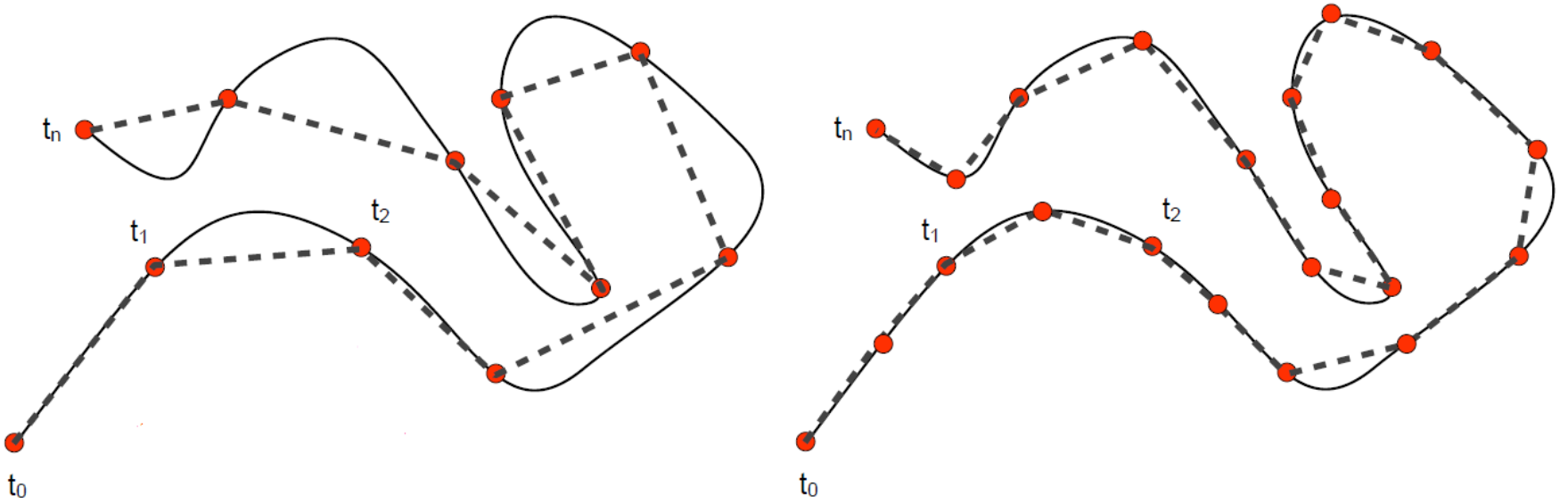


- 制造的必要性
  - 刀具轨迹只能走直线段和圆弧

曲线的离散：采样

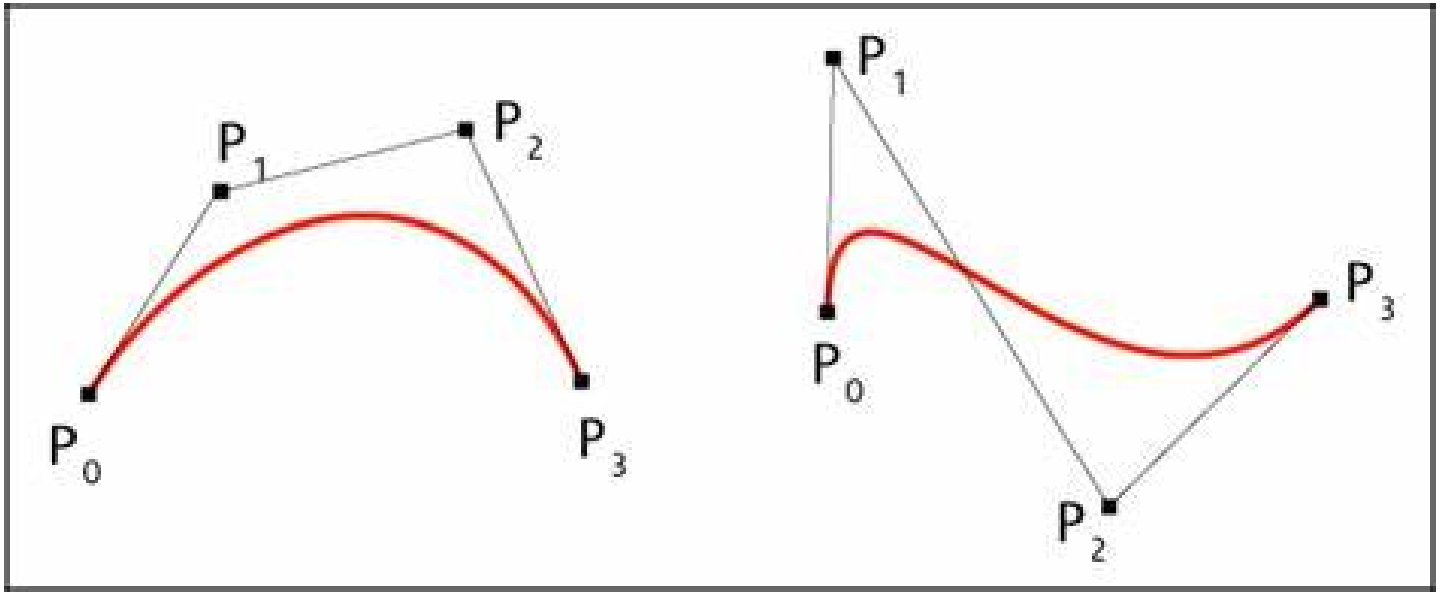
- Nyquist–Shannon采样定理

If a function x(t) contains no frequencies higher than B hertz, it is completely determined by giving its ordinates at a series of points spaced 1/(2B) seconds apart.



Bezier曲线的离散定理

- 曲线到弦的最大距离<控制顶点到弦的最大距离
- 给定误差，估计离散层级



离散曲线的几何量的计算

- 如果有连续表达，利用连续表达的曲线来计算

- 如无连续表达
  - 差分法：利用差分形式来近似微分属性
  - 拟合法：利用光滑函数来拟合估计属性

- Tylor展开及估计

重心坐标

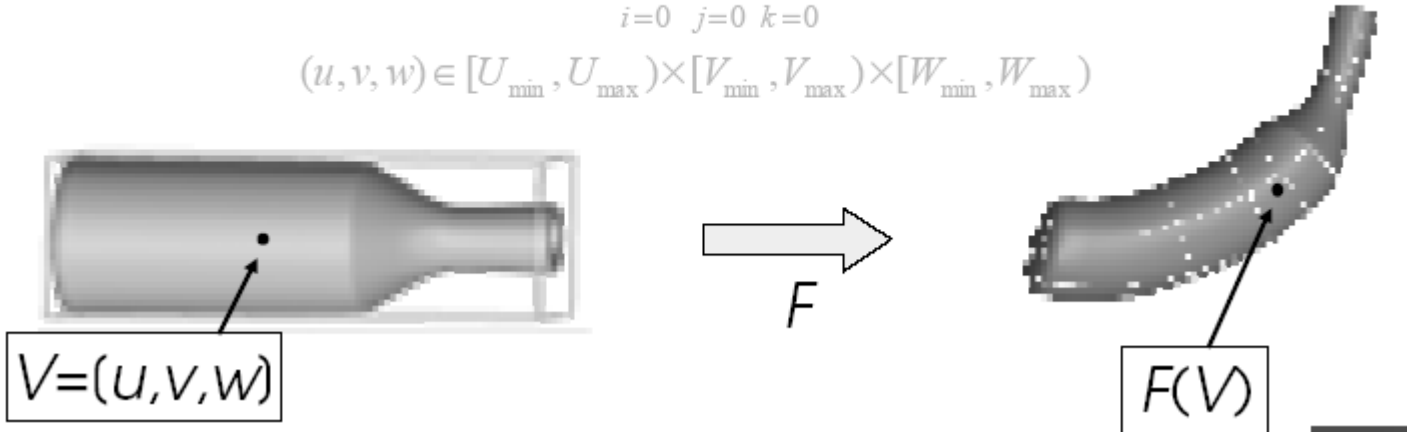
Free-form Deformation (FFD)

[Sederberg et al. 86]

- Embed the object into a domain that is more easily parametrized than the object.
- Advantages:
  - You can deform arbitrary objects
  - Independent of object representation

$$X_{new} = F(u,v,w) = \sum_{i=0}^{l}\sum_{j=0}^{m}\sum_{k=0}^{n} P_{ijk} B_i(u) B_j(v) B_k(w)$$

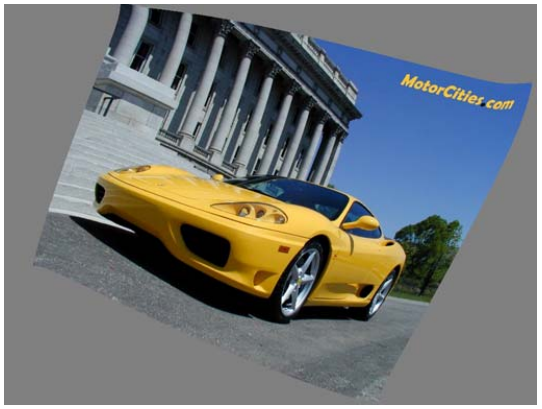
$$(u,v,w) \in [U_{\min}, U_{\max}) \times [V_{\min}, V_{\max}) \times [W_{\min}, W_{\max})$$



图像变形

- 交互： boundary editing

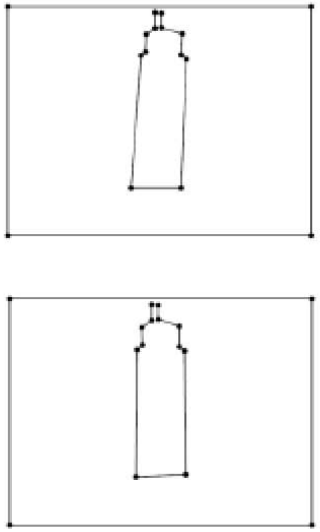
F_1



F_2

图像变形

original image



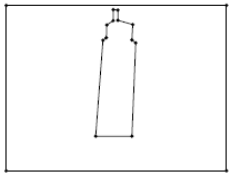
mask

warped image

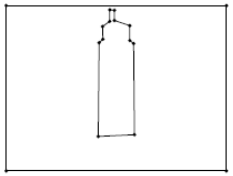
问题：给定一个包含物体的边界多边形，改变边界时，如何计算物体的变形？

即，内部点与边界点（控制顶点）之间的关联关系？

(a)



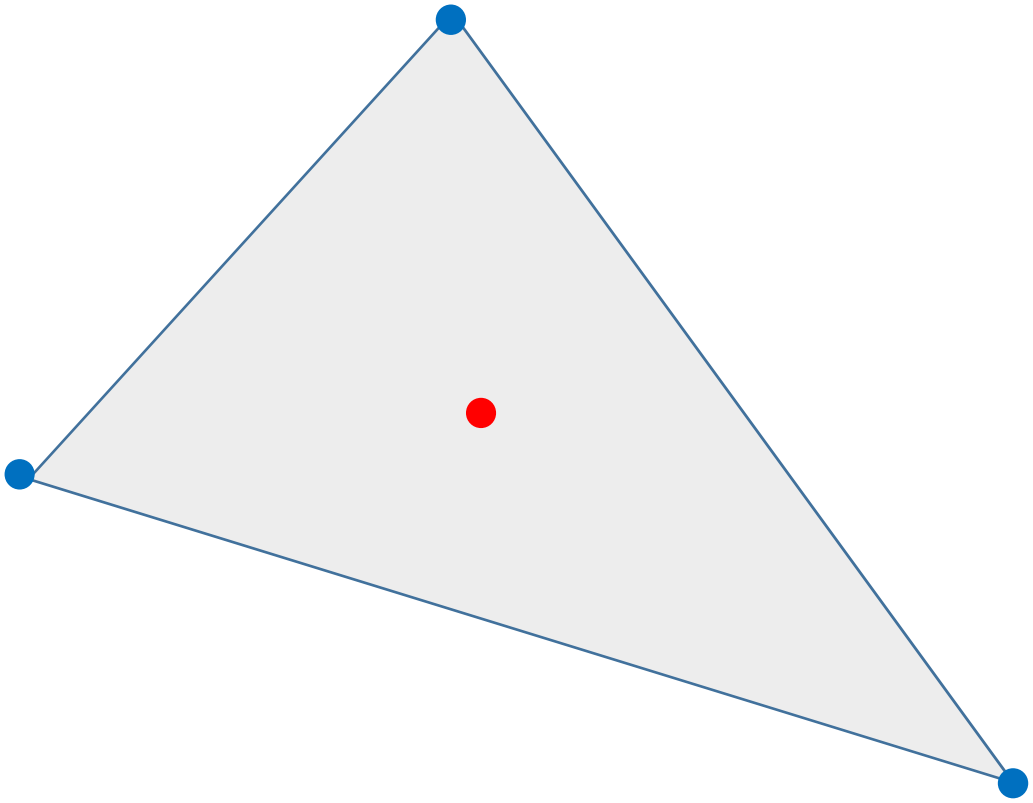
(b)



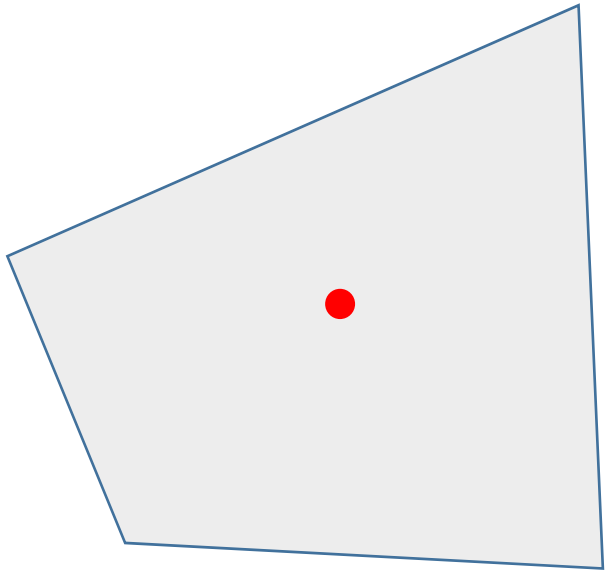
(c)

(d)

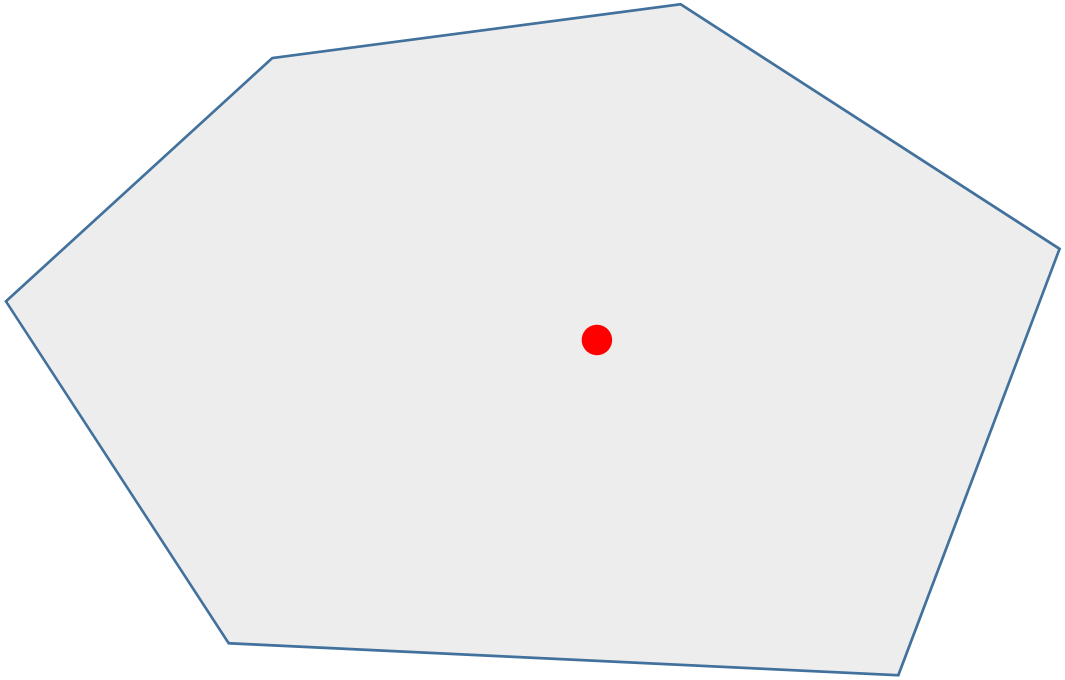
三角形的重心坐标



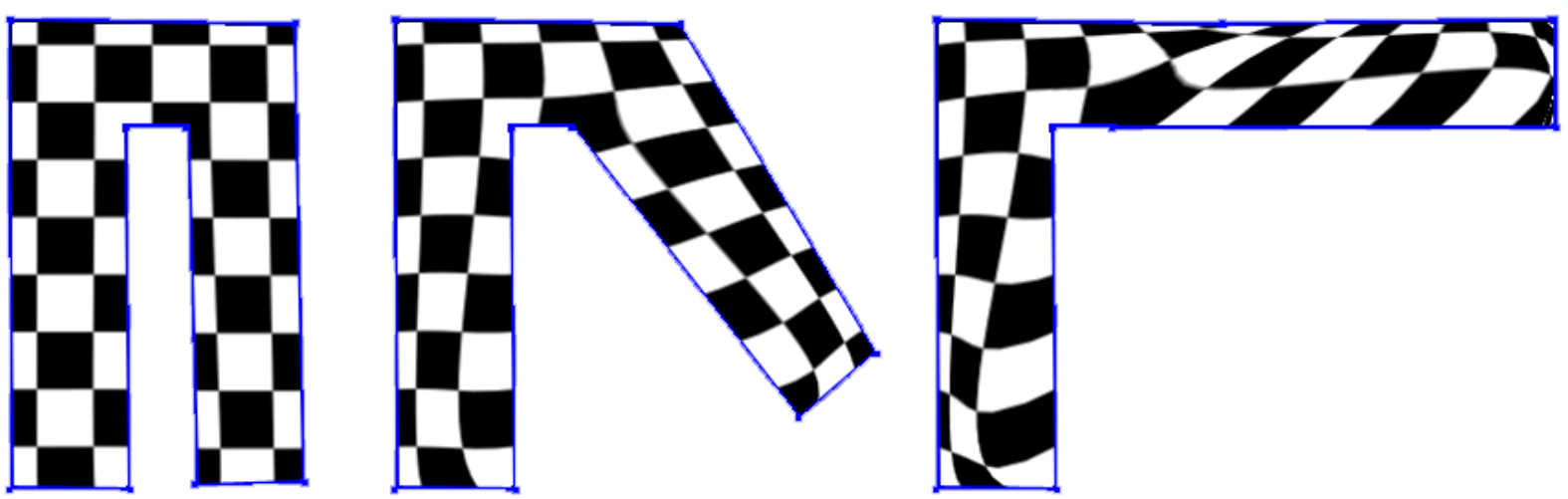
四边形?



多边形的重心坐标?



Warping with BC



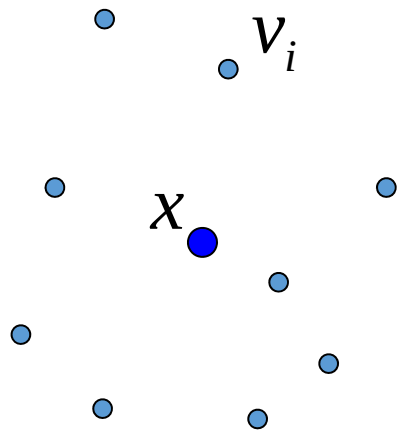
Coordinates

- Homogeneous coordinates
  - Given points $v_\Sigma = \{v_1, \cdots, v_i, \cdots\}$
  - Express a new point x as affine combination of v_Σ

$$x = \sum b_i v_i, \quad where \sum b_i = 1$$

  - b_i are called *homogeneous coordinates*
  - *Barycentric* if all

$$b_i \geq 0$$



Applications

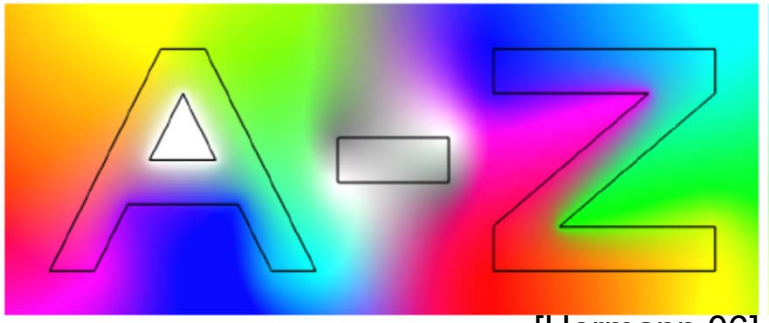
- Boundary interpolation

$$f(x) = \sum b_i f_i$$

  - Color/Texture interpolation
- Mapping

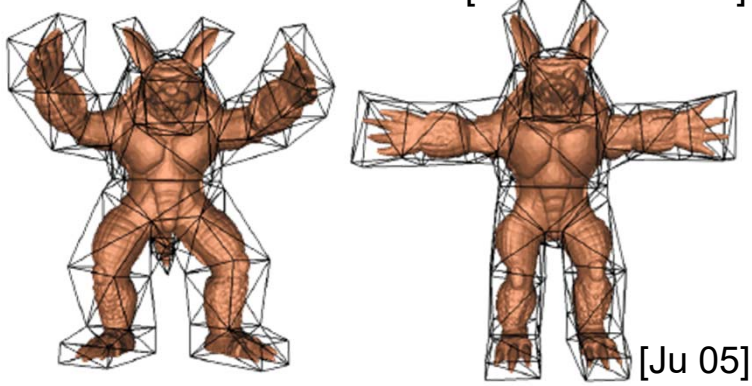
$$x' = \sum b_i v'_i$$

  - Shell texture
  - Image/Shape deformation



[Hormann 06]

[Porumbescu 05]



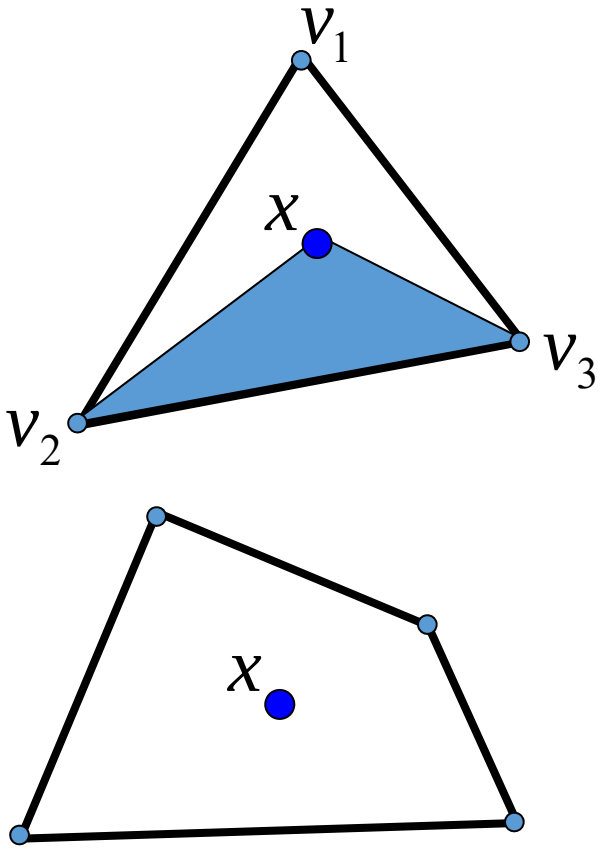
[Ju 05]

Coordinates In A Polytope

- Points v_Σ form vertices of a closed polytope
  - x lies inside the polytope
- Example: A 2D triangle
  - Unique (barycentric):

$$b_1 = \frac{A_{x v_2 v_3}}{A_{v_1 v_2 v_3}}$$

  - Can be extended to any N-D simplex
- A general polytope
  - Non-unique
  - The triangle-trick can not be applied.



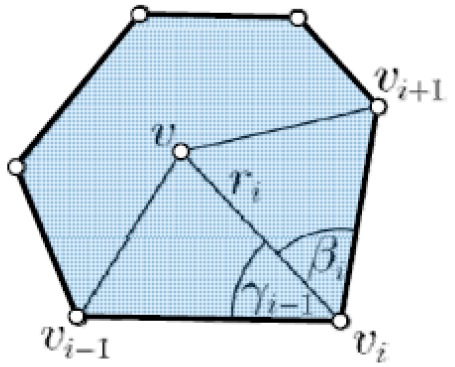
BC of 2D Polygons

- 2D Polygons
  - Wachspress [Wachspress 75][Loop 89][Meyer 02][Malsch 04]
    - Barycentric within convex shapes
  - Mean value [Floater 03][Hormann 06]
    - Homogeneous within any closed shape, barycentric within convex shapes and kernels of star-shapes
  - Discrete harmonic [Desbrun 02][Floater 06]
    - Homogeneous within convex shapes
- A general construction in 2D [Floater 06]
  - Complete: a single scheme that can construct all possible homogeneous coordinates in a convex polygon
  - Reveals a simple connection between known coordinates via a parameter
    - Wachspress
    - Mean value
    - Discrete harmonic

各种重心坐标的计算方法

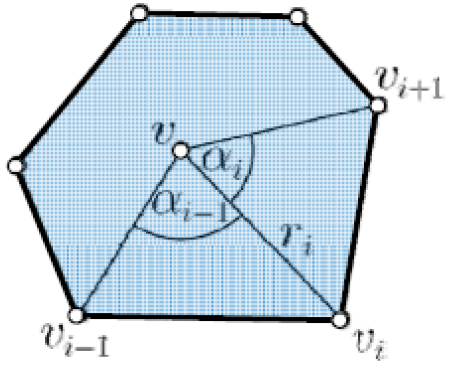
- ***Wachspress*** (WP) coordinates

$$w_i = \frac{\cot \gamma_{i-1} + \cot \beta_i}{{r_i}^2}$$



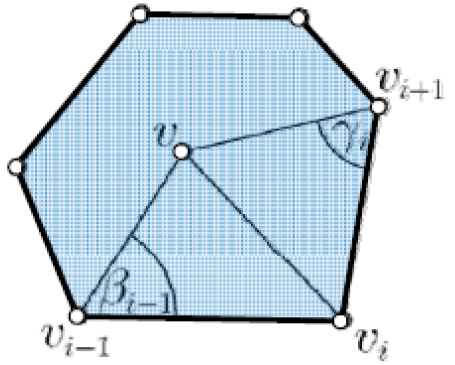
- ***mean value*** (MV) coordinates

$$w_i = \frac{\tan(\alpha_{i-1}/2) + \tan(\alpha_i/2)}{r_i}$$

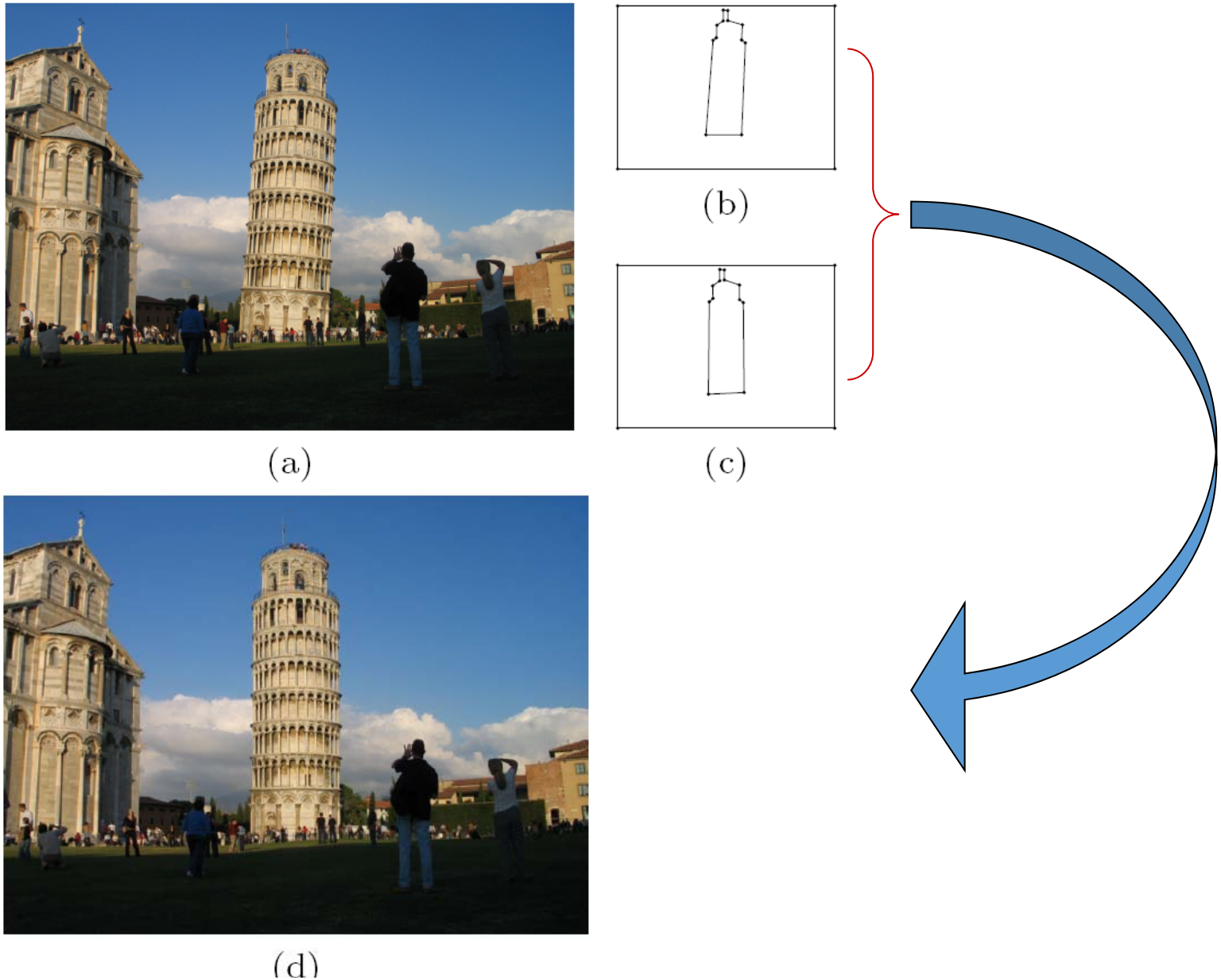


- ***discrete harmonic*** (DH) coordinates

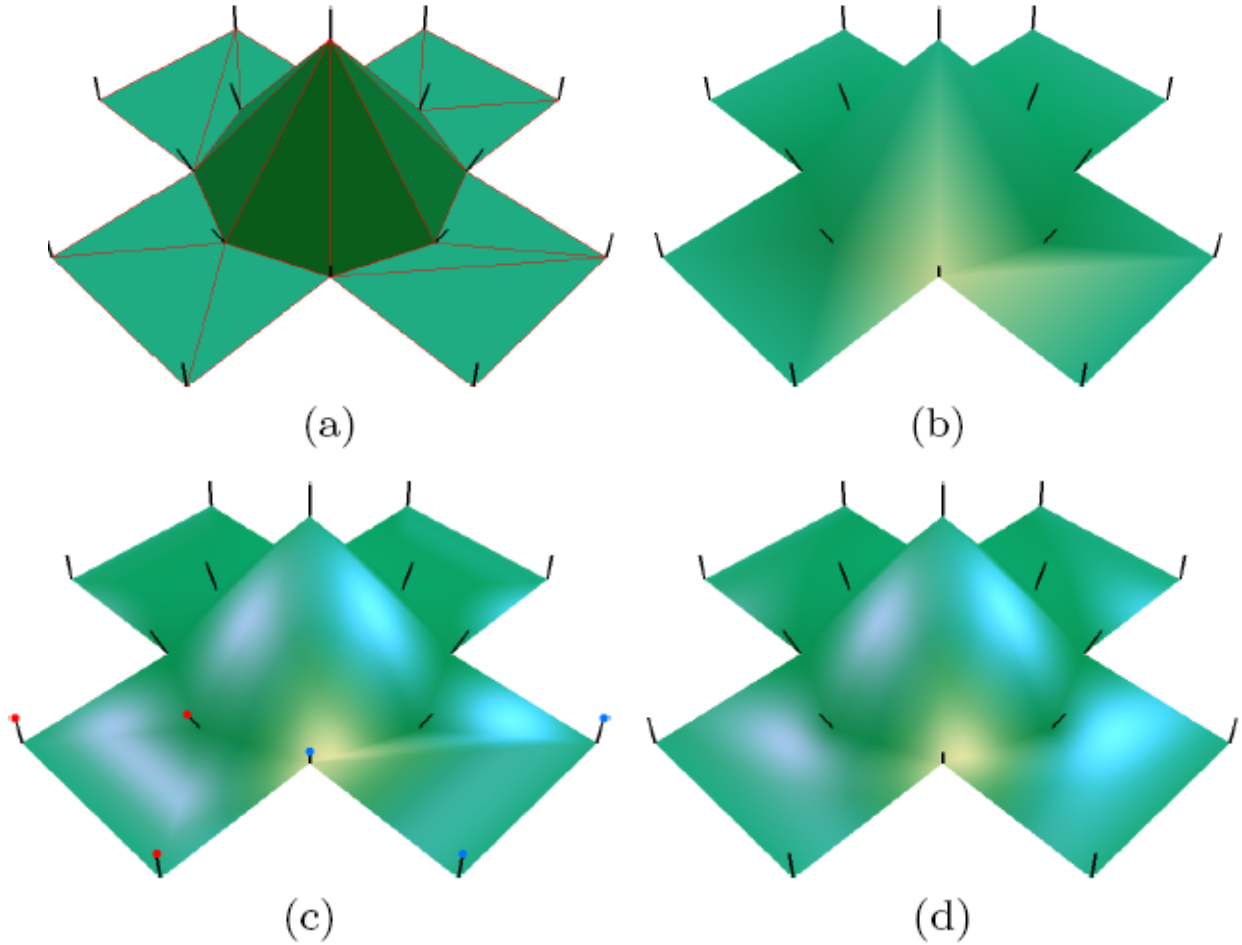
$$w_i = \cot \beta_{i-1} + \cot \gamma_i$$



- 1.imge warping

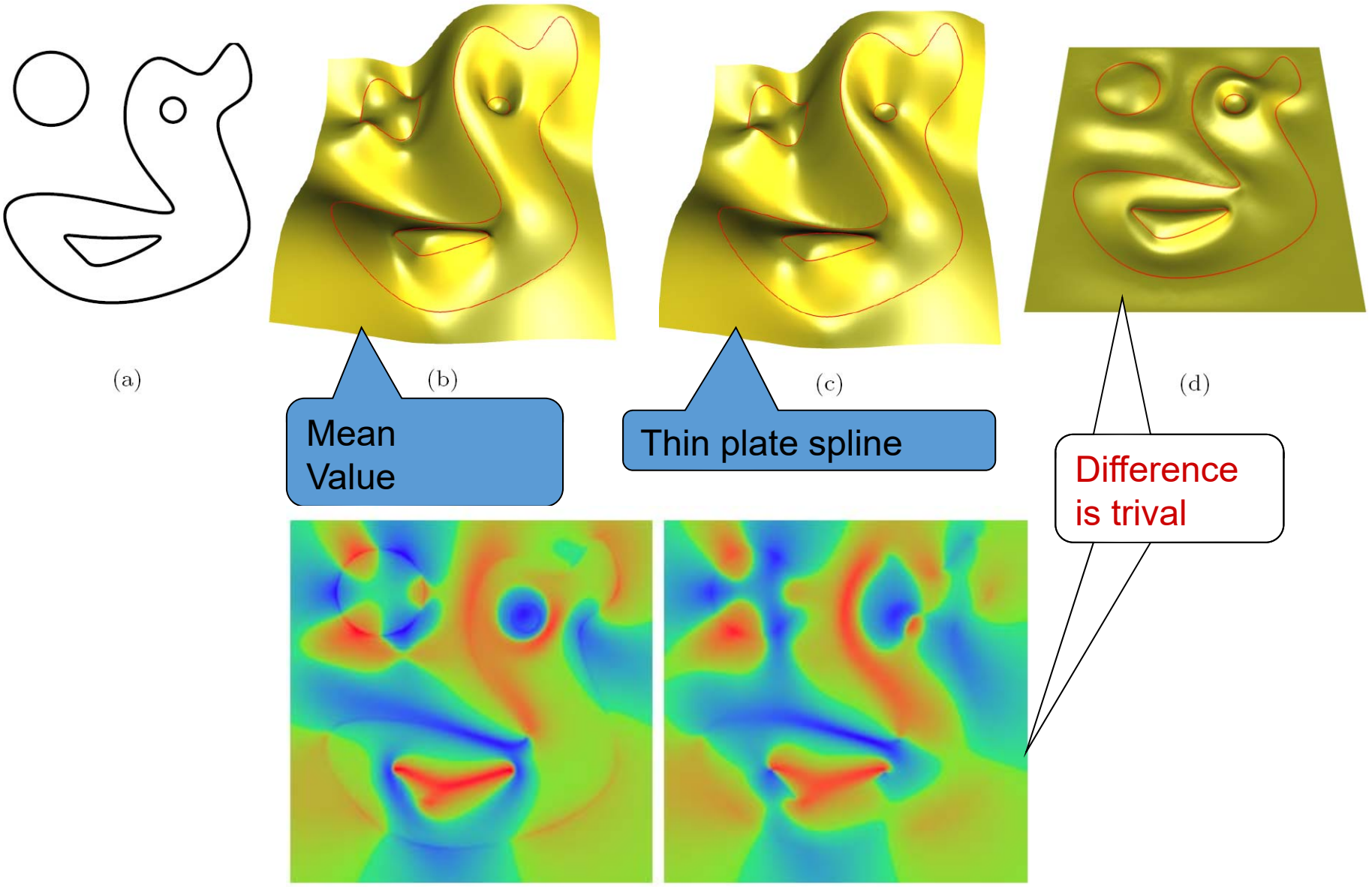


- 2. shading

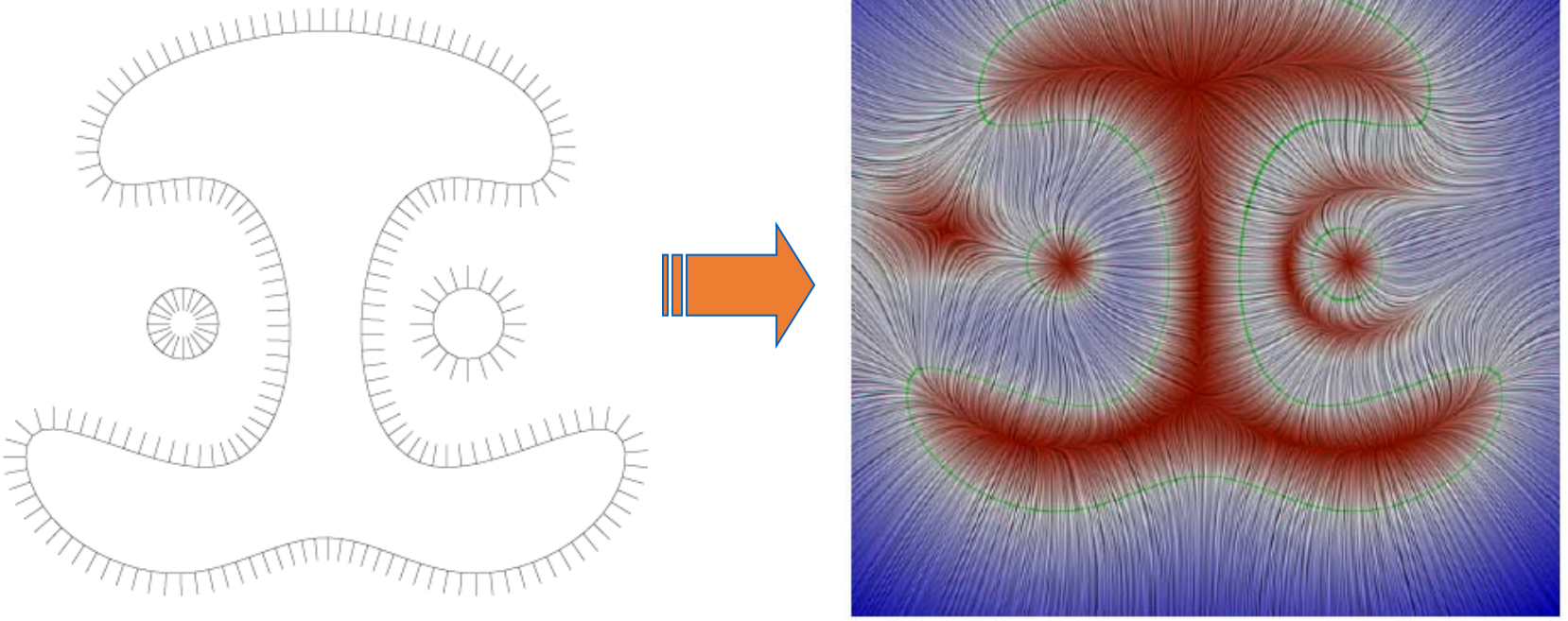


a.flat b.Gourand c. Phong d.using MV

3. Transfinite Interpolation: Interpolating height function to model a surface



- allow directly updating on interpolation when resampled.

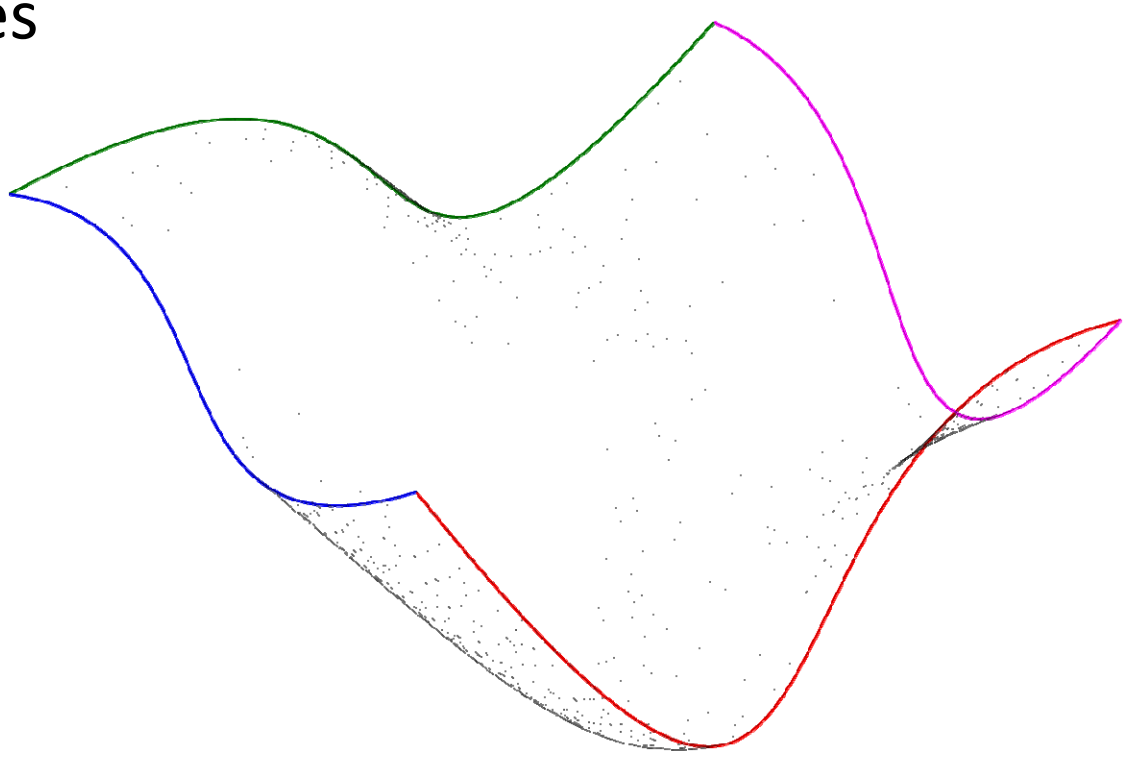


a. 3 curves with 100 sample vertices.

b. increase sample to 357 vertices using less than 10 sec.

Transfinite Interpolation

- 问题：给定4条边界曲线，构造插值这4条曲线的一张曲面
- Coons surfaces



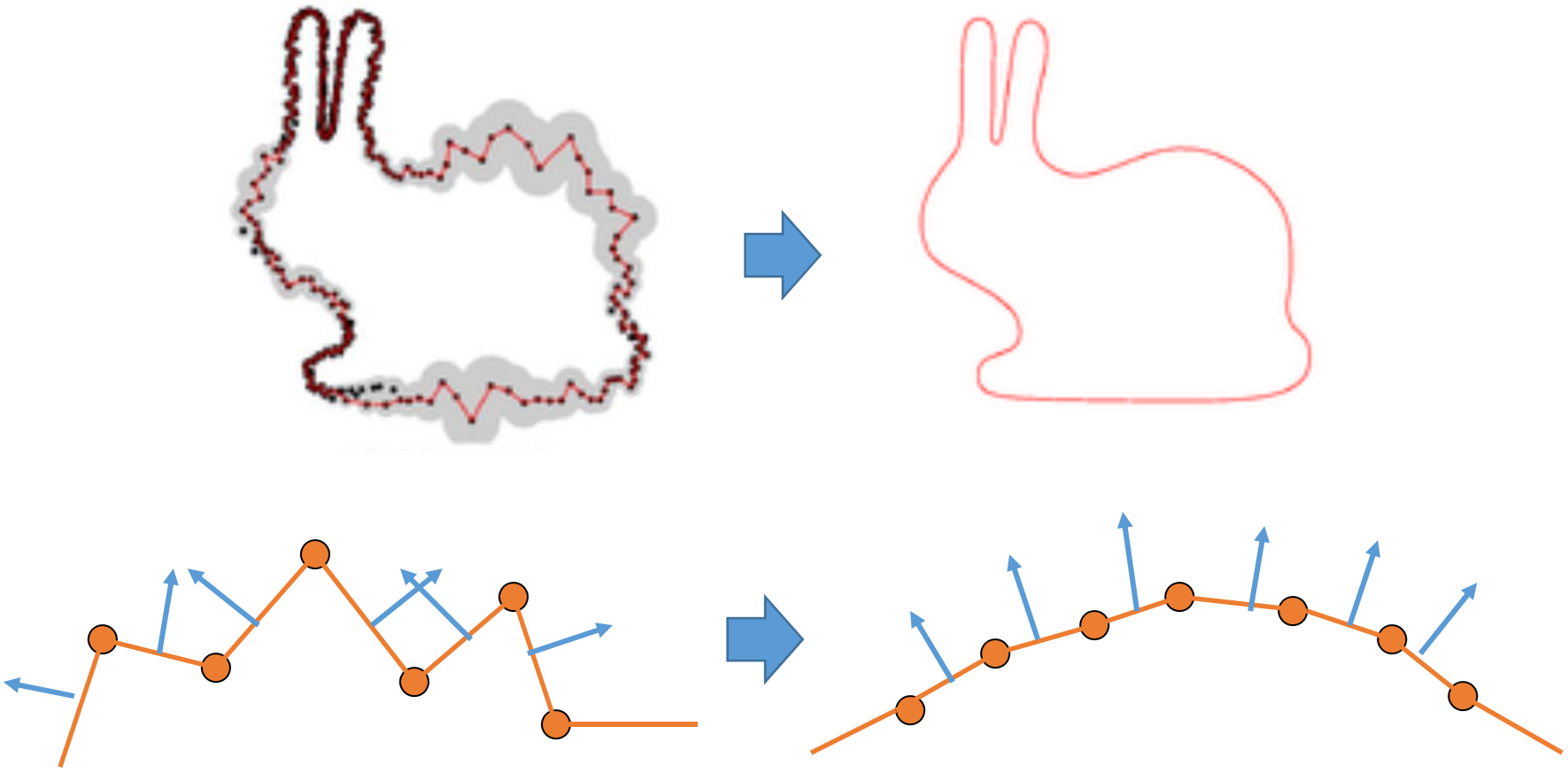
广义重心坐标的学习资料

- http://www.inf.usi.ch/faculty/hormann/barycentric

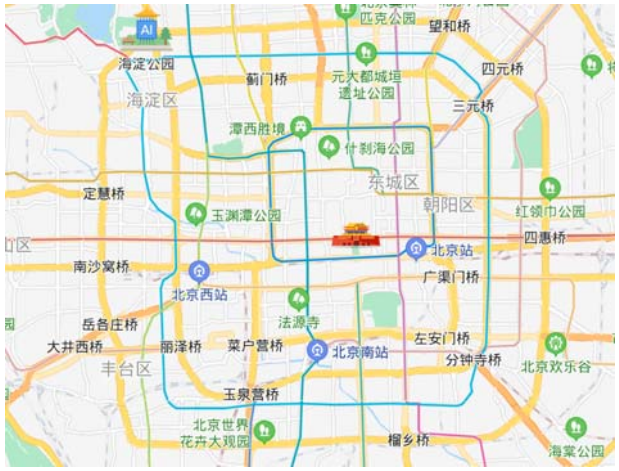
2D形状（离散曲线）处理

离散曲线的去噪/滤波

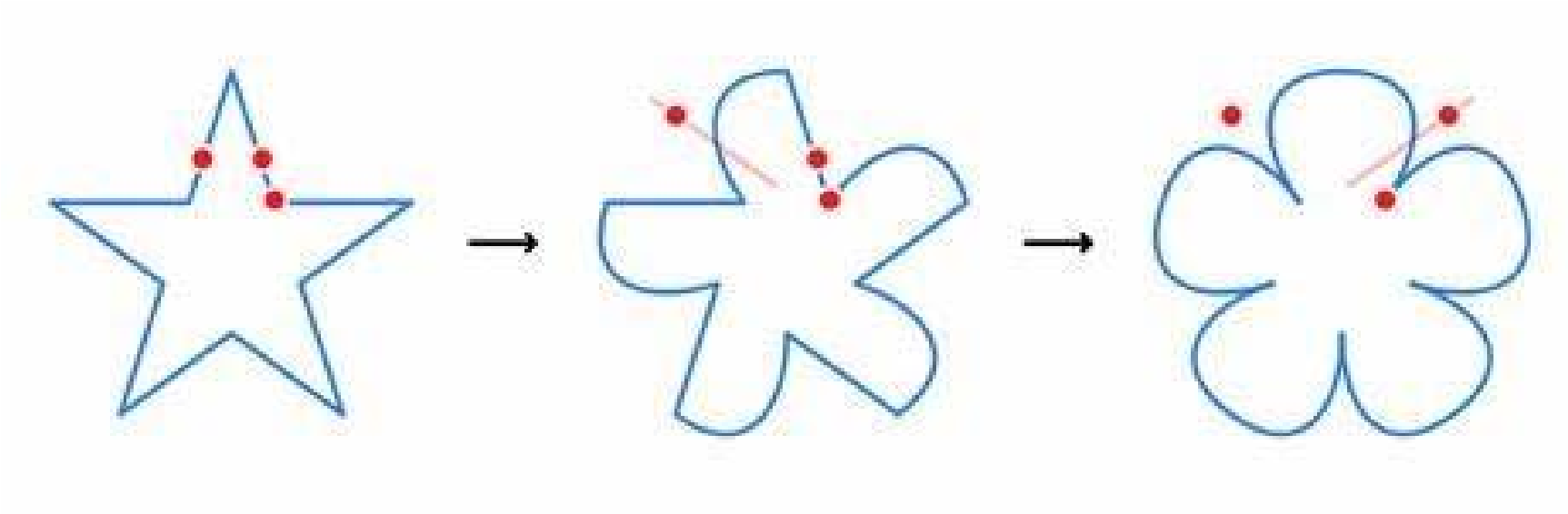
- Denoising, smoothing, fairing



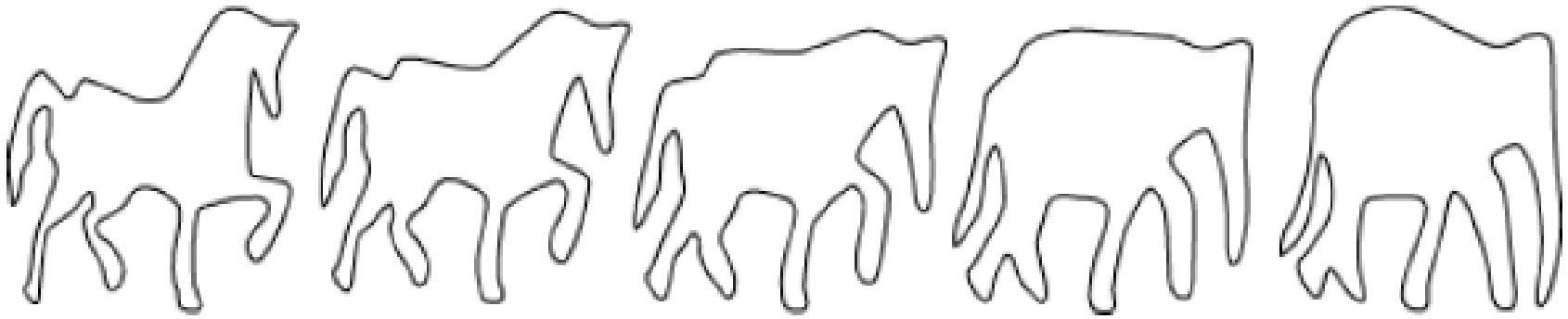
曲线简化(Simplification)



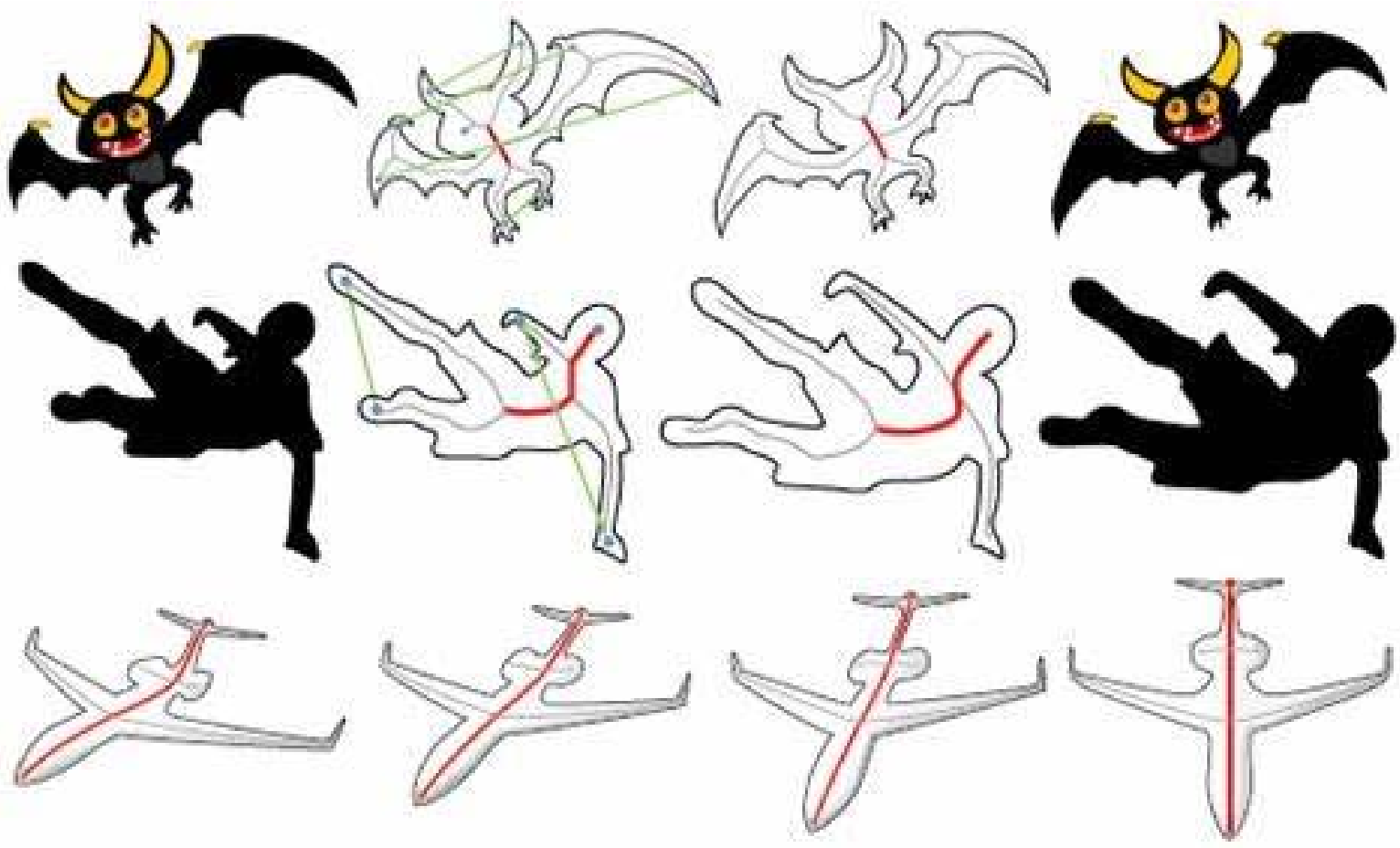
曲线编辑/变形(Editing/Deformation)



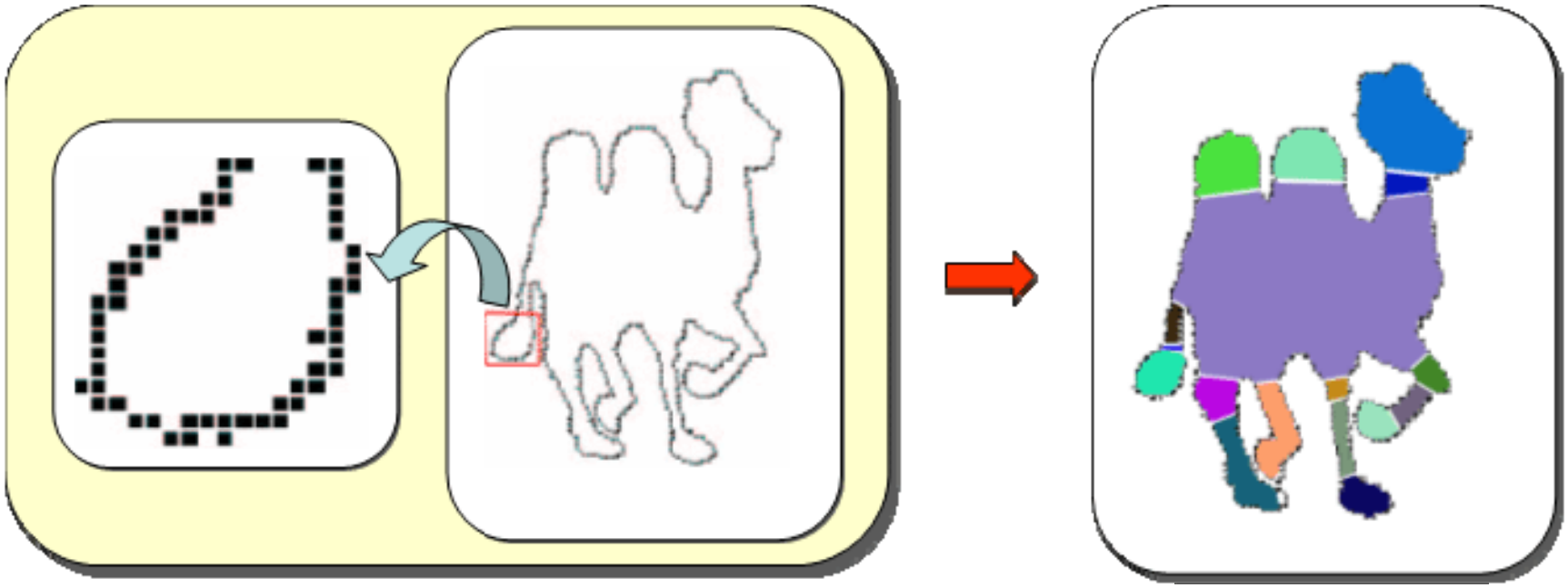
形状插值(Morphing)



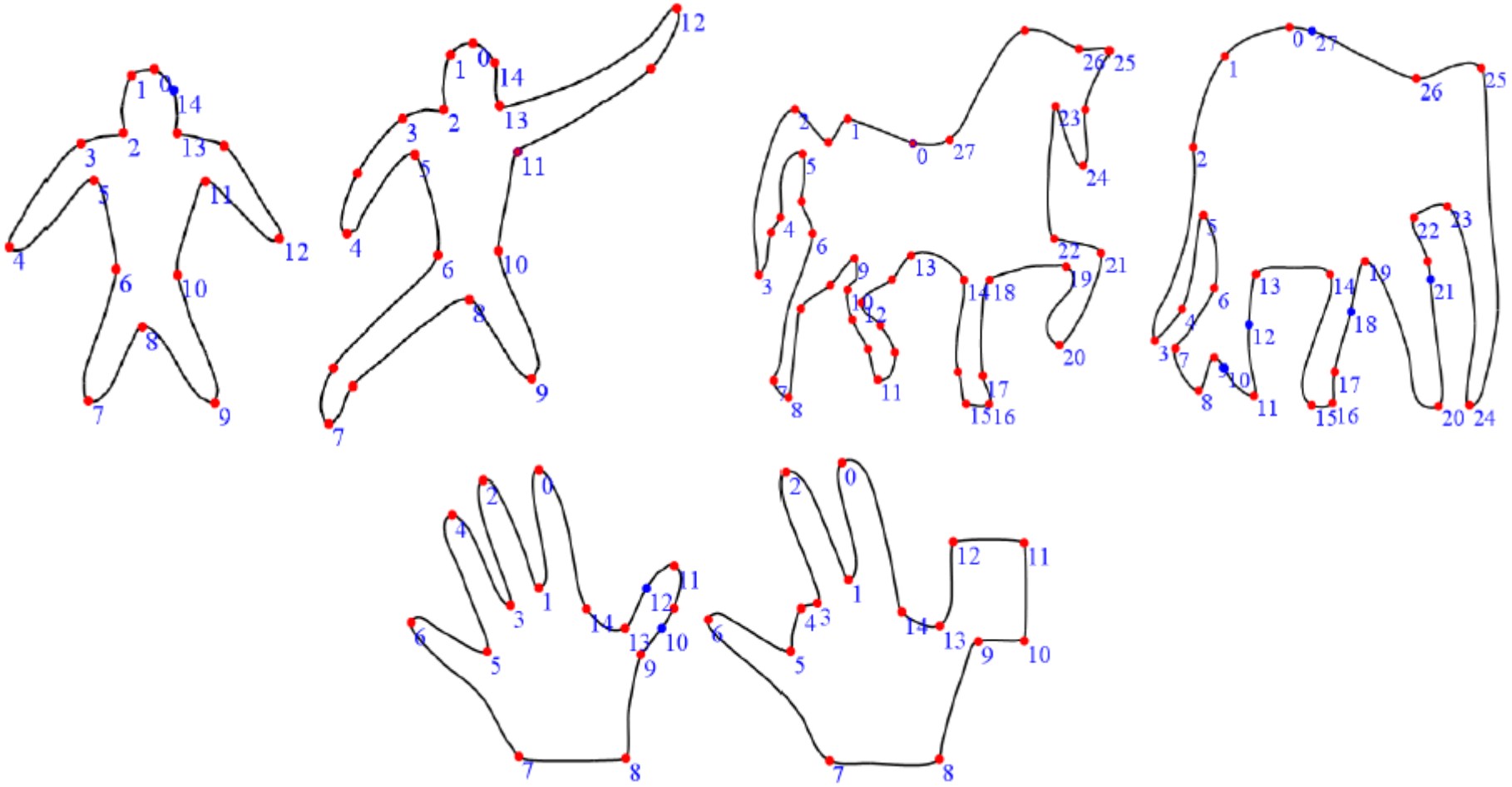
形状的对称性检测(Symmetry)



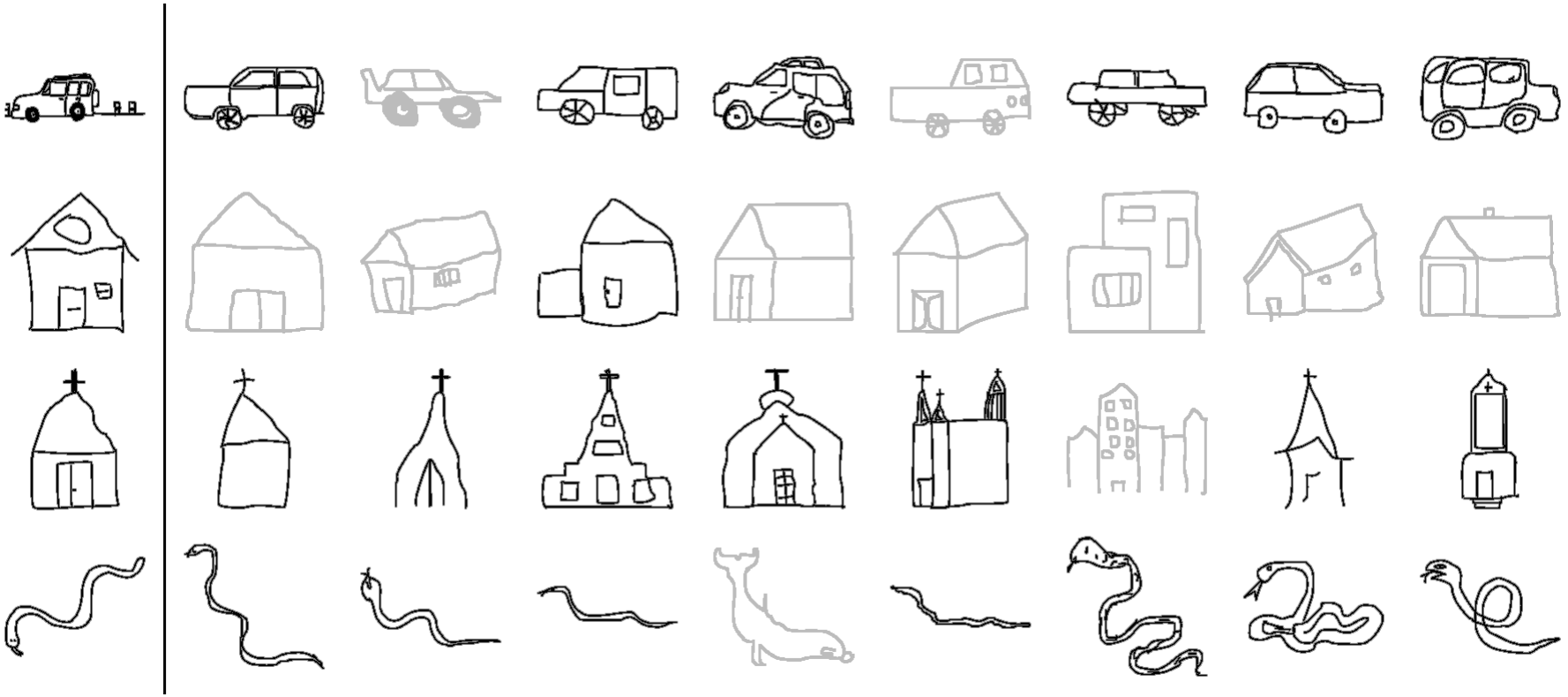
形状分割(Segmentation)



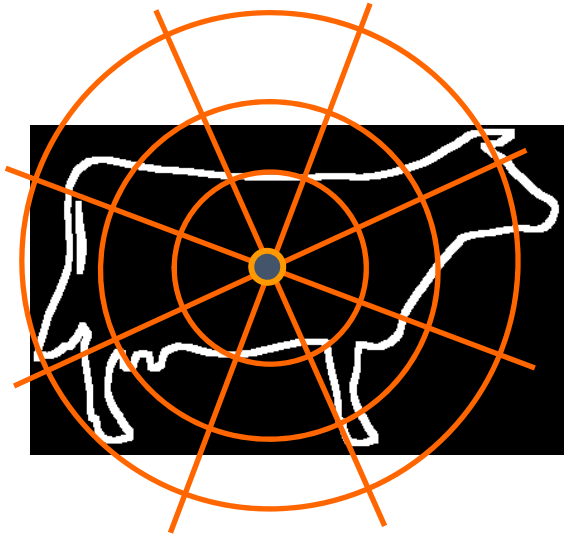
形状匹配(Matching/Correspondences)



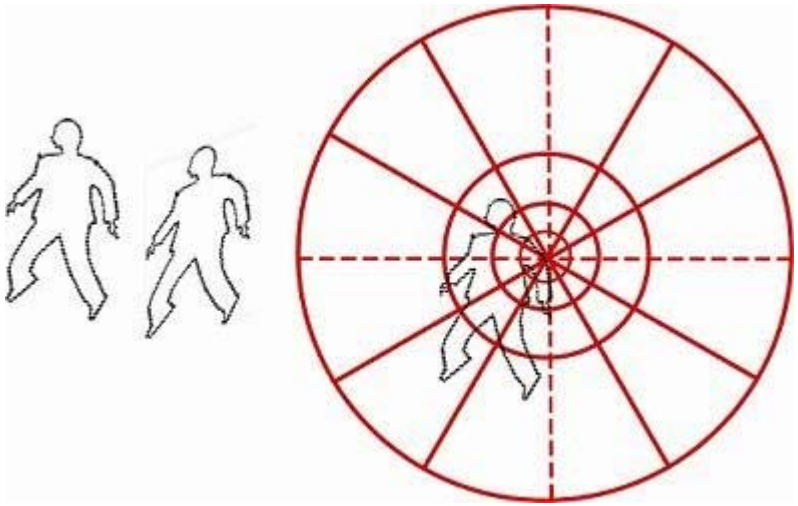
形状检索(Retrieval)



形状描述子(Descriptors)



Global descriptor



Local descriptor

中国科学技术大学

University of Science and Technology of China

谢　谢　！