# Artificial Intelligence (CS 452): Probability Exercises

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[ **1** ] This question involves combinatorics and basic probability.

a. We need to figure out how many different ways there are to choose 5 different cards, remembering that we don't care about the order (so, for example, the hand $\langle 3\spadesuit,\ 4\heartsuit,\ J\spadesuit,\ K\clubsuit,\ A\clubsuit\rangle$ is the same as $\langle A\clubsuit,\ 4\heartsuit,\ J\spadesuit,\ K\clubsuit,\ 3\spadesuit\rangle$). We get the number of ways of choosing $m$ things from a set of $n$ things (where the order of the $m$ things doesn't matter) using a general formula:

$$\binom{n}{m} = \frac{n!}{(n-m)!\,m!} = \frac{n \times (n-1) \times \cdots \times ((n-m)+1)}{m \times (m-1) \times \cdots \times 2 \times 1}$$

Thus our answer is:

$$\binom{52}{5} = \frac{52!}{47!\,5!} = \frac{52 \times 51 \times 50 \times 49 \times 48}{5 \times 4 \times 3 \times 2 \times 1} = 2,598,960$$

b. Each hand has equal probability, equal to $1/2,598,960 \approx 3.85e-07 = 0.000000385$

c. To get a royal flush, you need the Ace, King, Queen, Jack, and Ten of any of the four suits; thus, there are 4 possible royal flushes and the joint probability of getting any one of them is $4/2,598,960 \approx 1.54e-06 = 0.00000154$
To get 4 of a kind, you need 4 cards that are the same (2's, 3's, etc.), plus one extra card; thus, there are 13 basic combinations, 2 through Ace, and then 48 choices for the extra card in each case, for a total of $(13 \times 48) = 624$ possibilities. Thus the odds of getting any one of them is $624/2,598,960 \approx 2.4e-04 = 0.00024$

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[ **2** ] We can calculate the values using basic principles.

a. $P(toothache)$: we marginalize, adding up all those table entries where *toothache* is true, to get: $(0.108 + 0.012 + 0.016 + 0.064) = 0.2$.

b. $P(cavity)$: marginalize again, to get $(0.108 + 0.012 + 0.072 + 0.008) = 0.2$.

c. $P(ache \mid cavity) = P(ache \wedge cavity)/P(cavity) = (0.108 + 0.012)/0.2 = 0.12/0.2 = 0.6$

d. $P(cavity \mid ache \vee catch) = P(cavity \wedge (ache \vee catch))/P(ache \vee catch) = (0.108 + 0.012 + 0.072)/(0.108 + 0.012 + 0.016 + 0.064 + 0.072 + 0.144) = 0.192/0.416 \approx 0.462$

[ **3** ] For each problem, we begin by converting the table of balls and boxes into a table of probabilities.

a. This is done by dividing the number of occurrences of each event (i.e., a certain number of balls in a certain box) by the total number of occurrences period (i.e., the total number of balls in all the boxes, which is 30):

| | **Box 1** | **Box 2** | **Box 3** |
|---|---|---|---|
| **Red** | 0.067 | 0.133 | 0.1 |
| **White** | 0.1 | 0.067 | 0.133 |
| **Blue** | 0.2 | 0.1 | 0.1 |

We can now calculate the probabilities as follows:

a. $P(Box1 \,|\, Red) = P(Box1 \wedge Red)/P(Red) = 0.067/(0.067 + 0.133 + 0.1) = 0.067/0.3 \approx 0.222$.

b. $P(Box2 \,|\, Red) = P(Box2 \wedge Red)/P(Red) = 0.133/0.3 \approx 0.443$.

c. $P(Box3 \,|\, Red) = P(Box3 \wedge Red)P(Red) = 0.1/0.3 \approx 0.333$.

(Note that you could also get one of these by subtracting the other two from 1.0, since they all must add up to the latter.)

b. In this case, we do the same, but multiply the chance of choosing a box (1/3) by the probability of choosing a ball from that box; so for example, the chance of choosing a red ball from Box 1 is ($1/3 \times 2/11 \approx 0.061$).

| | **Box 1** | **Box 2** | **Box 3** |
|---|---|---|---|
| **Red** | 0.061 | 0.148 | 0.1 |
| **White** | 0.091 | 0.074 | 0.133 |
| **Blue** | 0.182 | 0.111 | 0.1 |

We can now calculate the probabilities as follows:

a. $P(Box1 \,|\, Red) = P(Box1 \wedge Red)/P(Red) = 0.061/(0.061 + 0.148 + 0.1) = 0.061/0.309 \approx 0.197$.

b. $P(Box2 \,|\, Red) = P(Box2 \wedge Red)/P(Red) = 0.148/0.309 \approx 0.479$.

c. $P(Box3 \,|\, Red) = P(Box3 \wedge Red)P(Red) = 0.1/0.309 \approx 0.324$.

Note how the probability goes down for Box 1 and Box 3, since in those boxes, red balls make up a lesser proportion of the total box, whereas in Box 2 it goes up, since there are proportionally more in there.

**[ 4 ]** We want to calculate the probability that the taxi is blue ($IB$) , given that it looks blue ($LB$); by Bayes' Rule (and normalization), we know:

$$P(IB \mid LB) = \frac{P(LB \mid IB)\, P(IB)}{P(LB)} = \frac{P(LB \mid IB)\, P(IB)}{P(LB \mid IB)\, P(IB) + P(LB \mid \neg IB)\, P(\neg IB)}$$

Now, in the first case, we do know the first thing, $P(LB \mid IB) = 0.75$, since we are 75% reliable. However, we have no way of knowing the prior probability that the taxi is blue, since we don't know anything about taxis (for instance, it would make a very big difference if we found out that *every* taxi in Athens was blue, obviously). Similarly, we can't calculate the probability that it looks blue without this information. However, once we know that 9 out of 10 taxis are green, we know that $P(IB) = 1 - P(\neg IB) = 0.1$. Thus we have:

$$P(IB \mid LB) = \frac{0.75 \times 0.1}{(0.75 \times 0.1) + (0.25 \times 0.9)} = \frac{0.075}{0.3} = 0.25.$$

Therefore, there is only a 25% chance that the taxi is blue, and it is in fact most likely green.