Artificial Intelligence (CS 131): Probability Exercises

[**1**]   Consider the domain of dealing 5-card poker hands from a standard deck of 52 cards:

a. How many atomic events are there in the joint probability distribution (i.e., how many different 5-card hands are there)?

b. What is the probability of any particular hand being dealt?

c. What is the probability of being dealt a royal flush? What about four of a kind?

[**2**]   You are given the following joint distribution over the occurrences of tooth-ache, the presence of cavities, and whether or not a dental pick catches on a tooth during a check-up:

	toothache		¬toothache	
	catch	¬*catch*	*catch*	¬*catch*
cavity	0.108	0.012	0.072	0.008
¬*cavity*	0.016	0.064	0.144	0.576

Calculate the following:

a. $P(\textit{toothache})$

b. $P(\textit{cavity})$

c. $P(\textit{toothache} \,|\, \textit{cavity})$

d. $P(\textit{cavity} \,|\, \textit{toothache} \vee \textit{catch})$

[**3**]   Suppose we randomly toss red, white, and blue balls into 3 boxes, and when we are done, the assortment of balls looks like this:

	Box 1	Box 2	Box 3
Red	2	4	3
White	3	2	4
Blue	6	3	3

Suppose someone takes a ball out of one of the boxes at random and shows it to you: it is red.

1. What is the probability that it came from each of the three boxes, if by "at random" we mean that they use some procedure that chooses any possible ball with uniform probability?

2. What is the probability that it came from each of the three boxes, if by "at random" we mean that they (a) first choose one of the boxes, uniformly at random, and then (b) choose a ball from that box uniformly at random?

[**4**]   Suppose you witness a nighttime hit-and-run accident involving a taxi in Athens. All taxis in Athens are blue or green. You swear, under oath, that the taxi was blue. Extensive testing shows that, under the dim lighting conditions, your ability to tell the difference between blue and green is 75% reliable. Is it possible to calculate the most likely color of the taxi? (*Hint*: distinguish carefully between the proposition that the taxi *is* blue and the proposition that it *appears to be* blue.) If it *is* possible, show the probability that the taxi is blue.

What about if you are given the extra knowledge that 9 out of 10 Athenian taxis are green? If it *is* possible now, show the probability that the taxi is blue.