TUFTS UNIVERSITY
Department of Computer Science

CS 131	Artificial Intelligence	Summer 2022
Midterm Exam 02 (Practice Exam)		05–09 August 2022

- **Please sign your name below (or write it to the top of each page of your work, if you are not solving on the exam paper). By doing so, you agree to be bound by Tufts policies on academic integrity.**
- **For this exam, you may use any notes you have taken, the Russell & Norvig text, and any materials distributed by the instructor (including lecture notes/videos and code samples). No other materials are to be used.**
- **This booklet contains 6 pages including the cover page.**
- **You have exactly 105 minutes (one hour, 45 minutes) to complete this exam *and* upload a PDF version to Gradescope. Be sure to leave yourself enough time for the latter steps.**
- **The maximum possible is 50.**

PROBLEM	SCORE
1	9
2	13
3	10
4	18
TOTAL	50

ANSWER KEY

1. **(12 pts.)** **TRUE OR FALSE.**

Indicate whether each of the following sentences is true or false, and provide a brief explanation.

a. (3 pts.) (T ___ F **X**) Whenever we want to calculate the probability that two things are both true, we multiply their prior probabilities: $P(A \wedge B) = P(A) \times P(B)$.

This is false in general, as it only applies where A and B are marginally independent of one another. When this is not the case, then the joint probability can be either higher than or lower than the simple product

b. (3 pts.) (T ___ F **X**) Let A_1 and A_2 be two variables in a Bayesian network. When there is no directed path between the two variables, they must be independent.

This is false in general. The rules for determining independence (via d-separation or otherwise) are given in terms of **undirected paths** between variables; the fact that there is no directed path does not always guarantee independence.

c. (3 pts.) (T **X** F ___) When designing a Bayes Net, the order in which we consider the variables affects the final size of the network, as it can cause CPTs to be larger or smaller depending upon variable ordering. Still, no matter how we construct it, any probabilistic query about our variables that we like can be answered using the information in the BN.

The order in which variables are added can affect things like the way nodes are connected by arrows, and therefore the size of the Conditional Probability Tables needed to specify it. However, Bayes Nets, in any configuration, still have the full expressive power of a complete joint probability specification, and so are sufficient to answer any probabilistic query we wish.

2. **(13 pts.)** **BASIC PROBABILITY.**

You are in charge of running the Tufts admissions office. Based on records from past years, you see that 90% of incoming students will pass their freshman year. Applicants are separated into three basic categories, $High, Middle, Low$, based on their test scores. Your records also show that among students who pass in freshman year, their test scores fall into those 3 categories with probabilities 0.3, 0.65, 0.05 respectively; among those who do not pass, the probability they had each kind of score are 0.05, 0.25, 0.7 respectively.

a. (5 pts.) What are the probabilities that students will have each category of test score, $P(High)$, $P(Middle)$, $P(Low)$?

Answer: we can calculate each probability by marginalization:

$$\begin{aligned} P(H) &= P(H \mid P)\ P(P) + P(H \mid F)\ P(F) \\ &= (0.3 \times 0.9) + (0.05 \times 0.1) \\ &= 0.27 + 0.005 = 0.275 \\ P(M) &= P(M \mid P)\ P(P) + P(M \mid F)\ P(F) \\ &= (0.65 \times 0.9) + (0.25 \times 0.1) \\ &= 0.585 + 0.025 = 0.61 \\ P(L) &= 1 - (P(H) + P(M)) = 1 - 0.885 = 0.115 \end{aligned}$$

b. (4 pts.) What is the probability that a student passes if they have a $High$ score?

Answer: we can calculate this using Bayes' Rule:

$$P(P \mid H) = \frac{P(H \mid P)\ P(P)}{P(H)} = \frac{0.3 \times 0.9}{0.275} = \frac{0.27}{0.275} \approx 0.982$$

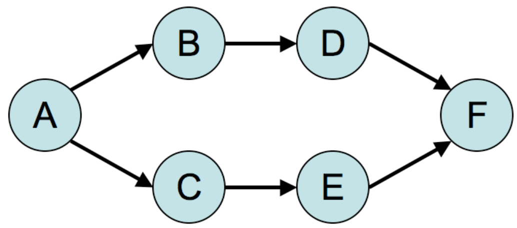
c. (4 pts.) What is the probability that a student will actually have a Low score and fail to pass their freshman year?

Answer: We can calculate this using the product rule:

$$P(L \wedge F) = P(L \mid F)\ P(F) = 0.7 \times 0.1 = 0.07$$

3. **(10 pts.)** **BAYES NETS.**

Consider the following Bayes Net:



a. (2 pts.) How many parameters are needed to specify the joint probability distribution using the above Bayes net, assuming each variable is binary (2-valued)?

Answer: since each variable is binary, each row of its truth-table needs only 1 value, and the rows correspond to all the combinations of its binary parents. Thus, the answer is $\sum_x 2^{|P(x)|}$, where $|P(x)|$ is the number of parents of variable x:

$$1 + 2 + 2 + 2 + 2 + 4 = 13$$

b. (4 pts.) Which of the following probabilistic relations are implied by the structure of the above Bayesian network? Justify your answer briefly using d-separation.

C and D are independent given B **X** Yes ___No

Reason: by the definition of d-separation, C and D are independent, since node B blocks one undirected path (type 1), and node F blocks the other (type 3).

C and F are independent given E ___Yes **X** No

Reason: while the node E blocks one path between C and F (d-separation, type 1), there is no node that blocks the other undirected path between them, and we cannot assume independence.

c. (4 pts.) Express $P(A \mid C, D)$ in terms of the probabilities directly available in the network. Simplify your answer as much as you can. (You do not need to calculate any normalization constants, α.)

Answer: Given C and D, A is independent of the nodes E and F, and so we can express our answer using only the terms:

$$\alpha\, P(A)\, P(C \mid A)\, P(D \mid B)\, P(B \mid A)$$

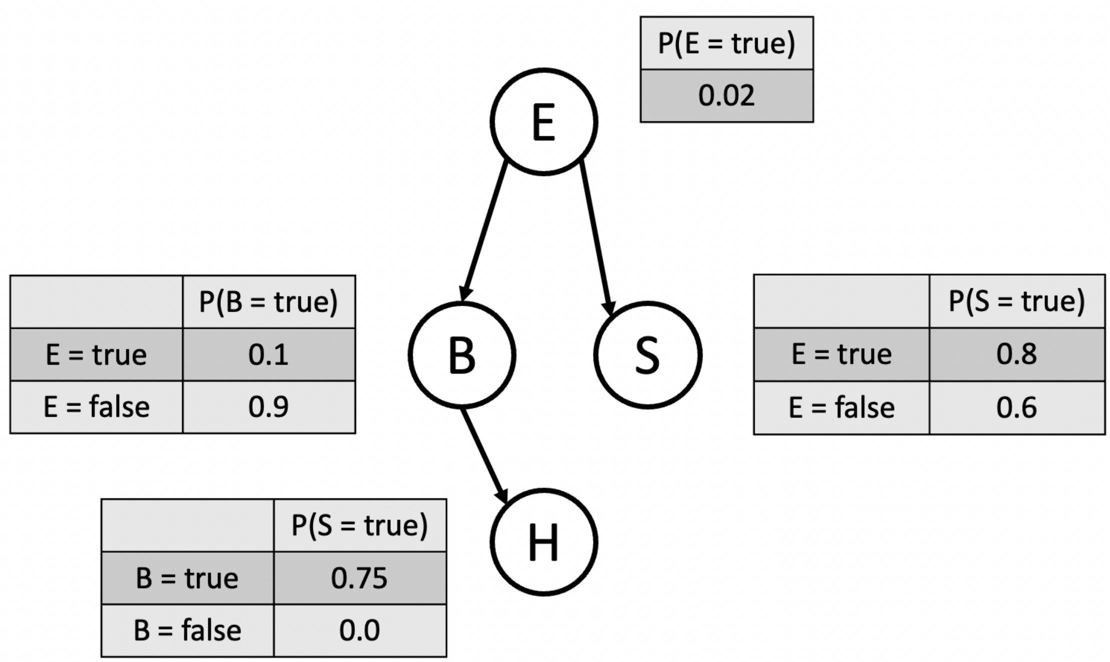
4. **(18 pts.) BAYES NETS (II).**

After escaping alive from the AI class and graduating, you take a high-paying job at a start-up in California. Unfortunately, your office was built on a fault line, which causes minor earthquakes about once every fifty days. If there is an earthquake, you would rather stay home, even if it gets you into trouble with your bosses, although you don't want to skip work too often. Earthquakes are hard to predict accurately on their own, but you have access to the Seismographic Society's web-page. Looking at their records, you see they correctly predicted 80% of earthquakes over the years; however, they also give false alarms 60% of the time.

On most mornings, you can also hear birds singing outside your window. However, the birds seem to sense when earthquakes are coming, and on days when this happens, there is a 90% chance they are silent. On other days, there is a 90% chance that they are singing; when they do so, you hear them 75% of the time (obviously, you *never* hear them when they do not sing).

a. (8 pts.) Draw a Bayes Net for this problem, including all Conditional Probability Tables and links between nodes.

Answer: If we use E to stand for *Earthquake*, S to stand for *Seismographic Society predicts an earthquake*, B to stand for *Birds sing*, and H to stand for *Hear birds singing*, then the BN for a given day's possible events is:



P(E = true)
0.02

	P(B = true)
E = true	0.1
E = false	0.9

	P(S = true)
E = true	0.8
E = false	0.6

	P(S = true)
B = true	0.75
B = false	0.0

b. (6 pts.) What is the probability of an earthquake given that no warning was issued by the Seismographic Society and you did not hear any birds? (Show all your work.)

Answer: We calculate as follows:

$$
\begin{aligned}
P(e \mid \neg s, \neg h) &= \frac{P(e, \neg s, \neg h)}{P(\neg s, \neg h)} \\
&= \frac{\sum_b P(e, \neg s, \neg h, b)}{\sum_e \sum_b P(\neg s, \neg h, e, b)} \\
&= \frac{P(\neg s \mid e)P(e) \sum_b P(\neg h \mid b)P(b \mid e)}{\sum_e P(\neg s \mid e)P(e) \sum_b P(\neg h \mid b)P(b \mid e)} \\
&= \frac{(0.2 \times 0.02)[(0.25 \times 0.1) + (1.0 \times 0.9)]}{(0.2 \times 0.02)[(0.25 \times 0.1) + (1.0 \times 0.9)] + (0.4 \times 0.98)[(0.25 \times 0.9) + (1.0 \times 0.1)]} \\
&= \frac{0.004 \times [0.025 + 0.9]}{0.004 \times [0.025 + 0.9] + 0.392 \times [0.225 + 0.1]} \\
&= \frac{0.0037}{0.0037 + 0.1274} = \frac{0.0037}{0.1311} \approx 0.028
\end{aligned}
$$

c. (4 pts.) Suppose you pay more attention and you actually hear the birds singing. What is the probability of an earthquake now?

Answer: We calculate as follows:

$$
\begin{aligned}
P(e \mid \neg s, h) &= \frac{P(e, \neg s, h)}{P(\neg s, h)} \\
&= \frac{\sum_b P(e, \neg s, h, b)}{\sum_e \sum_b P(\neg s, h, e, b)} \\
&= \frac{P(\neg s \mid e)P(e) \sum_b P(h \mid b)P(b \mid e)}{\sum_e P(\neg s \mid e)P(e) \sum_b P(h \mid b)P(b \mid e)} \\
&= \frac{(0.2 \times 0.02)[(0.75 \times 0.1) + (0.0 \times 0.9)]}{(0.2 \times 0.02)[(0.75 \times 0.1) + (0.0 \times 0.9)] + (0.4 \times 0.98)[(0.75 \times 0.9) + (0.0 \times 0.1)]} \\
&= \frac{0.0003 \times 0.075}{0.004 \times 0.075 + 0.392 \times 0.675} \\
&= \frac{0.0003}{0.0003 + 0.2646} = \frac{0.0003}{0.2649} = 0.0011325
\end{aligned}
$$