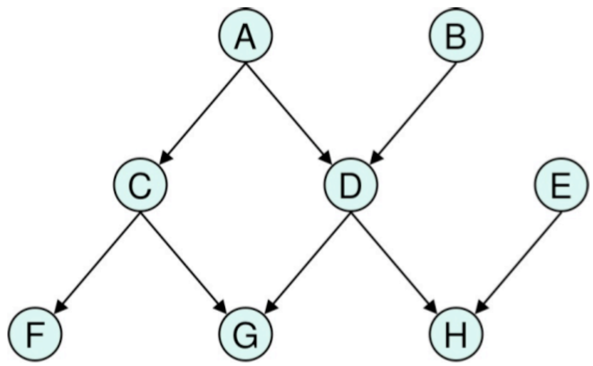
Artificial Intelligence (CS 131)
Bayes Net Exercises (Answer Key)

[**1**] (*from S. Zilberstein*) Consider the following Bayes Net:



a. There are 8 variables in the BN; if they are all Boolean (two-valued), the total joint probability distribution requires:

$$2^8 - 1 = 256 - 1 = 255 \text{ total values.}$$

For this BN, each CPT will have 2^n rows (where n is the number of parents of a given node), with 1 value per row (since every node is Boolean, we just need to give the probability $P(v = True)$, since $P(v = False) = 1 - P(v = True)$). The total is thus (taking nodes in order, $A \rightarrow H$):

$$1 + 1 + 2 + 4 + 1 + 2 + 4 + 4 = 19 \text{ total values.}$$

b. We can determine whether or not each of the probability relations given is determined by using the definition of d-separation (Lecture 13, slides 7–8).

i. $P(E \mid G) = P(E)$: **Yes**: every path from E to G is blocked in the 3rd way by H.

ii. $P(C \mid D) = P(C)$: **No**: the path $C - A - D$ is not blocked, and if we know D, it can tell us something about C.

iii. $P(C \mid D, A) = P(C \mid A)$: **Yes**: the bottom path from C to D is blocked in the 3rd way by G, and the top path is blocked in the 2nd way by A.

iv. $P(B \mid A, C) = P(B \mid A)$: **Yes**: the bottom path from B to C is blocked in the 3rd way by G, and the top path is blocked in the 2nd way by A.

v. $P(C, D \mid E) = P(C, D)$: **Yes**: every path from C or D to E is blocked in the 3rd way by H.

vi. $P(F \mid A, E, H) = P(F \mid A)$: **Yes**: the bottom path from F to E or H is blocked in the 3rd way by G, and the top path is blocked in the 2nd way by A.

vii. $P(A, C \mid D, E, H) = P(A, C \mid D)$ **Yes**: the bottom path from A or C to E or H is blocked in the 3rd way by G, and the top path is blocked in the 1st way by D.

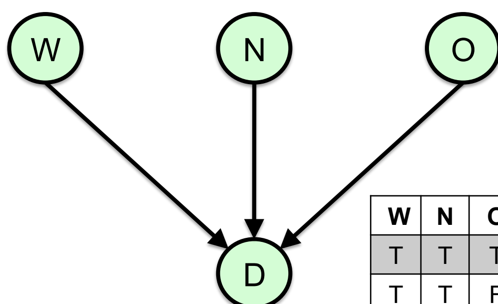
[**2**] (*from D. Precup*) In the Data problem, whether or not the next alien is carrying a weapon does not affect the overall probability that Data can overpower it, nor what sort of weapon these are; similarly we can see that whether Data overpowers the alien does not affect the sorts of weapons that are carried, and the sort of weapon does not affect whether they are carried or not, or whether Data overpowers the alien. That is, all three of the main variables are independent, and only affect whether or not Data is disabled.

a. We can thus build a BN as follows, letting the variables be:

- W: True if the alien is carrying a weapon.
- N: True if the weapon-type that the aliens carry is a neural disruptor.
- O: True if Data overpowers the alien.
- D: True if Data is disabled.

We then reason as follows. Whenever the alien is overpowered by Data, or does not have a weapon, then there is no chance Data is disabled. When the alien has a weapon, and Data does not overpower it, then the probability of being disabled is either 1.0 (when the weapon is a neural disruptor) or 0.1 (when the weapon is not a neural disruptor). The BN is therefore as follows:

P(W = True)	**P(N = True)**	**P(O = True)**
0.8	0.6	0.5



W	**N**	**O**	**P(D = True)**
T	T	T	0.0
T	T	F	1.0
T	F	T	0.0
T	F	F	0.1
F	T	T	0.0
F	T	F	0.0
F	F	T	0.0
F	F	F	0.0

b. To calculate the probability that Data is disabled (node D), given no other information, we need to conditionalize over that node's parents:

$$\begin{aligned}
P(D) &= \sum_W \sum_N \sum_O P(D \,|\, W, N, O)\, P(W, N, O) \\
&= \sum_W \sum_N \sum_O P(D \,|\, W, N, O)\, P(W)\, P(N)\, P(O) && \text{[Independence in BN]} \\
&= (1.0 \times 0.8 \times 0.6 \times 0.5) + (0.1 \times 0.8 \times 0.4 \times 0.5) \\
&= 0.24 + 0.016 = 0.256
\end{aligned}$$

c. The probability that Data is disabled, given that the alien actually has a weapon, is calculated by fixing ($W = True$) and conditionalizing on the *other parents* of D:

$$\begin{aligned}
P(D \,|\, W = T) &= \sum_N \sum_O P(D \,|\, W = T, N, O)\, P(N, O \,|\, W = T) \\
&= \sum_N \sum_O P(D \,|\, W = T, N, O)\, P(N)\, P(O) && \text{[Independence in BN]} \\
&= (1.0 \times 0.6 \times 0.5) + (0.1 \times 0.4 \times 0.5) \\
&= 0.3 + 0.02 = 0.32
\end{aligned}$$

d. The probability that the alien was not carrying a weapon, $P(W = False)$, given that Data is unharmed, $(D = False)$, is calculated using Bayes' Rule and conditionalization on the other possible values of the remaining parents of D:

$$
\begin{aligned}
P(W = F \mid D = F) &= \frac{P(D = F \mid W = F)\, P(W = F)}{P(D = F)} && \text{[Bayes' Rule]} \\
&= \frac{P(D = F \mid W = F)\, P(W = F)}{1 - P(D = T)} && \text{[Equivalent]} \\
&= \frac{P(D = F \mid W = F) \times 0.2}{0.744} \\
&= \frac{\left[\sum_N \sum_O P(D = F \mid W = F, N, O)\, P(N, O \mid W = F)\right] \times 0.2}{0.744} && \text{[Conditionalize]} \\
&= \frac{\left[\sum_N \sum_O P(D = F \mid W = F, N, O)\, P(N)\, P(O)\right] \times 0.2}{0.744} && \text{[Independence]} \\
&= \frac{\left[1.0 \times \left[(0.6 \times 0.5) + (0.6 \times 0.5) + (0.4 \times 0.5) + (0.4 \times 0.5)\right]\right] \times 0.2}{0.744} \\
&= \frac{1.0 \times 1.0 \times 0.2}{0.744} = 0.269
\end{aligned}
$$

The probability that the alien was carrying a neural disruptor, given that Data is unharmed is calculated similarly:

$$
\begin{aligned}
P(W = T, N = T \mid D = F) &= \frac{P(D = F \mid W = T, N = T)\, P(W = T, N = T)}{P(D = F)} \\
&= \frac{P(D = F \mid W = T, N = T)\, P(W = T)\, P(N = T)}{1 - P(D = T)} \\
&= \frac{P(D = F \mid W = T, N = T) \times 0.8 \times 0.6}{0.744} \\
&= \frac{\left[\sum_O P(D = F \mid O, W = T, N = T)\, P(O \mid W = T, N = T)\right] \times 0.48}{0.744} \\
&= \frac{\left[\sum_O P(D = F \mid O, W = T, N = T)\, P(O)\right] \times 0.48}{0.744} \\
&= \frac{\left[1.0 \times 0.5\right] \times 0.48}{0.744} = \frac{0.24}{0.744} = 0.323
\end{aligned}
$$