Quiz 4

Name: _*Solution*_____________________ Date: _____________________ I participated today: _____

1. What size matrix is required to fully describe the transformation a quantum operation has on an n-qubit system?

   $$2^n \times 2^n$$

2. When applying controlled Z, does it matter which qubit is the target and which is the control? In other words, does there exist some state $|\psi_1, \psi_2\rangle$ such that $CZ(\psi_1, \psi_2)$ does not produce the same state as $CZ(\psi_2, \psi_1)$?

   *The controlled Z is symmetrical; it flips the phase of the superposition term where **both** qubits are a $|1\rangle$. Therefore, it does not matter which is which. (Try proving this mathematically if you like.)*

3. In digital logic, a set of Boolean operators is *functionally complete* if its members can be used to implement any possible Boolean function. For example, the sets $\{AND, NOT\}$ and $\{NAND\}$ are both functionally complete. Does there exist a functionally complete set of quantum logic gates?

   Yes, for example, the set {CCNOT}. (Try implementing AND, NOT, and NAND using CCNOT; it is not difficult if you assume qubits can be initialized to $|1\rangle$ as well as $|0\rangle$.)

   Note that this is distinct from the notion of a universal gate set, which is a set of quantum logic gates that can get arbitrarily close to some desired quantum state with a finite number of steps. For example, {CCNOT} is not a universal gate set but {CCNOT, H} is.

4. In the superdense coding protocol, the sender seems to encode two bits of information into a single qubit. How is this possible?

   The superdense coding protocol uses an entangled pair of qubits, which means that the state of each qubit cannot be described separately, even when they are separated in space. So, though the encoding process only requires operations on one of the qubits, the information is really encoded in the two-qubit state.