Quantum Software Development

Lecture 3: Quantum Control Logic (cont.), Quantum Communication

February 7, 2024

Tufts University | School of Engineering

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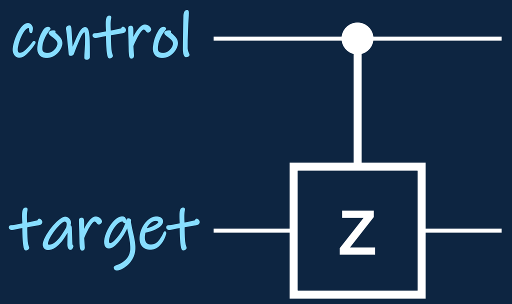
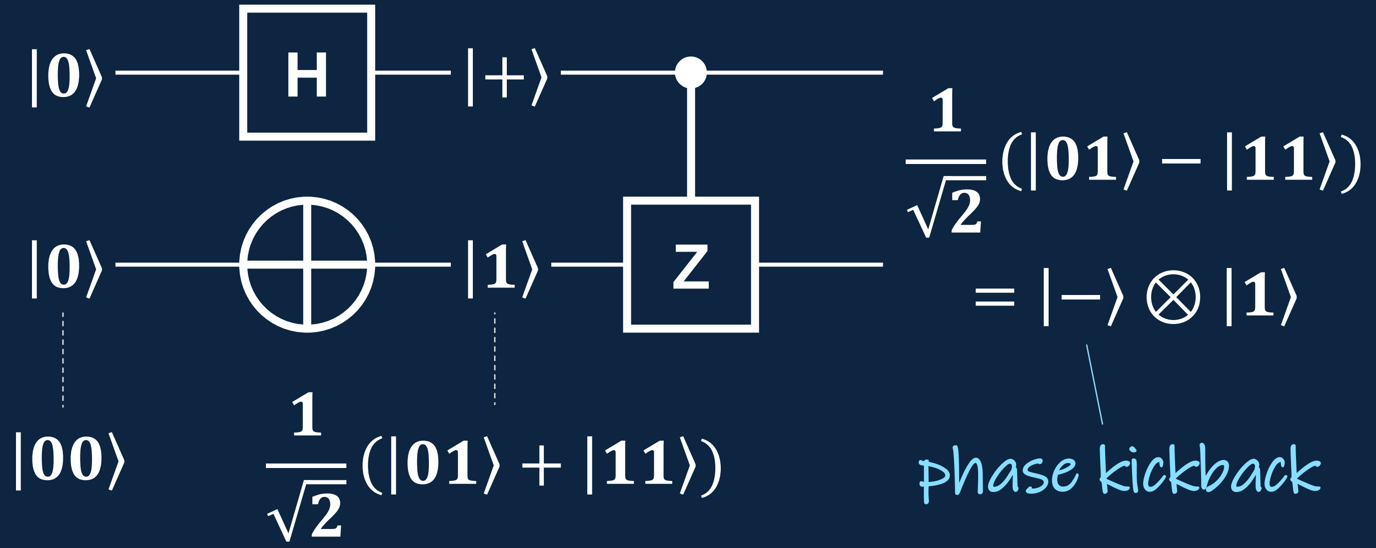
Quantum Control Logic (cont.)

What does the Controlled Z gate do?

CZ gate applies Z to target when control is $|1\rangle$



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$



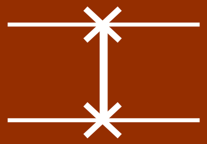
Controlled Z flips the phase of the superposition term where the control(s) and target is a $|1\rangle$.

Common Multi-Qubit Gates

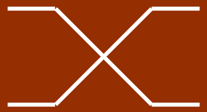
SWAP

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Swaps amplitudes of two qubits



OR



CNOT (CX)

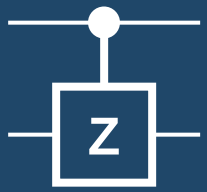
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Applies X to target when control is $|1\rangle$

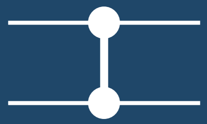
CZ

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Flips phase when both control and target are $|1\rangle$



OR



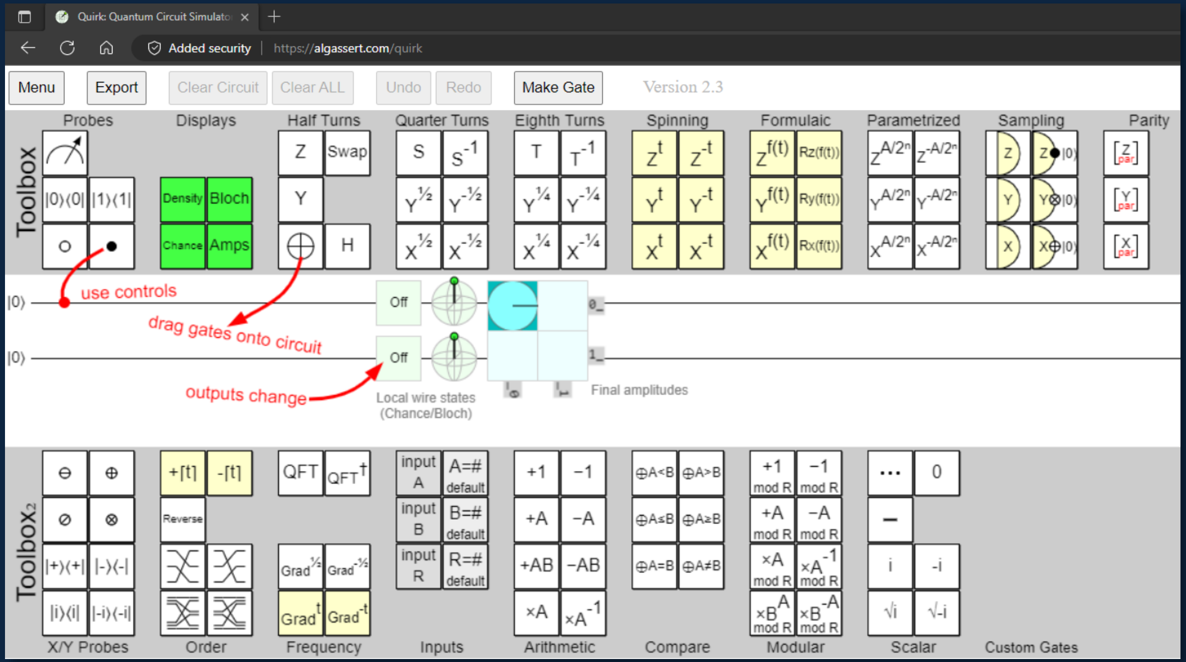
CCNOT (Toffoli)

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Applies X to target when both controls are $|1\rangle$

Try multi-qubit gates in Quirk.

- Go to https://algassert.com/quirk
- Apply controlled and SWAP gates to the first two qubits.
- Can you prepare a state that renders the Bloch sphere representation useless?
- What happens to the size of the amplitudes display as you add more qubits?



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Quantum Communication

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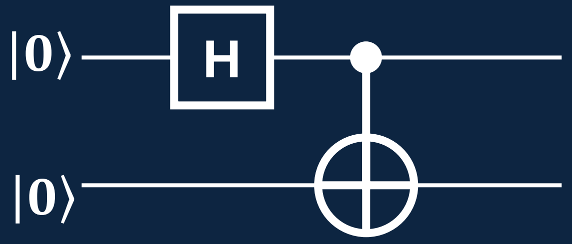
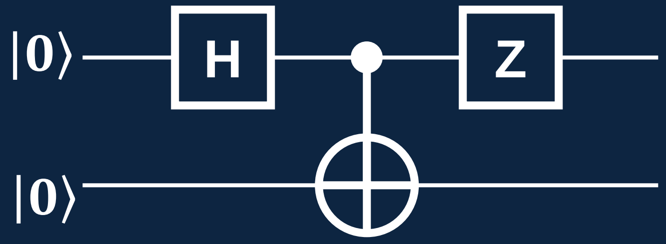
The Bell states are the four maximally entangled states for two qubits.

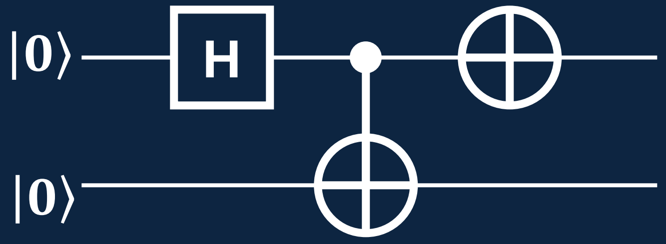
$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$



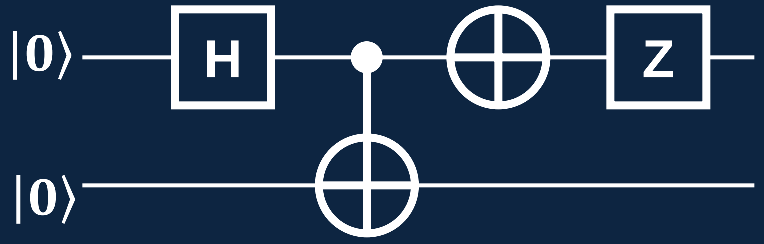
$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$



$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$



$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

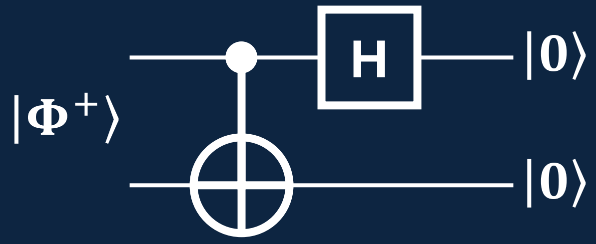


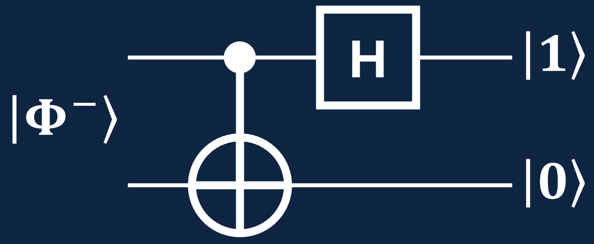
State	Extra Ops
$\|\Phi^+\rangle$	-
$\|\Psi^+\rangle$	X
$\|\Phi^-\rangle$	Z
$\|\Psi^-\rangle$	X, Z

Each of the Bell states correspond to a different value when “disentangled”.

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

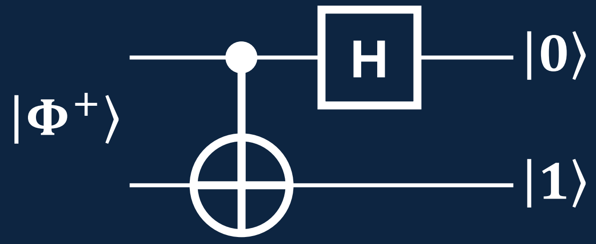
$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

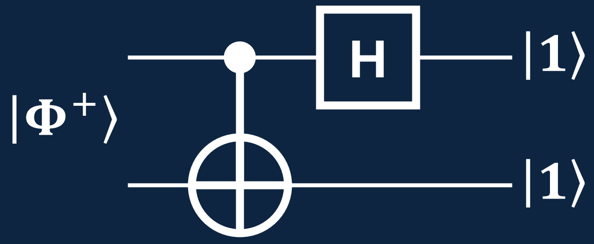




$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$





State	Value
$\lvert\Phi^+\rangle$	00
$\lvert\Psi^+\rangle$	01
$\lvert\Phi^-\rangle$	10
$\lvert\Psi^-\rangle$	11

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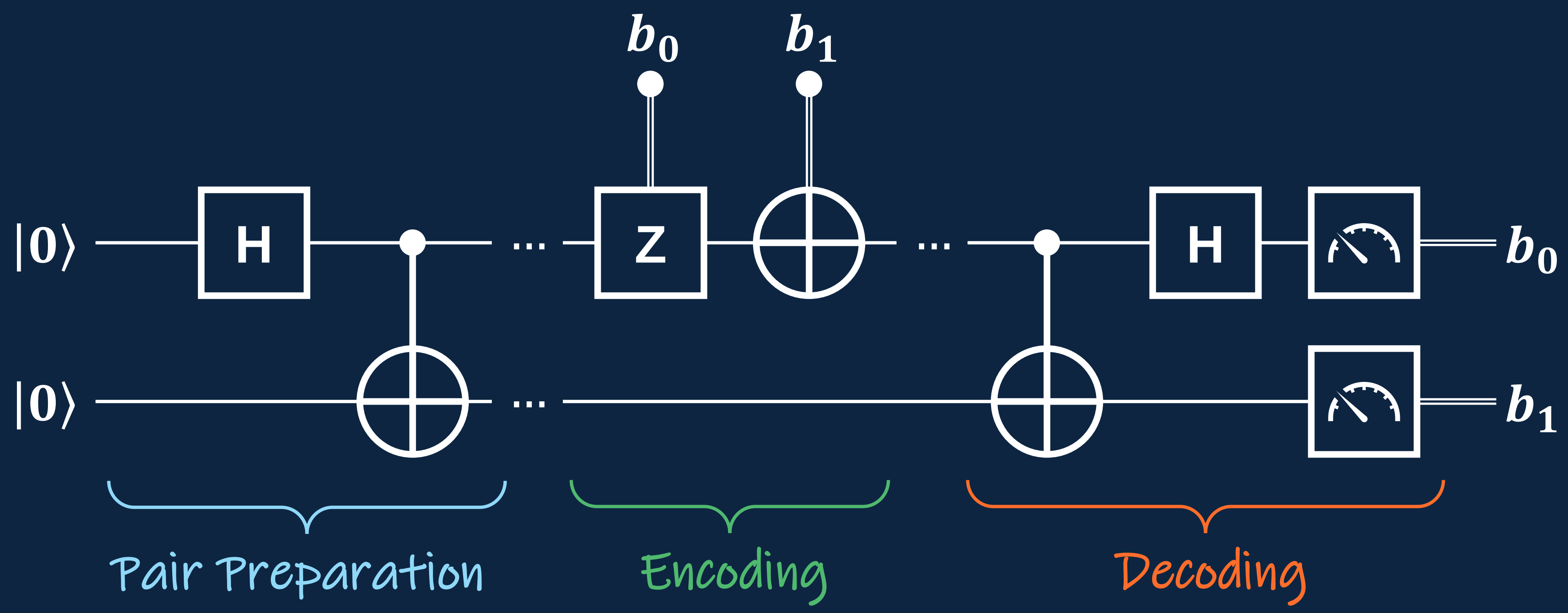
The superdense coding protocol allows two bits of information to be encoded into a pair of entangled qubits.



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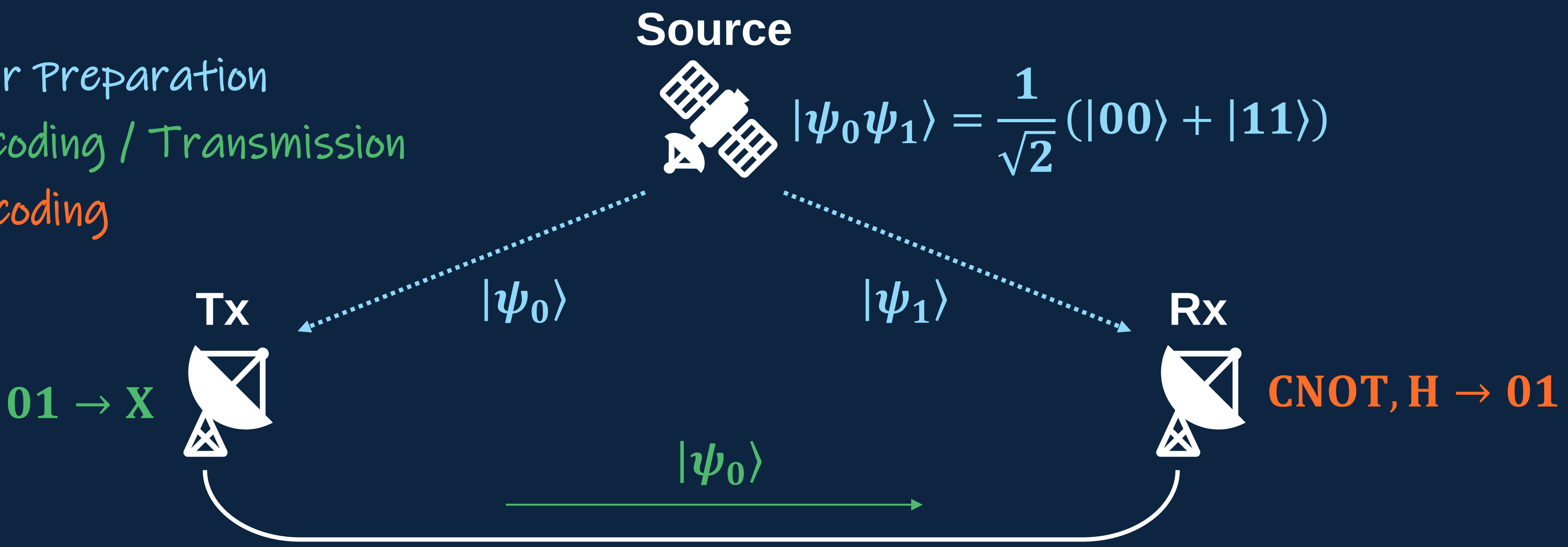
Quantum communication protocols are theoretically immune* from eavesdroppers.

***This does NOT mean systems that implement them cannot be hacked.**

1. Pair Preparation
2. Encoding / Transmission
3. Decoding



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