Midterm Exam

Name: ________________________________ Date: ______________________________

Instructions: For multiple choice questions, circle the best answer. For free response questions, circle your simplified answer. All questions are intended to have unambiguous solutions. If you think a question may be ambiguous, please ask for clarification. A single notes sheet is allowed for reference. Use of electronic devices is not permitted. Use the back of the page if you need additional workspace.

Scoring: One point is awarded for each question attempted, and one additional point for each correct answer. There are 15 questions, so there are 30 possible points total.

1. Which of the following statements is correct, regarding the difference between classical and quantum computation?

    a. Quantum computers are generally faster than conventional, classical computers.

    b. Classical computers process information using the principles of digital logic, while quantum computers process information using the principles of quantum mechanics.

    c. Since quantum computers can take advantage of quantum mechanics, they are better at correcting errors produced by quantum effects than classical computers.

    d. Quantum computers are like classical analog computers that process continuous values, but they are unlike classical digital computers that process binary digits.

2. Which of the following statements is correct, regarding the principle of superposition?

    a. A quantum state is the superposition (linear combination, or sum) of components that each correspond to a possible measurement outcome.

    b. A qubit in superposition rapidly oscillates between zero and one such that its measurement outcome cannot be predicted.

    c. When a qubit in superposition is measured, either a zero or one is observed probalistically, though the qubit remains in the superposition state.

    d. The amplitudes (coefficients) of the superposition terms in a quantum state are unconstrained.

3. Which of the following statements is correct, regarding the phenomenon of entanglement?

    a. Quantum entanglement implies that faster than light communication is possible, albeit technically very difficult to implement.

    b. Once two qubits are entangled, single-qubit gates cannot be applied to either of them.

    c. If a multi-qubit state can be described as entangled, that means measuring one of the qubits always fully determines the measurement outcomes of the other qubits.

    d. An entangled state cannot be expressed as the tensor product of independent state vectors.

4. Which of the following statements is correct, regarding the phenomenon of interference?

    a. Quantum interference effects can be simulated efficiently on a classical computer.

    b. The states of two different qubits can interfere with each other based on their phase, causing them to annihilate (destructive interference) or combine into a larger qubit (constructive interference).

    c. When a quantum operation is applied, it is distributed to each term in the superposition, and results with the same value interfere with each other (add together).

    d. Quantum interference is inherently noisy and nondeterministic, and it must be mitigated to improve coherence times and perform useful computation.

5. Which of the following statements is correct, regarding quantum communication?

    a. Quantum communication relies on entanglement and measurement to instantaneously transmit information from one party to another.

    b. A quantum communication network could be used to transmit quantum information without causing decoherence, i.e., preserving superposition.

    c. Since the security of quantum communication channels is based on the laws of physics, rather than the mathematics of an algorithm, certain quantum communication protocols are unhackable.

    d. Current proposals for quantum key distribution schemes could replace all classical information security infrastructure if implemented.

6. What is the probability of observing a $|1\rangle$ when measuring a qubit with the state $\frac{1}{\sqrt{3}}|0\rangle - \frac{i\sqrt{2}}{\sqrt{3}}|1\rangle$?

$$\left|-\frac{i\sqrt{2}}{\sqrt{3}}\right|^2 = \mathbf{\frac{2}{3}}$$

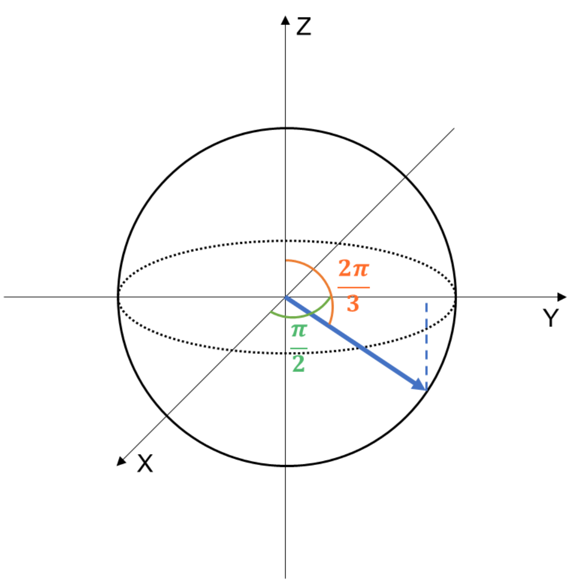
7. Let $|x\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix}, |y\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-i\end{bmatrix}$. What is the vector representation of the two-qubit state $|x, y\rangle$?

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix} \otimes \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-i\end{bmatrix} = \mathbf{\frac{1}{2}}\begin{bmatrix}\mathbf{1}\\\mathbf{-i}\\\mathbf{i}\\\mathbf{1}\end{bmatrix}$$

8. Consider the gate $T = \begin{bmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{bmatrix}$. What is $T|x\rangle$, where $|x\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + e^{\frac{-i\pi}{4}}|1\rangle\right)$?

$$\begin{bmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{bmatrix} \cdot \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ e^{\frac{-i\pi}{4}} \end{bmatrix} = \frac{\mathbf{1}}{\sqrt{\mathbf{2}}} \begin{bmatrix} \mathbf{1} \\ \mathbf{1} \end{bmatrix} = \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}(|\mathbf{0}\rangle + |\mathbf{1}\rangle)$$

9. What is the state of the qubit whose Bloch sphere coordinates are $(\theta, \varphi) = \left(\frac{2\pi}{3}, \frac{\pi}{2}\right)$, as shown below? (Give your answer in either vector or Dirac notation.)

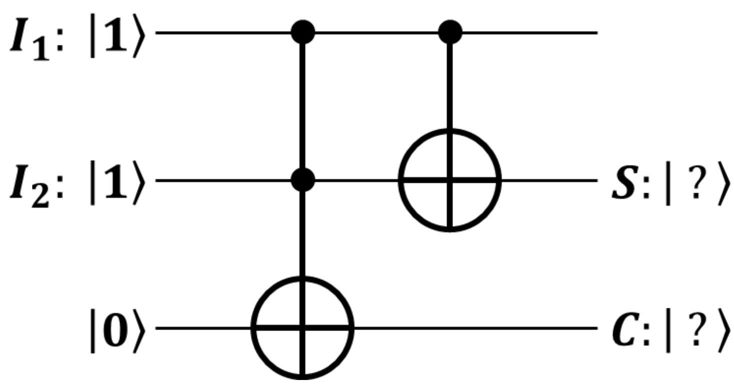


$$\begin{bmatrix} \cos\left(\frac{\pi}{3}\right) \\ e^{i\frac{\pi}{2}} \sin\left(\frac{\pi}{3}\right) \end{bmatrix} = \begin{bmatrix} \frac{\mathbf{1}}{\mathbf{2}} \\ \frac{\boldsymbol{i}\sqrt{\mathbf{3}}}{\mathbf{2}} \end{bmatrix}$$

10. Referring to the single-qubit state depicted in question 9, what single parametrized gate could be used to prepare that state, and what is the parameter value (angle of rotation)? (Assume a starting state of $|0\rangle$.)

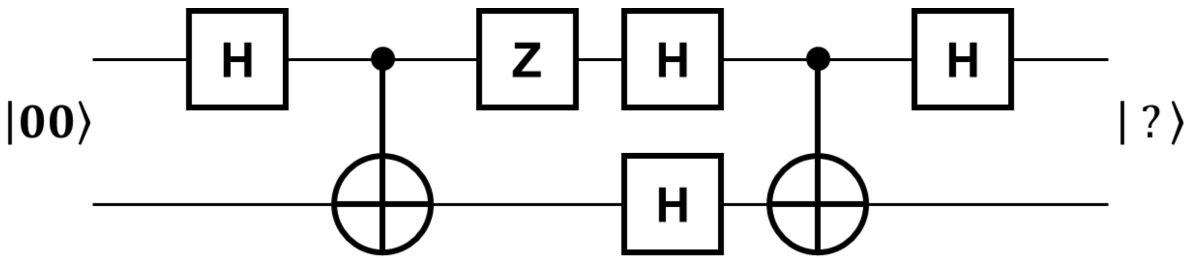
$$\boldsymbol{R_x}\left(-\frac{\mathbf{2\pi}}{\mathbf{3}}\right)\ or\ \boldsymbol{R_x}\left(\frac{\mathbf{4\pi}}{\mathbf{3}}\right)$$

11. What is the state of S and C after the circuit below has been run? (Note that I_1 and I_2 are initialized to $|1\rangle$.)



$$\boldsymbol{S}: |\mathbf{0}\rangle, \boldsymbol{C}: |\mathbf{1}\rangle$$

12. What is the state of the two-qubit system after the circuit below has been run?



(Note that this is a challenging problem.) After first H and CNOT, state is $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$*. Z makes it* $\frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$*. Hadamard transform makes it:*

$$\frac{1}{2} \cdot \frac{1}{\sqrt{2}} \left(\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} \right) = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

CNOT makes it $\frac{1}{\sqrt{2}}(|01\rangle + |11\rangle) = \frac{1}{\sqrt{2}}(|0\rangle + 1\rangle) \otimes |1\rangle$*. H makes it* $|\mathbf{01}\rangle$*.*

13. What is the output of the following Q# snippet? (Assume the appropriate scaffolding is in place to compile and run the code.)

```
use qubit = Qubit() {
    H(qubit);
    Z(qubit);
    H(qubit);
    Message($"{M(qubit)}");
}
```

a. “Zero”

b. “One”

c. “Zero” or “One” with 50% probability

d. Compile error

e. Runtime error

14. Consider the Q# operation below that takes an n-qubit register in a uniform superposition and flips the phase of the larger-valued half of the superposition terms. That is, it transforms the state:

$$\frac{1}{\sqrt{N}}\sum_{i=0}^{N-1}|i\rangle$$

into:

$$\frac{1}{\sqrt{N}}\left(\sum_{i=0}^{\frac{N}{2}-1}|i\rangle-\sum_{j=\frac{N}{2}}^{N-1}|j\rangle\right)$$

where $N = 2^n$.

```
/// # Summary
/// Transforms a register in uniform superposition into the state:
///
///   1/√N(|0> + |1> +  ... + |N/2-1> - |N/2> - |N/2 + 1> - ... - |N-1>)
///
/// where N = 2^(Length(register)).
///
/// # Input
/// ## register
/// A register of unknown length. All of its qubits are in the |+> state.
operation PhaseFlipLargerHalf (register : Qubit[]) : Unit {
    // TODO
}
```

Which of the following correctly implements the operation?

a. **`Z(register[0]);`**

b. `Z(register[Length(register)-1]);`

c. `ApplyToEach(Z, register[Length(register)/2 .. Length(register)-1]);`

d. `ApplyToEach(Z, register[1 .. 2 .. Length(register)-1]);`

e. The state cannot be prepared in Q#.

15. Consider the Q# operation below that flips a target qubit when the register value is $|01\rangle$, without modifying the register. That is, it transforms the state:

$$|register, target\rangle = a|00,0\rangle + b|01,0\rangle + c|10,0\rangle + d|11,0\rangle$$

into:

$$|register, target\rangle = a|00,0\rangle + b|01,1\rangle + c|10,0\rangle + d|11,0\rangle$$

for any $a, b, c, d \in \mathbb{C}$ such that $|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$.

```
/// # Summary
/// Flips a target qubit when the register value is |01>, without
/// modifying the register.
///
/// # Input
/// ## register
/// 2-qubit register in an unknown state.
///
/// ## target
/// Single qubit in the |0> state.
operation Check01 (register : Qubit[], target : Qubit()) : Unit {
    // TODO
}
```

Which of the following correctly implements the operation?

a.
```
if M(register[0]) == Zero and M(register[1]) == One {
     X(target);
 }
```

b.
```
CNOT(register[1], target);
```

c.
```
X(register[0]);
CNOT(register[0], target);
CNOT(register[1], target);
X(register[0]);
```

d.
```
CCNOT(register[0], register[1], target);
X(register[0]);
```

e.
```
X(register[0]);
Controlled X(register, target);
X(register[0]);
```