Final Project

Due Friday, May 3rd at 11:59 PM ET.

Assignment

The final project assignment is to study a quantum algorithm through software implementation and present your results in a video. How you approach the assignment is up to you, but a recommended breakdown is given below.

1. Select a quantum algorithm to study. Several options are provided in the last section.
   It's a good idea to check with the instructor if you are going to stray from the list.
2. Think through the high-level design of the program on paper.
   A good way to tell if you understand how your design works is by explaining it to someone else.
3. Write a Q# program that verifiably implements the algorithm.
   Write and run unit tests with the simulator to show your implementation works as expected.
4. Use the built-in resource estimator to obtain precise metrics for the computational resources required by your program, and/or run your program on a real quantum computer and capture the results.
5. Create a <10-minute video presenting your work to a technical audience. The grading rubric is provided in the next section.
   It is to your benefit if the video is accessible enough to be shared with others.

Grading

The project grade is based on how well your video answers the questions below. The questions are driven by what information is important to a typical viewer. Each question is scored out of 5 points. The total number of points is divided by 30 to calculate the final project score.

1. Who are you? What is the purpose of this video?
2. What is the background and context of what you're talking about? Am I (the viewer) going to be able to understand this?
3. What were you setting out to do? What challenge were you trying to overcome?
4. How did you approach it? What did you accomplish?
5. So what? Who cares? What is the significance of what you did?
6. What is the main takeaway? What did you learn?

Algorithm List

Each algorithm listed includes references to papers containing circuit diagrams that show how they could be implemented. For a more comprehensive survey, see https://quantumalgorithmzoo.org/.

Integer Arithmetic

Adder

This circuit adds two arbitrarily large integers together.

- Vedral, V. & Ekert, A. Quantum Networks for Elementary Arithmetic Operations. Phys. Rev. A 54, 147 (1996). https://doi.org/10.1103/PhysRevA.54.147 (Alternative: https://arxiv.org/abs/quant-ph/9511018v1)
- Luo, J., Zhou, RG., Luo, G. et al. *Traceable Quantum Steganography Scheme Based on Pixel Value Differencing.* Sci Rep 9, 15134 (2019). https://doi.org/10.1038/s41598-019-51598-8
- Ma Y., Ma, H. and Chu, P. *Demonstration of Quantum Image Edge Extration Enhancement Through Improved Sobel Operator.* IEEE Access, vol. 8, pp. 210277-210285, 2020. https://doi.org/10.1109/ACCESS.2020.3038891

Two's Complement

This circuit turns an integer into its negative version, represented using the standard two's complement form. This is used in combination with the adder above to form a subtractor.

- Luo, G., Zhou, RG. & Hu, W. *Efficient quantum steganography scheme using inverted pattern approach*. Quantum Inf Process 18, 222 (2019). https://doi.org/10.1007/s11128-019-2341-3
- Ma Y., Ma, H. and Chu, P. *Demonstration of Quantum Image Edge Extration Enhancement Through Improved Sobel Operator.* IEEE Access, vol. 8, pp. 210277-210285, 2020. https://doi.org/10.1109/ACCESS.2020.3038891

Reverse Parallel Subtractor

This is an in-place version of an integer subtractor that's integrated together into a nice, neat package.

- Zhou, R.-G., Hu, W., Luo, G., Liu, X., Fan, P. *Quantum realization of the nearest neighbor value inter- polation method for INEQR*. Quantum Inf. Process. 17, 166 (2018). https://doi.org/10.1007/s11128-018-1921-y

Cyclic / Positional Shift Transformation

This circuit is used to move the elements of an array up or down by some number of indices. For example, it could move everything 2 positions to the left or to the right. Elements will "wrap around", so if you move all of them one position to the right, the old last element will become the new first element.

- Le, P. Q., Iliyasu, A. M., Dong, F. & Hirota, K. *Strategies for designing geometric transformations on quantum images*. Theor. Comput. Sci. 412, 1406–1418 (2011). https://doi.org/10.1016/j.tcs.2010.11.029

- Luo, J., Zhou, RG., Luo, G. et al. *Traceable Quantum Steganography Scheme Based on Pixel Value Differencing*. Sci Rep 9, 15134 (2019). https://doi.org/10.1038/s41598-019-51598-8

Divider

This circuit performs integer division, producing both the quotient and the remainder.

- Khosropour, A., Aghababa, H. & Forouzandeh, B. *Quantum Division Circuit Based on Restoring Division Algorithm*. 2011 Eighth Int. Conf. Inf. Technol. New Gener. 3–6, (2011). https://doi.org/10.1109/ITNG.2011.177
- Thapliyal, H., Munoz-Coreas, E., T. S. S. Varun and T. S. Humble, "Quantum Circuit Designs of Integer Division Optimizing T-count and T-depth," in IEEE Transactions on Emerging Topics in Computing, vol. 9, no. 2, pp. 1045-1056, 1 April-June 2021. https://doi.org/10.1109/TETC.2019.2910870 (Alternative: https://arxiv.org/abs/1809.09732)

Thresholder / Comparator

This circuit has a few forms. In the "single register" mode, it takes a register and an ancilla qubit and "thresholds" the register, so for values where the register is greater than some specified integer, the ancilla will be flipped to 1. In the "double register" mode, it compares two registers and flags some ancilla qubits based on the result.

- Ma Y., Ma, H. and Chu, P. *Demonstration of Quantum Image Edge Extration Enhancement Through Improved Sobel Operator*. IEEE Access, vol. 8, pp. 210277-210285, 2020. https://doi.org/10.1109/ACCESS.2020.3038891
- Luo, G., Zhou, RG. & Hu, W. *Efficient quantum steganography scheme using inverted pattern approach*. Quantum Inf Process 18, 222 (2019). https://doi.org/10.1007/s11128-019-2341-3

QFT-Based Arithmetic

The paper below shows how to efficiently perform several arithmetic operations using the QFT.

- Ruiz-Perez, L. & Garcia-Escartin J. C. *Quantum arithmetic with the Quantum Fourier Transform*. Quantum Inf Process 16, 152 (2017). https://arxiv.org/abs/1411.5949

Classical Data Encoding

Flexible Representation of Quantum Images (FQRI)

This method uses a single register to represent the index of an element in a 1D array. The value of the element at that index is encoded as the amplitude of the index register. In other words, the entire classical array is stored in the amplitudes of the corresponding states at each index.

- Le, P. Q., Dong, F. & Hirota, K. *A flexible representation of quantum images for polynomial preparation, image compression, and processing operations*. Quantum Inf. Process. 10, 63–84 (2011). https://doi.org/10.1007/s11128-010-0177-y

Novel Enhanced Quantum Representation (NEQR) / Generalized Quantum Image Representation (GQIR)

NEQR stores classical data in two registers: an index and a value. The two are entangled together so the value register will contain the classical data value at index i when the index register is in state i. Formally, this will represent the classical data (call it c) in the superposition $|i, c[i]\rangle$ with uniform amplitudes for each term.

NEQR is limited to square images with a power of 2 as the width; GQIR is simply a generalized form that works with classical data of arbitrary sizes.

- Zhang, Y., Lu, K., Gao, Y. & Wang, M. *NEQR: A novel enhanced quantum representation of digital images.* Quantum Inf. Process. 12, 2833–2860 (2013). https://doi.org/10.1007/s11128-013-0567-z
- Jiang, N., Wang, J. & Mu, Y. *Quantum image scaling up based on nearest-neighbor interpolation with integer scaling ratio*. Quantum Inf Process 14, 4001–4026 (2015). https://doi.org/10.1007/s11128-015-1099-5

General Algorithms

Quantum Phase Estimation (QPE)

QPE is used to estimate the eigenvector of a quantum operation. This is a fundamentally useful thing in a wide variety of applications. For example, Shor's algorithm uses QPE to get the value of the period of modular exponentiation.

- Qiskit textbook section on QPE

Amplitude Amplification

This is a generalized form of Grover's algorithm that works when there's more than one "correct" answer to a problem. It amplifies the amplitude of all of the correct answers equally.

- Brassard, G., Hoyer, P., Mosca, M. & Tapp, A. *Quantum Amplitude Amplification and Estimation.* Quantum Computation and Quantum Information, Samuel J. Lomonaco, Jr. (editor), AMS Contemporary Mathematics, 305:53-74, 2002. https://doi.org/10.1090/conm/305/05215 (Alternative: https://arxiv.org/abs/quant-ph/0005055)

Amplitude Estimation / Quantum Counting

This is a general-purpose algorithm that retrieves the amplitude of a given state (or rather, the aggregated amplitude for all of the "correct" states in the amplitude amplification algorithm). It is a combination of Grover's and Shor's (or more accurately amplitude amplification and QPE), though the last paper in this list describes a way to do it without QPE.

This is also used to calculate the number of correct states, which is helpful in many computational fields. When used in this way, amplitude estimation is known as the quantum counting algorithm.

- Brassard, G., Hoyer, P., Mosca, M. & Tapp, A. *Quantum Amplitude Amplification and Estimation*. Quantum Computation and Quantum Information, Samuel J. Lomonaco, Jr. (editor), AMS

Contemporary Mathematics, 305:53-74, 2002. https://doi.org/10.1090/conm/305/05215 (Alternative: https://arxiv.org/abs/quant-ph/0005055)

- Stamatopoulos, N., Egger, D., Sun, Y. et al. *Option Pricing using Quantum Computers*. Quantum 4, 291 (2020). https://doi.org/10.22331/q-2020-07-06-291
- Suzuki, Y., Uno, S., Raymond, R. et al. *Amplitude estimation without phase estimation*. Quantum Inf Process 19, 75 (2020). https://doi.org/10.1007/s11128-019-2565-2

Variational Quantum Eigensolver

VQE is an algorithm with a lot of potential use cases. It can be used to solve optimization problems, including solving the structure and details of interesting new molecules. It is being investigated for materials science applications and is one of the most promising quantum algorithms for near-term usage because it requires a small number of qubits.

- Qiskit textbook section on VQE

Quantum Linear-Solving Algorithm (QLSA / HHL)

The Quantum Linear-Solving Algorithm, also named the HHL algorithm after its authors, uses a quantum computer to solve a system of linear equations. It works in a similar way to the classical step of Simon's algorithm, but on general-purpose equations. This has usage in many fields, from finance simulation to fluid dynamics.

- Qiskit textbook section on QLSA / HHL

Quantum Approximate Optimization Algorithm

QAOA is an interesting algorithm that finds an "approximately optimal" solution to a combinatorial optimization problem. You can adjust how "approximate" the solution is; the closer you get to the ideal option, the longer it'll take. So, if you just need something "good enough", this can find it for you in a reasonable amount of time.

- Qiskit Textbook Section on QAOA

Quantum Neural Networks

There are a lot of methods and algorithms to run a neural network on a quantum computer. The one below provides a hybrid method that uses a quantum subroutine as part of a feed-forward neural network. Note that you'll need to have some understanding of machine learning theory already to use this algorithm.

- Qiskit textbook section on hybrid quantum neural networks

Applied Algorithms

Variational Quantum Classifier

This is a machine-learning algorithm that provides a neural network with some data and asks it to make a binary choice based on the data. In other words, it attempts to sort the entity the data belongs to into one of two groups. For example, given a picture, tell whether it is a picture of a cat or not. Or, given this set of health metrics, tell whether or not this person is at high risk for a heart attack.

- Rodney David's introductory article on VQC

Quantum Kernel Estimation

This algorithm is used in machine learning to augment Support Vector Machines. It finds the kernel functions for general-purpose nonlinear classifier problems, which are used in pattern recognition systems to determine which group/label a particular input belongs to.

- Havlíček, V., Córcoles, A.D., Temme, K. et al. *Supervised learning with quantum-enhanced feature spaces*. Nature 567, 209–212 (2019). https://doi.org/10.1038/s41586-019-0980-2

Variational Quantum Linear Solver

This algorithm uses VQE to solve a linear system of equations. This is similar to HHL/QLSA, but can be run with much fewer qubits, so it's closer to being used practically.

- Qiskit textbook section on the VQLS algorithm

Fast Poisson Solver

The Poisson equation has applications across many areas of physics and engineering, such as the dynamic process simulation of ocean current. This paper presents a quantum algorithm for solving the Poisson equation, as well as a complete and modular circuit design. It's based on the QLSA/HHL but offers several improvements that reduce the overall circuit complexity.

- Wang, S., Wang, Z., Li, W. et al. *Quantum fast Poisson solver: the algorithm and complete and modular circuit design*. Quantum Inf Process 19, 170 (2020). https://doi.org/10.1007/s11128-020-02669-7

Least Squares solver

This algorithm solves the least squares problem that fits a curve to a set of data points. It uses HHL/QLSA, amplitude estimation, and Grover's algorithm together to do this faster than a classical computer.

- Shao, C., Xiang, H. *Quantum regularized least squares solver with parameter estimate*. Quantum Inf Process 19, 113 (2020). https://doi.org/10.1007/s11128-020-2615-9

Image Segmentation Filter

This algorithm provides a dual-threshold filter that clips values of an array that are either too low or too high, leaving only values that are in the middle. This is useful across a range of data processing tasks, such as audio or image processing.

- Yuan, S., Wen, C., Hang, B. et al. *The dual-threshold quantum image segmentation algorithm and its simulation*. Quantum Inf Process 19, 425 (2020). https://doi.org/10.1007/s11128-020-02932-x

JPEG Decompression

One of the hardest problems in quantum computing right now is finding an efficient way to encode classical data into a quantum superposition, so that we can take advantage of quantum data processing. This method uses the JPEG compression scheme to encode/compress the classical data and implements a JPEG decompressor on the quantum side to decompress it and retrieve the "original" data (though it is in a lossy form, so it's only an approximation of the original data). It's mainly used in image and video processing.

- Jiang, N., Lu, X., Hu, H. et al. *A Novel Quantum Image Compression Method Based on JPEG*. Int J Theor Phys 57, 611–636 (2018). https://doi.org/10.1007/s10773-017-3593-2

Steganography

Steganography is the practice of hiding a "secret" message within another "cover" (disguise) message, such as hiding secret text inside of an image that otherwise looks totally normal. These two papers offer different techniques for accomplishing this using quantum computers.

- Luo, G., Zhou, RG. & Hu, W. *Efficient quantum steganography scheme using inverted pattern approach*. Quantum Inf Process 18, 222 (2019). https://doi.org/10.1007/s11128-019-2341-3
- Luo, J., Zhou, RG., Luo, G. et al. *Traceable Quantum Steganography Scheme Based on Pixel Value Differencing*. Sci Rep 9, 15134 (2019). https://doi.org/10.1038/s41598-019-51598-8

Image Classification

This gives a way of classifying images (determining which set or label they belong to, such as "this is an airplane" vs. "this is a leopard") faster than classical machine learning systems. It uses the quantum comparator and amplitude estimation algorithms in clever ways to do it.

- Dang, Y., Jiang, N., Hu, H. et al. *Image classification based on quantum K-Nearest-Neighbor algorithm*. Quantum Inf Process 17, 239 (2018). https://doi.org/10.1007/s11128-018-2004-9

Edge Extraction Filter

This algorithm processes an image by setting all the pixels representing object edges to white, and other pixels to black. This is an important operation in computer vision and machine learning. The papers below provide ways to do this and claim that this is more accurate than classical methods.

- Zhou, RG., Liu, DQ. *Quantum Image Edge Extraction Based on Improved Sobel Operator*. Int J Theor Phys 58, 2969–2985 (2019). https://doi.org/10.1007/s10773-019-04177-6
- Ma Y., Ma, H. and Chu, P. *Demonstration of Quantum Image Edge Extration Enhancement Through Improved Sobel Operator*. IEEE Access, vol. 8, pp. 210277-210285, 2020. https://doi.org/10.1109/ACCESS.2020.3038891