Quiz 1

Name: _Solution_ ______________________ Date: ______________________ I participated today: ______

1. What is the difference between classical and quantum computation?

   Classical computers process information using the principles of digital logic, while quantum computers process information using the principles of quantum mechanics.

2. When are assignments due in a typical week? Are late assignments accepted?

   Assignments are due on Wednesdays at 4:30pm, with an automatic extension to 11:59pm. Assignments submitted up to one week late after the due date will be accepted for half credit. Assignments more than one week late will not be accepted.

3. Let $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$, $|x\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$. $A|x\rangle = ?$

$$A|x\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot \left(-\frac{1}{\sqrt{2}}\right) \end{bmatrix} = \begin{bmatrix} \frac{1}{2} + \frac{1}{2} \\ \frac{1}{2} - \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

4. Let $|x\rangle = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $|y\rangle = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$. Which of the following expressions denote the tensor product of $|x\rangle$ and $|y\rangle$? (Circle all that apply.)

   a. $|\boldsymbol{x}\rangle \otimes |\boldsymbol{y}\rangle$

   b. $|\boldsymbol{x}\rangle|\boldsymbol{y}\rangle$

   c. $|\boldsymbol{x}, \boldsymbol{y}\rangle$

   d. $|\boldsymbol{xy}\rangle$

   e. $\langle x|y\rangle$

   f. $\begin{bmatrix} \boldsymbol{x_1} \\ \boldsymbol{x_2} \end{bmatrix} \otimes \begin{bmatrix} \boldsymbol{y_1} \\ \boldsymbol{y_2} \end{bmatrix}$

   g. $\begin{bmatrix} \boldsymbol{x_1 y_1} \\ \boldsymbol{x_1 y_2} \\ \boldsymbol{x_2 y_1} \\ \boldsymbol{x_2 y_2} \end{bmatrix}$

   h. $x_1 y_1 + x_2 y_2$

5. What does it mean for a qubit to be "in superposition"?

   A qubit is said to be "in superposition" if its $|0\rangle$ *and* $|1\rangle$ *amplitudes are both nonzero.*

6. True or False: Measuring a qubit in superposition necessarily changes its state.

   True. If a qubit is in superposition, then measuring it would cause its state to change to a $|0\rangle$ *or* $|1\rangle$ *probabilistically.*

7. What is the probability of observing a $|1\rangle$ when measuring a qubit with the state $\frac{3}{5}|0\rangle + \frac{4}{5}|1\rangle$?

$$\left|\frac{4}{5}\right|^2 = \frac{16}{25} = 64\%$$

8. Which of the following could represent the state of a single qubit? (Circle all that apply.)

   a. $|\mathbf{0}\rangle$

   b. $|0\rangle + |1\rangle$

   c. $\frac{\mathbf{1}}{\sqrt{\mathbf{2}}}(|\mathbf{0}\rangle + \boldsymbol{i}|\mathbf{1}\rangle)$

   d. $\frac{\mathbf{1}}{\mathbf{2}}|\mathbf{0}\rangle - \frac{\sqrt{\mathbf{3}}}{\mathbf{2}}|\mathbf{1}\rangle$

   e. $2|0\rangle - \sqrt{3}\,|1\rangle$

   f. $\mathbf{cos}\left(\frac{\boldsymbol{\pi}}{\mathbf{12}}\right) \cdot |\mathbf{0}\rangle + \mathbf{sin}\left(\frac{\boldsymbol{\pi}}{\mathbf{12}}\right) \cdot |\mathbf{1}\rangle$

9. What is the difference between a digital logic gate and a quantum logic gate?

   A digital logic gate is defined with a truth table, while a quantum logic gate is defined with a matrix.

10. What is the result of applying an X gate to a qubit with the state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$? (Recall the X gate is defined as the matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.)

    There are multiple ways to look at this. One is to distribute the gate to each term of the superposition like this: $X\left[\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right] = \frac{1}{\sqrt{2}}(X|0\rangle + X|1\rangle) = \frac{1}{\sqrt{2}}(|1\rangle + |0\rangle)$*, i.e. the state is unchanged.*