# Quiz 3

Name: *Solution* ____________________ Date: ______________________ I participated today: _____

1. How many vector components are required to represent the state of a system of $n$ qubits? In other words, what is the dimensionality of an $n$-qubit state vector?

$$2^n$$

2. What is the complete state vector for two $|i\rangle$ qubits? (Recall that $|i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$.) Give your answer in Dirac notation.

$$|i\rangle \otimes |i\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\ i\end{bmatrix} \otimes \frac{1}{\sqrt{2}}\begin{bmatrix}1\\ i\end{bmatrix} = \frac{1}{2}\begin{bmatrix}1\\ i\\ i\\ -1\end{bmatrix} = \frac{1}{2}(|00\rangle + i|01\rangle + i|10\rangle - |11\rangle)$$

3. What is the meaning of the $n$-qubit state below? (Hint: Plug in small values for $n$ and write out the resulting superposition.)

$$\frac{1}{\sqrt{2^{n-1}}}\sum_{i=0}^{2^{n-1}-1}|2i\rangle$$

*This is a superposition of the even values of a register, i.e., the values where the last qubit is $|0\rangle$. For example, if $n = 2$, the expression evaluates to $\frac{1}{\sqrt{2}}(|0\rangle + |2\rangle) = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$.*

4. Suppose a register of $n$ qubits is in a uniform superposition $\frac{1}{\sqrt{2^n}}\sum_{i=0}^{2^n-1}|i\rangle$, and then a $Z$ gate is applied to each qubit. Which terms of the superposition have their sign flipped? (Hint: Again, try small values for $n$ and see what happens.)

*The terms with an odd number of ones have their sign flipped. That's because each Z gate flips the sign wherever the target qubit is a $|1\rangle$. For example, if $n = 2$, the start state is $\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$. After the Z gates are applied, we get $\frac{1}{2}(|00\rangle + (-1)|01\rangle + (-1)|10\rangle + (-1)(-1)|11\rangle) = \frac{1}{2}(|00\rangle - |01\rangle - |10\rangle + |11\rangle)$.*

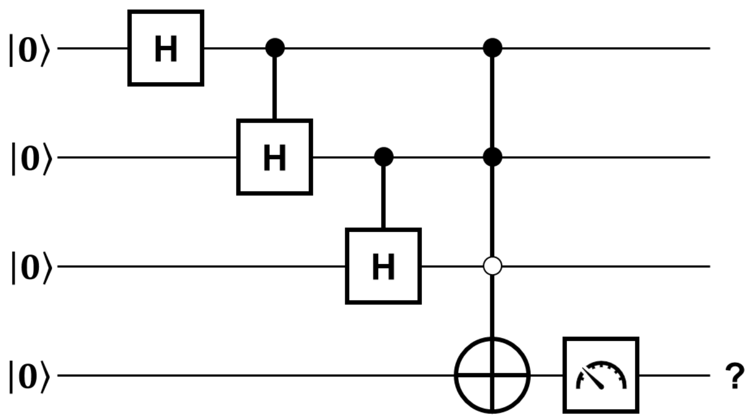
5. What does it mean for the states of two qubits to become "entangled"?

   *The states of two qubits become entangled when they cannot be described separately, i.e., factored into two independent state vectors.*

6. True or False: The state $\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle - |11\rangle)$ is an example of entanglement.

   *True. There is no way to factor this two-qubit state into two independent single-qubit states. When the first qubit is measured, if it is a* $|0\rangle$, *then the second qubit is a* $|+\rangle$; *if it is a* $|1\rangle$, *then the second qubit is a* $|-\rangle$.

7. What is the probability of measuring a $|1\rangle$ in the circuit below?




   *The state of the system before measurement is* $\frac{1}{\sqrt{2}}|0000\rangle + \frac{1}{\sqrt{4}}|1000\rangle + \frac{1}{\sqrt{8}}|1101\rangle + \frac{1}{\sqrt{8}}|1110\rangle$. *So, the probability of measuring a* $|1\rangle$ *on the fourth qubit is* $\frac{1}{8}$ *or 12.5%.*

8. Referring to the circuit above, suppose a $|1\rangle$ is actually measured. What is the state of the four-qubit system then?

   *Given a* $|1\rangle$ *was measured, the only possibility for the other qubits is* $|110\rangle$. *So, the state of the four qubits is* $|1101\rangle$.