E&M II (PHYS7120 - PHYS/ASTR 7130)

Homework #1

I. PROBLEM 1 (10 POINTS)

Define the integral $2\pi\delta(t) = \int_{-\infty}^{\infty} d\omega e^{-i\omega t}$ by a limit process using $e^{-\epsilon|\omega|}$, and show that the δ-function may be defined by the expression:

$$\delta(t) = \lim_{\epsilon \to 0} [\frac{\epsilon}{\pi(\epsilon^2 + t^2)}] \tag{1}$$

II. PROBLEM 2 (10 POINTS)

Find the Fourier transform of the triangular pulse $f(x) = h(1 - a|x|)$ for $|x| < 1/a$ and $f(x) = 0$ for $|x| > 1/a$
Note: That this function provides an alternative definition of the δ-function if $a \to \inf$ and $h = a$.

III. PROBLEM 3 (10 POINTS)

Polarization Problem - Solve Brau Exercise 4.3

IV. PROBLEM 4 (10 POINTS)

Polarization Problem - Solve Brau Exercise 4.4

V. PROBLEM 4 (10 POINTS)

Diffraction Problem - Solve Brau Exercise 9.8

Electrodynamics II (PHYS7120) / Radiative Processes (PHYS/ASTR 7130)

Homework #2

I. PROBLEM 2.1 (10 POINTS)

Solve Brau Exercise 4.11

Hints: As suggest for the exercise, start with equation (4.129). $\vec{r_0}$ is the position of the electron and you can write down the equation of motion for it using constant $\vec{v} = \vec{\beta}c$. Beginning of section 4.2.2. discusses solving (4.129) and can provide hints how to solve the problem.

II. PROBLEM 2.2 (10 POINTS)

Thomson scattering: Assume that we have two polarization directions of the incoming radiation defined by two orthogonal polarization unit vectors, $\hat{e_1} = \hat{e_\perp}$ and $\hat{e_2} = \hat{e_\|}$, that are perpendicular to the incoming wave vector $\vec{k_0}$. The angle between the incoming wave vector $\vec{k_0}$ and the scattered wave $\vec{k}$ is Θ and $\hat{e_\|}$ lies in the plane formed by $\vec{k}$ and $\vec{k_0}$.

Hint 1: To better picture the problem, make a drawing of $\vec{k_0}$, $\vec{k}$, $\hat{e_1} = \hat{e_\perp}$, $\hat{e_2} = \hat{e_\|}$, and scattering angle θ

Hint 2: Thomson scattering occurs when an EM plane wave interacts with a single free electron. You can assume the electron as point particle (charge q and mass m), which responds to the incoming plane wave and performs a non-relativist oscillation in the field of the incoming plane wave. The Lorentz force can be neglected when the field strength is weak, which is a reasonable assumption for this case.

The differential scattering cross section for the scattering process is given by

$$\frac{d\sigma_{scatt}}{d\Omega} = \left(\frac{q^2}{4\pi\epsilon_0 mc^2}\right)^2 \left|\hat{k} \times \hat{e}_0\right|^2 \tag{1}$$

you can start with this equation.

Hint 3: For unpolarized light you can assume that you have equal amounts of all polarization directions.

(a) Show that the total Thomson cross section of unpolarized light is given by $\sigma_T = \frac{8\pi}{3} r_c^2$, where $r_c = 2.82 \times 10^{-15}$ m is the classical particle radius.

(b)Find the degree of polarization defined as:

$$\Pi(\Theta) = \frac{\frac{d\sigma_\perp}{d\Omega} - \frac{d\sigma_\|}{d\Omega}}{\frac{d\sigma_\perp}{d\Omega} + \frac{d\sigma_\|}{d\Omega}} \tag{2}$$

(c) Make a plot of the angular dependence of the cross section and the degree of polarization. Hint: This should be a figure of Π and $\frac{d\sigma_{scatt}}{d\Omega}$ as function of θ.

III. PROBLEM 2.3 (10 POINTS)

Solve Brau Exercise 4.15

Hint: Count up the electrons dN_e using the differential with respect to momentum. At zero temperature the electrons settle into the lowest available energy states and fill the available levels up to some maximum value of the momentum p_F.

IV. PROBLEM 2.4 (10 POINTS)

The habitable zone is the area around a star where it is not too hot and not too cold for liquid water to exist on the surface of surrounding planets. Consider a star with surface temperature T and radius R. A good approximation for the radiation spectrum of the star is a blackbody. Assume there is a planet with radius r_p at a distance d from the star. The distance between the star and the planet is significantly larger than the stars radius ($d \gg R$).

Hint: You can obtain the flux using the luminosity. Start with the luminosity of the star, which is given by the Luminosity-Radius-Temperature relation $L = 4\pi R^2 k_B T^4$, which is valid for a blackbody. The luminosity defines the total energy radiated per unit time.

(a) What is the radiation flux F (energy per unit area per unit time) from the star at the planet's position?

(b) Assume the planet absorbs a fraction η of all the radiation reaching it, while the rest is reflected into space. What is the total energy L_p absorbed by the planet per unit time?

(c) Assume that there is no internal heating source in the planet and the total absorbed radiation reaches a new thermal equilibrium on the planet and is emitted back to space in the form of blackbody radiation. What is the temperature T_p on the surface of the planet?

(d) If the temperature range for the habitable zone is $T_1 < T < T_2$, what would be the range of distance for a planet to be in the habitable zone? Express your results in terms of T, R, η,T_1, and T_2.

(e) The Sun has a surface temperature of approximately $T = 5800$K and radius of $R = 6.96 \times 10^8$m. Let's take $T_1 \simeq 273$K, $T_2 \simeq 373$K, and $\eta = 0.7$. Compute the distance range of the habitable zone in Astronomical Units (AU). An Astronomical Unit is defined as the average distance between the Earth and the Sun (1 AU=1.496×10^{11}m). Is the Earth in the habitable zone? Interpret your result and comment what effects have been neglected, it might to think about the Earth and the Moon.

Electrodynamics II (PHYS7120) / Radiative Processes (PHYS/ASTR 7130)

Homework #3

I. PROBLEM 3.1 (10 POINTS)

A muon is moving along a trajectory $\vec{r}(t)$ in a vacuum. At an observation point $\vec{r_0}$ and time t_0, is it possible to have two values of retarded time t_{ret} corresponding to two different positions on the muon's trajectory?

a) If your answer is "yes", please give a concrete example, i.e., the form of $\vec{r}(t)$ and the situation for the two values of t_{ret}. If your answer is "no", please give your reasoning and proof.

b) Instead of a vacuum assume the process described above takes place in a homogeneous medium with an index of refraction $n > 1$. Describe if the situation is different for this case and repeat (a).

II. PROBLEM 3.2 (10 POINTS)

It is impossible for a spherically symmetric distribution of charge oscillating radially to radiate. Prove this using the following method. Take the current to be given by:

$$\vec{J}(\vec{r'}, t_R) = \vec{r'} f(r') e^{-i\omega t_R} \tag{1}$$

where f is a function only dependent on r' and the retarded time is t_R. Solve the problem by choosing an observation point along the z-axis to calculate the corresponding electromagnetic fields.

III. PROBLEM 3.3 (10 POINTS)

Solve Brau Problem 10.2

IV. PROBLEM 3.4 (10 POINTS)

Solve Brau Problem 10.3

Electrodynamics II (PHYS7120) / Radiative Processes (PHYS/ASTR 7130)

Homework #5

I. PROBLEM ANITA EXPERIMENTAL SIGNATURES (10 POINTS)

The ANITA long-duration balloon experiment [1] searches for neutrino events that generate particle showers in the Antartic ice.

a) Find the dominant polarization direction of a signal that is generated with a random polarization in the ice (n=1.3) and propagates to surface (air n=1.0) at a steep angle of 70°.

b) Find the polarization direction of a signal that is generated with a random polarization in the air and reflects of the ice at the Brewster angle.

c) Explain how the ANITA experiment can distinguish signals. Antropogenic (human made radio signals) are backgrounds to the searches of the high-energy neutrinos by ANITA, what can be done to distinguish antropogenic signals from neutrino signals ?

II. PROBLEM: CHARGE RELAXATION IN AN OHMIC MEDIUM (10 POINTS)

Show that the Fourier Transform of the current density is given by $\tilde{J}(\vec{r},\omega) = \tilde{\sigma}(\omega)\tilde{E}(\vec{r},\omega)$. Hint: see lecture notes from March 21, 2022.

III. PROBLEM: POWER DENSITY OF TRANSMITTED AND REFLECTED LIGHT BEAM (10 POINTS)

The transmission coefficient T and the reflection coefficient R measure that fraction of the total incident power that is transmitted and reflected, respectively. Since they measure electromagnetic power propagation (which is described by the Poynting vector), they are proportional to the magnitude squared of the electric field. However, there are also other constants that cannot be neglected which account for the power getting spread out over a larger area due to refraction. Find the power density of the transmitted and reflected light beam as function of the incident angle Θ_i. Assume a plane, linearly polarized, monochromatic electromagnetic wave and that media are not lossy and isotropic.

IV. PROBLEM: BRAU 7.13 (10 POINTS)

V. PROBLEM: BRAU 7.14 (10 POINTS)

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[1] P. W. Gorham *et al.* (ANITA), Phys. Rev. Lett. **121**, 161102 (2018), arXiv:1803.05088 [astro-ph.HE].

Electrodynamics II (PHYS7120) / Radiative Processes (PHYS/ASTR 7130)

Homework #4

I. PROBLEM 6.1 (10 POINTS)

II. PROBLEM 6.4 (10 POINTS)

III. PROBLEM 6.6 (10 POINTS)

IV. PROBLEM: EM WAVES IN CONDUCTING MATERIALS (10 POINTS)

In conducting materials, electromagnetic waves attenuate with distance. Consider a uniform, linear, isotropic medium, with dielectric constant ϵ, conductivity σ, and negligible magnetism ($\mu = 1$). The attenuating plane wave propagating in z-direction has the general form:

$$\vec{E}(\vec{r},t) = \vec{E}_0 e^{ikz-\kappa z-i\omega t} \tag{1}$$

$$\vec{H}(\vec{r},t) = \vec{H}_0 e^{ikz-\kappa z-i\omega t} \tag{2}$$

(a) Write down expressions for k and κ as function of ω.
(b) Relate the electric amplitude E_0 to the magnetic amplitude H_0.
Hints:
Look at the Maxwell equations and see what they imply ...
You can define a complex index of refraction $n(\omega) = \sqrt{\epsilon + \frac{i\sigma}{\epsilon_0 \omega}}$.

Electrodynamics II (PHYS7120) / Radiative Processes (PHYS/ASTR 7130)

Homework #6

I. PROBLEM: BRAU 10.13 (10 POINTS)

Hint: See also discussion in class on Wednesday April 6, 2022

II. PROBLEM: CHERENKOV RADIATION (10 POINTS)

1) A water-cherenkov detector (for example Super-Kamiokande) detects particles through the Cherenkov radiation emitted when relativistic charged particles propagate through the water ($n = 1.33$).

a) Determine the threshold energy for muons and protons.

b) Find the Cherenkov radiation angle for a 2 GeV electron.

c) Do you expect to observe any Chrenkov light from a charged kaon ($m_K^{\pm} = 497.6$ MeV) with energy of 550 MeV ?

d) Do you expect to observe any Chrenkov light from a 3 GeV neutron ? Explain.

2) Imaging air Cherenkov telescopes (IACTs) detect Cherenkov light from cosmic ray air showers or gamma-rays interacting in the Earth atmosphere. What is the maximum distance on the ground to observe a vertical air Cherenkov shower and what is the minimum energy a proton needs to have before Cherenkov light can be observed. Assume the air shower initiates 8.5 km above ground. The index of refraction of air can be approximated as n=1.0003.

III. PROBLEM: BRAU 10.22 (10 POINTS)