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AI Homework 5

Exercise 1

1. n persons in a room, and 365 days in a year.

a) What is the probability that at least two persons have the same birthday?
365! / ((365-n)! * (365^n))

b) Calculate this probability for n=50.

P(nobody shares birthday) = $\dfrac{\frac{365!}{315!}}{365^{50}}$ = 0.0296

1 – P(nobody shares birthday) = **0.9704**

c) **n = 23** for probability to be greater than 0.5

Tried n=25: $\dfrac{\frac{365!}{340!}}{365^{25}}$ Then, n=20: $\dfrac{\frac{365!}{345!}}{365^{20}}$ Then, **n=23:** $\dfrac{\frac{365!}{342!}}{365^{23}}$

2. 100 total hard drives, 20 defective. Quality control selects 2 hard drives to test without replacement.

a) 20/100 = 0.2
b) 19/99 = 0.1919
c) 1/5 * 19/99 = 0.03838

3. Fill in table: P(A,B) = P(A,B,c) + P(A,B,~c)

A	B	P(A, B, c)	P(A, B, ~c)	P(A, B)
T	T	0.25	0.15	0.4
T	F	0.05	0.1	0.15
F	T	0.05	0.2	0.25
F	F	0.1	0.1	0.2

P(A) = P(A,b) + P(A,~b) = 0.4 + 0.15 = 0.55

4. SFOT problem

A = Predict SFOT

B = Actual SFOT
$P(b|a) = ?$
$P(b) = 6/365$
$P(a|b) = 0.95$
$P(a|\sim b) = 0.15$

$P(b|a) = [P(a|b)*P(b)] / P(a)$

$P(a) = P(a,b) + P(a,\sim b)$
$= [P(a|b) * P(b)] + [P(a|\sim b) * P(\sim b)]$
$= [0.95*(6/365)] + [0.15*(359/365)]$
$= 0.01561643835 + 0.14753424657$
= **0.16315068492**

Exercise 2

1. $P(w,s,\sim r,c) = P(w|s,\sim r) * P(\sim r|c) * P(s|c) * P(c)$
$= .7 * .1 * .2 * .8 =$ **0.0112**

2. Handwritten

Exercise 2,

#2 $P(r|\sim w) = \frac{P(r,\sim w)}{P(\sim w)} = \frac{\sum_{c,s} P(\sim w, r, c, s)}{\sum_{c,s,r} P(\sim w, r, c, s)}$

chain top: $\sum_s P(\sim w|r,s,\cancel{c}) \cdot \sum_c P(r|c) \cdot \sum_s P(s|c) \sum_c P(c)$

$= ①(c,s)\quad ②(\sim c,s)\quad ③(c,\sim s)\quad ④(\sim c,\sim s)$

$①\ P(\sim w|r,s) \cdot P(r|c) \cdot P(s|c) \cdot P(c) = .05 \cdot .9 \cdot .2 \cdot .8 =$

$+ ②\ P(\sim w|r,s) \cdot P(r|\sim c) \cdot P(s|\sim c) \cdot P(\sim c) = .05 \cdot .2 \cdot .6 \cdot .2 =$

$+ ③\ P(\sim w|r,\sim s) \cdot P(r|c) \cdot P(\sim s|c) \cdot P(c) = .1 \cdot .9 \cdot .8 \cdot .8 =$

$+ ④\ P(\sim w|r,\sim s) \cdot P(r|\sim c) \cdot P(\sim s|\sim c) \cdot P(\sim c) = .1 \cdot .2 \cdot .8 \cdot .2 =$

$0.0072 + 0.0012 + .0576 + 0.0032 = 0.0692$

chain bottom: $P(\sim w) \sum_{s,c,r}$

$P(\sim w|r,s) \cdot P(r|c) \cdot P(s|c) \cdot P(c)$

top $+ ①(c,s,\sim r)\quad ②(\sim c,s,\sim r)\quad ③(c,\sim s,\sim r)\quad ④(\sim c,\sim s,\sim r)$

$①\ P(\sim w|\sim r,s) \cdot P(\sim r|c) \cdot P(s|c) \cdot P(c) = .3 \cdot .1 \cdot .2 \cdot .8 = 0.0048$

$②\ P(\sim w|\sim r,s) \cdot P(\sim r|\sim c) \cdot P(s|c) \cdot P(c) = .3 \cdot .8 \cdot .2 \cdot .6 = 0.0288$

$③\ P(\sim w|\sim r,\sim s) \cdot P(\sim r|c) \cdot P(\sim s|c) \cdot P(c) = 1 \cdot .1 \cdot .8 \cdot .8 = 0.064$

$④\ P(\sim w|\sim r,\sim s) \cdot P(\sim r|\sim c) \cdot P(\sim s|\sim c) \cdot P(\sim c) = 1 \cdot .8 \cdot .8 \cdot .2 = 0.128$

$P(\sim w) =$ bottom $=$ top $+$ all $= 0.0692 + 0.2256 = 0.2948$

$0.0692 / 0.2948 = \boxed{0.23435414}$

3. Handwritten

Exercise 2

#3 $P(\sim s|w) = \frac{P(\sim s, w)}{P(w)} = \frac{\sum_{c,r} P(w,r,c,\sim s)}{\sum_{c,r,s} P(w,r,c,\sim s)}$

chain top: $\sum_{c,r} P(w|r,\sim s) \cdot P(r|c) \cdot P(\sim s|c) \cdot P(c)$

① (c,r) ② $(\sim c,r)$ ③ $(c,\sim r)$ ④ $(\sim c,\sim r)$

① $P(w|r,\sim s) \cdot P(r|c) \cdot P(\sim s|c) \cdot P(c) = .9 \cdot .9 \cdot .8 \cdot .8 = 0.5184$

② $P(w|r,\sim s) \cdot P(r|\sim c) \cdot P(\sim s|\sim c) \cdot P(\sim c) = .9 \cdot .2 \cdot .4 \cdot .2 = 0.0144$

③ $P(w|\sim r,\sim s) \cdot P(\sim r|c) \cdot P(\sim s|c) \cdot P(c) = 0.0$

④ $P(w|\sim r,\sim s) \cdot P(\sim r|\sim c) \cdot P(\sim s|\sim c) \cdot P(\sim c) = 0.0$

top $= 0.0144 + 0.5184 = 0.5328$

chain bottom: $P(w) \sum_{s,c,r}$

Exercise 2, #2 showed that $P(\sim w) = 0.2948$

$P(w) = 1 - P(\sim w)$ $P(w) = 1 - 0.2948 = 0.7052$

$P(\sim s|w) = \frac{P(\sim s, w)}{P(w)} = \frac{0.5328}{0.7052} = \boxed{0.7555}$

4. Handwritten

Exercise 2

#4 $P(\sim s|w,r) = \frac{P(\sim s,w,r)}{P(w,r)} = \frac{\sum_c P(w,r,\sim s,c)}{\sum_{c,s} P(w,r,\sim s,c)}$

chain top: $\sum_c P(w|r,\sim s) \cdot P(r|c) \cdot P(\sim s|c) \cdot P(c)$

① c ② ~c

① $P(w|r,\sim s) \cdot P(r|c) \cdot P(\sim s|c) \cdot P(c) = .9 \cdot .9 \cdot .8 \cdot .8$

② $P(w|r,\sim s) \cdot P(r|\sim c) \cdot P(\sim s|\sim c) \cdot P(\sim c) = .9 \cdot .2 \cdot .4 \cdot .2$

$P(\sim s,w,r) =$ ① $= 0.5184 +$ ② $= 0.0144 = 0.5328$

chain bottom: $P(w,r) = \sum_{c,s} P(\sim s,w,r,c)$

top + ① (c, s) ② (~c, s)

③ $P(w|r,s) \cdot P(r|c) \cdot P(s|c) \cdot P(c) = .95 \cdot .9 \cdot .2 \cdot .8$

④ $P(w|r,s) \cdot P(r|\sim c) \cdot P(s|\sim c) \cdot P(\sim c) = .95 \cdot .2 \cdot .6 \cdot .2$

③ $= 0.1368 + 0.0228 = 0.1596$

bottom = top + 0.1596 = 0.6924 = $P(w,r)$

$P(\sim s|w,r) = \frac{P(\sim s,w,r)}{P(w,r)} = \frac{0.5328}{0.6924} = \boxed{0.7695}$

#5. $P(\sim s|w,r) = 1 - P(\sim s|w)$. The reason for this is with ~s being consistent, $P(w|\sim s,\sim r) = 0$. So, only r affects calculations.

Exercise 3

1. What quantity does a represent?

α = 1/P(b)

2. Write the factorized expression for P(J|b) after eliminating M.

$\alpha\ \Sigma(a,e,m)\ P(J|a)\ P(m|a)\ P(a|B,e)\ P(B)\ P(e)$
$= \alpha\ \Sigma(a)P(J|a)\ \Sigma(e)\ P(a|B,e)\ P(e)$
$= \alpha\ P(b)\Sigma(e)P(e)P(a|b,e)P(J|a)$

We can leave out $\Sigma(m)$ $P(m|a)$ because **m** is conditionally independent of J|a, and results in 1.

3. Write the condition probability table for variable A after eliminating variable E. (Work handwritten below.)

B	P(A)
T	0.94002
F	0.001578

4. Write the conditional probability table for J after eliminating A. (Work handwritten below.)

B	P(J)
T	0.849017
F	0.0513413

5. Eliminate variable B and write the probability table for J. What do these probabilities represent? (Work handwritten below.)

This represents the marginal probability of J:

P(J)
0.05213898

Work:

#3

B	P(A)
T	$.95 \cdot .002 + .94 \cdot (1-.002) = 0.0019 + 0.93812 = 0.94002$
F	$.29 \cdot .002 \cdot .001(1-.002) = 0.00058 + 0.000998 = 0.001578$

#4 cond prob of J after elim A

$P(J|B) = \sum_a P(J|A)P(A|B)$

① a ② ~a

$P(J|a) \cdot P(a|B) = .9 \cdot .94002 = 0.846018$

$+\ P(J|\sim a) \cdot P(\sim a|B) = 0.05 \cdot (1-0.94002) = 0.002999$

$P(J|B) = 0.849017$

$P(J|\sim B) = \sum_a P(J|A)P(A|\sim B)$ ① a ② ~a

$P(J|a) \cdot P(a|\sim B) = .9 \cdot 0.001578 = 0.0014202$

$P(J|\sim a) \cdot P(\sim a|\sim B) = 0.05 \cdot (1-0.001578) = 0.0499211$

$= 0.0513413$

#5. elim. var. B and write prob table for J.

J	P(J)
T	0.05213898
F	

$P(J) = \sum_B P(J|B) \cdot P(B)$

① B ② ~B

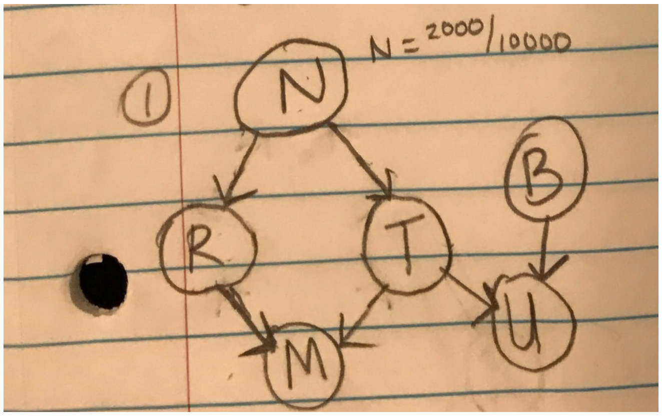
① $0.849017 \cdot .001 = 0.00084902$

$+$ ② $0.0513413 \cdot (1-.001) = 0.05128996$

0.05213898

Exercise 4

1. Draw Bayesian network (Handwritten)



2. Factorized expression for full joint distribution, P(N, R, T, M, U, B):
P(N) * P(R|N) * P(T|N) * P(M|R,T) * P(U,T,B) * P(B)

3. Conditional probability table for each node:

Marginal Probability of N:

N	P(N)
T	.2
F	.8

Conditional:

N	P(R)
T	.9
F	.3

Conditional:

N	P(T)
T	.6
F	.4

Conditional:

R	T	P(M)
T	T	.168
T	F	.042
F	T	.08
F	F	.042

Conditional:

B	T	P(U)
T	T	.5
T	F	.05
F	T	.3
F	F	.01

Marginal Probability of B:

B	P(B)
T	.05
F	.95

4. Calculate marginal probabilities for each node.

P(N) = .2
P(R) = .42
P(T) = .44
P(M) = .0749824
P(U) = .14312
P(B) = .05

Handwritten work:

$= P(N) \cdot P(R|N) \cdot P(T|N) \cdot P(M|R,T) \cdot P(U|T,B) \cdot P(B)$

3. conditional prob. table for each node.

N	P(N)
T	.2
F	.8

N	P(R)
T	$720+1080/2K = .9$
F	$960+1440/8K = .3$

N	P(T)
T	$1080+120/2K = .6$
F	$960+2240/8K = .4$

R	T	P(M)
T	T	$1680/10K = .168$
T	F	$420/10K = .042$
F	T	$800/10K = .08$
F	F	$420/10K = .042$

B	T	P(U)
T	T	.5
T	F	.05
F	T	.3
F	F	.01

B	P(B)
T	$500/10{,}000 = .05$
F	.95

4. Marginal prob for each node.

a) $\sum_N P(R|N) \cdot P(N)$

① N ② ~N $= P(R|N) \cdot P(N) + P(R|\sim N) \cdot P(\sim N) = \overbrace{.9 \cdot .2}^{.18} + \overbrace{.3 \cdot .8}^{.24} = .42$

$P(R) = .42 \quad P(\sim R) = .58$

b) $\sum_N P(T|N) \cdot P(N)$

$P(T|N) \cdot P(N) + P(T|\sim N) \cdot P(\sim N) = .6 \cdot .2 + .4 \cdot .8 = .12 + .32 = 0.4$

$P(T) = 0.44 \quad P(\sim T) = .56$

c) $P(M|R,T) \cdot P(R) \cdot P(T) \sum_{R,T}$

① R,T ② ~R,T ③ R,~T ④ ~R,~T

① $.168 \cdot .42 \cdot .44 = 0.0310464$

② $.08 \cdot .58 \cdot .44 = 0.020416$

③ $.042 \cdot .42 \cdot .56 = 0.0098784$

④ $.042 \cdot .58 \cdot .56 = 0.0136416$

P(M) Sum = 0.0749824

$P(\sim M) = 0.9250176$

d) $P(U|T,B) \cdot P(T) \cdot P(B)$ $\sum_{T,B}$

① T,B ② ~T,B ③ T,~B ④ ~T,~B

① $.5 \cdot .44 \cdot .05 = .011$

② $.05 \cdot .56 \cdot .05 = 0.0014$

③ $.3 \cdot .44 \cdot .95 = 0.1254$

④ $.01 \cdot .56 \cdot .95 = 0.00532$

P(U) = sum = 0.14312

Marginal Probabilities

$P(N) = .2$

$P(R) = .42$

$P(T) = .44$

$P(M) = .0749824$

$P(U) = .14312$

$P(B) = .05$