Variable Elimination - New Example

Target: $P(J|B)$ ← going to be a 2×2 matrix (true/false values for J and B)

$$P(J|B) = \frac{P(J,B)}{P(B)} = \alpha P(J,B) = \alpha \sum_a \sum_e \sum_m P(a,B,e,J,m)$$

$$= \alpha \sum_a \sum_e \sum_m P(J|a)\,P(m|a)\,P(a|B,e)\,P(B)\,P(e)$$

$$= \sum_a P(J|a) \sum_e P(a|B,e)\,P(e) \sum_m P(m|a)$$

(the $\sum_m P(m|a)$ term equals 1)

$f_1(J,A)$ $\quad f_2(A,B,E)$ $\quad f_3(E)$

Now - assign values to the factors

$$f_1(J,A) = \begin{bmatrix} P(j|a) & P(\neg j|a) \\ P(j|\neg a) & P(\neg j|\neg a) \end{bmatrix} = \begin{bmatrix} 0.90 & 0.10 \\ 0.05 & 0.95 \end{bmatrix}$$

← a true
← a false
↑ j true ↑ j false

$$f_2(A,B,E) = \begin{bmatrix} P(a|b,e) & P(a|b,\neg e) \\ P(a|\neg b,e) & P(a|\neg b,\neg e) \end{bmatrix}, \begin{bmatrix} P(\neg a|b,e) & P(\neg a|b,\neg e) \\ P(\neg a|\neg b,e) & P(\neg a|\neg b,\neg e) \end{bmatrix}$$

$$= \begin{bmatrix} 0.95 & 0.94 \\ 0.29 & 0.001 \end{bmatrix}, \begin{bmatrix} 0.05 & 0.06 \\ 0.71 & 0.999 \end{bmatrix}$$

← b true
← b false

↑ e true ↑ e false ↑ e true ↑ e false

a true a false

$$f_3(E) = \begin{bmatrix} P(e) \\ P(\neg e) \end{bmatrix} = \begin{bmatrix} 0.002 \\ 0.998 \end{bmatrix}$$

Variable Elimination - Continued

Now- create new factors using pointwise product

$$f_4(A,B) = \sum_e f_2(A,B,e) \times f_3(e)$$

$$= [f_2(A,B,e) \times f_3(e)] + [f_2(A,B,\sim e) \times f_3(\sim e)]$$

$$= \left[0.002 \times \begin{bmatrix} 0.95 & 0.05 \\ 0.29 & 0.71 \end{bmatrix}\right] + \left[0.998 \times \begin{bmatrix} 0.94 & 0.06 \\ 0.001 & 0.999 \end{bmatrix}\right]$$

(a true, a false) → e true; (a true, a false) → e false

$$= \begin{bmatrix} 0.94 & 0.060 \\ 0.0016 & 0.9984 \end{bmatrix}$$

← b true; ← b false

(a true, a false)

← no more e!! marginalized over e, now just in terms of a and b

$$f_5(J,B) = \sum_a f_1(J,a) \times f_4(a,B)$$

$$= [f_1(J,a) \times f_4(a,B)] + [f_1(J,\sim a) \times f_4(\sim a,B)]$$

$$= \begin{bmatrix} 0.90 \\ 0.10 \end{bmatrix} \times \begin{bmatrix} 0.94 \\ 0.0016 \end{bmatrix} + \begin{bmatrix} 0.05 \\ 0.95 \end{bmatrix} \times \begin{bmatrix} 0.060 \\ 0.9984 \end{bmatrix}$$

$$= \begin{bmatrix} 0.846 & 0.0014 \\ 0.094 & 0.00016 \end{bmatrix} + \begin{bmatrix} 0.0030 & 0.0499 \\ 0.0570 & 0.9485 \end{bmatrix}$$

← j true; ← j false

(b true, b false) → (b true, b false)

$$= \begin{bmatrix} 0.849 & 0.0513 \\ 0.1510 & 0.9487 \end{bmatrix} = \begin{bmatrix} p(j|b) & p(j|\sim b) \\ p(\sim j|b) & p(\sim j|\sim b) \end{bmatrix}$$

← j true; ← j false