Probalistic Graphic Model[1][2] Notes

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Contents

1 Introduction and Overview

Model: The model is a **declarative representation** of our understanding of the world. It's **declarative** means that the representation stands on its own, which means that we can look into it and make sense of it **aside from any algorithm** that we might choose to apply on.

1. Representation
   - Directed and undirected
   - Temperal and plate models
2. Inference
   - Exact and approximate
   - Decision making
3. Learning
   - Parameters and structure
   - With and without complete data

1.1 Distributions

Chapters 2.1.1 to 2.1.3.

1.2 Factors

Chapter 4.2.1.

1.3 Quiz

1. **Factor product.**

Let X, Y and Z be binary variables.

If $\phi_1(X,Y)$ and $\phi_2(Y,Z)$ are the factors shown below, compute the selected entries (marked by a '?') in the factor $\psi(X,Y,Z) = \phi_1(X,Y) \cdot \phi_2(Y,Z)$, giving your answer according to the ordering of assignments to variables as shown below.

Separate each of the 3 entries of the factor with spaces, e.g., an answer of

0.1 0.2 0.3

means that $\psi(1,1,1) = 0.1$, $\psi(1,2,1) = 0.2$, and $\psi(2,2,2) = 0.3$. Give your answers as exact decimals without any trailing zeroes.

X	Y	$\phi_1(X,Y)$
1	1	0.8
1	2	0.5
2	1	0.5
2	2	0.6

×

Y	Z	$\phi_2(Y,Z)$
1	1	0.2
1	2	0.2
2	1	0.9
2	2	1.0

=

X	Y	Z	$\psi(X,Y,Z)$
1	1	1	?
1	1	2	
1	2	1	?
1	2	2	
2	1	1	
2	1	2	
2	2	1	
2	2	2	?

Fig. 1: Exercise 01-01

2. Factor reduction.

Let X, Z be binary variables, and let Y be a variable that takes on values 1, 2, or 3.

Now say we observe $Y = 3$. If $\phi(X, Y, Z)$ is the factor shown below, compute the missing entries of the reduced factor $\psi(X, Z)$ given that $Y = 3$, giving your answer according to the ordering of assignments to variables as shown below.

As before, you may separate the 4 entries of the factor by spaces.

X	Y	Z	$\phi(X,Y,Z)$
1	1	1	14
1	1	2	60
1	2	1	40
1	2	2	27
1	3	1	42
1	3	2	85
2	1	1	4
2	1	2	59
2	2	1	54
2	2	2	3
2	3	1	96
2	3	2	30

X	Z	$\psi(X,Z)$
1	1	?
1	2	?
2	1	?
2	2	?

Fig. 2: Exercise 01-02

3. **Properties of independent variables.**

Assume that A and B are independent random variables. Which of the following options are always true? You may select 1 or more options.

- [ ] $P(B|A) = P(B)$
- [ ] $P(A, B) = P(A) \times P(B)$
- [ ] $P(A) = P(B)$
- [ ] $P(A) \neq P(B)$

Fig. 3: Exercise 01-03

4. **Factor marginalization.**

Let X, Z be binary variables, and let Y be a variable that takes on values 1, 2, or 3.

If $\phi(X, Y, Z)$ is the factor shown below, compute the entries of the factor

$\psi(Y, Z) = \sum_X \phi(X, Y, Z)$,

giving your answer according to the ordering of assignments to variables as shown below.

Separate the 4 entries of the factor with spaces, and do not add any extra trailing or leading zeroes or decimal points.

X	Y	Z	$\phi(X,Y,Z)$
1	1	1	68
1	1	2	95
1	2	1	65
1	2	2	63
1	3	1	57
1	3	2	5
2	1	1	40
2	1	2	40
2	2	1	14
2	2	2	78
2	3	1	16
2	3	2	89

Y	Z	$\psi(Y,Z)$
1	1	?
1	2	?
2	1	?
2	2	?
3	1	
3	2	

Fig. 4: Exercise 01-04

My answers for the Quiz:

01-01: 0.16 0.45 0.6

01-02: 42 85 96 30

01-03: choose 1 and 2

01-04: 190 163 70 207

2 Representation

2.1 Baysian Network (Directed Models)

2.1.1 Baysian Network Fundamentals

2.1.2 Semantics and Factorization

Chapters 3.2.1, 3.2.2. If you are unfamiliar with genetic inheritance, please watch this short Khan Academy video for some background.

2.1.3 Reasoning Patterns

Chapter 3.2.1.

2.1.4 Flow of Probabilistic Influence

Chapter 3.3.1.

2.1.5 Bayesian Networks: Independencies

Conditional Independence. Chapters 2.1.4, 3.1.

Independencies in Bayesian Networks. Chapter 3.2.2.

Naive Bayes. Chapter 3.1.3.

Bayesian Networks: Knowledge Engineering

Application - Medical Diagnosis Chapter 3.2: Box 3.D (p. 67).

2.2 Template Models for Bayesian Networks

Overview. Chapter 6.1.

Temporal Models - DBNs. Chapters 6.2, 6.3.

Temporal Models - HMMs. Chapters 6.2, 6.3.

Plate Models. Chapter 6.4.1.

2.3 Structured CPDs for Bayesian Networks

Overview. Chapters 5.1, 5.2.

Tree-Structured CPDs. Chapter 5.3.

Independence of Causal Influence. Chapter 5.4.

Continuous Variables. Chapter 5.5.

2.4 Markov Network Fundamentals (Undirected Models)

Pairwise Markov Networks. Chapter 4.1.

General Gibbs Distribution. Chapter 4.2.2.

Conditional Random Fields. Chapter 4.6.1.

Independencies in Markov Networks and Bayesian Networks

Independencies in Markov Networks. Chapter 4.3.1.

I-Maps and Perfect Maps. Chapter 3.3.4.

Local Structure in Markov Networks

Log-Linear Models. Chapter 4.4, p. 125.

Shared Features in Log-Linear Models. Chapter 4: Box 4.B (p. 112), Box 4.C (p. 126), Box 4.D (p. 127).

2.5 Decision Making

Maximum Expected Utility Chapter 22.1.1, 23.2.104, 23.4.1-2, 23.5.1

Utility Functions Chapter 22.2.1-3, 22.3.2, 22.4.2

Value of Perfect Information Chapter 23.7.1-2

2.6 Knowledge Engineering & Summary

Reference

[1] Daphne Koller. Probabilistic graphical models.

[2] Daphne Koller and Nir Friedman. *Probabilistic graphical models: principles and techniques.* MIT press, 2009.