Technische Universität Berlin

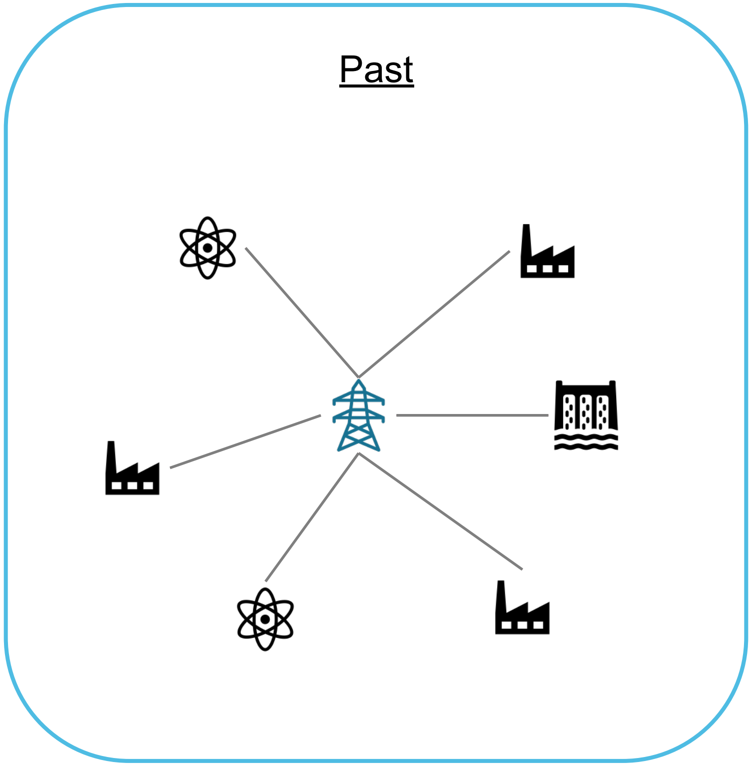
Modeling Decentralized Electricity Markets

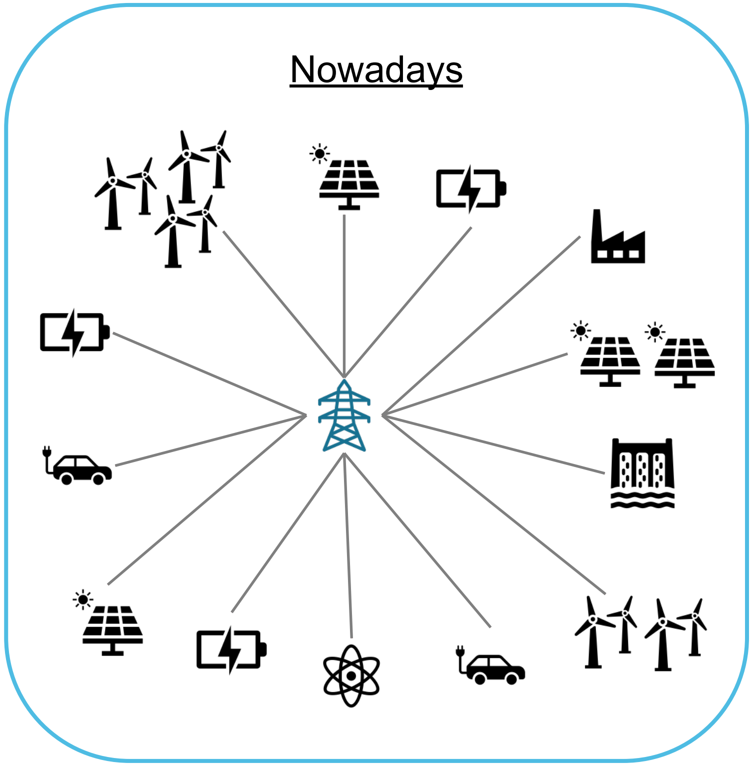
Solving Multi-Period Optimal Power Flow using Alternating Direction Method of Multipliers

Eric Rockstädt | Master Thesis

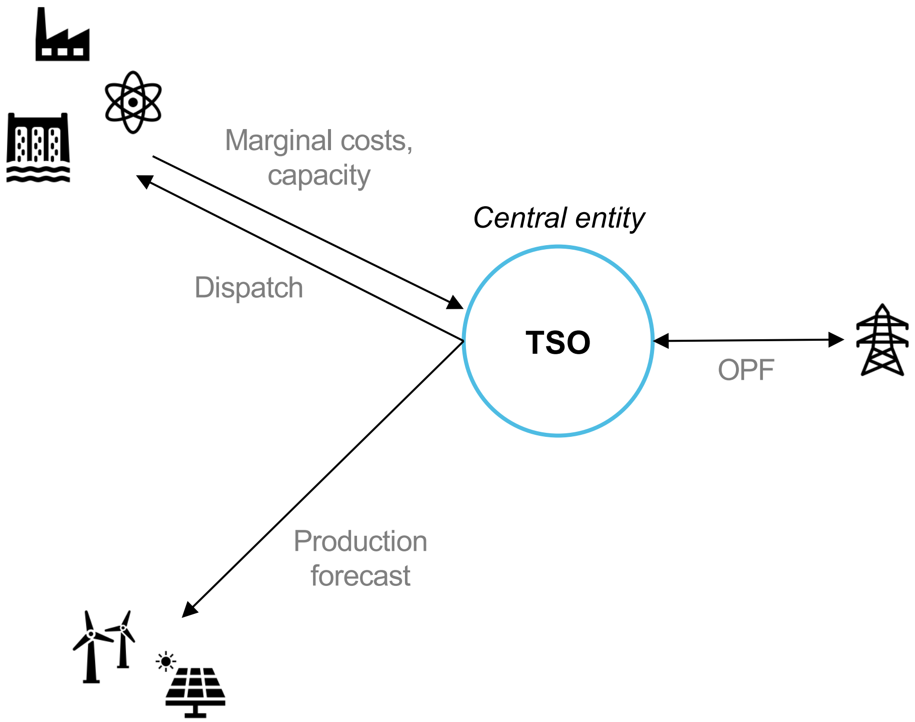
Agenda

1.1 Motivation





1.1 Motivation



Sharing of sensible information

Trend towards decentralized systems

Utilizing new algorithms and computational advantages

1.1 Motivation

Alternating Direction Method of Multipliers (ADMM)

- Solves distributed convex optimization problems
- Decomposes main problem into multiple subproblems
- Enables decentralization and parallelisation
- Fine-tuning of algorithm depends on one parameter
- Introduced by Gabay et al. in mid-1970s

1.1 Motivation

Alternating Direction Method of Multipliers (ADMM)

Typically, ADMM solves the following problem:

$$\begin{aligned} \min\,(x,z) \quad & f(x) + g(z) \\ \text{s.t.} \quad & \mathbf{A}x + \mathbf{B}z = c \end{aligned}$$

Complicating constraint

The corresponding augmented Lagrangian yields:

$$L_p(x,y,\lambda) = f(x) + g(z) + \lambda^T(\mathbf{A}x + \mathbf{B}z - c) + \frac{\gamma}{2}\|\mathbf{A}x + \mathbf{B}z - c\|_2^2$$

1.1 Motivation

Alternating Direction Method of Multipliers (ADMM)

The corresponding augmented Lagrangian yields:

$$L_p(x, y, \lambda) = f(x) + g(z) + \lambda^T(\mathbf{A}x + \mathbf{B}z - c) + \frac{\gamma}{2}\|\mathbf{A}x + \mathbf{B}z - c\|_2^2$$

The single iterations are:

$$x^{v+1} := \min\,(x) \quad f(x) + g(z^v) + (\lambda^v)^T(\mathbf{A}x + \mathbf{B}z^v - c) + \frac{\gamma}{2}\|\mathbf{A}x + \mathbf{B}z^v - c\|_2^2$$

$$z^{v+1} := \min\,(z) \quad f(x^v) + g(z) + (\lambda^v)^T(\mathbf{A}x^v + \mathbf{B}z - c) + \frac{\gamma}{2}\|\mathbf{A}x^v + \mathbf{B}z - c\|_2^2$$

$$\lambda^{v+1} := \lambda^v + \gamma(\mathbf{A}x^{v+1} + \mathbf{B}z^{v+1} - c)$$

1.2 Research Questions

1. Is it possible to implement a decentralized algorithm with the help of ADMM that can optimize an OPF?
2. Can the algorithm be extended by energy storage resources?
3. Are the results the same as for a centralized problem?

1.3 Current State of Research

Fundamentals

Conejo et al. (2006)

Decomposition techniques in mathematical programming: engineering and science applications

Boyd (2010)

Distributed Optimization and Statistical Learning via the Alternating Direction Method of Multipliers

Application

Xing et al. (2017)

Distributed algorithm for dynamic economic power dispatch with energy storage in smart grids

+ ADMM is utilized
+ Communication network is established
+ Storages are included
- Transmission network is neglected
- Software implementation is not published

Yang et al. (2019)

A Fully Decentralized Distribution Market Mechanism Using ADMM

+ Algorithm for a decentralized OPF
+ ADMM is utilized
- No storages are included
- Software implementation is not published
- Equations are not explained in detail

2.1 Modeling Framework

- **Storage**
  - Max power output/input
  - Max energy level
  - Marginal costs
- **Generator**
  - Max power output
  - Marginal costs
- **Transmission Line**
  - Max capacity
  - Susceptance
- **Node**
  - Demand

2.2 Mathematical Formulations

Centralized Problem

$$\min \sum_{t \in \mathcal{T}} \sum_{g \in \mathcal{G}} mc_g \cdot P_{g,t} + \sum_{s \in \mathcal{S}} mc_s \cdot (D_{s,t} + C_{s,t})$$

$$\begin{aligned}
\text{s.t.} \quad & 0 \leq P_{g,t} \leq \overline{p_g} && \forall g \in \mathcal{G}, t \in \mathcal{T} \\
& 0 \leq D_{s,t} \leq \overline{p_s} && \forall s \in \mathcal{S}, t \in \mathcal{T} \\
& 0 \leq C_{s,t} \leq \overline{p_s} && \forall s \in \mathcal{S}, t \in \mathcal{T} \\
& 0 \leq E_{s,t} \leq \overline{e_s} && \forall s \in \mathcal{S}, t \in \mathcal{T} \\
& E_{s,t} - E_{s,t-1} - C_{s,t} + D_{s,t} = 0 && \forall s \in \mathcal{S}, t \in \mathcal{T} \\
& \sum_{n \in \mathcal{N}} I_{n,t} = 0 && \forall t \in \mathcal{T} \\
& -\overline{f_l} \leq PTDF_{l,n} \cdot I_{n,t} \leq \overline{f_l} && \forall l \in \mathcal{L}, t \in \mathcal{T}, n \in \mathcal{N}
\end{aligned}$$

Complicating constraint

$$I_{n,t} = \sum_{g \in \mathcal{G}_n} P_{g,t} + \sum_{s \in \mathcal{S}_n} (D_{s,t} - C_{s,t}) - d_{n,t}$$

* See thesis for a detailed nomenclature

2.2 Mathematical Formulations

1. Remove inequality constraints

$$\min \sum_{t\in\mathcal{T}} \sum_{g\in\mathcal{G}} mc_g \cdot P_{g,t} + \sum_{s\in\mathcal{S}} mc_s \cdot (D_{s,t} + C_{s,t})$$

$$\begin{aligned}
\text{s.t.} \quad & 0 \leq P_{g,t} \leq \overline{p_g} && \forall g \in \mathcal{G}, t \in \mathcal{T} \\
& 0 \leq D_{s,t} \leq \overline{p_s} && \forall s \in \mathcal{S}, t \in \mathcal{T} \\
& 0 \leq C_{s,t} \leq \overline{p_s} && \forall s \in \mathcal{S}, t \in \mathcal{T} \\
& 0 \leq E_{s,t} \leq \overline{e_s} && \forall s \in \mathcal{S}, t \in \mathcal{T} \\
& E_{s,t} - E_{s,t-1} - C_{s,t} + D_{s,t} = 0 && \forall s \in \mathcal{S}, t \in \mathcal{T} \\
& \sum_{n\in\mathcal{N}} I_{n,t} = 0 && \forall t \in \mathcal{T} \\
& \sum_{n\in\mathcal{N}} PTDF_{l,n} \cdot I_{n,t} + U_{l,t} - \overline{f_l} = 0 && \forall l \in \mathcal{L}, t \in \mathcal{T} \\
& \sum_{n\in\mathcal{N}} K_{l,t} - PTDF_{l,n} \cdot I_{n,t} - \overline{f_l} = 0 && \forall l \in \mathcal{L}, t \in \mathcal{T}
\end{aligned}$$

Introduction slack variables

$$I_{n,t} = \sum_{g\in\mathcal{G}_n} P_{g,t} + \sum_{s\in\mathcal{S}_n} (D_{s,t} - C_{s,t}) - d_{n,t}$$

* See thesis for a detailed nomenclature

2.2 Mathematical Formulations

2. Apply Augmented Lagrangian Relaxation

$$
\begin{aligned}
\min \quad \sum_{t \in \mathcal{T}} \quad & \sum_{g \in \mathcal{G}} mc_g \cdot P_{g,t} + \sum_{s \in \mathcal{S}} mc_s \cdot (D_{s,t} + C_{s,t}) \\
& + \lambda_t \cdot \sum_{n \in \mathcal{N}} I_{n,t} \\
& + \frac{\gamma}{2} \cdot \| \sum_{n \in \mathcal{N}} I_{n,t} \|_2^2 \\
& + \sum_{l \in \mathcal{L}} \quad \mu_{l,t} \cdot (\sum_{n \in \mathcal{N}} PTDF_{l,n} \cdot I_{n,t} + U_{l,t} - \overline{f_l}) \\
& \qquad + \frac{\gamma}{2} \cdot \| \sum_{n \in \mathcal{N}} PTDF_{l,n} \cdot I_{n,t} + U_{l,t} - \overline{f_l} \|_2^2 \\
& \qquad + \rho_{l,t} \cdot (\sum_{n \in \mathcal{N}} K_{l,t} - PTDF_{l,n} \cdot I_{n,t} - \overline{f_l}) \\
& \qquad + \frac{\gamma}{2} \cdot \| \sum_{n \in \mathcal{N}} K_{l,t} - PTDF_{l,n} \cdot I_{n,t} - \overline{f_l} \|_2^2
\end{aligned}
$$

$$
\begin{aligned}
\text{s.t.} \quad & 0 \le P_{g,t} \le \overline{p_g} && \forall g \in \mathcal{G}, t \in \mathcal{T} \\
& 0 \le D_{s,t} \le \overline{p_s} && \forall s \in \mathcal{S}, t \in \mathcal{T} \\
& 0 \le C_{s,t} \le \overline{p_s} && \forall s \in \mathcal{S}, t \in \mathcal{T} \\
& 0 \le E_{s,t} \le \overline{e_s} && \forall s \in \mathcal{S}, t \in \mathcal{T} \\
& E_{s,t} - E_{s,t-1} - C_{s,t} + D_{s,t} = 0 && \forall s \in \mathcal{S}, t \in \mathcal{T}
\end{aligned}
$$

$$
I_{n,t} = \sum_{g \in \mathcal{G}_n} P_{g,t} + \sum_{s \in \mathcal{S}_n} (D_{s,t} - C_{s,t}) - d_{n,t}
$$

* See thesis for a detailed nomenclature

2.2 Mathematical Formulations

Decentralized Problem – Generator subproblem

$$
\begin{aligned}
\min \quad \sum_{t\in\mathcal{T}} \quad & mc_g \cdot P^v_{g,t} \\
& + P^v_{g,t} \cdot (\lambda_t + \sum_{l\in\mathcal{L}} \sum_{n\in\mathcal{N}} (\mu_{l,t} - \rho_{l,t}) \cdot PTDF_{l,n}) \\
& + \frac{\gamma}{2} \cdot \| P^v_{g,t} + \Theta_{\breve{n},t} - P^{v-1}_{g,t} + \Phi_{\breve{n},t} - \Psi_{\breve{n},t} - d_{\breve{n},t} \\
& \quad + \sum_{n\in\mathcal{N}\setminus\breve{n}} \Theta_{n,t} + \Phi_{n,t} - \Psi_{n,t} - d_{n,t} \|_2^2 \\
& + \sum_{l\in\mathcal{L}} \quad \frac{\gamma}{2} \cdot \| \sum_{n\in\mathcal{N}} PTDF_{l,n} \cdot I_{n,t} + U^v_{l,t} - \overline{f_l} \|_2^2 \\
& \quad + \frac{\gamma}{2} \cdot \| \sum_{n\in\mathcal{N}} K^v_{l,t} - PTDF_{l,n} \cdot I_{n,t} - \overline{f_l} \|_2^2 \\
& \quad + \frac{\gamma}{2} \cdot \| U^v_{l,t} - \Upsilon_{l,t} \|_2^2 \\
& \quad + \frac{\gamma}{2} \cdot \| K^v_{l,t} - \Gamma_{l,t} \|_2^2 \\
& + \frac{\gamma}{2} \cdot (P^v_{g,t} - P^{v-1}_{g,t})^2
\end{aligned}
$$

$$
\text{s.t.} \quad 0 \le P^v_{g,t} \le \overline{p_g} \qquad \forall t \in \mathcal{T}
$$

$$
I_{n,t} = \sum_{g\in\mathcal{G}_n} P_{g,t} + \sum_{s\in\mathcal{S}_n} (D_{s,t} - C_{s,t}) - d_{n,t}
$$

* See thesis for a detailed nomenclature

2.2 Mathematical Formulations

Decentralized Problem – Storage subproblem

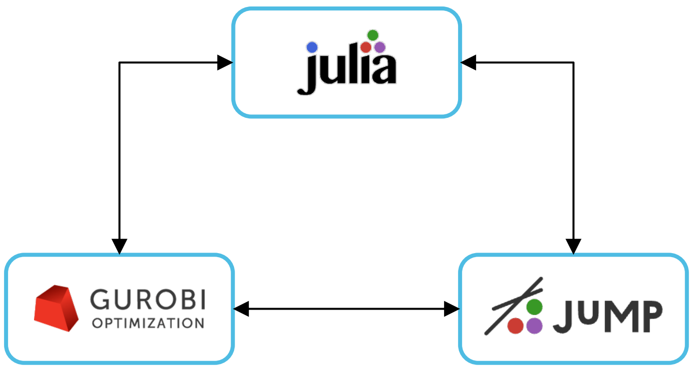
$$
\begin{aligned}
\min \sum_{t\in\mathcal{T}} \quad & mc_s \cdot (D^v_{s,t} + C^v_{s,t}) \\
& + (D^v_{s,t} - C^v_{s,t}) \cdot (\lambda_t + \sum_{l\in\mathcal{L}} \sum_{n\in\mathcal{N}} (\mu_{l,t} - \rho_{l,t}) \cdot PTDF_{l,n}) \\
& + \frac{\gamma}{2} \cdot \big\| \Theta_{\breve{n},t} + D^v_{s,t} + \Phi_{\breve{n},t} - D^{v-1}_{s,t} \\
& \qquad - C^v_{s,t} - (\Psi_{\breve{n},t} - C^{v-1}_{s,t}) - d_{\breve{n},t} \\
& \qquad + \sum_{n\in\mathcal{N}\setminus\breve{n}} \Theta_{n,t} + \Phi_{n,t} - \Psi_{n,t} - d_{n,t} \big\|_2^2 \\
& + \sum_{l\in\mathcal{L}} \quad \frac{\gamma}{2} \cdot \big\| \sum_{n\in\mathcal{N}} PTDF_{l,n} \cdot I_{n,t} + U^v_{l,t} - \overline{f_l} \big\|_2^2 \\
& \qquad + \frac{\gamma}{2} \cdot \big\| \sum_{n\in\mathcal{N}} K^v_{l,t} - PTDF \cdot I_{n,t} - \overline{K_l} \big\|_2^2 \\
& \qquad + \frac{\gamma}{2} \cdot \big\| U^v_{l,t} - \Upsilon_{l,t} \big\|_2^2 \\
& \qquad + \frac{\gamma}{2} \cdot \big\| K^v_{l,t} - \Gamma_{l,t} \big\|_2^2 \\
& + \frac{\gamma}{2} \cdot (D^v_{s,t} - D^{v-1}_{s,t})^2 \\
& + \frac{\gamma}{2} \cdot (C^v_{s,t} - C^{v-1}_{s,t})^2
\end{aligned}
$$

$$
\begin{aligned}
\text{s.t.} \quad & 0 \le D^v_{s,t} \le \overline{p_s} && \forall t \in \mathcal{T} \\
& 0 \le C^v_{s,t} \le \overline{p_s} && \forall t \in \mathcal{T} \\
& 0 \le E^v_{s,t} \le \overline{e_s} && \forall t \in \mathcal{T} \\
& E^v_{s,t} - E^v_{s,t-1} - C^v_{s,t} + D^v_{s,t} = 0 && \forall t \in \mathcal{T}
\end{aligned}
$$

$$
I_{n,t} = \sum_{g\in\mathcal{G}_n} P_{g,t} + \sum_{s\in\mathcal{S}_n} (D_{s,t} - C_{s,t}) - d_{n,t}
$$

* See thesis for a detailed nomenclature

2.3 Implementation of the Algorithm



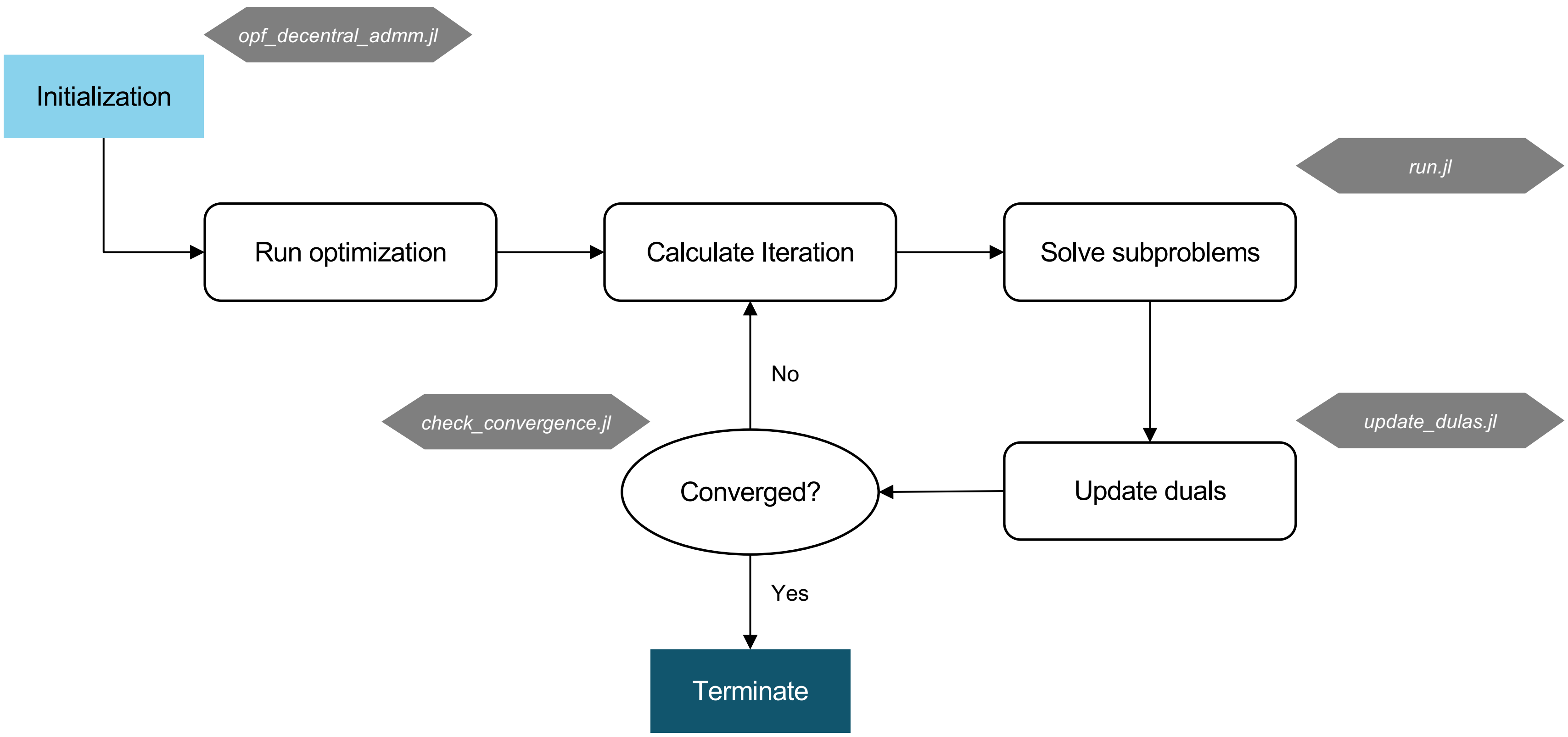
Published on github with MIT license

Divided into several units for better maintenance

Code well documented in thesis

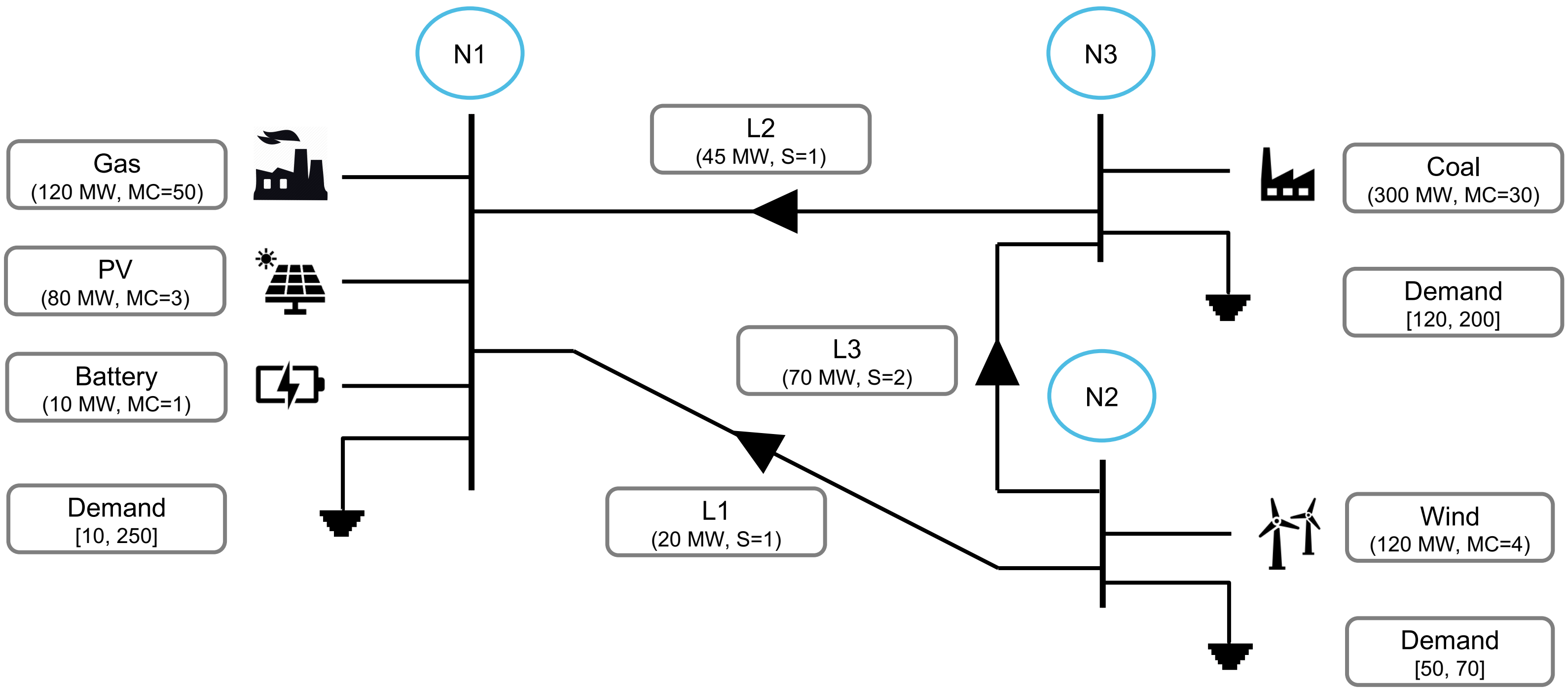
Can be easily extended

2.3 Implementation of the Algorithm



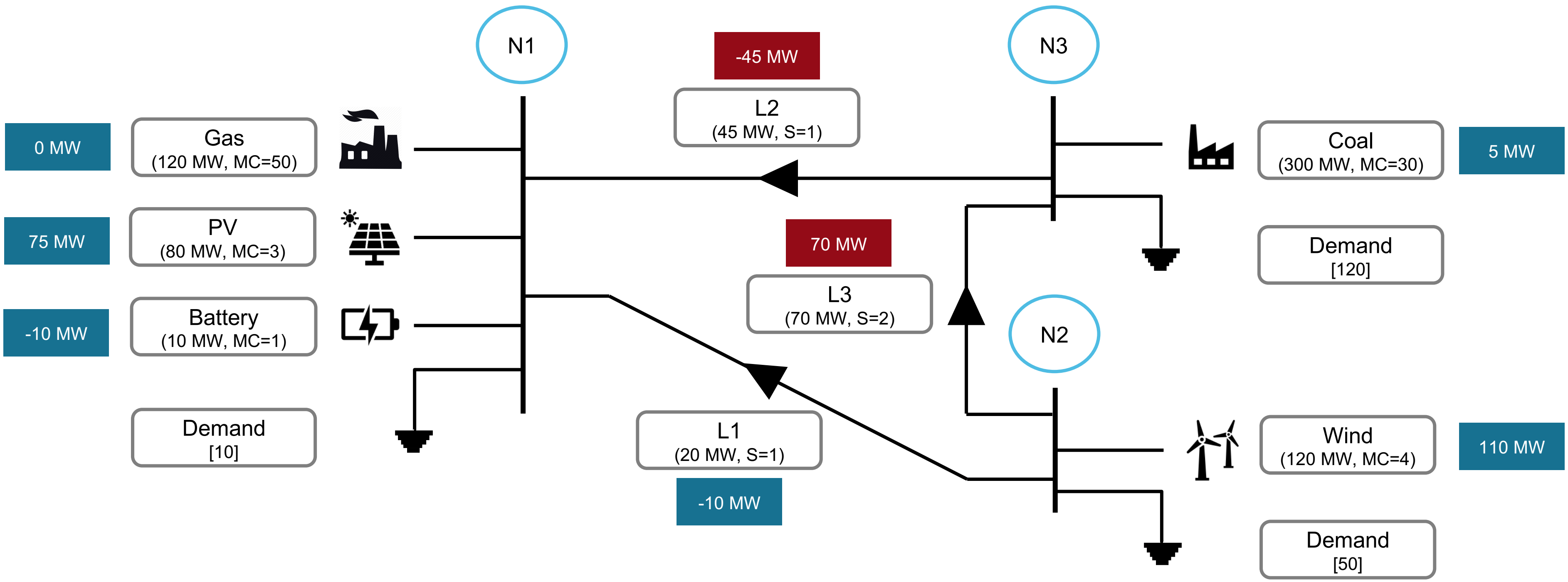
3. Results

Three Node System



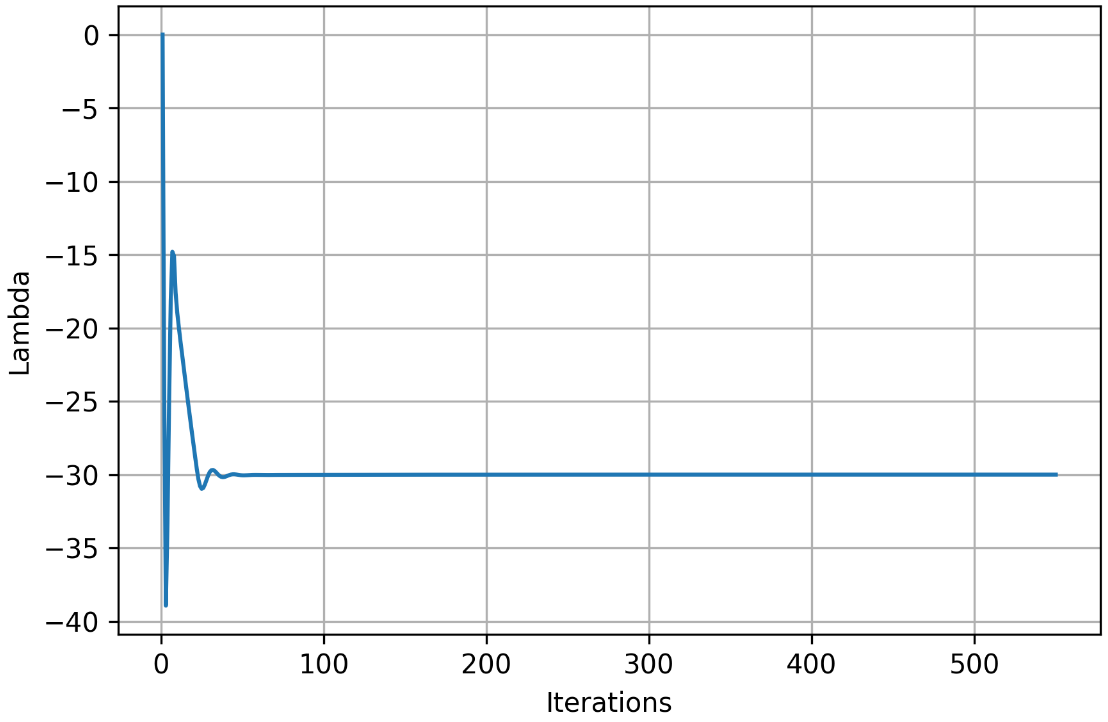
3. Results

Centralized Problem for t=1



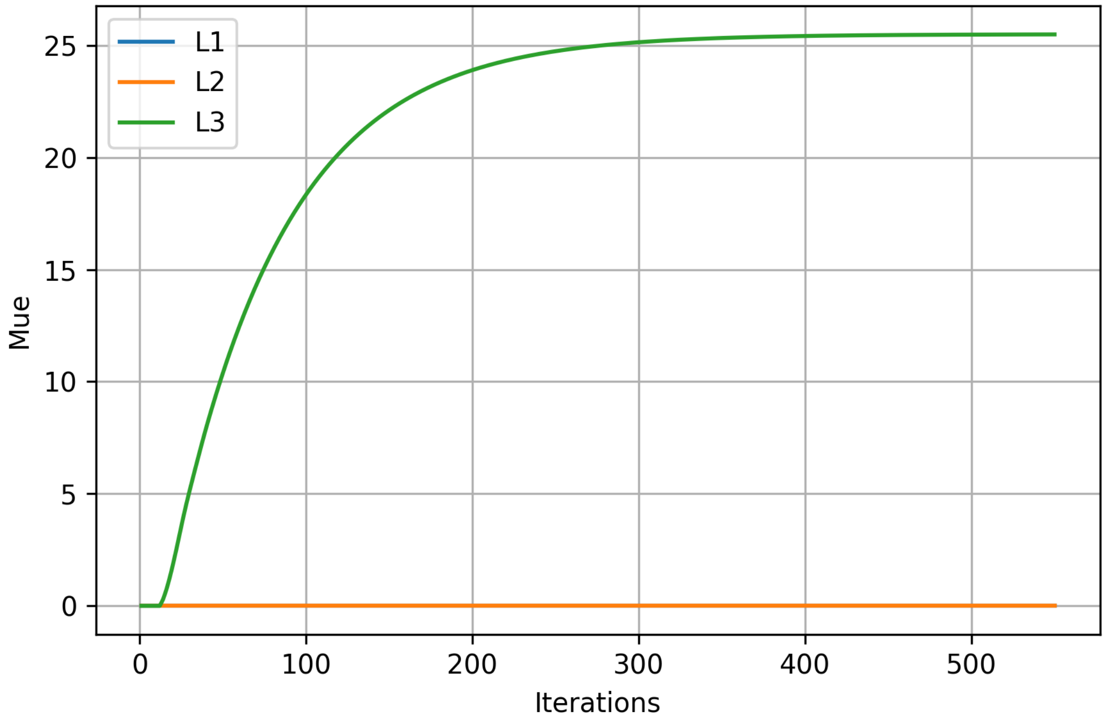
3. Results

Convergence of Lambda (System Balance) for t=1



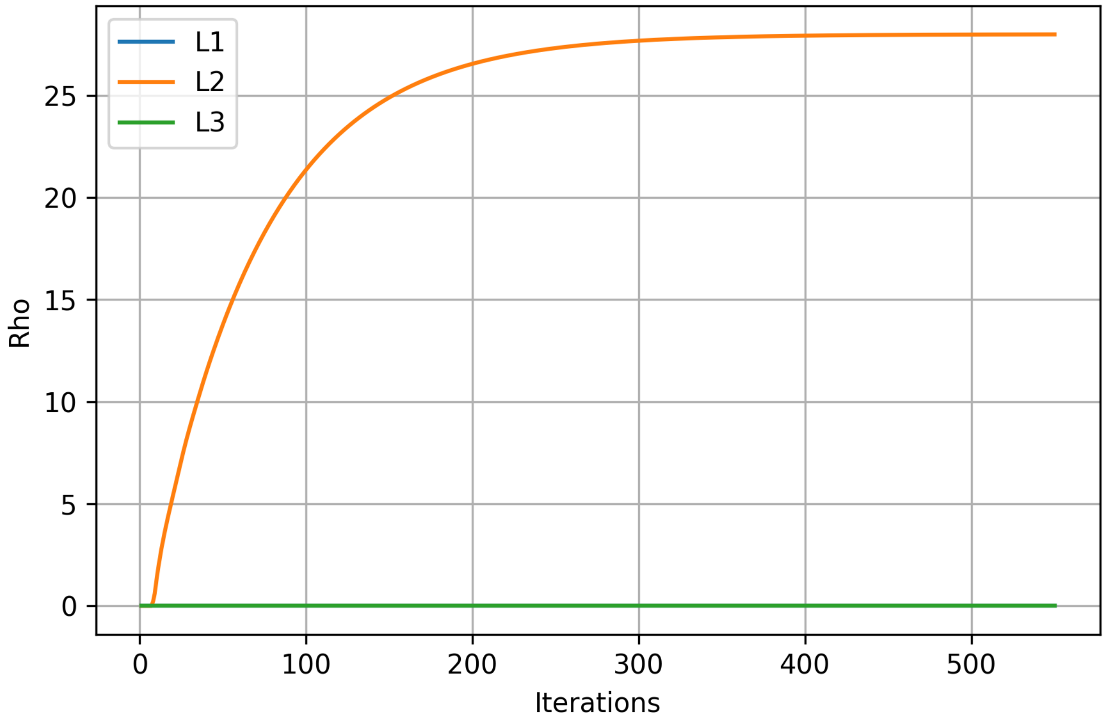
3. Results

Convergence of Mu (Upper Flow) for t=1



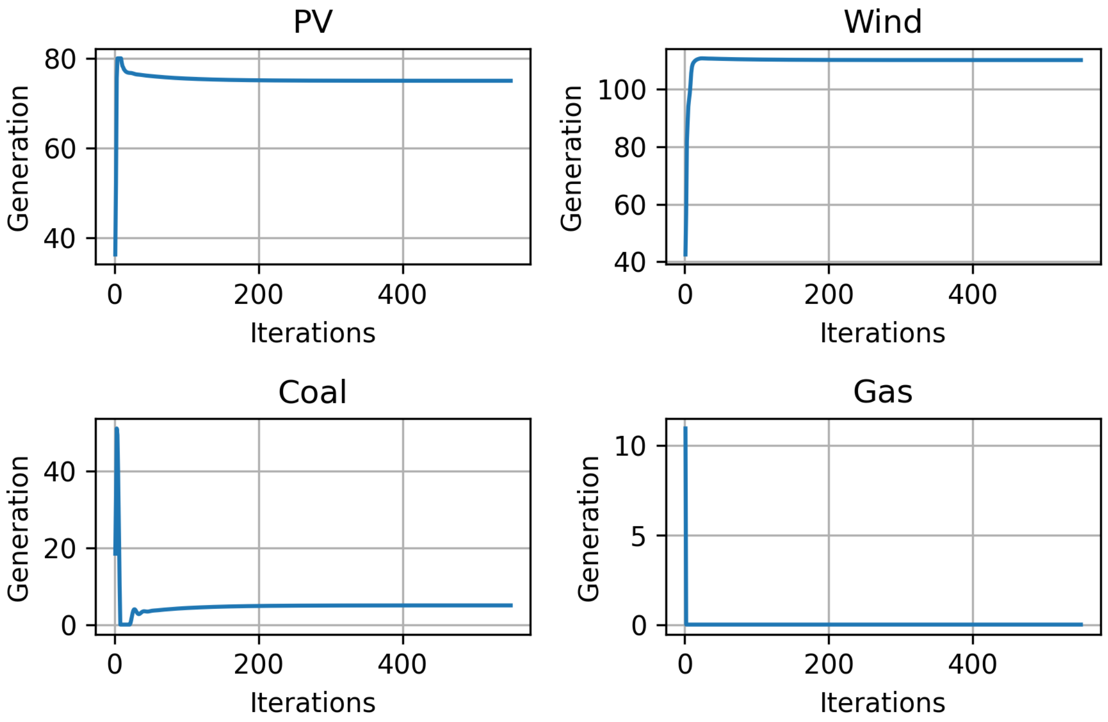
3. Results

Convergence of Rho (Lower Flow) for t=1



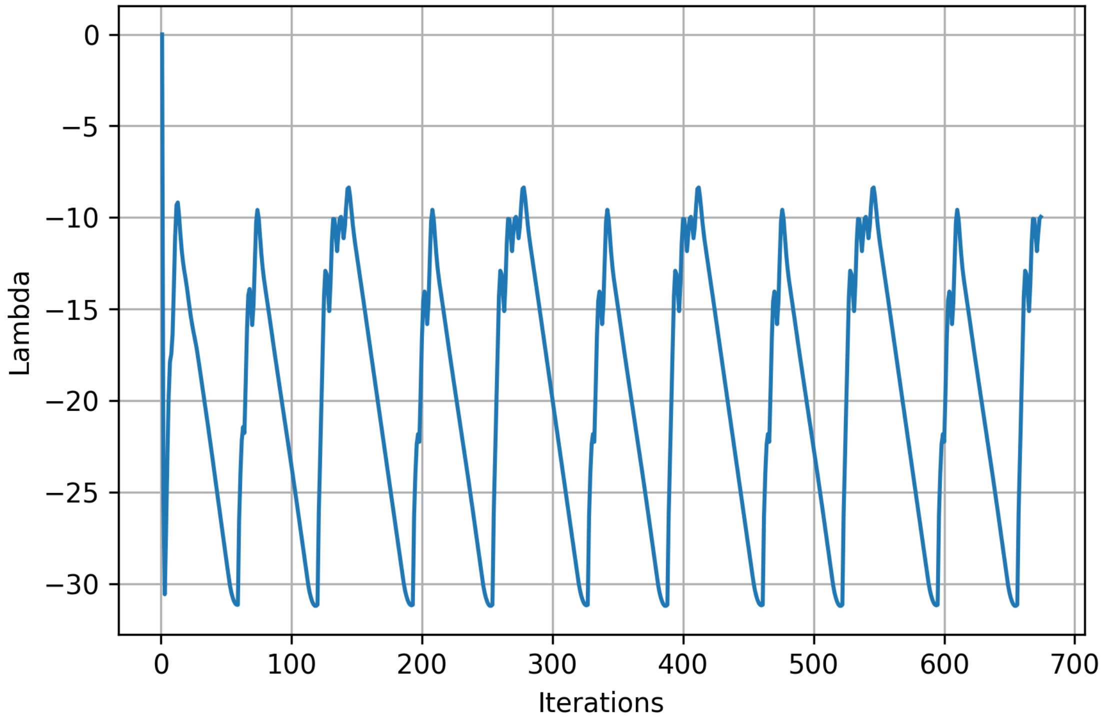
3. Results

Convergence of Generators for t=1



3. Results

Convergence problem – wrong weight for flow penalty terms



$$\frac{\gamma}{2} \longrightarrow 10$$

4. Conclusion

Implementation of decentralized OPF is possible

Identical results between central and decentral approach

Damping parameter γ influences convergence very strongly

Contributions

- Open-source package
- Well-documented derivation
- Detailed description of findings
- Algorithm can be further extended

Outlook

- Automate selection of damping parameter γ
- Investigate implementation of regulations
- Create financial incentives to increase network security
- Include a more sophisticated case study

Thanks for your attention!

2.2 Mathematical Formulations

Decentralized Problem – Update Duals

$$\lambda_t^{v+1} = \lambda_t^v + \gamma \cdot (\sum_{g \in \mathcal{G}} P_{g,t}^v + \sum_{s \in \mathcal{S}} D_{s,t}^v - \sum_{s \in \mathcal{S}} D_{s,t}^v - d_{n,t})$$

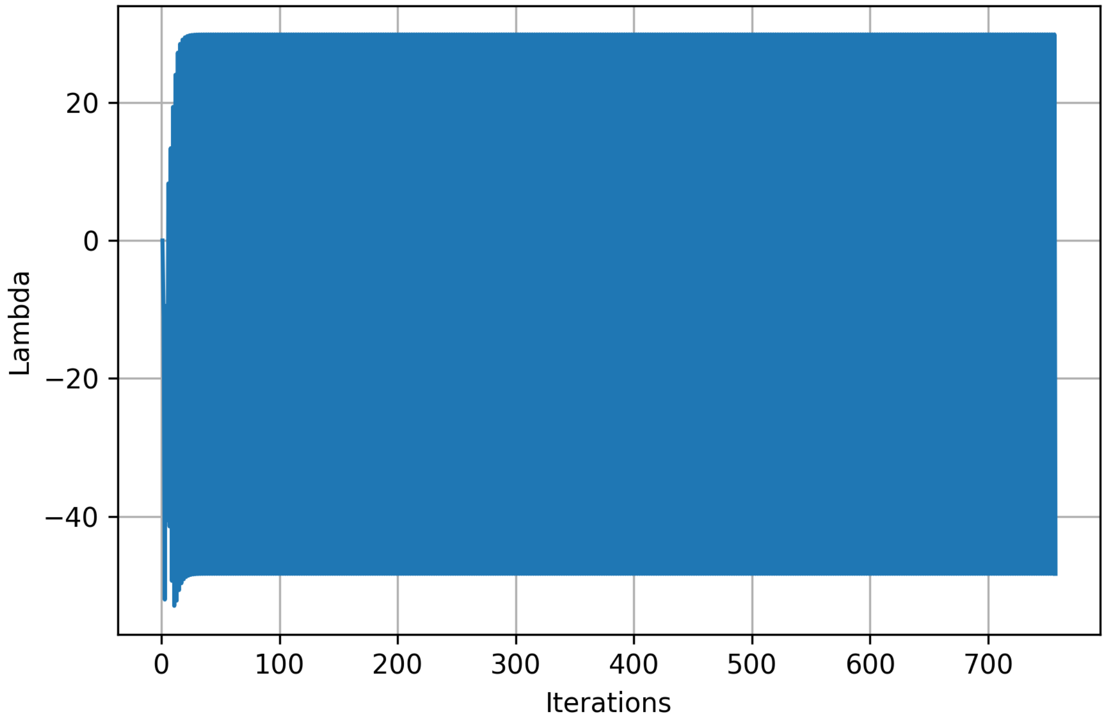
$$\rho_{l,t}^{v+1} = \rho_{l,t}^v + \gamma \cdot (\sum_{n \in \mathcal{N}} PTDF_{l,n} \cdot I_{n,t}^v + \Upsilon_{l,t}^v - \overline{f_l})$$

$$\mu_{l,t}^{v+1} = \mu_{l,t}^v + \gamma \cdot (\Gamma_{l,t}^v - \sum_{n \in \mathcal{N}} PTDF_{l,n} \cdot I_{n,t}^v - \overline{f_l})$$

* See thesis for a detailed nomenclature

3. Results

<u>Convergence problem – big damping parameter</u>



$\gamma = 0.5\ (0.3)$