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Contract Networks for Electric Power Transmission

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Abstract
A contract network extends the concept of a contract path to address the problem of loop flow and congestion in electric power transmission systems. A contract network option provides an internally consistent framework for assigning long-term capacity rights to a complicated electric transmission network. The contract network respects the special conditions induced by Kirchoff's Laws; accommodates thermal, voltage, and contingency constraints on transmission capacity; and can be adopted without disturbing existing methods for achieving an economic power dispatch subject to these constraints. By design, a contract network would maintain short-run efficiency through optimal spot-price determination of transmission prices. Through payment of congestion rentals, the contract network makes a long-term capacity-right holder indifferent between delivery of the power or receipt of payments in a settlement system.

Everybody talks about the weather, but nobody does anything about it.[1]

Introduction

The electric utility industry has entered a new era, one characterized by competition between utility-owned and independent power, by long-term movements of power from one region to another, and by the development of short-term markets in which many buyers shop for the lowest cost power (Joskow 1989, 125-208; Wilkinson 1989). This greater use of market forces, encouragement of new suppliers, and increasing reliance on economy power sales, plus recent precedents in power company mergers, have placed new demands on the electric power transmission systems throughout the world. Efficient use of the transmission system in a market context calls for changes in the institutions that govern transmission transactions. In the United States: "The most debated public policy issue involving the electric utility industry of the 1990s will likely be that of transmission access and the use of the bulk power transmission system" (*Public Utilities Fortnightly* 1990, 12).

One unfulfilled requirement for institutional reform is a design for a consistent system of (i) short-term use pricing and (ii) long-term firm transmission capacity rights that can accommodate the complex problems of loop flow in the presence of line thermal limits, bus voltage tolerances, and other constraints on the transmission system.

Everybody talks about loop flow, but nobody does anything about it (Federal Energy Regulatory Commission 1989; Kelly et al. 1987). Most prevailing firm transmission rights are specified in terms of "contract paths" or "interface transfer capabilities" that do not address the special conditions in electric networks.[2] The present paper suggests the use of a "contract network" as a basic building block of a market in power transmission. A contract network and the associated rights can accommodate a system for short-term efficient pricing and long-term firm use of a transmission network.

The next section outlines the basic requirements for firm transmission rights. A subsequent section summarizes the principal problems in describing the economics of electric power networks, including loop flow, thermal limits, voltage tolerances, and contingency constraints. With this background, we present the concept of a contract network and its associated firm transmission rights, and link this long-term definition to short-term efficient transmission prices determined consistent with existing network dispatch procedures. An example illustrates the general model. The final section outlines a few research and implementation questions raised by the prospect of designing a practical system based on a contract network model. An appendix summarizes supporting details.

Firm Transmission Rights

It is easy to think of the attractions of the electric power grid as a highway for delivering cheap power from distant sources. Located far from urban load centers, low cost hydro or coal plants send power over the grid and reduce the difficulty of constructing new facilities in environmentally more sensitive areas. But despite the appeal of low cost power, the principal functions of the electric power transmission grid continue to be in maintaining reliability and lowering generation capacity costs through system diversity and capacity sharing. Even without the attractions of cheap energy from distant sources, much of the existing electric power grid would be justified by the reliability benefits alone (Corey 1989). Hence, multiple uses motivate the grid operator's procedures and priorities.

Pressure for Transmission Rights

As for the interstate highway system, perhaps the best transmission policy might be to maintain excess capacity everywhere and eliminate any concerns about access and congestion. But the transmission system is becoming more like the downtown streets with all the problems of excess demand for a common property. Unfortunately, when traffic stalls on this highway, the whole system can come crashing down. With capacity limits and the associated congestion expected to continue, there is a need for a new definition of the rules of the road.

Traditionally, with the focus on reliability, management of the grid could and did operate successfully through a system of committees, a club of insiders who could arrange and would honor informal "gentlemen's agreements" to share the economic burdens and benefits. But, in recent years, the changing economic conditions have put this club under pressure. The rise of sustained long-distance economy power sales has increased the importance of the grid and raised the economic stakes. Now imperfections in the informal arrangements can have substantial economic impact, and the club meetings have become more contentious. The traditional club members—electric utilities—are searching for alternatives.

Furthermore, the related interest in greater use of market forces has included widespread entry of new participants. In some areas of the country, independent power producers (IPPs) or non-utility generators (NUGs) are expected to provide virtually *all* the new power generating capacity. During the recent period of excess generating capacity, these new participants have been accommodated as special cases. Fortunately, as special cases, they were small and few and, therefore, easy to absorb. But this is about to change, as the nation looks to expanding generating capacity. And the new players are clamoring for admission to the transmission club with assignment of meaningful transmission rights (Pierce 1990, 28-29). Often this pressure for opening the transmission grid includes an equal interest in developing a full-blown competitive market for new power sources: "the estimates of effective concentration are *extremely* sensitive to the transmission capacity assumption," (Schmalensee and Golub 1984, 21, emphasis in the original.). And this will virtually require open access to the transmission system:

> A third variation is open transmission access so that any potential supplier and any of the potential customers would be able to obtain the best prices. I do not believe it is possible to have competitive bidding for generation resources and customers without having some form of open transmission access (Casazza 1988, 17).

Finally, if there is any doubt that the rules are changing, recent regulatory decisions should settle the point that the status-quo system is not up to meeting the challenges of the new market. The Federal Energy Regulatory Commission (FERC) has conditioned approval of utility mergers on open access to the transmission grid, and many proposals have been offered as the best design for pricing and using the grid (FERC 1989). These new proposals highlight the several interconnected components of transmission operations and economics. But none of the proposals faces squarely the difficulty of defining, much less monitoring, firm transmission rights in the face of loop flow constraints. For example, the FERC in its "no fault" transmission policy as applied in the Public Service of Indiana case explicitly declined to examine the reasons for the existence of any transmission constraints (Lindsay and Hall 1990).

Efficient Use of the Electric Power System

The goal is to promote economic efficiency in the use of the electric power system. Economic dispatch of electric power plants connected through a transmission grid provides a natural starting point for discussion of efficient electricity markets. By definition, economic dispatch maximizes the benefits less the costs subject to the availability of plants and the constraints of the transmission network. The balance of costs and benefits sets prices equal to marginal costs reflecting both the direct costs of generation and the opportunity costs throughout the system. With this perspective, the work of Schweppe et al. develops the theory of spot pricing that respects the particular conditions of electric power transmission systems (Schweppe et al. 1988). Efficient short-run prices are consistent with economic dispatch, and, in principle, short-run equilibrium in a competitive market would reproduce both these prices and the associated power flows.

The availability of efficient short-run prices could provide a powerful tool for guiding the use of the electric power system. The theory of spot pricing identifies the competitive price at each bus. Efficient transmission of power from one bus to another would not be priced at anything higher than the difference in the spot prices at the respective buses. Hence, this difference is the natural equilibrium definition of the price of transmission, and a matrix of spot price differences across buses provides the framework for efficient transmission pricing. Efficient prices could motivate the use of a short-run market, or, more likely, central dispatch could be used in conjunction with an efficient pricing model and a settlement system to manage the appropriate financial transfers among the participants. The theory of efficient short-run power prices provides the well developed starting point for an efficient use of the transmission system (Schweppe et al. 1988).

Whatever the practice of short-run usage pricing, it must be integrated with a policy for long-term access and contracts for firm transmission service. From one perspective, under rather ambitious assumptions, the long-term market for power transmission could operate as a sequence of efficient short-term spot markets. The principal requirement would be for decreasing or, at least, constant returns to scale in the transmission system. Although full reliance on short-term markets might be attractive in finessing the need for a definition of long-term rights, even the narrow technical requirement of constant returns to scale is unlikely to be met in most cases (Read 1988; Read and Sell 1988). Therefore, only in an ideal world would we be likely to rely solely on the optimal long-run outcome arising from a series of short-run pricing decisions.

Hence creating a competitive long-term market presents a new set of complications. And the long-run market is the key to overall efficiency. The most important requirement is to provide the right incentives for location and construction of new generating facilities and new load centers. By comparison with the costs of poor choices on these major plant investment decisions, there would likely be small operating inefficiencies from any failure to adopt a perfect short-run transmission pricing model.

In addition to assigning rights to the existing transmission system, efficient expansion of the transmission system, especially in the presence of economies of scale, presents its own set of challenges. In the absence of property or contract rights for users of the system, expansion of a centrally operated grid used by many relatively small market participants would require in principle a cost-benefit analysis that might lead to a solution that would not be replicated in a fully decentralized market. And if there are large economies of scale, efficient use of the transmission system might well produce prices which would not cover the cost of the expansion.[3] This subject, the optimal design and expansion of the transmission network, is an important topic that may be simplified by creation of a system of contract rights, but it is separable from the focus of the present discussion. Here we assume that there is some mechanism for deciding on the design of the system and covering the total costs (typically through "club membership" or access fees), and we address the problem of defining rights and pricing the use of the system in the short and long term.

Requirements for Firm Transmission Rights

Our perspective is on developing a framework for long-term contracts that define firm rights to the transmission system. Experience suggests that investors in long-lived, fixed facilities of the type and scale of major electric power plants will be reluctant to make commitments with no more than a promise of being allowed to participate in a short-term spot market for transmission services. Practical development of long-term deals with the associated capacity and energy payments must include some form of firm right to power transmission. Ideally there will be an associated usage pricing mechanism that reinforces the incentives for open access, economic dispatch, and efficient secondary markets for long-term firm rights.

In addition, any system for transmission rights must meet other equally important criteria. Foremost is preservation of the reliability of power system operations. Any proposal for revising the current system must recognize and respect the real complications of day-to-day management of a power network. Just as with airlines and the air traffic control system, investment and pricing rules must respect the unrestricted operational authority of the system controllers.[4] A proposal which requires a major change in current short-term system operations will face possibly insurmountable institutional barriers.

Furthermore, a reasonable transmission allocation and pricing system should be decomposable by region and company, and it must meet the test of administrative feasibility. Finally, any transmission proposal must address the FERC concern over the existence and possible abuse of market power. Ideally a transmission protocol should be consistent with a competitive market and at least neutral with respect to the exercise of market power.

No current proposal meets all the tests, and the status quo is under pressure, especially on the open access and economic efficiency tests. But moving from the status quo presents a number of conceptual obstacles in defining transmission rights. What is the capacity of the system? How can we deal with loop flow and move beyond the fiction of the contract path? How can we preserve reliability and capture the benefits of efficient pricing? How can we allocate rights to the system?

Transmission Contract Path and Loop Flow

The problems created by "loop flow" are familiar to electrical engineers but often counterintuitive to others on first examination. Simply put:

> Because of the nature of ac [alternating current] transmission systems, energy transactions between two systems can cause flows in parallel transmission paths in other connected systems not directly involved in the transaction (North American Electric Reliability Council 1989, 36).

Electricity moves according to Kirchoff's laws, essentially following the path of least resistance. As a result of these physical laws, power moves across many parallel lines in often circuitous routes.[5] One of the most important economic implications of this prevalence of loop flow is that the power transmission highway is very unlike other highways, and analogies comparing other highways, railroads, or pipelines can be quite misleading.[6]

The most ubiquitous analogy is the contract path. In a rail line, it is easy to describe the route that a train will take, and the existence of parallel track is not a problem in writing a contract. The contract could specify a particular route and that path could be used. Likewise, it is easy to look at a map of an electric transmission system and find a path between a generating plant and a customer. And it is common practice to write a contract describing the power flow over that path. But it may not be so easy to see that path used:

> System economics often justify the use of transmission facilities among interconnecting utilities to establish "contract paths" for interchange transactions. In many

LOOP FLOW CAN AFFECT DISTANT THIRD PARTIES

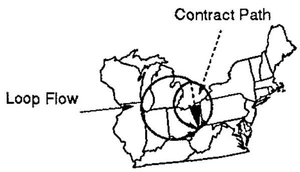

Figure 1

> instances, these facilities carry only a small fraction of the transaction, the balance of the transaction utilizing facilities in other systems (NAERC 1989, 36).

Hence, as illustrated in figure 1, the "contract path" is a fiction (FERC 1989). The actual flow of power may and often does diverge widely from the contract path. As a result, the supposed economics of the contract path may have little to do with the actual costs of the power transfer. Furthermore, these loop flows can affect third parties distant from the intended power flow, and, under the current rules, these third parties may and often do incur costs without compensation.

When loop flow is a small part of power economics, when informal swaps can balance out the effects over time, and when all the parties are members of the same transmission club, it is reasonable to employ the contract path fiction as a practical accommodation in crafting power contracts. These circumstances fit the past, and the contract path has been a workable fiction. But all these conditions are changing. And it is widely recognized that giving explicit attention to the economic effect of loop flow and the limitations of the contract path model presents one of the greatest challenges for designing a new power transmission regime.

The problem of loop flow is ubiquitous and can invalidate some of the most important elements of transmission agreements. For example, what is the capacity of the network? The difficulty of defining the transfer capability of the power system is closely related to the economic problems of loop flow.

> While "transfer capabilities" between one system and another are often quoted, it is understood by those who determine them, and those who use them, that these capabilities are approximations for a specific set of conditions and not firm values that apply at all times. Therefore, a published "transfer capability" should be regarded more as a typical or average value. The actual capability at any moment may be considerably higher or considerably lower (NAERC 1989, 41).

This ambiguity about the capacity of the network does not depend on the condition of the power lines. Even in normal periods of operation, when all the transmission lines are available, the transmission capacity from one region to another depends critically on the configuration of load and generation.

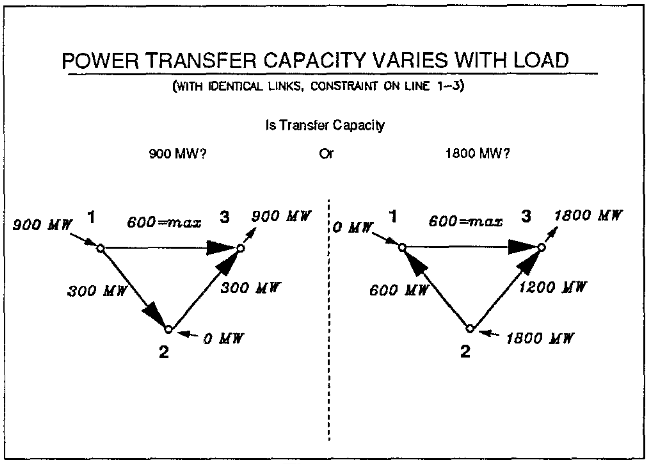

Figure 2

Consider the simple example in figure 2. This network consists of three buses and three lines. For the sake of illustration, we assume that the lines are identical except that a thermal constraint on the line between buses 1 and 3 limits the flow on that line to 600 MegaWatts (MWs). Now suppose that we consider buses 1 and 2 to be in the same region, perhaps the generating region, and bus 3 is the load region. What is the transfer capability from the generating region to the load region?

The two panels in figure 2 depict two different load patterns that exhaust the capacity of the constrained line. As seen by a comparison of the two panels, the estimate of the transfer capability depends on the configuration of the generation. In the left panel, the total demand at bus 3 is for 900 MWs, and these 900 MWs are provided by the generator at bus 1. The flow of power follows Kirchoff's laws. Since the path 1→2→3 is twice as long as the path 1→3, it has twice the resistance. Hence 600 MWs move along the path 13, while 300 MWs move along the parallel path 1→2→3. This is loop flow.[7] There is no power generated at bus 2 and none can be added there without violating the 600 MWs constraint on the line between 1 and 3. Because of the constraint, as long as we choose to generate 900 MWs at bus 1, we cannot satisfy any more demand at bus 3. In a real sense, therefore, the power transfer capability might be viewed as 900 MWs.

If demand increases at bus 3, there is no choice but to generate power at bus 2 and reduce the generation at bus 1; otherwise the power flow along 1→3 would exceed the maximum thermal limit. In the extreme, as shown in the right panel of figure 2, if demand rises to 1800 MWs, the only solution is to generate all the power at bus 2 and none at bus 1. Hence, the power transfer capability might be viewed as 1800 MWs.

In a real network, the conditions in the left panel of figure 2 might represent the economics and availability of generators at one time and the right panel would apply at another. And the announced transfer capability might be somewhere in between. But—"The actual capability at any moment may be considerably higher or considerably lower"—even when there has been no change in the transmission system. A contract for 600 MWs between 1 and 3 may have relied on a contract path along the direct connection, but the higher demand case would have precluded this use of the system and the contract right could not be honored. Evidently changing load patterns could play havoc with transmission rights expected to extend over many years.

This small system makes the interactions apparent, but, in a larger network, it can be difficult to assign the loop flows. This example illustrates the problems of defining and using transmission rights along a contract path. An alternative is to define the rights in terms of a contract network.

Defining Transmission Rights in a Contract Network

If electric power flows on every parallel path, then the definition of rights and contracts need a better approximation than the contract path. The contract network is an extension of the contract path that provides a framework for defining long-term rights while preserving short-run efficient use of the system.

Efficient Pricing

In the short-run, the problem of loop flow calls out for a process that will discipline use of the transmission system. In particular, the transmission protocol for a competitive market should (i) respect long-term capacity rights, (ii) force short-run users to recognize opportunity costs, and (iii) promote efficient trades to capture the changing economics of power loads and power generation. Those who have long-term rights should not be disadvantaged by the short-term problems of transmission congestion; those who create loop-flow congestion should pay the full cost induced by their use of the system; and those who have cheap power should be able to trade with those who have expensive power. In short—short-term transmission prices should be determined by optimal spot prices in the manner developed by Schweppe and others. Although these spot prices can vary significantly over time, our focus is on the locational differences in prices.

The transmission prices calculated according to optimal spot-pricing theory incorporate the marginal cost of generation, the marginal cost of losses, and the opportunity cost created by congestion in the system. The first two cost groups are straightforward. Economic dispatch calls for the use of the cheapest combination of power plants needed to meet the existing load. If all the plants and loads were located at the same place, then the plants would be dispatched in order of lowest to highest marginal cost. If plants are located in different places and power must travel over the transmission grid, then the losses of power in transmission should enter the economic dispatch calculation. But again, after adjusting for losses, economic dispatch calls for using the cheapest plants first, and optimal spot prices will be equal to marginal costs.

If the transmission grid is heavily loaded, however, bottlenecks may lead to congestion, and congestion will prevent full use of all the cheapest plants. Often referred to as "out-of-merit" dispatch, the constrained use of the plants creates a frequently significant opportunity cost that can be assigned to the constraints which induce the congestion. This opportunity cost should be included in the prices.

The congestion constraints arise in two principal forms. The first and easier to understand is the limit on the flow of power on an individual line. Just as in figure 2, the thermal capacity of a transmission line sets an upper limit on the flow of power on that line. And through the interactions of Kirchoff's laws, a line constraint affects every other flow in the network. A change in generation or load at any bus will have some effect on the flow on the constrained line; hence, the constraint can affect the opportunity costs at each bus. It is possible to calculate the congestion cost induced by any thermal constraint and thereby estimate the effect on the optimal spot prices throughout the network. The result for thermal constraints is a central part of the theory of spot pricing.

A second major source of congestion in a power network arises from voltage magnitude constraints at buses. In normal operations, or as an approximation of the more complicated worst-contingency analysis, voltage constraints define operating bounds which can limit the amount of power flowing on transmission lines.[8] Even when power flows do not approach the thermal limits of the system, and the transmission lines appear to have excess capacity, voltage limits can constrain the transfer capacity and must be included in the calculation of congestion costs.

Voltage constraints inevitably require attention to both the real and reactive power loads and transfers in the alternating current (AC) transmission system. Recall that real power (the power that lights our lamps) is measured in watts or MegaWatts and reactive power is measured in voltage-amperes-reactive or Vars and MegaVars (MVars). Power generation, load, and flow in an AC system are divided into both real and reactive power components. It is natural for us to speak of real power flow, but an examination of reactive power takes all but the electrical engineer into unfamiliar territory (Kelly et al. 1987). Even the engineers have been known to shy away from a physical interpretation of reactive power loads:

> Recall that reactive power (VArs) is a purely mathematical concept used to define how far the current is out of phase with the voltage. To best understand reactive-power compensation, simply accept the fact that VArs must be supplied to a transmission line to compensate for reactive power consumption. Do not try to understand the meaning of reactive power in physical terms (Reason 1989, 35).

Without voltage constraints, the principal matter of concern is the real power flow, and it is common practice to ignore the associated reactive power analysis. But voltage can be affected by both real and reactive power loads, and the interaction between the two is critical in determining both the induced limits on real power flows and the associated spot prices.[9] In this event, spot pricing now applies to both real and reactive power, and the associated transmission prices must be determined for both types of power.

Unfortunately, voltage limitations and the associated reactive power compensation problems are prevalent. For example, the "surprising" power shortages in New England and New York in 1988 were attributed in large part to voltage problems and the "hidden but critical" role of reactive power (Zorpette 1989, 46-47). Hence, it will not be enough to account for the congestion limits created by thermal limitations on transmission lines. Any new regime for transmission access and pricing must address the congestion problems created by reactive power compensation and voltage constraints. The most direct method is to establish transmission prices and contract rights in terms of both real and reactive power.

Capacity Rights

In other networks, long-term rights for use of the system can be defined, enforced and traded. For example, in allocating capacity in a gas transportation system, rights can be defined to send gas through individual bottlenecks in the pipeline system. Holders of these rights could use them, in which case their gas can be identified as having flowed through the bottleneck. And holders could sell the rights to others in a secondary market, in which case the buyer would have an equally well-defined asset. In a fully functioning market for the well-defined rights, prices in the short-run should reflect opportunity costs created by congestion at the bottleneck. The pipeline operator need only keep a list of the current capacity-right holders and verify that actual use of the system corresponds to the allocation of capacity. The secondary market would provide the trading opportunities and give all the participants the right incentives to pursue economically efficient exchanges (Hogan 1990).

Implicitly this system for gas pipelines or other networks exploits the one-for-one definition of the capacity rights. If you sell me one unit of capacity at a bottleneck, I have one more unit of capacity. There is no complication of loop flow. But as discussed earlier in the explanation of figure 2, the story is different in the case of an electric transmission network. Now there is no one-for-one trade, and the impact of different loads can be far from obvious.

However, it is not necessary to identify the trades in different uses of electric transmission capacity. As developed in the theory of spot pricing, the preferred definition of transmission and its associated pricing is from bus to bus. With transmission prices defined as the difference between the spot prices at the buses, we conceal the problems of loop flow in the calculation of the prices and marginal costs. The spot prices summarize all the information about the interactions in the network, and there is no need to define the transmission path.

The "buy-sell" model is an alternative interpretation of the efficient short-run pricing system that accommodates the loop flow problem. In the buy-sell model, the grid operator stands between power generators and power consumers. Ideally, the operator buys and sells power at the buses at the short-run efficient prices. One great attraction of this perspective is that there is no need to define transmission at all; users of the network never transmit power across the network, they merely sell at some nodes and buy at others. The problems of loop flow and congestion are then hidden in the internal operations of the network. All transmission is implicit. If the bus prices are short-run efficient, then the implied transmission prices are just as used here—the difference between the short-run prices at the buses.[10]

The principal difficulty with the buy-sell model as an institutional reform is in obtaining acceptance by the users of the network. If there is a strong grid with excess capacity, then the implicit transmission prices will be small and the users might be confident that the grid operator would be willing to buy and sell power at reasonable prices. But if congestion problems are large or there is uncertainty about the pricing in the buy-sell arrangement, investors might prefer a more traditional link between a plant and a customer with a well-defined transmission capability to move the power from source to destination. In this case, investors need a mechanism to protect against price changes through a definition of the capacity rights embedded in a transmission agreement.

As with the definition of transmission prices, the preferred definition of a transmission capacity right is from bus to bus, with no attention to the paths by which the power flows. Hence, in figure 2, we might define the generator at bus 1 as having a "right" to send 900 MWs to bus 3. But suppose conditions change, and it is cheaper or necessary to generate 1800 MWs at bus 2, as shown in the right panel in figure 2. How can we organize events such that the capacity-right holder located at bus 1 sells the right to transfer 900 MWs and the generator at bus 2 buys the right to transfer 1800 MWs?

One approach might be to allow for bilateral trades of capacity rights among users of the network. For the simple three-bus case, it is an easy matter to see the interactions among users of the system. But as the network gets more complicated than that shown in figure 2, there will be many intervening buses and lines. With rights for transmission capacity defined between bus pairs, the complexity of the interactions and required trades could eliminate all but a few one-for-one trades, and the problem of loop flow would return to the forefront. Obviously, the central grid operator, someone with an overview of the system, must be involved. But that involvement must be more than just keeping a list of the capacity-right holders.

In principle, the central operator would know or be able to calculate that 1 MW from 1 to 3 displaces 2 MWs from 2 to 3. Hence, one approach would be for all trades of capacity rights to be made through the central operator with the requirement that, before any power moves, the appropriate capacity must be obtained from the capacity-right holder. This form of a buy-sell model with the grid would allow efficient trades and give users the necessary information about opportunity costs. But it would place a substantial burden on the central operator, especially if there were many trades. Regrettably, it is normal for economic and load conditions to change frequently, so that there would be a requirement for many capacity trades.

An alternative would be to merge the short-term pricing and an implicit secondary market in capacity rights as part of a contract network. Here a one-step pricing method is available that will allow for implicit trades without imposing new demands on the operator of the transmission grid. In particular, given a designated "swing" bus, recall that the spot price at any bus is a combination of the system marginal cost of generation at the swing bus, the impact on losses, and the impact on congestion. Hence, for bus *i*:

$$\text{Bus Price}_i = \text{Generation} + \text{Losses}_i + \text{Congestion}_i.$$

The transmission price between any two buses can be defined as the difference in the spot prices. Consequently, the transmission price between bus *i* and bus *j* is

$$\text{Transmission Price}_{ij} = (\text{Losses}_j - \text{Losses}_i) + (\text{Congestion}_j - \text{Congestion}_i),$$

or

$$T_{ij} = T_{Lij} + T_{Cij}.$$

The first term, TL_{ij}, is just the incremental effect of transmission from bus *i* to bus *j* on the system losses and can be thought of as the operating cost of the transmission service. It would be normal to expect both long-term and short-term users to pay this cost.

The second term, T_{Cij}, is the incremental effect of transmission from bus *i* to bus *j* on the system congestion costs. This charge has an interpretation as the rent on the transmission capacity used by the power transmission from bus *i* to bus *j*. Since the congestion charge measures the opportunity cost, this is the maximum that others in the system would be willing to pay to purchase the capacity right. If a holder of a capacity right actually uses the transmission system, then, at the margin, efficiency would require that the right holder recognize that trades are available that would pay T_{Cij} for the capacity right between bus i and bus j. Similarly, if others use the transmission system and through loop flow prevent the capacity-right holder from transmitting power, then the T_{Cij} charge from the optimal transmission price is the minimum that the right holder should accept to "rent" the capacity to the actual users.

These interpretations of the congestion charge suggest a definition of the capacity right which would be consistent with successful operation of a secondary market. For any period, suppose that the loads and transmission flows follow economic dispatch and the corresponding spot prices are available consistent with economic use of the system. Then all users of the transmission system are charged the transmission prices T_{ij} according to their usage. But in addition, the owners of capacity rights receive a "rental" payment from the grid equal to T_{Cij}, the congestion charge, applied to their full capacity rights. Hence, everyone pays both the loss and the congestion charge, so everyone faces incentives for efficient short-run use of the system. And the capacity-right holder is compensated if loop flow or load conditions prevent full exercise of the capacity right.

If the right holder uses the full capacity right, then the congestion charge paid is just balanced by the rental payment, and the net cost of transmission is just the losses charge. This is true no matter what the load conditions or no matter how large the total transmission charge.[11]

But, just as importantly, in the presence of optimal spot prices, whenever the right holder is precluded from using the full capacity, the compensation received is just the amount needed to make the right holder indifferent between delivering the power or receiving the compensation.[12] In the latter case, the right holder can honor any long-term delivery commitments by using the rental payment or congestion fee to purchase expensive power at the point of destination. In other words, the calculation of the optimal spot prices for the contract network in the short run produces the same result as the ideal secondary market but without the requirement for explicit capacity trades.

At the margin, both the short-run user and the capacity-right holder would face the same incentives, but the capacity-right holder would also receive a rental payment that guarantees the economic viability of long-term power sale requirements. Hence, both the transmission capacity rights and the transmission prices can be defined between pairs of buses.[13] And there is no need to identify any transmission contract paths. The problems of loop flow are again handled in the calculation of the optimal spot prices for the contract network.

As reviewed in the appendix, the congestion charges could arise from thermal limits on lines, voltage constraints at buses, or a mixture of both. Since they derive from optimal spot prices, they are consistent with any efficient pricing mechanism. For example, they are the rents that would arise in the buy-sell model with the grid operator as the market maker, or these same prices would appear in a perfectly competitive decentralized market without transaction costs. But neither extreme is necessary. Any method which produces a reasonable approximation of spot-price differences can accommodate the transfer of rental payments for congestion that reflect the opportunity costs in the network.

Calculating Prices

As shown by Schweppe et al., optimal spot prices could be obtained as a byproduct of economic dispatch, and real-time spot pricing could be used to control the network (Schweppe et al. 1988). Such real-time spot-pricing would be a major innovation that would simplify much of the effort to achieve theoretical conformance with the principles of efficient use of the system. However, system operators already have well-developed methods for controlling the network, and they currently seek an economic dispatch within the limitations of a variety of explicit and implicit constraints. Given the importance of close control of the network to maintain reliability, there would be natural and legitimate objections to imposing dramatic changes in the dispatch methods in order to simplify estimation of transmission prices.

There is merit in improving the dispatch process, if this can be done. But the dispatch process is both complicated and critical in meeting the essential reliability standards of the transmission network. When trying to introduce the use of spot power prices for calculating efficient transmission prices, therefore, a less ambitious goal would be in order. One approach is to accept as given the results of the dispatch process and then allow for periodic or even ex post estimation of consistent spot prices, all the while accepting the actual system dispatch as an optimal balance of the underlying economics and constraints, but leaving the dispatch process undisturbed.[14] The dispatchers would be presumed to be solving a difficult problem, and the prices would be calculated to communicate the right incentives to the transmission customers.

Even when there is no central computer calculating the optimal prices, with these prices in turn guiding dispatch, there usually is enough information available to indicate the implied constraints on the network. This description of the results of the dispatch process depends on information that is familiar to dispatchers and can be specified without extensive calculation. And with this limited amount of information, there is an easy method for estimating prices under the assumption that the actual dispatch is the result of an optimization with respect to a set of constraints.

The additional information includes an identification of the binding constraints in the transmission system and the variable costs of operation of the marginal generating plants at critical buses. Typically, the system operator has a good deal of experience with the critical constraints in the transmission network and ready knowledge of the short-run variable costs of operation of plants. We need know only the identity of the constraints and do not require ex ante estimates of the limits on the flows or voltages.

For the costs at critical generating buses, even if they are not reported regularly, we can reasonably estimate these costs, which should be dominated by energy costs. At each bus, we will know the plants, or more precisely the units, that are running and those that are available but idle. Then the running cost of the most expensive plant in use but running at its upper limit provides a lower bound on the spot price at that bus. Apparently if the true spot price (including the import or export of power) were lower, the plant would not be dispatched. Likewise, the running cost of the least expensive plant not running at that bus provides an upper bound on the spot price at that bus. Apparently if the spot price were higher, the plant would be running. And in the likely event that the marginal plants are partially loaded, the incremental running cost provides both lower and upper bounds and thereby determines the spot price at that bus.

For locations where there is direct customer load, additional information may be available to obtain better bounds on the spot prices. For example, interruptible contracts might provide lower bounds if the supply had been interrupted, and upper bounds otherwise. Or there may be data available on outage costs to provide an upper bound on the spot price at any location. Obviously, the better the information and the tighter the bounds, the better the estimation of the spot price.[15]

This type of variable cost information may even be reported as part of a billing and payments scheme for generators. For instance, the commonly used split-savings systems in power pools depend on estimates of the marginal cost of all plants, both those run and those left idle. Hence, electric utilities have demonstrated the ability to provide acceptable estimates of the key information. In any event, collection or reasonable estimation of these data should be a modest additional burden when reporting the net loads at each bus. And given this additional information, as discussed in the appendix, transmission prices can be estimated ex post. This ex post method is particularly attractive as a transition approach for developing a new transmission regime. The ex post method allows the current dispatch operations to remain in place and calculates prices consistent with the actual usage by applying the marginal tests of economic dispatch.

Hence, the contract network can operate within the existing network control system, requiring a reform only in pricing, when combined with a settlement process to redistribute the transmission congestion payments. The redistributed congestion charge makes the capacity-right holder indifferent between transmitting the power and keeping the rental payment.

Example Contract Network

The simple three bus and three line network from figure 2 will serve to illustrate the pricing and payments in the contract network (Hogan 1990). Assume that this network has been accepted as a reasonable approximation of the major regions and connections for the underlying system. Furthermore, assume for simplicity that the load is always met, sometimes by operating expensive generation facilities, and that operators provide an economic dispatch subject to the constraints on the system. Then, after identifying the net loads at each bus and the binding constraints for the applicable contingency, either thermal constraints on lines or voltage constraints on buses, ex post prices can be calculated as the optimal spot prices.[16]

For purposes of the illustration, we concentrate on the case of a thermal limit on the line from bus 1 to bus 3. Figure 3 repeats the network but adds the bus prices. Here, the left panel describes both the initial load and the allocation of capacity rights. Hence, a generator at bus 1 is assumed to have the right to transfer 900 MWs of power to bus 3. The load at bus 3 is just satisfied by this transfer. Furthermore, the cost of power at bus 2 is assumed to be too high to be economic, so there is no need to transfer power from 2 to 3.

Under these conditions, the 600 MWs constraint on line 1 to 3 is not binding. As shown in the left panel of figure 3, the prices consist only of the charge for marginal losses. There is no congestion charge. Note that for the quadratic DC Load approximation, marginal losses are always linear in flow. The flow on path $1 \rightarrow 2 \rightarrow 3$ yields marginal losses of 0.075, and

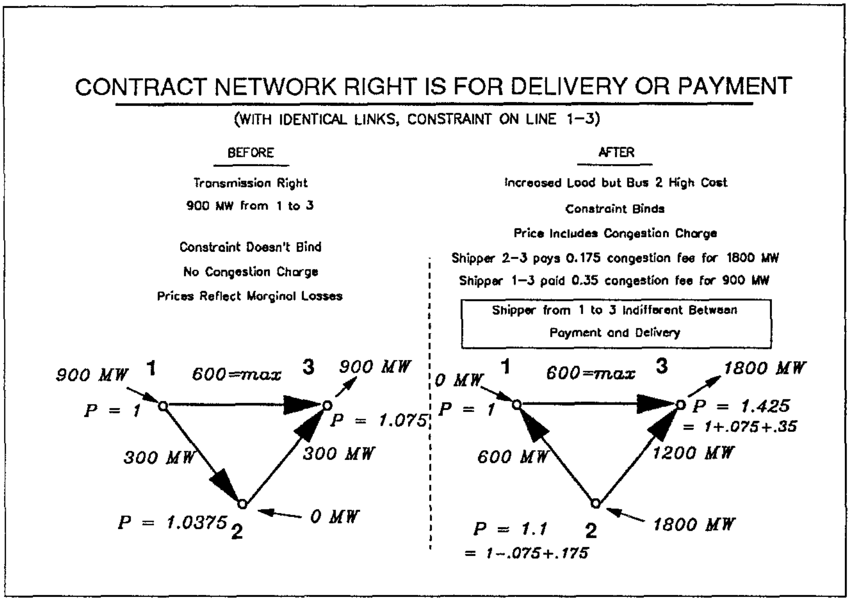

Figure 3

along path 1→3 the losses are also 0.075. The path 1→2→3 is twice as "long", but the path 1→3 has twice the flow. Hence, the marginal losses equalize along each path.

Normalizing for the price at bus 1 (the swing bus), the ex post optimal spot prices are 1.0375 at bus 2 and 1.075 at bus 3 (Hogan 1990). And the transmission charges are 0.075 (i.e., 1.075 - 1.0) for power transmitted from 1 to 3, and 0.0375 (i.e., 1.075 - 1.0375) for power transmitted from 2 to 3. The only power flowing is the 900 MWs moving from 1 to 3, so the total transmission payment is 900*0.075 = 67.5. The capacity-right holder pays this full transmission cost. And, since there is no congestion charge, the rental payment for the capacity right is exactly zero.

If the system operates only in the mode of the left panel of figure 3, then the capacity-right holder always enjoys full access to the system, pays only the operating cost to cover marginal losses, and there is no congestion rental payment. But suppose now that, after the assignment of capacity rights, there is a change in system economics and load conditions. Suppose now that the demand for power at bus 3 increases to 1800 MWs as shown in the right panel of figure 3. Here the constraint on line 1 to 3 is known to be binding. Furthermore, suppose the dispatch required use of relatively expensive generation at bus 2 and the system operators place a lower bound on the price at bus 2 of 1.1 relative to the price at the swing bus. We assume that the operators know or can estimate these three sets of information: the net loads at the three buses (0 MWs, 1800 MWs, -1800 MWs); the identity of the binding constraint (we know the constraint binds on the line from 1 to 3, but do not need to specify the exact flow on the line); and a bound on the price at one of the two nodes (price at bus 2 must be at least 1.1, the cost of the most expensive plant actually generating power at this bus).

Then the ex post price calculation determines the optimal spot prices as 1.0, 1.1, and 1.425. Furthermore, the same calculation separates the prices into the contribution of generation (1.0, by definition), marginal losses (0, -0.075, and 0.075), and congestion charges (0, 0.175, 0.35).

Why does the pricing model yield these new estimates? The marginal losses of (0, -0.075, and 0.075) can be verified by noting that the flow along 1→3 is the same as before, so the marginal losses along this line must not have changed. The reduction at bus 2 to the net of 0.925 is obviously the value such that the marginal losses from bus 2 to bus 3 equalize on either path with the new flows. In other words, for this dispatch to be optimal in the absence of the constraint, the cost of power at bus 2 would have to be not more than 0.925.

The relative price at the swing bus is 1.0 by definition. And the price 1.1 at bus 2 is determined by assuming that an expensive plant is running "out-of-merit" at a known relative price. The constraint induces a congestion charge at bus 2 of 0.175, (1.1-0.925=0.175). The remaining price at bus 3 is calculated in the pricing model. However, it can be verified by tracing the changes and costs savings if load at bus 3 reduces by 1 MW.

With a 1 MW reduction at bus 3, we would need 1 MW less at bus 2. In addition, the dispatcher would be able to substitute 1 MW at bus 1 for an additional 1 MW at bus 2 and still not violate the constraint of 600 MWs on the line 1→3. After this redispatch, the flows would be 1→3:600, 1→2:-599, 2→3:1199. The resulting net loads would be: Bus 1 at 1 MW; Bus 2 at 1798 MWs; and Bus 3 at -1799 MWs. We would save 2.2 at bus 2, pay 1.0 more at bus 1. In addition, we would save 0.075 of losses on 1→2. Since the flow of line 2→3 is twice that on line 1→2, the marginal losses are also twice, yielding a loss saving of 0.15 on 2→3. The total saving is 2.2-1.0+0.075+0.15 = 1.425. And this 1.425 is just the price at bus 3. Hence the price represents the full opportunity cost with the optimal dispatch in the presence of the constraints.

With this calculation, which generalizes to a more complicated network, we determine the congestion rentals as the differences between the generation and loss-only prices (1.0, 0.925, and 1.075) and the full bus prices (1.0, 1.1, and 1.425). From these spot prices, we can derive the optimal transmission charges as:

$$T_{13} = (1.425-1.0) = T_{L13} + T_{C13} = (1.075-1.0) + (0.35-0),$$

or

$$T_{13} = 0.425 = T_{L13} + T_{C13} = 0.075 + 0.35,$$

and

$$T_{23} = (1.425-1.1) = T_{L23} + T_{C23} = (1.075-0.925) + (0.35-0.175),$$

or

$$T_{23} = 0.325 = T_{L23} + T_{C23} = 0.15 + 0.175.$$

However, the capacity-right holder is unable to use the 900 MWs transfer right. As examination of the congestion charges shows, the rental payment is just the amount needed to honor the 900 MWs delivery requirement. The actual user pays 0.325 for each of 1800 MWs shipped from bus 2 to bus 3. The congestion charge included in this transmission price is 0.175 for a total rental payment of 0.175 * 1800 = 315.0. And this rental payment is just equal to the payment that the grid makes to the capacity-right holder of the 0.35 congestion rental from 2 to 3 for the 900 MWs, or 0.35 * 900 = 315.0.

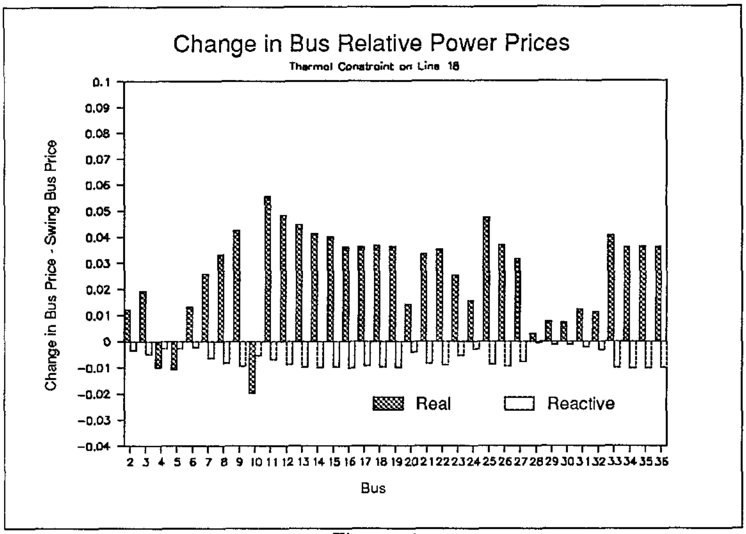

Figure 4

Furthermore, if the market is in equilibrium at the economic dispatch, which is an explicit assumption of the optimal spot-price calculation, then the cost of delivered power at bus 3 is 1.425. Presumably, the generator at bus 1 has an obligation to meet 900 MWs of the 1800 MWs load at bus 3. At the current prices, the generator is indifferent between actually generating at bus 1, delivering the power and paying the transmission price of 0.425, or purchasing the power at bus 3 at a price of 1.425. The capacity-right holder still retains the rental of 315, which makes the effective costs of transmission equal to the cost of losses only. Viewed another way, the right holder from 1 to 3 is indifferent to using the system

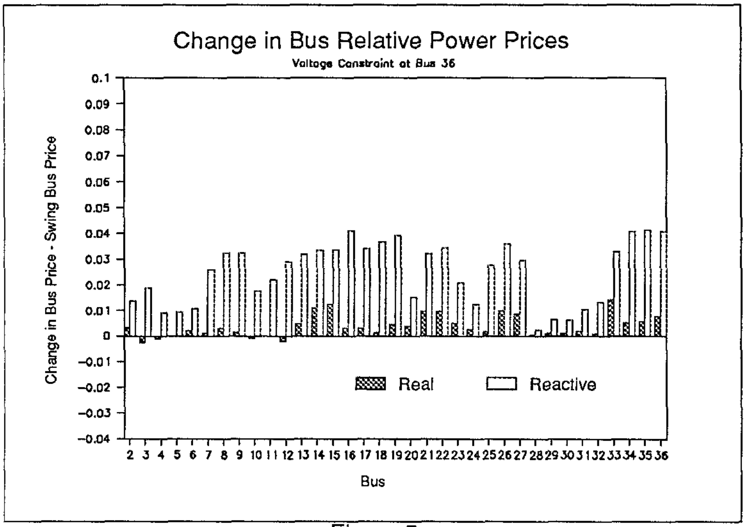

Figure 5

and paying the marginal cost of losses, for an effective delivered price of 1.075; or purchasing the power at bus 3, which, after accounting for the rental payment of 0.35, also yields an effective price of 1.075.

This three bus example illustrates the key principles of the contract network. From here, it is a straightforward matter to extend the application to a larger, more realistic network. For example, consider figures 4 and 5 which summarize the changes in the short-run bus prices induced by recognizing a congestion constraint in a large network aggregated to 36 buses and 52 links. Hence, the values shown are the errors in the bus and transmission prices that would be obtained if we failed to account properly for congestion interactions throughout the system. The prices are measured relative to the cost of generation. The data in figure 4 are for a thermal constraint on one line that induced a 5% increase in the cost of generation at one bus, and the figure shows the resulting changes in all the other prices. Similarly, figure 5 shows the same type of information but now with a voltage constraint and less than a 1% out-of-merit dispatch at a key load center. Yet as shown in the figures, these small deviations from the unconstrained cases produce large changes in the marginal prices at most buses in the system. And the errors, in the range of several percent of the cost of generation, could have a significant effect on the evaluation of the relative economics of power plant location decisions.

Research and Implementation Questions

Any implementation of a contract network would raise a number of questions. As with other applications of marginal-cost pricing, the general framework would accommodate many alternative formulations based on the same underlying principles. The innovations would require institutional changes in either the national grids of other countries or the Balkanized grid in the United States. Rate design and capacity expansion decisions would change in the presence of a system of capacity rights (Brown and Barnich 1991, 12-16). And the description of the contract network goal leaves open transition questions such as the treatment of existing rights.

The contract network definition of capacity rights requires some allocation of those rights. In principle, any feasible allocation would be compatible with the ex post pricing system and reallocation of congestion rents. Furthermore, the contract network framework appears neutral with respect to the choice between voluntary systems preferred by transmission owners (Rueger 1990, 36-37) and the default mandatory access sought by new entrants (Penn 1990, 26-30). In practice, the assignment of initial rights will be a complicated task. The existing transmission system has been paid for by existing users, and there will be many assertions of grandfathered rights to the current system.

A full investigation of the alternative methods of allocation, recognizing both historical obligations and the pressure for economic efficiency, is a separate topic that goes beyond the present discussion. However, there is one natural alternative that follows from the same observations that support the development of the contract network. Given the use of the contract network to integrate the calculations of transmission capacity usage and the impacts of loop flow, it is natural to suggest the use of a similar mechanism for allocating capacity.

Suppose that all possible pairs of generation and consumption options are included implicitly or explicitly; then a set of bids for transmission capacity from bus to bus could be identified with an associated maximum price that could be paid for each right. The generic model used in calculating the short-term prices and congestion rents could be adapted with a new objective function, namely maximizing the value of the selected bids, to determine a combination of bids which (i) would be feasible, (ii) would recognize the interactions of loop flow, and (iii) would provide the most efficient allocation of transmission rights.

Use of a contract network, with or without the formal bidding model for allocation of initial capacity, leaves open important questions in the design of a complete transmission protocol. For example, refunding the congestion rents leaves the grid with only the rents on losses, which are not likely to cover the fixed costs of the existing transmission system. This will create the need for the design of a system of access fees, operating in parallel with the short-term pricing of losses and congestion in the contract network.

For similar reasons, it would be unusual for the natural incentives associated with the contract network to align the required decisions for optimal expansion of the transmission grid. In the presence of economies of scale, the expectation would be that the overall benefits of expanding the grid would be large, but might not be sufficient to justify the expansion for recipients of the new capacity rights based solely on the revenues that would be collected through the congestion rents. Although the contract network would provide property rights to eliminate the loop flow related problems of the commons, free riders might wait for others to expand the grid in the hope of relying on the subsequent unconstrained spot market. In this case, a cost-benefit analysis and public oversight through regulation would typically be required to support system expansion decisions. This remains as a major issue that needs further attention to develop a better empirical understanding of the requirements for transmission capacity expansion.

The related problems of cost-based rate regulation for transmission may be simplified by the likely excess of average over marginal costs. This might provide the opportunity to accommodate a contract network pricing system that reflects short-run opportunity costs and a long-term access charge that nets out any aggregate opportunity rents. In this circumstance, there would be cost-based rates with the proper incentives for use of the transmission system.

The idealized contract network with the market equilibrium assumptions implies that we can calculate the efficient prices. However, it is easy to imagine the existence of pockets of market power where participants in the system might be able to manipulate price and cost data to their advantage. It remains as a subject for future research to determine the robustness of the contract network operations in the face of varying degrees of market power.[17] However, as mentioned above, the difficulties with a contract network should be no more severe than for the common split-savings systems which have been an accepted practice for many years. Furthermore, by providing a viable long-term transmission right, a contract network could help mitigate market power.

Similarly, the contract-network framework builds on the theory of spot-pricing from Schweppe et al., but the specific requirement is only for calculation of the *differences* in the efficient prices at the buses. Hence, at least within the network if not at the boundaries, actual power sales might be based on any of a number of contract or regulatory provisions that deviate from marginal cost pricing. Assuming economic dispatch, it is an open question as to how much these power pricing rules can differ from pure spot pricing without subverting the incentives for efficient transmission.

Contrary to conventional practice, as for instance in the gas pipeline capacity allocation model, reliability or priority categories appear to be moot in a contract network as defined here. Ordinarily priority categories address the problem of specific performance. If the capacity is unavailable, the lower priority capacity-right holder is curtailed and enters the secondary market to purchase alternative supplies or capacity rights. But as the mechanism for accommodating the problems of loop flow, the contract network rights integrate specific performance and the putative result of trades in a secondary market. Any capacity-right holder keeps a rental payment if curtailment is necessary, and the rental payment is always determined by the value of the constrained capacity in the short-term market. Hence, the contract network appears to be an alternative to the use of priority categories or reliability blocks. It remains to be determined if the institutions for a new transmission protocol can develop without the customary use of priority categories.

The basic contract-network model is designed for a single capacity profile with coincident requirements for transmission capacity. It may be that a single allocation of capacity is sufficient for a transmission system, or there may be a preference for different allocations according to expectations about different coincident peaks. Issues such as the appropriate period of coverage, remain as empirical questions that would be addressed best in the context of particular transmission grids.

It is clear that adopting a transmission pricing regime that replaces contract paths with contract networks will require major institutional change. In many countries, the national grids could implement a contract-network framework. In the United States, the Federal Energy Regulatory Commission, or some similar body, will be necessarily involved in designing and enforcing the pricing mechanisms and settlements process. Connections between various power pools will need to be monitored with appropriate pricing of sales of power between grids. Although, under certain conditions, it is possible to decompose the transmission pricing by subnetwork, it will be necessary to give attention to the institutional mechanisms for achieving this pricing change.

Conclusion

A contract network provides an internally consistent framework for defining long-term capacity rights to a complicated electric transmission network consistent with the principles of a competitive market. By design, a contract network would maintain short-run efficiency through optimal spot-price calculation of transmission prices. Through the payment of congestion rentals, the contract network makes the capacity-right holder indifferent between delivery of the power or receipt of payments in a settlement system. The contract network respects the special conditions induced by Kirchoff's laws and the prevalence of loop flow, thermal limits, voltage constraints, and contingencies. Furthermore, a contract network approach can be adopted without necessarily disturbing existing methods for achieving an economic power dispatch subject to these constraints.

Appendix

Overview

The contract network framework provides an economically efficient ex post transmission pricing system that is (i) consistent with an optimal dispatch of the network and (ii) will accommodate assignment of long-term transmission capacity rights. The ex post feature is designed to avoid a necessity to redesign network control procedures. This appendix summarizes the basic results (Hogan 1990; Bell 1988; Wood and Wollenberg 1984, 75; Bergen 1986; Elgerd 1982; Anderson and Fouad 1977; Cuthbert 1987); outlines an ex post linearized version of the optimal dispatch model that can be used for determining transmission prices; and relates the resulting transmission prices to a contract network definition of capacity rights with congestion payments for rental of those capacity rights. The focus is on the problems of loop flow and the spatial distribution of prices at a given time.[18]

Optimal Power Flow Model

An alternating current (AC) electrical network can be described as a set of buses connected by transmission lines which carry both real and reactive power flows. The real power flows are measured in MegaWatts (MWs), and the reactive power flows are measured in MegaVoltAmperesReactive (MVars). The Var is the product of voltage and current, which is the same unit as the watt; the notational difference is maintained to distinguish between real and reactive power.

The flow of power in an AC electric network can be described by a system of nonlinear equations known as the AC load flow model. If there are n_B buses, let $y = g - d$ be the $2 * n_B$ vector of bus net real and reactive power injections, i.e., generation minus demand. The notation here follows the development of the "DC" Load Flow model. The sign convention implies that increasing y increases costs or decreases net benefits. The DC Load flow refers to the real power half of the nonlinear AC load flow model. Under the maintained assumptions, there is a weak link between the reactive power and real power halves of the full problem. And the real power flow equations have the same general form as the direct current flow equations in a purely resistive network; hence the name "DC Load Flow" (Schweppe et al. 1988). If voltage constraints and the associated reactive power are important, then we require the full AC model and spot pricing theory (Caramanis et al., 1982).

Given the configuration of the network consisting of the buses and lines, with the associated resistances and reactances—assumed fixed for the period—Kirchoff's laws and conservation of power at each bus determine the constraints on net injections that balance system generation; losses, $L(y)$; the flow on each line, $K(y)$; and the voltage at each bus, $J(y)$.

The optimal dispatch problem is to choose the net injections or loads, typically by controlling the dispatch of power plants, in order to achieve the maximum net benefits. For our present purposes, we define abstract benefit function, $B(d)$, with cost function, $C(g)$, and assume that the system operating constraints can be characterized as limits on the average power flows through the lines, $z_{\max}$, limits on the voltage magnitudes at the buses, $V_{\max}$, and Kirchoff's laws (Caramanis et al. 1982; Feinstein et al. 1988). For simplicity here, ignore any lower bounds or other constraints on power injections which can be accommodated, mutatis mutandis.

Hence, if E is the 2 by $2{*}n_B$ elementary matrix which sums the real and reactive loads, then the generic optimal dispatch problem is:

$$\underset{d,g}{Max}\, B(d) - C(g)$$

s.t.

$$Eg - Ed - L(g-d) = 0,$$

$$K(g-d) \leq z_{\max},$$

$$J(g-d) \leq V_{\max}.$$

Given a solution to this problem, there is an associated pricing problem that connects marginal costs, congestion, and the marginal benefits or equilibrium prices. Under the usual regularity assumptions, including continuous differentiability of B and C, the pricing model follows from the first-order conditions of the optimal dispatch problem, using the appropriate constraint multipliers, θ, μ, and τ, respectively. The first-order conditions obtained from the local linear programming outer linearization or the first order Kuhn-Tucker conditions for the constrained optimization problem include (Gribik et al. 1990; Caramanis et al. 1982; Ray 1987):

$$\nabla C^t = \nabla B^t = (E - \nabla L)^t\theta - \nabla K^t\mu - \nabla J^t\tau.$$

The bus prices are $p \triangleq \nabla B^t$, which are equal to the marginal costs, ∇C^t. Hence, we have the decomposition of bus prices to include the price of marginal generation, in $p_G = E^t\theta$, the effect of losses, with $p_L = -\nabla L^t\theta$, and congestion, in $p_C = -(\nabla K^t\mu + \nabla J^t\tau)$, with the negative signs determined by the sign convention in defining net injections. Therefore, the bus prices satisfy:

$$p = p_G + p_L + p_C.$$

Thus, given a solution to the optimal dispatch problem, including the constraint multipliers, the dual prices obtain in the usual way. In the event that a formal optimization is employed with a full and explicit specification of the benefit and cost functions, the necessary dual variables will be available as a byproduct of the optimal solution. In principle, this solution could arise through a tatonnement process in a real-time auction.

In many applications, no such formal optimization is available and the functions B and C are not explicitly accessible. Typically system operators exercise necessary judgment in the actual dispatch to respect constraints and costs that are quite real even though they may be difficult to incorporate in the existing computer programs. For instance, intertemporal interactions can have a complicated impact on the generation cost function, represented only approximately in the usual static optimal dispatch model.

To avoid the possibly fatal necessity to create a more complicated dispatch model, or tamper with the existing dispatch process, transmission pricing can exploit the implicit prices consistent with the actual dispatch, determined after the fact. Typically the dispatch process produces a great deal of information available to characterize bounds on the marginal costs at particular buses. For example, under the assumption of optimality, the running cost of any operating plant provides a lower bound on the marginal cost, and hence the price, at that bus. With these bounds and a limited set of additional information, it is straightforward to determine the values of the multipliers and describe ex post the relationships among the prices.

In many cases, there will be a unique solution to the constraints and therefore only one consistent set of prices. For example, if there are no binding constraints, the conditions on marginal losses fully determine the bus prices. In the case of more than one feasible solution for the multipliers, typically created by a piece-wise linear approximation of the cost function—yielding a vertical segment on the supply curve, resolve any ambiguity by choosing the multipliers and the prices p through the linear program that minimizes the resulting rents.[19]

One goal is to find the minimum set of information needed in order to calculate such prices. From inspection of the pricing equation, it is clear that we need the description of the network with the associated resistances and reactances, plus the net loads at each bus, from which to calculate the various derivatives. In addition, we need to identify the binding network constraints, for which, by complementary slackness, the multipliers may be nonzero. For each binding constraint, we require a binding upper or lower bound on the prices at the buses.

The pragmatic operating assumption of the contract network framework is that this information is or could be readily available as a byproduct of the dispatch process. The information required, especially for the estimated bounds on prices at key buses, is similar to that produced routinely in split-savings settlement systems commonly used in power pools. With this information, it is a straightforward matter to obtain the ex post prices. These bus prices provide the essential ingredients in defining transmission prices and rights in the contract network.

Defining Transmission Rights, Prices, and Rents

Given the system loads with the corresponding set of optimal constraint prices (θ,μ,τ), define the bus prices for real and reactive power in terms of a price reflecting the cost of generation, p_G, marginal losses, p_L, and a congestion price reflecting the marginal rents on capacity constraints, p_C.

Transmission is defined as simultaneous input of power into one bus and output of the same amount of power at another bus. Obtain the short-run price of transmission as the difference between the prices at the two buses. Hence the price of power transmission is a matrix with the typical element for transmission from bus i to bus j as $t_{ij} = p_j - p_i$, which nets out the common cost of generation and includes only the effect of losses and congestion.

With this definition of a transmission price, there is flow from bus to bus, but no need to define artificially the path followed in the network. Hence, there is a contract network with no need for a contract path.

If T is the matrix of real power transmission prices, then the transmission analogs to the bus power price decompositions are

$$T = T_L + T_C.$$

All users of transmission pay the transmission prices T, either directly for "transmission" or indirectly through purchase and sale of power at the various buses.

A transmission capacity right is defined as the right to put power in one bus and take out the same amount of power at another bus in the network. We assume that the simultaneous use of all the allocated rights is feasible. However, in the contract network, we amend the definition of a capacity right to allow for either specific performance or receipt of an equivalent rental payment.

In effect, the holders of long-term transmission rights are deemed to have acquired the right to use the system and ex post pay only the short-run cost of losses *or* to receive a rental payment for use of their rights by others. This rental payment is computed using the capacity congestion components of the network transmission prices, T_C, based on the capacity right rather than the actual usage. With ex post pricing, this is the central idea of the transmission line "shareholding" model when extended to a network (Read 1988).

To clarify, suppose the capacity-right holder has 300 MWs of transmission rights between bus i and bus j. Then in every period the payment to the right-holder is $300t_{Cij}$. To the extent that the right-holder also uses the transmission system, say to transmit 100 MWs, the right holder pays, $100t_{ij}$. For this 100 MWs, the net cost is only the cost of losses, $100t_{Lij}$, consistent with the capacity right. For the remaining 200 MWs, the rental payment is available to defray the cost of purchasing 200 MWs at bus j in order to satisfy a delivery contract.

If the system is economically dispatched, and the prices of all transmission and power sales are short-run efficient, then apparently this rental payment will make the capacity-right holder just indifferent between (i) purchasing power at bus j or (ii) specific performance in actually shipping the additional power from bus i to bus j at the current transmission prices. With specific performance, the shipper would pay the transmission price, which would include losses and the rental payment. But if the shipper instead purchases power at the competitive price at the destination, the rental payment as defined just compensates for the increased price of the power.

Curtailments, Contingencies, and Other Extensions

The ex post pricing model follows from the definition of the generic optimal dispatch model. This model, with bounds on real power flows and voltage magnitudes, can accommodate a variety of special conditions that complicate operational control of the transmission grid. Here we illustrate such extensions by considering special cases or interpretations for curtailments, contingency analysis, and displacement, which are subsumed in principle under the generic formulation.

Curtailments. In the normal operation of the grid, load patterns may create bottlenecks of such importance that the optimal dispatch choice may include interruption or curtailment of service to customers in a certain region. Typically, this service curtailment is an option of last resort, but the pricing system should be able to accommodate this load condition.

The economic implication of using curtailments only as a last resort is the assignment of a high value on meeting total load. Hence, the natural mechanism for including the effect of transmission-induced curtailments is to specify the opportunity cost of the curtailment by assigning a high price for any such outage. This outage penalty would enter the pricing model as a lower bound on the ex post spot price at the bus with the curtailed load. This high curtailment price would affect in turn the estimate of the marginal values of the constraints which caused the bottleneck. And these constraint values would propagate throughout the network to modify all the prices. If the curtailed users are transmission capacity-right holders, then the congestion payments would compensate them for the curtailment consistent with the assumed outage penalty value.

Contingencies. Normal operation of the transmission grid depends on worst-case contingency analysis. Under typical operating conditions, the flow of power on the lines and the voltages at buses are both far from any sustained limits for the current configuration of the network. Rather than the current conditions, however, the system operators are concerned with the conditions that will exist in the event of an occurrence of the worst contingency.

For instance, suppose a line goes down. Immediately all the flows on the system will rearrange according to Kirchoff's laws for this new network with the missing line.[20] To a first approximation, the standard operating criterion is to maintain the dispatch so that with the present loads at the buses, the line flows and voltages resulting from the worst contingency will be within the acceptable limits.

Identifying the worst contingency and solving the optimal dispatch problem in real time, ex ante, is a difficult and complex task. But once the solution is obtained, by definition, it should be easy to identify the constraints and flows, ex post, and formulate the generic pricing problem. All that is required is that we identify the binding constraints in the worst contingency that apparently determined the limits of the actual dispatch, and to use the network description for this contingency in calculating the appropriate prices. Hence, in addition to the information on the bounds on prices, it will be necessary to identify the anticipated contingency that constrained the actual dispatch.

Strictly speaking, the calculation of the losses and the required generation at the swing bus should be made for the pre-contingency configuration of the network in order to capture the actual cost of the losses. But the constraints on the flows and the voltage should reflect the contingency conditions. In other words, suppose that k indexes the possible contingency conditions, $k = 1,2,...,n$. For each contingency, there is a corresponding formulation of Kirchoff's laws and system operating limits. The formulation of the generic optimal dispatch problem might be restated as:

$$\underset{d,g}{Max}\, B(d) - C(g)$$

s.t.

$$Eg - Ed - L(g-d) = 0,$$

$$K^k(g-d) \le z_{\max}, \text{ for all } k,$$

$$J^k(g-d) \le V_{\max}, \text{ for all } k.$$

Assuming the dispatchers solve this complicated problem, after the fact we need only identify the binding constraints. If this information is available, then the elements of ∇K

and ∇J would be obtained from the binding contingency configuration; and the elements of ∇L from pre-contingency configuration. Similar modifications of the generic optimal power flow model would include the effect on prices of contingency conditions such as the loss of a bus, failure of a generator, and so on, to allow for a restricted change in net loads in the event of the contingency.

Displacement. The allocation bidding model (Hogan 1992) and the definition of contract rights implicitly include the possibility of displacement transmission where transmission rights and flows, moving in opposite directions, in effect cancel each other. This presents no special difficulty once we note the implications of the displacement flow from "expensive" to "cheap" power sources. In the event that such "flow" is deemed to have occurred, the interpretation is that the transmission price is negative. Naturally, this means that the putative transmission is reducing either losses, congestion, or both. Hence, the user pursuing such a transaction should be paid by the grid. In the event that the congestion charge was negative for this transaction, the other users of the grid in effect would be paying the displacement transmission for the benefit of increasing the overall transmission capacity.

Revenue Adequacy

The contract network pricing model uses congestion payments as the rental fee for use of the capacity rights. In the case of specific performance, where the transmission capacity-right holder actually uses the capacity, the payment of the full rental fee ensures that the marginal opportunity cost of transmission is the true total cost, but the average net cost of transmission use is determined solely by marginal losses. However, if the capacity-right holder does not use the capacity, then under the optimal dispatch assumption the rental payment is at least enough to ensure that the net cost to purchase power at the destination is no more than the cost of own-generation and transmission. This definition of the capacity right in the contract network, in effect for either specific performance or receipt of a congestion rental payment, is designed to make the capacity-right holder indifferent between the two outcomes.[21]

The transmission grid operator stands between the parties, collecting congestion payments from the users of the system and disbursing congestion rentals to the holders of the capacity rights. It is of interest to the grid operator, therefore, to determine the adequacy of the revenue to cover the obligations to the capacity-right holders. Will the total congestion payments in the contract network be sufficient to cover the rental obligations?

This is a special case of the larger question of the total revenue adequacy of any transmission pricing scheme. It is well known that transmission construction enjoys economies of scale, and these economies imply that at the optimal scale the short-run marginal cost can be below the average cost. Hence a one-part, marginal-cost pricing system would not generate the revenues needed to cover the cost of expansion. Restoration of revenue adequacy would depend on a multi-part pricing system with fixed and variable charges. However, this full analysis is beyond the present scope; here we address the narrower question of the short-run revenue adequacy of the transmission congestion payments system.

Assume the capacity rights have been fully allocated, or any residual rights are construed as belonging to the grid operator. In the important special case of the "DC load" approximation for real power only with a given set of binding constraints, there is a happy result that follows from the linearity of the approximate line flows as functions of the bus loads; namely, when the same constraints bind the total congestion payments made by the actual users of the grid equal the total capacity rental payments that must be made to the capacity-right holders. Hence, in the DC load approximation for real power flows, there is exact revenue adequacy for the congestion payments. And, as usual, for the losses component there is a rental payment to the grid because marginal losses exceed average losses.

The general case for the full AC contract network model is more complicated, but the congestion payments should parallel the loss payments in making a net contribution to the grid. Suppose the actual dispatch is y^* and the capacity rights are y. The congestion prices are given by $p_C^t = -(\mu^t \nabla K + \tau^t \nabla J)$, evaluated at y^*. Assuming all the net load is transmission subject to congestion rental payments, the actual payments to the grid equal $p_C^t(d^* - g^*) = -p_C^t y^* = (\mu^t \nabla K + \tau^t \nabla J)y^*$, and the payment obligations from the grid would be $(\mu^t \nabla K + \tau^t \nabla J)y$, with ∇K and ∇J evaluated at y^*. Then revenue adequacy would require that $(\mu^t \nabla K + \tau^t \nabla J)\,(y - y^*) \le 0$.

Given that $\mu, \tau \ge 0$, it would be enough to demonstrate that for the binding constraints, denoted as $<\,>$, we have $<\nabla K>(y - y^*) \le 0$ and $<\nabla J>(y - y^*) \le 0$, for any feasible y and the actual dispatch y^*. A sufficient condition for these inequalities would be convexity of the set of feasible net loads. In particular, if any convex combination of y and y^*, say $y^o = y^* + t(y - y^*)$ for $0 \le t \le 1$, is a feasible load, then it must meet the bounds on voltages and average flows at least for the binding upper bounds at y^*. Hence, by Taylor's Theorem without remainder applied to the continuously differentiable K, for the binding constraints we would have $<K(y^o)> = <K(y^*)> + t<\nabla K>(y - y^*) \le <K(y^*)> = <z_{\max}>$, with ∇K evaluated at a point between y^o and y^*. By continuity, therefore, $<\nabla K>(y - y^*) \le 0$, evaluated at y^*. Similarly, $<\nabla J>(y - y^*) \le 0$.

Monotonicity of K and J as functions of y would guarantee $K(y^o)$ between $K(y)$ and $K(y^*)$, and $J(y^o)$ between $J(y)$ and $J(y^*)$, and this would imply that the feasible set of net loads is convex. We conjecture that at least over the normal range of variation, an AC load flow is convex or monotonic in this sense.[22] In other words, if monotonicity is true, moving continuously from one load pattern to another results in a movement of the line flows and voltages in the same direction as the ultimate objective. An alternative motivation that is intuitively consistent with the physics would imply convexity directly; in other words, if two load patterns are feasible, then their average should also be feasible. In this case the total congestion payments to the grid operator will be at least as large as the total payment obligations from the grid operator. Given the nonlinearity of the AC load model, the bounds on the payments may be strict and there will be greater payments in than payments out.[23] In all the examples examined to date, there is a net contribution to the grid operator (Hogan 1990). This contribution, along with the contribution from losses, could be used as part of the overall payments to defray the fixed costs of transmission.

Notes

I have benefitted from repeated conversations on transmission pricing with members of the Harvard Utility Forum, colleagues and clients at Putnam, Hayes & Bartlett, and many others including Robert Arnold, Homer Brown, Douglas Bohi, Roger Bohn, Bernard Cherry, Charles Cicchetti, Ron Clark, Gordon Corey, James Cunningham, Charles Davies, Roderick Deane, Kevin Devine, Ken Fleming, Richard Flynn, Mark Friese, James Groelinger, George Gross, Kenneth Haase, George Hall, Steve Henderson, Steve Herod, Mike Hewlett, Eric Hirst, Terry Howson, Robert Irwin, Joseph Keppinger, Henry Lee, William Lindsay, Cathy Mannion, David Marshall, John Macadam, James Malinowski, John Meyer, Mike Moy, Thomas Milburn, Ray Orson, Howard Pifer, Douglas Powell, Grant Read, Martin Rosevear, Bart Smith, Charles Stalon, Irwin Stelzer, Donald Stock, Hodson Thornber, Keith Turner, Max Wilkinson, and Lee Wilson. The idea of using contract networks for defining long-term rights grew out of intensive discussions with Sarah Johnson, Thomas Parkinson, Larry Ruff, and Michael Schnitzer. Support has come in part from the Harvard Utility Forum. The author is a consultant on electric transmission issues for Duquesne Light Company, the British National Grid Company, and Electricorp of New Zealand. The views presented in this paper are not necessarily attributable to any of those mentioned, and the remaining errors are solely the responsibility of the author.

1. Often attributed to Groucho Marx, but earlier from Charles Dudley Warner, *Hartford Courant*, Editorial, August 24, 1897.

2. One partial exception is the Texas region, ERCOT, which uses network flow analysis to evaluate wheeling and transmission under certain simplifying assumptions to calculate impacts throughout a network. The Texas AC system is electrically isolated from the rest of the United States.

3. With a single price for use of the system, this is the familiar bridge toll setting problem where the total benefits as measured by consumer surplus outweigh the costs of the bridge, but the optimal short run price would not produce enough revenue to cover those same costs.

4. Eric Hirst, Oak Ridge National Laboratories, suggested the analogy.

5. The FERC report, 1989, contains an excellent discussion of the problems of loop flow.

6. John Meyer, Kennedy School of Government, Harvard, cautions that highway systems can be more complicated. Although an individual vehicle can be routed along a particular path, the collection of all vehicles will respond to congestion by spilling over onto all parallel paths. Hence the results here for electric networks are more general and may have applications in other networks.

7. Here we are ignoring losses and use the conventional DC load approximation for purpose of the illustration. Since the lines are identical, path $1\rightarrow2\rightarrow3$ has twice the resistance of path $1\rightarrow3$, which makes it easy to verify the power flows.

8. Power system operators monitor operations to protect the system in the case of sudden loss of a transmission line, generator and so on. The response to a contingency is rapid, hence operators must preserve enough excess capacity to guarantee reliability even when a key component is suddenly removed. The worst-contingency sets the limit on the system. This worst case may be a thermal or a voltage constraint. See the appendix for a further discussion of contingency analysis and the estimation of prices in a contract network.

9. It is well known that real power flows are largely determined by the difference in voltage angles across lines, and the flow of reactive power is determined by the differences in voltage magnitudes across lines. This often leads to a "decoupled" analysis of power flows, and this allows the development of spot prices for real power without considering reactive power loads. Furthermore, computational algorithms for solving the AC load flow equations often exploit a similar procedure. However, the decoupling on flows via the differences in angles and magnitudes does not apply for the angles or the voltage magnitudes themselves, and calculation of induced constraints and spot prices depends importantly on recognizing the interactions among real power, reactive power, and voltage magnitude. See the appendix for more details.

10. This "buy-sell" model is close to the design adopted for the British national grid, but the British pricing system for buses does not recognize congestion constraints or the marginal costs of out-of-merit dispatch. Hence the implicit transmission pricing does not yet provide efficient incentives for location of new generation plants. This transmission pricing scheme is of course subject to further revision in the still incomplete transition in the privatization of the British electric system.

11. For simplicity, the focus here is on the congestion charge, which is the greatest source of price variability. However, as proposed by Larry Ruff, the ideas can be extended to treatment of increased or decreased charges for marginal losses.

12. This use of a contract to substitute payment for specific performance has a close analogies in the power supply contracts in the evolving British market. Sales are to and from the grid, but side payments between producers and consumers provide a near perfect hedge that make the economics look like a direct sale from the individual producer to the customer. With the introduction of transmission constraints and pricing, the capacity contract provides the analogous financial hedge against price changes. Of course, the capacity right is strictly a financial instrument and confers no physical control over the network. To emphasize that there is no attempt to interfere with operations, alternative terminology might be "non exclusive rights to compensation" as suggested in New Zealand by Grant Read, or "congestions contracts" as described in the United Kingdom by Putnam, Hayes & Bartlett, Ltd.

13. Other equivalent formulations could be developed. For instance, each right between any two buses could be decomposed into a pair of rights relative to a reference bus. In this formulation, all "transmission" would be to or from the reference bus. This would permit such simplifications as assigning generators rights to the reference bus and customers rights from the reference bus.

14. Roger Bohn, MIT, points out that ex post pricing of transmission services would be similar to the familiar ex post pricing through fuel adjustment clauses in the United States.

15. Schweppe et al. argue for the robustness of spot pricing even when the perfect optimal solution is not available: "The fact that the true (prices) may not be calculated does not destroy the value of implementing a spot price based energy marketplace. The actual value calculated will be much closer to the true values than the present-day flat or time-of-use rates, etc. The goal of implementing the spot price based energy marketplace is to improve the coupling between the utility and its customers, not to achieve theoretical optimality." F. C. Schweppe, M. C. Caramanis, R. D. Tabors, and R.E. Bohn, *Spot Pricing of Electricity*. Kluwer Academic Publishers, Norwell, MA (1988): 97.

16. See the appendix for a description of the method of ex post calculation of the prices given loads and the location of any binding constraints.

17. See for example the discussion in M. Einhorn, "Electricity Wheeling and Incentive Regulation," *Journal of Regulatory Economics*. Vol. 2 (1990): 173-189. Abstracting from the problems of networks and loop flow, Einhorn develops non-uniform price cap models that provide incentives for profit-maximizing utilities to provide the welfare-maximizing transmission capacity, even in the presence of market power. However, as Einhorn notes, the method depends in part on the ease of analysis of the wheeling customers' profits and may be "difficult if many wheeling customers appear," as is the case in the typical network.

18. The bible for the development and summary of the theory of spot market pricing is F. C. Schweppe, M. C. Caramanis, R. D. Tabors, and R.E. Bohn, *Spot Pricing of Electricity*. Kluwer Academic Publishers, Norwell, MA (1988). Earlier W. Vickrey, "Responsive Pricing of Public Utility Services," *Bell Journal of Economics and Management Science*. Vol. 2, No. 1, (1971): 337-346, proposed a variant of spot pricing to achieve efficiency in utility markets. R. E. Bohn, M. C. Caramanis, and F. C. Schweppe, "Optimal Pricing in Electrical Networks Over Space and Time," *Rand Journal of Economics*. Vol. 15, No. 3 (Autumn 1984):360-376, presented a succinct summary of the analysis of spot pricing for real power flows with illustrative data on the range of prices over space and time. And M. C. Caramanis, R. E. Bohn, and F. C. Schweppe, "Optimal Spot Pricing: Practice and Theory," *IEEE PAS*. Vol. PAS-101, No. 9 (September 1982): 3234- 3245, develops optimal spot pricing for both real and reactive power; this yields two spot prices, one for each type of power. The National Regulatory Research Institute report, K. Kelly, J. S. Henderson, and P. A. Nagler, *Some Economic Principles for Pricing Wheeled Power, NRRI-87-7*. Columbus OH, (August 1987), provides an excellent introduction and overview of the principal technological, economic, and institutional factors underlying the spot pricing model. In a series of similarly lucid analyses that extend the Schweppe at al. framework to consider the problem of economies of scale in transmission, E. G. Read and D. P. M. Sell, addressed the issue of transmission pricing in the New Zealand context, "Pricing and Operation of Transmission Services: Short Run Aspects," Report to Trans Power, Canterbury University and Arthur Young, New Zealand, (October 1988); E. G. Read, "Pricing of Transmission Services: Long Run Aspects," Report to Trans Power, Canterbury University, New Zealand, October 1988; E. G. Read and D. P. M. Sell, "A Framework for Transmission Pricing," Report to Trans Power, Arthur Young, New Zealand, December 1988. See also National Regulatory Research Institute, K. Kelly (ed.), *Nontechnical Impediments to Power Transfers NRRI-87-8*. Columbus OH, September 1987. In similar work with more descriptive information on the industry, the Office of Technology Assessment, *Electric Power Wheeling and Dealing: Technological Considerations for Increasing Competition*, Washington, DC, (May 1989), reviews the technological and institutional developments for power wheeling in the United States, but does not address the details in spot pricing. H. Asano, "Demand-Side Management by Real-Time Pricing for Electric Power Service," Energy Modeling Forum, Stanford University, CA, Draft, June 1989; and H. Outhred, C.H. Bannister, R.J. Kaye, Y.B. Lee, D. Sutanto, R. Manimaran, "Electricity Pricing: Optimal Operation

and Investment by Industrial Customers," *Energy Policy*. Vol. 16, No. 4 (August 1988): 384-393, are examples of recent analyses modeling the response of customers to spot pricing. For a summary of both the promise and critiques of spot pricing of electricity as a practical means of market regulation, see S. C. Littlechild, "Spot Pricing of Electricity: Arguments and Prospects," *Energy Policy*. Vol. 16, No. 4 (August 1988): 398-403. M. C. Caramanis, R. E. Bohn, and F. C. Schweppe, "System Security Control and Optimal Pricing of Electricity," *Electrical Power and Energy Systems*. Vol. 9, No. 4 (October 1987): 217-224, extends spot pricing through the use of price-quantity markets for eliciting efficient response from customers in providing reserve capacity and interruptible demands in order to meet rapid changes in security requirements of a network. S.S. Oren, S.A. Smith, R.B. Wilson, and H. Chao, *Selected Papers in Priority Service Methods*. Electric Power Research Institute, Palo Alto, CA, (August 1987) discuss the use of offer prices when customers can demand as much as they want at the ex ante offer price, as opposed to a negotiated or ex post price calculation, and relate the optimal offer prices to the use of block prices that achieve most of the efficiency benefits.

19. Note that this is not the same minimization as in the dual problem, where the choice is over the dual variables given the marginal prices p.

20. Here we abstract from the transient stability problems in moving from one equilibrium flow to another. Note that this formulation assumes the constraints can be specified acceptably in terms of the limiting conditions on the equilibrium flow after the contingency.

21. The mechanics could be different than this either-or interpretation. All actual users of the grid could pay the full transmission price, including the capacity-right holder. The capacity-right holder would always receive the full rental payment at the current congestion price. In the case of specific performance for the capacity-right holder, the net transmission cost is reduced to the cost of losses. And if the capacity-right holder cannot use the capacity, the rental payment covers the increased cost of purchase at the destination, with the net cost the same as for specific performance.

22. Note that this implies quasi-convexity but not necessarily convexity of K and J as functions of y, and convexity of the feasible net loads. Apparently convexity of the set of feasible net loads, y, is a conjecture that is plausible, unproven in general, but consistent with experimental evidence after many years of use of load flow models; see W. F. Tinney and D. I. Sun, *Optimal Power Flow: Research and Code Development*. Electric Power Research Institute, Palo Alto, CA, (February 1987): 2-4.

23. Recognize that this requires that all users of the transmission grid pay the short-term transmission price. There are no exemptions, such as for "native" load. Of course, the native load customers could be assigned the capacity-rights, in which case their net cost of transmission is always limited to the cost of losses.

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