# Assessing the Price of Water using a Two-Stage Linear Programming Model

Philipp Herpich *(347934)*, Alexandra Krumm *(357329)*, Christoph Weyhing *(388965)*,
Lukas Wolff *(349434)* and Bobby Xiong *(362923)*

***Abstract*—Commonly, marginal costs of hydropower plants in energy market models are set to zero, neglecting the semi-dispatchable property of hydropower plants with a reservoir. In this paper we present a Two-Stage Linear Programming framework to determine the minimum price a hydro power plant is operating at over the course of a year. In the first stage we use an economic dispatch model to determine electricity prices with an hourly resolution. In the second stage, we model a price-taking, profit maximizing agent operating a hydro power plant. From the resulting actions of the agent we can infer the minimum price the agent is willing to dispatch at.**

**We provide mathematical formulations for both stages and showcase the model simulating the German electricity market for a 365-day period.**

***Index Terms*—hydropower, linear programming, German electricity market**

## ACRONYMS

EEG Erneuerbare-Energien-Gesetz.
EEX European Energy Exchange.
LP Linear Program.
MC Marginal Cost.
O&M Operation & Maintenance.
OTC Over-the-Counter.
PSP Pumped Storage Plant.
PV Photovoltaics.
RET Renewable Energy Technologies.
RoR Run-of-River.

## I. INTRODUCTION

RENEWABLE Energy Technologies (RET) have negligible variable costs due to the lack of primary energy costs. However, this is only valid, if the primary energy source is non-dispatchable, i.e. in the case of solar and wind. For a hydropower plant equipped with a reservoir, the cost for the primary energy carrier (water stored in the reservoir) can be expressed by its opportunity costs. These costs result from the fact, that power demand at a certain time can only be satisfied, if the stored energy level is sufficient. Hence, the opportunity costs equal the lost revenue in one period. Consequently, the operator of a hydropower plant aims to maximize its profit at all times, given a projected, but uncertain power demand and generation dispatch of other market participants.

We use our Two-Stage Linear Program (LP) to determine the minimal energy price at which a hydropower plant operator is willing to produce and analyzed whether this price changes over time.

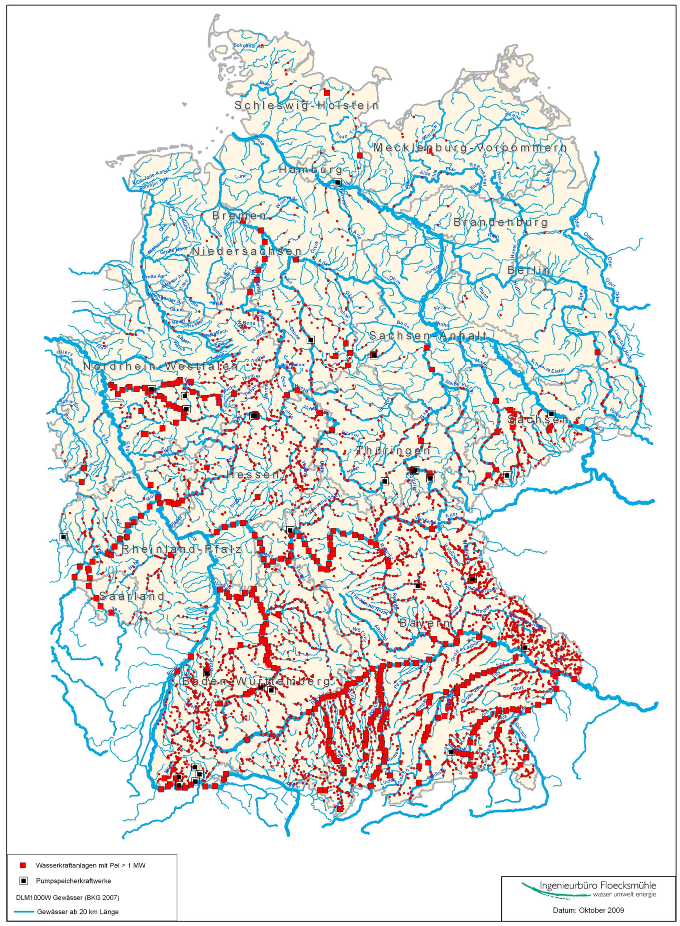


Fig. 1. Topology of German hydropower plants
Source: Ingenieurbüro Flöcksmühle via UBA (2015).

### A. Status quo: Hydropower in Germany

Hydropower plants transform the potential/kinetic energy of the water in electricity. Three different types of hydropower plants are used in conventional energy systems. The Run-of-River (RoR) type simply generates electricity through the water that flows in the river without further installations to dispatch the inflow and hence the energy supply. In order to store the energy, reservoir type hydropower plants, are equipped with a dam which decouples the inflow of water from the immediate energy generation. The third type of hydropower plant is the Pumped Storage Plant (PSP), which possesses an upper and lower basin (IHA 2018). The power plant then can either pump water from the lower basin to the upper by using electricity or generate electricity by running water in the opposite direction. The system is a more less independent cycle and does not rely on inflows from a river. The RoR and reservoir hydropower type depend on the inflows of rivers which are spatially different. As most of these power plants in Germany are located in the south (see fig. 1), within or close to the Alps the inflows are mainly determined by the melting of the glaciers in the summer. A majority of the dams is hence not only used to shift the energy generation, but also to enable protection from flooding. Some dams also face additional constraints such as minimum levels in summer time to be used as touristic attractions.

In many cases, reservoir power plants and PSP are combined into a group of power plants in which the PSP fills the reservoir (e.g. power plant group *Sellrain-Silz*). The energy stored within a reservoir or PSP (upper basin) is determined by the volume of the water and the height difference between the turbine and the reservoir. The corresponding generation is limited to the installed capacity of the turbine. In Germany, there are 14.7 GW installed of which 3.9 GW are RoR, 9.3 GW PSP and 1.5 GW reservoir power plants (BNetzA 2018). In Germany, operators can choose between different revenue options. They either receive payments according to the Erneuerbare-Energien-Gesetz (EEG) (Renewable Energy Sources Act) which grants a fixed payment for hydropower plants or sell the electricity independently at the European Energy Exchange (EEX) or via Over-the-Counter (OTC)-trades. The EEG reform from 2017 no longer guarantees fixed payments - new projects now receive payments according to their bids at a previous auction (BMWi 2017). Due to hydrological, environmental and economic restrictions, the potential is nearly exploited and hence an increase of capacity can only be achieved by retrofit. In the past, the investments in hydropower constantly declined from 2009 on and are now with 20 million in 2017 by far the renewable energy technology with the least investments (BMWi 2018).

Especially, new projects with small power plants face difficulties to finance themselves via the EEG (Keuneke 2015, 32). In 2017, only 27.5% of the power plants did receive payments from the EEG (own calculations based on 50hertz et al. (2016) and BMWi (2018)). Larger power plants can obtain higher revenues by engaging on the EEX. For our further analysis we consider the rather large reservoir power plant *Roßhaupten* (Uniper 2018) in South Germany as a representative power plant to draw on realistic data. In our model, we assume that the total electricity generated by the hydropower plant is traded on the day-ahead market.

### B. Literature review

Several studies assess the development of models to predict the short-term usage of hydropower plants under different circumstances, assumptions and objectives. Approaches and parameters on the optimum installation of small hydropower plants are analyzed in the study of Mishra et al. (2011). Pérez-Díaz et al. (2010) developed a model solving a short-term scheduling problem for revenue maximization by using dynamic programming for a hydropower plant selling generated power within a pool-based electricity market. A two-phase multistage stochastic model is developed by Séguin et al. (2017) to bridge inflow uncertainty and predict unit commitment and loading. Lu et al. (2015) presented a model for short-term hydro generation scheduling and applied it to the cascade hydropower plants in Xiluodu and Xiangjiaba in China.

## II. Model

In the course of this research, we implemented a Two-Stage LP Model to determine the price of water. The first stage of the model is an economic dispatch model which mimics the German electricity market and generates a cost minimal dispatch plus the corresponding hourly electricity prices. Subsequently, a profit maximizing agent represented by a LP, takes the price output from the first stage as an exogenous input in the second stage of the model and tries to optimize its dispatch with a given storage constraint and inflow profile. The actions of the agent allow us to draw a conclusion about the minimal price for that a hydropower plant operator is willing to generate power.

The following section gives an overview of assumptions we have taken for our models and mathematical formulations for both stages.

### A. Sets, parameters and variables

For clarity and readability, variables are capitalized while parameters (exogenously set) are written in lowercase.

| | |
|---|---|
| $\mathcal{D}$ | Set of dispatchable technologies: $d$ |
| $\mathcal{N}$ | Set of nondispatchable technologies: $n$ |
| $\mathcal{S}$ | Set of storages: $s$ |
| $\mathcal{T}$ | Set of time slices in h: $t \in \{1...T\}$ |
| $\mathcal{X}$ | Set of all technologies: $x$ |
| $\eta_x$ | Efficiency of storage |
| $\eta_x$ | Efficiency of technology in $MWh_{el}/MWh_{th}$ |
| $\lambda_x$ | $CO_2$ factor of technology in $t/MWh_{th}$ |
| $a_n$ | Availability/max. output of nondispatchable tech. |
| $c^{CO_2}$ | $CO_2$ price in €/t |
| $c_t^{el}$ | Price of electricity in time slice in €/$MWh_{el}$ |
| $c_x^{MC}$ | Marginal cost of technology in €/$MWh_{el}$ |
| $c_x^{OM,f}$ | Fixed O&M cost of technology in €/MW (installed) |
| $c_x^{OM,v}$ | Variable O&M cost of technology in €/$MWh_{el}$ |
| $c_x^{fuel}$ | Fuel cost of technology in €/$MWh_{th}$ |
| $i_t$ | Water inflow in time slice in $m^3$/h |
| $l_0$ | Initial reservoir level in $m^3$ |
| $l_{max}$ | Maximum level of reservoir in $m^3$ |
| $l_s$ | Energy storage capacity of storage in $MWh_{el}$ |
| $l_{s,0}$ | Initial level of storage in $MWh_{el}$ |
| $p_x$ | Installed power of technology in $MW_{el}$ |
| $z_t$ | Load in time slice in $MWh_{el}$ |
| $\vartheta$ | Outflow of hydro power plant in $m^3/MWh_{el}$ |
| $D_{t,s}$ | Energy demand by storage $s$ in $t$ by technology in $MWh_{el}$ |
| $G_t^{hyp}$ | Generated energy in $t$ by hydro power plant in $MWh_{el}$ |
| $G_{t,x}$ | Generated energy in $t$ by technology in $MWh_{el}$ |
| $L_{t,s}$ | Storage level for storage $s$ in $t$ in $MWh_{el}$ |

### *B. Assumptions*

To facilitate our modeling approach, we used a set of assumptions which are listed below:

i For both stages of our model we used a *copper plate* approach, i.e. ignoring grid and power flow constraints completely. This is common practice and can be found in other models for the German energy market as well, e.g. Zerrahn and Schill (2017).

ii We are modeling a closed market without any possibilities of energy export or import.

iii In the first stage of our model, the installed capacities for each technology are fixed over time.

iv In both stages of the model we neglect ramping and unit commitment constraints, due to the capacity aggregation of power plants by technology in the first stage of the model. Ramping and unit commitment constraints are negligible for hydropower plants.

v In the second stage, we set prices as exogenous input to the model ignoring the impact the production of our hydropower plant would have on the price. We base this assumption on the relatively small capacity of our hydropower plant when compared to the average hourly electricity demand of the German market. Further notes on this topic can be found in section V.

vi Lastly, we assume perfect foresight in the first and second stage of our model. Hence, the solar and wind input data is known for the whole modeling time frame. In the second stage, the profit maximizing agent receives all hourly energy prices upfront.

### *C. First Stage – Economic dispatch*

Based on the results of our economic dispatch model, the electricity price for each hour of the year is obtained. The model formulation for our economic dispatch, which minimizes the total cost of energy production, is represented by equations (1)-(11).

The objective function (1) minimizes total generation cost summed up over all technologies. (2) ensures that the energy balance holds. The energy generation of each dispatchable technology is limited by their installed capacity (3). Furthermore, constraint (4) ensures that the energy generation of each nondispatchable technology is limited by their availability depending on the weather conditions. The energy generation always has to be positive, as (5) guarantees.

The constraints (6)-(11) describe the behavior of the storage technologies (in this paper PSP). Equations (6) and (7) ensure, that the storage maximally produces as much energy as it stores and does not exceed its power capacity. (8) describes the installed energy storage capacity while (9) and (10) set the initial and final storage level. The energy balance of the storage is described in (11): The level of the current time period is the net sum of the previous level, energy generation, and demanded energy by the storage times the storage efficiency.

$$min \sum_t \sum_x \left( \mathrm{c}_x^{MC} G_{t,x} - \mathrm{c}_x^{OM,f} \mathrm{p}_x \right) \tag{1}$$

$$s.t. \sum_x G_{t,x} = \mathrm{z}_t + \sum_s D_{t,s} \qquad \forall t \in \mathcal{T} \tag{2}$$

$$G_{t,d}/1\mathrm{h} \leq \mathrm{p}_d \qquad \forall t \in \mathcal{T}, d \in \mathcal{D} \tag{3}$$

$$G_{t,n}/1\mathrm{h} \leq \mathrm{a}_n \mathrm{p}_n \qquad \forall t \in \mathcal{T}, n \in \mathcal{N} \tag{4}$$

$$G_{t,x} \geq 0 \qquad \forall t \in \mathcal{T}, n \in \S \tag{5}$$

$$G_{t,s} \leq L_{t,s} \qquad \forall t \in \mathcal{T}, s \in \mathcal{S} \tag{6}$$

$$G_{t,s}/1\mathrm{h} \leq \mathrm{p}_s \qquad \forall t \in \mathcal{T}, s \in \mathcal{S} \tag{7}$$

$$L_{t,s} \leq \mathrm{l}_s \qquad \forall t \in \mathcal{T}, s \in \mathcal{S} \tag{8}$$

$$L_{1,s} = \mathrm{l}_{s,0} \qquad \forall s \in \mathcal{S} \tag{9}$$

$$L_{1,s} \leq L_{T,s} \qquad \forall s \in \mathcal{S} \tag{10}$$

$$L_{t-1,s} - G_{t-1,s} + \eta_s D_{t-1,s} = L_{t,s} \ \forall t \in \mathcal{T}, s \in \mathcal{S} \tag{11}$$

The marginal costs $\mathrm{c}_x^{MC}$ for each technology are calculated via equation (12).

$$\mathrm{c}_x^{MC} = \frac{\mathrm{c}_x^{fuel} + \mathrm{c}^{CO_2} \lambda_x}{\eta_x} + \mathrm{c}_x^{OM,v} \tag{12}$$

### *D. Second Stage – Hydropower plant operation*

In the second stage, we model the dispatch of the hydropower plant operator as a reaction to the market price in the respective time period with the objective to maximize profit. To implement this behavior, the objective function (13) maximizes the sum of hourly generated power times the corresponding electricity price. The electricity price of each hour is the price of the last generating technology that is part of the dispatch from (1)-(11) (merit order).

$$max \quad \mathrm{c}_t^{el} G_t^{hyp} \tag{13}$$

$$s.t. \quad L_1 = \mathrm{l}_0 \tag{14}$$

$$L_T - \vartheta G_T^{hyp} + \mathrm{i}_T = \mathrm{l}_0 \tag{15}$$

$$L_t \leq \mathrm{l}_{\max} \qquad \forall t \in \mathcal{T} \tag{16}$$

$$L_{t-1} - \vartheta G_t^{hyp} + \mathrm{i}_{t-1} = L_t \qquad \forall t \in \mathcal{T} \tag{17}$$

$$G_t^{hyp} \leq \mathrm{g}_{\max} \qquad \forall t \in \mathcal{T} \tag{18}$$

$$L_t \geq \mathrm{l}_{\min}, G_t \geq 0 \qquad \forall t \in \mathcal{T} \tag{19}$$

The initial and final storage level are given exogenously and have to be equal as represented by (14) & (15). Constraint (16) ensures that the storage level never exceeds the maximum storage capacity. (17) sets the storage level at any time $t$ equal to the sum of the storage level and the inflow of the previous period $t-1$, minus the generated energy in period $t$. Maximum generation in each period is limited by (18) and equations in (19) define the domain of the variables.

## III. DATA

THE following section provides a detailed overview of all data sources that were used as inputs for our model.

### A. First Stage – Economic dispatch

To create an accurate representation of the German electricity market for the first stage of our model, we gathered technology, market and weather data from various sources and merged it together.

Our technology data is mainly based on open source data from DIW (2013, 79; 2014, 70) and includes up to date information about fuel costs, emission factors and plant efficiencies for all major (dispatchable) technologies (see table II) which are currently used for electricity production on the German market. Due to the nature of nondispatchable technologies (or Renewable Energy Technologies (RET)), i.e. solar PV, offshore wind, and onshore wind, only the fixed Operation & Maintenance (O&M) costs are considered (see table I). Variable costs for RET are assumed to be zero.

TABLE I
O&M COSTS PER TECHNOLOGY

| Technology | Variable O&M costs (€/MWh) | Fixed O&M costs (€/MW) |
|---|---|---|
| Biomass | 0 | 100000 |
| Hydro Reservoir | 0 | 20000 |
| Hydro RoR | 0 | 60000 |
| Hard coal | 6 | 60000 |
| Lignite | 7 | 30000 |
| Natural gas | 4 | 20000 |
| Oil | 3 | 6000 |
| Nuclear | 17 | 0 |
| Solar | 0 | 25000 |
| Wind offshore | 0 | 80000 |
| Wind onshore | 0 | 35000 |

To get the required market data, we draw block-unit accuracy data on installed capacities from the German Federal Association for Networks (BNetzA 2018). We aggregate this data by technology and age (less than 15 years, $15-30$ years, $30-45$ years, older than 45 years). Based on DIW (2014, 70), efficiency decreases as the age of a technology increases (see table II).

Due to the versatile characteristics of PSP, we model it endogenously with generation and storage fully implemented. The current aggregated storage capacity in Germany for PSP is estimated to be $40\,\mathrm{GWh}$ (Bundestag 2017, 8).

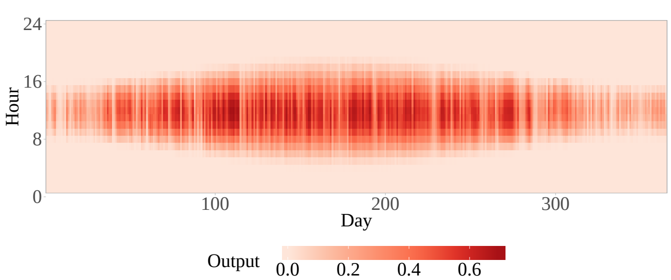


Fig. 2. Annual solar PV output in Germany (2017)
Source: Own illustration based on Pfenninger and Staffell (2016).

In order to keep our simulation as accurate as possible and account for seasonal and intraday fluctuations, we based our availability for renewables on empirical open-source time

TABLE II
PARAMETERS OF DISPATCHABLE AND NONDISPATCHABLE TECHNOLOGIES

| Technology | Capacity ($\mathrm{MW_{el}}$) | Efficiency ($\mathrm{MW_{el}/MW_{th}}$) | Fuel cost (€/$\mathrm{MW_{th}}$) | $CO_2$ factor ($\mathrm{tCO_2/MW_{th}}$) |
|---|---|---|---|---|
| Biomass | 7354 | 0.455 | 0 | 0 |
| Hydro Reservoir | 1543 | 0.9 | 0 | 0 |
| Hydro RoR | 3947 | 0.9 | 0 | 0 |
| Hard coal 1 | 6900 | 0.45 | 8.35 | 0.337 |
| Hard coal 2 | 3900 | 0.425 | 8.35 | 0.31 |
| Hard coal 3 | 9200 | 0.375 | 8.35 | 0.30 |
| Hard coal 4 | 2700 | 0.35 | 8.35 | 0.295 |
| Lignite 1 | 2900.25 | 0.424 | 3.1 | 0.399 |
| Lignite 2 | 6900 | 0.4 | 3.1 | 0.38 |
| Lignite 3 | 7500 | 0.352 | 3.1 | 0.37 |
| Lignite 4 | 2700 | 0.328 | 3.1 | 0.36 |
| Natural gas 1 | 10686.2 | 0.585 | 22.73 | 0.201 |
| Natural gas 2 | 6562.3 | 0.54 | 22.73 | 0.198 |
| Natural gas 3 | 5457.5 | 0.45 | 22.73 | 0.195 |
| Natural gas 4 | 1449.7 | 0.4 | 22.73 | 0.192 |
| Oil 1 | 300 | 0.456 | 45.82 | 0.266 |
| Oil 2 | 700 | 0.442 | 45.82 | 0.26 |
| Oil 3 | 1100 | 0.414 | 45.82 | 0.25 |
| Oil 4 | 500 | 0.4 | 45.82 | 0.24 |
| Nuclear | 9516 | 0.332 | 3 | 0 |
| Solar PV | 40716 | - | - | - |
| Wind offshore | 4132 | - | - | - |
| Wind onshore | 45460 | - | - | - |

series data for solar PV (Pfenninger and Staffell 2016) and wind (Staffell and Pfenninger 2016)[1].

All time-series data sets are calibrated to January 1, 00:00 of a 365-day long year to preserve intertemporal correlations between the different technologies. Significant higher solar PV output as well as longer output durations can be observed during the summer days (see fig. 2).

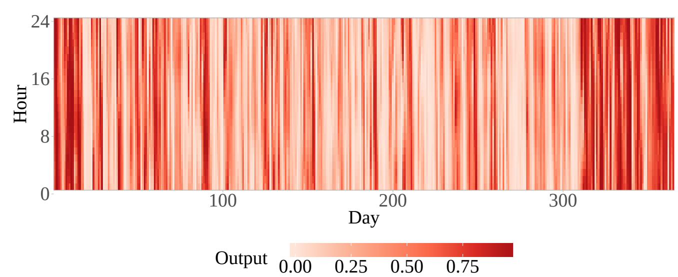


Fig. 3. Annual wind offshore output in Germany (2017)
Source: Own illustration based on Staffell and Pfenninger (2016).

As for offshore (see fig. 3) and onshore wind (see fig. 4), output is highly fluctuating over time. It can be observed, that wind output, duration and frequency increase during the winter days.

We obtain an empirical demand profile for the German electricity market with an hourly resolution from OPSD (2018), a free and open data platform for power system modeling. In 2017, the total average load accounts for $57.618\,\mathrm{GW}$ and peak load for $78.758\,\mathrm{GW}$. The average demand during winter (October 1 to March 31) is $10.9\,\%$ higher than during summer time (April 1 to September 30). As displayed in fig. 5, a significantly higher load can load can be observed during business days as the data reflects household and industry demand combined.

[1]Hourly simulations are available on renewables.ninja.

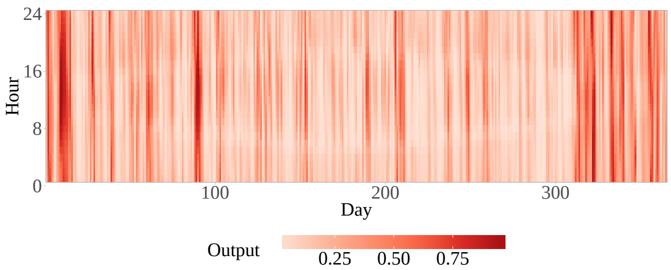


Fig. 4. Annual wind onshore output in Germany (2017)
Source: Own illustration based on Staffell and Pfenninger (2016).

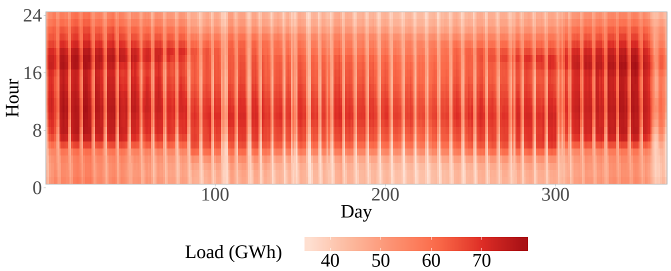


Fig. 5. Annual load in Germany (2017)
Source: Own illustration based on OPSD (2018).

### B. Second Stage – Hydropower plant operation

To model the behavior of the hydropower plant operator, we require plant related properties, such as the capacity, water throughput, and reservoir related properties, including volume and inflow (see fig. 6) with an hourly resolution.

Due to the limited availability of above-mentioned information, the data is drawn from publications of the operator (Uniper 2018, 6) using ScanIt[2].

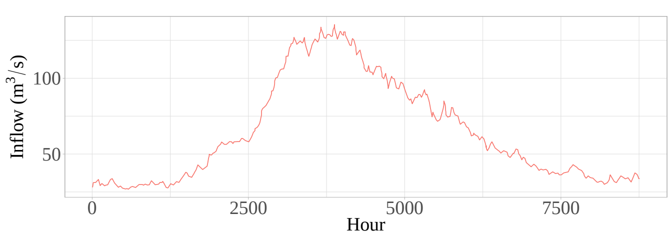


Fig. 6. Annual inflow at the hydropower plant Roßhaupten
Source: Own illustration based on Uniper (2018, 6).

Further technical parameters of the modeled hydropower plant *Roßhaupten* (Uniper 2018, 5) are displayed in table III, below.

TABLE III
PARAMETERS OF THE HYDROPOWER PLANT

| | |
|---|---|
| Maximum Power | 45.5 MW |
| Capacity | $168 \times 10^6\,\mathrm{m}^3$ |
| Initial level | $100 \times 10^6\,\mathrm{m}^3$ |
| Minimum level | $68 \times 10^6\,\mathrm{m}^3$ |
| Throughput | $11\,868\,\mathrm{m}^3/\mathrm{MWh}$ |

[2]ScanIt is a program for extracting data from scientific graphs.

## IV. Results

THE results for the model and data presented in II and III are discussed in this section. The first stage is required to obtain the input price data for the second stage.

### A. First stage

Our first stage of the model returns dispatch per technology as well as resulting prices with an hour resolution. Figure 7 displays the resulting dispatch aggregated weekly. Biomass,

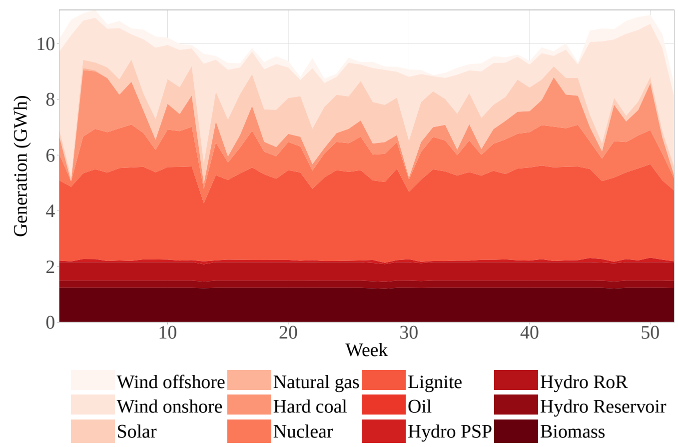


Fig. 7. Economic dispatch – weekly aggregated
Source: Own illustration.

Hydro Reservoir and Hydro RoR generate electricity at a constant level, since their marginal costs are zero. Also Wind offshore, Wind onshore and Solar have zero marginal costs, but their output highly relies on exogenous availability patterns. Solar output is highest during spring, summer, and autumn, while wind shows higher availability in winter. These RET mostly operate at their maximum available generation limits. Subsequently, Lignite, Nuclear, Hard coal and Natural gas are used to supply the residual load. The total generation follows the exogenous demand time series: In summer, energy demand is lower than in winter (see fig. 7). Based on the hourly

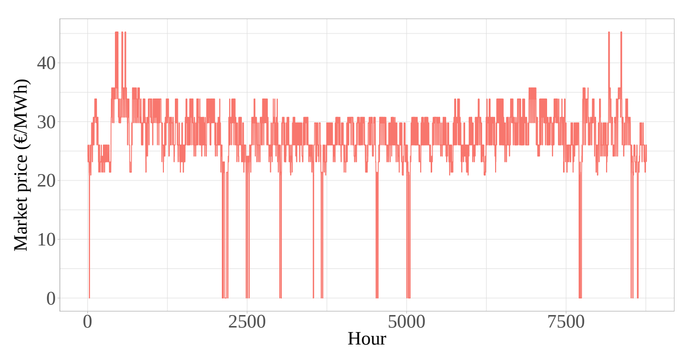


Fig. 8. Market price of the economic dispatch
Source: Own illustration.

dispatch from the first stage, the hourly electricity market prices are obtained (see fig. 8). The prices are equal to the marginal cost of the most expensive technology that produces energy during the corresponding hour (merit order). High price peaks occur in winter, as a result of higher energy demand due to low temperatures and decreased availability of renewables. Analogously, time periods with a market price of 0 €/MWh occur in spring and summer. Low price levels at the end and beginning of the year result from reduced industrial and economic activities during Christmas holidays.

### B. Model validation

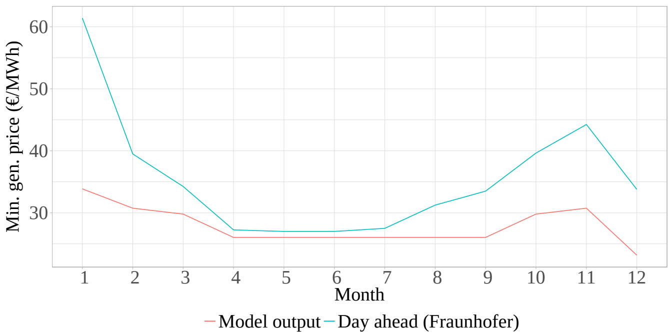


Fig. 9. Monthly minimal generation prices
Source: Own illustration.

To have a benchmark for the resulting price time-series of the first stage of our model, we used the hourly day-ahead market prices time-series from EPEX Fraunhofer ISE (2017) for validation and comparison purposes. For our research question, an accurate approximation of the minimum dispatch price of the hydropower plant operator and the changes of the minimum dispatch price over time are most important. We chose to compare minimum monthly prices because a monthly resolution enables us to detect trends over time while also hiding upward outliers. As can be seen in(fig.9), the minimum dispatch price trend for the price output from the first stage closely resembles the trend of the minimum dispatch price when running the second stage model with the EPEX data. Deviations can be explained via the assumptions and simplifications of the model such as ignoring import and export and ignoring grid and power flow constraints.

### C. Second stage

In figures 10 and 11, the resulting energy generation of the hydropower plant for the economic dispatch prices is displayed on a monthly and hourly resolution. Both figures show that energy generation between April and September is higher than between October and March. A peak can be observed in May and July due to the high water inflow from snow melt and rain. Alterations of the reservoir capacity or initial level change the peak value but not the shape of the peak itself.

The level of the hydropower plant reservoir strongly varies during the year as shown in figure 12. After an increase in January, the storage level drops to the lower limit of the reservoir of $5.73\,\mathrm{GW\,h}$ ($\doteq 68 \times 10^6\,\mathrm{m}^3$). Starting in the end of April ($\approx 2600\,\mathrm{h}$), the level rises from the lower limit to the upper limit of the reservoir of $14.16\,\mathrm{GW\,h}$ ($\doteq 168 \times 10^6\,\mathrm{m}^3$). The level stays close to the upper limit of the reservoir until the beginning of October ($7000\,\mathrm{h}$). Afterwards it slowly returns to its initial level.

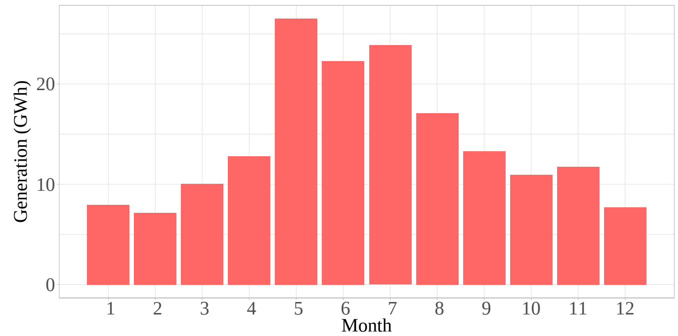


Fig. 10. Monthly generation of the hydropower plant
Source: Own illustration.

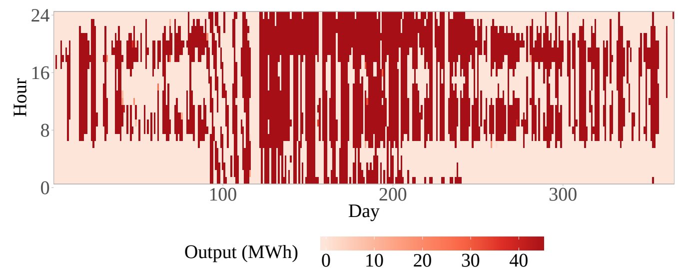


Fig. 11. Hourly generation of the hydropower plant
Source: Own illustration.

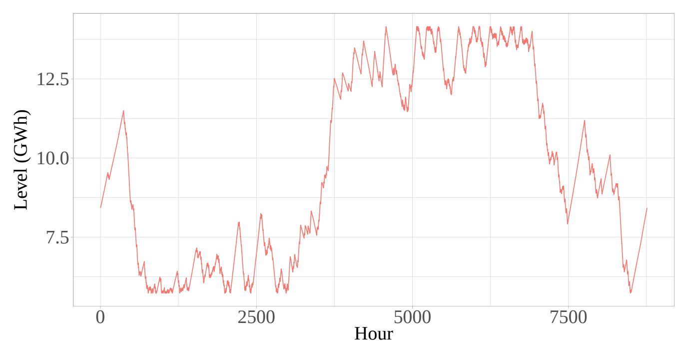


Fig. 12. Storage level of the hydropower plant
Source: Own illustration.

Figure 9 shows the lowest monthly price at which the hydropower plant dispatches. For both price time-series (economic dispatch and day-ahead market price data from EPEX, Fraunhofer ISE 2017), the minimum price level is higher in the months from October to March than from April to September, except for December. This behavior can be explained with the electricity price curves (for the economic dispatch, see figure 8): Both price curves show almost exclusively high price peaks in the months from October to March. During these months general electricity prices are higher due to higher demand and lower availability of solar PV. Hence, the hydropower plant operator can be more selective and the minimum dispatch price shifts upwards. Furthermore, the water inflow is much higher from April to September than during the rest of the year, which makes a higher throughput necessary to prevent an overflow of the reservoir, as can be seen in figure 11. Hence, the power plant operator produces electricity even when prices are lower. December is an exception, since during the Christmas holidays, industrial power demand significantly decreases, which leads to less demand and subsequently lower electricity prices. Due to that, the hydropower operator is "forced" to produce power for lower prices in December.

From figures 8-12 and 6 two basic principles for the operation of a hydropower plant can be derived:

i During periods with higher average electricity prices, generation should be increased and reserves should be reduced or depleted.
ii Storage reserves should be filled up or kept at the same level in periods with high inflow.

The examined hydropower plant shows the aforementioned behavior: At the beginning of the year, energy is produced during the high price periods until the reservoir level decreases to the lower limit. In spring and summer, when prices are lower, high water inflow is saved for high-price seasons at the end of the year, which causes the reservoir level to increase towards the upper limit. Finally, storage level decreases again to the lower limit, as the hydropower plant dispatches to maximize profit. To which extent this behavior can be observed depends on the amount and the yearly distribution of water inflow and the reservoir capacity. For example, the examined reservoir at Forggensee produces a lot of energy in summer even when prices are low, since the reservoir capacity is not capable to store all the water for high price periods later in the year.

The initial storage level has a negligible influence on the overall behavior of the hydropower plant operator and the minimum price. However, January and December are strongly influenced by the binding equation (15) that forces the final storage level to be equal to the initial level. The higher the initial level, the lower the minimum price level in January, since the high prices make a dispatch more attractive. Consequently, a low initial level leads to a high minimum price level, because the low reservoir level is only used during high-price peaks. In December, an inverse effect occurs. Higher initial levels force the hydropower plant operator to save water to reach the initial level again. As a result, the minimum price level in December is higher. Lower initial levels lead to lower minimum price levels in December.

## V. Further research

The current modeling approach of a Two-Stage LP Model is based on the assumption that the hydropower plant operator has no impact on the electricity price due to the relatively low generation. However, our model neglects the fact that there is more than one hydropower plant operator in the German market with all of the operators having the same goal and similar conditions and constraints. In Germany, the combined capacity of all hydropower plants plus PSP approximately sums to 10 GW. Clearly, the combined output of all hydropower plant operators may have an impact on the price of electricity in certain periods.

One option to reflect the aggregated market power of the hydropower plant operators would be to use a game theoretic approach such as a Stackelberg game. Within a Stackelberg setting, the market could be modeled as the leader with the hydropower plant operators being the followers. In our Two-Stage LP Model, the hydropower plant operator of the second stage did not compete with the market for demand, but simply accepted the set price and could always dispatch. This assumption is reasonable for a single hydropower plant operator but is unrealistic when considering the combined action of all hydropower plant operators. We propose to model the remaining market as a leader, because the combined capacity of all other technologies exceeds that of the hydropower plant operators by far. A Stackelberg competition would model the interdependency between the actions of all remaining market participants and the hydropower plant operators. Additionally, the difference in market power would be taken into account by the sequential setup.

However, using a Stackelberg model would require comprehensive research on how to aggregate the storage capacities of the individual hydropower plants correctly. Simply adding up does not suffice, because the storage capacity of one hydropower plant cannot be utilized in another plant. Hence, storage levels for each individual plant would have to adhere to general constraints. Doing so is not only challenging from a modeling perspective, but would also require extensive efforts to gather all relevant data on a power plant level for all hydropower plants in Germany.

In addition, a thorough analysis of uncertainty on the optimal solution for power plant operators would be an interesting topic for further research. Uncertainty about future electricity prices can arise from uncertainty about future demand, behavior of other market participants or availability of nondispatchable technologies. In order to account for uncertainty a stochastic or robust programming approach could be used.

The Stackelberg model, as well as an analysis of the impact of uncertainty on the behavior of the operating agent were out of the scope of this analysis, but we hope this discussion initiates further research in this area.

## VI. Conclusion

In this paper, we presented an approach on how to assess the minimum price at which a hydropower plant operator is willing to operate, given realistic storage constraints and inflow conditions. We developed a Two-Stage LP Model to first model the German electricity market and then determine the response of a hydropower plant operator to the resulting prices. The resulting electricity prices from the first stage resemble the corresponding EPEX hourly day-ahead market prices. The first stage can be used to simulate resulting price time-series for different sets of inputs.

With the second stage of our model, we determined the optimal actions of a hydropower plant operator given the fixed input prices. We identified that the minimum price at which the operator is willing to dispatch, changes over time and is higher in the winter than in the summer. This can be explained by higher electricity prices and the low water inflow to the hydropower plant during the winter season (see fig. 6). Additionally, we found out that the minimum prices are also sensitive to initial and final level constraints.

In most energy market models, hydropower plants are considered to have zero marginal costs (e.g. Zerrahn and Schill 2017). This neglects the fact that hydropower plant operators are taking actions to maximize profit and do not dispatch to certain prices unless storage limitations are forcing them to. We believe that our results can be utilized to get a better understanding of how hydropower plants operate and foster a discussion about how to take minimum dispatch prices of those operators into account.

Every data source we used is listed in the references. Both optimization models were written in Julia using the JuMP[3] package and Gurobi as underlying solver. All data transformation and plotting were either done in Julia or R[4] and all code we used for our work can be found in our public Git repository: https://gitlab.tu-berlin.de/xiong.bobby/ew-mod-2018/.

## References


50hertz, Amprion, Tennet, and TransnetBW. 2016. Prognose der EEG-Umlage 2017 nach AusglMechV. https://www.netztransparenz.de/portals/1/Content/EEG-Umlage/EEG-Umlage%202017/20161014_Veroeffentlichung_EEG-Umlage_2017.pdf.

BMWi. 2017. *Erneuerbare-Energien-Gesetz 2017 (EEG 2017)*. Bundesministerium für Wirtschaft und Energie.

– . 2018. Zeitreihen zur Entwicklung der Erneuerbaren Energien in Deutschland. Visited on 09/24/2018. https://www.erneuerbare-energien.de/EE/Navigation/DE/Service/Erneuerbare_Energien_in_Zahlen/Zeitreihen/zeitreihen.html.

BNetzA. 2018. *Kraftwerksliste der Bundesnetzagentur - Stand: 02.02.2018*. Daten aus Monitoring. Bonn, Germany: Bundesnetzagentur.

Bundestag, W. D. D. 2017. *Entwicklung der Stromspeicherkapazitäten in Deutschland von 2010 bis 2016*. WD 8 - 3000 - 083/16. Berlin, Germany: Deutscher Bundestag.

DIW. 2013. *Current and Prospective Costs of Electricity Generation until 2050*. Data Documentation 68. Berlin, Germany: Deutsches Institut für Wirtschaftsforschung (DIW).

– . 2014. *Electricity Sector Data for Policy-Relevant Modeling: Data Documentation and Applications to the German and European Electricity Markets*. Data Documentation 72. Berlin, Germany: Deutsches Institut für Wirtschaftsforschung (DIW).

– . 2017. *Electricity, Heat and Gas Sector Data for Modelling the German System*. Data Documentation 92. Berlin, Germany: Deutsches Institut für Wirtschaftsforschung (DIW).

Fraunhofer ISE. 2017. Stromproduktion und Börsenstrompreise in Deutschland. Visited on 09/24/2018. https://www.energy-charts.de/price_de.htm.

International Hydropower Association. 2018. Types of hydropower. Visited on 09/20/2018. https://www.hydropower.org/types-of-hydropower.

Keuneke. 2015. *Marktanalyse zur Vorbereitung von Ausschreibungen: Vorhaben IId, Wasserkraft*.


[3]Julia for Mathematical Optimization

[4]R is an open source language for statistical computing and evaluation


Lu, P., Zhou, J., Wang, C., Qiao, Q., and Mo, L. 2015. "Short-term hydro generation scheduling of Xiluodu and Xiangjiaba cascade hydropower stations using improved binary-real coded bee colony optimization algorithm.". *Energy Conversion and Management* 91: 19–31.

Mishra, S., Singal, S., and Khatod, D. 2011. "Optimal installation of small hydropower plant—A review.". *Renewable and Sustainable Energy Reviews* 15 (8): 3862–3869.

Open Power System Data. 2018. Data package: Timeseries DE. Visited on 09/22/2018. https://data.open-power-system-data.org/time_series/.

Pérez-Díaz, J. I., Wilhelmi, J. R., and Arévalo, L. A. 2010. "Optimal short-term operation schedule of a hydropower plant in a competitive electricity market.". *Energy Conversion and Management* 51 (12): 2955–2966.

Pfenninger, S. and Staffell, I. 2016. "Long-term patterns of European PV output using 30 years of validated hourly reanalysis and satellite data.". *Energy* 114: 1251–1265.

Séguin, S., Fleten, S.-E., Côté, P., Pichler, A., and Audet, C. 2017. "Stochastic short-term hydropower planning with inflow scenario trees.". *European Journal of Operational Research* 259 (3): 1156–1168.

Staffell, I. and Pfenninger, S. 2016. "Using bias-corrected reanalysis to simulate current and future wind power output.". *Energy* 114: 1224–1239.

Umweltbundesamt. 2015. Nutzung von Flüssen: Wasserkraft. Visited on 09/22/2018. https://www.umweltbundesamt.de/themen/wasser/fluesse/nutzung-belastungen/nutzung-von-fluessen-wasserkraft.

Uniper. 2018. *Staudamm Roßhaupten: Erneuerung der Dammdichtung*. Press event on 21.03.2018. Lechbruck am See, Germany: Uniper SE.

Zerrahn, A. and Schill, W.-P. 2017. "Long-run power storage requirements for high shares of renewables: review and a new model.". *Renewable and Sustainable Energy Reviews* 79: 1518–1534.