Application of Variance Reduction Techniques for Sequential Sampling to Stochastic Unit Commitment in Microgrids

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***Abstract*—Stochastic approaches have been widely used to address unit commitment problems with uncertain variables. In these approaches, accuracy of the random estimator as well as good performance of the stochastic program remains a challenge. This study presents a model for a two-staged unit commitment problem with uncertainty. It utilizes Monte Carlo sampling to account for uncertainty and implements the L-shaped Method to decompose the two stages. Variance reduction techniques are introduced. The effect of sequential sampling as a measure of keeping samples sizes moderate through quality assessment is analysed. The results indicate that variance reduction techniques improve the estimation quality significantly, if certain conditions are not breached. A large energy storage may increase variance. The use of sequential sampling drastically reduces computation time while maintaining reasonably good results. Especially Anithetic Variates sampling in combination with averaged two-replication procedure produced good results, whereas Latin Hypercube Sampling did not yield conclusive results.**

***Index Terms*—stochastic unit commitment, variance reduction, sequential sampling, L-shaped method, microgrid**

I. Introduction

A microgrid is a cluster of loads, energy generation units and energy storage systems that are coordinated to reliably supply electricity. From the viewpoint of the main grid into which it is typically integrated, a microgrid is an individual unit that responds to control signals [1]. Microgrids are frequently utilized in rural contexts, developing countries and military premises [2]. They are often characterized by uncertainty of components. When variable energy resources, such as photovoltaic or wind power are included these are a source of uncertainty. Furthermore, the distribution of loads is typically uncertain. In our approach no variable energy resources are included and hence only loads are uncertain.

This paper models unit commitment (UC) decisions in a microgrid. While the economic dispatch problem finds an optimal operating policy of generation units to serve the demand, the UC problem answers the more complicated question of which combination of generation resources should be used to supply a load most economically. To ”commit” a generating unit in this context means to turn it on. The commitment decision is made for time spans of 24 hours to one week. The economic dispatch is a subproblem of the UC solution procedure [3].

The modeled microgrid is a representation of a real-life application. It consists of the components thermal generation resource (TGR), uncertain load, energy storage resource (ESR) and electric vehicles (EVs) that can be plugged into the microgrid and charged or discharged. It interacts with the host power system one-directional by purchasing energy from the distribution company. We represent realistic conditions by introducing a market structure with a forward contract and a real-time market. The purchase decision for the forward contract needs to be made before the uncertainty of the load values unravels. Hence, we are dealing with the expected value of loads when making a purchase decision in the forward-contract.

In the following, we look at an optimization problem where the objective function takes the form of an expectation $\mathbb{E}[f(x, \tilde{l})]$ that contains the complicated function f. It is hence not practicable to solve it for a fixed x or $x \in X$. Instead, we are approximating the optimal solution with the sample average approximation (SAA) method that is based on Monte Carlo sampling techniques [4], [5]. [4] describes the SAA method as first generating a random sample $\xi_1, ...\xi_N$ of size N and then approximating with the sample average problem

$$\min_{x \in X} \left\{ \hat{f}_N(x) = N^{-1} \sum_{j=1}^{N} F(x, \xi_j) \right\}.$$

The SAA method combined with deterministic algorithms shows good efficiency for two-staged stochastic optimization problems [6], [7].

The utilization of variance reduction techniques in unit commitment is widely found in literature. Wu and Shahidehpour [8] generate Monte Carlo samples with Antithetic Variates

(AV) to accelerate the convergence of results as random numbers for prediction of day-ahead electricity market clearing price forecasts with a hybrid time-series and an adaptive wavelet neural network model. Morshed et al. [9] consider load uncertainty through a similar sampling method with AV for generating random numbers to study the uncertainties of plug-in electric vehicles, photovoltaic and wind energy in a probabilistic optimal power flow approach. The sample variance reduction method Latin Hypercube Sampling (LHS) accounts for uncertainties of wind and solar power plants in a flexible microgrid model with fast reserve supply by Alirezazadeh et al. [10]. Also Shi et al. [11] make use of LHS to generate uncertain day-ahead net power scenarios in a microgrid but combine it with a backward scenario reduction technique for less computation to find a stochastic bidding strategy in a bi-level optimization problem. Wu et al. [12] calculate the cost of power system reliability in a unit commitment problem using LHS but point out that other variance reduction techniques such as AV, control variates or importance sampling are also applicable.
To the best knowledge of the authors the application of sequential sampling in unit commitment problems constitutes a gap in research so far. Xie et al. [13] propose an optimization and selection approach that includes an optimization search and best candidate decision selection for power grid scheduling with high shares of wind power generation. They identify a best candidate solution through the ranking-and-selection method and allocate more computational budget to it to improve performance assessment and computation time. It is similar to sequential sampling in its goal of improving computation time and performance but takes a fundamentally different approach.

This paper answers four central questions. Firstly, it shows how a microgrid can be operated optimally under uncertainty with the stochastic UC model. Secondly, the model addresses the question of efficiency of microgrid operations in a two-staged problem. Furthermore, the paper demonstrates the influence of variance reduction techniques in unit commitment. Lastly, the model addresses the aforementioned research gap. The relatively new topic of sequential sampling for stochastic programming is covered theoretically in several works [14], [15], [16], [5]. However, the application of the sequential sampling method to a unit commitment setting together with an analysis of its perks and drawbacks has not been studied yet.

The subsequent Section II further describes the modeled problem and its mathematical formulation. Section III explains how the L-shaped method and multiprocessing were applied to solve the problem. Section IV presents the modelling results and discusses them. Finally, section V concludes the report with a summary of the conducted work and an outlook.

II. Problem Description

In the following a mathematical formulation of the problem and its nomenclature is given. Additionally, the sampling approach for the SAA method is presented together with the applied variance reduction techniques AV and LHS. Also, the conceptual framework of sequential sampling is introduced. The problem formulation is based on [17].

Nomenclature

Parameters

$\lambda_{FW}, \lambda_{RT}$	Electricity retail price of forward, realtime contract [\$/kWh]
c_g^u, c_g^p	Static [\$/h], linear [\$/kWh] fuel cost parameter of plant g
$E_\sigma^{max}[h]$	Maximum energy level of ESR σ [kWh]
$E_\sigma^{min}[h]$	Minimum energy level of ESR σ [kWh]
$l[h]$	Total load [kW]
p_g^{max}	Maximum power output of plant g [kW]
$p_\sigma^i[h]$	Minimum charging power ESR σ [kW]
$p_\sigma^w[h]$	Maximum withdrawal power ESR σ [kW]
R_g	Maximum ramping gradient of plant g [kW]
$T_g^\downarrow$	Minimum down-time of plant g
$T_g^\uparrow$	Minimum up-time of plant g

Sets and Indices

H	Set of time steps
ESR	Set of energy storage resources
h	Index of hourly time step
σ_s	Index of energy storage
σ_{EV}	Index of electric vehicle storage

Variables

$E_\sigma[h]$	Energy level of ESR σ [kWh]
$p_g[h]$	Power generated by plant g [kW]
p_σ^{net}	Power output of ESR σ [kW]
$p_{FW}[h], p_{RT}[h]$	Energy purchased in forward, real-time contract [kW]
$u_g[h]$	Commitment status of plant g

A. Problem Formulation

This subsection describes the mathematical formulation of the two-stage stochastic UC problem. Equation (1) describes the first-stage objective function, minimizing start-up generator costs, costs of electricity purchased through the forward contract as well as the expectation of the undetermined cost of the second stage $\mathbb{E}[F(x,\tilde{l})]$. First-stage decision variables are the plant commitment status u_g and the electricity purchased in the forward contract p_{FW}. Attached to the first-stage problem is constraint (3), which guarantees that no energy is sold in the forward contract, and constraints (4) and (5), which ensure the minimum down and up time of the TGR respectively. The following objective functions and constraints account for $\forall h \in H$, $\forall \sigma \in ESR$ unless stated otherwise.

$$\min \left\{ \sum_{h \in H} [c_g^u u_g[h] + \lambda_{FW}[h] p_{FW}[h]] + \mathbf{E}[F(x, \tilde{l})] \right\} \quad (1)$$

subject to

$$u_g[h] \in \{0, 1\} \quad (2)$$

$$p_{FW}[h] \geq 0 \quad (3)$$

$$u_g[h-1] - u_g[h] \leq 1 - u_g[\nu], \forall \nu \in \mathbb{N} \text{ such that } h \leq \nu \leq min\{h - 1 + T_g^{\downarrow}, H\} \quad (4)$$

$$u_g[h] - u_g[h-1] \leq u_g[\nu], \forall \nu \in \mathbb{N} \text{ such that } h \leq \nu \leq min\{h - 1 + T_g^{\uparrow}, H\} \quad (5)$$

The second stage objective function (6) aims to minimize generation costs and real-time electricity purchase costs for a realized specific instance of load l^v. The first-stage decisions $(u_g[h])^*$ and $(p_{FW}[h])^*$ are contained in the vector x^*. Equations (7) and (8) constrain the generation gradient and maximum. Constraint (9) ensures that electricity is only bought via the real-time market. The load balance (10) expresses that the sum of energy from the forward contract in the first stage, energy generated, energy purchased in real-time and energy withdrawn from the storage units must fulfill the energy demand in every hour.

$$F(x^*, l^v) := \min \left\{ \sum_{h \in H} [c_g^p p_g[h] + \lambda_{RT}[h] p_{RT}[h]] \right\} \quad (6)$$

subject to

$$-R_g \leq p_g[h] - p_g[h-1] \leq R_g \quad (7)$$

$$0 \leq p_g[h] \leq (u_g[h]) * p_g^{max} \quad (8)$$

$$p_{RT}[h] \geq 0 \quad (9)$$

$$p_g[h] + (p_{FW}[h])^* + p_{RT}[h] + \sum_{\sigma \in ESR} p_\sigma^{net} \quad \geq \quad l^v[h] \quad (10)$$

The constraints regarding both the energy storage and the electric vehicle are attached to the second-stage problem. Equations (11) and (13) constrain the energy stored and storage power. Equation (12) describes the storage balance.

$$E_\sigma^{min}[h] \leq E_\sigma[h] \leq E_\sigma^{max}[h] \quad (11)$$

$$E_\sigma[h] = E_\sigma[h-1] - p_\sigma^{net}[h] \cdot 1h \quad (12)$$

$$-p_\sigma^i[h] \leq p_\sigma^{net} \leq p_\sigma^w[h] \quad (13)$$

B. Sampling Techniques

By drawing N samples of the uncertain load $\tilde{l}$, each with the respective probability of $\frac{1}{N}$, the first-stage decision problem is reformulated to a discrete problem through SAA. Samples are typically drawn via the Monte Carlo method. To achieve better accuracy of the estimator, the sample size in crude Monte Carlo sampling can be increased. However, complex optimization problems are computationally demanding, even with modest sample sizes. To increase the accuracy of the random estimator with the same sample sizes, two variance reduction techniques are examined. They promise to yield lower variance and thus higher accuracy compared to crude Monte Carlo sampling.

The AV method aims to reduce variance by creating an antithetic for every sample obtained. This is usually done via drawing $\frac{N}{2}$ samples s from a uniform distribution $U(0,1)$, creating the antithetic samples $s' = 1 - s$ and finally applying the inverse cumulative distribution function to (s, s') to transform them to samples of the original distribution (ξ, ξ'). AV results in an even sampling from the distribution, which in turn reduces variance [18].

The aim of LHS is to produce samples which represent the probability distribution as a whole in order to lower the variance. This is achieved by dividing the distribution into N parts of equal probability, drawing a random sample from each part and finally shuffling these samples [18].

C. Sequential Sampling

Even with AV and LHS the problem remains to obtain reasonably good results with regard to variance and objective value while not exceeding modest sample sizes. Therefore, in sequential sampling the sample size is increased iteratively, starting with a low sample size. After each iteration the candidate solution is examined with respect to the objective value and the standard deviation. If a certain threshold is met, the sequence stops, yielding a plausible objective value with a still modest sample size [15].

In the sequential sampling procedure by [16] the performance of a previously generated candidate solution x_k is compared to the optimal solution x^* on a sample set of size n_k in iteration k. If the gap of the respective objective values is smaller than a certain threshold, the algorithm stops and returns x_k as the approximately best solution. Otherwise it moves onto the next candidate solution while increasing the sample size $n_k > n_{k-1}$. Additionally the sampling method can be complemented with AV or LHS.

III. Solution Methodology

The following section describes the solution methodology for the stochastic two-stage optimization problem and the implementation of multiprocessing in order to achieve modest computation times.

A. L-shaped Method

The L-shaped method is an algorithm for solving two-staged stochastic optimization problems. It is based on Benders decomposition. In a problem, where decisions do not occur simultaneously, but decisions in later periods are dependent on previous ones, Benders decomposition splits the problem into a master problem with first-stage variables and a sub problem with second-stage variables. For initialization, the master problem is solved. Afterwards, the dual of the sub problem is calculated with the given first-stage decisions. If the dual is infeasible or unbounded, an infeasibility cut is added to the master problem with a constraint. If the dual yields an optimal value, an optimality cut is attached to the master problem. Subsequently, an upper and a lower bound for the objective value are computed. The problem is solved by iterating between master and sub problem [19].

The L-shaped method largely follows the same pattern as shown in figure 1. However, in one iteration N sub problems are solved one after the other. Their weighted output is then added to the single master problem via cuts. In the implementation of this paper, no feasibility cuts are generated, because the problem is always feasible, as an infinite amount of electricity can be bought at any given time. The algorithm terminates if the difference between upper and lower bound is smaller than the threshold value ϵ.

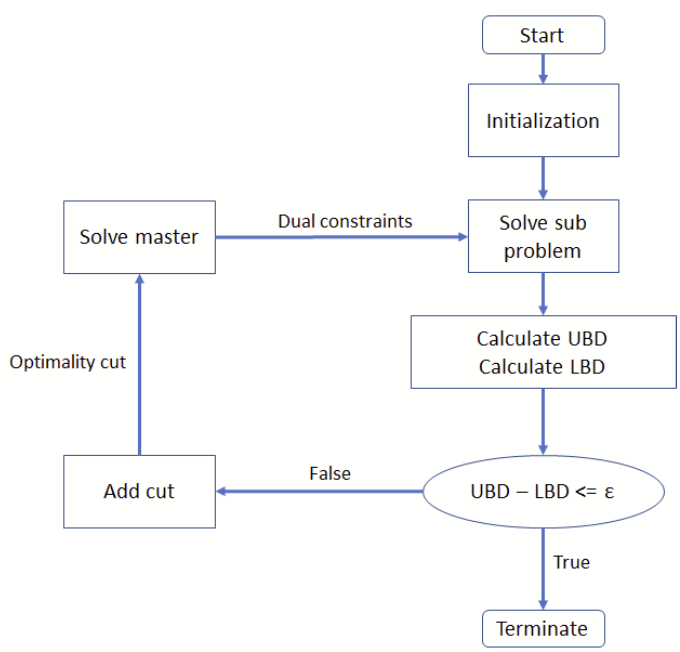

Fig. 1: L-Shaped method flow chart [own illustration]

B. Multiprocessing

When solving the L-shaped method for N samples, the second-stage sub problems are solved independent of each other. This peculiarity can be exploited to speed up the computation time of the L-shaped method via multiprocessing. Each sub problem calculation is assigned to a single core, thereby parallelizing the computation bulk of the L-shaped method. Afterwards the results for each sub process are gathered and fed back to the master problem. Multiprocessing in the L-shaped method was implemented via the python module pathos [20] [21]. A speed up of the computation time by the factor 4 was observed.

IV. Results

After briefly presenting data for different configurations that will be used as case studies, this section discusses results of the optimization problem with respect to sample size, sampling method and size of the cumulative energy storage. The section concludes with the impact of sequential sampling.

A. Case Study Data

Two different cases are modeled to illustrate different aspects. The first case includes an ESR while the second case includes an ESR an EVs.

In both cases the microgrid is connected to the main grid but can only purchase electricity. The prices for energy in the forward and real-time contract are $\lambda_{FW} = 0.2\$/kWh$ and $\lambda_{RT} = 0.3\$/kWh$ respectively. A TGR with a maximum power of $p_g^{max} = 12kW$ and maximum ramping gradient of $R_g = 5kW$ is present and not running at hour 0. Its commitment cost is $c_g^u = 2.12\$$ and its running cost is $c_g^p = 0.128\$/kWh$. The influence of demand uncertainty is reflected by the uncertain load vector l of all 24 hours. The vector consists of the mean value of the random variable for each hour that is normally distributed and independent from other hours. Its variance is equal to $\frac{1}{3}$ of its mean value. Figure 2 depicts the load profile over the course of a day.

The ESR in the first case has a maximum charging and discharging power of $p_{\sigma_s} = 10kW$ and an energy storage capability of $E_{\sigma_s}^{max} = 5kWh$.

In the second case the ESR characteristics remain as in case one. Additionally, there are three EVs which are equipped with vehicle-to-grid technology. They each have a maximum charging and discharging power of $p_{\sigma_{EV}} = 11kW$ and an energy storage capability of $E_{\sigma_{EV}}^{max} = 38kWh$. We assume that each EV is plugged into a charging station at 30% state of charge (SoC) at the beginning of the eighth hour and must be at 60% SoC at the end of the seventeenth hour when it is plugged out. During plug-in the SoC must be less than 80% and more than 20%.

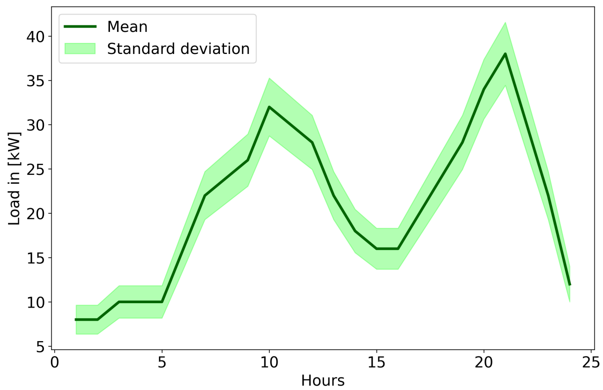

Fig. 2: Load values and standard deviation for 24 hours

B. Results Variance Reduction Techniques

The results in this section were obtained by first solving the L-shaped method for sample sets of several configurations and afterwards evaluating the resulting fixed first-stage decision variables on a test sample with a size of $N = 1000$.

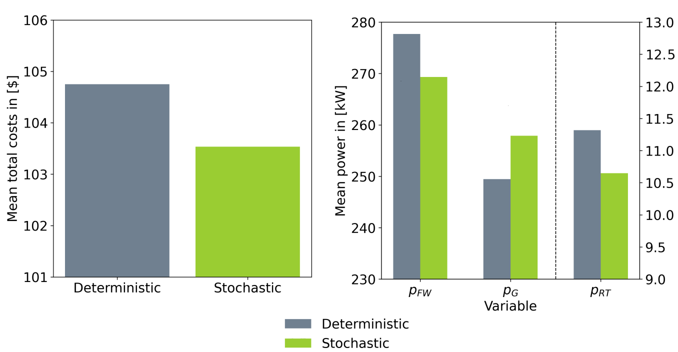

Fig. 3: Stochastic vs. deterministic UC (No energy storage, N=1000)

The effects depicted in figure 3 are examplary for the advantage of stochastic UC compared to deterministic UC. The stochastic approach yields a lower objective value by a modest margin. Moreover, the impact on decision variables is significant, as less electricity is bought in favor of the cheaper TGR.

Figure 4 shows the results of the first case for evaluating first-stage variables from different configurations. The results depict the mean objective value and mean standard deviation for a cumulative energy storage of 5 kWh. Sampling with either AV or LHS yields a significantly lower mean objective value compared to crude Monte Carlo sampling. The standard deviation of samples obtained via variance reduction techniques is also lower in most cases. Increasing the sample size to large sizes , e.g. 10^5, is not justified when looking at the resulting objective value and standard deviation.

Figure 5 depicts the second case with EVs. As they must have a higher SoC when plugging out of the microgrid, more energy is needed in total and the objective value is expected

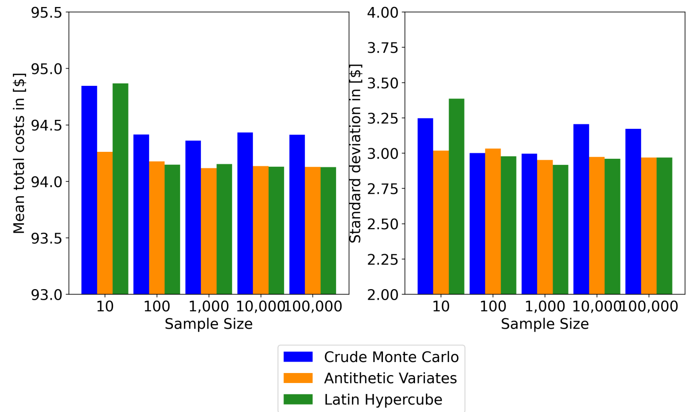

Fig. 4: Objective value and standard deviation for the first case

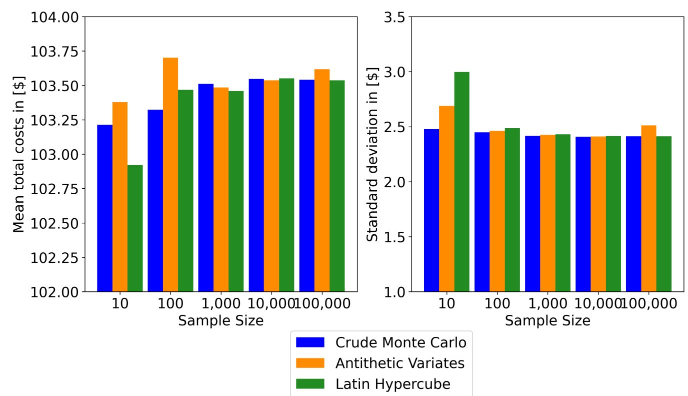

Fig. 5: Objective value and standard deviation for the second case

to increase compared to the first case. However, the results show an increasing objective value with increasing sample size irrespective of the sampling technique. The standard deviation is also not decreasing with increasing sample size. This is probably caused by the structure of the ESR constraints. Introducing a large energy storage opens up a plethora of possibilities for the solver to finish in a local but not a global minimum, by increasing the solution space. Furthermore, because of equality constraints in the energy level equation as well as negative recourse vector in the feedback from the sub problem and a consequential loss of monotonicity, variance reduction techniques such as AV may not work as intended anymore [22].

C. Results Sequential Sampling

The following section presents the results of utilizing a sequential sampling method which includes AV and LHS in the context of a stochastic UC. The implementation of the procedure is based on [5] and [15] and aims to find a high quality solution within a maximum of 10 iterations. Based on this target iteration, a confidence intervall (CI) of $1-\alpha = 1-0.1$ and an inflation factor of $a_k = \frac{1}{k}$, the following parameters were chosen to allow modest computation times and samples sizes [15]. See table I for further details.

TABLE I: Important parameters for sequential sampling

h	ϵ'	p^*	c_p	α
0.008	0.22	$2.7 * 10^{-2}$	9.7667	0.1

For quality solution assessment, single replication procedure (SRP) and averaged two-replication procedure (A2RP) were exploited to estimate the gap as well as the sample variance (SV) [14]. As recommended by [5], A2RP should be preferred over SRP since the latter underestimates the optimality gap for small sample sizes and yields low quality solutions. Table II shows that A2RP always leads to better results in any case. Secondly, using variance reduction techniques like AV provides better results. The best results in terms of smallest gap, SV and CI, were obtained for AV and A2RP. However, utilizing LHS in sequential sampling and in the context of a stochastic UC does not yield expected results. In comparison to AV, the estimators were much higher.

TABLE II: Results for sequential sampling

Sampling method	Estimator	Gap	SV	CI
Crude Monte Carlo	SRP	0.02537	10.1134	$[0, 1.0331]$
Crude Monte Carlo	A2RP	0.0199	11.2885	$[0, 0.9392]$
AV	SRP	0.0106	4.8648	$[0, 0.8187]$
AV	A2RP	0.0009	4.6081	$[0, 0.6723]$

V. Conclusion

This paper examined the quality improvements for stochastic UC. A stochastic UC problem for a small microgrid was modeled and solved with the L-shaped method. To improve the quality of the objective value estimation, different sampling techniques were used. In general AV and LHS performed better than crude Monte Carlo sampling. The results worsened with increasing sample size, when an ESR, especially a high capacity one, was introduced to the model. This is particularly true for the variance reduction techniques. Furthermore sequential sampling was applied to guarantee a high quality solution and a good trade-off between computation time and sample size. Hereby, using AV and A2RP yields the best estimators underpinning the results from [5]. In combination with LHS no conclusive results could be obtained from sequential sampling.

Several assumptions limit the impact of this work. It was assumed that each hour of the load vector is normally distributed and independent of other hours. This needs to be verified. There is no price and no efficiency losses associated with the ESR in the model. Realistic economical parameters for ESR and EV charging and consideration of losses could improve the applicability of the results.

Further research could investigate how conditions for variance reduction techniques can be maintained in UC applications including ESR. Moreover, questions remain open in the application of sequential sampling in UC models. Future work could focus on the combination of variance reduction techniques, sequential sampling and UC problems. Especially the mechanics between LHS and sequential sampling in UC settings leave room for further research.

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APPENDIX
SOFTWARE DOCUMENTATION

The following sections explains our modeling framework which was written in python. The source code can be accessed via https://gitlab-edu.aot.tu-berlin.de/yurdakul/ses_opt.

In order to provide a framework which can be used for several use cases, a package called `seqsuc` was created. This package contains all relevant files for creating and solving a L-shape method or run a sequential sampling method. In the `parameter.py` file, the Parameter object is defined which includes all relevant parameter for the model set-up, like the costs or characteristics of the TGR. Once created, this object is passed to any other object in the framework to access the parameters. In future, this object could be expanded by read in functions of parameters to even increase the dynamic behaviour of the framework. Secondly, the `uc_model.py` file contains three functions to create a master, sub and test problem. These functions mainly utilize the pyomo framework to create the corresponding optimization problems. Thus, the LShapeMethod object uses these functions to set up a L-shape method. This object is defined in the file `l_shape.py` and provides further methods like solving the optimization problem or running a test. While creating a LShapeMethod, one can define the sample size, the sampling method and IO options. Lastly, the file `seq_sampling.py` inhabits the SequentialSampling object which is used to run a sequential method as explained above. The object uses the LShapeMethod object to run the optimization problems. It also provides options to set the sampling and estimator methods while initializing the object. Both objects contain several private functions which are explained in detail in the source code. All files within the package use the `helpers` package which is located inside of `seqsuc`. This helper package contains a lot of functions which are used by the defined objects, for example printing IO messages to the terminal. It also contains the functions to create the samples according to Crude Monte Carlo, AV and LHS. The `main.py` file illustrates an example on how to use the framework. The repository also includes a conda environment specifying the used python packages. We used python in the version 3.9.

All the computation work was done on a computer with the following specifications:

- 2,3 GHz Quad-Core Intel Core i5 processor 8th generation
- 16 GB 2133 MHz LPDDR3 RAM

The optimization problems were solved utilizing the Gurobi solver (9.0.3). Hereby, we used the free academic license.