

Smart Energy Systems  
Winter 2020-2021

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# Optimization Project Group Final Presentation

supervised by Ogün Yurdakul

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# Agenda

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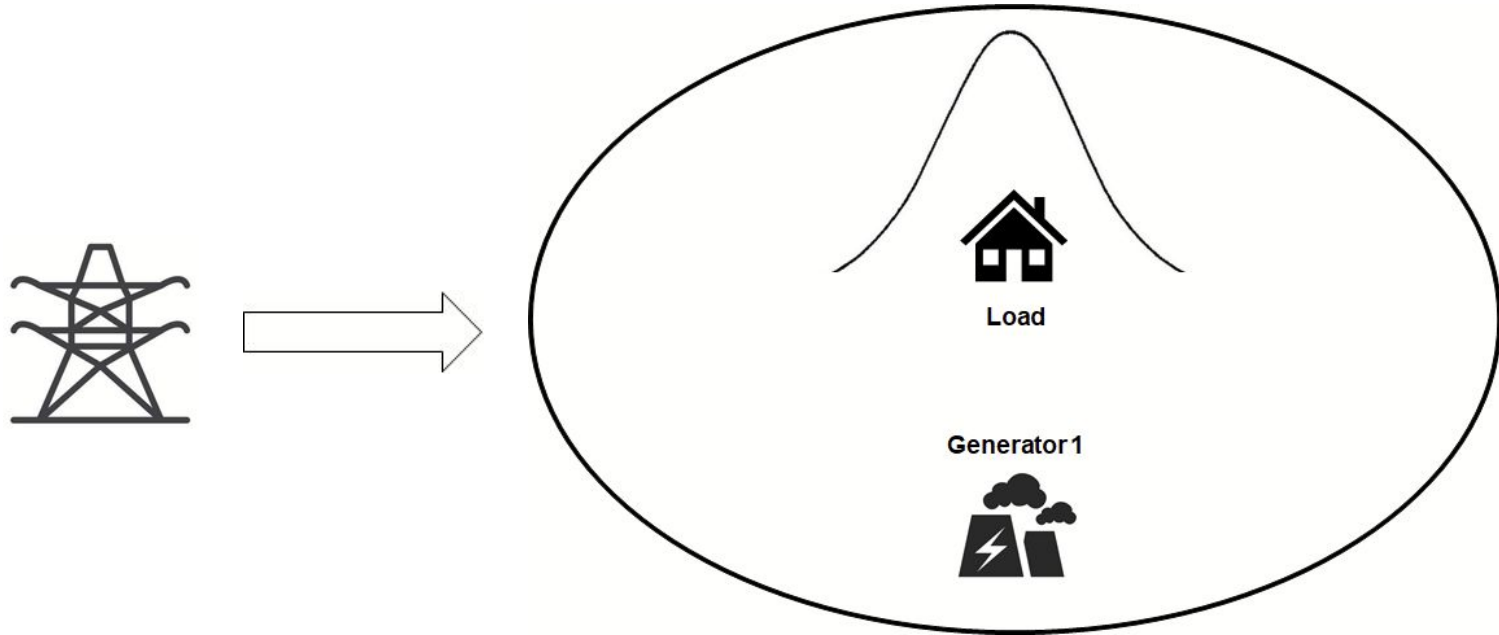
1. Introduction
2. Problem description
3. Solution methodology
4. Results
5. Conclusion
6. Outlook

1. How can we optimally operate a microgrid under uncertainty?
2. How can we ensure efficiency of the operation in a two-staged problem?
3. How can we leverage variance reduction techniques in this modelling approach?

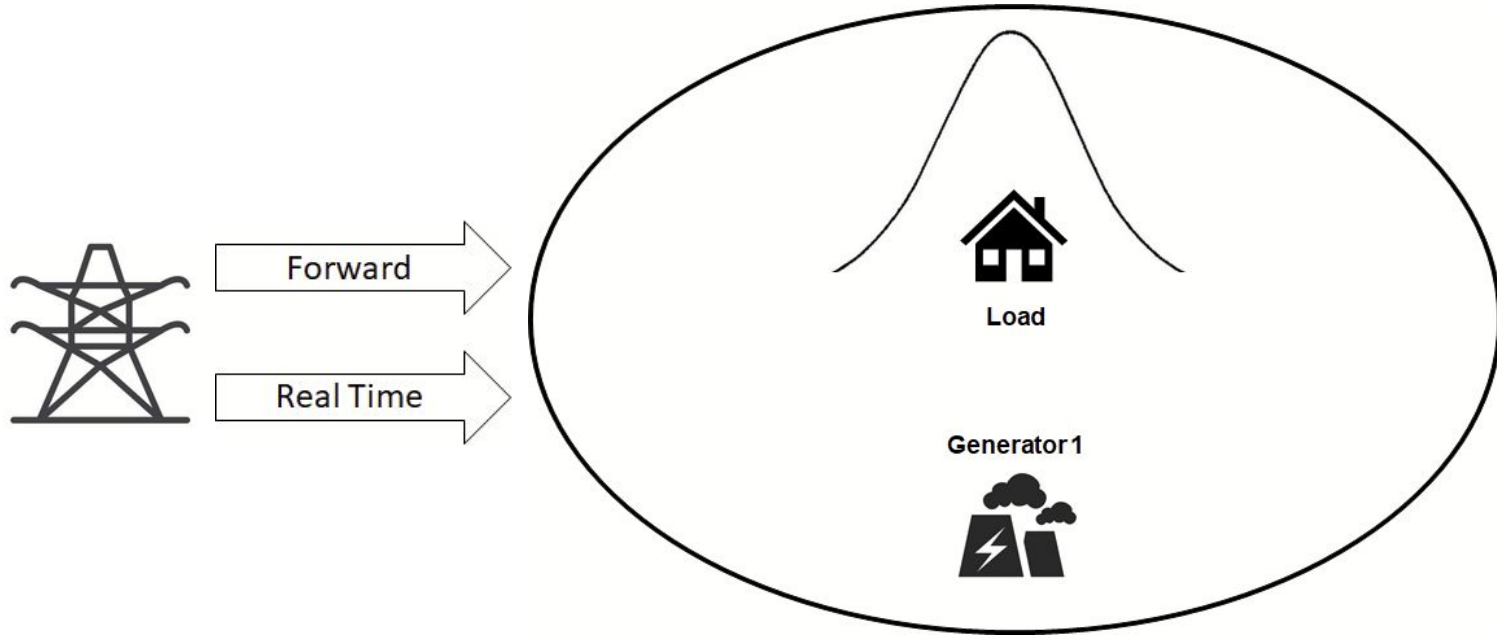
- Short-term planning of a microgrid is determined by unit commitment (UC) and economic dispatch decisions
- Sets start-up and shut-down of thermal generation resources while minimizing costs
- Based on expected load, equipment limitations, and operational policies

# Problem description | Model set-up

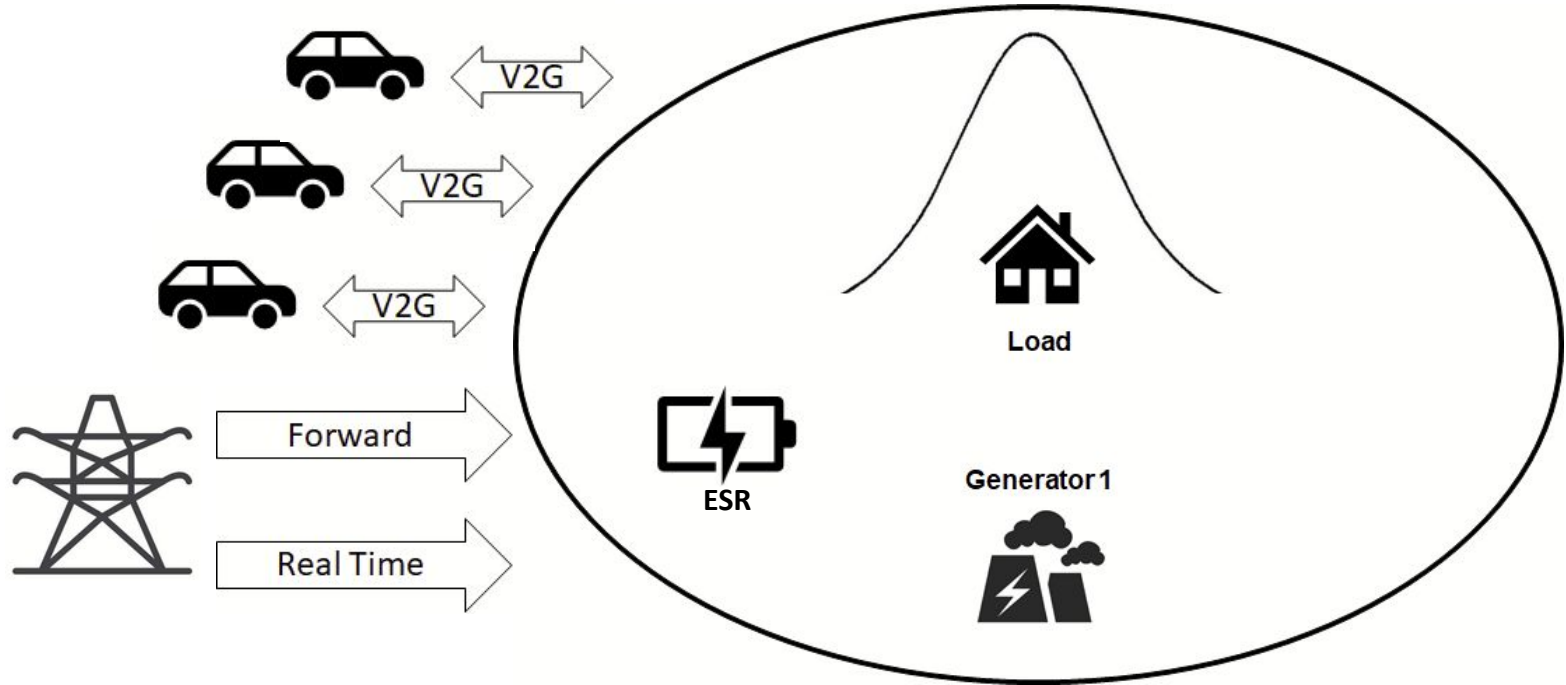
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# Problem description | Model set-up



# Problem description | Model set-up



# Problem description | Objective function



$$\min_{u_G, p_{FW}, p_G, p_{RT}} \left( \sum_{h \in H} c_G^u * u_G[h] + \lambda_{FW} * p_{FW}[h] + E[c_G^p * p_G[h] + \lambda_{RT} * p_{RT}[h]] \right)$$

$c_G^u$	Fixed costs of generator	0.0000212 \$/h	$\lambda_{RT}$	Price of real time contract (RT)	0.3 \$/kWh
$u_G[h]$	Unit commitment of generator in hour $h$	-	$p_{RT}[h]$	Power purchased from RT in hour $h$	kW
$\lambda_{FW}$	Price of forward contract (FW)	0.25 \$/kWh	$c_G^p$	Linear costs of generator	0.128 \$/kWh
$p_{FW}[h]$	Power purchased from FW in hour $h$	kW	$p_G[h]$	Power generation of generator in hour $h$	kW

Source: Yurdakul et al. (2020)



# Problem description | Generator



## Characteristics:

$[p_G]^m$	Minimum power output	0 kW
$[p_G]^M$	Maximum power output	12 kW
$T_G^\uparrow$	Minimum uptime	3 h
$T_G^\downarrow$	Minimum downtime	4 h
$R_G$	Ramping	5 kW

## Assumptions:

- Start generation with 0 kW

## Constraints:

$$u_G[h] * [p_G]^m \leq p_G[h] \leq u_G[h] * [p_G]^M$$

$$-R_G \leq p_G[h] - p_G[h-1] \leq R_G$$

$$u_G[h] - u_G[h-1] \leq u_G[\nu], \forall \nu \in N \text{ such that } h \leq \nu \leq \min\{h-1 + T_G^\uparrow, H\}$$

$$u_G[h-1] - u_G[h] \leq 1 - u_G[\nu], \forall \nu \in N \text{ such that } h \leq \nu \leq \min\{h-1 + T_G^\downarrow, H\}$$

$$\forall h \in H = \{0, \dots, 24\}$$

Source: Yurdakul et al. (2020)

# Problem description | Energy storage resource (ESR)



## Characteristics:

$E_{\sigma_S}^M$	Minimum storage level	0 kWh
$E_{\sigma_S}^m$	Maximum storage level	5 kWh
$p_{\sigma_S}^w$	Maximum withdrawal power	10 kW
$p_{\sigma_S}^i$	Minimum charging power	10 kW
$E_{\sigma_S}[h]$	Storage level in hour $h$	kWh

## Constraints:

$$E_{\sigma_S}^m \leq E_{\sigma_S}[h] \leq E_{\sigma_S}^M$$

$$E_{\sigma_S}[h] = E_{\sigma_S}[h-1] - P_{\sigma_S}^{net}[h] * 1h$$

$$-p_{\sigma_S}^i \leq P_{\sigma_S}^{net}[h] \leq p_{\sigma_S}^w$$

$$\forall h \in H$$

## Assumptions:

- No losses, hence efficiency 100 %
- No costs
- Initialize an empty storage

Source: Yurdakul et al. (2020)

# Problem description | Electrical vehicle (EV)



## Characteristics:

$E_{\sigma_S}^M$	Minimum storage level	0 kWh
$E_{\sigma_S}^m$	Maximum storage level	38 kWh
$p_{\sigma_S}^w$	Maximum withdrawal power	11 kW
$p_{\sigma_S}^i$	Minimum charging power	11 kW

## Assumptions:

- Based on ESR
- Plugged in at 30 % at hour 7
- Plugged out at 60 % at hour 17
- State of charge between 20 % and 80 %

## Constraints:

$$0.2 * E_{\sigma_S}^M \leq E_{\sigma_S}[h] \leq 0.8 * E_{\sigma_S}^M \forall h \in \{7, \dots, 17\}$$

$$E_{\sigma_S}[7] = 0.3 * E_{\sigma_S}^M - P_{\sigma_S}^{net}[7]$$

$$E_{\sigma_S}[h] = E_{\sigma_S}[h-1] - P_{\sigma_S}^{net}[h] * 1h \forall h \in \{8, \dots, 17\}$$

$$E_{\sigma_S}[17] = 0.6 * E_{\sigma_S}^M$$

$$-p_{\sigma_S}^i \leq P_{\sigma_S}^{net}[h] \leq p_{\sigma_S}^w \forall h \in H$$

$$P_{\sigma_S}^{net}[h] = 0, E_{\sigma_S}[h] = 0 \forall h \in H \setminus \{7, \dots, 17\}$$

Source: Yurdakul et al. (2020)

# Problem description | Load

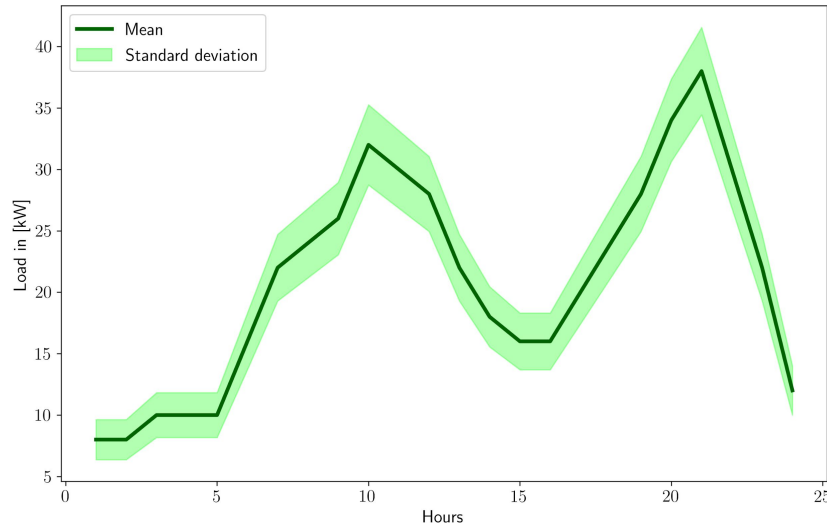


## Constraints:

$$p_G[h] + p_{FW}[h] + p_{RT}[h] + \sum_{\sigma_S \in ESR} P_{\sigma_S}^{net}[h] \geq L[h] \quad \forall h \in H$$

$L[h]$  Load value in hour  $h$        $kW$

## Characteristics:



## Assumptions:

- Hours independent, normally distributed
- Variance =  $\frac{1}{3}$  \* mean

Source: Yurdakul et al. (2020)

# Problem description | DAI-Labors testbed



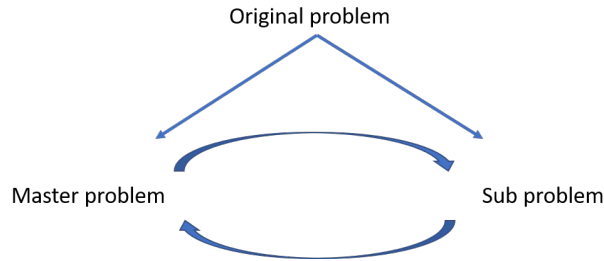
- Optimization methods
  - Represent DAI-Labors testbed
  - Applicable to real life example
  - Scalable

$$\begin{aligned} \min & (c^T x) \\ \text{s.t. } & Ax \leq b \\ & x > 0 \end{aligned}$$

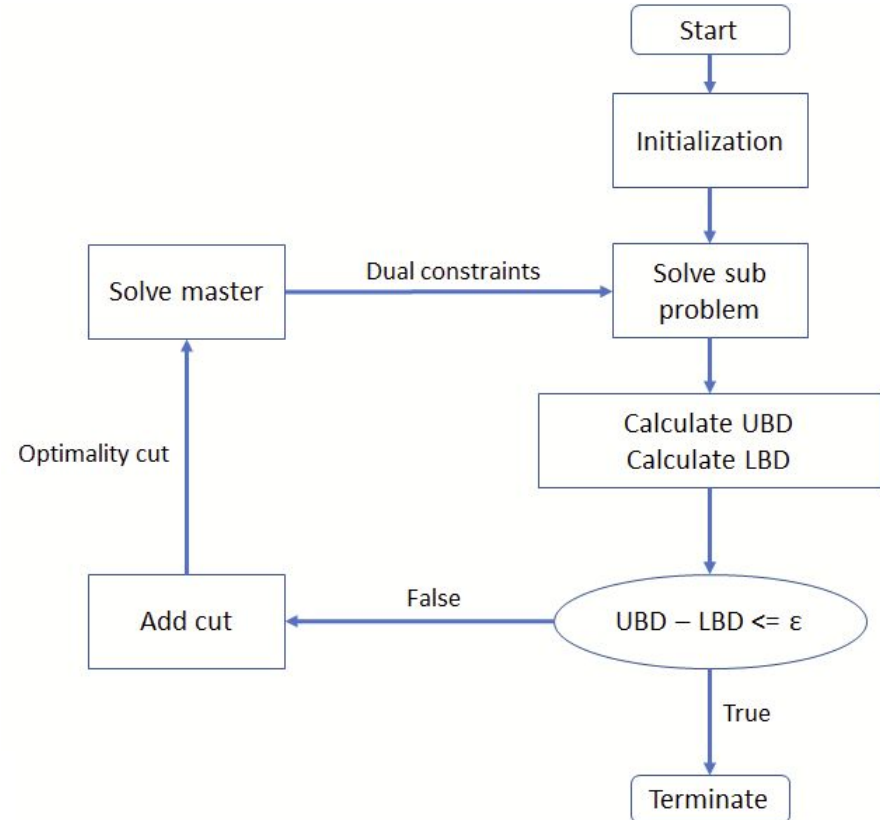
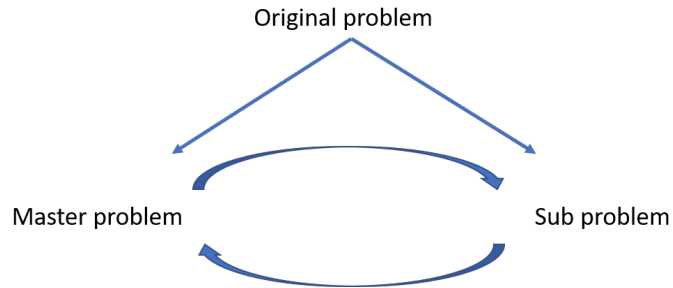
Solving for all samples simultaneously increases **computation time**



Split problem into **two stages** and solve iteratively

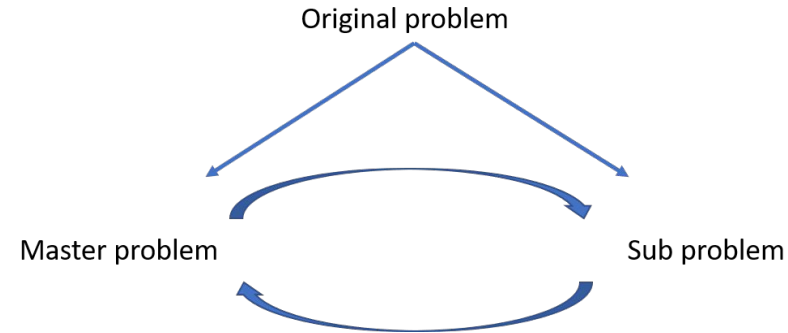


# Solution methodology | L-shape method



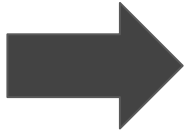
# The L-shaped Method

- decomposition into master and sub problem
- solve subproblem: complicating variables are treated as parameters to get a candidate solution
- insert optimality cut into the master problem
- optimality cut is a proxy for the 1st stage decision on 2nd stage costs
- master problem: lower bound (less constraints)
- sub problem: upper bound
- optimal solution when upper and lower bound are sufficiently close
- L-shaped method:
  - uncertainty
  - multiple subproblems





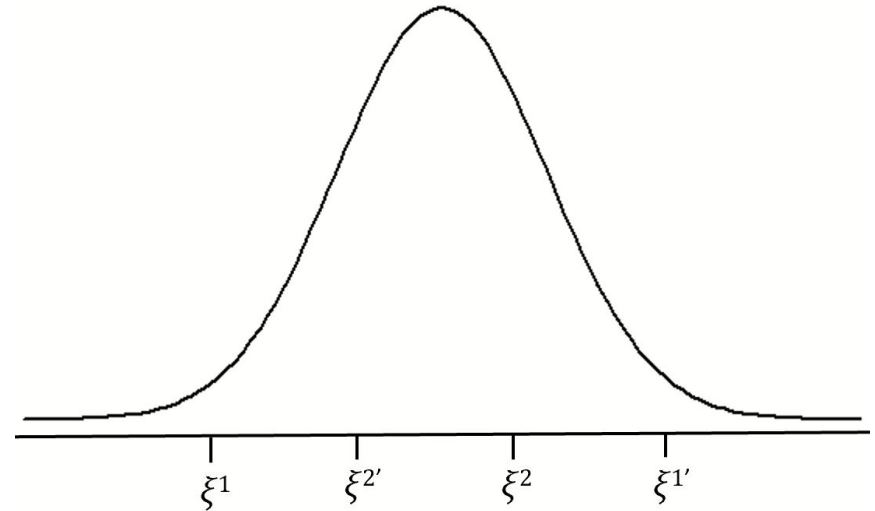
- Goal: Decrease variance to get more accurate estimator of the mean
- Increasing sample size? ⚡ computation time
- Better: improving Monte Carlo samples through variance reduction techniques



1. **Latin Hypercube Sampling**
2. **Antithetic Variates**

Source: Homem-de-Mello & Bayraksan (2016)

- Idea: exploit correlations by pairing negatively correlated random variables
1. Create random samples with  $N = \frac{1}{2}$  sample size from a normal distribution (general case: from uniform distribution)
  2. Calculate antithetics: mean values - (random samples - mean values)
  3. Join the random samples and its antithetics to create the full sample

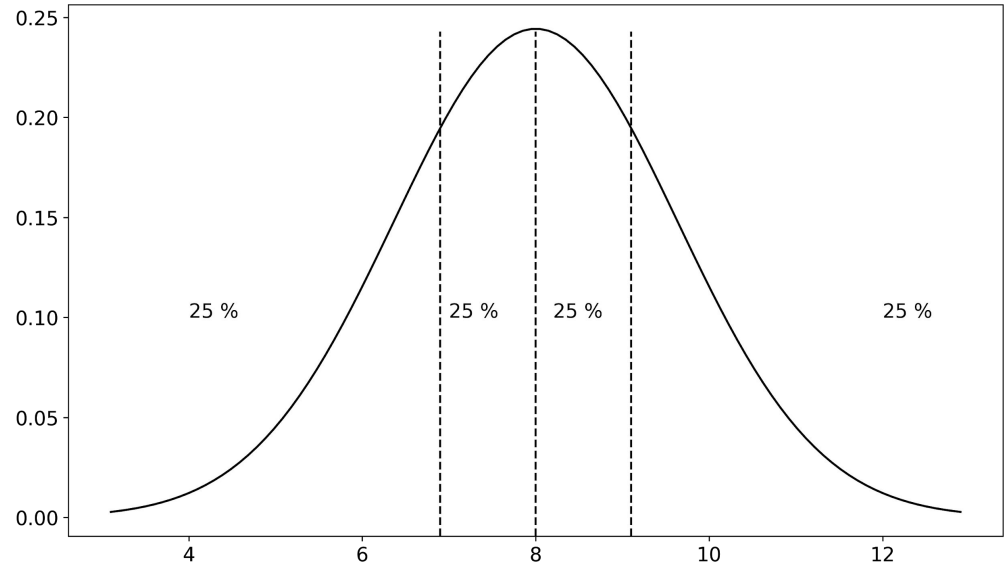


Source: based on Homem-de-Mello & Bayraksan (2016)

# Latin hypercube | Implementation



1. For each hour, divide distribution into  $N$  parts of equal probability ( $N$  = sample size)
2. Draw a random sample from each part
3. Shuffle hourly sets
4. Create random vector from hourly sets



e.g. sample size = 4

Source: based on Homem-de-Mello & Bayraksan (2016)

## Antithetic Variates

1. Create random samples with  $N = \frac{1}{2}$  sample size from a normal distribution
2. Calculate antithetics: mean values - (random samples - mean values)
3. Join the random samples and its antithetics to create the full sample

## Latin Hypercube Sampling

1. For each hour, divide distribution into  $N$  parts of equal probability ( $N$  = sample size)
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1. compare deterministic with stochastic approach (relation forward / real time)
  2. Mean, variance, objective value (MS2)
  3. Sample size & SD (MS3) → higher sample size decreases variance, computation time increases
  4. MS 4: Variance reduction techniques & sample size
  5. Final: Variance reduction techniques & sample size with new components
  6. Computation time/ multiprocessing
  - 7.

# Scrum board

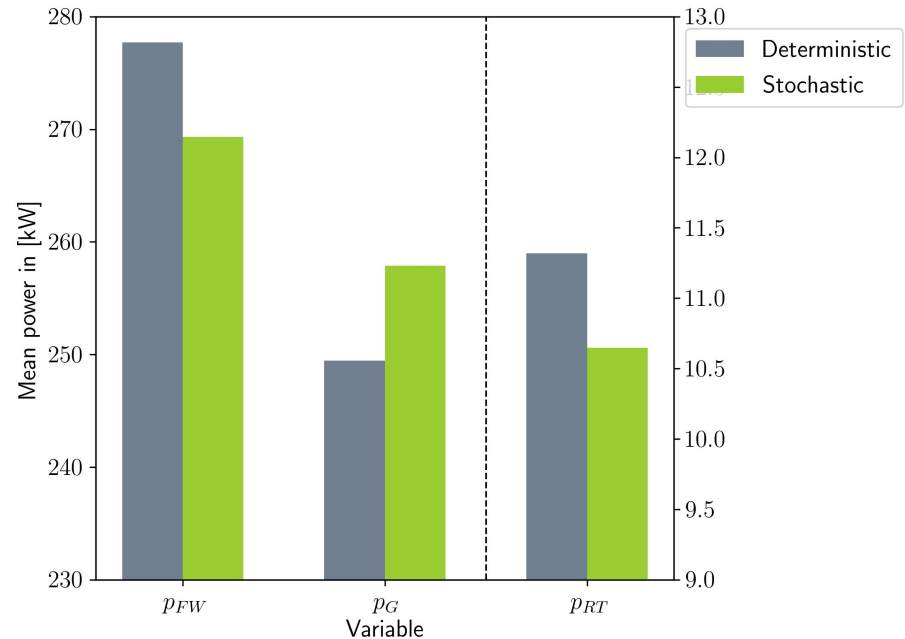
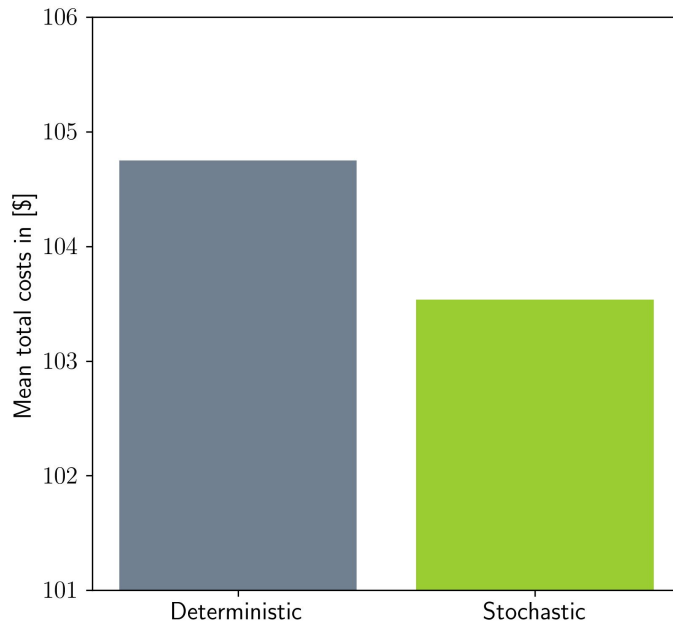


Ogün Yurdakul > ses\_opt > Issue Boards

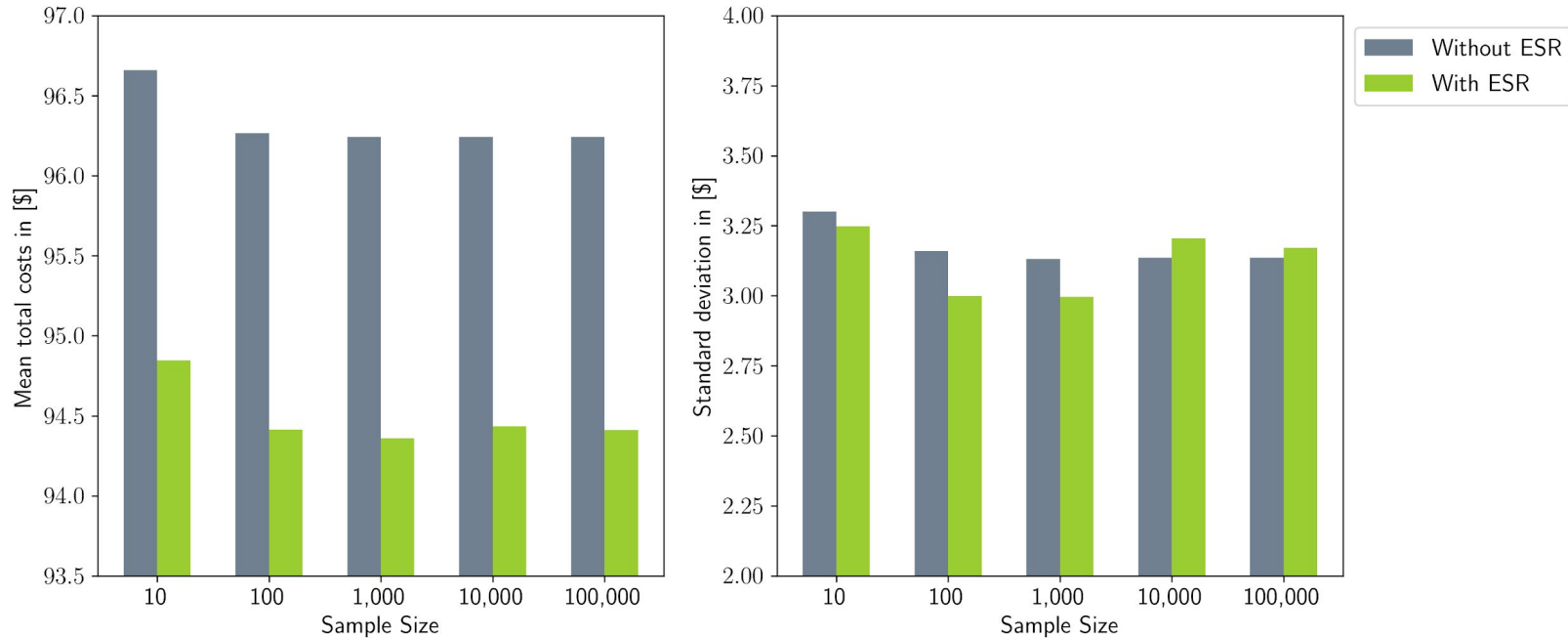
Development ▾ Search or filter results... Add list ↕

Development	To Do	Doing	Review	Closed
	Sequential sampling #26			Implement electric vehicles #25
				Antithetic variates implementation #27
				Latin Hypercube sampling #28
				Monte Carlo sampling (Task 1) #19
				Calculate mean and variance of optimal cost (Task 1) #20
				Add thermal generation resource, minimum uptime and downtime and ramping constraint (Task 2) #21
				Run model with new constraints (Task 2) #22
				Add ESR and run model (Task 3)

# Results | Deterministic vs. stochastic

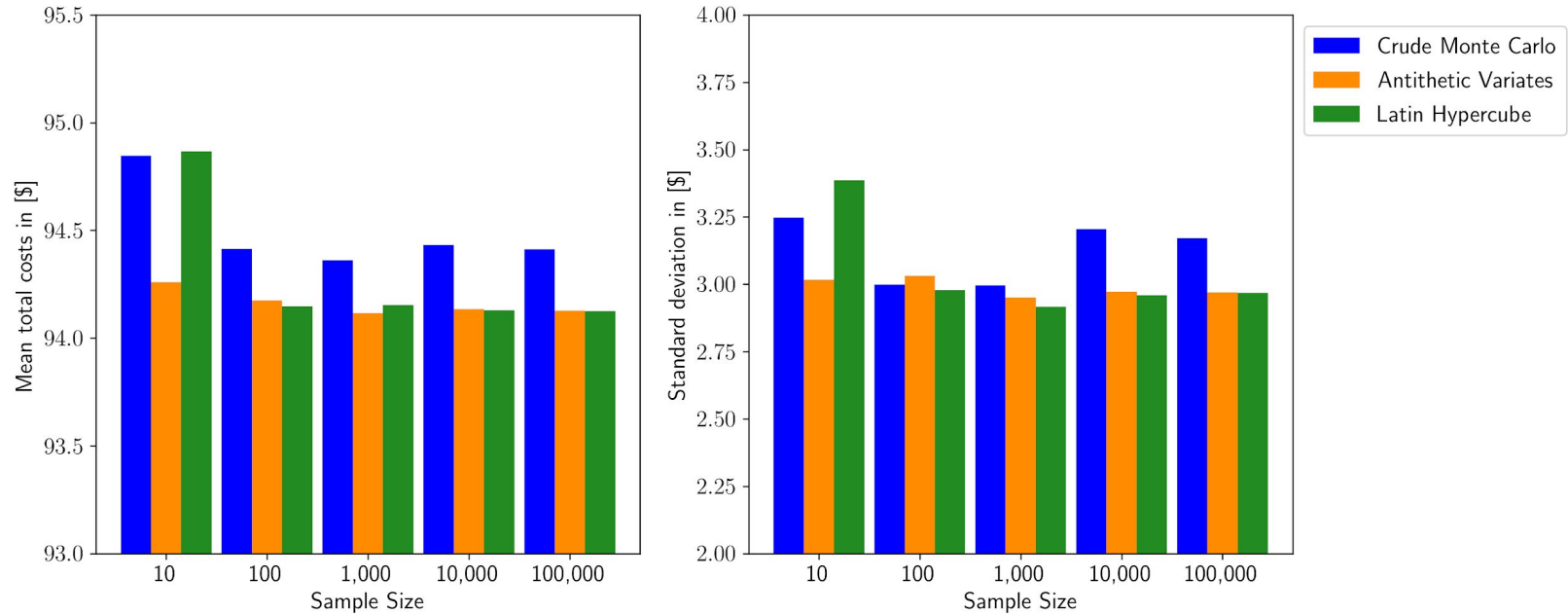


# Results | Sample size

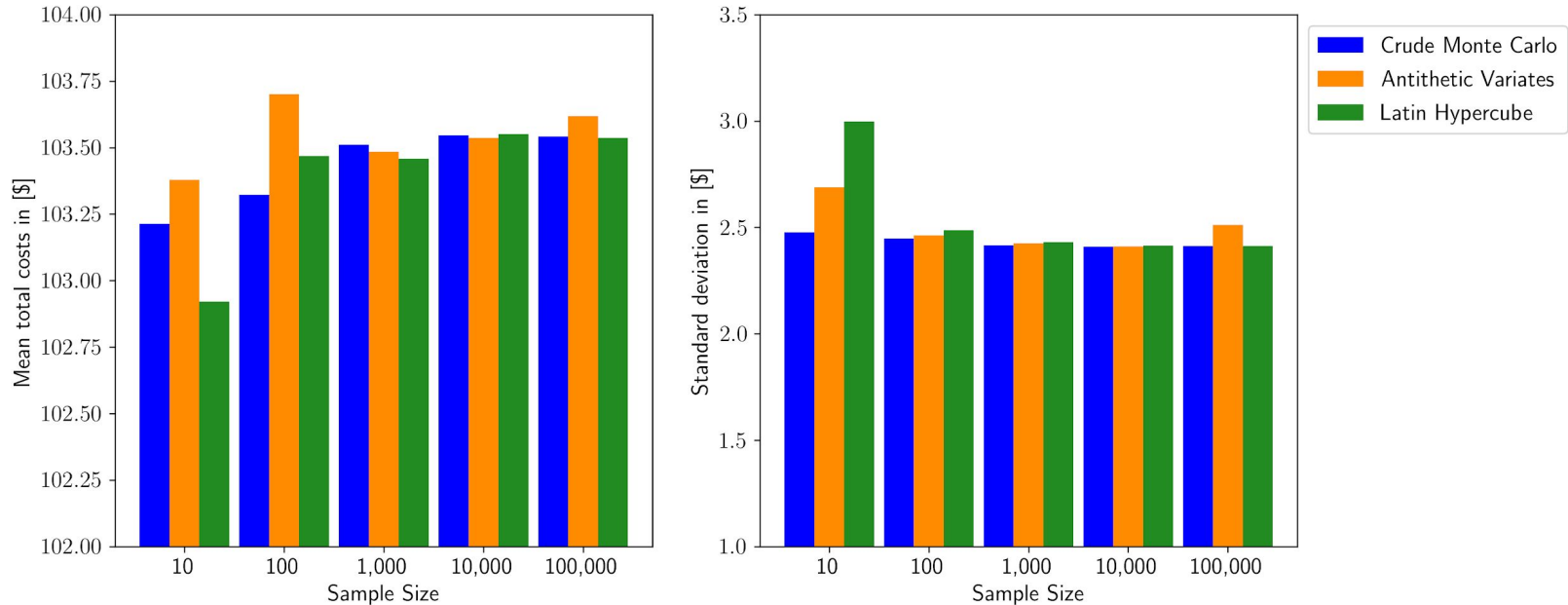




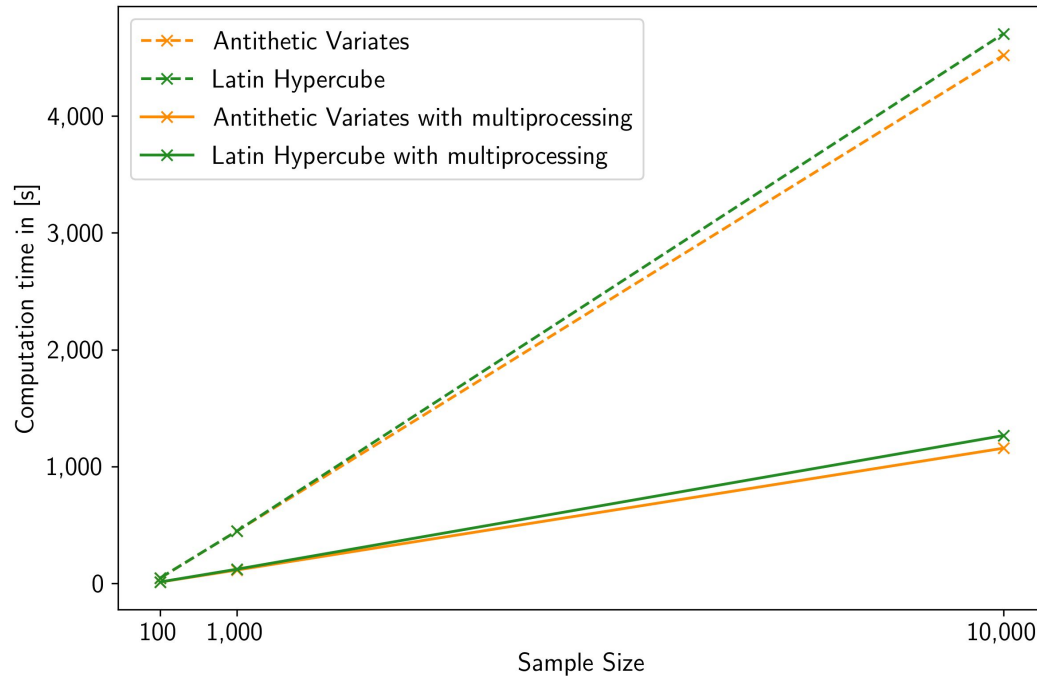
# Results | Sampling techniques



# Results | Sampling techniques



Problematic: Equalities in constraints, negative recourse vector from second stage



## Measures to improve runtime

- Remove unnecessary constraints
  - ↓ 5 %
- Execution via terminal
  - ↓ 20 %
- Enable multiprocessing
  - ↓ 75 %

- Stochastic approach yields better objective value
- Variance reduction techniques decrease the standard deviation in scenarios with no ESR
  - This effect does not yield with ESR: constraints violate the pre-conditions for Antithetic Variates and Latin Hypercube Sampling (no monotonicity)
- Structure of L-shaped method enables parallelization of tasks, facilitating multiprocessing to decrease processing time

- Economic mechanisms
- Distribution and variability of load throughout seasons
- Further application of variance reduction techniques in optimization problems
- Avoidance of overly conservative sample sizes while assuring quality of solution → sequential sampling

Birge, J. R., and Louveaux, F., *Introduction to stochastic programming*. Springer Science & Business Media, 2011.

Conejo, A. J., Castillo, E., Minguez, R., and Garcia-Bertrand, R., *Decomposition techniques in mathematical programming: engineering and science applications*. Springer Science & Business Media, 2006.

Homem-de-Mello T. and Bayraksan, G., *Scenario Generation and Sampling Methods*, Lecture Slides, 2016. Accessible via [https://www.youtube.com/watch?v=RkUdWL\\_3KLA](https://www.youtube.com/watch?v=RkUdWL_3KLA)

Yurdakul et al., *Quantification of the Impact of GHG Emissions on Unit Commitment in Microgrids*, in IEEE PES T&D-LA 2020.



# Thanks for your attention

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