
Tutorial Sheet 5: Graph Theory

- (1) Draw these graphs.
(a) K_7 (b) $K_{1,8}$ (c) $K_{4,4}$ (d) C_7 (e) W_7 (f) Q_4 .
- (2) For which values of n are these graphs bipartite?
(a) K_n (b) C_n (c) W_n (d) Q_n .
- (3) Let n be a positive integer. Show that a subgraph induced by a nonempty subset of the vertex set of K_n is a complete graph.
- (4) How many subgraphs with at least one vertex does K_3 have?
- (5) For which values of n are these graphs regular?
(a) K_n (b) C_n (c) W_n (d) Q_n .
- (6) Find an adjacency matrix for each of these graphs.
(a) K_n (b) C_n (c) W_n (d) $K_{m,n}$ (e) Q_n .
- (7) Find a self-complementary simple graph with five vertices.
- (8) How many nonisomorphic simple graphs are there with five vertices and three edges?
- (9) What is the product of the incidence matrix and its transpose for an undirected graph?
- (10) Show that every connected graph with n vertices has at least $n - 1$ edges.
- (11) Show that if a connected simple graph G is the union of the graphs G_1 and G_2 , then G_1 and G_2 have at least one common vertex.
- (12) Show that a simple graph G with n vertices is connected if it has more than $(n-1)(n-2)/2$ edges.
- (13) For which values of n do these graphs have an Euler circuit?
(a) K_n (b) C_n (c) W_n (d) Q_n .
- (14) Show that a bipartite graph with an odd number of vertices does not have a Hamilton circuit.
- (15) Show that K_5 is nonplanar.
- (16) Suppose that a connected bipartite planar simple graph has e edges and v vertices. Show that $e \leq 2v - 4$ if $v \geq 3$.
- (17) Suppose that a connected planar graph has 30 edges. If a planar representation of this graph divides the plane into 20 regions, how many vertices does this graph have?