Chapter 2. Units

I often say that when you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind; it may be the beginning of knowledge, but you have scarcely, in your thoughts, advanced to the stage of science, whatever the matter may be
– William Thomson (Lord Kelvin)

All engineered systems require measurements for specifying the size, weight, speed, etc. of objects as well as characterizing their performance. Understanding the application of these units is the single most important objective of this textbook because it applies to all forms of engineering and everything that one does as an engineer. Understanding units is **far** more than simply being able to convert from feet to meters or vice versa; combining and converting units from different sources is a challenging topic. For example, if building insulation is specified in units of BTU inches per hour per square foot per degree Fahrenheit, how can that be converted to thermal conductivity in units of Watts per meter per degree C? Or can it be converted? Are the two units measuring the same thing or not? (For example, in a new engine laboratory facility that was being built for me, the natural gas flow was insufficient… so I told the contractor I needed a system capable of supplying a minimum of 50 cubic feet per minute (cfm) of natural gas at 5 pounds per square inch (psi). His response was "what's the conversion between cfm and psi?" Of course, the answer is that there is no conversion; cfm is a measure of flow rate and psi a measure of pressure. One might as well be asking what's the conversion between kilograms and miles.) Engineers must struggle with these misconceptions every day.

Base units

Engineers in the United States are burdened with two systems of units and measurements: (1) the *English* or *USCS* (US Customary System) ☹ and (2) the *metric* or *SI* (Système International d'Unités) ☺. Either system has a set of *base units* , that is, units which are defined based on a standard measure such as a certain number of wavelengths of a particular light source. These base units include:

- Length (meters (m), centimeters (cm), millimeters (mm); feet (ft), inches (in), kilometers (km), miles (mi))
  - 1 m = 100 cm = 1000 mm = 3.281 ft = 39.37 in
  - 1 km = 1000 m
  - 1 mi = 5280 ft
- Mass (lbm, slugs, kilograms); (1 kg = 2.205 lbm = 0.06853 slug) (lbm = "pounds mass")
- Time (seconds; the standard abbreviation is "s" not "sec") (same units in USCS and SI!)
- Electric current (really electric charge in units of coulombs [abbreviation: 'coul'] is the base unit and the derived unit is current = charge/time) (1 coulomb = charge on 6.241506 x 10^{18} electrons) (1 ampere [abbreviation: amp]= 1 coul/s)

Moles are often reported as a fundamental unit, but it is not; it is just a bookkeeping convenience to avoid carrying around factors of 10^{23} everywhere. The choice of the number of particles in a mole of particles is completely arbitrary; by convention Avogadro's number is defined by N_A = 6.0221415 x 10^{23}, the units being particles/mole (or one could say individuals of any kind, not limited just to particles, e.g. atoms, molecules, electrons or students).

Temperature is frequently interpreted as a base unit but again it is not, it is a *derived unit*, that is, one created from combinations of base units. Temperature is essentially a unit of energy divided by Boltzman's constant. Specifically, the average kinetic energy of an ideal gas particle in a 3-dimensional box is 1.5kT, where k is Boltzman's constant = 1.380622 x 10^{-23} J/K (really (Joules/particle)/K; every textbook will state the units as just J/K but you'll see below how useful it is to include the "per particle" part as well). Thus, 1 Kelvin is the temperature at which the kinetic energy of an ideal gas (and **only** an ideal gas, not any other material) molecule is 1.5kT =2.0709 x 10^{-23} J.

The ideal gas constant ($\Re$) with which you are very familiar is simply Boltzman's constant multiplied by Avogadro's number, *i.e.*

$$\Re = kN_A = \left(1.38\times10^{-23}\frac{J}{particle\ K}\right)\left(6.02\times10^{23}\frac{particle}{mole}\right) = 8.314\frac{J}{mole\ K}\frac{cal}{4.184J} = 1.987\frac{cal}{mole\ K}$$

(Equation 1)

In the above equation, note that we have multiplied and divided units such as Joules as if they were numbers; this is valid because we can think of 8.314 Joules as 8.314 x (1 Joule) and additionally we can write (1 Joule) / (1 Joule) = 1. Extending that further, we can think of (1 Joule)/(1 kg m^2/s^2) = 1, which will be the basis of our approach to units conversion – multiplying and dividing by 1 written in different (and sometimes odd-looking) forms. Note also the value of the "hidden unit" 'particle' in the above equation. I find it extremely useful to include such units because the real units aren't J/K; if you have 2 particles you'll have twice as much energy (J) at the same value of K (temperature), so the real units ARE in fact J/(particle K).

Why does this discussion apply only for an ideal gas? By definition, ideal gas particles have only kinetic energy and negligible potential energy due to inter-molecular attraction; if there is potential energy, then we need to consider the total internal energy of the material (E, units of Joules) which is the sum of the microscopic kinetic and potential energies, in which case the temperature for any material (ideal gas or not) is defined as

$$T = \left(\frac{\partial U}{\partial S}\right)_{V=const.}$$

(Equation 2)

where U is the internal energy of the material (units J), S is the entropy of the material (units J/K) and V is the volume. This intimidating-looking definition of temperature, while critical to understanding thermodynamics, will not be needed in this course. (Until you read this you thought you understood temperature because of its common usage and a handy device called a thermometer; in fact, temperature is quite difficult to understand. The one thing you should understand is that it's the driving force for heat transfer, that is, heat must always flow from a higher to a lower temperature and never the reverse.)

Derived units

Derived units are units created from combinations of base units; there are an infinite number of possible derived units. Some of the more important/common/useful ones are:

- Area = length^2; 640 acres = 1 mile^2, or 1 acre = 43,560 ft^2
- Volume = length^3; 1 ft^3 = 7.481 gallons = 28,317 cm^3; also 1 liter = 1000 cm^3 = 61.02 in^3
- Velocity = length/time
- Acceleration = velocity/time = length/time^2 (standard gravitational acceleration on earth = g = 32.174 ft/s^2 = 9.806 m/s^2)
- Force = mass * acceleration = mass*length/time^2
  - 1 kg m/s^2 = 1 Newton = 0.2248 pounds force (pounds force is usually abbreviated lbf and Newton N) (equivalently 1 lbf = 4.448 N)
- Energy = force x length = mass x length^2/time^2
  - 1 kg m^2/s^2 = 1 Joule (J)
  - 778 ft lbf = 1 British thermal unit (BTU)
  - 1055 J = 1 BTU
  - 1 J = 0.7376 ft lbf
  - 1 calorie = 4.184 J
  - 1 dietary calorie = 1000 calories
- Power (energy/time = mass x length^2/time^3)
  - 1 J/s = 1 kg m^2/s^3 = 1 Watt
  - 746 W = 550 ft lbf/sec = 1 horsepower
- Heat capacity = J/moleK or J/kgK or J/mole˚C or J/kg˚C (see note below)
- Pressure = force/area
  - 1 N/m^2 = 1 Pascal
  - 101325 Pascal = 101325 N/m^2 = 14.696 lbf/in^2 = 1 standard atmosphere
- Current = charge/time (1 amp = 1 coul/s)
- Voltage = energy/charge (1 Volt = 1 J/coul)
- Capacitance = amps / (volts/s) (1 farad = 1 coul^2/J)
- Inductance = volts / (amps/s) (1 Henry = 1 J s^2 / coul^2)
- Resistance = volts/amps (1 ohm = 1 volt/amp = 1 Joule s / coul^2)
- Torque = force x lever arm length = mass x length^2/time^2 – same as energy but one would usually report torque in Nm (Newton meters), not Joules, to avoid confusion.
- Radians, degrees, revolutions – these are all dimensionless quantities, but must be converted between each other, i.e. 1 revolution = 2π radians = 360 degrees.

Special consideration 1: pounds force vs. pounds mass

By far the biggest problem with USCS units is with mass and force. The problem is that pounds is both a unit of mass AND force. These are distinguished by lbm for pounds (mass) and lbf for pounds (force). We all know that W = mg where W = weight, m = mass, g = acceleration of gravity. So

$1 \text{ lbf} = 1 \text{ lbm x g} = 32.174 \text{ lbm ft/s}^2$ (Equation 3)

Sounds ok, huh? But wait, now we have an extra factor of 32.174 floating around. Is it also true that

$1 \text{ lbf} = 1 \text{ lbm ft/s}^2$

which is analogous to the SI unit statement that

$1 \text{ Newton} = 1 \text{ kg m/s}^2$ (Equation 4)

No, 1 lbf cannot equal 1 lbm ft/s^2 because 1 lbf equals 32.174 lbm ft/sec^2. So what unit of mass satisfies the relation

$1 \text{ lbf} = 1 \text{ (mass unit) ft/s}^2$?

This mass unit is called a "slug" believe it or not. With use of equation (2) it is apparent that

$1 \text{ slug} = 32.174 \text{ lbm} = 14.59 \text{ kg}$ (Equation 5)

Often when doing USCS conversions, it is convenient to introduce a conversion factor called g_c; by rearranging Equation 3 we can write

$$g_c = \frac{32.174 \text{ lbm ft}}{\text{lbf s}^2} = 1$$ (Equation 6).

Since Equation 2 shows that $g_c = 1$, one can multiply and divide any equation by g_c as many times as necessary to get the units into a more compact form *(an example of "why didn't somebody just say that?")*. Keep in mind that **any** units conversion is simply a matter of multiplying or dividing by 1, e.g.

$$\frac{5280 \text{ ft}}{\text{mile}} = 1;\ \frac{1 \text{ kg m}}{\text{N s}^2} = 1;\ \frac{778 \text{ ft lbf}}{\text{BTU}} = 1;\ \textit{etc.}$$

For some reason 32.174 lbm ft/ lbf s^2 has been assigned a special symbol called g_c even though there are many other ways of writing 1 (e.g. 5280 ft / mile, 1 kg m / N s^2, 778 ft lbf / BTU) all of which are also equal to 1 but none of which are assigned special symbols.

If this seems confusing, I don't blame you. That's why I recommend that even for problems in which the givens are in USCS units and where the answer is needed in USCS units, first convert everything to SI units, do the problem, then convert back to USCS units. I disagree with some authors who say an engineer should have "native fluency" in both systems; it is somewhat useful but not necessary. The second example in the next sub-section below uses the approach of converting to SI, do the problem, and convert back to USCS. The third example shows the use of USCS units employing g_c.

Special consideration 2: temperature

Many difficulties also arise with units of temperature. There are four temperature scales in "common" use: Fahrenheit, Rankine, Celsius (or Centigrade) and Kelvin. Note that one speaks of "degrees Fahrenheit" and "degrees Celsius" but just "Rankines" or "Kelvins" (without the "degrees").

T (in units of ˚F) = T (in units of R) - 459.67
T (in units of ˚C) = T (in units of K) - 273.15
1 K = 1.8 R
T (in units of ˚C) = [T (in units of ˚F) – 32]/1.8,
T (in units of ˚F) = 1.8[T (in units of ˚C)] + 32
Water freezes at 32˚F / 0˚C, boils at 212˚F / 100˚C

Special note *(another example of "that's so easy, why didn't somebody just say that?")*: when using units involving temperature (such as heat capacity, units J/kg˚C, or thermal conductivity, units Watts/m˚C), one can convert the temperature in these quantities these to/from USCS units (e.g. heat capacity in BTU/lbm˚F or thermal conductivity in BTU/hr ft ˚F) simply by multiplying or dividing by 1.8. You don't need to add or subtract 32. Why? Because these quantities are really derivatives with respect to temperature (heat capacity is the derivative of internal energy with respect to temperature) or refer to a temperature gradient (thermal conductivity is the rate of heat transfer per unit area by conduction divided by the temperature gradient, dT/dx). When one takes the derivative of the constant 32, you get zero. For example, if the temperature changes from 84˚C to 17˚C over a distance of 0.5 meter, the temperature gradient is (84-17)/0.5 = 134˚C/m. In Fahrenheit, the gradient is [(1.8*84 +32) – (1.8*17 + 32)]/0.5 = 241.2˚F/m or 241.2/3.281 = 73.5˚F/ft. The important point is that the 32 cancels out when taking the difference. So *for the purpose of converting between ˚F and ˚C in units like heat capacity and thermal conductivity,* one can use 1˚C = 1.8˚F. That **doesn't** mean that one can just skip the + or – 32 whenever one is lazy.

Also, one often sees thermal conductivity in units of W/m˚C or W/mK. How does one convert between the two? Do you have to add or subtract 273? And how do you add or subtract 273 when the units of thermal conductivity are not degrees? Again, thermal conductivity is heat transfer per unit area **per unit temperature gradient**. This gradient could be expressed in the above example as (84˚C-17˚C)/0.5 m = 134˚C/m, or in Kelvin units, [(84 + 273)K – (17 + 273)K]/0.5 m = 134K/m and thus the 273 cancels out. So one can say that 1 W/m˚C = 1 W/mK, or 1 J/kg˚C = 1 J/kgK. And again, that **doesn't** mean that one can just skip the + or – 273 (or 460, in USCS units) whenever one is lazy.

Examples of the use (and power) of units

Example 1

An object has a weight of 300 lbf at earth gravity. What is its mass in units of lbm?

$$F = ma \Rightarrow m = \frac{F}{a} = \frac{F}{a}(1) = \frac{F}{a}(g_c) = \frac{300\ lbf}{32.174\frac{ft}{s^2}}\left(\frac{32.174\ lbm\ ft}{lbf\ s^2}\right) = 300\ lbm$$

This shows that an object that weighs 300 lbf at earth gravity has a mass of 300 lbm. At any other gravity level, its mass would still be 300 lbm but its weight would be different, but in all cases this weight would still be calculated according to F = ma (force = mass x acceleration) or, specifically for weights, we can use W = mg (weight = mass x acceleration of gravity).

Example 2

What is the weight (in lbf) of one gallon of air at 1 atm and 25˚C? The molecular mass of air is 28.97 g/mole = 0.02897 kg/mole.

Ideal gas law: $PV = n\Re T$

(P = pressure, V = volume, n = number of moles, $\Re$ = universal gas constant, T = temperature)

Mass of gas (m) = moles x mass/mole = $n\mathcal{M}$ ($\mathcal{M}$ = molecular mass)

Weight of gas (W) = mg, where g = acceleration of gravity = 9.81 m/s^2

Combining these 3 relations: $W = PV\mathcal{M}g/\Re T$

$$W = \frac{PVMg}{\Re T} = \frac{\left(1\text{ atm}\frac{101325\text{ N/m}^2}{\text{atm}}\right)\left(1\text{ gal}\frac{\text{ft}^3}{7.481\text{ gal}}\left(\frac{\text{m}}{3.281\text{ ft}}\right)^3\right)\left(\frac{0.02897\text{ kg}}{\text{mole}}\right)\left(\frac{9.81\text{ m}}{\text{s}^2}\right)}{\left(\frac{8.314\text{ J}}{\text{mole K}}\right)(25+273)\text{K}}$$

$$W = 0.0440\frac{\left(\frac{\text{N}}{\text{m}^2}\right)\left(\text{m}^3\right)\left(\frac{\text{kg}}{\text{mole}}\right)\left(\frac{\text{m}}{\text{s}^2}\right)}{\left(\frac{\text{J}}{\text{mole K}}\right)\text{K}} = 0.0440\frac{(\text{N})(\text{m})(\text{kg})\left(\frac{\text{m}}{\text{s}^2}\right)}{\text{J}} = 0.0440\frac{(\text{N})\left(\frac{\text{kg m}^2}{\text{s}^2}\right)}{\text{J}}$$

$$W = 0.0440\text{ N}\left(\frac{1\text{ lbf}}{4.448\text{ N}}\right) = 0.00989\text{ lbf} \approx 0.01\text{ lbf}$$

Note that it's easy to write down all the formulas and conversions. The tricky part is to check to see if you've actually gotten all the units right. In this case I converted everything to the SI system first, then converted back to USCS units at the very end – which is a pretty good strategy for most problems. The tricky parts are realizing (1) the temperature must be an absolute temperature, i.e. Kelvin not ˚C, and (2) that moles are not the same as mass, so you have to convert using $\mathcal{M}$. If in doubt, how do you know whether to multiply or divide by $\mathcal{M}$? Check the units!

Example 3

A car with a mass of 3000 lbm is moving at a velocity of 88 ft/s. What is its kinetic energy (KE) in units of ft lbf? What is its kinetic energy in Joules?