

# MATHEMATICAL ANALYSIS II

## INTEGRAL CALCULUS

SEVER ANGEL POPESCU

To my family.

.....

.....

To those who really try to put their lives in the Light of  
Truth.

.....

Dedicated to the memory of my late Professor and  
Mentor, Dr. Doc. Nicolae Popescu.

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