UNIT-1
Wave Optics

☆ **Interference of Light Waves :-**
The phenomenon of modification in intensity due to two waves of same frequency and constant phase diff. in the region of superposition is called interference.

When two light waves of same frequency travel approximately in the same direction and have a phase diff. that remains constant with time, the resultant intensity of light is not distributed uniformly in space. This non-uniform distribution of light due to the superposition of wave is called interference. At some points the intensity is maximum and the interference is constructive and at some other point the intensity is minimum and the interference is called destructive interference.

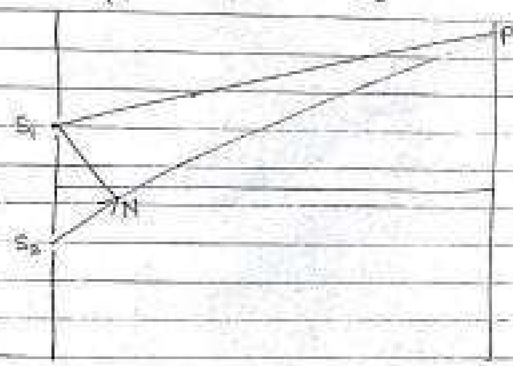
☆ **Coherent Sources :-** The two sources are said to be coherent if they emit light waves of same frequency or wave length, nearly the same amplitude and having a constant phase diff.

they are produced by:
(i) by the division of wave front
ex- Fresnel's Biprism
(ii) by the division of amplitude
ex- Newton's Rings

★ **Phase difference and Path difference :-**
The diff. b/w the optical paths of the two rays which are in constant phase diff. with each other.

The corresponding phase diff is given as

$$\delta = \frac{2\pi}{\lambda} \times \text{path diff.}$$



S_2N = Path diff.
$\Rightarrow S_2P - S_1P$

★ **Analytical treatment of interference (Conditions for interference and intensity) :-**

Consider a monochromatic source of light 'S' emitting waves of wavelength 'λ' and two narrow pin holes 'S_1' and 'S_2' equidistant from 'S' and behaving as two coherent sources.

Let a_1, a_2 are the amplitude of waves coming from S_1 and S_2 respectively and 'y_1' and 'y_2' are the displacement of the two waves.

$y_1 = a_1 \sin\omega t \qquad y_2 = a_2 \sin(\omega t + \delta)$

Hence, the resultant displacement.
$y = y_1 + y_2$
$\Rightarrow y = a_1 \sin\omega t + a_2 \sin(\omega t + \delta)$
$\Rightarrow y = a_1 \sin\omega t + a_2 \sin\omega t$ [illegible]

$\Rightarrow Y = \sin\omega t\,(a_1 + a_2 \cos\delta) + a_2 \cos\omega t \sin\delta$ — (I)

Now, let us assume

$a_1 + a_2 \cos\delta = R\cos\theta$ — (II)

$a_2 \sin\delta = R\sin\theta$ — (III)

Then,

$\Rightarrow y = R\sin\omega t\cos\theta + R\cos\omega t\sin\theta$

$\Rightarrow y = R(\sin\omega t\cos\theta + \cos\omega t\sin\theta)$

$\Rightarrow y = R\sin(\omega t + \theta)$ — (IV)

On squaring (II) & (III) and adding we get,

$\Rightarrow a_1^2 + a_2^2\cos^2\delta + 2a_1a_2\cos\delta = R^2\cos^2\theta$

& $\Rightarrow R^2\sin^2\theta = a_2^2\sin^2\delta$

$\Rightarrow R^2\cos^2\theta + R^2\sin^2\theta = a_1^2 + a_2^2(\cos^2\delta + \sin^2\delta) + 2a_1a_2\cos\delta$

$\Rightarrow R^2(\cos^2\theta + \sin^2\theta) = a_1^2 + a_2^2$

$\Rightarrow R^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\delta$

$I = a_1^2 + a_2^2 + 2a_1a_2\cos\delta$

when

$a_1 = a_2 = a$

$\Rightarrow I = a^2 + a^2 + 2a^2\cos\delta$

$\Rightarrow I = 2a^2 + 2a^2\cos\delta$

$\Rightarrow I = 2a^2(1 + \cos\delta)$

$\Rightarrow I = 2a^2 \times 2\cos^2\frac{\delta}{2}$

$\Rightarrow \boxed{I = 4a^2\cos^2\frac{\delta}{2}}$

★ Conditions for constructive and destructive interference :-

(i) Constructive Interference (Bright Fringe)

Intensity is maximum when $\cos\delta = 1$

$\therefore I_{max} = a_1^2 + a_2^2 + 2a_1a_2$

$\Rightarrow I_{max} = (a_1 + a_2)^2$

when $a_1 = a_2 = a$, then

$\Rightarrow I_{max} = 4a^2$

For phase diff.

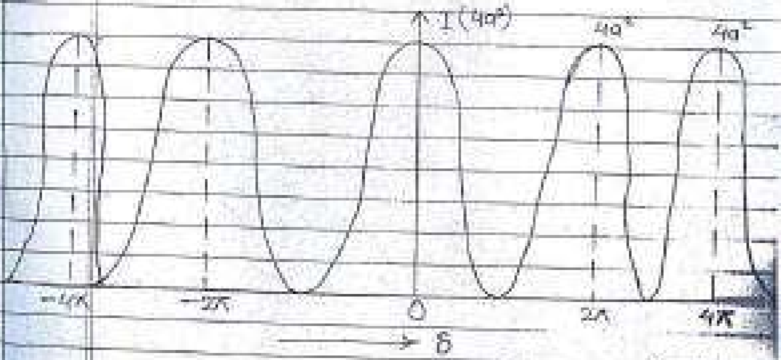
$\cos\delta = 1$

$\delta = 2n\pi$

$n = 0, 1, 2, 3, 4, \ldots$

$\delta = 0, 2\pi, 4\pi, 6\pi, \ldots$

Graph,



For path diff.,

$\therefore$ Phase diff. $= \frac{2\pi}{\lambda} \times$ (path diff.)

$$\delta = \frac{2\pi}{\lambda} \times \text{path diff.}$$

$$2n\pi = \frac{2\pi}{\lambda} \times \text{path diff}$$

$$\therefore \text{Path diff.} = n\lambda$$

For dark fringes,

$I = a_1^2 + a_2^2 + 2a_1 a_2 \cos\delta$

(II) Destructive interference / Dark Fringes:

For minima,

$\cos\delta = -1$

$I_{min} = a_1^2 + a_2^2 - 2a_1 a_2$

$I_{min} = (a_1 - a_2)^2$

When $a_1 = a_2 = a$

$\boxed{I_{min} = 0}$

For phase diff.,

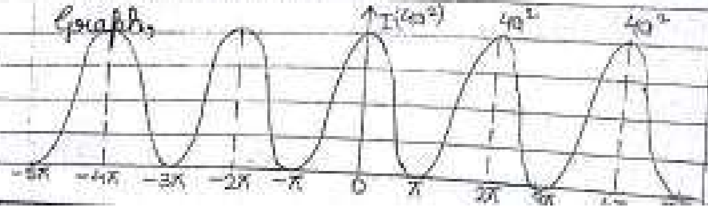
$\cos\delta = -1$

$\delta = (2n+1)\pi$

$n = 0, 1, 2, 3, \ldots$

$\lambda = \pi, 3\pi, 5\pi, 7\pi, \ldots$

Graph,



For path diff.,

$\therefore$ Phase diff. $= \frac{2\pi}{\lambda} \times$ path diff.

$$\delta = \frac{2\pi}{\lambda} \times \text{path diff}$$

$$(2n+1)\pi = \frac{2\pi}{\lambda} \times \text{path diff}$$

$$\text{path diff.} = (2n+1)\frac{\lambda}{2}$$

* Average Intensity :-

$$I_{av} = \frac{\int_0^{2\pi} I\, d\delta}{\int_0^{2\pi} d\delta}$$

$$I_{av} = \frac{\int_0^{2\pi} (a_1^2 + a_2^2 + 2a_1 a_2 \cos\delta)\, d\delta}{\int_0^{2\pi} d\delta}$$

$$I_{av} = \frac{\left[a_1^2\delta + a_2^2\delta + 2a_1 a_2 \sin\delta\right]_0^{2\pi}}{\left[\delta\right]_0^{2\pi}}$$

$$I_{av} = \frac{a_1^2\, 2\pi + a_2^2\, 2\pi}{2\pi}$$

$$I_{av} = \frac{\cancel{2\pi}(a_1^2 + a_2^2)}{\cancel{2\pi}}$$

$$I_{av} = a_1^2 + a_2^2$$

Thus, it is clear that the intensity at the bright points is $4a^2$ and at the dark points it is 0, the average intensity is $2a^2$, hence we can say that the energy is never destroyed but it is only transferred from point of minimum intensity to maximum intensity. In a complete cycle the intensity varies from 0 to $4a^2$ and average is $2a^2$. It means that whatever energy apparently disappears at minima is actually present at maxima.

Thus, there is no violation of law of conservation of energy during the phenomenon of interference.

* Fresnel's Biprism :-

It is a device to obtain two coherent sources to produce sustained interference. It is a combination of two prism of very small refracting angle placed base to base. Actually it is constructed as a single prism with one of its angle about 179° and the other two about 30' each.

* Production of Fringes :-

In narrow adjustable slit S is illuminated by a monochromatic light. the light from the source is allowed to fall symmetrically on the biprism placed at a small from S.

When light from the source falls on the lower portion of biprism, it appears to come from S_2. Similarly, the light falling on the upper portion bend downwards and appears to come from S_1.

Hence, S_1 and S_2 behave as two coherent sources

* Theory of interference fringes :-

The point is equidistant from S_1 and S_2 therefore the path difference b/w the two waves reaching at O from S_1 and S_2 is 0. Hence the point O will of maximum intensity.

Now, consider a point P at a distance x from point O.

The waves are reaching at this point from S_1 and S_2.

$\therefore PQ = x - \frac{d}{2}$, $PR = x + \frac{d}{2}$

By using ΔS_2PR,

$(S_2P)^2 = (PR)^2 + (S_2R)^2$

$(S_2P)^2 = D^2 + \left(x + \frac{d}{2}\right)^2$ —— ①

By using ΔS_1PQ,

$(S_1P)^2 = (S_1Q)^2 + (PQ)^2$

$(S_1P)^2 = D^2 + \left(x - \frac{d}{2}\right)^2$ —— ②

From ① − ② ; we get

$(S_2P)^2 - (S_1P)^2 = \cancel{D^2} + \cancel{x^2} + \frac{d^2}{4} + xd - \cancel{D^2}$

$- \cancel{x^2} - \frac{d^2}{4} + xd$

$(S_2P)^2 - (S_1P)^2 = 2xd$

$(S_2P - S_1P)(S_2P + S_1P) = 2xd$

$S_2P - S_1P = \frac{2xd}{S_2P + S_1P}$

Assume $S_2P \approx S_1P \approx D$

$\therefore S_2P - S_1P = \frac{xd}{D}$

$\therefore \boxed{\delta = \frac{2\pi}{\lambda}\left(\frac{xd}{D}\right)}$

★ Expression for fringe width :-

- For bright fringe,

$\frac{xd}{D} = n\lambda$

$x = \frac{n\lambda D}{d}$

$n = 0$, $x_0 = 0$

$n = 1$, $x_1 = \frac{\lambda D}{d}$

$n = 2$, $x_2 = \frac{2\lambda D}{d}$

$n = 3$, $x_3 = \frac{3\lambda D}{d}$

$\boxed{x_2 - x_1 = \frac{\lambda D}{d}}$

- For dark fringe,

$\frac{xd}{D} = (2n+1)\frac{\lambda}{2}$

$x = \frac{(2n+1)\lambda D}{2d}$

$n = 0$, $x_0 = \frac{\lambda D}{2d}$

$n = 1$, $x_1 = \frac{3\lambda D}{2d}$

$n = 2$, $x_2 = \frac{5\lambda D}{2d}$

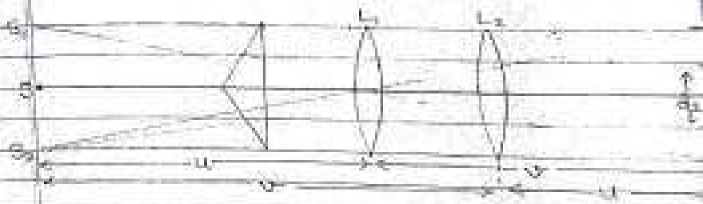
$$\therefore \boxed{x_2 - x_1 = \frac{\lambda D}{d}}$$

Thus the fringe width increase with the increase in wavelength of light distance of screen and source and by bringing two coherent sources close together.

$$\boxed{\bar{X} = \frac{\lambda D}{d}}$$

★ **Determination of the distance b/w two virtual sources (Displacement method) :-**

For this purpose we can use the displacement method. In this method the distance b/w the slit and the eye piece is kept more then four times the focal length of a convex lens so that the two positions of the lens to form the images of S_1 and S_2 are founded.



Now by applying the magnification formula,

$$\frac{d_1}{d} = \frac{v}{u} \quad ---- \text{(1)}$$

$$\frac{d_2}{d} = \frac{u}{v} \quad ---- \text{(2)}$$

From (1) × (2)

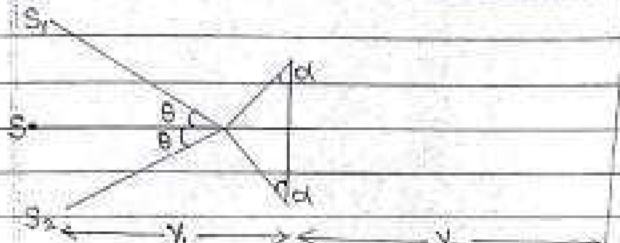
$$\frac{d_1 d_2}{d^2} = 1$$

$$d^2 = d_1 d_2$$

$$\boxed{d = \sqrt{d_1 d_2}}$$

★ **Fringes with white light by using biprism :-**

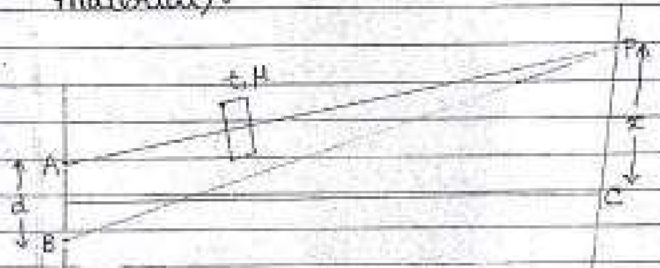
If the source of monochromatic is s[illegible] by a source of white light then th[illegible] interference pattern consist of a fe[illegible] coloured fringes with a central band.



$$\theta = (\mu - 1)d$$
$$d = 2(\mu - 1)\alpha Y_1$$

where, μ = refractive index

★ Displacement of fringes (thickness of a thin sheet of a transparent material):-



Suppose A and B are two virtual coherent sources. The point C is equidistant from A and B. When a transparent plate of thickness 't' and refractive index 'μ' is introduced in path of one of the beams the fringes which were originally at [illegible] to point P. The time taken by [illegible] from D to P in air is same as time taken by the wave from A [illegible] partly through air & partly through the plate.

Suppose C_0 is the velocity of light [in] air and C is the velocity of light in medium.

Time taken by wave $= \frac{BP}{C_0} = \frac{(AP-t)}{C_0} + \frac{t}{C}$

× by C_0 both sides

$$\boxed{\frac{C_0}{C} = \mu}$$

$$BP = (AP - t) + t\mu$$
$$BP - AP = (\mu - 1)t$$

If point P was originally occupied by the nth bright fringe then the path difference be,

$$(\mu - 1)t = n\lambda$$

from $x = \frac{n\lambda D}{d}$

$$n\lambda = \frac{xd}{D}$$

$$\boxed{(\mu - 1)t = \frac{xd}{D}}$$

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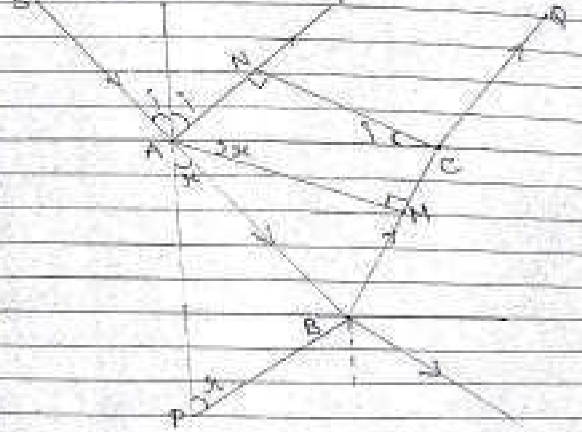
Interference in thin films :-

Due to reflected light -



Consider a transparent film of thickness 't' and refractive index 'μ'. A ray of light SA incident on the upper surface of the film is partly reflected along AT and partly refracted along AB. At the point B a part of it is reflected along BC and finally along CQ. The path diff. b/w the two rays AT & CQ can be calculated.

Draw CN normal to AT and AM ⊥ to BC. Also produce CB to meet AP.

The optical path diff..

$x = \mu(AB + BC) + CQ - (AN + NT)$

$x = \mu(AB + BC) - AN$ —— ① $\{\because CQ = NT\}$

Now, by using Snell's law,

$\mu = \dfrac{\sin i}{\sin r}$

$\mu = \dfrac{AN/AC}{CM/AC}$

$\mu = \dfrac{AN}{CM}$

$AN = \mu CM$ —— ②

From ①,

$x = \mu(AB + BC) - \mu CM$ {From ②}

$x = \mu(AB + BC - CM)$ —— ③

$x = \mu(PC - CM)$

$x = \mu PM$ —— ④

$AB = PB$

$\therefore PB + BC = PC$

$x = \mu(PB + BC - CM)$

$x = \mu(PC - CM)$

In △ APM,

$\cos r = \dfrac{PM}{AP} = \dfrac{PM}{2t}$

$PM = 2t \cos r$

From ④

$\boxed{x = 2\mu t \cos r}$