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A heuristic algorithm for a single vehicle static bike sharing rebalancing problem

Fábio Cruz[a,*], Anand Subramanian[a], Bruno P. Bruck[b], Manuel Iori[b]

[a] *Centro de Informática, Universidade Federal da Paraíba, CEP 58059-900, João Pessoa, Brazil*
[b] *DISMI, University of Modena and Reggio Emilia, 42122, Reggio Emilia, Italy*

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ABSTRACT

The static bike rebalancing problem (SBRP) concerns the task of repositioning bikes among stations in self-service bike-sharing systems. This problem can be seen as a variant of the one-commodity pickup and delivery vehicle routing problem, where multiple visits are allowed to be performed at each station, i.e., the demand of a station is allowed to be split. Moreover, a vehicle may temporarily drop its load at a station, leaving it in excess or, alternatively, collect more bikes from a station (even all of them), thus leaving it in default. Both cases require further visits in order to meet the actual demands of such station. This paper deals with a particular case of the SBRP, in which only a single vehicle is available and the objective is to find a least-cost route that meets the demand of all stations and does not violate the minimum (zero) and maximum (vehicle capacity) load limits along the tour. Therefore, the number of bikes to be collected or delivered at each station must be appropriately determined in order to respect such constraints. We propose an iterated local search (ILS) based heuristic to solve the problem. The ILS algorithm was tested on 980 benchmark instances from the literature and the results obtained are competitive when compared to other existing methods. Moreover, our heuristic was capable of finding most of the known optimal solutions and also of improving the results on a number of open instances.

1. Introduction

The task of repositioning a commodity from one location to another is a well-known problem arising in different contexts such as logistics, transportation, and various disciplines, notably industrial engineering and operations management. A practical application arises in self-service bike sharing systems (BSS), which are becoming increasingly popular in recent years. Users rent bikes and return them at stations distributed over a region. In such systems, each station has an inventory with a load capacity, an initial number of bikes, and consequently a number of free slots where users can return bikes to the system. Throughout the day, some stations may have no bike to be rented or free slots to store returned bikes. Therefore, an attempt to avoid this scenario, which is unpleasant for users, is to determine an initial acceptable number of bikes (and free slots) at each station. This task can be done based on demand history and peaks at each station [1,4,34,43]. A vehicle with limited load capacity then periodically collects and delivers bikes across different stations so as to rebalance the system.

Alternatives to the street traffic are important not only because of its impact in urban congestion, but also in the environment, commuting, and so on. The emerging worldwide BSS are proving to be an effective solution to mitigate the effects of traffic issues in large urban centers Detailed information on several bike sharing systems worldwide can be found on the interactive bike sharing world map available at http://bikesharingmap.com. By August of 2016, there were approximately 1, 392,170 bikes and pedelecs (bikes assisted by a small electric motor) being used worldwide. According to the website, in 2015 there were at least 1005 cities with an operating BSS and 324 cities with programs under planning or construction. One of the most famous systems is the *Vélib*' in Paris, with 1800 stations and more than 20,000 bikes.

The activity of repositioning bikes among stations on a regular basis is called rebalancing, and this is done by one or more vehicles that move bikes from one station to another in order to restore its inventory to the initial desired configuration. As per DeMaio [10], good rebalancing systems are present in successful bike sharing programs, and since the vehicles move back and forth across an urban area, a vehicle routing optimization can be utilized.

The rebalancing is either static, performed when nearly no bikes are being used, or dynamic, which is done while the system is still in use. The static bike rebalancing problem (SBRP) is motivated by the fact that very few bikes are being used at night.

* Corresponding author.
E-mail addresses: fabiocba@di.ufpb.br (F. Cruz), anand@ci.ufpb.br (A. Subramanian), bruno.p.bruck@gmail.com (B.P. Bruck), manuel.iori@unimore.it (M. Iori).

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In this work we consider the single vehicle SBRP, which is clearly $\mathcal{NP}$-hard, because it includes, among others, the classical traveling salesman problem (TSP) as a special case. The SBRP can be seen as a variant of the one-commodity pickup and delivery TSP [20,21], with the difference that multiple visits are allowed to be performed at each station, i.e., the demand of a station is allowed to be split. Moreover, a vehicle may arbitrarily drop its load at a station, leaving it in excess or, alternatively, collect more bikes from a station (even all of them), thus leaving it in default. Both cases require further visits in order to meet the actual demands of such station. This strategy of allowing a station to act as a buffer or a temporary depot is denoted as *temporary operation* (i.e., temporary pickup and temporary dropoff). Finally, visits to balanced stations are optional for the SBRP. Salazar-González and Santos-Hernández [36] considered a similar yet different problem, in which an upper limit is imposed on the number of visits to the customers and to the depot, and the single vehicle that performs the rebalancing is not forced to leave the depot with an empty load.

An increasing number of works regarding bike sharing systems and related issues, such as the balancing of their stations, has been published over the last years. Exact approaches for multiple vehicle SBRPs were suggested by Dell'Amico et al. [8], Di Gaspero et al. [11–13], Kloimüllner et al. [24], Raviv et al. [35]. Moreover, Alvarez-Valdes et al. [1], Dell'Amico et al. [9], Forma et al. [16], Papazek et al. [28,29], Raidl et al. [31], Rainer-Harbach et al. [32,33] also addressed different types of multiple vehicle SBRPs, but with heuristics.

Several exact [2,6,14,15,36] and heuristic [22,27] algorithms were proposed for single vehicle SBRPs. Furthermore, in contrast to static rebalancing, there are relatively few works related to dynamic rebalancing [4,5,7,23,37].

The works of Chemla et al. [6] and Erdoğan et al. [14] were the only ones to consider the same variant dealt in the present paper. Chemla et al. [6] proposed a mathematical formulation over an extended graph, where each station is replicated according to an upper bound on the number of visits. Due to its visible intractability, two relaxations were developed. The authors also presented among other contributions, a polynomial algorithm to compute optimal bike displacements for a given sequence of vertices, which is useful to determine if a route is feasible or not, as well as tabu search heuristics and a branch-and-cut algorithm that solves a relaxation of the problem. Erdoğan et al. [14] proposed the first exact method for the problem, which consists of a branch-and-cut algorithm that makes use of no-good cuts (also known as Benders combinatorial cuts), and they reported optimal solutions for instances with up to 60 stations.

Despite the advances on the development of efficient exact approaches for SBRPs, heuristic methods still appear to be more suitable for dealing with medium and large size instances of this challenging class of problems. This work proposes a hybrid iterated local search (ILS) based heuristic for the single vehicle SBRP considered in Chemla et al. [6] and Erdoğan et al. [14]. Hybridized ILS algorithms, especially when combined with randomized variable neighborhood descent (RVND), revealed to be very effective when solving a large variety of vehicle routing problems [9,30,39,40,44], including those involving only a single vehicle [3,41].

The algorithm that was developed combines successful ingredients from previous works with some problem-specific procedures suggested in Chemla et al. [6] to improve the solution quality, as well as to check if a solution is infeasible. We also implemented several perturbation mechanisms and the impact of each possible combination on the solution quality and CPU time are measured by extensive computational experiments on a subset of challenging test-problems. The results obtained on 980 benchmark instances from the literature show that our algorithm is competitive when compared to other methods, and a number of new best known solutions is reported. We also conduct an analysis on how the performance of the algorithms in terms of solution quality and CPU time varies according to the number of stations and the vehicle capacity.

The remainder of the paper is organized as follows. Section 2 presents a formal problem definition. Section 3 describes the proposed heuristic algorithm. Section 4 reports and discusses the computational results, and Section 5 contains the concluding remarks.

2. Problem description

Let n be the number of stations, $V = \{1, \ldots, n\}$ be the set of vertices representing their locations (station 0 represents the depot), and A be the set of arcs in a complete and directed graph $G = (V \cup \{0\}, A)$. For each arc $a_{(i,j)} \in A$, there is a cost c_a, satisfying the triangular inequality ($c_{(i,j)} + c_{(j,k)} \geq c_{(i,k)}, \forall\, i, j, k \in V$).

For each $i \in V$, let $p_i \in \mathbb{Z}$ be the amount of bikes initially stored, $p'_i \in \mathbb{Z}$ be the number of bikes requested by i after the service is performed, and $d_i = p'_i - p_i$ be the total demand. When $d_i > 0$ and $d_i < 0$, we assume that $i \in V$ is a delivery and a pickup station, respectively. A station $i \in V$ may have no demand ($p_i = p'_i$) and in this case the visit becomes optional. Each station $i \in V$ has a capacity $q_i \in \mathbb{Z}$ and the depot is assumed to have no bikes, i.e., $q_0 = p_0 = p'_0 = 0$. Finally, let $Q \in \mathbb{Z}$ be the vehicle capacity.

The objective is to find a least-cost route that starts and ends at the depot, visits each station with non-zero demand at least once, meets the demands of all stations (i.e., the initial load p_i is changed to the target demand p'_i, $\forall\, i \in V$), and does not violate the minimum (zero) and maximum (Q) load limits. Therefore, the number of bikes to be collected or delivered at each visit to a station should be appropriately determined in order to respect such constraints.

Finally, stations may serve to perform temporary operations, either as a temporary depot or a temporary buffer, i.e., supply more bikes than their initial demand or hold more bikes (without exceeding its inventory load capacity), and in both cases have their demand satisfied in later visits.

Fig. 1 shows a graphical representation of an optimal solution for the benchmark instance n20q10D (n=20 and Q=10). The nodes are distributed according to the spatial coordinates of the stations. The positive and negative values next to the nodes are the number of bikes collected and delivered, respectively. The arcs and their associated values represent the vehicle traveling to the next station in the sequence and the vehicle load, respectively. For example, the vehicle delivers 2 bikes in the first visit to station 12, collects 10 at station 10, returns to 12 to deliver 6 more (meeting the demand of 8) and then travels to station 14 with a load of 4 bikes.

3. Proposed heuristic

ILS iteratively alternates between local search (intensification) and perturbation (diversification) mechanisms with a view of finding high quality solutions. In our case, we embed a variable neighborhood descent (VND) [26] based procedure in the local search phase of the metaheuristic. As in previous works (e.g., [30,39]), the neighborhoods of our algorithm are examined in a random manner during the search (RVND).

As opposed to most of the former ILS implementations cited in Section 1, infeasible solutions are temporary accepted after the application of perturbation moves, not only for the sake of diversification, but also as an attempt to escape from local optimal solutions. This modification, sometimes referred to as *strategic oscillation* (see, e.g., [17–19]), was crucial for the favorable performance of the heuristic when dealing with the single vehicle SBRP considered here, which appears to be more challenging to solve than other VRPs where ILS was successfully applied to obtain high quality solutions by only considering the feasible search space.

The proposed hybrid heuristic, called ILS_{SBRP}, combines multiple restarts, local search, perturbations mechanisms, and a repair phase. Fig. 2 illustrates the flowchart of ILS_{SBRP}. For each of the I_R restarts, a feasible initial solution is generated using a simple greedy randomized

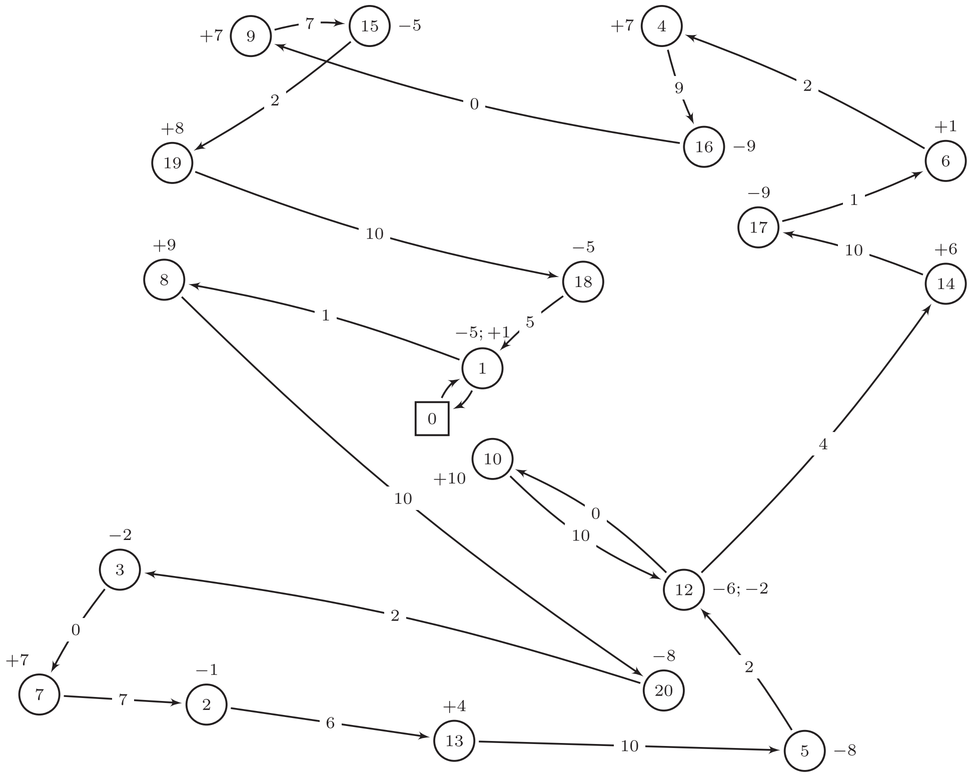


Fig. 1. Representation of optimal solution with value 5989 for instance n20q10D.

constructive algorithm (see Section 3.3). Next, local search, perturbation and repair procedures are successively applied until the stopping criterion is met, that is, when the number of consecutive attempts to escape from a local optimal solution reaches I_{ILS} trials. Because perturbation moves are allowed to produce infeasible solutions, we implemented a procedure called AddUnbalancedVertex (see [6] and Section 3.2 below for details), which includes additional visits to stations whose demands are not exactly met, with the aim of repairing such solutions. Nevertheless, there is no guarantee that a solution will be feasible after applying this procedure. When an infeasible solution is not totally repaired and the local search does not find a move that leads to a feasible solution, then the infeasible solution is disregarded and the perturbation procedure is called. Note that perturbation is always applied over the best solution of the current multi-start iteration. Finally, ILS_{SBRP} returns the best solution found among all restarts.

3.1. Solution representation

A solution for the single vehicle SBRP considered in the present work can be represented as a sequence of visits to stations, starting and ending at the depot, along with the amount of bikes collected or delivered at each visit.

Three vectors are used as data structures to store: (i) the route, where the first and last element are fixed at 0, i.e., the depot; (ii) the operation performed by the vehicle during a visit, where negative and positive values indicate the amount of bikes delivered and collected, respectively; and (iii) the vehicle load during the route.

As in Chemla et al. [6], a flow network is used to check in polynomial time whether or not a solution is feasible, with respect to bike displacements and vehicle capacity, given a sequence of vertices representing visits to stations. A detailed explanation can be found in Appendix A.

We also use another data structure which consists of a key-value map composed by n+1 elements that store the number of visits performed at each station. This is useful, for example, to check whether a solution includes all stations with non-zero demand. Note that information held in (ii) is extracted from the computed bike displacements when solving the max-flow problem (see Appendix A). From such, it is possible to derive, in linear time, the vehicle loads in (iii) by the adding or subtracting the bike displacements at each visit.

3.2. Repairing infeasible solutions

As already mentioned, infeasible solutions are allowed after perturbations. We therefore re-implemented the procedure called AddUnbalancedVertex in [6], which tries to repair a solution by adding stations to the route. More precisely, both the most unbalanced station in excess and in default, i and j, respectively, are selected and three moves are possible: (i) adding arcs (j,i) and (i,j) after the existing visit to j; (ii) adding arcs (i,j) and (j,i) after the existing visit to i; (iii) if both i and j are not in the sequence, adding (i,j) at the end of the sequence, before returning to the depot.

For example, let us consider a scenario where stations i=12 and j=14 are the most unbalanced. More precisely i has initially 20 bikes and a demand of −10, i.e., a pickup station, while j is initially holding 3 out of 10 (target) bikes, i.e., a delivery station with demand 7. An infeasible solution is presented in Fig. 3a, where the referred stations are not balanced, that is, their demands are not met, since only 4 bikes were collected in station 12 and 4 bikes were delivered at station 14. Fig. 3b shows a modified solution, where after the addition of arcs (14, 12) and (12, 14), a new and feasible configuration of bike displacements were determined by means of the maximum flow check (see Appendix A). We can observe that the second visit complements the first one, meeting the demand of both stations: the vehicle deliveries 1 bike at station 14, collects the remaining 7 at station 12, now balanced, and finally meets the demand of station 14 by delivering 6 more bikes.

It is worth emphasizing that the AddUnbalancedVertex procedure does not necessary lead to a feasible solution. However, in general, experimental results showed that such procedure has a high level of success in fully repairing infeasible solutions.

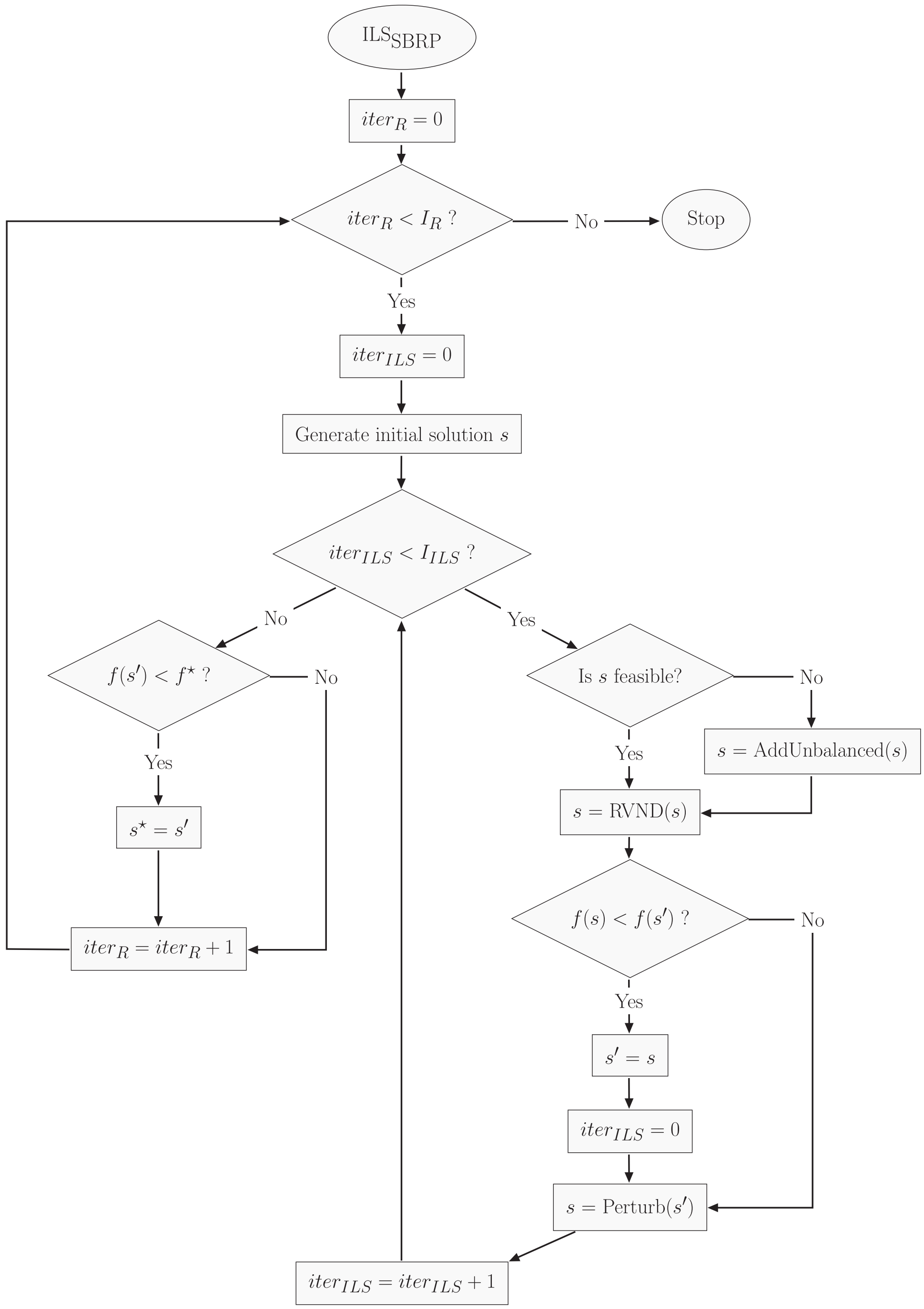


Fig. 2. $\mathrm{ILS}_{\mathrm{SBRP}}$ flowchart.

3.3. Constructive procedure

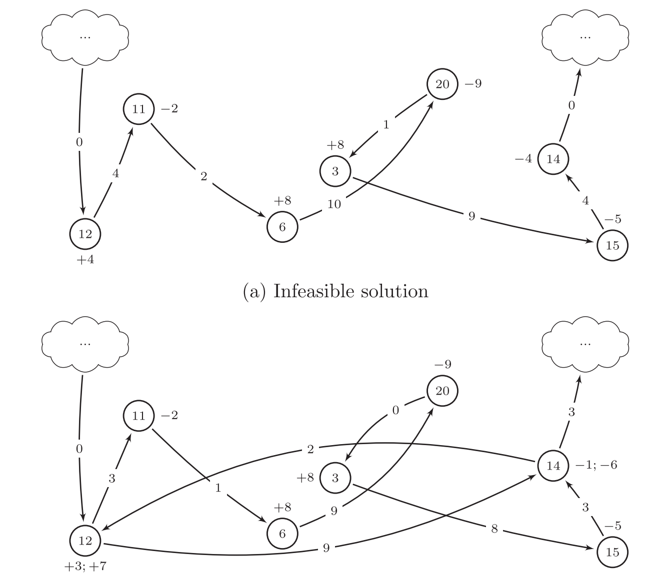
The pseudocode of the greedy randomized constructive procedure is presented in Algorithm 1. The algorithm stores and maintains a list of open vertices (*OV*) corresponding to stations whose demands are still not fully met. Stations without demand are also included in this list. In order to ensure a level of diversity during the process of generating an initial solution, *OV* is randomly shuffled (line 4).

The algorithm follows a greedy procedure by selecting the first vertex to be inserted at the end of the route (before the depot) whose demand is completely met by a single visit without violating the load limits ([0, Q]). Next, the vehicle load is updated and the station that was inserted into the partial solution is removed from *OV* (lines 8–12).

However, it may come to a point where no station can be fully served in a single visit, either because the vehicle has not enough bikes to deliver, or the residual capacity is not sufficient to collect the required bikes at once. Hence, a split becomes necessary. The second part of the constructive procedure (lines 13–17) iterates over *OV* searching for a station whose demand maximizes the number of bikes that can be delivered or collected. Ties are broken according to the nearest insertion criterion. The station demand and vehicle load are updated after the insertion. Next, the algorithm restarts from line 5 and the entire insertion procedure is repeated until *OV* becomes empty. Note that the generated initial solution is always feasible.

Algorithm 1. Initial solution constructive procedure.

1: Procedure GenerateInitialSolution
2: $Q' \leftarrow Q$



(a) Infeasible solution

(b) Feasible solution after additional visits to unbalanced stations 12 and 14

Fig. 3. Handling an infeasible solution by considering additional visits to unbalanced stations.

```
    Solution ← ∅
    OV ← List randomly shuffled with stations where d_i ≠ 0+random ones with d_i=0
    repeat
      inserted ← false
      for all i ∈ OV do
        if d_i ≤ Q′ or Q − Q′ ≥ d_i then
          Solution ← Solution ∪ i
          Update vehicle capacity and remove i from OV
          inserted ← true
          break
      if not inserted then
        for all j ∈ OV do
          compute exchange_j
        i ← max{exchange_j | j ∈ exchange}
        Solution ← Solution ∪ i
      update OV
      update Q′
    until OV ≠ ∅
    return Solution
    end GenerateInitialSolution.
```

3.4. Local search

Initial and perturbed solutions are possibly improved by means of an RVND based procedure during the local search. RVND consists of systematically examining different types of neighborhoods in a random manner. In particular, if the best neighbor consists of an improving move, then the search may continue from any of the existing neighborhoods (including the last one that has been explored) at random. Otherwise, a different neighborhood other than those that did not succeed in finding an improving move is randomly selected. The procedure ends when all neighborhoods fail to refine the current solution. Only feasible moves are accepted.

The following six neighborhood structures were implemented.

- Reinsertion — $N^{(1)}$: A station is removed and then reinserted in another position of the sequence.
- Or-opt2 — $N^{(2)}$: Two consecutive stations are removed and then inserted in another position.
- Or-opt3 — $N^{(3)}$: Three consecutive stations are removed and then inserted in another position.
- 2-opt — $N^{(4)}$: Two non-adjacent arcs are removed from the sequence and then two new ones are inserted. In other words, a subsequence of the tour is reversed.
- Swap — $N^{(5)}$: Permutation of two stations.
- Suppression — $N^{(6)}$: Given a sequence $L = i_0, i_1, \ldots, i_k$, a suppression list is composed of visits to stations i_j, $\forall j \in \{1, \ldots, k-1\}$, such that $p'_{i_j} = p_{i_j}$ (zero demand) or $p'_{i_j} \neq p_{i_j}$ and i_j is visited more than once in the tour. The best move, if any, consists in selecting one station to be removed from L so that the solution cost is minimized and the resulting new sequence L' is feasible. For example, Fig. 4b shows the removal of an additional visit to station 2, thus modifying the subsequence 2, 6, 2, 9, 0 to 2, 6, 9, 0. This neighborhood was originally proposed by Chemla et al. [6], but the authors considered all stations.

The first five are well-known TSP neighborhood structures, while the last is a problem-specific neighborhood. Fig. 4a depicts an initial solution and Fig. 4b–g illustrate modified solutions that were obtained after changing the previous one by means of one of the neighborhoods described above. For example, Fig. 4d shows a solution in which a 2-opt move was applied over the solution shown in Fig. 4c. For ease of presentation, values of pickup/delivery operations as well as the vehicle load are omitted.

3.5. Perturbation mechanisms

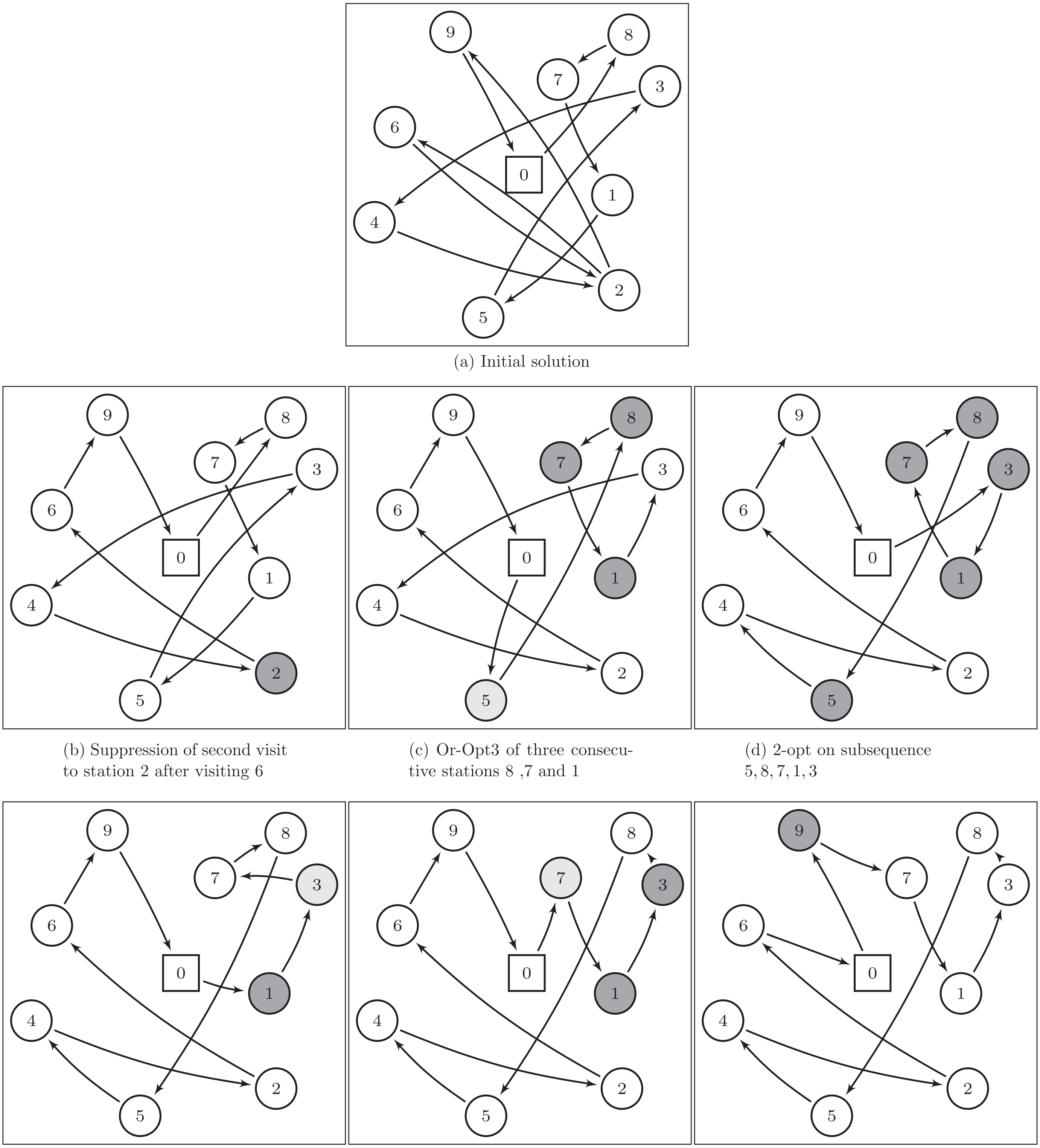
One of the four mechanisms described below is selected at random whenever the algorithm enters the perturbation phase.

- AddBuffer — $P^{(1)}$: An additional visit to a station is included, expecting to act as buffer, using the cheapest insertion criterion. Unrouted stations are inserted twice using the same criterion [6].
- AddStations — $P^{(2)}$: This perturbation mechanism generalizes the previous one in the sense of allowing multiple visits to be added in the solution, but with a different insertion criterion. More precisely, an additional visit (or two, in the case of unrouted stations) to up to three random stations are included towards the end of the route. Here we only consider stations that are visited at most once. Adjacent visits to the same station are forbidden.
- Double-Bridge — $P^{(3)}$: Introduced by Martin et al. [25] for the TSP, this perturbation consists of a permutation of two subsequences. As a result, four arcs are removed and four new ones are added so as to generate a new sequence.
- Suppression — $P^{(4)}$: A suppression move (see Section 3.4) is applied at random, but in this case the resulting modified sequence is allowed to be infeasible.

Fig. 5 shows an example of perturbations applied over a (supposedly) local optimal solution (Fig. 5a). Fig. 5b shows the AddBuffer perturbation, when an additional visit to station 7 is performed expecting it to act as a buffer. In Fig. 5c, the perturbation AddStations is applied by adding two random visits: one to station 8 and another one to station 6. In Fig. 5d and a Double-Bridge move is applied by interchanging subsequence 6,4,1 with subsequence 7,9.

4. Computational experiments

The $\mathrm{ILS}_{\mathrm{SBRP}}$ algorithm was coded in C++(g++4.6.4) and the computational tests were carried on an Intel®Core® i7-3770 with 3.40 GHz and 16 GB of RAM running Ubuntu 14.04. Only a a single thread was used during the experiments.



(a) Initial solution

(b) Suppression of second visit to station 2 after visiting 6

(c) Or-Opt3 of three consecutive stations 8 ,7 and 1

(d) 2-opt on subsequence 5, 8, 7, 1, 3

(e) Swap between station 1 and station 3

(f) Or-Opt2 of two consecutive stations 1 and 3

(g) Reinsertion of station 9 before station 7

Fig. 4. Example regarding the application of neighborhood structures.

4.1. Instances

The benchmark instances used to test the proposed algorithm are those suggested by Hernández-Pérez and Salazar-González [20], which were originally created for the one-commodity pickup and delivery traveling salesman problem. The benchmark contains instances ranging from 20 to 500 customers (stations), and vehicle capacities ranging from 10 to 1000. For each pair of problem size and vehicle capacity, there are 10 instances named from A to J and, for each vertex i, there is a demand $d_i \in [-10, 10]$. Chemla et al. [6] and Erdoğan et al. [14] only reported results for a subset of instances of the referred benchmark. Therefore, in order to compare our results with theirs, we

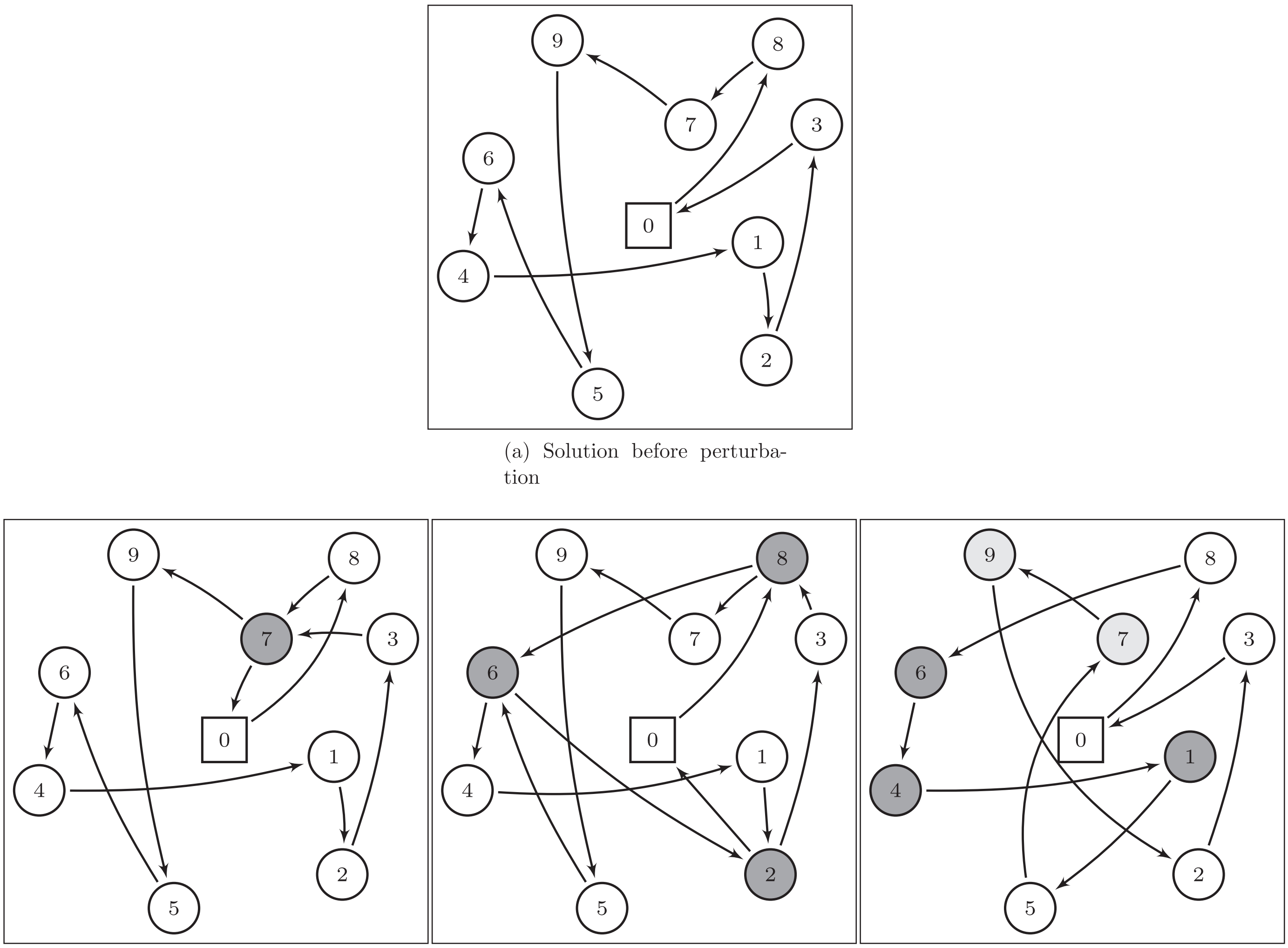


Fig. 5. Example regarding the application of perturbation mechanisms.

Table 1
Impact of different combinations of the perturbation mechanisms.

Perturbations used	Avg. gap (%)	Time (s)
$P^{(1)}$	2.43	244.40
$P^{(2)}$	1.08	773.49
$P^{(3)}$	1.04	498.24
$P^{(4)}$	1.56	249.88
$P^{(1)} + P^{(2)}$	1.27	506.45
$P^{(1)} + P^{(2)} + P^{(3)}$	1.05	514.41
$P^{(1)} + P^{(2)} + P^{(3)} + P^{(4)}$	1.11	431.43
$P^{(1)} + P^{(2)} + P^{(4)}$	1.25	417.30
$P^{(1)} + P^{(3)}$	1.21	371.44
$P^{(1)} + P^{(3)} + P^{(4)}$	1.30	331.70
$P^{(1)} + P^{(4)}$	1.78	248.69
$P^{(2)} + P^{(3)}$	0.91	630.29
$P^{(2)} + P^{(3)} + P^{(4)}$	0.99	505.70
$P^{(2)} + P^{(4)}$	1.08	503.75
$P^{(3)} + P^{(4)}$	1.29	341.32

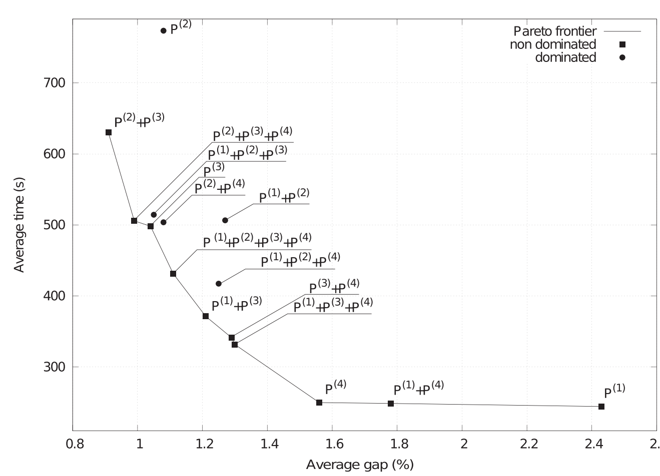


Fig. 6. Pareto efficient frontier.

tested ILS_{SBRP} for all instances considered in at least one of the two works (see Section 4.4). Furthermore, to compute the initial and final targets as well as the load capacity for each station, the same procedures adopted by such authors were employed: for each vertex i, $p_i = \alpha \times 10$, $p_i = \alpha \times (10 + d_i)$, $q_i = \alpha \times 20$, where α is an input parameter, and experiments were conducted with $\alpha = 1$ and $\alpha = 3$. In order to properly compare our results with those in Chemla et al. [6] and Erdoğan et al. [14], we adopted their same convention of rounding down the values of the cost matrix to the nearest integer (floor), although we noticed that this can cause slight violations of the triangle

Table 2
Aggregate average results per instance group for $n \in \{20, 30, 40, 50, 60\}$ and $\alpha = 1$.

Instance group	Erdoğan et al. [14]		Chemla et al. [6]				ILS_{SBRP}				
	Gap (%)	Time (s)	UB1 Gap (%)	UB1 Time (s)	UB2 Gap (%)	UB2 Time (s)	Avg. Gap (%)	Best Gap (%)	Avg. Time (s)	Avg. TT_{UB2}	Avg. NV
n20q10	0.00	0.35	2.57	2.75	0.00	3.53	0.06	0.00	5.84	0.38	20.10
n20q15	0.00	0.30	–	–	–	–	0.00	0.00	2.13	–	19.19
n20q20	0.00	0.13	–	–	–	–	0.00	0.00	0.94	–	18.65
n20q25	0.00	0.15	–	–	–	–	0.00	0.00	0.63	–	18.62
n20q30	0.00	1.96	2.47	2.22	0.39	2.38	0.00	0.00	0.62	0.01	18.66
n20q35	0.00	1.12	–	–	–	–	0.00	0.00	0.46	–	18.70
n20q40	0.00	1.22	–	–	–	–	0.00	0.00	0.46	–	18.65
n20q45	0.00	1.13	1.52	2.71	0.01	2.83	0.00	0.00	0.45	0.01	18.62
n20q1000	0.00	0.83	2.43	2.92	0.00	3.04	0.00	0.00	0.45	0.01	18.62
Avg.	0.00	0.80	2.25	2.65	0.10	2.95	0.01	0.00	1.33	0.10	18.87
n30q10	0.00	6.22	–	–	–	–	0.02	0.00	28.08	–	30.14
n30q15	0.00	3.87	–	–	–	–	0.12	0.00	10.29	–	28.49
n30q20	0.00	163.59	–	–	–	–	0.02	0.02	5.06	–	27.84
n30q25	0.00	5.61	–	–	–	–	0.00	0.00	2.36	–	27.53
n30q30	0.00	82.20	–	–	–	–	0.00	0.00	1.85	–	27.53
n30q35	0.00	293.27	–	–	–	–	0.02	0.00	1.89	–	27.70
n30q40	0.00	584.62	–	–	–	–	0.00	0.00	1.61	–	27.62
n30q45	0.00	221.69	–	–	–	–	0.01	0.00	1.47	–	27.62
n30q1000	0.00	190.20	–	–	–	–	0.00	0.00	1.42	–	27.64
Avg.	0.00	172.36	–	–	–	–	0.02	0.00	6.00	–	28.01
n40q10	0.00	124.80	3.56	151.92	0.09	1752.75	0.09	0.04	56.89	14.02	39.76
n40q15	0.00	25.55	–	–	–	–	0.01	0.00	21.78	–	37.53
n40q20	0.00	14.72	–	–	–	–	0.01	0.00	9.33	–	37.01
n40q25	0.03	723.88	–	–	–	–	0.04	0.03	5.79	–	36.83
n40q30	0.00	36.56	4.05	92.72	0.00	93.87	0.00	0.00	4.43	0.06	36.79
n40q35	0.00	38.66	–	–	–	–	0.00	0.00	3.36	–	36.77
n40q40	0.00	70.65	–	–	–	–	0.00	0.00	3.13	–	36.79
n40q45	0.00	74.28	4.69	93.63	0.61	94.82	0.00	0.00	2.99	0.04	36.83
n40q1000	0.00	70.17	5.07	112.48	0.77	114.54	0.00	0.00	2.95	0.03	36.82
Avg.	0.00	131.03	4.34	112.69	0.37	514.00	0.02	0.01	12.29	3.54	37.24
n50q10	0.79	1198.48	–	–	–	–	0.30	0.23	210.98	–	49.64
n50q15	0.43	1970.12	–	–	–	–	0.29	0.23	75.07	–	46.71
n50q20	0.00	295.45	–	–	–	–	0.05	0.00	35.85	–	46.11
n50q25	0.00	272.82	–	–	–	–	0.00	0.00	25.52	–	45.71
n50q30	0.00	177.40	–	–	–	–	0.00	0.00	13.18	–	45.47
n50q35	0.24	1461.09	–	–	–	–	0.16	0.16	10.61	–	45.63
n50q40	0.01	1408.94	–	–	–	–	0.01	0.01	8.83	–	45.48
n50q45	0.00	1221.33	–	–	–	–	0.00	0.00	8.35	–	45.41
n50q1000	0.11	1909.76	–	–	–	–	0.09	0.09	6.70	–	45.44
Avg.	0.18	1101.71	–	–	–	–	0.10	0.08	43.90	–	46.18
n60q10	1.24	3924.62	13.62	412.18	2.57	4533.96	0.67	0.57	419.95	35.15	60.21
n60q15	0.51	1957.50	–	–	–	–	0.30	0.27	140.29	–	56.48
n60q20	0.00	1285.03	–	–	–	–	0.00	0.00	72.99	–	55.65
n60q25	0.07	943.42	–	–	–	–	0.07	0.07	44.32	–	55.13
n60q30	0.13	1252.65	7.17	416.50	0.57	911.58	0.08	0.07	29.63	2.46	55.22
n60q35	0.13	1096.98	–	–	–	–	0.06	0.05	22.52	–	55.01
n60q40	0.22	2607.91	–	–	–	–	0.19	0.19	21.10	–	55.33
n60q45	0.15	2795.20	7.11	410.29	1.78	427.01	0.16	0.15	19.85	0.43	55.42
n60q1000	0.18	2816.42	7.14	413.87	1.76	515.42	0.18	0.18	16.28	0.49	55.31
Avg.	0.29	2075.52	8.76	413.21	1.67	1596.99	0.19	0.17	87.44	9.63	55.97

inequality. As a consequence of such violations, some stations might be visited an additional time with no pickup nor delivery services being performed, serving only as a shortcut to arrive to another station.

4.2. Impact of the perturbation mechanisms

In this section we are interested in evaluating the impact of the perturbation mechanisms described in Section 3.5, that is, AddBuffer ($P^{(1)}$), AddStations ($P^{(2)}$), Double-Bridge ($P^{(3)}$), and Suppression ($P^{(4)}$). In view of this, we selected a subset of 30 challenging instances for performing the experiments. These instances were chosen according to the largest gap values between the upper bounds obtained by our method on preliminary experiments and the lower bounds reported in Erdoğan et al. [14]. We ran ILS_{SBRP} 10 times for each of the 30 instances considering all possible combinations of perturbations. For this testing we arbitrarily adopted $I_{\mathcal{R}}$=10 and $I_{ILS} = n$.

Table 1 shows the impact of each combination over the average gaps (computed as $(UB - LB)/LB$), where UB is the solution found by ILS_{SBRP} and LB is the lower bound computed by Erdoğan et al. [14]) and CPU times required by ILS_{SBRP} to run to completion. From the results presented in such table, we were capable of deriving the Pareto efficient frontier from each combination that is not dominated by any other in neither solution quality nor computational time, as shown in Fig. 6. We can see that the combination $P^{(2)} + P^{(3)} + P^{(4)}$ belongs to the frontier and has a good compromise between solution quality and CPU time. Therefore, we herein decided to adopt this configuration for the perturbation mechanisms.

4.3. Parameter tuning

The main ILS_{SBRP} parameters to be calibrated are the number of restarts ($I_{\mathcal{R}}$) and the maximum number of consecutive ILS iterations

Table 3
Aggregate average results per instance group for $n \in \{20, 30, 40, 50, 60\}$ and $\alpha = 3$.

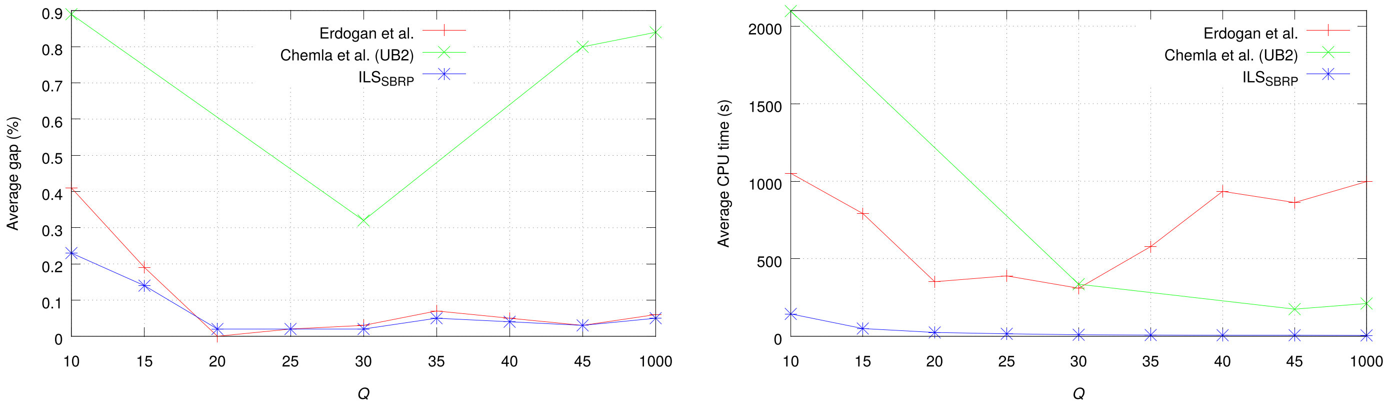
Instance group	Erdoğan et al. (2015)		Chemla et al. [6]				ILS$_{SBRP}$				
	Gap (%)	Time (s)	UB1 Gap (%)	UB1 Time (s)	UB2 Gap (%)	UB2 Time (s)	Avg. Gap (%)	Best Gap (%)	Avg. Time (s)	Avg. TT$_{UB2}$	Avg. NV
n20q10	0.00	0.48	0.73	40.71	0.00	51.72	0.02	0.00	219.94	18.77	39.38
n20q15	0.00	0.32	–	–	–	–	0.01	0.00	46.86	–	29.27
n20q20	0.00	0.37	–	–	–	–	0.01	0.00	32.96	–	27.16
n20q25	0.00	0.39	–	–	–	–	0.00	0.00	12.10	–	22.29
n20q30	0.00	0.35	3.93	3.29	0.00	4.44	0.07	0.00	5.69	0.28	20.03
n20q35	0.00	0.33	–	–	–	–	0.16	0.05	4.46	–	19.71
n20q40	0.00	0.28	–	–	–	–	0.00	0.00	2.72	–	19.34
n20q45	0.00	0.31	3.39	2.67	0.00	3.62	0.00	0.00	2.12	0.05	19.17
n20q1000	0.00	0.93	3.47	2.63	0.00	2.63	0.00	0.00	0.45	0.01	18.60
Avg.	0.00	0.42	2.88	12.33	0.00	15.60	0.03	0.01	36.37	4.78	23.88
n30q10	0.00	153.85	–	–	–	–	0.04	0.00	1115.36	–	57.39
n30q15	0.00	65.36	–	–	–	–	0.03	0.00	206.32	–	42.47
n30q20	0.00	8.16	–	–	–	–	0.03	0.00	133.46	–	39.38
n30q25	0.00	10.27	–	–	–	–	0.10	0.03	65.47	–	33.97
n30q30	0.00	9.91	–	–	–	–	0.02	0.00	28.78	–	30.12
n30q35	0.00	9.37	–	–	–	–	0.22	0.00	24.29	–	29.83
n30q40	0.00	5.92	–	–	–	–	0.08	0.05	11.85	–	28.32
n30q45	0.00	2.96	–	–	–	–	0.14	0.00	9.96	–	28.49
n30q1000	0.00	221.30	–	–	–	–	0.02	0.00	1.39	–	27.59
Avg.	0.00	54.12	–	–	–	–	0.08	0.01	177.43	–	35.28
n40q10	0.00	235.42	4.76	408.07	0.39	3983.91	0.07	0.00	2619.68	457.03	73.67
n40q15	0.00	28.83	–	–	–	–	0.05	0.01	390.64	–	53.45
n40q20	0.00	62.22	–	–	–	–	0.01	0.00	331.38	–	50.29
n40q25	0.00	177.39	–	–	–	–	0.06	0.01	136.27	–	43.57
n40q30	0.00	108.31	4.64	101.43	0.00	1524.21	0.10	0.04	57.86	19.28	39.63
n40q35	0.00	304.70	–	–	–	–	0.13	0.03	55.48	–	39.35
n40q40	0.00	21.68	–	–	–	–	0.01	0.00	34.54	–	38.30
n40q45	0.00	25.74	5.14	113.43	0.00	219.71	0.01	0.00	21.41	3.29	37.49
n40q1000	0.00	80.91	4.58	91.08	0.30	95.60	0.00	0.00	2.97	0.03	36.81
Avg.	0.00	116.13	4.78	178.50	0.17	1455.86	0.05	0.01	405.58	119.91	45.84
n50q10	0.99	2693.41	–	–	–	–	0.33	0.19	3579.49	–	95.30
n50q15	0.00	1702.17	–	–	–	–	0.06	0.03	1503.56	–	69.44
n50q20	0.89	3085.39	–	–	–	–	0.53	0.42	1374.12	–	64.04
n50q25	0.59	2024.55	–	–	–	–	0.34	0.24	544.63	–	54.96
n50q30	0.46	1345.21	–	–	–	–	0.38	0.29	215.59	–	49.75
n50q35	1.27	4212.45	–	–	–	–	0.79	0.59	201.46	–	49.60
n50q40	0.45	3545.41	–	–	–	–	0.45	0.39	143.50	–	48.49
n50q45	0.23	2057.76	–	–	–	–	0.23	0.18	74.94	–	46.75
n50q1000	0.10	1938.98	–	–	–	–	0.09	0.09	6.66	–	45.41
Avg.	0.55	2511.70	–	–	–	–	0.36	0.27	849.33	–	58.19
n60q10	2.83	3718.02	46.25	401.91	5.97	4524.10	0.63	0.30	3602.77	39.70	115.50
n60q15	2.73	2932.58	–	–	–	–	0.41	0.36	2613.02	–	85.09
n60q20	0.39	3772.60	–	–	–	–	0.26	0.19	2619.83	–	78.76
n60q25	2.63	3636.12	–	–	–	–	0.59	0.46	1153.57	–	67.26
n60q30	2.88	4702.35	13.48	412.35	5.06	4533.51	0.74	0.57	430.87	5.75	60.34
n60q35	2.42	4795.69	–	–	–	–	0.87	0.66	418.48	–	60.05
n60q40	0.46	3829.99	–	–	–	–	0.37	0.31	244.74	–	57.60
n60q45	0.26	2223.52	15.14	413.41	2.04	3840.61	0.26	0.23	136.54	7.77	56.44
n60q1000	0.18	2940.39	7.47	414.81	1.94	483.46	0.18	0.17	15.97	0.11	55.26
Avg.	1.64	3616.81	20.59	410.62	3.75	3345.42	0.48	0.36	1248.42	13.33	70.70

without improvement over the current local optimal solution (I_{ILS}). Here we set I_R=10, as in Silva et al. [39], where the authors put forward a multi-start ILS that was capable of obtaining state-of-the-art results for the split-delivery VRP.

In previous works, such as those mentioned in Section 1, the value of I_{ILS} was tuned based on the instance size. In VRPs (e.g., [30,39,40,44]), this parameter is usually set as a function of the number of customers and vehicles. In TSPs (e.g., [38,41]), I_{ILS} was set only as a function of the number of customers (or jobs). We decided to use the same rationale as in Silva et al. [38], by setting $I_{ILS} = \max\{I_{min}, \beta \times n\}$, where I_{min} and β are input parameters. For the latter we set $\beta = 4$, as in Subramanian et al. [42]. Note that I_{min} is more important for small size instances and its role is to prevent low values for I_{ILS}, which in this case may lead to an insufficient number of ILS iterations required for obtaining high quality solutions. We then tested several values for I_{min}, more specifically, 100, 120, 140, 160, and 180. For each of them, we ran ILS$_{SBRP}$ 10 times for all instances containing 20 and 30 stations. The average results obtained suggest that $I_{min} = 160$ seems to provide a good compromise between solution quality and CPU time, since the algorithm managed to find almost all best known solutions in a relatively short amount of time when using this value. Therefore, we set $I_{ILS} = \max\{160, 4 \times n\}$.

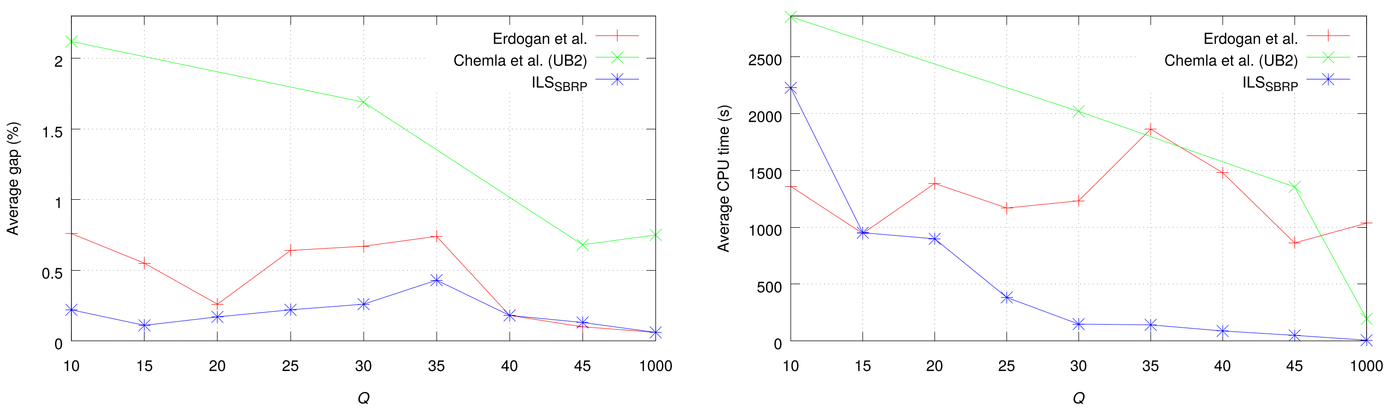
4.4. Comparison with the literature

ILS$_{SBRP}$ was executed 10 times for each instance with a time limit of 1 h. The upper bounds (best heuristic solution values) found by our algorithm are compared with those determined by two versions of the tabu search heuristic of Chemla et al. [6]. The first one (TS1) starts from an initial solution generated by a greedy procedure, while the second version (TS2) receives the solution produced by their branch-and-cut algorithm (over a relaxation of the problem) as initial solution.



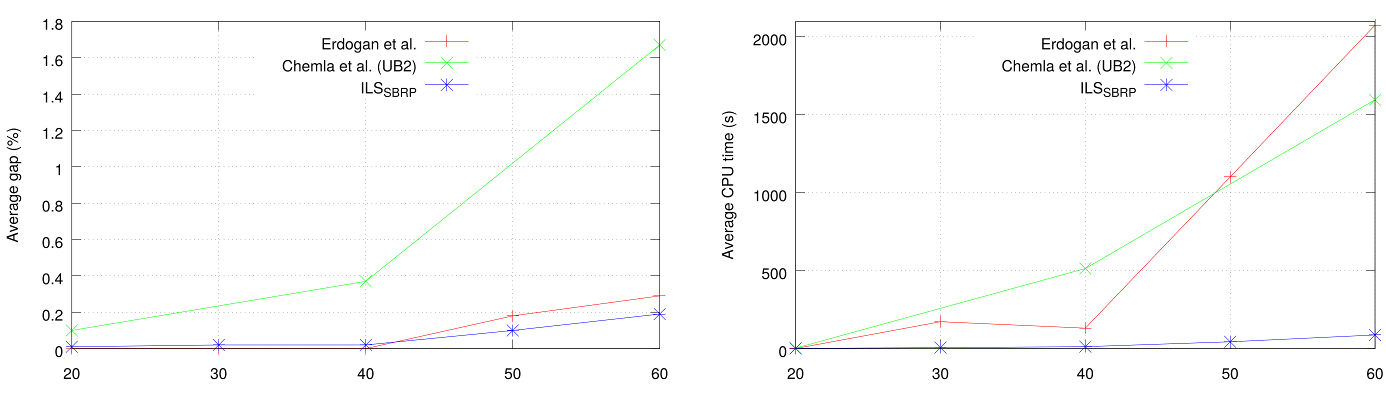
(a) Average gap (%) (b) Average CPU time (s)

Fig. 7. Average gap (%) and average CPU time (s) per Q and $\alpha = 1$.



(a) Average gap (%) (b) Average CPU time (s)

Fig. 8. Average gap (%) and average CPU time (s) per Q and $\alpha = 3$.

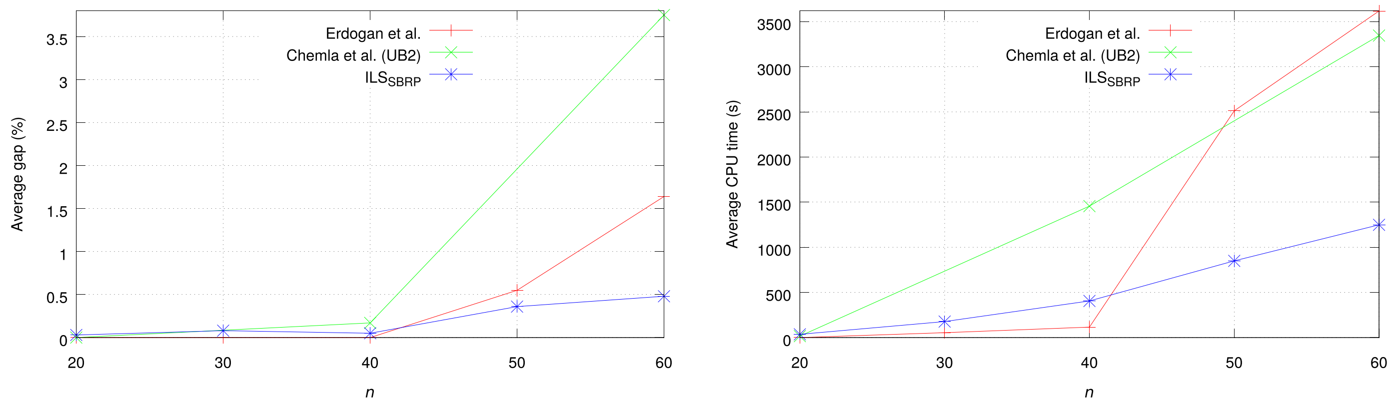


(a) Average gap (%) (b) Average CPU time (s)

Fig. 9. Average gap (%) and average CPU time (s) per n and $\alpha = 1$.

Detailed results of TS1 and TS2 are available at http://cermics.enpc.fr/~meuniefr/SVOCPDP.html. We use the same notation adopted in [6], and hence we call UB1 the best solution value found by TS1 and UB2 the best solution value found considering both TS1 and TS2. A comparison is also performed with the lower and upper bounds obtained by the exact Branch-and-cut algorithm of Erdoğan et al. [14]. Regarding the benchmark instances, Chemla et al. [6] considered $n \in \{20, 40, 60, 100\}$ and $Q \in \{10, 30, 45, 1000\}$, whereas Erdoğan et al. [14] considered $n \in \{20, 30, 40, 50, 60\}$ and $Q \in \{10, 15, 20, 25, 30, 35, 40, 45, 1000\}$.

Chemla et al. [6] ran their experiments on an AMD Athlon 5600+2.8 GHz with 16 GB of RAM, while Erdoğan et al. [14] performed their testing on an Intel i7 3.60 GHz and 8 GB of RAM. On the one hand, because the hardware performance of the first appears to be quite inferior to the second, as well as to our intel i7 3.40 GHz, we decided to estimate an approximation factor based on the single thread rating values reported in https://www.cpubenchmark.net/compare.php?cmp[]=86&cmp[]=896, so as to better compare the



(a) Average gap (%) (b) Average CPU time (s)

Fig. 10. Average gap (%) and average CPU time (s) per n and α = 3.

Table 4
Summary of the performance of the best solutions aggregated by n.

		Erdoğan et al. (2015)	ILS_{SBRP}			
		#Opt	**#Opt**	**#Impr.**	**#Equal**	**#Worse**
$\alpha = 1$	20	90	90	0	90	0
	30	90	89	0	89	1
	40	89	87	0	88	2
	50	83	83	5	85	0
	60	72	70	12	76	2
	Total	424	419	17	428	5
$\alpha = 3$	20	90	89	0	89	1
	30	90	87	0	87	3
	40	90	83	0	83	7
	50	69	63	17	67	6
	60	60	55	27	58	5
	Total	399	377	44	384	22

runtime performance of the methods. According to the referred website, the AMD Athlon 5600+2.8 GHz is roughly 2.43 times slower than our processor. We thus report the original CPU time values of Chemla et al. [6] divided by a factor of 2.43. On the other hand, since the machine used in [14] is rather equivalent to ours, perhaps even slightly faster, we decided to consider the original runtime values reported by the authors.

4.4.1. Results for instances with up to 60 stations

The aggregate average results for instances containing 20, 30, 40, 50, and 60 stations are reported in Tables 2 and 3, where **Instance group** denotes the set of 10 instances of a particular group (for example, group n20q10 contains 10 instances with n=20 and Q=10); **UB1 Gap (%)**, **UB2 Gap (%)**, and **Gap (%)** correspond to the gap between UB1, UB2, and the upper bound found by Erdoğan et al. [14], respectively, and the lower bound reported in [14]; **Time (s)**, **UB1 Time (s)**, and **UB2 Time (s)** indicate, respectively, the CPU time in seconds spent by Erdoğan et al. [14], TS1, and TS2, where the last two are scaled to our processor as mentioned above; **Avg. Gap (%)** and **Best Gap (%)** are the gaps of the average solution and the best solution, respectively, found by ILS_{SBRP} over the 10 runs with respect to the lower bounds in [14]; **Avg. Time (s)** is the average CPU time in seconds spent by ILS_{SBRP} to completion over the 10 runs; **Avg.** $\mathbf{TT_{UB2}}$ **(s)** denotes the average time over the 10 runs to find or improve the best heuristic solution found in Chemla et al. [6] (UB2); and **Avg. NV** is the average number of visits of the final solutions found by ILS_{SBRP}. Detailed results are available at http://arxiv.org/pdf/1605.00702v2.pdf, including the best solution found when considering all experiments.

From Table 2, it can be observed that the quality of the solutions found by ILS_{SBRP}, as well as those obtained by the algorithm of Erdoğan et al. [14], are visibly superior than the ones determined by the tabu searches of Chemla et al. [6], especially TS1. Such superiority becomes even more prominent for α=3, as presented in Table 3. Also, the average CPU times spent by ILS_{SBRP} to find or improve the best solutions reported by Chemla et al. [6] (UB2) are rather small in most cases, sometimes only a matter of relatively few seconds, except for the instance group n40q10 when α=3, where the proposed algorithm required more CPU time.

In addition, assuming the same values for the demands, the smaller the vehicle capacity, the larger the relative number of visits. This increases the size of the tour, thus affecting the number of operations performed during the local search, and possibly the number of ILS iterations, as more moves are required to be evaluated.

Figs. 7 and 8 show how the average gaps and CPU times of each method vary according to the value of Q. We omit the results of TS1 because the associated gaps are quite inferior when compared to those obtained by the other algorithms. While the average gaps of ILS_{SBRP} tend to be larger for very small values of Q, the average CPU time decreases as the value of Q increases, both for $\alpha = 1$ and $\alpha = 3$. A similar behavior regarding the CPU time performance can be observed for the heuristic of Chemla et al. [6], as opposed to the algorithm of Erdoğan et al. [14], which does not seem to have a consistent pattern when considering this aspect.

Figs. 9 and 10 illustrate the behavior of the average gaps and CPU times of the algorithms as the number of stations increases. Overall, the quality of the solutions found by ILS_{SBRP} and by the algorithm of Erdoğan et al. [14] are equivalent except for n=60 and $\alpha = 3$, where the former clearly outperforms the latter. Moreover, there is a considerable increase on the CPU time for both methods from the literature for $n > 40$, in contrast to ILS_{SBRP}, whose increase appears to be more moderate. However, this was somewhat expected since the CPU effort of the algorithms of Erdoğan et al. [14] and of Chemla et al. [6] tend to increase exponentially with increasing values of n.

Finally, Table 4 shows a summary of the best solutions found by the proposed algorithm, where **#Opt** denotes the number of optimal solutions, **#Impr.** corresponds to the number of improved solutions, **#Equal** indicates the number of equal solutions, and **#Worse** is the number of worse solutions. The results of Chemla et al. [6] were not included in these tables because they are dominated by those obtained by our heuristic for all instances. For $\alpha = 1$, ILS_{SBRP} found 419 of the 424 known optimal solutions and improved the result of 17 of the 26 instances that remain open. As for $\alpha = 3$, ILS_{SBRP} achieved 377 out of 399 proven optimal solutions and improved the best known solution of

Table 5
Detailed results for n=100 and $\alpha = 1$.

Instance	Chemla et al. [6]						ILS$_{SBRP}$						
	UB1	UB1 Gap (%)	UB1 Time (s)	UB2	UB2 Gap (%)	UB2 Time (s)	Avg. Sol.	Avg. Gap (%)	Best Sol.	Best Gap (%)	Avg. Time (s)	Avg. TT$_{UB2}$ (s)	Avg. NV
n100q10A	14921	45.58	681.15	13273	29.50	4805.44	11283.20	10.09	11258	9.84	3558.39	3.23	97.30
n100q10B	17658	59.29	470.81	14981	35.14	4588.93	12669.20	14.29	12609	13.75	3600.31	5.28	102.20
n100q10C	17138	44.75	617.06	15636	32.07	4758.60	13251.00	11.92	13224	11.69	3600.47	4.72	100.70
n100q10D	19278	57.05	489.29	16586	35.12	4620.98	13832.80	12.69	13783	12.29	3600.54	3.23	98.40
n100q10E	16867	69.73	368.51	12513	25.91	4485.81	10974.90	10.44	10954	10.23	3385.98	6.10	105.70
n100q10F	14759	46.88	486.42	12621	25.60	4603.72	11226.20	11.72	11191	11.37	3600.31	5.26	101.30
n100q10G	16772	63.48	380.01	13820	34.71	4498.96	11186.60	9.04	11160	8.78	3347.66	2.40	100.10
n100q10H	15941	45.21	393.16	14863	35.39	4511.70	12339.70	12.41	12308	12.12	3600.40	4.02	106.70
n100q10I	17799	50.46	485.19	16602	40.34	4603.72	13540.70	14.46	13469	13.86	3600.81	4.15	104.60
n100q10J	20044	75.98	459.71	14988	31.59	4578.66	12491.30	9.67	12462	9.41	3600.54	3.72	96.90
Avg.	–	55.84	483.13	–	32.54	4605.65	–	11.67	–	11.33	3549.54	4.21	101.39
n100q30A	12175	62.41	518.46	8033	7.16	4634.94	7820.00	4.31	7820	4.31	225.57	1.47	91.00
n100q30B	11066	48.70	537.77	10223	37.37	4657.54	8094.50	8.77	8094	8.76	570.56	0.26	93.50
n100q30C	12106	50.14	430.96	9149	13.47	4549.08	8505.70	5.49	8503	5.46	480.66	0.70	91.80
n100q30D	11317	45.52	589.95	9690	24.60	4706.43	8339.90	7.24	8336	7.19	602.54	0.52	91.10
n100q30E	10446	35.42	451.91	8479	9.92	4569.21	7992.90	3.62	7986	3.53	403.74	1.03	95.90
n100q30F	11960	61.14	373.85	8281	11.57	4497.73	8028.60	8.17	8020	8.05	316.18	1.24	93.40
n100q30G	11290	46.31	490.94	8872	14.97	4609.88	8075.00	4.64	8075	4.64	246.69	0.24	90.00
n100q30H	11144	42.19	387.82	8944	14.12	4503.89	8257.00	5.35	8257	5.35	630.06	0.52	97.00
n100q30I	12112	45.32	450.68	9189	10.25	4568.80	8674.60	4.08	8652	3.81	547.99	1.61	96.50
n100q30J	10636	44.70	560.37	9014	22.63	4678.90	7923.00	7.79	7923	7.79	475.18	0.36	90.40
Avg.	–	48.19	479.27	–	16.61	4597.64	–	5.95	–	5.89	449.92	0.80	93.06
n100q45A	8494	18.12	757.56	8103	12.69	4876.10	7632.00	6.14	7632	6.14	179.88	0.12	91.00
n100q45B	8838	27.33	391.93	8020	15.54	4508.82	7660.00	10.36	7660	10.36	260.68	0.36	92.50
n100q45C	10056	30.21	435.07	8270	7.09	4551.96	7993.00	3.50	7993	3.50	189.28	0.97	90.40
n100q45D	9442	27.15	614.19	8535	14.94	4729.85	7914.70	6.58	7900	6.38	318.62	0.27	88.20
n100q45E	10258	39.47	613.77	7864	6.92	4729.85	7835.00	6.53	7835	6.53	171.33	7.82	94.10
n100q45F	10348	37.36	485.60	7817	3.76	4602.90	7731.00	2.62	7731	2.62	175.23	1.07	93.30
n100q45G	9856	31.59	668.00	8286	10.63	4791.88	7864.00	4.99	7864	4.99	139.25	0.10	90.00
n100q45H	9506	25.94	481.49	7796	3.28	4597.56	7740.00	2.54	7740	2.54	266.65	5.06	94.00
n100q45I	10334	32.23	426.85	8667	10.90	4543.33	8042.20	2.90	8037	2.84	288.97	0.21	96.10
n100q45J	9021	27.53	536.95	7860	11.12	4656.31	7588.50	7.28	7566	6.96	260.60	0.41	89.70
Avg.	–	29.69	541.14	–	9.69	4658.86	–	5.34	–	5.29	225.05	1.64	91.93
n100q1000A	8447	18.81	682.38	8199	15.32	4803.38	7453.00	4.83	7453	4.83	132.76	0.10	92.00
n100q1000B	8669	22.76	543.52	8183	15.88	4664.11	7491.00	6.08	7491	6.08	172.13	0.09	93.30
n100q1000C	8692	15.81	647.05	8673	15.56	4766.82	7898.50	5.24	7895	5.19	118.31	0.09	90.10
n100q1000D	10116	41.08	489.71	8363	16.63	4605.78	7572.80	5.61	7565	5.50	163.68	0.11	88.80
n100q1000E	8922	17.62	355.78	8071	6.40	4472.26	7771.00	2.44	7771	2.44	125.79	0.09	94.30
n100q1000F	10348	42.47	483.54	8053	10.87	4602.49	7648.00	5.30	7648	5.30	129.92	0.17	93.10
n100q1000G	10954	45.12	432.19	8516	12.82	4549.49	7817.50	3.57	7813	3.51	96.81	0.06	90.10
n100q1000H	9275	22.86	397.68	7786	3.14	3374.94	7593.00	0.58	7593	0.58	185.62	0.36	94.00
n100q1000I	10145	31.58	399.32	8461	9.74	4517.45	7975.00	3.44	7975	3.44	118.25	0.13	96.00
n100q1000J	9178	30.27	446.57	8655	22.84	4565.93	7315.00	3.82	7315	3.82	117.66	0.06	89.40
Avg.	–	28.84	487.77	–	12.92	4492.27	–	4.09	–	4.07	136.09	0.13	92.11

44 out of 51 open instances.

4.4.2. Results for instances with 100 stations

Tables 5 and 6 illustrate the detailed results found by our algorithm and the tabu searches of Chemla et al. [6] for every instance containing 100 stations. In this case, the gaps are computed with respect to the lower bound reported in [6]. The results obtained show that ILS$_{SBRP}$ clearly outperforms the methods from the literature, both in terms of solution quality and CPU time. In general, the proposed heuristic was capable of significantly improving the best known solution of all instances.

In addition, when considering $\alpha = 1$, ILS$_{SBRP}$ required, on average, at most 8 seconds to find or improve the best results of Chemla et al. [6]. In some cases, such as those involving $Q \geq 30$, our algorithm spent, on average, only a fraction of a second to achieve a superior solution than the best one from the literature. For $\alpha = 3$, more time was required, on average, to accomplish the same purpose, but mostly for Q=10.

4.5. Impact of not using stations as buffers

In this section we evaluate the impact on the solution quality of forbidding the use of temporary operations, i.e., forbidding stations to serve as buffers along the route. Fig. 11a–e show the average gaps between the values of best solutions found by ILS$_{SBRP}$ on the new problem variant and on the original one. For brevity, we have only considered the instances with $\alpha = 1$. As somewhat expected, we can observe that the fact of not allowing temporary operations leads to worse solutions, with average gaps ranging from 0.38% for n=30 and Q=15 to 3.87% for n=60 and Q=10.

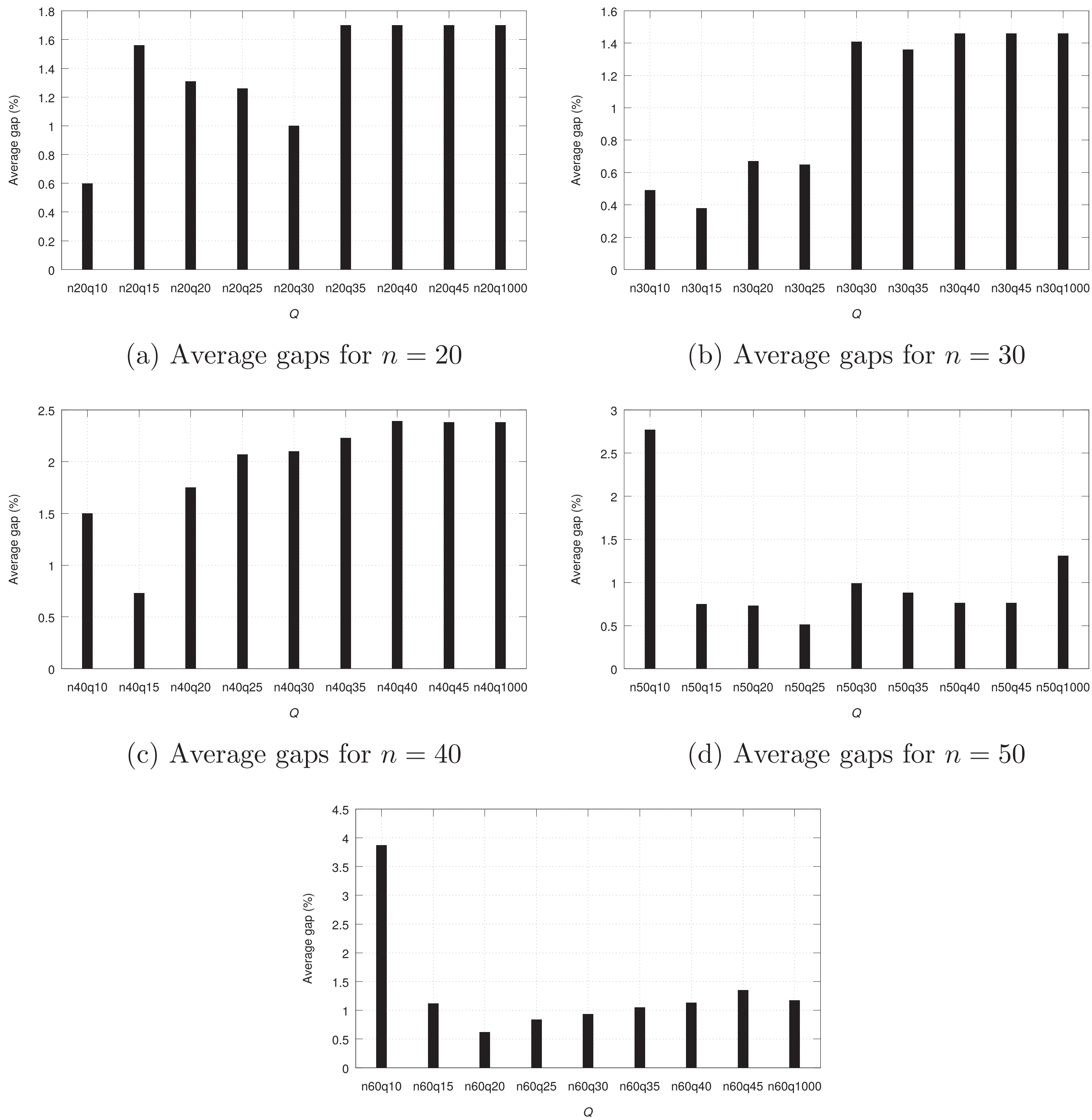
Table 6
Detailed results for n=100 and $\alpha = 3$.

Instance	Chemla et al. [6]						ILS_{SBRP}						
	UB1	UB1 Gap (%)	UB1 Time (s)	UB2	UB2 Gap (%)	UB2 Time (s)	Avg. Sol.	Avg. Gap (%)	Best Sol.	Best Gap (%)	Avg. Time (s)	Avg. TT_{UB2} (s)	Avg. NV
n100q10A	36057	60.84	5133.28	28277	26.14	9301.11	24121.60	7.60	24014	7.12	3610.42	22.11	97.30
n100q10B	47107	71.99	7200.56	35199	28.51	11434.53	29709.60	8.47	29438	7.48	3631.88	49.10	102.20
n100q10C	50606	72.67	486.01	35779	22.08	4682.60	31802.50	8.51	31540	7.62	3623.25	67.37	100.70
n100q10D	47489	51.18	7768.73	37972	20.88	12010.51	33846.70	7.75	33654	7.14	3641.22	92.56	98.40
n100q10E	41002	75.89	776.05	30222	29.65	4938.55	25092.60	7.64	24917	6.89	3613.53	23.73	105.70
n100q10F	43544	86.70	459.30	28488	22.14	4660.42	25307.10	8.51	25176	7.94	3619.96	50.33	101.30
n100q10G	38539	69.26	454.37	28822	26.58	4629.19	24877.20	9.26	24642	8.23	3616.24	34.89	100.10
n100q10H	44411	67.69	587.89	33853	27.83	4816.94	29039.70	9.65	28794	8.72	3615.54	63.92	106.70
n100q10I	48727	65.76	5187.92	37199	26.54	9423.95	32380.00	10.15	32023	8.94	3643.19	62.52	104.60
n100q10J	45590	66.76	355.78	34086	24.68	4543.33	29373.50	7.44	29147	6.61	3630.50	71.32	96.90
Avg.	–	68.87	2840.99	–	25.50	7044.11	–	8.50	–	7.67	3624.57	53.79	101.39
n100q30A	16110	55.73	441.64	13366	29.20	4560.58	11278.00	9.02	11258	8.83	3550.25	2.40	91.00
n100q30B	18739	68.61	346.74	15537	39.80	4464.04	12650.70	13.83	12605	13.41	3600.72	4.28	93.50
n100q30C	18871	61.46	437.53	16116	37.89	4577.84	13231.50	13.21	13224	13.14	3600.42	3.55	91.80
n100q30D	18262	49.67	722.23	16419	34.56	4865.01	13786.00	12.98	13783	12.96	3600.72	3.61	91.10
n100q30E	16867	69.83	368.10	13118	32.08	4486.23	10984.10	10.59	10954	10.29	3194.65	3.51	95.90
n100q30F	14838	46.49	453.55	13674	35.00	4569.62	11245.80	11.02	11191	10.48	3600.31	2.21	93.40
n100q30G	16772	69.37	381.66	13262	33.92	4505.12	11183.30	12.93	11160	12.70	3404.20	3.92	90.00
n100q30H	15609	40.53	543.52	14495	30.50	4660.00	12332.80	11.03	12296	10.70	3600.52	4.03	97.00
n100q30I	19159	57.85	266.63	16620	36.93	4410.22	13519.20	11.38	13469	10.97	3600.51	3.48	96.50
n100q30J	20044	78.22	459.71	15423	37.13	4578.66	12513.00	11.26	12462	10.81	3600.43	3.17	90.40
Avg.	–	59.78	442.13	–	34.70	4567.73	–	11.73	–	11.43	3535.27	3.42	93.06
n100q45A	11372	36.43	717.71	10694	28.30	4833.78	9229.40	10.73	9192	10.28	1597.41	1.78	91.00
n100q45B	16114	74.68	409.59	14520	57.40	4526.08	10233.70	10.93	10209	10.67	2955.64	0.99	92.50
n100q45C	14817	50.09	516.00	13243	34.15	4633.30	10871.50	10.12	10815	9.55	3093.03	1.55	90.40
n100q45D	15845	62.58	424.79	15845	62.58	4540.87	11143.90	14.34	11103	13.92	3562.47	1.03	88.20
n100q45E	11628	32.52	602.68	11400	29.92	4718.34	9521.70	8.51	9498	8.24	1687.03	1.60	94.10
n100q45F	12821	50.90	446.16	12243	44.10	4742.17	9437.70	11.08	9398	10.62	1663.30	1.09	93.30
n100q45G	14829	71.60	436.30	10827	25.29	4553.60	9445.00	9.30	9445	9.30	1215.65	2.09	90.00
n100q45H	13072	42.35	652.80	12319	34.15	4768.87	10226.60	11.37	10206	11.14	2391.88	2.09	94.00
n100q45I	14366	46.89	387.00	14366	46.89	4503.07	10864.10	11.08	10841	10.85	3066.06	1.68	96.10
n100q45J	13989	53.75	494.64	11850	30.24	4611.12	10138.10	11.43	10131	11.35	2469.49	2.09	89.70
Avg.	–	52.18	508.77	–	39.30	4643.12	–	10.89	–	10.59	2370.20	1.60	91.93
n100q1000A	9402	29.43	469.57	8017	10.36	4590.16	7457.20	2.66	7453	2.60	117.51	0.08	92.00
n100q1000B	8793	25.68	506.96	7595	8.55	4627.14	7491.00	7.07	7491	7.07	182.14	0.82	93.30
n100q1000C	9312	21.43	422.74	8554	11.55	4541.69	7904.30	3.08	7895	2.96	118.19	0.07	90.10
n100q1000D	9832	33.73	531.61	7595	3.31	4649.73	7574.00	3.02	7565	2.90	154.08	26.25	88.80
n100q1000E	8922	15.87	355.78	8071	4.82	707.85	7771.00	0.92	7771	0.92	119.50	0.30	94.30
n100q1000F	9371	28.57	721.41	8783	20.50	4838.30	7648.00	4.93	7648	4.93	124.09	0.06	93.10
n100q1000G	10954	46.16	431.78	8219	9.67	4549.08	7816.50	4.30	7813	4.25	103.84	0.18	90.10
n100q1000H	8829	20.19	549.28	8488	15.55	4666.17	7593.00	3.37	7593	3.37	174.74	0.08	94.00
n100q1000I	10664	40.10	306.07	8149	7.06	4421.73	7975.00	4.78	7975	4.78	124.87	0.38	96.00
n100q1000J	8311	20.04	534.49	7976	15.20	4650.56	7315.00	5.65	7315	5.65	111.68	0.12	89.40
Avg.	–	28.12	482.97	–	10.66	4224.24	–	3.98	–	3.94	133.06	2.83	92.11

5. Concluding remarks

In this work we proposed a hybrid ILS algorithm that was especially designed to solve a challenging single-vehicle SBRP variant. Extensive computational experiments were conducted on 980 instances from the literature ranging from 20 to 100 stations. The results were compared with those reported in Chemla et al. [6] and Erdoğan et al. [14]. For the 900 instances containing up to 60 stations, the proposed heuristic, called ILS_{SBRP}, was capable of finding 796 out of 823 known optimal solutions (97%) and improving the result of 61 out of 77 open instances (79%). Our algorithm only failed to be at least equal to the best known solution in 27 instances (3%). In addition, the average gap of the average solutions found by ILS_{SBRP} and the lower bound reported in [14], for each instance group, was always smaller than 0.7%, thus ratifying the robustness of our heuristic. As for the 80 instances involving 100 stations, ILS_{SBRP} outperformed the best heuristics available for the problem by considerably improving the best known solution for all instances.

Future work may include the development of an enhanced procedure to verify whether or not the solution is feasible. Currently, this is the most time consuming part of the algorithm, where we use a relatively costly max-flow based procedure [6] for performing this task. Hence, any improvement on this procedure could possibly lead to an improvement on the CPU time. One possible alternative is to devise cheaper heuristic procedures to identify infeasible solutions so as to avoid unnecessary calls to the max-flow routine. Also, other type of hybridizations could be experimented by combining, for example, efficient exact algorithms with the heuristic suggested in this work. Finally, ILS_{SBRP} could be extended to tackle most realistic cases involving multiple vehicles and time constraints. We believe that it



(a) Average gaps for $n = 20$ (b) Average gaps for $n = 30$

(c) Average gaps for $n = 40$ (d) Average gaps for $n = 50$

(e) Average gaps for $n = 60$

Fig. 11. Average gaps between the original scenario and the one where temporary operations are forbidden, grouped by Q and $\alpha = 1$.

would be also interesting to further investigate the impact of temporary pickups/drop-offs of bikes, as these operations are quite disliked by the system operators.

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Appendix A. Checking feasibility

Let $L = i_1, i_2, \ldots, i_k$ be a sequence of vertices, where $i_1 = i_k = 0$. A directed graph can be built using p_i, p'_i, and q_i for each i in the sequence, as follows:

- Let s be the source of the flow network, and for each vertex i representing the first occurrence of each station in the sequence let us define a set of arcs u_i with capacity p_i;
- Let t be the sink of the flow network, and for each i' representing the last occurrence of each station in the sequence let us define a set of

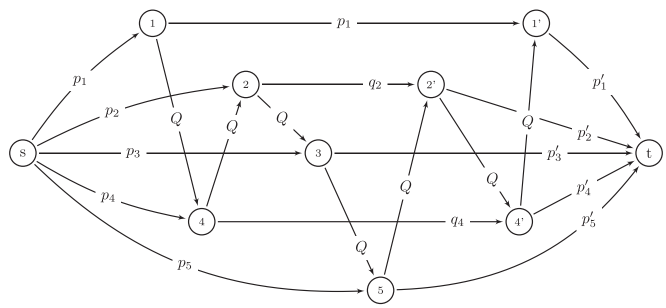

Fig. A1. Flow network for feasibility checking.

arcs w_i, with capacity p'_i;

- For each $j = 2, \ldots, k-1$ let us define an arc $b_{j,j+1}$ with capacity Q; and
- If a station i is visited more than once, let us define an arc $d_{e,e+1}$ with capacity q_i, between the eth and $(e+1)$th visits to i.

By computing an s–t maximum flow, one can find optimal bike displacements along the sequence L. For each station i, let us define $\hat{p}_i$ and $\hat{p}'_i$, respectively, as the resulting s–t flow on arcs u_i and w_i. Flow on arcs $b_{j,j+1}$ indicates the number of bikes from i_j to i_{j+1} and flow on arcs $d_{e,e+1}$ denotes the quantity of bikes remaining in a station i after the eth visit and before the $(e+1)$th visit. Fig. A1 depicts a flow network for a sequence $L = 0, 1, 4, 2, 3, 5, 2, 4, 1, 0$, where s and t correspond to the depot.

Chemla et al. [6] states that sequence L induces a feasible solution when $\hat{p}_i = p_i$, for each station i in the sequence. Also, if a vertex i' is not in L, then $\hat{p}_{i'} = \hat{p}'_{i'} = 0$.

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