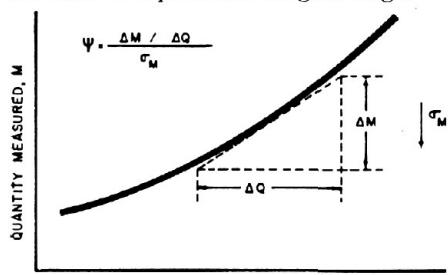


Evaluation of Analytical Methods by the Sensitivity Criterion

Sensitivity is a measure of performance useful in the evaluation of methods of test. Its application to analytical chemistry can be pictured in terms of an analogy with a radio receiver. Thus, an analytical method receives a signal in the form of a sample, and noise in the form of experimental error. The method detects and amplifies the desired signal (component to be determined) and yields a message in the form of a measured value. Sensitivity, in terms of this analogy, is a measure that increases directly with amplification and inversely with noise.

IN THE classical methods of analytical chemistry, a proportional relationship usually exists between the quantity measured and the amount of the desired component in the sample. For example, in the determination of zinc, the amount of this metal is directly proportional to the quantity of zinc oxide, zinc sulfate, or other zinc compound actually measured. The two factors determining the merit of such procedures are the experimental error and the ratio of the equivalent weight of the zinc compound to that of zinc. It is well recognized that the experimental error should be small and the ratio of equivalent weights large.



The sensitivity, ψ , of method M for the determination of component Q is the slope of the tangent AM/AQ divided by the standard deviation σ_M . Sensitivity is essentially equivalent to the reciprocal of the standard deviation of the calculated values of Q.

Recently, a single quantity called "sensitivity" has been introduced to combine these two factors in a single measure for the merit of a test (1). In cases where there is a proportional relationship between measured and desired quantities, sensitivity is the proportionality factor (ratio of equivalent weights) divided by the standard deviation (experimental error).

Bound Styrene in Synthetic Rubber

With the introduction of physical methods in analytical chemistry, the quantity measured is frequently not proportional nor even linearly related to the quantity desired. For example, refractive index is used to determine bound styrene in styrene-butadiene rubbers (SBR). The relationship between refractive index and bound styrene is approximated by a quadratic equation. Thus, there is no single conversion factor analogous to the conversion of zinc compound to zinc. Further, the experimental error may vary with the bound styrene content. In such cases, the sensitivity or merit of the test cannot be expressed by a single value for the entire range of bound styrene contents. Nevertheless, the sensitivity of the method for any bound styrene content can be determined.

Suppose it is desired to determine the sensitivity at some given value of bound styrene. A tangent is drawn to the curve at that value (Figure 1). The slope of the tangent is, of course, the analog to the ratio of equivalent weights in the zinc example. Thus, the sensitivity is the slope of the tangent divided by the standard deviation of the refractive index measurements. Mathematically:

$$\text{Sensitivity} = \psi = \frac{\text{slope}}{\text{std. dev.}} = \frac{d\eta/dS}{\sigma_\eta}$$

where η denotes refractive index and S the bound styrene content.

Elemental Analyses Avoided

One of the principal advantages of the sensitivity criterion is the possibility of comparing two methods without knowledge of the relationship of either one to the characteristic being determined—for example, density and refractive index can both be used to measure the bound styrene of SBR. Application of the sensitivity criterion makes it unnecessary to perform the laborious elemental analyses required to establish the styrene contents of a series of calibration samples. Consider two samples of styrene-butadiene rubber that differ by a small but definite amount of bound

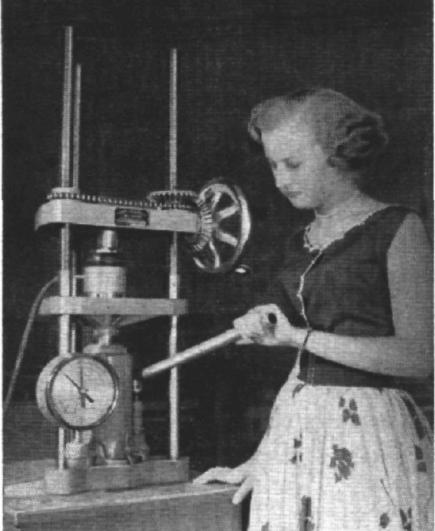
R. D. STIEHLER, chief, testing and specifications section, National Bureau of Standards since 1948, is a native of Springfield, N. Y. He studied at Rensselaer Polytechnic Institute, (Bachelor's degree), Johns Hopkins University (Ph.D.), California Institute of Technology (National Research Council Fellow), and Johns Hopkins University (research fellow in ophthalmology). In 1939 he became a research chemist at B. F. Goodrich; in 1942 he set up the Quartermaster Corps Climatic Research Laboratory in Massachusetts and in 1943 became a technical assistant on the RFC's synthetic rubber program. In 1946 he joined NBS as coordinator of work on synthetic rubber. He is author or co-author of more than 20 scientific papers and a member of AMERICAN CHEMICAL SOCIETY, AAAS, ASTM, American Society for Quality Control, and Institution of the Rubber Industry.

JOHN MANDEL, statistician in the Division of Organic and Fibrous Materials, National Bureau of Standards, obtained a master's degree in chemistry at the University of Brussels, in 1937, and studied mathematical statistics at Columbia University in 1943 and 1944. In his native Belgium, he was a research chemist with the Société Belge de Recherches et d'Etudes. He has done chemical research and development with Foster D. Snell and with the B. G. Corp. Since 1947 he has written numerous papers dealing with both statistical methodology and its application to physical and chemical experimentation.



R. D. Stiehler (right) and John Mandel

REPORT FOR ANALYSTS



Courtesy Esso Research & Engineering Co.

THE IDEAL PRESS FOR MAKING KBr PELLETS FOR INFRARED SPECTROSCOPIC ANALYSIS

Also for forming pellets for x-ray and other types of spectroscopic analysis

- 20 Ton Capacity Hydraulic Press
- Accurate alignment in operation
- Guided moving platen
- Sturdy 3 column construction
- Even pressure distribution for uniformly dense pellets
- Self-contained
- Bench mounted
- Hand operated
- 9" x 9" platen area
- 22" max. vertical opening, adjustable

A multi-purpose unit with quickly adjusted vertical opening, by handwheel, permits many different set-ups in minimum time.

Used around the world for RESEARCH, CONTROL & TESTING.

Available accessories include: Heating and Cooling Platens; Temp. Controls, Auxiliary low pressure guages; Fast air closing; Extruding units; Testing units, etc.

Available in 30 and 50 ton sizes.

Write for Bulletin.

**LOOMIS ENGINEERING
& MANUFACTURING CO.**
133 So. 14th St., Newark 7, N. J.

Circle No. 18A on Readers' Service Card, page 77A



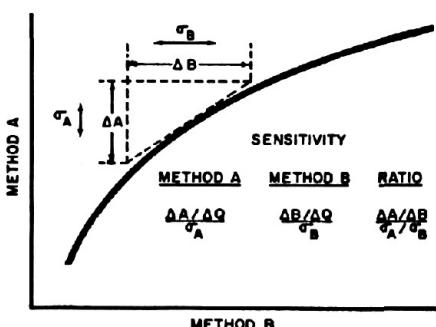
Refractive index measurements are one method used to study sensitivity

styrene. Repetitive measurements of refractive index and density are made on each to ascertain the standard deviation of each type of measurement. The other pertinent data are summarized in the following tabulation:

	Sample <i>X</i>	Sample <i>Y</i>	Difference, <i>X - Y</i>
Bound styrene (unknown)	S_x	S_y	ΔS
Refractive index	η_x	η_y	$\Delta \eta$
Density	ρ_x	ρ_y	$\Delta \rho$

Since the sensitivity of refractive index measurements is $\frac{\Delta \eta / \Delta S}{\sigma \eta}$ and that of density measurements is $\frac{\Delta \rho / \Delta S}{\sigma \rho}$, the ratio of these sensitivities, $\frac{\Delta \eta / \Delta \rho}{\sigma \eta / \sigma \rho}$, is a measure of their relative merit. If the ratio exceeds unity, refractive index is a more sensitive measure of bound styrene; if it is less than unity, density is the more sensitive.

If a series of samples covering a range of bound styrene contents had been used in place of only two samples, a relationship between refractive index and density could be determined without a



Relative merit of two methods suitable for determining component Q is given by the ratio of their sensitivities.

knowledge of the actual bound styrene content of any sample. From this relationship, the relative sensitivity of the two methods could be ascertained for any particular sample by using the following equation (see also Figure 2):

$$\frac{\psi \eta}{\psi \rho} = \frac{d\eta/d\rho}{\sigma \eta / \sigma \rho}$$

The case just cited shows that a knowledge of the bound styrene content is not necessary to compare the two methods. Nevertheless, it is possible in this instance to determine the relation of each method separately with bound styrene, by laborious elemental analysis. Using these relationships the error in refractive index as well as density can be converted into uncertainty measures for the calculated bound styrene values. This procedure is essentially equivalent to measuring their sensitivities, as the absolute sensitivity of a method is equivalent to the reciprocal of the standard deviation of the desired calculated component, in this case bound styrene calculated from either density or refractive index measurements.

Sensitivity Criterion Where No Absolute Method Exists

In many instances there is no absolute method for determining the desired quantity. In these cases, there is no alternative to the use of the sensitivity criterion for the comparison of different methods of test. For example, it is necessary to control the molecular characteristics of the polymer in synthetic rubber production. These characteristics cannot be precisely defined but include such parameters as molecular weight distribution, branching, etc. In SBR production, these characteristics are controlled through measurements of Williams plasticity or Mooney viscosity. If it is assumed that both methods measure the same molecular characteristics, the use of the sensitivity criterion appraises the one which is more responsive to change in the product. As physical methods of test become more widespread in industry, instances of this type are bound to be more prevalent. The determination of their relative sensitivities is the simplest and most objective means for establishing their relative merit.

Another advantage of sensitivity as a measure of merit is its independence of the scale of measurement. Many measurements of physical or chemical properties can be expressed in more than one scale. For example, light absorption can be measured on either an absorbance or percentage transmittance scale. Since these two scales are not proportionally related, both standard deviation and coefficient of variation are dependent on

the scale in which the measurement is expressed. In such cases, the usual measures of reproducibility, such as standard deviation, average deviation, or coefficient of variation, are unsuited for evaluating test methods. For example, the scales on both the Williams plasticity and the Mooney viscosity instruments are graduated in arbitrary units. If one of these instruments had a larger coefficient of variation than the other, it would suffice to make an appropriate nonlinear change in scale (such as logarithmic exponential, or raising to an appropriate power) to reverse the order of superiority of the two instruments from the viewpoint of coefficient of variation. Sensitivity, on the other hand, is invariant, no matter what transformation is made in the scale of measurement.

Spectrophotometric Applications

Sensitivity has the further advantage of focusing attention on the importance of the behavior of the experimental error in any type of measurement. For example, in spectrophotometric work, when Beer's law is applicable, it is customary to select concentrations in the neighborhood of 37% transmittance for the measurement of transmittance or absorbance.

The reason for this choice is the fact that the quantity $\frac{1}{c} \times \frac{dc}{dI}$, where c is concentration and I is transmitted intensity, is smallest when $\frac{I}{I_0} = e^{-1} = 0.37$, provided that the equation expressing Beer's law, $\frac{I}{I_0} = e^{-kc}$, holds. When this line of reasoning is used, the underlying assumption—viz., the constancy of the error of transmittance measurements over the entire range of transmittance values—is not immediately apparent. The sensitivity criterion can be applied to this problem in the following manner:

In the above formulation of Beer's law, the concentration in the cell c results generally from dilution of the original unknown concentration c_0 , by a dilution factor, D . Thus, $c = \frac{c_0}{D}$. The problem is to select D in such a way that the sensitivity of the measurement, $\frac{I}{I_0}$, is maximized. Since

$$\frac{I}{I_0} = e^{-Kc} = e^{-K\frac{c_0}{D}}$$

the slope of $\frac{I}{I_0}$ with respect to c_0 (neglecting the sign) is:

$$\frac{K}{D} e^{-K\frac{c_0}{D}}$$



The Westphal balance is useful in making density determinations

The sensitivity is therefore:

$$\frac{\frac{K}{D} e^{-K\frac{c_0}{D}}}{\sigma I/I_0}$$

The maximum of this function, with respect to D , is obtained when the numerator is the maximum, provided that the denominator is constant. It is easily found that this maximum occurs when $D = kc_0$ —i.e., when $\frac{I}{I_0} = e^{-1} = 0.37$.

But if the behavior of the error term is different—i.e., if the standard deviation of transmittance varies systematically from one end of the scale to the other—the maximum value of the sensitivity no longer occurs at the transmittance value of 37%.

The sensitivity of a method applies to a single determination. The average of n determinations has a sensitivity increased by the factor \sqrt{n} . Thus, if method A has a sensitivity twice that of method B, a single determination by method A is equivalent to four determinations by method B. Such information is especially valuable in the over-all evaluation of alternative methods of measurement, which should be based not only on their technical merit but also on economic considerations. The over-all cost to obtain analytical information with a desired degree of confidence is equal to the cost per determination multiplied by the number of determinations necessary to attain this confidence. Thus, sensitivity provides an objective means of making rational decisions in the choice of methods of measurement.

Literature Cited

- (1) Mandel, J., Stiehler, R. D., *J. Research Natl. Bur. Standards* 53, 155 (1954), RP 2527; *Natl. Bur. Standards Tech. News Bull.* 40, 139 (1956).

For chemists who are never satisfied...



Photograph Courtesy of the Fleischmann Laboratories, Standard Brands Incorporated

S & S Analytical Filter Papers make possible new standards of precision

New techniques in industry now demand added precision in the analytical laboratory—especially in quality control. S & S Analytical Filter Papers offer unique and exclusive advantages to the chemist who is never satisfied.

FOR QUANTITATIVE GRADES — less than 0.007% ash; 97-98% alpha cellulose content. **FOR ALL GRADES**—Standardization for consistent accuracy. Selection from a wide range of types.

Every year many chemists who were once satisfied with other brands standardize their procedure on S & S Analytical Filter Papers for consistent accuracy lot after lot, large selection of grades, and the confidence that goes with using the finest. And the finest costs no more.

Even if you are satisfied with your present filtration products, you owe it to yourself and your work to try S & S Analytical Filter Papers and filtration specialties and compare. Send for the free S & S Analysis Kit sampler described below. It costs you nothing, and there is no obligation.

FREE Analysis Kit

Sampler with large assortment of 11 cm circles in wide selection of grades. Several quantitative papers included in sampler also are available in sheets or strips for chromatography and electrophoresis.

Carl Schleicher & Schuell Co.
Keene, New Hampshire, Dept. AC-47
Please send FREE S & S Analysis Kit SAMPLER.

(Name)	(Position)
(Company)	
(Address)	
(City)	(State)
<input type="checkbox"/> Also send FREE S & S Quick Reference Catalog.	

Circle Nos. 19 A-1, 19 A-2 on Readers' Service Card, page 77 A

