Problems with Correlations in Relative Data and a Proposed Alternative

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November 3, 2015

The Papers

Proportionality: A Valid Alternative to Correlation for Relative Data
David Lovell, Vera Pawlowsky-Glahn, Juan Jos Egozcue, Samuel Marguerat, and Jrg Bhler
PLoS Computational Biology 11(3) (2015)

Proportions, Percentages, PPM: Do the Molecular Biosciences Treat Compositional Data Right?
Lovell DR, Muller W, Taylor JM, Zwart AB, Helliwell CA
Compositional data analysis: Theory and applications p193-207 (2011)

Section 1

Introduction to the Problem

What is Relative Data? –Also called "Compositional Data"

- Compositional data are vectors of non-negative components showing the *relative* weight or importance of a set of *parts in a total*
- The total sum of a compositional vector is considered irrelevant, or an artifact of the sampling procedure.
- No individual component can be interpreted isolated from the other. A composition carries no absolute information on increment/decrement of mass.

Is *my* data relative??

Relative data arises naturally in many biological measurements:

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Then your data might be relative!

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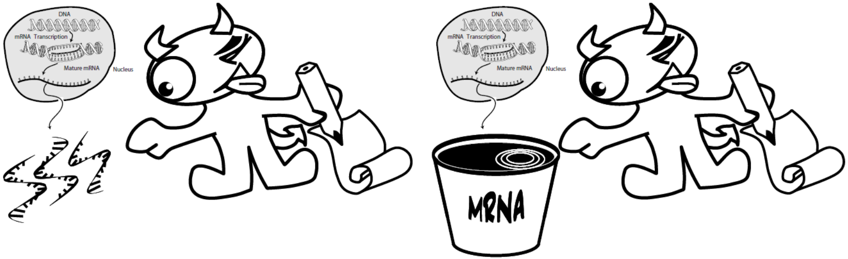
Long answer: It depends...

In certain cases it doesn't matter much but in others it matters a lot.

The ‘Omics Imp’

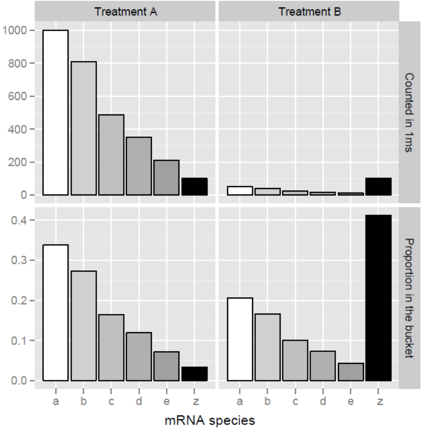
Lovell et al.



- On the left the imp tallies sequences as they are produced in a fixed time period
- On the right the imp counts the sequences in some fixed size bucket
    - Data on the right are parts of a total

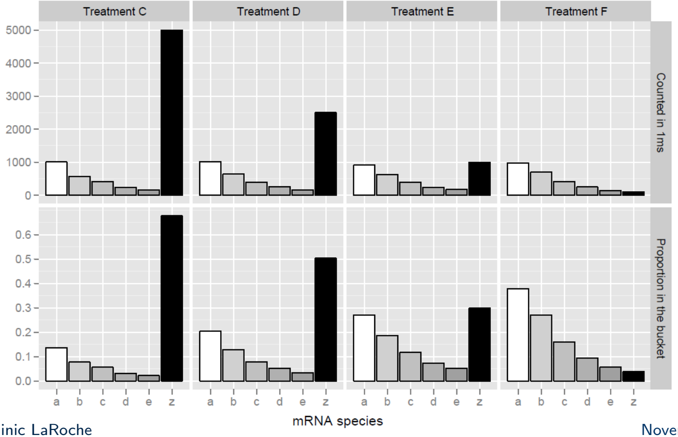
Relative data can be misleading

Example 1.



Relative data can be misleading

Example 2.



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- $c_3 \gg c_1, c_2$
  - Here $var\left(c_3\right)$ dominates the composition
  - Correlation is biased high as $var\left(c_3\right) \uparrow$

David Lovell's Take

Yes, I am going to show a slide show during a slide show.

David Lovell scaring you about correlating relative data

A little background before the proposed alternative: Model 2 regression

Also known as Standardized Major Axis (SMA) Regression

- Traditional regression has 1 random variable: $Y = \beta_0 + \beta X + \epsilon$
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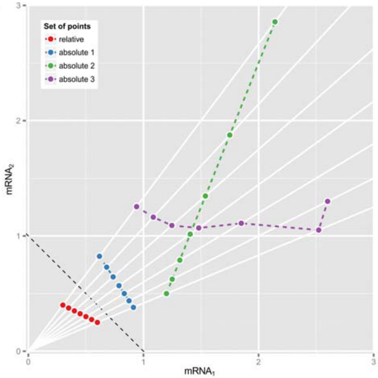
- SMA regression gives the relationship between to random variables
  - Accounts for the random error in both variables
  - Slope estimate: $\beta = \frac{sd(Y_1)}{sd(Y_2)}$

Correlations on Relative Data



Correlations on relative data tell us absolutely nothing about the relationship between the absolute abundances.

The proposed alternative to correlation

The authors propose “proportionality”, ϕ, as a substitute for correlation.

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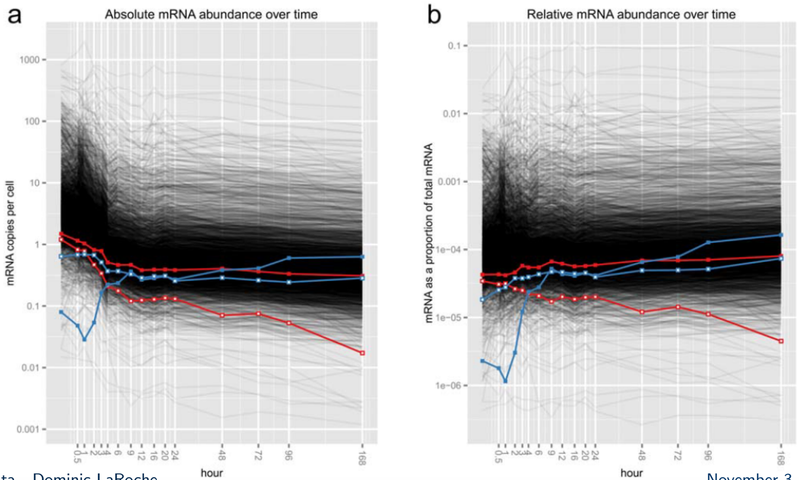
- They drop the unnecessary term to get ϕ:

$$\phi(log(x), log(y)) = (1 + \beta^2 - 2\beta|r|)$$

Benefits of proportionality

- Derived from Aitchison's log-ratio variance
- Composed of two established metrics of association
- However, ϕ is not symmetric like ρ

Yeast Example



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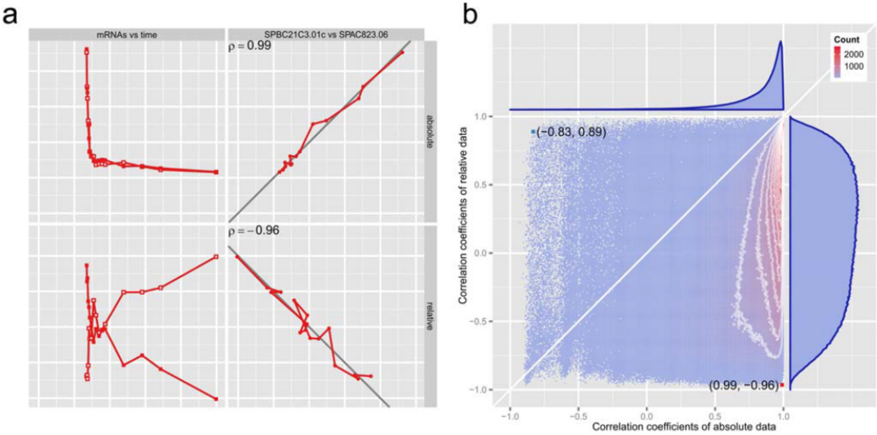

When can I mostly ignore the relative nature of my data?

Relative data aren't always a problem:

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- Only using univariate statistics (e.g. variance)
- log-transformation can *help* (due to the properties of the log)

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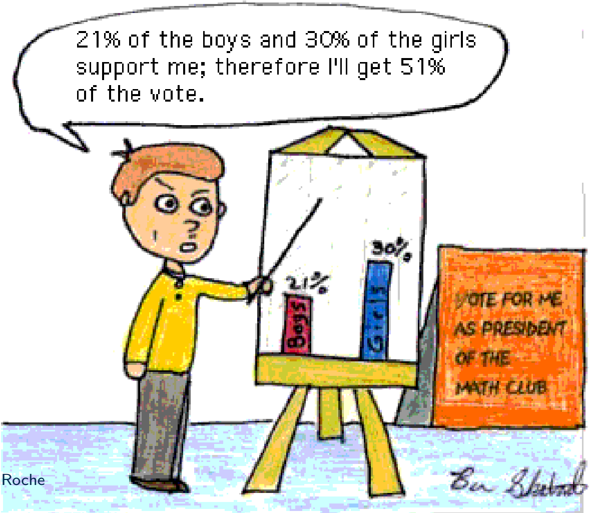
Do you feel lucky? -*Dirty Harry*

Questions?


21% of the boys and 30% of the girls support me; therefore I'll get 51% of the vote.
21%
Boys
30%
Girls
VOTE FOR ME AS PRESIDENT OF THE MATH CLUB