1 Important Results

Sequence Convergence: If $a_1 ... a_n$ converge to a as n $\to \infty$ then:

$$\lim_{n\to\infty}\left(1+\frac{a_n}{n}\right)^n = e^a,\ \lim_{n\to\infty}\left(1-\frac{a_n}{n}\right)^n = e^{-a},\ \sum\frac{a^x}{x!} = e^a,\ \sum\frac{xa^x}{x!} = ae^a \text{ and, } \sum\frac{x^2a^x}{x!} = ae^a$$

$$\sum\frac{(1-a)^x}{x!} = e^{1-a}-1,\ \int_{-\infty}^{\infty} e^{\frac{x^2}{2}} = \sqrt{2\pi}??$$

Stirling's Formula:

$$n! \approx \sqrt{2\pi}n^{n+(1/2)}e^{-n}$$

Binomial Results

$$\sum_{k=0}^{\alpha}\binom{\alpha}{k}z^k = (1+z)^\alpha, (|z|<1),\ \sum_{k=0}^{n}\binom{n}{k} = 2^n,\ \sum_{k=0}^{n}\binom{k}{m} = \binom{n+1}{m+1}$$

Geometric Series

$$\sum_{k=0}^{n-1}ar^k = a\frac{1-r^n}{1-r},\ \sum_{k=0}^{\infty}ar^k = \frac{a}{1-r},\ \text{for } |r|<1,\ \sum_{k=0}^{\infty}kr^k = \frac{1}{(1-r)^2},\ \sum_{k=a}^{b}r^k = \frac{r^a-r^{b+1}}{1-r}$$

Differentiating Under the Integral:
if f(x,θ), a(θ), and b(θ) are diferentiable with respect to θ, then

$$\frac{d}{d\theta}\int_{a(\theta)}^{b(\theta)} f(x,\theta)dx = f(b(\theta),\theta)\frac{d}{d\theta}b(\theta) - f(a(\theta),\theta)\frac{d}{d\theta}a(\theta) + \int_{a(\theta)}^{b(\theta)}\frac{\delta}{\delta\theta}f(x,\theta)dx$$

If a(θ) and b(θ) are constant the first two terms are 0 and only the final term is needed. This result can be applied if $f(x,\theta)$ is differentiable for all values of θ.

Gamma Results:

$$\Gamma(\alpha) = \int_0^{\infty} t^{\alpha-1}e^{-t}dt$$

$$B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt = \frac{(x-1)!(y-1)!}{(x+y-1)!} = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

$\Gamma(\alpha+1) = \alpha\Gamma(\alpha)$, $\alpha > 0$
$\Gamma(n) = (n-1)!$
$\Gamma(1/2) = \sqrt{\pi}$
$f(t) = \frac{t^{\alpha-1}e^{-t}}{\Gamma(\alpha)}$, $0 < t < \infty$, is a PDF.
$\int_0^\infty x^{\alpha-1}e^{-x/\beta}dx = \Gamma(\alpha)\beta^\alpha$.
For any constants a and b: $P(a < X_{\alpha,\beta} < b) = \beta(f(a|\alpha,\beta) - f(b|\alpha,\beta)) + P(a < X_{\alpha-1,\beta} < b)$,(Can use this repeatedly to get to an integral that can be evaluated analytically if α is an integer)
For $X \sim$gamma(α,β), $\bar{X} \sim$gamma$(n\alpha,\beta/n)$
$(n-1)S^2 \sim$Gamma$(\frac{n-1}{2},2)$ and $S^2 \sim$Gamma$(\frac{n-1}{2},\frac{2}{n-1})$

Normal Results:
$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-z^2/2}dz = 1$
$\int_0^{\infty} e^{-z^2/2}dz = \sqrt{\frac{\pi}{2}}$
Stein's Lemma: If $X \sim n(\theta, \sigma^2)$, $E\left[g(x)(X-\theta)\right] = \sigma^2 E(g'(X))$
The sum of normal random variables is distributed N($\sum \mu_i$, $\sum \sigma_i^2$)
Sums abd differences of independent normal r.v.'s are independent normal r.v's if the vars are equal.
$\bar{X}$ ~n($\mu, \sigma^2/n$) and $S^2 \sim \chi^2_{n-1}$ are independent random variables.
For $X_j \sim n(\mu_j, \sigma_j^2)$ indep. and constants a_{ij} and b_{rj} with j=1...m, i=1...k, and r=1...m where $k+m \leq n$, define:

$U_i = \sum_{j=1}^{n} a_{ij}X_j$, i=1...k
$V_r = \sum_{j=1}^{n} b_{rj}X_j$, r=1...m

a. U_i and V_r are indep. iff $Cov(U_i, V_r) = 0$ where $Cov(U_i, V_r) = a_{ij}b_{rj}\sigma_j^2$

b. The random vectors U and V are indep. iff U_i is indep. of V_r for all pairs i,r

For X_i ~n(μ_i, σ_i^2), $Z = \sum(ax_i + b_i) \sim n\left(\sum(a_i\mu_i + b_i), \sum a_i^2\sigma_i^2\right)$
For $Y_i = \beta x_i + \epsilon$ where $\epsilon \sim$ n(0,σ^2), Y ~n($\beta x, \sigma^2$)

Poisson Results:
Recursive Poisson formula: $P(X = x+1|\lambda) = \frac{\lambda}{x+1}P(X = x)$
If X ~Poisson(θ) and Y ~Poisson(λ) are independent then, $X + Y = Z$ ~Poisson($\theta + \lambda$).

χ^2 **Results**:
For any function $h(x)$, $E\left[h(\chi_p^2)\right] = pE\left(\frac{h(\chi^2_{p+2})}{\chi^2_{p+2}}\right)$
The sum of $\chi^2_{p_i}$ random variables is distributed $\chi^2_{\sum p_i}$

F Results:
If $X \sim F_{p,q}$ then $1/X \sim F_{q,p}$
If $X \sim t_q$ then $X^2 \sim F_{1,q}$
IF $X \sim F_{p,q}$ then $\frac{(p/q)X}{1+(p/q)X} \sim \beta(p/2, q/2)$

Beta Results:

$$B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt = \frac{(x-1)!(y-1)!}{(x+y-1)!} = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

t Results:
$t \sim \frac{X}{\sqrt{V/p}}$, where X ~n(0,1) and $V \sim \chi^2_p$
$t_p^2 = \frac{X^2/1}{\chi^2_p/p} = \frac{\chi^2/1}{\chi^2_p/p} \sim F_{1,p}$