Problem Set 1: Demand Estimation
ECON 853 Fall 2022
Due Sep 30, 2022

This problem set is designed to help you understand the nuts and bolts of discrete choice demand models and imperfect competition. In this exercise, we will use simulated fake data so that you know the true distributions and then we will try alternative approaches to estimation. Please read the full problem set before starting, as it may save some heart-ache.

Market Simulation

Demand

Consumer i's preferences for product j in market m are assumed to take the following form:

$$
\begin{aligned}
U_{ijm} &= X_{jm}\beta - \alpha_i p_{jm} + \xi_{jm} + \epsilon_{ijm} \\
\alpha_i &= \alpha + \sigma_\alpha v_{ip} \\
U_{i0} &= 0
\end{aligned}
\tag{1}
$$

where X_{jm} is the vector of characteristics of product j in market m. In our example, it is a three dimensional vector with the first element being a constant. p_{jm} denotes prices and ξ_{jm} is the product-market specific demand shock.

We assume that the consumer taste shocks are iid with distributions

- $v_{ip} \sim LN(0,1)$, where LN is the lognormal distribution

- ϵ_{ijm} is drawn from a Type I Extreme Value distribution.

Supply

All products are produced by single-product firms. The markets are regional, while the firms are national. The marginal cost of producing product j in market m is given by:

$$
MC_{jm} = \gamma_0 + \gamma_1 W_j + \gamma_2 Z_{jm} + \eta_{jm}
\tag{2}
$$

W_j is a common cost shifter for firm j across all markets. Z_{jm} are the market specific supply shifters. We assume that $\eta_{jm} \sim N(0,1)$.

Data

The researcher observes product characteristics for each market (X), its price in each market (p) and market shares, which we denote by s_{jm}. She also observes common and market specific cost shifters - W and Z. The demand parameters of interests are $\theta = (\beta, \alpha, \sigma_\alpha)$. We will also be interested in estimating the supply-side parameters γ in the latter part of the pset.

Problem 0: Simulate Data

Generate fake data - $< X_{jm}, p_{jm}, s_{jm}, W_j, Z_{jm} >$ - for $j = 3$ and $m = 100$ assuming

1. Product characteristics are exogenous and are drawn from the following distributions.

   - $X_{jm} = (X_{1jm}, X_{2jm}, X_{3jm})$, with $X_{1jm} = 1$ (a constant).
   - $X_{2jm} \sim U[0, 1]$ and $X_{3jm} \sim N(0, 1)$

2. Demand shocks are iid, with $\xi_{jm} \sim N(0, 1)$

3. $W_j \sim N(0, 1)$, and $Z_{jm} \sim N(0, 1)$.

4. True Parameters are $\beta = (5, 1, 1)$, $\alpha = 1$, $\sigma_\alpha = 1$, and $(\gamma_0, \gamma_1, \gamma_2) = (2, 1, 1)$.

5. Firms behave oligopolistically in each market, that is, $\frac{p_{jm} - MC_{jm}}{p_{jm}} = \frac{-1}{\frac{d \ln s_{jm}}{d \ln p_{jm}}}$.

   (You would need to derive the market shares s_{jm} for this part as a function of mean utilities $\{\delta_{jm}\}_{j=1,2,3}$ where $\delta_{jm} = X_{jm}\beta - \alpha_i p_{jm} + \xi_{jm}$. You should use the expression for s_{jm} along with the stated relation between lerner index and elasticities to solve for equilibrium prices and shares.)

Problem 1: BLP and Hausman Instruments

1. Consider the following moment conditions: $E[\xi|X] = 0$, $E[\xi|p] = 0$ and $E[\xi|W, Z] = 0$.

   (a) Given you know the true model, which of these moment conditions are valid? Which are relevant? Why?

(b) Can you use BLP and Hausman instruments in this setting? Discuss their pros and cons for identification of demand side parameters.

2. Estimate θ a.la BLP using competitor characteristics $E[\xi_{jm}|X_{-j,m}] = 0$, and cost shifters $E[\xi_{jm}|W_j, Z_{jm}] = 0$ as instruments.

   (a) Write down the moments and construct your objective function.

   (b) Estimate θ. Report the point estimate for each parameter.

   (c) Write the expressions for own–price and cross–price elasticities of demand. Also write pseudocode that describes how you would write a function to calculate the elasticities.

   (d) Write the expression for consumer surplus. Also write pseudocode that describes how you would write a function to calculate consumer surplus.

3. Suppose you estimated θ assuming incorrectly that $E[\xi|p] = 0$ in each market. How would you expect the parameter estimates to compare to true values and the ones obtained using instruments?

Problem 2: Adding the supply-side

1. Estimate the mark-ups and hence marginal costs, given the estimated θ (Note that the lerner index of a firm always equals the elasticity of demand *faced by the firm*):

   (a) Write down marginal costs under the three pricing assumptions: 1) perfect competition, 2) perfect collusion, and 3) oligopoly.

   (b) Compute marginal costs a make a boxplot for each version of marginal costs, including the truth.

2. Suppose you now wanted to estimate θ and γ jointly while assuming that $E[\xi, \eta|X, W] = 0$.

   (a) Write pseudo-code for estimating θ and γ under each of the assumptions in part 1(a).

   (b) How would you use your estimates from 2(a) to test the three assumptions in part 1(a)?

Problem 3: Merger exercise

1. Suppose firm 1 and firm 2 plan to merge.

   (a) Write down the merged firm's pricing equation.

   (b) Explain the terms in the equation for the merged firm's prices.

   (c) Simulate the merger and compare prices pre and post merger.