ECON 8853 — Industrial Organization   Fall 2022

Problem Set 1 — Demand Estimation

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Problem 0: Simulate Data

For a multinomial logit model with random coefficients, we know that the market shares

$$s_{ijm} = \frac{e^{\delta_{jm}+\mu_{ijt}}}{1+\sum_k e^{\delta_{km}+\mu_{ikt}}} \tag{1}$$

Therefore

$$\frac{ds_{ijm}}{dp_{jm}} = -\alpha_i s_{ijm}(1-s_{ijm}) \tag{2}$$

Integrate over consumer characteristics to get

$$\frac{ds_{jm}}{dp_{jm}} = -\int_v \alpha_i s_{ijm}(1-s_{ijm})dP_v^*(v) \approx -\frac{1}{ns}\sum_{i=1}^{ns} \alpha_i s_{ijm}(1-s_{ijm}) \tag{3}$$

The stated relation between Lerner Index and elasticities tells us (after simplifying the elasticities)

$$\frac{ds_{jm}}{dp_{jm}} = -\frac{s_{jm}}{p_{jm}-MC_{jm}} \tag{4}$$

Combine (3) and (4) we get

$$p_{jm} \approx MC_{jm} + \frac{1}{\frac{1}{ns}\sum_{i=1}^{ns} \alpha_i s_{ijm}(1-s_{ijm})} s_{jm} \tag{5}$$

Define

$$\boldsymbol{p_m} = \begin{pmatrix} p_{1m} \\ \vdots \\ p_{Jm} \end{pmatrix} \tag{6}$$

$$\boldsymbol{s_m} = \begin{pmatrix} s_{1m} \\ \vdots \\ s_{Jm} \end{pmatrix} \tag{7}$$

$$\boldsymbol{MC_m} = \begin{pmatrix} MC_{1m} \\ \vdots \\ MC_{Jm} \end{pmatrix} \tag{8}$$

where

$$MC_{jm} = \gamma_0 + \gamma_1 Wj + \gamma_2 Z_{jm} + \eta_{jm} \tag{9}$$

$$\boldsymbol{\Omega_m} = \begin{pmatrix} \omega_{1m} & & \\ & \ddots & \\ & & \omega_{Jm} \end{pmatrix} \tag{10}$$

where

$$\omega_{jm} = \frac{1}{\frac{1}{ns}\sum_{i=1}^{ns} \alpha_i s_{ijm}(1 - s_{ijm})} \tag{11}$$

Therefore, we know

$$\boldsymbol{p_m} = \boldsymbol{MC_m} + \boldsymbol{\Omega_m s_m} \tag{12}$$

Note also that

$$s_{jm} = \int_v \frac{e^{\delta_{jm}}}{1 + \sum_k e^{\delta_{km}}} dP_v^*(v) \approx \frac{1}{ns}\sum_{i=1}^{ns} \frac{e^{\delta_{jm}}}{1 + \sum_k e^{\delta_{km}}} \tag{13}$$

Here

$$\boldsymbol{\delta_m} = \begin{pmatrix} \delta_{1m} \\ \vdots \\ \delta_{Jm} \end{pmatrix} \tag{14}$$

where

$$\delta_{jm} = \boldsymbol{X_{jm}\beta} - \alpha_i p_{jm} + \xi_{jm} \tag{15}$$

To back-up prices, we look for the roots of $G(\boldsymbol{p_m}) = \boldsymbol{MC_m} + \boldsymbol{\Omega_m s_m} - \boldsymbol{p_m}$ for each market.

Problem 1: BLP and Hausman Instruments

1

(a) The first moment condition $E[\xi|X] = 0$ is invalid because firms choose their observed and unobserved product characteristcs together. Hence, products with a certain observed characteristic profile may be more likely to have high unobserved quality. The second moment condition $E[\xi|p] = 0$ is also invalid because prices are a function of marginal cost and the markup term, and the markup term is determined in part by unobserved product quality.

Finally, the use of more traditional instruments in the moment condition $E[\xi|W, Z]$ is valid, as firms do not choose product characteristics based on cost shifters.

(b) We can use BLP instruments (i.e., characteristics of other products in the same market) because they are both relevant and valid. Hausman instruments are valid when markets experience shocks independently, but they are invalid when shocks are correlated between markets, so the question of whether Hausman instruments are valid or not depends entirely on the particular empirical context.

2

We estimate the following

$$\min_{\theta} \|s(x, p, \delta(x, p\xi, \theta_1 \mid \theta_2)) - S\| \tag{16}$$

where $[\theta_1 = \sigma_\alpha, \theta_2 = (\alpha, \boldsymbol{\beta})]$. Following BLP and Nevo, we exploit the fact that the parameters enter respectively in a non-linear and linear fashion to simplify the computation.
Given an initial guess of $(v_{ip}, \boldsymbol{\delta}^0, \theta_2^0, \Phi)$, we search over θ_1 to minimize the GMM objective function

$$\omega(\theta_1, \theta_2^0)' Z \Phi^{-1} Z' \omega(\theta_1, \theta_2^0) \tag{17}$$

where $Z = [X_1\, X_2\, X_3\, BLP_1\, BLP_2\, W\, Z]$ is the vector of instruments. Our system of equations is over-identified, as we are using 4 instruments for the price. The BLP instruments are constructed by summing products $J - j$ characteristics (excluding the intercept) for all j products.
For each candidate of θ_1, we find the optimal $\boldsymbol{\delta}$ implied by the contraction mapping proposed in BLP and compute the GMM objective function.
Having found the non-linear parameter, we estimate a 2-stage efficient GMM over the linear parameters θ_2.
The estimates we obtain are

β_1	β_2	β_3	α	σ_α
5.281	1.437	1.02	-0.75	1.35

Table 1: BLP Estimates

The reasons we stil get noisy estimates of the true parameters can be: (i) numerical error propagation from bounding shares/marginal costs; (ii) approximation error through simulation of the integrals; (iii) consistency, as the number of products is very small and we need a sufficiently large J to get closer to the true estimates (according to BLP asymptotic results); (iv) bad in-sample 'luck' due to the chosen seed (and random draws). Overall, though, the direction and the magnitudes approximately replicate the true values.

Total consumer surplus should be

$$CS = \sum_{i=1}^{ns} \sum_{j=1}^{J} \sum_{m=1}^{M} (U_{ijm} - p_{jm}) \tag{18}$$

When we have the optimal $\boldsymbol{\delta}$, we can again back out demand shock $\boldsymbol{\xi}$, and we get estimate for $\boldsymbol{U}$, which combined with the price vector $\boldsymbol{p}$, gives us the estimate for total consumer surplus.

3

The demand elasticities are

$$
\eta_{jkm} = \frac{\partial s_{jm}}{\partial p_{km}} \frac{p_{km}}{s_{jm}} \tag{19}
$$

$$
= \begin{cases} -\frac{p_{jm}}{s_{jm}} \int_{v_{ip}} a_i s_{ijm}(1 - s_{ijm}) dF(v) \text{ if } j = k \\ \frac{p_{km}}{s_{jm}} \int_{v_{ip}} a_i s_{ijm} s_{ikm} dF(v) \quad \text{o.w.} \end{cases} \tag{20}
$$

$$
\approx \begin{cases} -\frac{p_{jm}}{s_{jm}} \frac{1}{ns} \sum_{ns} a_i s_{ijm}(1 - s_{ijm}) \text{ if } j = k \\ \frac{p_{km}}{s_{jm}} \frac{1}{ns} \sum_{ns} a_i s_{ijm} s_{ikm} \quad \text{o.w.} \end{cases} \tag{21}
$$

$$
\tag{22}
$$

we can estimate the following elasticities using observed prices and shares and approximate the integral by simulation (the logic is very similar to Problem 0).

4

If we were to estimate the model using prices assuming orthogonality with respect to ξ_{jm}, we would end up having biased and inconsistent estimates. The direction of the bias depends on the covariance between ξ_{jm} and p_{jt}. We performed a similar exercise in Problem Set 0: positive covariance implies a downward bias, while negative covariance implies the opposite.

Problem 2: Adding the Supply-Side

1

(a) Under perfect competition

$$
MC_{jm} = MR_{jm} = p_{jm} \tag{23}
$$

Under perfect collusion, single product firms work together as if all products belong to one firm. The profit maximization problem is

$$
\max_{p_{1m},\cdots,p_{Jm}} \sum_{j=1,\cdots,J} (p_{jm} - MC_{jm}) s_{jm} \tag{24}
$$

The FOC is

$$
s_{jm} + \sum_{k=1,\cdots,J} (p_{km} - MC_{km}) \frac{ds_{km}}{dp_{jm}} = 0 \tag{25}
$$

which is equivalent to the matrix form

$$
\boldsymbol{s_m} + \boldsymbol{\Delta_m}(\boldsymbol{p_m} - \boldsymbol{MC_m}) = \boldsymbol{0} \tag{26}
$$

where $\boldsymbol{s_m}$, $\boldsymbol{p_m}$, and $\boldsymbol{MC_m}$ are defined in Problem 0, and

$$\boldsymbol{\Delta_m} = \begin{pmatrix} \frac{ds_{1m}}{dp_{1m}} & \cdots & \frac{ds_{Jm}}{dp_{1m}} \\ \vdots & \ddots & \vdots \\ \frac{ds_{1m}}{dp_{Jm}} & \cdots & \frac{ds_{Jm}}{dp_{Jm}} \end{pmatrix} \tag{27}$$

$$\frac{ds_{jm}}{dp_{km}} = \begin{cases} \int_{v_i} \alpha_i s_{ijm} s_{km} dP(v_i), & \text{if } k \neq j \\ -\int_{v_i} \alpha_i s_{ijm}(1 - s_{ijm}) dP(v_i), & \text{if } k = j \end{cases} \tag{28}$$

From the above equation we get

$$\boldsymbol{MC_m} = \boldsymbol{p_m} + \boldsymbol{\Delta_m^{-1}} \boldsymbol{s_m} \tag{29}$$

Under oligopoly, single product firm j maximizes its own profit

$$\max_{p_{jm}} (p_{jm} - MC_{jm}) s_{jm} \tag{30}$$

The FOC is

$$s_{jm} + (p_{jm} - MC_{jm}) \frac{ds_{jm}}{dp_{jm}} = 0 \tag{31}$$

which gives

$$MC_{jm} = p_{jm} + \frac{1}{\frac{ds_{jm}}{dp_{jm}}} s_{jm} \tag{32}$$

Finally, the mark-ups are given by

$$p_{jm} - MC_{jm} = \begin{cases} 0, & \text{perfect competition} \\ -\boldsymbol{\Delta_m^{-1}}[\boldsymbol{j}, :]\boldsymbol{s_m}, & \text{perfect collusion} \\ -\frac{1}{\frac{ds_{jm}}{dp_{jm}}} s_{jm}, & \text{oligopoly} \end{cases} \tag{33}$$

From the markup we can backup the marginal cost, given that we observe prices, shares and we can compute the share elasticities (through the optimal δ we computed in the BLP estimation).
From what we understand, the scope of the exercise is to assess the effect that misspecification can have on marginal cost estimates under different conduct assumptions. We simulated the data under the true world of oligopoly conduct, and we are assessing how marginal costs differ if we assume a different (wrong) conduct.
The marginal cost distributions are

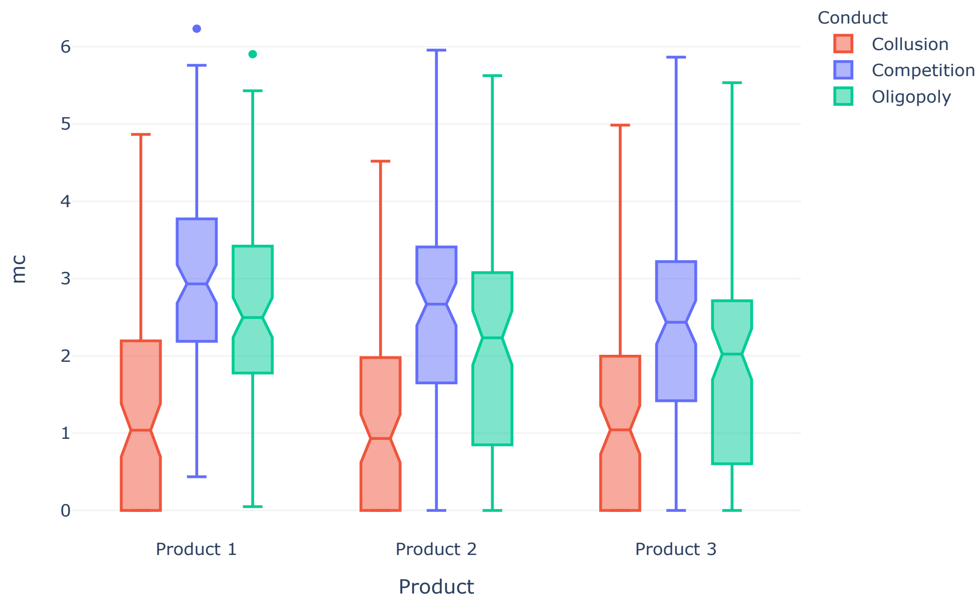

Figure 1: Marginal Costs Under Different Conducts

2

(a) In addition to demand moments, we add supply moments in the GMM.
We now have a multiple (two) equation GMM with the following moment conditions:

$$\begin{bmatrix} \mathbb{E}\left(\xi_{jm} Z_{jm}\right) \\ \mathbb{E}\left(\eta_{jm} Z_{jm}\right) \end{bmatrix} = 0_{2dim(Zjm)\times 1} \tag{34}$$

The two equations share the same instruments given the assumptions, so the model is still over-identified.
We cannot estimate the system equation by equation since the choice of the weighting matrix will affect our results. We should still be able to jointly estimate it by searching over the non linear parameters in the first equation and then perform a 2-step efficient GMM estimating the two equations together, which enter linearly in the problem.
The choice of the dependent variable depends entirely on the assumptions about market conduct, so the code would be the same with just different marginal costs.
(b) It is not clear to us what this question is asking: if we estimate the model under each of these three specifications, testing it again different models will almost always accept the null.

Problem 3: Merger Exercise

Let

$$\Omega_m = \begin{pmatrix} \frac{1}{|\eta_{11m}|} & \frac{1}{|\eta_{12m}|} & 0 \\ \frac{1}{|\eta_{21m}|} & \frac{1}{|\eta_{22m}|} & 0 \\ 0 & 0 & \frac{1}{|\eta_{33m}|} \end{pmatrix}$$

Then, $p_{1m} = MC_{1m} + \Omega s_{1m}$, and $p_{2m} = MC_{2m} + \Omega s_{2m}$. In each equation, the matrix Ω includes the off-diagonal terms in the (1,2) and (2,1) place because products one and two are priced as if owned jointly. This means that the optimal price for each product factors in the effect on the other product's share.

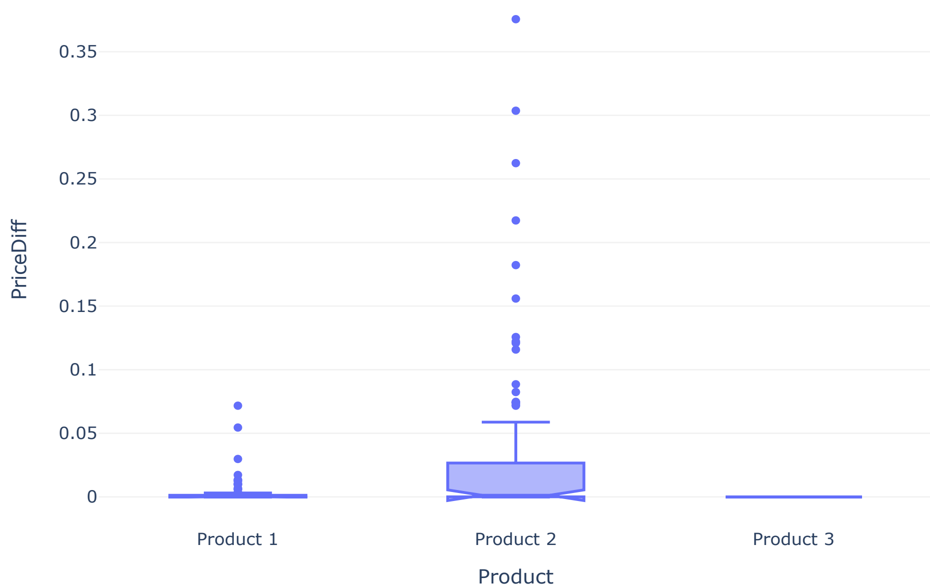

Figure 2: Price Differences Post-Merger

Prices of product 1 and 2 increase as the market share increases, while prices of product 3 stays the same as firm 3 keeps its original market share.