Problem Set 0: Warm-up

ECON 535, Fall 2022
Due: Friday, Septmeber 9

The goal of this problem set is to help you get started on coding in the language of your choice. The problem set presents you with a simple model - a system which takes parameter values as inputs and gives a data generating process as output. The first part of the problem set asks you to simulate the model given the true parameter values. That is, it asks you to generate fake data assuming that you know the data generating process. The second part of the questions asks you to recover the parameter values using the fake data.

You can work on assigments for this class in groups of up to three. You should take this as an opportunity to practice managing projects efficiently and exercise version control via Git. The TA will hold a special session introducing Git.

1 Demand and Supply

Here, we lay down a very simple model of demand and supply that you might have seen many times. Consumer preferences at any period t are represented by the following demand curve -

$$\ln Q_t = \beta_1 - \beta_2 \ln P_t + \epsilon_t^D$$

where ϵ_t^D represent demand shocks. The production side of the economy is represented by the following supply curve

$$\ln P_t = \delta \ln Q_t + \ln \mu - \ln a_t$$

where $\ln a_t$ represent supply shocks. We assume that demand and supply shocks are i.i.d with

$$\epsilon_t^D \sim N(0, \sigma^D);$$

$$\ln a_t \sim N(0, \sigma^S).$$

In this question, we use upper case letters to refer to a random variable and lower case letters to their realisations. We use bold letters whenever we want to denote vectors.

1.1 Generate Data

1. Fix parameter values $(\beta_1, \beta_2, \delta, \mu, \sigma^D, \sigma^S) = (0, 2, 0.2, 0, 1, 1)$
2. Generate a vector of $T = 50$ draws each for demand shocks ϵ_t^D and supply shocks a_t.
3. Derive log prices as function of the demand/supply shocks and parameters. Given the shock realisations, generate the price data.
4. Use the price vector and either of the price-quantity relations to generate the quantity data.
5. We have generated $\boldsymbol{w}$ where $w_t \equiv (q_t, p_t)$. Plot the empirical distributions for p_t and q_t.

1.2 Recover Parameters

In this part, you should treat the fake data as the data observed by the econometrician.We are interested in estimating the demand elasticity β_2.

1.2.1 OLS/IV

1. Report the OLS estimate of β_2 by regressing $\ln q_t$ on $\ln p_t$. Call it β_2^{OLS}.
2. Under what condition is the OLS estimate consistent? Given you know the full model of data generation - is the consistency condition met?
3. Suppose that we observe a component of the supply shocks - Z_t such that $\ln a_t = \gamma \ln Z_t + \ln a_t^u$.[1] Given $\gamma > 0$ and $E(\epsilon_t^D . \ln Z_t) = 0$, propose an alternative methodlogy to get a consistent estimate of β_2.

1.2.2 Method of Moments

Simulate fake data for Z. Define random variables $W = (P, Q, Z)$ and fake data as $w_t = (p_t, q_t, z_t)$. Define the vector of random variables $g(W, \theta)$ such that

$$
\begin{aligned}
g^1(W,\theta) &= \ln Z . \ln Q \\
g^2(W,\theta) &= \ln Z . \ln P
\end{aligned}
$$

where θ is the set of unknown parameters. Define $g(\boldsymbol{w}, \theta)$ as empirical realisations of $g(W, \theta)$.

Denote the population moments and their empirical analogs by $E(g(W,\theta))$ and $\tilde{E}(g(\boldsymbol{w},\theta))$ respectively.

1. What is the value of the above population moments given θ.
2. The above moments are analytically easy to compute. But in the class, we will encounter problems where moments of interest involve a complicated integral. Therefore, here we will approximate population moments by simulating the data generating process multiple times.
   (a) Fix θ. Simulate the model $S = 100$ times. For each simulation $s = 1 \ldots S$, you get $w_t^s = (q_t^s, p_t^s)$
   (b) Compute the simulated moments
   $$E^{sim}(g(W,\theta)) = \frac{1}{S}\sum_{s=1}^{S} \tilde{E}(g(\boldsymbol{w}^s, \theta))$$
   Report the simulation error - $E(g(W,\theta)) - E^{sim}(g(W,\theta))$ for each moment.
   (c) Nest steps (a) and (b) in a function
   $$f(\theta) \equiv E^{sim}(g(W,\theta))$$
   .
3. Compute the empirical moments $\tilde{E}(g(\boldsymbol{w},\theta))$ from fake data $\boldsymbol{w}$. Note that in this example, empirical moments do not depend on θ.

[1] For completness, lets assume $\ln Z_t \sim N(o, \sigma^Z)$ and $\ln a_t^u \sim N(0, \sigma^S - \gamma^2 * \sigma^Z)$

4. Search over $\theta = \beta_2$ to minimise the distance (you can choose a norm of your choice) between i) actual and empirical moments, and ii) simulated and emprical moments. That is, report

$$\hat{\theta} = \arg\min_{\theta} \|E(g(W,\theta) - \tilde{E}(g(\boldsymbol{w},\theta))\|$$
$$\hat{\theta}^{sim} = \arg\min_{\theta} \|E^{sim}(g(W,\theta) - \tilde{E}(g(\boldsymbol{w},\theta))\|$$

   Note: When you simulate the model, you essentially simulate $N(0,1)$ supply and demand shocks[2] random variable. You should keep these simulation draws for shocks fixed whenever evaluating the function at a new θ.

5. In the above question, in theory, does choice of $D(.)$ matter? Why or why not?

6. The above moments identify β_2. Can you come up with moment conditions that will identify the supply elasticity δ?

[2] One can always scale up the realisations of the shocks by a constant σ to get draws for a $N(0,\sigma^2)$ random variable.