ECON 8853 — Industrial Organization Fall 2022

Problem Set 1 — Demand Estimation

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Problem 0: Simulate Data

For a multinomial logit model with random coefficients, we know that the market shares

$$s_{ijm} = \frac{e^{\delta_{jm}}}{1 + \sum_k e^{\delta_{km}}} \tag{1}$$

Therefore

$$\frac{ds_{ijm}}{dp_{jm}} = -\alpha_i s_{ijm}(1 - s_{ijm}) \tag{2}$$

Integrate over consumer characteristics to get

$$\frac{ds_{jm}}{dp_{jm}} = -\int_v \alpha_i s_{ijm}(1 - s_{ijm}) dP_v^*(v) \approx -\frac{1}{ns} \sum_{i=1}^{ns} \alpha_i s_{ijm}(1 - s_{ijm}) \tag{3}$$

The stated relation between Lerner Index and elasticities tells us

$$\frac{ds_{jm}}{dp_{jm}} = -\frac{s_{jm}}{p_{jm} - MC_{jm}} \tag{4}$$

Combine (3) and (4) we get

$$p_{jm} \approx MC_{jm} + \frac{1}{\frac{1}{ns} \sum_{i=1}^{ns} \alpha_i s_{ijm}(1 - s_{ijm})} s_{jm} \tag{5}$$

Define

$$\boldsymbol{p_m} = \begin{pmatrix} p_{1m} \\ \vdots \\ p_{Jm} \end{pmatrix} \tag{6}$$

$$\boldsymbol{s_m} = \begin{pmatrix} s_{1m} \\ \vdots \\ s_{Jm} \end{pmatrix} \tag{7}$$

$$\boldsymbol{MC_m} = \begin{pmatrix} MC_{1m} \\ \vdots \\ MC_{Jm} \end{pmatrix} \tag{8}$$

where

$$MC_{jm} = \gamma_0 + \gamma_1 W j + \gamma_2 Z_{jm} + \eta_{jm} \tag{9}$$

$$\boldsymbol{\Omega_m} = \begin{pmatrix} \omega_{1m} & & \\ & \ddots & \\ & & \omega_{Jm} \end{pmatrix} \tag{10}$$

where

$$\omega_{jm} = \frac{1}{\frac{1}{ns}\sum_{i=1}^{ns} \alpha_i s_{ijm}(1 - s_{ijm})} \tag{11}$$

Therefore, we know

$$\boldsymbol{p_m} = \boldsymbol{MC_m} + \boldsymbol{\Omega_m s_m} \tag{12}$$

Note also that

$$s_{jm} = \int_v \frac{e^{\delta_{jm}}}{1 + \sum_k e^{\delta_{km}}} dP_v^*(v) \approx \frac{1}{ns}\sum_{i=1}^{ns} \frac{e^{\delta_{jm}}}{1 + \sum_k e^{\delta_{km}}} \tag{13}$$

Here

$$\boldsymbol{\delta_m} = \begin{pmatrix} \delta_{1m} \\ \vdots \\ \delta_{Jm} \end{pmatrix} \tag{14}$$

where

$$\delta_{jm} = \boldsymbol{X_{jm}\beta} - \alpha_i p_{jm} + \xi_{jm} \tag{15}$$

Problem 1: BLP and Hausman Instruments

1

(a) The first moment condition $E[\xi|X] = 0$ is invalid because firms choose their observed and unobserved product characteristcs together. Hence, products with a certain observed characteristic profile may be more likely to have high unobserved quality. The second moment condition $E[\xi|p] = 0$ is also invalid because prices are a function of marginal cost and the markup term, and the markup term is determined in part by unobserved product quality.

Finally, the use of more traditional instruments in the moment condition $E[\xi|W, Z]$ is valid, as firms do not choose product characteristics based on cost shifters.

(b) We can use BLP instruments (i.e., characteristics of other products in the same market) because they are both relevant and valid. Hausman instruments are valid when markets experience shocks independently, but they are invalid when shocks are correlated between markets, so the question of whether Hausman instruments are valid or not depends entirely on the particular empirical context.

2

Problem 2: Adding the Supply-Side

1

(a) Under perfect competition

$$MC_{jm} = MR_{jm} = p_{jm} \tag{16}$$

Under perfect collusion, single product firms work together as if all products belong to one firm. The profit maximization problem is

$$\max_{p_{1m},\cdots,p_{Jm}} \sum_{j=1,\cdots,J} (p_{jm} - MC_{jm})s_{jm} \tag{17}$$

The FOC is

$$s_{jm} + \sum_{k=1,\cdots,J} (p_{km} - MC_{km}) \frac{ds_{km}}{dp_{jm}} = 0 \tag{18}$$

which is equivalent to the matrix form

$$\boldsymbol{s_m} + \boldsymbol{\Delta_m}(\boldsymbol{p_m} - \boldsymbol{MC_m}) = \boldsymbol{0} \tag{19}$$

where $\boldsymbol{s_m}$, $\boldsymbol{p_m}$, and $\boldsymbol{MC_m}$ are defined in Problem 0, and

$$\boldsymbol{\Delta_m} = \begin{pmatrix} \frac{ds_{1m}}{dp_{1m}} & \cdots & \frac{ds_{Jm}}{dp_{1m}} \\ \vdots & \ddots & \vdots \\ \frac{ds_{1m}}{dp_{Jm}} & \cdots & \frac{ds_{Jm}}{dp_{Jm}} \end{pmatrix} \tag{20}$$

$$\frac{ds_{jm}}{dp_{km}} = \begin{cases} \alpha_i s_{jm} s_{km}, & \text{if } k \neq j \\ -\alpha_i s_{jm}(1 - s_{jm}), & \text{if } k = j \end{cases} \tag{21}$$

From the above equation we get

$$\boldsymbol{MC_m} = \boldsymbol{p_m} + \boldsymbol{\Delta_m^{-1}} \boldsymbol{s_m} \tag{22}$$

Under oligopoly, single product firm j maximizes its own profit

$$\max_{p_{jm}} (p_{jm} - MC_{jm})s_{jm} \tag{23}$$

The FOC is

$$s_{jm} + (p_{jm} - MC_{jm}) \frac{ds_{jm}}{dp_{jm}} = 0 \tag{24}$$

which gives

$$MC_{jm} = p_{jm} + \frac{1}{\frac{ds_{jm}}{dp_{jm}}} s_{jm} \tag{25}$$

Finally, the mark-ups are given by

$$p_{jm} - MC_{jm} = \begin{cases} 0, & \text{perfect competition} \\ -\boldsymbol{\Delta_m^{-1}[j,:]s_m}, & \text{perfect collusion} \\ -\frac{1}{\frac{ds_{jm}}{dp_{jm}}} s_{jm}, & \text{oligopoly} \end{cases} \tag{26}$$

2

(a) In addition to demand moments, we add supply moments in the GMM.

Problem 3: Merger Exercise

Let

$$\Omega = \begin{pmatrix} -\frac{\partial q_1}{\partial p_1} & -\frac{\partial q_2}{\partial p_1} & 0 \\ -\frac{\partial q_2}{\partial p_1} & -\frac{\partial q_2}{\partial q_2} & 0 \\ 0 & 0 & -\frac{\partial q_3}{\partial p_3} \end{pmatrix}.$$

Then, $p_{1m} = MC_{1m} + \Omega^{-1}s_{1m}$, and $p_{2m} = MC_{2m} + \Omega^{-1}s_{2m}$. In each equation, the matrix Ω includes the off-diagonal terms in the (1,2) and (2,1) place because products one and two are priced as if owned jointly. This means that the optimal price for each product factors in the effect on the other product's share.