Problem Set 1 ECON8861

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1. Resulting net saving policy function $a'(s,a) - a$ and consumption function $c(s,a)$

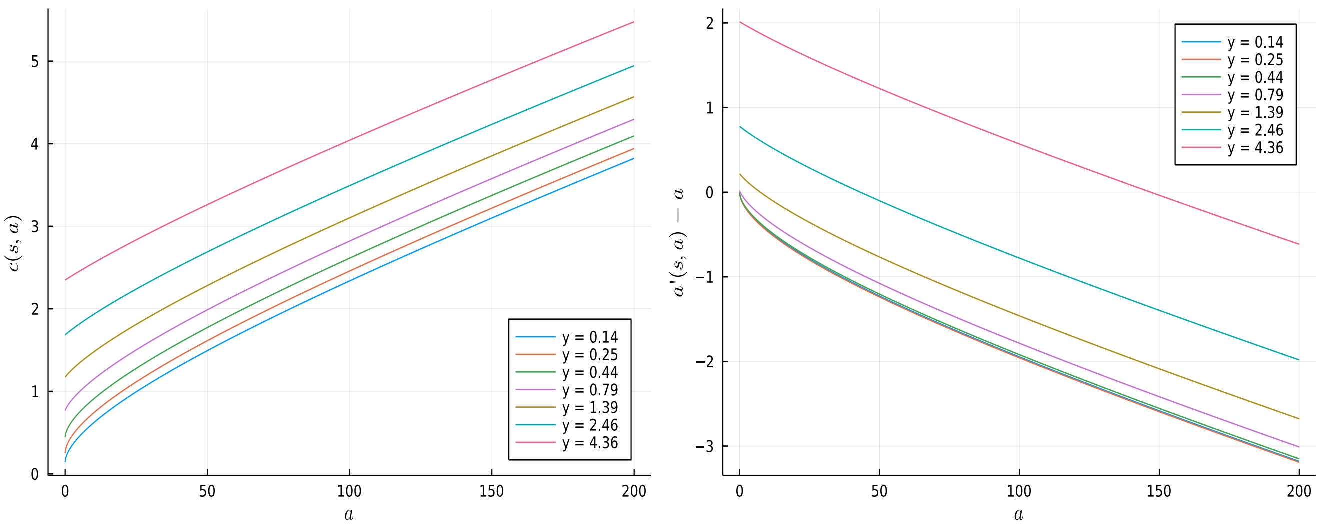

Figure 1: (Left) Consumption policy; (Right) Net saving policy

2. Conditional CDF of assets $\sum_{a=\underline{a}}^{k} D(s,a)$

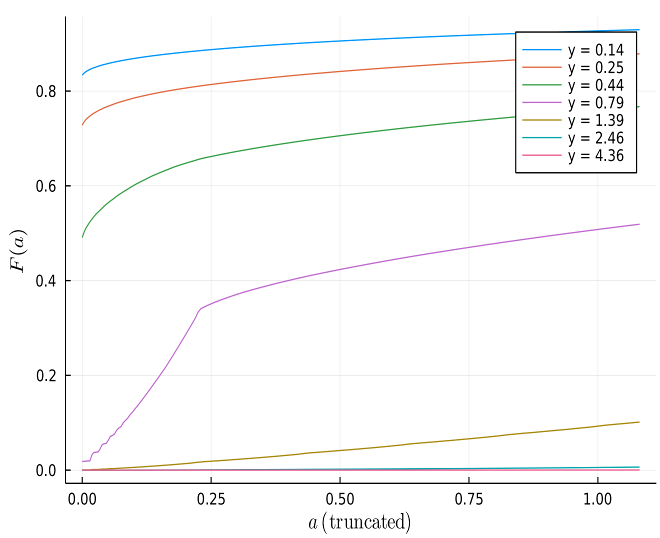

Figure 2: Conditional Asset CDFs

3. Conditional MPCs using forward differences both at the boundary and in the interior. Central differences might have lower truncation error.

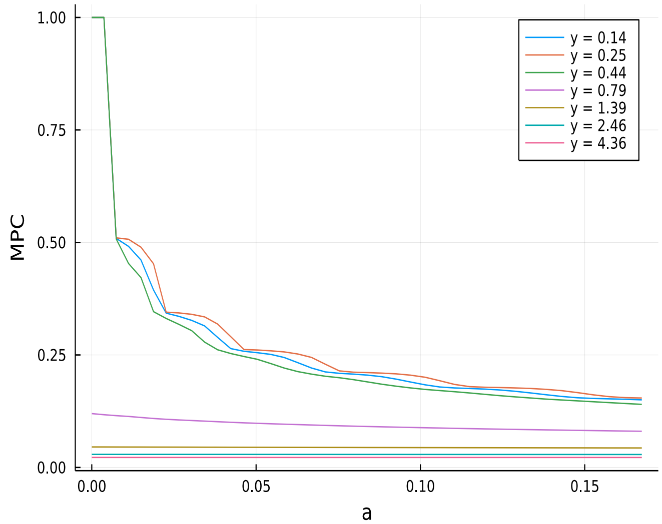

Figure 3: Conditional MPCs

4. Aggregate consumption and asset (rounded to 2nd digit)

   - $\overline{MPC} := \sum_{s,a} D(s,a)\frac{\partial c(s,a)}{\partial a} = 0.24$
   - $A := \sum_{s,a} D(s,a)\frac{a}{4Y} = 1.47$

5. Comparative static over β

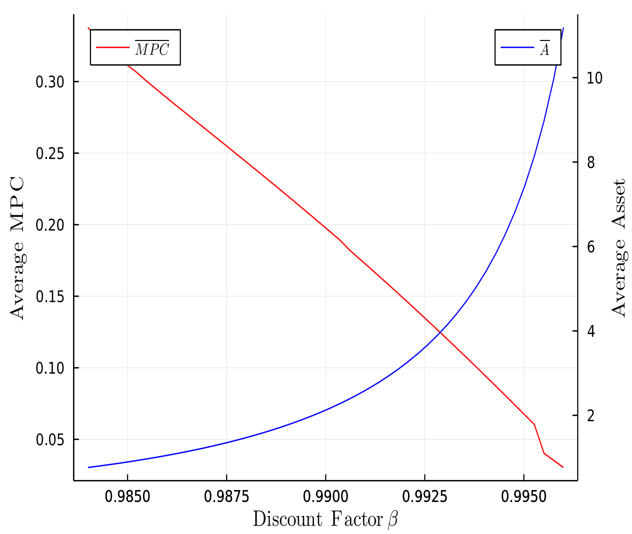

Figure 4: Comparative static over β

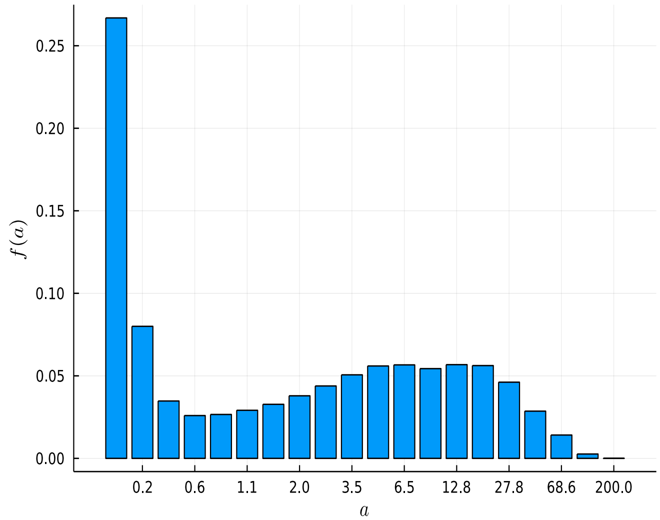

Figure 5: Cross-sectional asset distribution

6. Cross-sectional asset distribution
   Here I constructed a custom function that bins the asset grid, but the bins are uneven. I will work on it in the next days to adjust it to even bins (as far as I know, Julia does not have a built-in function that re-scales and bins the data as in Python)

Problem 2
From the problem

$$V(s, a|Y) = \max \mathbb{E} \left\{ \sum \beta^t \frac{c_t^{1-\gamma}}{1-\gamma} \right\}$$

$$c_t + a_{t+1} = (1+r)a_t + sY$$

$$a_{t+1} \geq \underline{a}$$

we arrive to $\int a'(s, a) dD(s, a)$ after the forward iteration. Both $c(s, a), a'(s, a)$ depend on $(Y, \underline{a})$ and other parameters.
Since the distribution is invariant, we can conclude it's homogeneous of degree zero in its arguments. We need to show then that the asset policy function is homogeneous of degree one in $(Y, \underline{a})$.
The budget constraint implicitly depends on the credit constraint, and if we scale it and the income it by a positive constant τ, for the budget constraint to hold at the optimal policies we will have to scale them by the same amount, leaving the Euler equation unchanged and so its optimum.