Problem Set ECON8862

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1 Estimate Q Models and Euler Equations for the Capital Stock by panel GMM

For the Q-model, the equation we want to estimate in the Q-model of investment is

$$\frac{I_t}{K_t} = \frac{1}{b} E_t \left(\beta \frac{V_{t+1}}{K_{t+1}} - 1 \right).$$

(a) The results from the standard regression with firm fixed effects is

Table 1: Q-model of investment

	(1) Q-model	(2) Augmented-Q-Model
Average Q	0.000249***	0.000269***
	(15.91)	(15.02)
Cashflow Rate $(t-1)$		0.0218***
		(5.65)
Constant	0.245***	0.245***
	(145.61)	(125.15)
FE	Firm	Firm
N	20357	16214

t statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

from which we can conclude that the average Q is positively associated to investement rate, and a positive and significant effect of cashflow on investment rate suggesting the presence of financially constrained firms.

(b) The results with the LSDV/GMM with firm and year fixed effects look like this

Table 2: LSDV and GMM Estimation

	(1) LSDV	(2) GMM
Average Q	0.000305***	-0.000108
	(15.94)	(-0.34)
Cashflow Rate $(t-1)$	0.0201***	0.0721
	(5.04)	(0.90)
Constant	0.244***	
	(118.32)	
FE	Firm, Year	Firm, Year
N	15779	13131

t statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

from which we can see that the LSDV estimator preserves sign an d significance, while the GMM gives unreliable estimates.

(c) I run a series of estimators (difference and system GMM) with and without the finite sample correction for the standard errors. The results look like this

Table 3: Difference v. System GMM

	(1) d-GMM-1	(2) s-GMM-1	(3) d-GMM-2-Wind	(4) d-GMM-2	(5) s-GMM-2-Wind	(6) s-GMM-2
Average Q	-0.000108	0.000181	-0.000389	-0.000389**	0.0000892	0.000304***
	(-0.52)	(1.01)	(-1.11)	(-2.64)	(0.46)	(6.57)
Cashflow Rate $(t-1)$	0.0721	0.0136	0.126	0.126	-0.0107	-0.0120
	(1.27)	(0.25)	(0.86)	(1.82)	(-0.20)	(-0.72)
Constant		0.281***			0.280***	0.272***
		(29.50)			(30.33)	(53.31)
sargan	137.1	205.2	137.1	137.1	205.2	456.1
sarganp	2.88e-10	6.17e-16	2.88e-10	2.88e-10	6.17e-16	1.12e-28
hansen		125.8	81.08	81.08	125.8	233.1
hansenp		0.0000186	0.00268	0.00268	0.0000186	0.000556
ar1	-41.41	-19.43	-18.80	-19.54	-19.55	-19.84
ar1p	0	4.41e-84	7.14e-79	4.72e-85	4.12e-85	1.31e-87
ar2	-0.472	-1.015	0.161	0.162	-0.953	-1.365
ar2p	0.637	0.310	0.872	0.871	0.340	0.172
N	13131	16356	13131	13131	16356	16356

t statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

where I also reported the Arellano-Bond test statistic, the Hansen and the Sargan test statistic and p-values for over-identified restrictions. The results are not robust nor significant, which suggests a further future inspection of the data looking for potential cleaning issues. In general, we see a positive effect of cashflow rate and a almost null effect of average Q.

(d) The Sargan-Hansen test statistic always lead to reject the null hypothesis, implying that the overidentifying restrictions are not valid. In other words, the instruments fail to satisfy the orthogonality condition assumed by the test.
For the Arellano-Bond test, as expected, the output above presents strong evidence against the

null hypothesis of zero autocorrelation in the first-differenced errors at order 1. By looking at the results for the $AR(2)$ in first differences, the null hypothesis of no serial correlation cannot be rejected which implies that the moment conditions are not valid.

(e) The results from interacting an indicator variable for firm size are the following

Table 4: GMM Estimation with Indicator for Small Firm

	(1) Q-model	(2) d-GMM-1	(3) s-GMM-1	(4) d-GMM-2-Wind	(5) d-GMM-2	(6) s-GMM-2-Wind	(7) s-GMM-2
Cashflow Rate$_{t-1}$	0.0541**	0.933**	0.330	1.641**	1.641***	0.514	0.115
	(2.82)	(2.65)	(1.36)	(2.67)	(3.53)	(1.50)	(0.73)
Small firm$_{t-1}$	-0.00358	0.206	0.199**	0.0488	0.0488	0.267*	0.229***
	(-0.41)	(0.82)	(2.68)	(0.15)	(0.17)	(2.40)	(5.25)
Small firm$_{t-1}$ $\times$ Cashflow Rate$_{t-1}$	-0.0370	-0.848*	-0.323	-1.515*	-1.515**	-0.509	-0.0392
	(-1.90)	(-2.39)	(-1.27)	(-2.45)	(-3.22)	(-1.42)	(-0.24)
Average Q$_{t-1}$	-0.000150	0.00288*	0.00258**	0.00347	0.00347*	0.00357*	0.00107
	(-1.76)	(2.00)	(2.86)	(1.43)	(2.08)	(2.14)	(1.61)
Small firm$_{t-1}$ $\times$ Average Q$_{t-1}$	0.0000669	-0.00352*	-0.00256**	-0.00428	-0.00428*	-0.00358*	-0.00100
	(0.78)	(-2.26)	(-2.76)	(-1.64)	(-2.52)	(-2.08)	(-1.48)
Constant	0.271***		0.113*			0.0643	0.126***
	(53.58)		(2.43)			(0.89)	(4.61)
N	16211	13131	16356	13131	13131	16356	16356

t statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The results seem to be counter-intuitive, though the low significance of the results is worrisome. It is most likely due to the sample quality, which might need further inspection.
Indeed, the marginal effect of being a small firm of an increase in the cashflow rate seems to be reducing the total investment rate, and I find a similar result for average Q.

(f) The Euler equation I am estimating is

$$\frac{I_t}{K_t} + \frac{1}{b} = E_t \left\{ \beta \frac{(1+\lambda_{t+1})}{(1+\lambda_t)} \left[\frac{1}{b}\pi_{t+1} + \frac{1}{2}\left(\frac{I_{t+1}}{K_{t+1}}\right)^2 + (1-\delta)\left(\frac{I_{t+1}}{K_{t+1}} + \frac{1}{b}\right)\right]\right\}$$

For perfect competition, assume

- $\beta = 0.95$
- $\delta = 0.05$
- $\frac{1}{b}$ is the constant
- π_{t+1} is the operating income
- $\frac{(1+\lambda_{t+1})}{(1+\lambda_t)}$ is equal to the cashflow rate

Table 5: Euler Equation GMM (Perfect Competition)

	(1) d-GMM-1	(2) d-GMM-2	(3) s-GMM-1	(4) s-GMM-2
$\frac{\beta}{b}\frac{(1+\lambda_{t+1})}{(1+\lambda_t)}\pi_{t+1}$	-0.000910	-0.000914	-0.0000470	-0.0000432
	(-1.63)	(-1.66)	(-0.14)	(-0.13)
$\frac{\beta}{1}\frac{(1+\lambda_{t+1})}{(1+\lambda_t)}\left(\frac{I_{t+1}}{K_{t+1}}\right)^2$	0.330	0.342	1.128*	1.084*
	(0.53)	(0.63)	(2.34)	(2.31)
$\beta\frac{(1+\lambda_{t+1})}{(1+\lambda_t)}(1-\delta)\left(\frac{I_{t+1}}{K_{t+1}}+\frac{1}{b}\right)$	-0.0150	-0.0134	-0.353	-0.334
	(-0.29)	(-0.27)	(-0.93)	(-0.93)
Constant			0.214*	0.242***
			(2.25)	(8.06)
N	10510	10510	13135	13135

t statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

For imperfect competition, assume

- $\beta = 0.95$
- $\delta = 0.05$
- $\frac{1}{b}$ is the constant
- π_{t+1} is the operating income given by $\frac{\alpha}{\mu}\frac{Y_{t+1}}{K_{t+1}}$, where $\mu = 1.3$ from literature consensus.
- $\frac{(1+\lambda_{t+1})}{(1+\lambda_t)}$ is equal to the cashflow rate

Table 6: Euler Equation GMM (Imperfect Competition)

	(1) d-GMM-1	(2) d-GMM-2	(3) s-GMM-1	(4) s-GMM-2
$\frac{\beta}{b}\frac{(1+\lambda_{t+1})}{(1+\lambda_t)}\pi_{t+1}$	-0.00523	-0.00521	-0.00517	-0.00501
	(-1.08)	(-1.11)	(-0.93)	(-0.95)
$\frac{\beta}{1}\frac{(1+\lambda_{t+1})}{(1+\lambda_t)}\left(\frac{I_{t+1}}{K_{t+1}}\right)^2$	0.447	0.478	1.258*	1.241
	(0.65)	(0.93)	(2.20)	(1.81)
$\beta\frac{(1+\lambda_{t+1})}{(1+\lambda_t)}(1-\delta)\left(\frac{I_{t+1}}{K_{t+1}}+\frac{1}{b}\right)$	-0.0113	-0.0104	-0.349	-0.338
	(-0.22)	(-0.21)	(-0.93)	(-0.96)
Constant			0.220*	0.247***
			(2.32)	(5.83)
N	10510	10510	13135	13135

t statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

2 Model Solution

In the model

- α is the capital share (usually approximately 0.33
- δ is the depreciation rate of capital and usually is around $0.03 - 0.05$
- p is the intensity in capital accumulation
- r is the real interest rate
- σ is the standard deviation of technology shocks
- ρ is the persistence of technology
- η_0, η_1 govern the shape of the dividend function
- γ_c, γ_f govern the shape of the adjustment cost function

The cases presented in the model imply (i) no friction due to adjustment costs, (ii) no frictions due to fixed costs and (iii) frictionless model. The case we will use is (ii).
The policy function does cross the 45 degree lines and looks like this

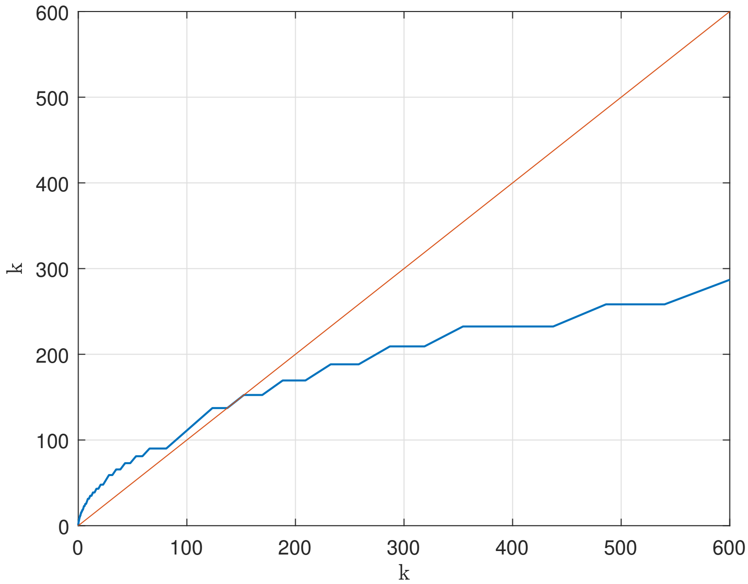

Figure 1: Policy Function

The value function looks like the following

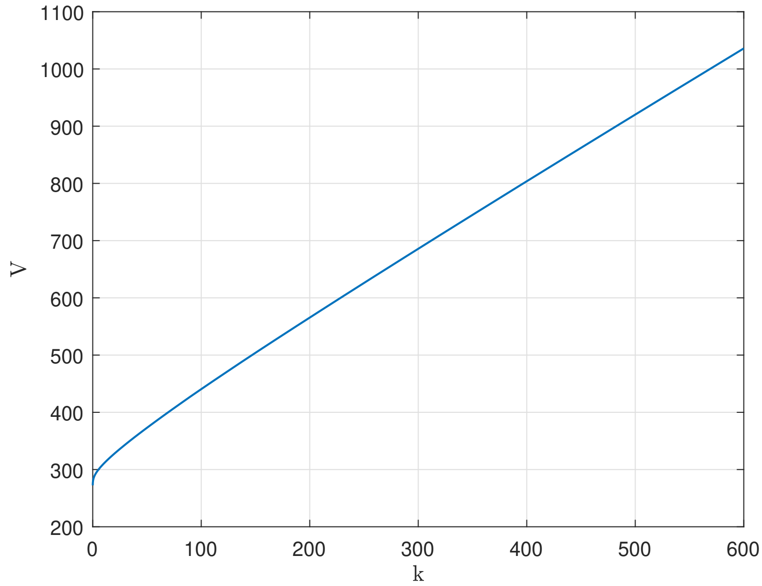

Figure 2: Value Function

3 The Ergodic Distribution of the Model

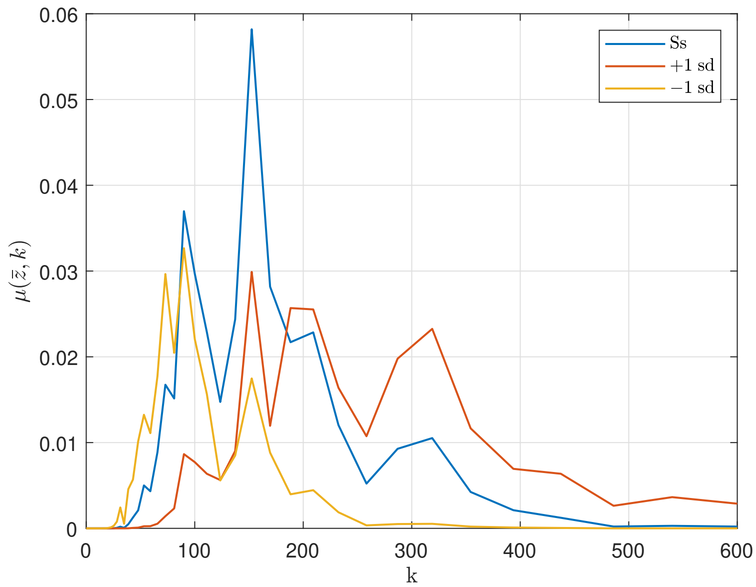

Figure 3: Ergodic Distribution

As we can see, in a better state the distribution puts more mass on higher values of capital, while in a worse one the mass of capital features mass points at lower levels. The moments are

Moment	$E(I/K)$	$\sqrt{\text{Var}(I)}$	$E(D/K)$
Value	3.1347	120.3852	-1.075

Table 7: Ergodic Distributions Moments

4 Estimation

The first simulation looks like the following

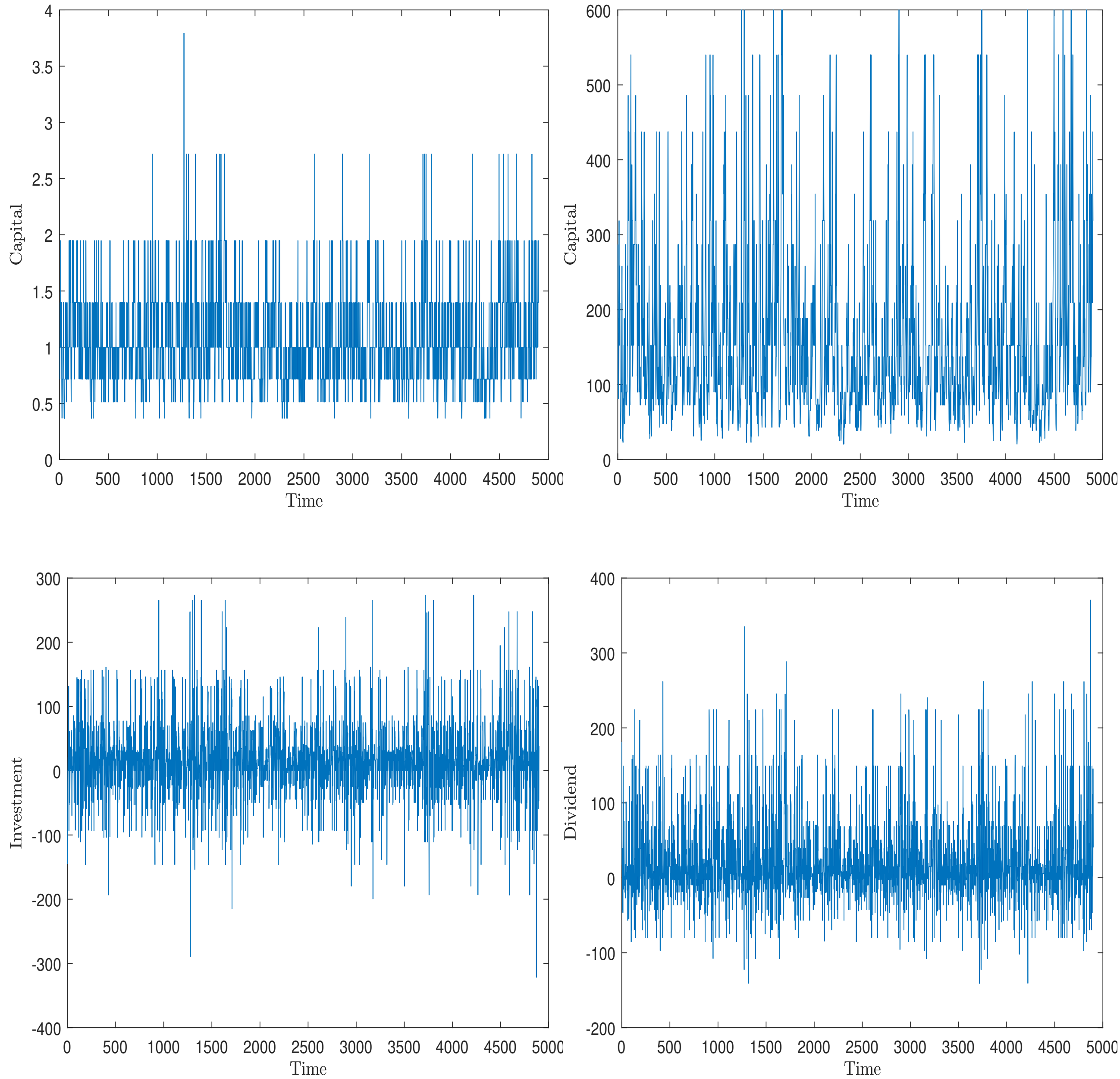

Figure 4: Similated Time Series

The results from the SMM estimation are the following

Parameter	γ	α	δ
Value	0.047104	0.70235	0.056093
Moment	$E(I/K)$	$\sqrt{\text{Var}(I)}$	$E(D/K)$
Value	0.08353	70.9389	0.090815

Table 8: Matched Moments/Parameters