ECON8862: Monetary Theory II, Fabio Schiantarelli

Lecture 7: Beyond BGG
Financial Frictions in Quantitative Business Cycle Models: New Contributions

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1 Introduction

Fundamentally, Bernanke, Gertler and Gilchrist (1997) (BGG henceforth) has the following mechanism: risk neutral entrepreneurs are assumed to guarantee a predetermined safe rate of return to the lenders and to absorb all the aggregate risk in the economy; negative shocks decrease entrepreneurial net worth, raise the external finance premium and the negative effect of the original shock is amplified.

Actually households do not lend directly to firms, but deposit their money in an intermediary that invests in many projects and diversifies away idiosyncratic risk (but not aggregate risk). Free entry generates zero profits, so that the rate of return on deposits equals the rate of return on the intermediary's loan portfolio. Essentially, the intermediary is only a pass-through entity.

What happens if the lenders' returns are not predetermined but are generated by an optimal contract? In other terms, what happens when one goes beyond guaranteeing depositors in the intermediary a return that is independent of the state of the economy, as in a standard deposit contract? Will shocks be amplified or attenuated? It turns out that once contracts allow for forward-looking behavior and are determined optimally, shocks are not amplified but dampened, as in the standard financial accelerator model, but dampened.[1] In this respect, House (2006) finds that under optimal contracts with complete sets of instruments amplification is dampened in mechanism with adverse selection, compare to the case where firms can issue debt. Dmitriev and Hoddenbagh (2017) show that the same result holds in costly state enforcement environment.

What should we make out of all this? Standard debt contracts provide predetermined returns to depositors and financial intermediaries. Optimal contracts, in the sense described above, are not very common. Therefore, the remaining challenge is to explain why that is the case.

[1] Carlstrom, Fuerst and Paustian (2016); Dmitriev and Hoddenbagh (2017) (DH and CFP, respectively) derive the results derive in this lecture independently. We rely mostly on the DH presentation and simulations.

2 Myopic Contingent Contract (MCC) and BGG.

Carlstrom, Fuerst and Paustian (2016) and Dmitriev and Hoddenbagh (2017) point out that interest rate on deposits (the rate that financial intermediaries pay to households) in BGG is predetermined. Since financial intermediaries are perfectly competitive, their aggregated returns on loans to entrepreneurs are equal to predetermined deposit rate plus monitoring costs. Even though monitoring costs depend on the realization of shocks at $t+1$, they are small. In DH and CFP neither the deposit rate that household receive, nor aggregated returns on loans for financial intermediaries are predetermined. We will use BGG and CF notation from the previous lecture.

As before, the expected return for entrepreneurs is

$$E\left\{ (\int_{\bar{\omega}_{t+1}^j}^{\infty} \omega R_{t+1}^k Q_t K_{t+1}^j dF(\omega) - (1 - F(\bar{\omega}_{t+1}^j))\bar{\omega}_{t+1}^j) R_{t+1}^k Q_t K_{t+1}^j \right\} \tag{1}$$

or more simply $f(\bar{\omega}_{t+1}^j) = \int_{\bar{\omega}_{t+1}^j}^{\infty} \omega R_{t+1}^k Q_t K_{t+1}^j dF(\omega) - (1 - F(\bar{\omega}_{t+1}^j))\bar{\omega}_{t+1}^j$.

The deposit rate (households' return) is defined by

$$g(\bar{\omega}_{t+1}^j) R_{t+1}^k Q_t K_{t+1}^j = R_{t+1}(Q_t K_{t+1}^j - N_t^j) \tag{2}$$

where $g(\bar{\omega}_{t+1}^j) = \{[1 - F(\bar{\omega}_{t+1}^j)]\bar{\omega}_{t+1}^j + (1-\mu)\int_0^{\bar{\omega}_{t+1}^j} \omega dF(\omega)\}$.

Now R_{t+1} is non-predetermined and should satisfy a more general Euler equation:

$$U_{c,t} = \beta \mathbb{E}[U_{c,t+1} R_{t+1}]. \tag{3}$$

If we plug in (2) into the Euler equation (3), we obtain

$$U_{c,t} = \beta \mathbb{E}_t \left[U_{c,t+1} g(\bar{\omega}_{t+1}^j) R_{t+1}^k \frac{Q_t K_{t+1}^j}{Q_t K_{t+1}^j - N_t^j} \right], \tag{4}$$

We can take capital out of expectation and obtain

$$U_{c,t}(Q_t K_{t+1}^j - N_t^j) = \beta \mathbb{E}_t \left[U_{c,t+1} g(\bar{\omega}_{t+1}^j) R_{t+1}^k Q_t K_{t+1}^j \right], \tag{5}$$

We can then formulate a Lagrangian

$$\mathcal{L}^{MCC} = \mathbb{E}_t \left\{ f(\bar{\omega}_{t+1}^j) R_{t+1}^k Q_t K_{t+1}^j + \lambda_t \left(U_{c,t}(Q_t K_{t+1}^j - N_t^j) - \beta \left[U_{c,t+1} g(\bar{\omega}_{t+1}^j) R_{t+1}^k Q_t K_{t+1}^j \right] \right) \right\}. \tag{6}$$

Using the Euler equation for the safe interest rate, we obtain $\beta R_{t+1} \mathbb{E}_t U_{c,t+1} = U_{c,t}$. Let's plug in the formula for the deposit rate (2) and obtain:

$$U_{c,t} = \beta \mathbb{E}_t U_{c,t+1} g(\bar{\omega}^j_{t+1}) R^k_{t+1} \frac{Q_t K^j_{t+1}}{Q_t K^j_{t+1} - N^j_t}. \tag{7}$$

Then the Lagrangian for BGG will be

$$\mathcal{L}^{BGG} = \mathbb{E}_t \bigg\{ f(\bar{\omega}^j_{t+1}) R^k_{t+1} Q_t K^j_{t+1} + \lambda_{t+1} \bigg(U_{c,t}(Q_t K^j_{t+1} - N^j_t) - \beta \mathbb{E}_t U_{c,t+1} g(\bar{\omega}^j_{t+1}) R^k_{t+1} Q_t K^j_{t+1} \bigg) \bigg\}. \tag{8}$$

Notice that in BGG the participation constraint is given state by state, while for the Myopic Contingent Contract (MCC) it is just "an average" constraint. For BGG, at each state in $t+1$ the productivity cutoff should move to guarantee the safe rate of return for households, so that share of households goes up for low R^k_{t+1} and down for high R^k_{t+1}. As a result, for every realization of R^k_{t+1}, there is a Lagrangian multiplier λ_{t+1}. The situation is very different with a MCC. In such case, at every state households can get paid any deposit rate entrepreneurs choose to pay, but on average households should get their reservation utility, otherwise they would just consume instead of lending. Hence, the reservation utility constraint is the only constraint for the contract in Carlstrom, Fuerst and Paustian (2016), and that is why the Lagrangian multiplier has timing λ_t.

We search for the maximum of the Lagrangian, and take the first order conditions with respect to $\bar{\omega}^j_{t+1}$ and K^j_{t+1}. One can prove that optimality is

$$BGG: \qquad \frac{\kappa_{t+1} \mathbb{E}_t \{ R^k_{t+1} f(\bar{\omega}_{t+1}) \}}{U_{c,t}} = -\mathbb{E}_t \bigg\{ \frac{f'(\bar{\omega}_{t+1})}{g'(\bar{\omega}_{t+1}) \beta \mathbb{E}_t U_{c,t+1}} \bigg\}, \tag{9}$$

$$MCC: \qquad \frac{\kappa_{t+1} \mathbb{E}_t \{ R^k_{t+1} f(\bar{\omega}_{t+1}) \}}{U_{c,t}} = -\frac{f'(\bar{\omega}_{t+1})}{g'(\bar{\omega}_{t+1})} \frac{1}{\beta U_{c,t+1}}. \tag{10}$$

Notice that for BGG we have only one optimality condition, but there are also an infinite number of participation constraints which allow to find optimal productivity cutoffs and leverage. For MCC there is only one constraint, but an infinity of optimality conditions, that allow to find cutoffs and leverage.

3 Dmitriev Hoddenbagh 2017, Published Version of Carlstrom, Fuerst and Paustian 2016

The utility of a risk neutral entrepreneur is given by:

$$V_t^e(j) = (1-\gamma)\sum_{s=1}^{\infty}\gamma^s \mathbb{E}_t C_{t+s}^e \tag{11}$$

DH allow for both risk neutral and risk averse entrepreneurs with a more general utility function $V_t^e(j) = (1-\gamma)\sum_{s=1}^{\infty}\gamma^s \frac{\mathbb{E}_t\left\{(C_{t+s}^e)^{1-\rho}\right\}}{1-\rho}$.[2] For the case of a risk neutral entrepreneur, one can show that:

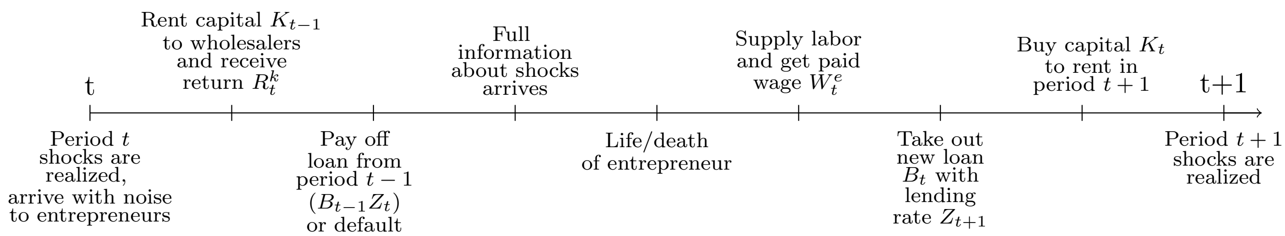
$$V_t^e = (1-\gamma)\left[N_t(\Psi_t - 1) + \sum_{s=1}^{\infty}\gamma^s \mathbb{E}_t\{W_{t+s}^e(\Psi_{t+s}-1)\}\right] \tag{12}$$

where

$$\Psi_t = 1 + \gamma \mathbb{E}_t\{\kappa_{t+1} f(\bar{\omega}_{t+1}^j) R_{t+1}^k \Psi_{t+1}\}. \tag{13}$$

$(1-\gamma)\Psi_t$ reflects the expected consumption from one dollar received by the entrepreneur in period t before it was decided whether he would die or survive. $(1-\gamma)(\Psi_t - 1)$ reflects the expected consumption from one dollar received by the entrepreneur in period t, conditional that does not die in period t. Let us look at the timeline of entrepreneurs in greater detail.

Figure 1: Timeline for Entrepreneurs with Information Noise



Now we need to formulate a Lagrangian for forward looking entrepreneurs. Again, the form of participation constraint is the same, since nothing has changed for household, while the objective function

[2] This utility function is also used by Candian and Dmitriev (2020) who analyze more in dept the consequences of risk aversion.

is different now. As before, we have two cases for predetermined and non-predetermined cases.

$$\mathcal{L}^{BGG,dyn} = \mathbb{E}_t\Big\{(1-\gamma)\Big[N_t^j(\Psi_t - 1) + \sum_{s=1}^{\infty}\gamma^s \mathbb{E}_t\{W_{t+s}^e(\Psi_{t+s}-1)\}\Big] + \tag{14}$$

$$\lambda_{t+1}\Big(U_{c,t}(Q_t K_{t+1}^j - N_t^j) - \beta\mathbb{E}_t U_{c,t+1} g(\bar{\omega}_{t+1}^j) R_{t+1}^k Q_t K_{t+1}^j\Big)...\Big\},$$

$$\mathcal{L}^{DH} = \mathbb{E}_t\Big\{(1-\gamma)\Big[N_t^j(\Psi_t - 1) + \sum_{s=1}^{\infty}\gamma^s \mathbb{E}_t\{W_{t+s}^e(\Psi_{t+s}-1)\}\Big] + \tag{15}$$

$$\lambda_t\Big(U_{c,t}(Q_t K_{t+1}^j - N_t^j) - \beta\Big[U_{c,t+1} g(\bar{\omega}_{t+1}^j) R_{t+1}^k Q_t K_{t+1}^j\Big]\Big)\Big\}. \tag{16}$$

then optimality conditions would be just

$$\text{BGG Dynamic}: \qquad \frac{\kappa_{t+1}\mathbb{E}_t\{R_{t+1}^k f(\bar{\omega}_{t+1})\Psi_{t+1}\}}{U_{c,t}} = -\mathbb{E}_t\Big\{\frac{f'(\bar{\omega}_{t+1})\Psi_{t+1}}{g'(\bar{\omega}_{t+1})\beta\mathbb{E}_t U_{c,t+1})}\Big\}, \tag{17}$$

$$\text{DH or CFP}: \qquad \frac{\kappa_{t+1}\mathbb{E}_t\{R_{t+1}^k f(\bar{\omega}_{t+1})\Psi_{t+1}\}}{U_{c,t}} = -\frac{f'(\bar{\omega}_{t+1})}{g'(\bar{\omega}_{t+1})}\frac{\Psi_{t+1}}{\beta U_{c,t+1}}. \tag{18}$$

Notice that for predetermined deposit rates for households, the F.O.C. with forward looking entrepreneurs have Ψ_{t+1} under expectation on the right and left hand side of equations, so that it would drop out in the first order approximation.

4 Log-linearization

The log-linearization of BGG optimality condition gives

$$\mathbb{E}_t\hat{R}_{t+1}^k - \mathbb{E}_t\hat{R}_{t+1} = \nu_\kappa\hat{\kappa}_{t+1} + \nu_\sigma\hat{\sigma}_{\omega,t} \tag{19}$$

$$\hat{R}_{t+1} - \mathbb{E}_t\hat{R}_{t+1} = 0 \tag{20}$$

where the constant $\nu_\kappa = \frac{\frac{g_{\omega\omega}}{g_\omega} - \frac{f_{\omega\omega}}{f_\omega}}{-\frac{f_\omega}{f} + \frac{g_\omega}{g} + \frac{f_{\omega\omega}}{f_\omega} - \frac{g_{\omega\omega}}{g_\omega}}\frac{1}{\kappa-1}$ and $\nu_\sigma = \frac{-\frac{g_{\sigma\omega}}{g}\left(\frac{f_{\omega\omega}}{f_\omega} - \frac{g_{\omega\omega}}{g_\omega} - \frac{f_\omega}{f}\right) + \frac{g_\omega}{g}\left(\frac{f_{\omega\sigma}}{f_\omega} - \frac{g_{\omega\sigma}}{g_\omega} - \frac{f_{\sigma\omega}}{f}\right)}{\frac{f_{\omega\omega}}{f_\omega} - \frac{g_{\omega\omega}}{g_\omega} - \frac{f_\omega}{f} + \frac{g_\omega}{g}}\sigma_\omega$. Again, when deposit rates are predetermined, it does not matter, whether entrepreneurs are myopic or not to the first order approximation. Log-linearization of the optimal contract, (18), and the participation constraint (5) gives

$$\mathbb{E}_t\hat{R}_{t+1}^k - \mathbb{E}_t\hat{R}_{t+1} = \nu_\kappa\hat{\kappa}_{t+1} + \nu_\sigma\hat{\sigma}_{\omega,t} \tag{21}$$

$$\hat{R}_{t+1} - \mathbb{E}_t\hat{R}_{t+1} = \hat{R}_{t+1}^k - \mathbb{E}_t R_{t+1}^k - \tilde{\alpha}\Big[\sigma(\hat{C}_{t+1} - \mathbb{E}_t\hat{C}_{t+1}) + \hat{\Psi}_{t+1} - \mathbb{E}_t\hat{\Psi}_{t+1}\Big] \tag{22}$$

$$\hat{\Psi}_{t+1} = \epsilon_N\mathbb{E}_{t+1}\Big\{(\kappa-1)(\hat{R}_{t+2}^k - \hat{R}_{t+2}) + \hat{R}_{t+2}^k + \nu_\Psi\hat{\sigma}_{\omega,t+1} + \hat{\Psi}_{t+2}\Big\} \tag{23}$$

where $\nu_\Psi = \frac{f_\sigma - g_\sigma\frac{f_\omega}{g_\omega}}{f}\sigma_\omega$. One can see from (22) that under the optimal contract, the surprise to lenders' returns depends not only on surprises to capital returns and consumption, but on *future* capital returns and *future* financial premiums as well. In the case of the myopic contingent contract we would have (22) without Ψ_{t+1}, so that the channel with future capital returns and financial premium would be absent, while the depositors' insurance channel $-\tilde{\alpha}\left[\sigma(\hat{C}_{t+1} - \mathbb{E}_t\hat{C}_{t+1})\right]$ will be still present. This will amplify the financial accelerator because lenders want to be paid a higher interest rate when the consumption of households is low and marginal utility is high. This further reduces entrepreneurs' net worth, increases the external finance premium and lowers investment.

In the forward-looking contract, if entrepreneurs expect higher future financial premiums or future returns to capital, they prefer to pay the lender a lower interest rate following a negative shock because one unit of net worth becomes more valuable (the second term in the square bracket). This is because, when a crisis hits and decreases entrepreneur's net worth, expected future financial premiums will rise because net worth changes are likely to be persistent. This makes it optimal for entrepreneurs to pay a smaller rate now in order to stabilizes their net worth. This effect may be large enough to more than compensate the increase in the finance premium due to the decrease in net worth caused by the negative shock. As a result, the main channel for the financial accelerator, the volatility in net worth, may be diminished when entrepreneurs are forward looking relative to BGG. Ultimately, the precise results will depend upon the calibration of the DSGE in which the financial contract is inserted.

5 Simulations

In Dmitriev and Hoddenbagh (2017) the different contracts described above are embedded in a New Keynesian DSGE model with price rigidity and the added feature of allowing for both entrepreneurial risk neutrality ($\sigma_e = 0$) and risk aversion ($\sigma_e = 1$). It turns out that risk aversion does not make a large difference as forward looking entrepreneurs behave as if they are risk averse even when their utility function is linear in wealth. Specifically, in their quantitative analysis DH compare three allocations: the competitive equilibrium under the BGG contract; the competitive equilibrium under the optimal contract with risk-neutral entrepreneurs; and the competitive equilibrium under the optimal contract with risk-averse entrepreneurs. They plot impulse responses for shocks to technology, the nominal interest rate (monetary policy shock) and idiosyncratic volatility. Impulse response functions for the MCC cases are not plotted. This is because the financial accelerator is reinforced for technology and monetary shocks relative to the BGG case, where the lending rate is state-contingent and entrepreneurs are risk-neutral and myopic.

Impulse responses for a persistent 1% technology shocks with sticky prices are plotted in Figure 2. Entrepreneurs, who are forward-looking and risk-neutral entrepreneurs, "look through" the initial negative returns to capital that last for roughly 4 years and become positive after that, which ensures that the present discounted value of capital accumulation is more profitable and drives investment up. Furthermore, the stabilizing influence of forward-looking entrepreneurs offsets the consumption insurance channel. As a result, the output response under the optimal contract with risk neutrality and the BGG contract are essentially the same. This coincidence does not hold, however, outside of the particular calibration employed here. When one allows the shock to be less persistent, the optimal risk-neutral

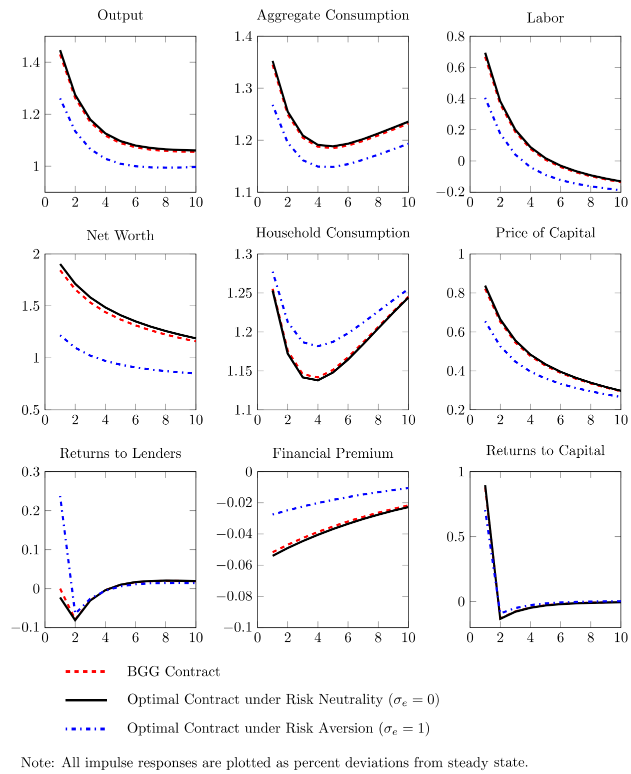


Figure 2: Impulse Response Functions, technology shock. Adapted from Dmitriev and Hoddenbagh (2017). See the article for the parameters used in the calibration.

contract weakens the financial accelerator relative to the BGG contract. When entrepreneurs are risk-averse, the optimal contract weakens somewhat the impact of technology shocks with respect to the risk-neutral case and the BGG contract. This is because risk-averse entrepreneurs seek to stabilize net worth as much as possible in order to smooth their consumption flows. This particular insurance motive is instead absent when entrepreneurs are risk-neutral. If there were no capital adjustment costs, returns to capital would always be positive following a positive productivity shock, which would significantly increase the present discounted value of capital and incentivize entrepreneurs to accumulate more net worth in these states, thereby increasing investment and giving rise to amplification.

The difference between the three allocations is clear in Figure 3, where the IRFs for a 1% nominal interest rate shock is plotted assuming that prices are sticky. Due to the lower persistence of the monetary shock relative to the technology shock, the price of capital depreciates back to its steady state level quickly after an initial increase. As a result, accumulating capital after a positive monetary shock is more costly, as capital returns are positive in the first period but become negative thereafter. Under the BGG contract, the deposit rate is not responsive to the shock since it is predetermined. This, in turn, increases net worth on impact because returns to capital increase, pushing down the financial premium and reducing

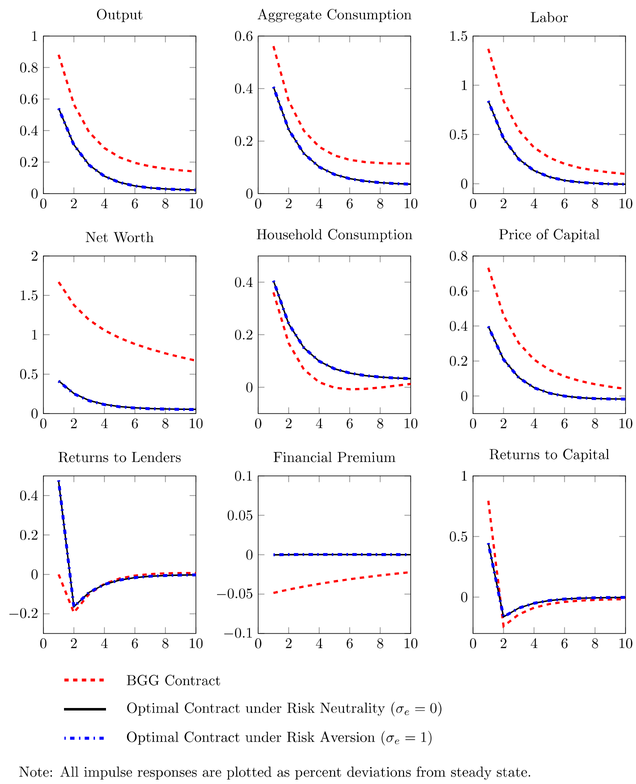


Figure 3: Impulse Response Functions, monetary shock. Adapted from Dmitriev and Hoddenbagh (2017). See the article for the parameters used in the calibration.

borrowing costs. Compared to the BGG contract, low capital returns and low borrowing costs generate a strong incentive for forward-looking entrepreneurs under the optimal contract to borrow more and pay lenders a higher rate, which reduces net worth and thus muffles the financial accelerator. Because net worth is already stable for these entrepreneurs, a higher level of risk aversion as virtually no effect on the economy's response to the monetary shock. Both forward-looking risk-neutral/averse entrepreneurs thus stabilize consumption and production to a similar extent under the optimal contract, leading to modest amplification in both cases.

Finally, Figure 4 displays impulse response for a one standard deviation increase in unobserved idiosyncratic volatility σ_ω (this can be seen as a "risk shock"). One can see that accumulating capital after a risk shock is more profitable than in the steady state: capital returns are negative in the first period but become positive after that. Under the BGG contract, the deposit rate fails to react to the disturbance because it is predetermined, as before. Together with the negative returns to capital on impact, this dynamic puts downward pressure on net worth, driving up the financial premium and increasing borrowing costs. Note that such contractionary effects are instead markedly less important under the optimal contract. With respect to the BGG contract, high capital returns and high borrowing costs

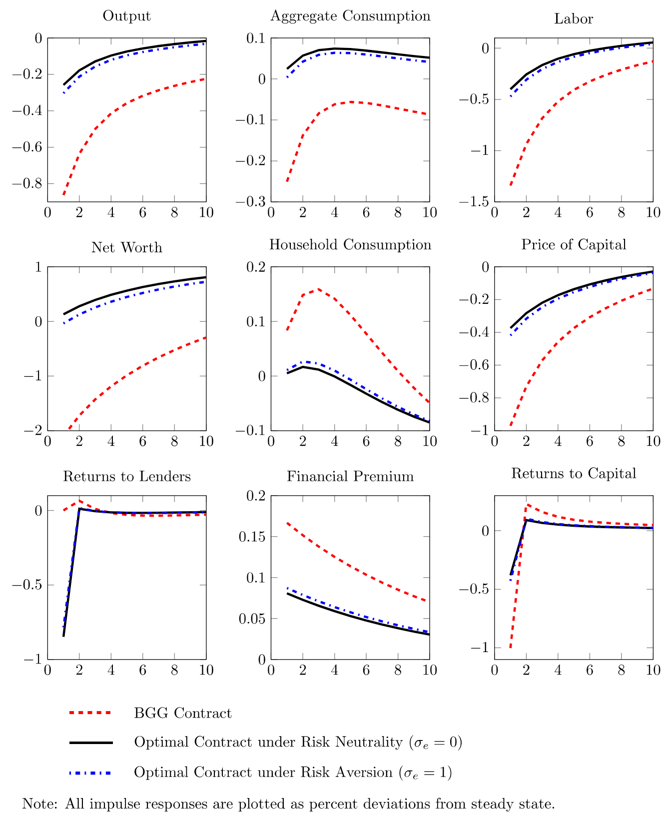


Figure 4: Impulse Response Functions, risk shock. Adapted from Dmitriev and Hoddenbagh (2017). See the article for the parameters used in the calibration.

generate a strong incentive for forward-looking entrepreneurs under the optimal contract to borrow less and repay lenders at a lower rate, which drives down the external financial premium and dampens the financial accelerator. Because net worth is relatively stable for forward-looking risk-neutral entrepreneurs, higher risk aversion has little effect on the dynamic response of the economy to the risk shock. Again, forward-looking entrepreneurs, either risk-neutral or risk-averse, stabilize consumption and output to a similar extent under the optimal contract, which dampens the amplification effect of the accelerator in both cases.

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6 Appendix

6.1 From V_t^e to Ψ_t.

Let's imagine that there are no wages, then in this case we have

$$V_t^e(j) = (1-\gamma)\sum_{s=1}^{\infty}\gamma^s \mathbb{E}_t C_{t+s}^e = (1-\gamma)\sum_{s=1}^{\infty}\gamma^s \mathbb{E}_t N_{t+s}^e \tag{24}$$

now we go to

$$E_t N_{t+1}^e(j) = N_t(j)\kappa_t \mathbb{E}_t[f(\bar{\omega}_{t+1}^j)R_{t+1}^k] \tag{25}$$

If we plug expression for net worth to the value function we would get

$$V_t^e(j) = (1-\gamma)\Big[\gamma \mathbb{E}_t N_{t+1}^e + \gamma^2 \mathbb{E}_t N_{t+2}^e ...\Big] = (1-\gamma)\mathbb{E}_t\Big[\gamma N_t(j)\kappa_t f(\bar{\omega}_{t+1}^j)R_{t+1}^k +$$
$$\gamma^2 N_t(j)\kappa_t f(\bar{\omega}_{t+1}^j)R_{t+1}^k \kappa_{t+1} f(\bar{\omega}_{t+2}^j)R_{t+2}^k + \gamma^3 N_t(j)\kappa_t f(\bar{\omega}_{t+1}^j)R_{t+1}^k \kappa_{t+1} f(\bar{\omega}_{t+2}^j)R_{t+2}^k \kappa_{t+2} f(\bar{\omega}_{t+3}^j)R_{t+3}^k ...\Big]$$

in the absence of wages

$$V_t^e = (1-\gamma)N_t(\Psi_t - 1) \tag{26}$$

where

$$\Psi_t = 1 + \gamma \mathbb{E}_t\{\kappa_t f(\bar{\omega}_{t+1}^j)R_{t+1}^k \Psi_{t+1}\}. \tag{27}$$

Homework: check expression for the value function with wages.

6.2 BGG Contract

In the BGG contract, the lender is guaranteed a fixed rate of return. In this case, the entrepreneur's Lagrangian will be

$$\mathcal{L}^{BGG} = (1-\gamma)\mathbb{E}_t\Big\{N_t\kappa_t R_{t+1}^k g(\bar{\omega}_{t+1}) + \lambda_{t+1}\Big[\beta \mathbb{E}_t\Big\{U_{C,t+1}\Big\}k_t R_{t+1}^k h(\bar{\omega}_{t+1}) - (k_t - 1)U_{C,t}\Big]\Big\}.$$

The entrepreneur's first order conditions with respect to κ_t and $\bar{\omega}_{t+1}$ are:

$$\frac{\partial \mathcal{L}^{BGG}}{\partial \kappa_t} = N_t \mathbb{E}_t\Big\{R_{t+1}^k g(\bar{\omega}_{t+1})\Big\} - \mathbb{E}_t\Big\{\lambda_{t+1}\Big\}\frac{U_{C,t}}{\kappa_t} = 0 \tag{28}$$

$$\frac{\partial \mathcal{L}^{BGG}}{\partial \omega_{t+1}} = N_t\kappa_t R_{t+1}^k g'(\bar{\omega}_{t+1}) + \lambda_{t+1}\beta \mathbb{E}_t\Big\{U_{C,t+1}\Big\}\kappa_t R_{t+1}^k h'(\bar{\omega}_{t+1}) = 0$$
$$= N_t g'(\bar{\omega}_{t+1}) + \lambda_{t+1}\beta \mathbb{E}_t\Big\{U_{C,t+1}\Big\}h'(\bar{\omega}_{t+1}) = 0 \tag{29}$$

If we plug in $\frac{\partial \mathcal{L}^{BGG}}{\partial \omega_{t+1}}$ into $\frac{\partial \mathcal{L}^{BGG}}{\partial \kappa_t}$, we find

$$N_t \mathbb{E}_t\Big\{R_{t+1}^k g(\bar{\omega}_{t+1})\Big\} = \mathbb{E}_t - \Big\{\frac{N_t g'(\bar{\omega}_{t+1})}{h'(\bar{\omega}_{t+1})\beta \mathbb{E}_t U_{c,t+1}}\Big\}\frac{U_{C,t}}{\kappa_t}. \tag{30}$$

Rearranging, simplifying and substituting in the stochastic discount factor yields:

$$\kappa_t \mathbb{E}_t \left\{ R^k_{t+1} g(\bar{\omega}_{t+1}) \right\} = -\mathbb{E}_t \left\{ \frac{g'(\bar{\omega}_{t+1})}{h'(\bar{\omega}_{t+1})} \right\} \frac{1}{\mathbb{E}_t \Lambda_{t,t+1}}. \tag{31}$$

6.3 MCC Contract

For MCC contract, the entrepreneur's Lagrangian is

$$\mathcal{L}^{MCC} = (1-\gamma) \left\{ N_t \kappa_t \mathbb{E}_t \left[R^k_{t+1} g(\bar{\omega}_{t+1}) \right] + \lambda_t \left[\mathbb{E}_t \left(\beta U_{C,t+1} k_t R^k_{t+1} h(\bar{\omega}_{t+1}) \right) - (k_t - 1) U_{C,t} \right] \right\}.$$

The first order conditions for κ_t and $\bar{\omega}_{t+1}$ are

$$\frac{\partial \mathcal{L}^{MCC}}{\partial \kappa_t} = (1-\gamma) \left[N_t \mathbb{E}_t \left\{ R^k_{t+1} g(\bar{\omega}_{t+1}) \right\} + \lambda_t \left(\mathbb{E}_t \left\{ \beta U_{C,t+1} R^k_{t+1} h(\bar{\omega}_{t+1}) - U_{C,t} \right\} \right) \right] = 0,$$
$$\frac{\partial \mathcal{L}^{MCC}}{\partial \bar{\omega}_{t+1}} = (1-\gamma) \left[N_t \kappa_t R^k_{t+1} g'(\bar{\omega}_{t+1}) + \lambda_t \beta U_{C,t+1} \kappa_t R^k_{t+1} h'(\bar{\omega}_{t+1}) \right] = 0.$$

Rearranging these first order conditions, solving in terms of λ_t and setting them equal to each other yields

$$\kappa_t \mathbb{E}_t \left\{ R^k_{t+1} g(\bar{\omega}_{t+1}) \right\} = -\frac{g'(\bar{\omega}_{t+1})}{h'(\bar{\omega}_{t+1})} \frac{1}{\Lambda_{t,t+1}}. \tag{32}$$