Review of Economic Dynamics 24 (2017) 43–65

Contents lists available at ScienceDirect

Review of Economic Dynamics

www.elsevier.com/locate/red

The financial accelerator and the optimal state-dependent contract ☆

Mikhail Dmitriev [a,*], Jonathan Hoddenbagh [b,*]

[a] Department of Economics, Florida State University, 288 Bellamy Building, Tallahassee, FL 32306, United States
[b] School of Advanced International Studies, Johns Hopkins University, 1717 Mass Ave NW, Washington, DC 20003, United States

ARTICLE INFO

Article history:
Received 25 September 2015
Received in revised form 22 December 2016
Available online 3 January 2017

JEL classification:
E32
E44

Keywords:
Financial accelerator
Financial frictions
Risk
Optimal contract
Agency costs

ABSTRACT

In the financial accelerator literature pioneered by Bernanke et al. (1999) entrepreneurs are myopic and risk-neutral, and loans have a predetermined rate of return by assumption. We relax these assumptions and derive the optimal state-dependent loan contract for forward-looking risk-averse entrepreneurs. We show that financial frictions deliver less amplification under the optimal state-dependent contract.

1. Introduction

In one of the foundational papers in the literature on financial frictions in macroeconomic models, Bernanke et al. (1999) — hereafter BGG — derive a contract between risk-averse lenders and risk-neutral borrowers in the costly state verification (CSV) framework of Townsend (1979).[1] The BGG model is widely used because of its ability to generate a financial accelerator which amplifies and propagates the impact of technology and monetary shocks in a dynamic New Keynesian framework. The underlying loan contract in BGG, which has become the standard contract for CSV models of

☆ We are indebted to Susanto Basu, Ryan Chahrour, Fabio Ghironi, Peter Ireland, and Fabio Schiantarelli for guidance and suggestions at every stage of the project. We thank the editor, Vincenzo Quadrini, and two anonymous referees for very helpful comments, as well as Chuck Carlstrom, Tim Fuerst, Nobuhiro Kiyotaki, Sami Alpanda, Ron Alquist, Huixin Bi, Sanjay Chugh, Ian Christensen and Kim Huynh. We thank Simon Gilchrist for his insightful discussion of the paper at the November 8, 2013 Green Line Macro Meeting, and Thomas Philippon for his insightful discussion of the paper at the Housing-Urban-Labor-Macro Conference at NYU Stern on February 28, 2014. We also thank seminar participants at the Bank of Canada, Boston College, the Federal Reserve Bank of Dallas, Florida State, Santa Clara, the University of Miami, the 2014 Canadian Economic Association meeting, the Fall 2014 Midwest Macro meeting and the Winter 2015 Eastern Economic Association meeting for valuable comments. This paper previously circulated under the titles "Utility Maximizing Entrepreneurs and the Financial Accelerator" and "The Financial Accelerator and the Optimal Lending Contract".

* Corresponding authors.
E-mail addresses: mdmitriev@fsu.edu (M. Dmitriev), hoddenbagh@jhu.edu (J. Hoddenbagh).
URLs: http://www.mikhaildmitriev.org (M. Dmitriev), https://sites.google.com/site/jonathanhoddenbagh/ (J. Hoddenbagh).

[1] A non-exhaustive list of some important early contributions in this literature include Bernanke and Gertler (1989) and Carlstrom and Fuerst (1997).

http://dx.doi.org/10.1016/j.red.2016.12.003
1094-2025/

financial frictions, assumes returns for lenders are predetermined and borrowers are risk-neutral and myopic, such that they maximize expected consumption in the next period only, without considering consumption in subsequent periods.[2]

We relax these assumptions and derive the optimal history-independent loan contract in the CSV model, which we call the *optimal contract* for simplicity.[3] Following early criticism of BGG by Chari (2003), we allow returns to the lender to vary with the aggregate state of the economy. Second, we introduce forward-looking entrepreneurs who maximize the present discounted value of all future consumption instead of next period expected consumption. Third, we consider a more general constant relative risk aversion (CRRA) preference specification for entrepreneurs that nests risk neutrality, which is assumed in BGG, as a special case.

We embed the optimal loan contract in a standard dynamic New Keynesian model, similar in all respects to the model in BGG, and derive three main conclusions. First, under the optimal contract, regardless of the degree of entrepreneurial risk aversion, business cycle amplification is smaller than under the BGG contract. Second, shocks to the cross-sectional variance of entrepreneurs' idiosyncratic productivity — so-called "risk" shocks — have little to no impact on the real economy when loan contracts are optimal, in contrast with the BGG contract. While recent work by Christiano et al. (2013) emphasizes the importance of risk shocks in driving business cycles, we demonstrate that risk shocks provide amplification only when the loan contract is non-contingent, regardless of the degree of lender's risk aversion. Third, we show that the financial accelerator in the CSV framework is dependent on three key characteristics: a predetermined lending rate, loose monetary policy and extremely persistent technology shocks. We conduct a number of robustness tests in the online appendix and find that the removal of any one of these characteristics weakens or eliminates the financial accelerator.

1.1. Overview of the model and intuition

Our model consists of entrepreneurs who borrow money from a representative household and purchase capital to use in production. Entrepreneurs are identical ex ante but differ depending on the ex post realization of an idiosyncratic productivity shock. Both agents have full information about the distribution of idiosyncratic shocks ex ante. Borrowers observe the realization of their idiosyncratic shock, but lenders do not: they must pay monitoring costs to observe it.

In the BGG contract, risk-neutral borrowers guarantee a predetermined safe rate of return to lenders in order to maximize returns on their equity. As a result, borrowers absorb all risk in the economy. It should be noted that this is an assumption and not an equilibrium condition. Because of this assumption, negative shocks decrease entrepreneurs' net worth which tightens financial constraints during recessions. The fall in output from a negative shock is further exacerbated by the decline in entrepreneurial net worth, which raises the cost of borrowing and creates a vicious circle of further declines in capital prices, net worth and investment. This results in the financial accelerator: the BGG contract amplifies macroeconomic fluctuations in a dynamic stochastic general equilibrium (DSGE) model.

As we mentioned previously, three key assumptions underpin the BGG contract and the subsequent literature that utilizes CSV frictions to generate a financial accelerator effect: (1) lenders returns are predetermined; (2) entrepreneurs are myopic — they maximize their expected *next period* consumption, rather than the expected discounted stream of *all future* consumption; and (3) entrepreneurs are risk-neutral. To gain deeper understanding of the mechanisms at play in generating the financial accelerator, we first relax the above assumptions in isolation and discuss the implications. We then relax all three assumptions simultaneously to construct the optimal contract.

When lenders' returns are predetermined as in BGG, we find that to a first order approximation the equilibrium loan contract is robust to alternative assumptions on entrepreneurial myopia or risk aversion. Because the predetermined lending rate is chosen in period t to satisfy the lender's Euler equation in that specific period without the possibility of revisions in period $t+1$, the lender's stochastic discount factor — which is invariant to the degree of entrepreneurial myopia or risk aversion — determines the rate of return. Thus, under a predetermined lending rate the equilibrium contract is identical regardless of whether entrepreneurs are forward-looking or myopic, risk-neutral or risk-averse, and the financial accelerator remains intact.

In contrast, when lenders' returns can vary with the aggregate state of the economy as Chari (2003) argued they should, the degree of entrepreneurial myopia and risk aversion matters a great deal. For example, when entrepreneurs are myopic and risk-neutral, the financial accelerator is stronger under a state-contingent lending rate than under a predetermined lending rate. Why is this so? Myopic risk-neutral entrepreneurs sell as much insurance to the representative household as they can because insurance does not affect their expected next period consumption. Risk-averse households prefer a state-contingent rate of return that is negatively correlated with household consumption. In recessions, households desire a higher rate of return because their marginal utility of consumption is high, and vice versa in booms. A state-contingent lending rate thus insulates households from fluctuations in consumption but exposes myopic entrepreneurs to larger swings in net worth. During a recession, the provision of insurance leads to very tight financial constraints for myopic entrepreneurs,

[2] Note that the BGG contract is optimal given the assumptions of a predetermined lending rate and entrepreneurs who maximize next period's expected consumption.

[3] To be precise, we derive the optimal one-period contract with deterministic monitoring. An excellent list of references for partial equilibrium multi-period contracts includes Monnet and Quintin (2005) for stochastic monitoring, Wang (2005) for deterministic monitoring, Cole (2013) for self-enforcing stochastic monitoring, and Popov (2014, 2016) who studies the impact of enforcement frictions on optimal loan contracts as well as optimal dynamic contracts under costly state verification.

even tighter than under a predetermined lending rate, as they must pay a higher lending rate due to the fall in household consumption. During a boom the opposite occurs. Thus, when entrepreneurs are risk-neutral and myopic, state-contingent lending rates actually strengthen the financial accelerator, leading to larger amplification of technology and monetary shocks. Myopic risk-neutral entrepreneurs miss good investment opportunities on a consistent basis because they do not take the future flow of capital returns into account when making investment decisions.

On the other hand, in a state-dependent loan contract forward-looking risk-neutral entrepreneurs sell less insurance to households because they are concerned not only about next period expected consumption but expected consumption in all future periods, which is affected by insurance claims. Forward-looking entrepreneurs desire high net worth in states of the world where the financial premium is high because capital returns are higher and borrowing is more costly. For instance, suppose that ex post there is a shock which suddenly decreases entrepreneurial net worth. Lower net worth today means that the financial premium is higher today and in the future. Forward-looking entrepreneurs thus find it profitable to enter into an ex ante loan contract that stipulates a lower lending rate in low net worth states, and a higher lending rate in high net worth states. This interplay between movements in net worth and the financial premium leads forward-looking risk-neutral entrepreneurs to behave in a “risk-averse” manner because they want to avoid borrowing in states with a high financial premium. This diminishes fluctuations in net worth that result from technology, monetary and risk shocks and thus dampens the financial accelerator. The degree to which forward-looking entrepreneurs dampen the financial accelerator depends in part on the strength of the CSV frictions and the persistence of the shocks in question. If there is no costly state verification so that financial frictions are absent, forward-looking entrepreneurs will ignore concerns about the financial premium and provide as much insurance as possible, generating large amplification.

What happens when we relax the assumption that entrepreneurs are risk-neutral? Under a state-contingent lending rate, higher entrepreneurial risk aversion dampens the financial accelerator. As entrepreneurial risk aversion approaches one (logarithmic CRRA utility), the distinction between myopic and forward-looking entrepreneurs matters less and less for determining the optimal state-contingent loan contract. Risk-averse entrepreneurs, whether myopic or forward-looking, effectively buy insurance from households by agreeing to large repayments in a boom and small repayments in a recession. When entrepreneurial risk aversion is equal to one, we prove that the state-contingent loan contract is mathematically identical for forward-looking and myopic entrepreneurs. The optimal loan contract with risk-averse entrepreneurs delivers smaller amplification for all shocks — technology, monetary and risk shocks — than the BGG contract. Ultimately then, we show that the optimal contract, which considers a state-contingent lending rate and forward-looking risk-averse entrepreneurs, dampens the financial accelerator.

We also find that risk shocks have little effect on the real economy and give the wrong comovement between macroeconomic aggregates when contracts are optimal. This contrasts with Christiano et al. (2013), who employ the BGG contract and emphasize the importance of risk shocks in generating business cycle fluctuations. Under the BGG contract, higher cross-sectional variance of entrepreneurs' idiosyncratic productivity causes an increase in defaults leading to a decline in the price of capital and consequently net worth. However, if returns to lenders are not predetermined and entrepreneurs are forward-looking, they realize that lower net worth implies higher financial premiums and more costly borrowing in the future. Therefore, forward-looking entrepreneurs desire more net worth in these states and thus negotiate lower returns to lenders, which stabilizes the response of net worth to the shock. As a result, under the optimal contract the financial accelerator is severely dampened for risk shocks.

1.2. Related literature

The CSV framework remains one of the most widely used methods for embedding financial frictions in DSGE models. The bulk of the literature follows the BGG framework and employs myopic entrepreneurs with fixed rate lending contracts. A non-exhaustive survey of recent work in this area includes Christiano et al. (2013), Christensen and Dib (2008), Ottonello (2015), Fernandez-Villaverde (2010), and Fernandez-Villaverde and Ohanian (2010).

In a related paper, Candian and Dmitriev (2016) study the behavior of risk-averse entrepreneurs that do not have access to risk-sharing technology within the entrepreneurial family and are thus exposed to both idiosyncratic and aggregate risk. They find that risk shocks do not generate strong amplification even when interest rates are predetermined as in BGG. Chugh (2013) also studies risk shocks in the BGG environment and concludes that cross-sectional firm level evidence provides little empirical support for the presence of large risk shocks.

Recent work by Carlstrom et al. (2016), hereafter CFP, simultaneously and independently derives the dynamically optimal contract for forward-looking, risk-neutral entrepreneurs, which is one of the cases we present here. CFP focus on the social planner's problem and the relative social efficiency of the optimal contract vis-a-vis the BGG contract for technology and monetary shocks, while we focus on the following question: do optimal contracts mitigate the financial accelerator or not? In contrast with CFP, we compare the model with frictions against a frictionless benchmark to examine the role of optimal contracts in amplifying and propagating business cycle fluctuations. We also study the impact of risk shocks, which are absent in CFP. Finally, we consider varying degrees of entrepreneurial risk aversion, while CFP focus on the risk-neutral case.

The CSV approach is not the only way to model financial frictions. In a related paper (Dmitriev and Hoddenbagh, 2014) we investigate the effect of optimal state-contingent contracts in a model with costly state enforcement frictions a la Kiyotaki and Moore (1997). In this alternative environment, we find that optimal state-contingent contracts severely dampen the amplification response from technology and monetary shocks. Again, our results demonstrate that amplification in the costly state enforcement literature is generated via the assumption of a predetermined lending rate (as in Gertler and Kiyotaki, 2010 and Gertler and Karadi, 2011). Cao and Nie (2016) also find that market incompleteness is more important than the underlying financial frictions in generating a financial amplification channel. An exception is presented by Jermann and Quadrini (2012), who allow both debt and equity contracts and achieve amplification by introducing adjustment costs between these instruments and ruling out other instruments. In the adverse selection framework, House (2006) extends Stiglitz and Weiss (1981) and also shows that financial frictions amplify business cycles only when returns for lenders are predetermined. When contracts are contingent or allow both debt and equity, financial frictions actually stabilize business cycles.

In contrast to macroeconomic models with agency costs, Di Tella (2015) investigates the role of optimal state-contingent contracts in a model with no labor and flexible prices, and finds that risk shocks do generate amplification. Risk shocks in Di Tella (2015) decrease the capital stock which leads to lower output and a recession. In models with agency costs, the aggregate capital stock moves slowly over the business cycle, so that risk shocks affect the economy only through changes in hours worked resulting from markup fluctuations. As a result, the mechanism delivering amplification from risk shocks is very different in agency cost models relative to Di Tella's model. In our framework households use labor supply adjustments to smooth consumption, and thus are more willing to take risk upon themselves — particularly when entrepreneurs are forward-looking and want to smooth their net worth. In equilibrium the optimal contract dampens negative risk shocks overall due to higher hours worked by lenders, who receive lower returns on loans which also stabilizes borrowers' net worth. If the labor supply becomes less elastic, households find it more difficult to use their labor supply to smooth consumption and are less willing to sell insurance to entrepreneurs. Since amplification in agency cost models works through fluctuations in hours worked, risk shocks and monetary shocks have almost no impact on the economy when labor supply is inelastic.

In summary, the bulk of the evidence suggests that the ability of financial frictions to amplify business cycle fluctuations is dependent on non-contingent loan contracts across a wide class of models, including the CSV, costly state enforcement and adverse selection frameworks common in the literature.

2. The optimal loan contract in partial equilibrium

Our main theoretical contribution in this paper is to introduce forward-looking risk-averse entrepreneurs into an otherwise standard CSV model of financial frictions. In this section we outline the key differences between the dynamically optimal loan contract chosen by forward-looking entrepreneurs and the contingent and non-contingent loan contracts chosen by myopic entrepreneurs in a partial equilibrium setting. Here we assume that entrepreneurs take the price of capital and the expected return to capital as given. In Section 3 we endogenize these variables in general equilibrium.

At time t, entrepreneur j purchases capital $K_t(j)$ at a unit price of Q_t. At time $t+1$, the entrepreneur rents this capital to perfectly competitive wholesale goods producers. The entrepreneur uses his net worth $N_t(j)$ and a loan $B_t(j)$ from the representative lender to purchase capital:

$$Q_t K_t(j) = N_t(j) + B_t(j).$$

After buying capital, the entrepreneur is hit with an idiosyncratic shock $\omega_{t+1}(j)$, a log-normal random variable with distribution $\log(\omega(j)) \sim \mathcal{N}(-\frac{1}{2}\sigma_\omega^2, \sigma_\omega^2)$ and mean of one. Capital returns, $R_{k,t+1}$, are exogenous here in partial equilibrium, but will be endogenous in general equilibrium and subject to aggregate shocks. The entrepreneur j is able to deliver $Q_t K_t(j) R_{k,t+1}\omega_{t+1}(j)$ units of assets.

Following BGG, we assume entrepreneurs die with constant probability $1-\gamma$. Different from BGG we allow entrepreneurs to have positive risk aversion σ_e. To make the framework tractable, we allow for complete insurance markets between entrepreneurs. Each entrepreneur works only in the first period and earns a wage W_t^e, which is invested in a common mutual fund with a family of other entrepreneurs to protect members of the family from idiosyncratic risk. Entrepreneurial net worth thus depends only on aggregate returns and the initial invested wage. Dying entrepreneurs consume all of their operational equities. If entrepreneurs survive they do not consume anything and reinvest their net worth. The entrepreneur's intertemporal utility function is

$$V_t^e(j) = \frac{(1-\gamma)}{1-\sigma_e}\mathbb{E}_t\left\{\sum_{s=1}^{\infty}\gamma^s\big(N_{t+s}(j)\big)^{1-\sigma_e}\right\} \tag{1}$$

where $N_t(j)$ is entrepreneur j's net worth. Risk neutrality is attained by setting $\sigma_e = 0$. The timeline for entrepreneurs is plotted in Fig. 1.

2.1. Borrower and lender payoffs

The contract between the lender and borrower follows the familiar CSV framework. We assume that the lender cannot observe the realization of idiosyncratic shocks to entrepreneurs unless he pays monitoring costs μ which are a fixed percentage of total assets. Given this friction, the borrower offers the lender a contract with a state-contingent interest rate Z_{t+1} subject to macroeconomic conditions.

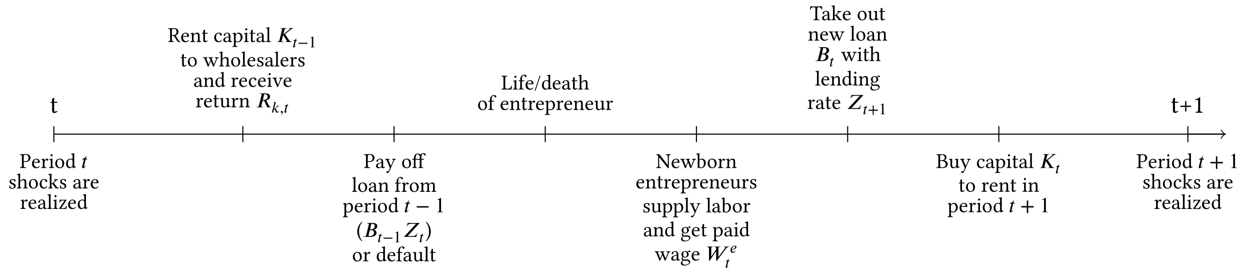


Fig. 1. Timeline for entrepreneurs.

The entrepreneur repays the loan only when it is profitable for the family to do so. In particular, the entrepreneur will repay the loan only if, after repayment, he has more assets than liabilities. We define the cutoff productivity level $\bar{\omega}_{t+1}$, also known as the bankruptcy threshold, as the minimum level of productivity necessary for an entrepreneur to repay the loan:

$$\underbrace{B_t(j)Z_{t+1}(j)}_{\text{Cost of loan repayment}} = \underbrace{\bar{\omega}_{t+1}R_{k,t+1}Q_tK_t(j).}_{\text{Minimum revenue for loan repayment}}$$

If $\omega_{t+1}(j) < \bar{\omega}_{t+1}$ the entrepreneur defaults and enters bankruptcy; if $\omega_{t+1}(j) \geq \bar{\omega}_{t+1}$ he repays the loan. The cutoff productivity level allows us to express the dynamics of aggregate net worth for the entrepreneurial family:

$$N_{t+1} = Q_tK_tR_{k,t+1}\left[\int_{\bar{\omega}_{t+1}}^{\infty} \omega f(\omega)d\omega - \bar{\omega}_{t+1}(1 - F(\bar{\omega}_{t+1}))\right] + W^e_{t+1}, \tag{2}$$

where f is the probability density function and F is the cumulative distribution function of the log-normal distribution of idiosyncratic productivity. Note that for a particular entrepreneur j, net worth does not include the wages of entrepreneurs joining the family mutual fund for the first time. Net worth for entrepreneur j can thus be expressed as

$$N_{t+1}(j) = Q_tK_t(j)R_{k,t+1}(1 - \Gamma_{t+1}), \tag{3}$$

where

$$\Gamma_{t+1} = \int_0^{\bar{\omega}_{t+1}} \omega f(\omega, \sigma_{\omega,t})d\omega + \bar{\omega}_{t+1}\Big[1 - F(\bar{\omega}_{t+1}, \sigma_{\omega,t})\Big]$$

is the gross share of revenue that goes to lenders before they pay monitoring costs, and σ_ω is the standard deviation of the logarithm of idiosyncratic productivity. We define leverage, κ_t, as the value of the entrepreneur's existing capital stock divided by current net worth:

$$\kappa_t(j) \equiv \frac{Q_tK_t(j)}{N_t(j)}. \tag{4}$$

Using individual net worth dynamics (3), we can express the objective function for forward-looking entrepreneurs (1) recursively[4]:

$$V^e_t(j) = \frac{1-\gamma}{1-\sigma_e}(N_t(j))^{1-\sigma_e}(\Psi_t - 1), \tag{5}$$

where

$$\Psi_t = 1 + \gamma\mathbb{E}_t\left\{\kappa_t^{1-\sigma_e}R_{k,t+1}^{1-\sigma_e}(1-\Gamma_{t+1})^{1-\sigma_e}\Psi_{t+1}\right\}. \tag{6}$$

Ψ_t is the entrepreneur's normalized utility per unit of net worth following the idiosyncratic productivity draw in period t (and before entrepreneurial life or death), while $\Psi_t - 1$ is the entrepreneur's normalized utility per unit of net worth prior to the idiosyncratic productivity draw in period t.

The gross rate of return for the lender, R_{t+1}, also depends on the productivity cutoff. For idiosyncratic realizations above the cutoff, the lender will be repaid the full amount of the loan $B_t(j)Z_{t+1}(j)$. For idiosyncratic realizations below the cutoff, the entrepreneur will enter bankruptcy and the lender will pay monitoring costs μ and take over the entrepreneur's assets, ending up with $[1-\mu)K_t(j)R_{k,t+1}h(\omega_{t+1}(j)]$. More formally, the lender's ex post return is

[4] We derive the recursive expression for the entrepreneur's objective function in Appendix A.

$$B_t(j)R_{t+1}(j) = \begin{cases} B_t(j)Z_{t+1}(j) & \text{if } \omega_{t+1}(j) \geq \bar{\omega}_{t+1}, \\ (1-\mu)K_t(j)R_{k,t+1}\omega_{t+1}(j) & \text{if } \omega_{t+1}(j) < \bar{\omega}_{t+1}. \end{cases}$$

Taking into account that loans to entrepreneurs are perfectly diversifiable, the lender's aggregate return on loans R_{t+1} is defined as

$$(K_t Q_t - N_t)R_{t+1} \equiv Q_t K_t R_{k,t+1}(\Gamma_{t+1} - \mu G_{t+1}), \tag{7}$$

where $G_{t+1} = \int_0^{\bar{\omega}_{t+1}} \omega f(\omega)d\omega$ is the share of total entrepreneurial assets under default. We define the financial premium as the ratio of the rental rate on capital to the lender's rate of return:

$$\text{Financial Premium} \equiv \frac{R_{k,t+1}}{R_{t+1}}.$$

2.2. Solving for the loan contracts: the BGG contract, the Myopic Contingent Contract (MCC) and the optimal contract

We solve for three loan contracts: the BGG contract, the myopic contingent contract (hereafter denoted MCC), and the optimal contract. The BGG contract assumes the lending rate is predetermined and entrepreneurs are myopic. The MCC allows for a state-contingent lending rate but maintains the assumption that entrepreneurs are myopic rather than forward-looking. We solve for the MCC as it isolates the impact of introducing a state-contingent lending rate on the financial accelerator. The optimal contract allows for a state-contingent lending rate and assumes entrepreneurs are forward-looking.

Our solution for all three contracts nests entrepreneurial risk aversion as well as the risk-neutral case. To attain risk neutrality we simply set $\sigma_e = 0$, while $\sigma_e > 0$ signals risk aversion, with larger positive values denoting stronger risk aversion. We also show solutions for the frictionless benchmark, which sets monitoring costs equal to zero ($\mu = 0$). In each of the propositions below we explicitly solve for each contract in the most general case, followed by the risk-neutral case and the frictionless benchmark with zero monitoring costs.

The differences between the three loan contracts arise from two sources: the lender's participation constraint and the borrower's objective function. First, the lender's participation constraint in BGG differs from the participation constraint in the MCC and the optimal contract. The participation constraint arises from the household Euler equation and stipulates the minimum rate of return that entrepreneurs must offer to lenders to receive a loan. In BGG, the participation constraint has the following form:

$$\mathbb{E}_t\left\{\Lambda_{t,t+1}\right\}R_{t+1} = 1, \tag{8}$$

where

$$\Lambda_{t,s} \equiv \beta^s \frac{U_{C,t+s}}{U_{C,t}} \tag{9}$$

is the household (*i.e.* shareholder) intertemporal marginal rate of substitution, also known as the household stochastic discount factor. Under this participation constraint, entrepreneurs pay a constant safe rate of return to the lenders, R_{t+1}, which ignores the risk-averse representative household's desire for consumption insurance. In contrast, the participation constraint for the MCC and the optimal contract is

$$\mathbb{E}_t\left\{\Lambda_{t,t+1}R_{t+1}\right\} = 1. \tag{10}$$

The above expression implies that households prefer a state-contingent rate of return that is negatively correlated with household consumption. Quite simply, households like consumption insurance. In recessions, households desire a higher rate of return because their marginal utility of consumption is high, and vice versa in booms.

Second, the borrower's objective function in BGG and MCC differs from the objective function which gives rise to the optimal contract. Entrepreneurs in BGG and MCC maximize next period net worth, defined in equation (2). If we substitute the expression for leverage from (4) into (2), we have the entrepreneur's objective function in BGG and MCC:

$$\max_{\kappa_t, \bar{\omega}_{t+1}} \mathbb{E}_t\left\{\left[\kappa_t N_t(j) R_{k,t+1}\left(\int_{\bar{\omega}_{t+1}}^{\infty} \omega f(\omega)d\omega - \bar{\omega}_{t+1}(1 - F(\bar{\omega}_{t+1}))\right)\right]^{1-\sigma_e}\right\}. \tag{11}$$

In contrast, under the dynamically optimal contract entrepreneurs maximize utility, given by (1). As we have mentioned before, utility maximizing entrepreneurs are concerned not only about current capital returns but also future capital returns and future financial premiums.

We now have all of the ingredients necessary to set up the entrepreneur's optimization problem and solve for the three different loan contracts: (1) the BGG contract; (2) the MCC contract; and, (3) the optimal contract.

Proposition 1. *To solve for the BGG contract, entrepreneurs choose their state contingent cutoff $\bar{\omega}_{t+1}$ and leverage $\kappa_t(j)$ to maximize next period net worth (11) subject to (3), (7) and (8). The solution to this problem is given by*

$$\mathbb{E}_t\left\{(1-\Gamma_{t+1})^{1-\sigma_e}R_{k,t+1}^{1-\sigma_e}\right\} = \mathbb{E}_t\left\{\frac{(1-\Gamma_{t+1})^{-\sigma_e}\Gamma_{\omega,t+1}}{\mathbb{E}_t\{\Lambda_{t,t+1}\}\kappa_t(\Gamma_{\omega,t+1}-\mu G_{\omega,t+1})}R_{k,t+1}^{-\sigma_e}\right\}. \tag{12}$$

Under log utility ($\sigma_e = 1$) equation (12) simplifies to

$$1 = \mathbb{E}_t\left\{\frac{\Gamma_{\omega,t+1}}{\mathbb{E}_t\{\Lambda_{t,t+1}\}\kappa_t(\Gamma_{\omega,t+1}-\mu G_{\omega,t+1})R_{k,t+1}(1-\Gamma_{t+1})}\right\}.$$

Under the frictionless model with zero monitoring costs ($\mu = 0$) equation (12) simplifies to

$$\mathbb{E}_t\left\{(1-\Gamma_{t+1})^{1-\sigma_e}R_{k,t+1}^{1-\sigma_e}\right\} = \mathbb{E}_t\left\{\frac{(1-\Gamma_{t+1})^{-\sigma_e}}{\mathbb{E}_t\{\Lambda_{t,t+1}\}\kappa_t}R_{k,t+1}^{-\sigma_e}\right\}.$$

Proof. See Appendix B.1. □

Corollary 1. *Log-linearization of the BGG optimality condition (12) and the BGG participation constraint (8) gives*

$$\mathbb{E}_t\hat{R}_{k,t+1} - \mathbb{E}_t\hat{R}_{t+1} = \nu_\kappa\hat{\kappa}_t + \nu_\sigma\hat{\sigma}_{\omega,t}, \tag{13}$$

$$\hat{R}_{t+1} - \mathbb{E}_t\hat{R}_{t+1} = 0, \tag{14}$$

where $\nu_\kappa = \dfrac{\frac{\Gamma_{\omega\omega}}{\Gamma_\omega} - \frac{\Gamma_{\omega\omega}-\mu G_{\omega\omega}}{\Gamma_\omega-\mu G_\omega}}{\frac{\Gamma_{\omega\omega}}{\Gamma_\omega} - \frac{\Gamma_{\omega\omega}-\mu G_{\omega\omega}}{\Gamma_\omega-\mu G_\omega} + \frac{\Gamma_\omega}{1-\Gamma} + \frac{\Gamma_\omega-\mu G_\omega}{\Gamma-\mu G}}\dfrac{1}{\kappa-1}$ *and* $\nu_\sigma = \dfrac{-\frac{\Gamma_\sigma-\mu G_\sigma}{\Gamma-\mu G}\left(\frac{\Gamma_{\omega\omega}}{\Gamma_\omega} - \frac{\Gamma_{\omega\omega}-\mu G_{\omega\omega}}{\Gamma_\omega-\mu G_\omega} + \frac{\Gamma_\omega}{1-\Gamma}\right) + \frac{\Gamma_\omega-\mu G_\omega}{\Gamma-\mu G}\left(\frac{\Gamma_{\omega\sigma}}{\Gamma_\omega} - \frac{\Gamma_{\omega\sigma}-\mu G_{\omega\sigma}}{\Gamma_\omega-\mu G_\omega} + \frac{\Gamma_\sigma}{1-\Gamma}\right)}{\frac{\Gamma_{\omega\omega}}{\Gamma_\omega} - \frac{\Gamma_{\omega\omega}-\mu G_{\omega\omega}}{\Gamma_\omega-\mu G_\omega} + \frac{\Gamma_\omega}{1-\Gamma} + \frac{\Gamma_\omega-\mu G_\omega}{\Gamma-\mu G}}$.

Proof. See Appendix C.1 and Appendix C.2. □

Equation (13) shows that in the BGG contract the entrepreneur's leverage depends on next period's expected financial premium, defined in log deviations from steady state as expected returns to capital ($\mathbb{E}_t\{\hat{R}_{k,t+1}\}$) minus expected returns to the household ($\mathbb{E}_t\{\hat{R}_{t+1}\}$). Equation (14) shows that lenders returns (deposit rate) are predetermined. We prove in Appendix B.4 that when lenders' returns are predetermined, to a first order approximation the loan contract is identical regardless of whether entrepreneurs are forward-looking or myopic, risk-neutral or risk-averse.[5]

Proposition 2. *To solve for the MCC contract, entrepreneurs choose their state contingent cutoff $\bar{\omega}_{t+1}$ and leverage $\kappa_t(j)$ to maximize (11) subject to (3), (7) and (10). The solution to this problem is given by*

$$\frac{\kappa_t\Lambda_{t,t+1}\mathbb{E}_t\left\{R_{k,t+1}^{1-\sigma_e}(1-\Gamma_{t+1})^{1-\sigma_e}\right\}}{R_{k,t+1}^{-\sigma_e}(1-\Gamma_{t+1})^{-\sigma_e}} = \frac{\Gamma_{\omega,t+1}}{\Gamma_{\omega,t+1}-\mu G_{\omega,t+1}}. \tag{15}$$

Under log utility ($\sigma_e = 1$) equation (15) simplifies to

$$\Lambda_{t,t+1}\kappa_t R_{k,t+1}(1-\Gamma_{t+1}) = \frac{\Gamma_{\omega,t+1}}{\Gamma_{\omega,t+1}-\mu G_{\omega,t+1}}.$$

Under the frictionless model with zero monitoring costs ($\mu = 0$) equation (15) simplifies to

$$\frac{\kappa_t\Lambda_{t,t+1}\mathbb{E}_t\left\{R_{k,t+1}^{1-\sigma_e}(1-\Gamma_{t+1})^{1-\sigma_e}\right\}}{R_{k,t+1}^{-\sigma_e}(1-\Gamma_{t+1})^{-\sigma_e}} = 1.$$

Proof. See Appendix B.2. □

Corollary 2. *Log-linearization of the MCC optimality condition (15) and the MCC participation constraint (10) gives*

$$\mathbb{E}_t\hat{R}_{k,t+1} - \mathbb{E}_t\hat{R}_{t+1} = \nu_\kappa\hat{\kappa}_t + \nu_\sigma\hat{\sigma}_{\omega,t}, \tag{16}$$

$$\hat{R}_{t+1} - \mathbb{E}_t\hat{R}_{t+1} = \hat{R}_{k,t+1} - \mathbb{E}_t\hat{R}_{k,t+1} + \tilde{\alpha}\left[-\sigma_e(\hat{R}_{k,t+1} - \mathbb{E}_t\hat{R}_{k,t+1}) + \hat{\Lambda}_{t,t+1} - \mathbb{E}_t\hat{\Lambda}_{t,t+1}\right], \tag{17}$$

[5] This is also true when the lending rate (Z) is predetermined, rather than the lender's return (R). The general equilibrium behavior of the model under predetermined R and predetermined Z is similar, so we only report results for predetermined R.

where $\tilde{\alpha} = \frac{\frac{\Gamma_\omega - \mu G_\omega}{\Gamma - \mu G}}{\frac{\Gamma_{\omega\omega}}{\Gamma_\omega} - \frac{\Gamma_{\omega\omega} - \mu G_{\omega\omega}}{\Gamma_\omega - \mu G_\omega} + \frac{\sigma_e \Gamma_\omega}{1-\Gamma}}$.

Proof. See Appendix C.1 and Appendix C.3. □

Corollary 2 clearly illustrates the differences between the BGG contract and the MCC contract. In equation (17), lender's returns depend on capital returns and household consumption, both elements which are missing in the BGG contract. For standard calibrations, $\tilde{\alpha}$ takes a value between five and six and the entrepreneurial risk aversion parameter σ_e is equal to one, so that lender's returns are very sensitive to the consumption level and the consumption insurance channel dominates the response to capital returns. When consumption is high, the lending rate declines; when consumption is low the lending rate increases. The negative covariance between the lender's consumption and the lender's returns reflects the nature of insurance, which amplifies the impact of shocks to the economy. Note that as entrepreneurs become more risk-averse (as σ_e increases), the impact of the consumption insurance channel declines.

Now that we have described the BGG and MCC contracts in detail, we turn our attention to the optimal contract. As discussed above, the optimal contract takes the consumption insurance channel from MCC and adds forward-looking entrepreneurs.

Proposition 3. *To solve for the optimal contract, entrepreneurs choose their state contingent cutoff $\bar{\omega}_{t+1}$ and leverage $\kappa_t(j)$ to maximize (1) subject to (3), (7) and (10). The solution to this problem is given by*

$$\frac{\kappa_t \Lambda_{t,t+1} \mathbb{E}_t \left\{ R_{k,t+1}^{1-\sigma_e} (1-\Gamma_{t+1})^{1-\sigma_e} \Psi_{t+1} \right\}}{\Psi_{t+1} R_{k,t+1}^{-\sigma_e} (1-\Gamma_{t+1})^{-\sigma_e}} = \frac{\Gamma_{\omega,t+1}}{\Gamma_{\omega,t+1} - \mu G_{\omega,t+1}}. \tag{18}$$

Under log utility ($\sigma_e = 1$) equation (18) simplifies to

$$\Lambda_{t,t+1} \kappa_t R_{k,t+1} (1-\Gamma_{t+1}) = \frac{\Gamma_{\omega,t+1}}{\Gamma_{\omega,t+1} - \mu G_{\omega,t+1}}.$$

Under the frictionless model with zero monitoring costs ($\mu = 0$) equation (18) simplifies to

$$\frac{\kappa_t \Lambda_{t,t+1} \mathbb{E}_t \left\{ R_{k,t+1}^{1-\sigma_e} (1-\Gamma_{t+1})^{1-\sigma_e} \Psi_{t+1} \right\}}{\Psi_{t+1} R_{k,t+1}^{-\sigma_e} (1-\Gamma_{t+1})^{-\sigma_e}} = 1.$$

Proof. See Appendix B.3. □

Corollary 3. *Log-linearization of the optimal contract, (18), and the participation constraint (10) gives*

$$\mathbb{E}_t \hat{R}_{k,t+1} - \mathbb{E}_t \hat{R}_{t+1} = \nu_\kappa \hat{\kappa}_t + \nu_\sigma \hat{\sigma}_{\omega,t}, \tag{19}$$

$$\hat{R}_{t+1} - \mathbb{E}_t \hat{R}_{t+1} = \hat{R}_{k,t+1} - \mathbb{E}_t \hat{R}_{k,t+1} - \tilde{\alpha} \left[\sigma_e (\hat{R}_{k,t+1} - \mathbb{E}_t \hat{R}_{k,t+1}) - (\hat{\Lambda}_{t,t+1} - \mathbb{E}_t \hat{\Lambda}_{t,t+1}) + \hat{\Psi}_{t+1} - \mathbb{E}_t \hat{\Psi}_{t+1} \right], \tag{20}$$

$$\hat{\Psi}_t = \epsilon_N \mathbb{E}_t \left\{ (1-\sigma_e)[(\kappa - 1)(\hat{R}_{k,t+1} - \hat{R}_{t+1}) + \hat{R}_{k,t+1} + \nu_\Psi \hat{\sigma}_{\omega,t}] + \hat{\Psi}_{t+1} \right\}, \tag{21}$$

where $\nu_\Psi = \frac{(\Gamma_\sigma - \mu G_\sigma) \frac{\Gamma_\omega}{\Gamma_\omega - \mu G_\omega} - \Gamma_\sigma}{1-\Gamma} \sigma_\omega$ *and* $\epsilon_N = \gamma \kappa^{1-\sigma_e} (1-\Gamma)^{1-\sigma_e} R_k^{1-\sigma_e}$.

Proof. See Appendix C.1 and Appendix C.4. □

We see from (20) that under the optimal contract with risk-aversion less than one, the surprise to lender's returns depends not only on surprises to capital returns and consumption, as in the MCC contract, but *future* capital returns and *future* financial premiums as well. If entrepreneurs are more optimistic about expected future financial premiums or future returns to capital, they prefer to pay the lender a lower interest rate because one unit of net worth becomes more valuable. Corollary 3 thus illustrates the strong stabilizing mechanism of the optimal contract. When a crisis hits and decreases entrepreneur's net worth, expected future financial premiums will rise. But entrepreneurs will also pay lenders a smaller deposit rate, which stabilizes their net worth. As a result, the main channel for the financial accelerator — volatility in net worth — is diminished when entrepreneurs are forward-looking. As risk-aversion increases, lenders' returns become less sensitive to future capital returns and future financial premiums. Under log utility for entrepreneurs ($\sigma_e = 1$), future capital returns and financial premiums have no effect on lender's returns as $\hat{\Psi}_t = 0$. This is why the optimal contract is identical for forward-looking and myopic entrepreneurs when $\sigma_e = 1$. However, under higher entrepreneurial risk-aversion

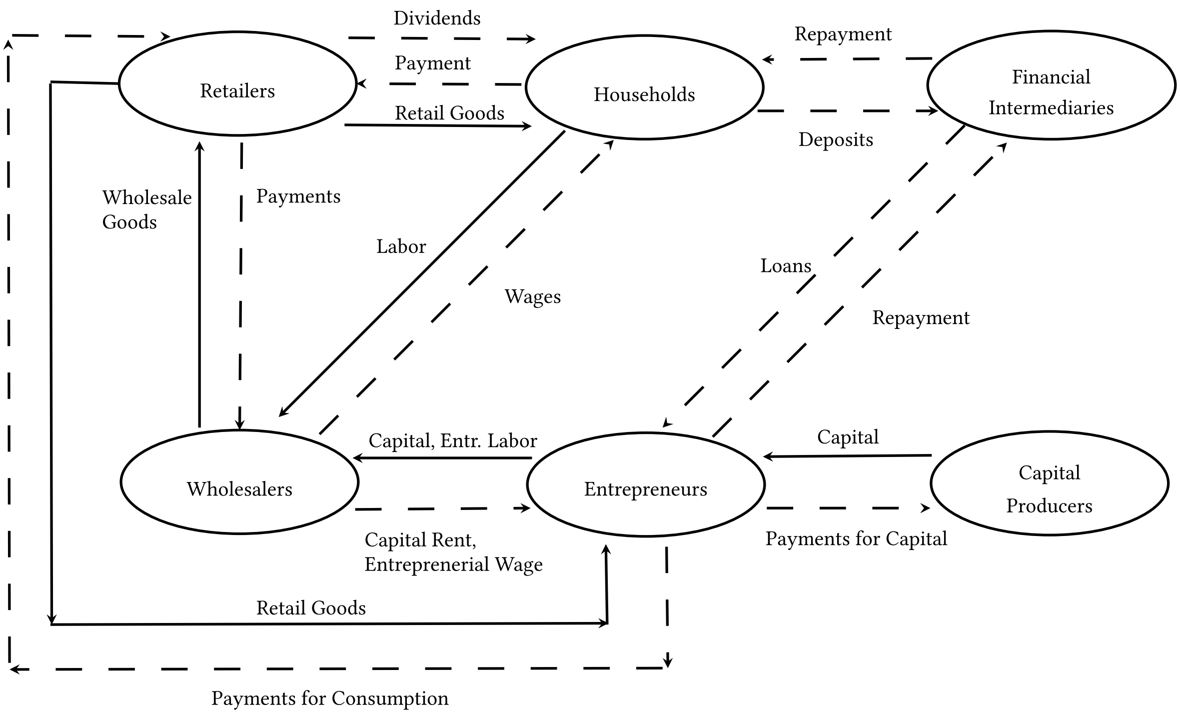


Fig. 2. Overview of the model.

($\sigma_e > 1$), increases in capital returns have a less positive effect on lenders' returns, as the insurance motive for entrepreneurs dominates that for households in the standard calibration with household risk aversion $\sigma = 1$.

Although we have taken a partial equilibrium view here, Corollaries 1–3 remain unchanged in the general equilibrium setting. In both partial and general equilibrium, leverage and the deposit rate are determined by the paths of capital returns and consumption. Therefore, the intuition provided by Corollaries 1–3 holds in general equilibrium.

3. The model in general equilibrium

We now embed the three loan contracts in a standard dynamic New Keynesian model. There are six agents in our model: households, entrepreneurs, financial intermediaries, capital producers, wholesalers and retailers. Entrepreneurs buy capital from capital producers and rent it to perfectly competitive wholesalers, who sell their goods to monopolistically competitive retailers. Retailers costlessly differentiate the wholesale goods and sell them to households at a markup over marginal cost. Retailers have price-setting power and are subject to Calvo (1983) price rigidities. Households bundle the retail goods in CES fashion into a final consumption good. A graphical overview of the model is provided in Fig. 2. The dotted lines denote financial flows, while the solid lines denote real flows (goods, labor, and capital).

3.1. Households

The representative household maximizes its utility by choosing the optimal path of consumption, labor and money:

$$\max \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s \left[\frac{C_{t+s}^{1-\sigma}}{1-\sigma} + \zeta \log\left(\frac{M_{t+s}}{P_{t+s}}\right) - \chi \frac{H_{t+s}^{1+\eta}}{1+\eta} \right] \right\}, \tag{22}$$

where C_t is household consumption, M_t/P_t denotes real money balances, and H_t is household labor effort. The budget constraint of the representative household is

$$C_t = W_t H_t - T_t + \Pi_t + R_t \frac{D_t}{P_t} - \frac{D_{t+1}}{P_t} + \frac{M_{t-1} - M_t}{P_t} + \frac{B_{t-1} R_t^n - B_t}{P_t}, \tag{23}$$

where W_t is the real wage, T_t is lump-sum taxes, Π_t is profit received from household ownership of final goods firms distributed in lump-sum fashion, D_t are deposits in financial intermediaries (banks) that pay a contingent nominal gross interest rate R_t, and B_t are nominal bonds that pay a gross nominal non-contingent interest rate R_t^n.

Households maximize their utility (22) subject to the budget constraint (23) with respect to deposits, labor, nominal bonds and money, yielding four first order conditions:

$$U_{C,t} = \beta \mathbb{E}_t \left\{ R_{t+1} U_{C,t+1} \right\}, \tag{24}$$

$$U_{C,t} = \beta R_t^n \mathbb{E}_t \left\{ \frac{U_{C,t+1}}{\pi_{t+1}} \right\}, \tag{25}$$

$$W_t U_{C,t} = \chi H_t^\eta, \tag{26}$$

$$U_{C,t} = \zeta \frac{1}{m_t} + \beta \mathbb{E}_t \left\{ \frac{U_{C,t+1}}{\pi_{t+1}} \right\}. \tag{27}$$

We define the gross rate of inflation as $\pi_{t+1} = P_{t+1}/P_t$, and real money balances as $m_t = M_t/P_t$.

3.2. Retailers

The final consumption good is made up of a basket of intermediate retail goods which are aggregated together in CES fashion by the representative household:

$$C_t = \left(\int_0^1 c_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

Demand for retailer i's unique variety is

$$c_{it} = \left(\frac{p_{it}}{P_t} \right)^{-\varepsilon} C_t,$$

where p_{it} is the price charged by retail firm i. The aggregate price index is defined as

$$P_t = \left(\int_0^1 p_{it}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}.$$

Each retail firm chooses its price according to Calvo (1983) in order to maximize net discounted profit. With probability $1-\theta$ each retailer is able to change its price in a particular period t. Retailer i's objective function is

$$\max_{p_{it}^*} \sum_{s=0}^{\infty} \theta^s \mathbb{E}_t \left\{ \Lambda_{t+s} \frac{p_{it}^* - P_{t+s}^w}{P_{t+s}} \left(\frac{p_{it}^*}{P_{t+s}} \right)^{-\varepsilon} Y_{t+s} \right\},$$

where P_t^w is the wholesale goods price. The first order condition with respect to the retailer's price p_{it}^* is

$$\sum_{s=0}^{\infty} \theta^s \mathbb{E}_t \left\{ \Lambda_{t,s} (p_{it}^*/P_{t+s})^{-\varepsilon} Y_{t+s} \left[p_{it}^* - \frac{\varepsilon}{\varepsilon-1} P_{t+s}^w \right] \right\} = 0. \tag{28}$$

From this condition it is clear that all retailers which are able to reset their prices in period t will choose the same price $p_{it}^* = P_t^* \quad \forall i$. The price level will evolve according to

$$P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \tag{29}$$

Dividing the left and right hand side of (29) by the price level gives

$$1 = \left[\theta \pi_{t-1}^{\varepsilon-1} + (1-\theta)(p_t^*)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}, \tag{30}$$

where $p_t^* = P_t^*/P_t$. Using the same logic, we can normalize (28) and obtain

$$p_t^* = \frac{\varepsilon}{\varepsilon-1} \frac{\sum_{s=0}^{\infty} \theta^s \mathbb{E}_{t-1} \left\{ \Lambda_{t,s} (1/p_{t+s})^{-\varepsilon} Y_{t+s} p_{t+s}^w \right\}}{\sum_{s=0}^{\infty} \theta^s \mathbb{E}_{t-1} \left\{ \Lambda_{t,s} (1/p_{t+s})^{-\varepsilon} Y_{t+s} \right\}}, \tag{31}$$

where $p_{t+s}^w = \frac{P_{t+s}^w}{P_t}$ and $p_{t+s} = P_{t+s}/P_t$.

3.3. Wholesalers

Wholesale goods are produced by perfectly competitive firms and then sold to monopolistically competitive retailers who costlessly differentiate them. Wholesalers hire labor from households and entrepreneurs in a competitive labor market at real wage W_t and W_t^e and rent capital from entrepreneurs at rental rate R_t^r. Note that capital purchased in period t is used in period $t+1$. Following BGG, the production function of the representative wholesaler is given by

$$Y_t = A_t K_{t-1}^{\alpha} (H_t)^{(1-\alpha)\Omega} (H_t^e)^{(1-\alpha)(1-\Omega)},$$

where A_t denotes aggregate technology, K_t is capital, H_t is household labor, H_t^e is entrepreneurial labor, and Ω defines the relative importance of household labor and entrepreneurial labor in the production process. Entrepreneurs inelastically supply one unit of labor, so that the production function simplifies to

$$Y_t = A_t K_{t-1}^{\alpha} H_t^{(1-\alpha)\Omega}. \tag{32}$$

We can express the price of the wholesale good in terms of the price of the final good. In this case, the price of the wholesale good will be

$$\frac{P_t^w}{P_t} = \frac{1}{\mathcal{X}_t}, \tag{33}$$

where $\mathcal{X}_t$ is the variable markup charged by final goods producers. The objective function for wholesalers is then given by

$$\max_{H_t, H_t^e, K_{t-1}} \frac{1}{\mathcal{X}_t} A_t K_{t-1}^{\alpha} (H_t)^{(1-\alpha)\Omega} (H_t^e)^{(1-\alpha)(1-\Omega)} - W_t H_t - W_t^e H_t^e - R_t^r K_{t-1}.$$

Here wages and the rental price of capital are in real terms. The first order conditions with respect to capital, household labor and entrepreneurial labor are

$$\frac{1}{\mathcal{X}_t} \alpha \frac{Y_t}{K_{t-1}} = R_t^r, \tag{34}$$

$$\frac{\Omega}{\mathcal{X}_t} (1-\alpha) \frac{Y_t}{H_t} = W_t, \tag{35}$$

$$\frac{\Omega}{\mathcal{X}_t} (1-\alpha) \frac{Y_t}{H_t^e} = W_t^e. \tag{36}$$

3.4. Capital producers

The perfectly competitive capital producer transforms final consumption goods into capital. Capital production is subject to adjustment costs, according to

$$K_t = I_t + (1-\delta) K_{t-1} - \frac{\phi_K}{2} \left(\frac{I_t}{K_{t-1}} - \delta \right)^2 K_{t-1}, \tag{37}$$

where I_t is investment in period t, δ is the rate of depreciation and ϕ_K is a parameter that governs the magnitude of the adjustment cost. The capital producer's objective function is

$$\max_{I_t} K_t Q_t - I_t,$$

where Q_t denotes the price of capital. The first order condition of the capital producer's optimization problem is

$$\frac{1}{Q_t} = 1 - \phi_K \left(\frac{I_t}{K_{t-1}} - \delta \right). \tag{38}$$

3.5. Lenders

One can think of the representative lender in the model as a perfectly competitive bank which costlessly intermediates between households and borrowers. The role of the lender is to diversify the household's funds among various entrepreneurs. The bank takes nominal household deposits D_t and loans out nominal amount B_t to entrepreneurs. In equilibrium, deposits will equal loanable funds ($D_t = B_t$). Households, as owners of the bank, receive a state contingent real rate of return R_{t+1} on their "deposits" — which equals the rate of return on loans to entrepreneurs.[6] Households choose the optimal lending rate according to their first order condition with respect to deposits:

[6] Note that lenders are not necessary in the model, but we follow BGG and MCC in positing a perfectly competitive financial intermediary between households and borrowers.

$$\beta \mathbb{E}_t \left\{ \frac{U_{C,t+1}}{U_{C,t}} R_{t+1} \right\} = \mathbb{E}_t \left\{ \Lambda_{t,t+1} R_{t+1} \right\} = 1.$$

As we discussed above, the lender prefers a return that co-varies negatively with household consumption in order to smooth consumption.

3.6. Entrepreneurs

We described the entrepreneur's maximization problem in detail in Section 2. Entrepreneurs choose their cutoff productivity level and leverage according to: (12) for the BGG contract; (15) for the MCC contract; and (18) for the dynamically optimal contract.

Wholesale firms rent capital at rate $R^r_{t+1} = \frac{\alpha Y_t}{\mathcal{X}_t K_{t-1}}$ from entrepreneurs. After production takes place entrepreneurs sell undepreciated capital back to capital goods producers for the unit price Q_{t+1}. Aggregate returns to capital are then given by

$$R_{k,t+1} = \frac{\frac{1}{\mathcal{X}_t}\frac{\alpha Y_{t+1}}{K_t} + Q_{t+1}(1-\delta)}{Q_t}. \tag{39}$$

Consistent with the partial equilibrium specification, entrepreneurs die with probability $1-\gamma$, which implies the following dynamics for aggregate net worth:

$$N_{t+1} = \gamma N_t \kappa_t R_{k,t+1} g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) + W^e_{t+1}. \tag{40}$$

3.7. Goods market clearing

We have goods market clearing:

$$Y_t = C_t + I_t + G_t + C^e_t + \mu G(\bar{\omega}_t, \sigma_{\omega,t-1}) R^k_t Q_{t-1} K_{t-1}, \tag{41}$$

where $\mu G(\bar{\omega}) = \int_0^{\bar{\omega}} \mu f(\omega)\omega d\omega$ is the fraction of capital returns that go to monitoring costs, paid by lenders.

3.8. Monetary policy

The central bank conducts monetary policy by choosing the nominal interest rate R^n_t. In Section 4 we employ the nominal interest rate rule used in BGG:

$$\log(R^n_t) - \log(R^n) = \rho^{R^n}\left(\log(R^n_{t-1}) - \log(R)\right) + \xi \pi_{t-1} + \epsilon^{R^n}_t \tag{42}$$

where ρ^{R^n} and ξ determine the relative importance of the past interest rate and past inflation in the central bank's interest rate rule. Shocks to the nominal interest rate are given by ϵ^{R^n}.

The monetary policy rule in BGG targets past inflation, unlike the conventional Taylor rule which targets current inflation:

$$\log(R^n_t) - \log(R^n) = \rho^{R^n}\left(\log(R^n_{t-1}) - \log(R)\right) + \xi \pi_t + \rho^Y\left(\log(Y_t) - \log(Y_{t-1})\right) + \epsilon^{R^n}_t. \tag{43}$$

We consider the conventional Taylor rule in the online appendix to examine the robustness of the financial accelerator to more hawkish monetary policy. The conventional Taylor rule is more aggressive in eliminating markup fluctuations and moving the economy toward the flexible price allocation than the BGG policy rule.

3.9. Shocks

The shocks in the model follow a standard AR(1) process. The AR(1) processes for technology, government spending and idiosyncratic volatility are given by

$$\log(A_t) = \rho^A \log(A_{t-1}) + \epsilon^A_t, \tag{44}$$

$$\log(G_t/Y_t) = (1-\rho^G)\log(G_{ss}/Y_{ss}) + \rho^G \log(G_{t-1}/Y_{t-1}) + \epsilon^G_t, \tag{45}$$

$$\log(\sigma_{\omega,t}) = (1-\rho^{\sigma_\omega})\log(\sigma_{\omega,ss}) + \rho^{\sigma_\omega}\log(\sigma_{\omega,t-1}) + \epsilon^{\sigma_\omega}_t, \tag{46}$$

where ϵ^A, ϵ^G and ϵ^{σ_ω} denote exogenous shocks to technology, government spending and idiosyncratic volatility, and G_{ss} and $\sigma_{\omega,ss}$ denote the steady state values for government spending and idiosyncratic volatility respectively. Recall that σ^2_ω is the variance of idiosyncratic productivity, and σ_ω is thus the standard deviation of idiosyncratic productivity. Nominal interest rate shocks are defined by the BGG Rule in (42) or the Taylor rule in (43).

3.10. Equilibrium

The model has 20 endogenous variables and 20 equations. The endogenous variables are: Y, H, C, Λ, C^e, W, W^e, I, Q, K, R^n, R_k, R, p^*, $\mathcal{X}$, π, N, $\bar{\omega}$, κ and Z. The equations defining these endogenous variables are: (4), (7), (9), (18), (24), (25), (26), (27), (30), (31), (32), (33), (35), (36), (37), (38), (39), (40), (41) and (42). The exogenous processes for technology, government spending and idiosyncratic volatility follow (44), (45) and (46) respectively. Nominal interest rate shocks are defined by the monetary policy rule in (42) or (43).

4. Quantitative analysis

In the impulse response analysis that follows we employ a first-order approximation of the model. As a robustness check we considered a second-order approximation of the model and found that the amplification response following risk shocks was identical to the first-order approximation. Log-linearization of the model does not result in certainty equivalence because the steady state, even though deterministic in the aggregate sense, still features non-zero volatility of idiosyncratic productivity. Every entrepreneur is still exposed to significant idiosyncratic risk in the steady state, and changes in idiosyncratic risk therefore have a first-order effect. Because first-order effects are present, second-order effects are of marginal importance. Researchers focus on second-order effects in cases where there is no first-order effect, which would be the case in a standard Real Business Cycle model.

4.1. Calibration

Our baseline calibration largely follows BGG. We set the discount factor $\beta = 0.99$, the household risk aversion parameter $\sigma = 1$ so that utility is logarithmic in consumption, and the elasticity of labor equal to 3 ($\eta = 1/3$). The share of capital in the Cobb–Douglas production function is $\alpha = 0.35$. Investment adjustment costs are $\phi_k = 10$ to generate an elasticity of the price of capital with respect to the investment capital ratio of 0.25. Quarterly depreciation is $\delta = 0.025$. Monitoring costs are $\mu = 0.12$. The death rate of entrepreneurs is $1 - \gamma = 0.0272$, yielding an annualized business failure rate of three percent. The idiosyncratic productivity term, $\log(\omega(j))$, is assumed to be log-normally distributed with variance of 0.28. The weight of household labor relative to entrepreneurial labor in the production function is $\Omega = 0.99$.

For price-setting, we assume the Calvo parameter $\theta = 0.75$, so that only 25% of firms can reset their prices in each quarter, meaning the average length of time between price adjustments is four quarters. As our baseline, we follow the BGG monetary policy rule and set the autoregressive parameter on the nominal interest rate to $\rho^{R^n} = 0.9$ and the parameter on past inflation to $\xi = 0.11$. Note that in the online appendix we also consider a conventional Taylor rule where the central bank targets current inflation rather than past inflation. For the conventional Taylor rule, we set $\rho^{R^n} = 0$, $\xi = 1.5$ and $\rho^Y = 0.5$. We follow BGG and set the persistence of the shocks to technology and government spending at $\rho^A = 0.999$ and $\rho^G = 0.95$. For risk shocks, we follow Christiano et al. (2013) and set the persistence of idiosyncratic volatility at $\rho^{\sigma_\omega} = 0.9706$ and the distribution of the shocks equal to $\epsilon_t^{\sigma_\omega} \sim N(0, 0.0283)$. In our standard calibration, we consider two settings for entrepreneurial risk aversion: risk neutrality ($\sigma_e = 0$) and log risk aversion ($\sigma_e = 1$). In the online appendix we study household and entrepreneurial risk aversion below one ($\sigma = \sigma_e = 0.5$) and above one ($\sigma = \sigma_e = 2$).

Following BGG, we consider a one percent technology shock and a 25 basis point shock (in annualized terms) to the nominal interest rate. For the risk shock, we allow the standard deviation of idiosyncratic productivity to increase by one percentage point, from 0.28 to 0.29.

4.2. Quantitative comparison: BGG and the optimal contract

In our quantitative analysis we compare three allocations: the competitive equilibrium under the BGG contract; the competitive equilibrium under the optimal contract with risk-neutral entrepreneurs ($\sigma_e = 0$); and the competitive equilibrium under the optimal contract with risk-averse entrepreneurs ($\sigma_e = 1$). We plot impulse responses for shocks to technology, the nominal interest rate and idiosyncratic volatility and explain the results in this section. Although we do not plot impulse responses for the MCC case, note that the financial accelerator is strengthened for technology and monetary shocks relative to the BGG case when the lending rate is state-contingent and entrepreneurs are myopic and risk-neutral.

Fig. 3 shows impulse responses for an extremely persistent one percent technology shock when prices are sticky. Forward-looking risk-neutral entrepreneurs "look through" the initial negative returns to capital that last for roughly 18 quarters (extending beyond the time scale of the impulse responses shown) and become positive thereafter, ensuring the present discounted value of capital accumulation is more profitable, which drives investment up. The stabilizing influence of forward-looking entrepreneurs cancels out the consumption insurance channel under this calibration, such that the output response under the optimal contract with risk neutrality and the BGG contract coincide almost exactly. In general, this coincidence does not hold outside of the particular calibration employed here. When the shock is less persistent, the optimal risk-neutral contract dampens the financial accelerator relative to the BGG contract. When entrepreneurs are risk-averse, the optimal contract dampens the impact of technology shocks relative to the risk-neutral case and the BGG contract. Risk-averse entrepreneurs want to stabilize net worth as much as possible in order to smooth their consumption flows, an insurance motive which is lacking when entrepreneurs are risk-neutral. If there were no capital adjustment costs, returns

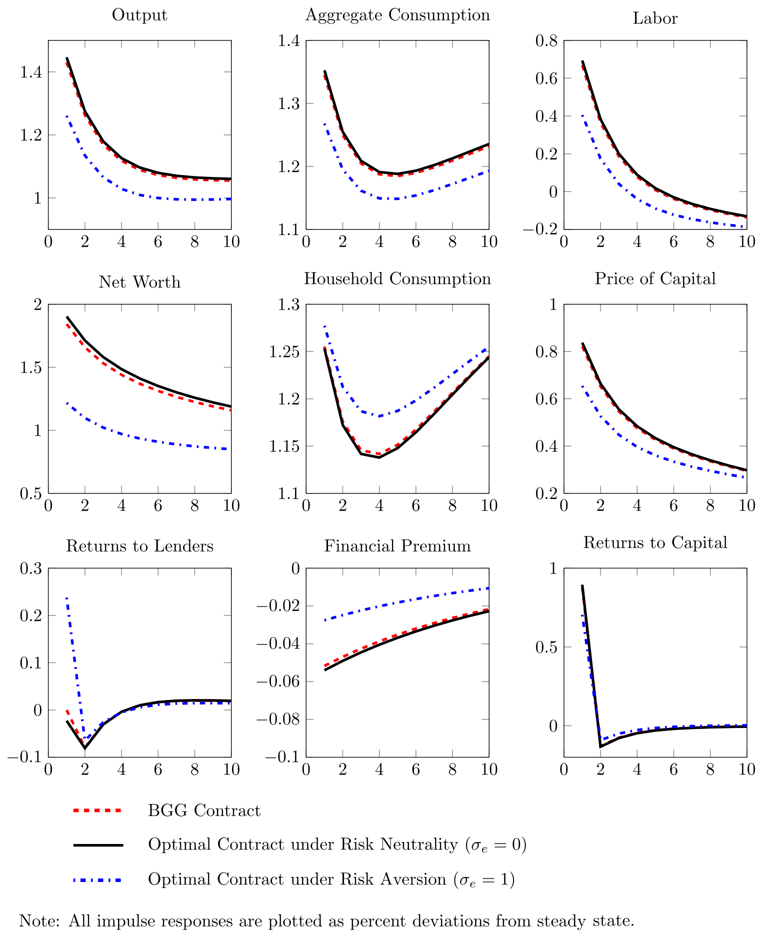
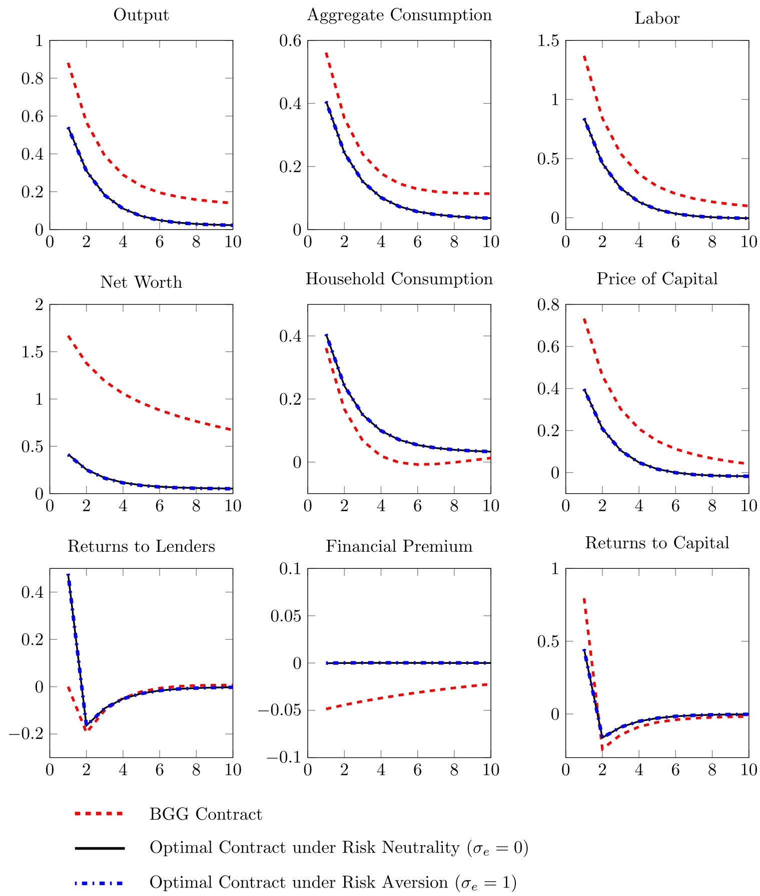


Fig. 3. Technology shock.

to capital would always be positive following a positive productivity shock, which would significantly raise the present discounted value of capital and lead entrepreneurs to accumulate more net worth in these states, pushing up investment and amplification.

The difference between the three allocations is quite acute in Fig. 4, which plots impulse responses for a one percent shock to the nominal interest rate when prices are sticky. Because the impact of the monetary shock is less persistent than the impact of the technology shock, the price of capital depreciates back to its steady state value relatively quickly after an initial rise. Accumulating capital after a positive monetary shock is thus more costly, since capital returns are positive in the first period but negative thereafter. Under the BGG contract the deposit rate does not respond to the shock at all because it is predetermined, which drives up net worth on impact because returns to capital increase, pushing down the financial premium and reducing the cost of borrowing. Relative to the BGG contract, low capital returns and low borrowing costs create a strong incentive for forward-looking entrepreneurs under the optimal contract to borrow more and repay lenders at a higher rate, which pushes up the financial premium until it reaches zero. Since net worth is already quite stable for forward-looking risk-neutral entrepreneurs, higher risk aversion has little impact on the dynamic response of the economy to monetary shocks. Forward-looking risk-neutral and risk-averse entrepreneurs thus stabilize consumption and output to a similar magnitude under the optimal contract, leading to small amplification in both cases.

In Fig. 5 we plot impulse responses for a one standard deviation increase in unobserved idiosyncratic volatility σ_ω, what we defined earlier as a risk shock. Accumulating capital after a risk shock is more profitable than in the steady state, since capital returns are negative in the first period but positive thereafter. Under the BGG contract the deposit rate does not respond to the shock at all because it is predetermined; coupled with negative returns to capital on impact, this drives down net worth, pushes up the financial premium and increases the cost of borrowing. These recessionary effects are much more muted under the optimal contract. Relative to the BGG contract, high capital returns and high borrowing costs create a strong incentive for forward-looking entrepreneurs under the optimal contract to borrow less and repay lenders at a lower rate, which pushes down the financial premium. Since net worth is relatively stable for forward-looking risk-neutral entrepreneurs, higher risk aversion has little impact on the dynamic response of the economy to risk shocks.



Note: All impulse responses are plotted as percent deviations from steady state.

Fig. 4. Monetary policy shock.

Forward-looking entrepreneurs, whether risk-neutral or risk-averse, stabilize consumption and output to a similar magnitude under the optimal contract, diminishing amplification in both cases.

5. Conclusion

Three key assumptions underpin the benchmark loan contract in the literature on CSV frictions in macroeconomic models: (1) returns to lenders are predetermined, and entrepreneurs are (2) myopic and (3) risk-neutral. We contribute to this literature by relaxing all three assumptions and allowing for a state-contingent lending rate with forward-looking risk-averse entrepreneurs. The resulting equilibrium loan contract, which we call the optimal contract, generates smaller amplification than the standard predetermined loan contract for technology shocks, monetary shocks and risk shocks. The financial accelerator is thus dampened under the optimal contract.

In the online appendix we show that even under the fixed rate loan contract of BGG, the strength of the financial accelerator depends on extremely persistent shocks and loose monetary policy. Stationary shocks or hawkish monetary policy dampen the financial accelerator considerably and in some cases reverse the accelerator such that financial frictions stabilize macroeconomic fluctuations. The financial accelerator works primarily through the demand channel by amplifying markup fluctuations. Increased net worth leads to lower financial premiums, cheaper borrowing, and more investment. As a result, the capital price rises which further increases net worth, inducing even higher investment that facilitates a further decline in markups, which ultimately raises output. This virtuous circle is undercut by hawkish monetary policy, which stabilizes markups, dampening the output response to technology and monetary shocks. Similarly, in models with nominal rigidities stationary technology shocks increase markups. Higher markups decrease returns to capital and shrink net worth, thereby reversing the financial accelerator. Thus, when central banks fight inflation aggressively or when shocks are stationary, financial frictions actually stabilize business cycle fluctuations.

These results reveal the fragility of the financial accelerator to alternative assumptions on the lending rate, entrepreneurial preferences, shock persistence and monetary policy. The most important takeaway is that CSV frictions do not always generate a financial accelerator and may in some cases generate a financial decelerator which stabilizes the economy's output response to technology, monetary and risk shocks.

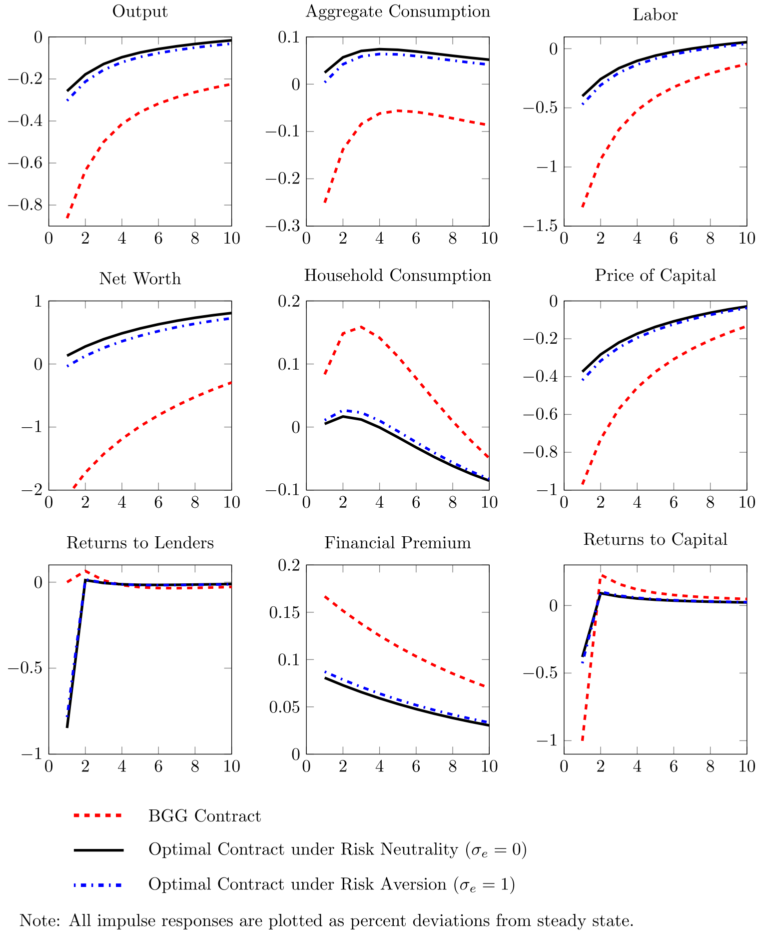


Fig. 5. Idiosyncratic volatility shock.

Appendix A. Recursive formulation of the entrepreneur's objective function

Our goal in this section is to prove that (5) is the recursive formulation of the entrepreneur's objective function (1). We begin by substituting the expression for Ψ_t from (6) into (5), which yields:

$$V_t^e(j) = \frac{1-\gamma}{1-\sigma_e}(N_t^{1-\sigma_e}(j))\gamma\mathbb{E}_t\left\{\kappa_t^{1-\sigma_e}R_{k,t+1}^{1-\sigma_e}(1-\Gamma_{t+1})^{1-\sigma_e}\Psi_{t+1}\right\}. \tag{A.1}$$

Rolling (6) one period forward gives an expression for Ψ_{t+1}:

$$\Psi_{t+1} = 1 + \gamma\mathbb{E}_{t+1}\left\{\kappa_{t+1}^{1-\sigma_e}R_{k,t+2}^{1-\sigma_e}(1-\Gamma_{t+2})^{1-\sigma_e}\Psi_{t+2}\right\}. \tag{A.2}$$

Substitute (A.2) into (A.1) to obtain:

$$V_t^e(j) = \frac{1-\gamma}{1-\sigma_e}N_t^{1-\sigma_e}(j)\gamma\mathbb{E}_t\left(\kappa_t^{1-\sigma_e}R_{k,t+1}^{1-\sigma_e}(1-\Gamma_{t+1})^{1-\sigma_e}\left[1+\gamma\mathbb{E}_{t+1}\left\{\kappa_{t+1}^{1-\sigma_e}R_{k,t+2}^{1-\sigma_e}(1-\Gamma_{t+2})^{1-\sigma_e}\Psi_{t+2}\right\}\right]\right). \tag{A.3}$$

Substitute the expression for net worth in period $t+1$,

$$N_{t+1}(j) = N_t(j)\kappa_t R_{k,t+1}(1-\Gamma_{t+1}) \tag{A.4}$$

into (A.3) to obtain:

$$V_t^e(j) = \frac{\gamma(1-\gamma)}{1-\sigma_e}\mathbb{E}_t\left\{N_{t+1}^{1-\sigma_e}(j)\right\} + \frac{\gamma(1-\gamma)}{1-\sigma_e}\mathbb{E}_t\left[N_{t+1}^{1-\sigma_e}(j)\gamma\mathbb{E}_{t+1}\left\{\kappa_{t+1}^{1-\sigma_e}R_{k,t+2}^{1-\sigma_e}(1-\Gamma_{t+2})^{1-\sigma_e}\Psi_{t+2}\right\}\right]. \tag{A.5}$$

Substitute (A.2) into (A.5) to further simplify:

$$V_t^e(j) = \frac{\gamma(1-\gamma)}{1-\sigma_e}\mathbb{E}_t\left\{N_{t+1}^{1-\sigma_e}(j)\right\} + \frac{\gamma(1-\gamma)}{1-\sigma_e}\mathbb{E}_t\left\{N_{t+1}^{1-\sigma_e}(j)(\Psi_{t+1}-1)\right\}. \tag{A.6}$$

Now substitute $V_{t+1}^e(j) = \frac{1-\gamma}{1-\sigma_e}\mathbb{E}_t\left\{N_{t+1}^{1-\sigma_e}(j)(\Psi_{t+1}-1)\right\}$ into (A.6) to derive the expression below and complete the proof:

$$V_t^e(j) = \frac{1-\gamma}{1-\sigma_e}\gamma\mathbb{E}_t\left\{N_{t+1}^{1-\sigma_e}(j)\right\} + \gamma\mathbb{E}_t V_{t+1}^e(j). \tag{A.7}$$

Equation (A.7) is the recursive formula for the original value function given in equation (1).

Appendix B. Solving the optimization problem for each loan contract

B.1. Proof of Proposition 1: the BGG contract

In the BGG contract, the lender is guaranteed a predetermined rate of return. In this case, the entrepreneur's Lagrangian will be

$$\mathcal{L}^{BGG} = \mathbb{E}_t\left\{\frac{N_t(j)^{1-\sigma_e}\kappa_t^{1-\sigma_e}R_{k,t+1}^{1-\sigma_e}(1-\Gamma_{t+1})^{1-\sigma_e}}{1-\sigma_e} + \lambda_{t+1}\left[\beta\mathbb{E}_t U_{c,t+1}\kappa_t R_{k,t+1}(\Gamma_{t+1}-\mu G_{t+1}) - (\kappa_t-1)U_{c,t}\right]\right\}.$$

The entrepreneur's first order conditions with respect to leverage κ_t and the productivity cutoff $\bar{\omega}_{t+1}$ are:

$$\frac{\partial\mathcal{L}^{BGG}}{\partial\kappa_t} = N_t(j)^{1-\sigma_e}\mathbb{E}_t\left\{\kappa_t^{-\sigma_e}R_{k,t+1}^{1-\sigma_e}(1-\Gamma_{t+1})^{1-\sigma_e}\right\} - \mathbb{E}_t\left\{\lambda_{t+1}\right\}\frac{U_{C,t}}{\kappa_t} = 0,$$

$$\frac{\partial\mathcal{L}^{BGG}}{\partial\bar{\omega}_{t+1}} = -N_t(j)^{1-\sigma_e}\kappa_t^{1-\sigma_e}R_{k,t+1}^{1-\sigma_e}(1-\Gamma_{t+1})^{-\sigma_e}\Gamma_{\omega,t+1} + \lambda_{t+1}\beta\mathbb{E}_t\left\{U_{C,t+1}\right\}\kappa_t R_{k,t+1}(\Gamma_{\omega,t+1}-\mu G_{\omega,t+1}) = 0.$$

If we separate λ_{t+1} on the right hand side from $\frac{\partial\mathcal{L}^{BGG}}{\partial\bar{\omega}_{t+1}}$, take expectations and then combine with $\frac{\partial\mathcal{L}^{BGG}}{\partial\kappa_t}$, we find:

$$\frac{N_t(j)^{1-\sigma_e}\mathbb{E}_t\left\{\kappa_t^{-\sigma_e}R_{k,t+1}^{1-\sigma_e}(1-\Gamma_{t+1})^{1-\sigma_e}\right\}}{\frac{U_{c,t}}{\kappa_t}} = \mathbb{E}_t\left\{\frac{N_t(j)^{1-\sigma_e}\kappa_t^{1-\sigma_e}R_{k,t+1}^{1-\sigma_e}(1-\Gamma_{t+1})^{-\sigma_e}\Gamma_{\omega,t+1}}{\beta\mathbb{E}_t\left\{U_{C,t+1}\right\}\kappa_t R_{k,t+1}(\Gamma_{\omega,t+1}-\mu G_{\omega,t+1})}\right\}. \tag{B.1}$$

Rearranging, simplifying and substituting in the stochastic discount factor yields

$$\mathbb{E}_t\left\{(1-\Gamma_{t+1})^{1-\sigma_e}R_{k,t+1}^{1-\sigma_e}\right\} = \mathbb{E}_t\left\{\frac{(1-\Gamma_{t+1})^{-\sigma_e}\Gamma_{\omega,t+1}}{\mathbb{E}_t\{\Lambda_{t,t+1}\}\kappa_t(\Gamma_{\omega,t+1}-\mu G_{\omega,t+1})}R_{k,t+1}^{-\sigma_e}\right\}. \tag{B.2}$$

B.2. Proof of Proposition 2: the MCC contract

In the MCC contract, the entrepreneur's Lagrangian is

$$\mathcal{L}^{MCC} = \mathbb{E}_t\left\{\frac{(1-\gamma)N_t(j)^{1-\sigma_e}\kappa_t^{1-\sigma_e}R_{k,t+1}^{1-\sigma_e}(1-\Gamma_{t+1})^{1-\sigma_e}}{1-\sigma_e}\right\}$$
$$+ \lambda_t\mathbb{E}_t\left\{\beta U_{c,t+1}\kappa_t R_{k,t+1}(\Gamma_{t+1}-\mu G_{t+1}) - (\kappa_t-1)U_{c,t}\right\}.$$

The entrepreneur's first order conditions with respect to κ_t and $\bar{\omega}_{t+1}$ are:

$$\frac{\partial\mathcal{L}^{MCC}}{\partial\kappa_t} = (1-\gamma)\mathbb{E}_t\left\{N_t(j)^{1-\sigma_e}\kappa_t^{-\sigma_e}R_{k,t+1}^{1-\sigma_e}(1-\Gamma_{t+1})^{1-\sigma_e}\right\} + \lambda_t\left(\mathbb{E}_t\left\{\beta U_{C,t+1}R_{k,t+1}(\Gamma_{t+1}-\mu G_{t+1}) - U_{C,t}\right\}\right) = 0,$$

$$\frac{\partial\mathcal{L}^{MCC}}{\partial\bar{\omega}_{t+1}} = (1-\gamma)\left[-N_t(j)^{1-\sigma}\kappa_t^{-\sigma_e}R_{k,t+1}^{1-\sigma_e}\Gamma_\omega(1-\Gamma_{t+1})^{-\sigma_e} + \lambda_t\beta U_{C,t+1}\kappa_t R_{k,t+1}(\Gamma_{\omega,t+1}-\mu G_{\omega,t+1})\right] = 0.$$

Rearranging these first order conditions, solving in terms of λ_t and setting them equal to each other yields

$$\frac{\kappa_t^{1-\sigma_e}\mathbb{E}_t\left\{R_{k,t+1}^{1-\sigma_e}(1-\Gamma_{t+1})^{1-\sigma_e}\right\}}{\kappa_t^{-\sigma_e}R_{k,t+1}^{-\sigma_e}(1-\Gamma_{t+1})^{-\sigma_e}} = \frac{\Gamma_\omega}{\Gamma_{\omega,t+1}-\mu G_{\omega,t+1}}\frac{1}{\Lambda_{t,t+1}}. \tag{B.3}$$

In the frictionless model monitoring costs are set to zero ($\mu = 0$), so that the optimality condition becomes

$$\frac{\kappa_t \mathbb{E}_t \left\{ R_{k,t+1}^{1-\sigma_e} (1-\Gamma_{t+1})^{1-\sigma_e} \right\}}{R_{k,t+1}^{-\sigma_e} (1-\Gamma_{t+1})^{-\sigma_e}} = \frac{1}{\Lambda_{t,t+1}}. \tag{B.4}$$

B.3. Proof of Proposition 3: the optimal contract with forward-looking entrepreneurs

Under the optimal contract, the forward-looking entrepreneur's Lagrangian has the following form (if we divide the value function by $(1-\gamma)N_t(j)$ as a scaling factor):

$$\mathcal{L}^{Opt} = \frac{(1-\gamma)}{1-\sigma_e} \mathbb{E}_t \left\{ N_t(j)(\Psi_t - 1) + \sum_{i=0}^{\infty} \lambda_{t+i} \left[\beta U_{c,t+i+1} \kappa_{t+i} R_{k,t+i+1} (\Gamma_{t+1} - \mu G_{t+1}) - (\kappa_{t+i} - 1) U_{c,t+i} \right] \right\},$$

where Ψ_t is defined in (6). The entrepreneur's first order condition with respect to leverage κ_t is

$$\begin{aligned} \frac{\partial \mathcal{L}_t^{Opt}}{\partial \kappa_t} =& 0 \\ =& (1-\gamma) \mathbb{E}_t \left\{ N_t(j)^{1-\sigma_e} \gamma \kappa_t^{-\sigma_e} R_{k,t+1}^{1-\sigma_e} (1-\Gamma_{t+1})^{1-\sigma_e} \Psi_{t+1} + \lambda_t \Big(\beta U_{C,t+1} R_{k,t+1} (\Gamma_{t+1} - \mu G_{t+1}) - U_{C,t} \Big) \right\}. \end{aligned}$$

The entrepreneur's first order condition with respect to the productivity cutoff $\bar{\omega}_{t+1}$ is

$$\begin{aligned} \frac{\partial \mathcal{L}_t^{Opt}}{\partial \bar{\omega}_{t+1}} =& 0 \\ =& (1-\gamma) \mathbb{E}_t \Bigg\{ - N_t(j)^{1-\sigma_e} \gamma \kappa_t^{1-\sigma_e} R_{k,t+1}^{1-\sigma_e} (1-\Gamma_{t+1})^{-\sigma_e} \Gamma_{\omega,t+1} \Psi_{t+1} \\ & + \lambda_t \Big[\beta U_{C,t+1} \kappa_t R_{k,t+1} (\Gamma_{\omega,t+1} - \mu G_{\omega,t+1}) \Big] \Bigg\}. \end{aligned}$$

We then move λ_t to the right hand side of both first order conditions and divide the equations by each other to obtain

$$-\frac{\kappa_t R_{k,t+1}^{-\sigma_e} (1-\Gamma_{t+1})^{-\sigma_e} \Gamma_{\omega,t+1} \mathbb{E}_{t+1} \Psi_{t+1}}{\mathbb{E}_t \left\{ R_{k,t+1}^{1-\sigma_e} (1-\Gamma_{t+1})^{1-\sigma_e} \Psi_{t+1} \right\}} = \frac{\beta \kappa_t U_{C,t+1} (\Gamma_{\omega,t+1} - \mu G_{\omega,t+1})}{\beta \mathbb{E}_t \left\{ U_{C,t+1} \kappa_t R_{k,t+1}^{1-\sigma_e} (\Gamma_{t+1} - \mu G_{t+1}) \right\} - \kappa_t U_{C,t}}$$

We use the participation constraint for lenders to simplify the denominator of the right hand side. After rearranging and simplifying, we get

$$\frac{\kappa_t \mathbb{E}_t \left\{ R_{k,t+1}^{1-\sigma_e} (1-\Gamma_{t+1})^{1-\sigma_e} \Psi_{t+1} \right\}}{R_{k,t+1}^{-\sigma_e} (1-\Gamma_{t+1})^{-\sigma_e}} = -\frac{\Gamma_{\omega,t+1}}{\Gamma_{\omega,t+1} - \mu G_{\omega,t+1}} \frac{\Psi_{t+1}}{\Lambda_{t,t+1}}. \tag{B.5}$$

In the frictionless model monitoring costs are set to zero ($\mu = 0$), such that $-\frac{g_\omega}{h_\omega} = 1$ and the optimality condition becomes

$$\frac{\kappa_t \mathbb{E}_t \left\{ \Psi_{t+1} R_{k,t+1}^{1-\sigma_e} (1-\Gamma_{t+1})^{1-\sigma_e} \right\}}{\Psi_{t+1} R_{k,t+1}^{-\sigma_e} (1-\Gamma_{t+1})^{-\sigma_e}} = \frac{1}{\Lambda_{t,t+1}}. \tag{B.6}$$

B.4. The BGG contract with forward-looking entrepreneurs

For a predetermined lending rate, the non-myopic entrepreneur's Lagrangian has the following form if we divide the value function by $(1-\gamma)N_t(j)$ as a scaling factor:

$$\mathcal{L} = \frac{1-\gamma}{1-\sigma_e} \mathbb{E}_t \left\{ N_t(j)^{1-\sigma_e} (\Psi_t - 1) + \sum_{i=0}^{\infty} \lambda_{t+i+1} \left[\beta \mathbb{E}_t \left\{ U_{c,t+i+1} \right\} \kappa_{t+i} R_{k,t+i+1} (\Gamma_{t+1} - \mu G_{t+1}) - (\kappa_{t+i} - 1) U_{c,t+i} \right] \right\}.$$

The entrepreneur's first order condition with respect to leverage κ_t is

$$\begin{aligned}\frac{\partial \mathcal{L}_t}{\partial \kappa_t} =& 0\\ =&(1-\gamma)\mathbb{E}_t\Bigg\{N_t(j)^{1-\sigma_e}\gamma\kappa_t^{-\sigma_e}(1-\Gamma_{t+1})^{1-\sigma_e}R_{k,t+1}^{1-\sigma_e}\Psi_{t+1}\\ &+\lambda_{t+1}\Big(\beta\mathbb{E}_t\{U_{C,t+1}\}R_{k,t+1}(\Gamma_{t+1}-\mu G_{t+1})-U_{C,t}\Big)\Bigg\},\end{aligned} \tag{B.7}$$

where we have used the fact that $\frac{\partial\Psi_t}{\partial\kappa_t}=\mathbb{E}_t\Big\{\gamma\kappa_t^{-\sigma_e}(1-\Gamma_{t+1})^{1-\sigma_e}R_{k,t+1}^{1-\sigma_e}\Psi_{t+1}\Big\}$ and $\frac{\partial\Psi_{t+i}}{\partial\kappa_t}=0$ for $i=1,2,\ldots$ The entrepreneur's first order condition with respect to the productivity cutoff $\bar{\omega}_{t+1}$ is

$$\begin{aligned}\frac{\partial \mathcal{L}_t}{\partial \bar{\omega}_{t+1}} =&(1-\gamma)\mathbb{E}_t\Bigg\{-N_t(j)^{1-\sigma_e}\gamma\kappa_t^{1-\sigma_e}(1-\Gamma_{t+1})^{-\sigma_e}\Gamma_{\omega,t+1}R_{k,t+1}^{1-\sigma_e}\Psi_{t+1}\\ &+\lambda_{t+1}\Big[\beta\mathbb{E}_t\Big\{U_{C,t+1}\Big\}\kappa_t R_{k,t+1}(\Gamma_{\omega,t+1}-\mu G_{\omega,t+1})\Big]\Bigg\}=0,\end{aligned}$$

where we have used the fact that $\frac{\partial\Psi_t}{\partial\bar{\omega}_{t+1}}=-\kappa_t^{1-\sigma_e}(1-\Gamma_{t+1})^{-\sigma_e}\Gamma_{\omega,t+1}R_{k,t+1}^{1-\sigma_e}\Psi_{t+1}$ and $\frac{\partial\Psi_{t+i}}{\partial\bar{\omega}_{t+1}}=0$ for $i=1,2\ldots$ One can express λ_{t+1} in the equation $\frac{\partial\mathcal{L}}{\partial\bar{\omega}_{t+1}}=0$ as a function of other variables, and substitute the result into $\frac{\partial\mathcal{L}}{\partial\kappa_t}=0$. Then, using the participation constraint to simplify, we obtain

$$\kappa_t\mathbb{E}_t\Bigg\{\Psi_{t+1}R_{k,t+1}^{1-\sigma_e}(1-\Gamma_{t+1})^{1-\sigma_e}\Bigg\}=\mathbb{E}_t\Bigg\{\frac{(1-\Gamma_{t+1})^{-\sigma_e}\Gamma_{\omega,t+1}R_{k,t+1}^{-\sigma_e}}{\Gamma_{\omega,t+1}-\mu G_{\omega,t+1}}\frac{\Psi_{t+1}}{\mathbb{E}_t\Lambda_{t,t+1}}\Bigg\}. \tag{B.8}$$

It is trivial to show that log-linearization of the BGG contract with myopic or non-myopic, risk-neutral or risk-averse, entrepreneurs gives an identical optimality condition. However, this identity does not hold for higher order approximations. Under the frictionless model monitoring costs equal zero ($\mu=0$), and the optimality condition simplifies to:

$$\frac{\kappa_t\mathbb{E}_t\Bigg\{(1-\Gamma_{t+1})^{1-\sigma_e}R_{k,t+1}^{1-\sigma_e}\Psi_{t+1}\Bigg\}}{\mathbb{E}_t\Bigg\{(1-\Gamma_{t+1})^{-\sigma_e}R_{k,t+1}^{-\sigma_e}\Psi_{t+1}\Bigg\}}=\frac{1}{\mathbb{E}_t\Lambda_{t,t+1}}. \tag{B.9}$$

Appendix C. Log-linearization of the lending contracts

C.1. Log-linearization of the common optimality condition

We begin by log-linearizing the common optimality condition for each contract. The non-linear participation constraint and FOC are, respectively:

$$\beta\mathbb{E}_t\Bigg\{U_{C,t+1}\Bigg\}\kappa_t R_{k,t+1}(\Gamma_{t+1}-\mu G_{t+1})-(\kappa_t-1)U_{C,t}=0, \tag{C.1}$$

$$\mathbb{E}_t\Bigg[(1-\Gamma_{t+1})^{1-\sigma_e}R_{k,t+1}^{1-\sigma_e}\Bigg]=\mathbb{E}_t\Bigg[\frac{(1-\Gamma_{t+1})^{-\sigma_e}\Gamma_{\omega,t+1}}{\mathbb{E}_t\Lambda_{t,t+1}\kappa_t(\Gamma_{\omega,t+1}-\mu G_{\omega,t+1})}R_{k,t+1}^{-\sigma_e}\Bigg]. \tag{C.2}$$

In their linearized form, these become:

$$-\sigma\Big(\mathbb{E}_t\hat{C}_{t+1}-\hat{C}_t\Big)+\hat{R}_{k,t+1}+\frac{\Gamma_\omega-\mu G_\omega}{\Gamma-\mu G}\bar{\omega}\hat{\omega}_{t+1}+\frac{\Gamma_\sigma-\mu G_\sigma}{\Gamma-\mu G}\sigma_\omega\hat{\sigma}_{\omega,t}=\frac{1}{\kappa-1}\hat{\kappa}_t, \tag{C.3}$$

$$\begin{aligned}\hat{\kappa}_t+\mathbb{E}_t R_{k,t+1}-\frac{\Gamma_\omega}{1-\Gamma}\bar{\omega}\mathbb{E}_t\hat{\omega}_{t+1}-\frac{\Gamma_\sigma}{1-\Gamma}\sigma_\omega\hat{\sigma}_{\omega,t}=&-\mathbb{E}_t\hat{\Lambda}_{t,t+1}+\Bigg(\frac{\Gamma_{\omega\omega}}{\Gamma_\omega}-\frac{\Gamma_{\omega\omega}-\mu G_{\omega\omega}}{\Gamma_\omega-\mu G_\omega}\Bigg)\bar{\omega}\mathbb{E}_t\hat{\omega}_{t+1}\\ &+\Bigg(\frac{\Gamma_{\omega\sigma}}{\Gamma_\omega}-\frac{\Gamma_{\omega\sigma}-\mu G_\omega}{\Gamma_\omega-\mu G_\omega}\Bigg)\sigma_\omega\hat{\sigma}_{\omega,t}.\end{aligned} \tag{C.4}$$

Now we take the expected value of the participation constraint (C.3) and obtain:

$$\mathbb{E}_t\hat{\Lambda}_{t,t+1}+\mathbb{E}_t\hat{R}_{k,t+1}+\frac{\Gamma_\omega-\mu G_\omega}{\Gamma-\mu G}\bar{\omega}\mathbb{E}_t\hat{\omega}_{t+1}+\frac{\Gamma_\sigma-\mu G_\sigma}{\Gamma-\mu G}\sigma_\omega\hat{\sigma}_{\omega,t}=\frac{1}{\kappa-1}\hat{\kappa}_t. \tag{C.5}$$

Define $\hat{\triangle}_t=\hat{R}_{k,t}-\hat{R}_t$, and rewrite the system as:

$$\frac{1}{\kappa-1}\hat{\kappa}_t - \frac{\Gamma_\sigma - \mu G_\sigma}{\Gamma - \mu G}\sigma_\omega \hat{\sigma}_{\omega,t} - \mathbb{E}_t \hat{\Delta}_{t+1} = \frac{\Gamma_\omega - \mu G_\omega}{\Gamma - \mu G}\bar{\omega}\mathbb{E}_t \hat{\omega}_{t+1},$$
$$\hat{\kappa}_t + \mathbb{E}_t \hat{\Delta}_{t+1} - \left(\frac{\Gamma_{\omega\sigma}}{\Gamma_\omega} - \frac{\Gamma_{\omega\sigma} - \mu G_{\omega\sigma}}{\Gamma_\omega - \mu G_\omega} + \frac{\Gamma_{\sigma_\omega}}{1-\Gamma}\right)\sigma_\omega \hat{\sigma}_{\omega,t} = \left(\frac{\Gamma_{\omega\omega}}{\Gamma_\omega} - \frac{\Gamma_{\omega\omega} - \mu G_{\omega\omega}}{\Gamma_\omega - \mu G_\omega} + \frac{\Gamma_\omega}{1-\Gamma}\right)\bar{\omega}\mathbb{E}_t \hat{\omega}_{t+1}. \tag{C.6}$$

Now we can set these two equations equal to each other and eliminate ω:

$$\hat{\kappa}_t + \mathbb{E}_t \hat{\Delta}_{t+1} - \left(\frac{\Gamma_{\omega\sigma}}{\Gamma_\omega} - \frac{\Gamma_{\omega\sigma} - \mu G_{\omega\sigma}}{\Gamma_\omega - \mu G_\omega} + \frac{\Gamma_\sigma}{1-\Gamma}\right)\sigma_\omega \hat{\sigma}_{\omega,t} =$$
$$\frac{\frac{\Gamma_{\omega\omega}}{\Gamma_\omega} - \frac{\Gamma_{\omega\omega} - \mu G_{\omega\omega}}{\Gamma_\omega - \mu G_\omega} + \frac{\Gamma_\omega}{1-\Gamma}}{\frac{\Gamma_\omega - \mu G_\omega}{\Gamma - \mu G}}\left(\frac{1}{\kappa-1}\hat{\kappa}_t - \frac{\Gamma_\sigma - \mu G_\sigma}{\Gamma - \mu G}\sigma_\omega \hat{\sigma}_{\omega,t} - \mathbb{E}_t \hat{\Delta}_{t+1}\right). \tag{C.7}$$

We can rearrange this to obtain:

$$\mathbb{E}_t \hat{\Delta}_{t+1} = \mathbb{E}_t\left\{\hat{R}_{k,t+1} - \hat{R}_{t+1}\right\} = \nu_\kappa \hat{\kappa}_t + \nu_\sigma \sigma_\omega \hat{\sigma}_{\omega,t} \tag{C.8}$$

where $\nu_\kappa = \frac{\frac{\Gamma_{\omega\omega}}{\Gamma_\omega} - \frac{\Gamma_{\omega\omega} - \mu G_{\omega\omega}}{\Gamma_\omega - \mu G_\omega}}{\frac{\Gamma_{\omega\omega}}{\Gamma_\omega} - \frac{\Gamma_{\omega\omega} - \mu G_{\omega\omega}}{\Gamma_\omega - \mu G_\omega} + \frac{\Gamma_\omega}{1-\Gamma} + \frac{\Gamma_\omega - \mu G_\omega}{\Gamma - \mu G}}\frac{1}{\kappa-1}$, $\nu_\sigma = \frac{-\frac{\Gamma_\sigma - \mu G_\sigma}{\Gamma - \mu G}\left(\frac{\Gamma_{\omega\omega}}{\Gamma_\omega} - \frac{\Gamma_{\omega\omega} - \mu G_{\omega\omega}}{\Gamma_\omega - \mu G_\omega} + \frac{\Gamma_\omega}{1-\Gamma}\right) + \frac{\Gamma_\omega - \mu G_\omega}{\Gamma - \mu G}\left(\frac{\Gamma_{\omega\sigma}}{\Gamma_\omega} - \frac{\Gamma_{\omega\sigma} - \mu G_{\omega\sigma}}{\Gamma_\omega - \mu G_\omega} + \frac{\Gamma_\sigma}{1-\Gamma}\right)}{\frac{\Gamma_{\omega\omega}}{\Gamma_\omega} - \frac{\Gamma_{\omega\omega} - \mu G_{\omega\omega}}{\Gamma_\omega - \mu G_\omega} + \frac{\Gamma_\omega}{1-\Gamma} + \frac{\Gamma_\omega - \mu G_\omega}{\Gamma - \mu G}}$.

C.2. Corollary 1: log-linearization of the BGG lending rate

The log-linear lending rate in BGG is given by:

$$\hat{R}_{t+1} - \mathbb{E}_t \hat{R}_{t+1} = 0. \tag{C.9}$$

C.3. Corollary 2: log-linearization of the MCC contract

The log-linearized MCC contract is obtained by setting $\hat{\Psi}_{t+1} = 0$ in the optimal contract equation below (C.16).

C.4. Corollary 3: log-linearization of the optimal contract

In their linearized form, the participation constraint and FOC for the optimal contract become:

$$\hat{R}_{t+1} = -\frac{1}{\kappa-1}\hat{\kappa}_t + \frac{\Gamma_\omega - \mu G_\omega}{\Gamma - \mu G}\bar{\omega}\hat{\omega}_{t+1} + \frac{\Gamma_\sigma - \mu G_\sigma}{\Gamma - \mu G}\sigma_\omega \hat{\sigma}_{\omega,t} + \hat{R}_{k,t+1}, \tag{C.10}$$

$$\hat{\kappa}_t + (1-\sigma_e)(\mathbb{E}_t R_{k,t+1} - \frac{\Gamma_\omega}{1-\Gamma}\bar{\omega}\mathbb{E}_t \bar{\omega}_{t+1}) + \sigma_e(\hat{R}_{k,t+1} - \frac{\Gamma_\omega}{1-\Gamma}\bar{\omega}\hat{\omega}_{t+1}) - \frac{\Gamma_\sigma \sigma_\omega}{1-\Gamma}\hat{\sigma}_{\omega,t} = \tag{C.11}$$
$$-\hat{\Lambda}_{t,t+1} + \left(\frac{\Gamma_{\omega\omega}}{\Gamma_\omega} - \frac{\Gamma_{\omega\omega} - \mu G_{\omega\omega}}{\Gamma_\omega - \mu G_\omega}\right)\bar{\omega}\hat{\omega}_{t+1} + \left(\frac{\Gamma_{\omega\sigma}}{\Gamma_\omega} - \frac{\Gamma_{\omega\sigma} - \mu G_{\omega\sigma}}{\Gamma_\omega - \mu G_\omega}\right)\sigma_\omega \hat{\sigma}_{\omega,t} + \hat{\Psi}_{t+1} - \mathbb{E}_t \hat{\Psi}_{t+1}.$$

Now we substitute the participation constraint into the optimality condition and obtain:

$$\hat{\kappa}_t + (1-\sigma_e)\mathbb{E}_t R_{k,t+1} + \sigma_e \hat{R}_{k,t+1} + \hat{\Lambda}_{t,t+1} - \left(\frac{\Gamma_{\omega\sigma}}{\Gamma_\omega} - \frac{\Gamma_{\omega\sigma} - \mu G_{\omega\sigma}}{\Gamma_\omega - \mu G_\omega} + \frac{\Gamma_\sigma}{1-\Gamma}\right)\sigma_\omega \hat{\sigma}_{\omega,t} =$$
$$(1-\sigma_e)\frac{\frac{\Gamma_\omega}{1-\Gamma}}{\frac{\Gamma_\omega - \mu G_\omega}{\Gamma - \mu G}}\left(\mathbb{E}_t \hat{R}_{t+1} + \frac{1}{\kappa-1}\hat{\kappa}_t - \frac{\Gamma_\sigma - \mu G_\sigma}{\Gamma - \mu G}\sigma_\omega \hat{\sigma}_{\omega,t} - \mathbb{E}_t \hat{R}_{k,t+1}\right)$$
$$+ \frac{\frac{\Gamma_{\omega\omega}}{\Gamma_\omega} - \frac{\Gamma_{\omega\omega} - \mu G_{\omega\omega}}{\Gamma_\omega - \mu G_\omega} + \frac{\sigma_e \Gamma_\omega}{1-\Gamma}}{\frac{\Gamma_\omega - \mu G_\omega}{\Gamma - \mu G}}\left(\hat{R}_{t+1} + \frac{1}{\kappa-1}\hat{\kappa}_t - \frac{\Gamma_\sigma - \mu G_\sigma}{\Gamma - \mu G}\sigma_\omega \hat{\sigma}_{\omega,t} - \hat{R}_{k,t+1}\right) + \hat{\Psi}_{t+1} - \mathbb{E}_t \hat{\Psi}_{t+1}. \tag{C.12}$$

Using $\frac{\frac{\Gamma_\omega}{1-\Gamma}}{\frac{\Gamma_\omega - \mu G_\omega}{\Gamma - \mu G}} = \kappa - 1$, we can simplify the previous expression to:

$$(1-\sigma_e)\mathbb{E}_t R_{k,t+1} + \sigma_e \hat{R}_{k,t+1} + \hat{\Lambda}_{t,t+1} - \left(\frac{\Gamma_{\omega\sigma}}{\Gamma_\omega} - \frac{\Gamma_{\omega\sigma} - \mu G_{\omega\sigma}}{\Gamma_\omega - \mu G_\omega} + \frac{\Gamma_\sigma}{1-\Gamma}\right)\sigma_\omega \hat{\sigma}_{\omega,t} =$$
$$+ (1-\sigma_e)(\kappa-1)\left(\mathbb{E}_t \hat{R}_{t+1} - \frac{\Gamma_\sigma - \mu G_\sigma}{\Gamma - \mu G}\sigma_\omega \hat{\sigma}_{\omega,t} - \mathbb{E}_t \hat{R}_{k,t+1}\right)$$

$$+\frac{\frac{\Gamma_{\omega\omega}}{\Gamma_\omega}-\frac{\Gamma_{\omega\omega}-\mu G_{\omega\omega}}{\Gamma_\omega-\mu G_\omega}+\frac{\sigma_e\Gamma_\omega}{1-\Gamma}}{\frac{\Gamma_\omega-\mu G_\omega}{\Gamma-\mu G}}\left(\hat{R}_{t+1}-\frac{\Gamma_\sigma-\mu G_\sigma}{\Gamma-\mu G}\sigma_\omega\hat{\sigma}_{\omega,t}-\hat{R}_{k,t+1}\right)$$
$$+\frac{\frac{\Gamma_{\omega\omega}}{\Gamma_\omega}-\frac{\Gamma_{\omega\omega}-\mu G_{\omega\omega}}{\Gamma_\omega-\mu G_\omega}}{\frac{\Gamma_\omega-\mu G_\omega}{\Gamma-\mu G}}\frac{1}{\kappa-1}\hat{\kappa}_t+\hat{\Psi}_{t+1}-\mathbb{E}_t\hat{\Psi}_{t+1}. \tag{C.13}$$

We use the expression for leverage to obtain:

$$(1-\sigma_e)\mathbb{E}_tR_{k,t+1}+\sigma_e\hat{R}_{k,t+1}+\hat{\Lambda}_{t,t+1}-\left(\frac{\Gamma_{\omega\sigma}}{\Gamma_\omega}-\frac{\Gamma_{\omega\sigma}-\mu G_{\omega\sigma}}{\Gamma_\omega-\mu G_\omega}+\frac{\Gamma_\sigma}{1-\Gamma}\right)\sigma_\omega\hat{\sigma}_{\omega,t}-(\hat{\Psi}_{t+1}-\mathbb{E}_t\hat{\Psi}_{t+1})=$$
$$(1-\sigma_e)(\kappa-1)\left(\mathbb{E}_t\hat{R}_{t+1}-\frac{\Gamma_\sigma-\mu G_\sigma}{\Gamma-\mu G}\sigma_\omega\hat{\sigma}_{\omega,t}-\mathbb{E}_t\hat{R}_{k,t+1}\right)+$$
$$\frac{\frac{\Gamma_{\omega\omega}}{\Gamma_\omega}-\frac{\Gamma_{\omega\omega}-\mu G_{\omega\omega}}{\Gamma_\omega-\mu G_\omega}+\frac{\sigma_e\Gamma_\omega}{1-\Gamma}}{\frac{\Gamma_\omega-\mu G_\omega}{\Gamma-\mu G}}\left(\hat{R}_{t+1}-\frac{\Gamma_\sigma-\mu G_\sigma}{\Gamma-\mu G}\sigma_\omega\hat{\sigma}_{\omega,t}-\hat{R}_{k,t+1}\right)+\frac{\frac{\Gamma_{\omega\omega}}{\Gamma_\omega}-\frac{\Gamma_{\omega\omega}-\mu G_{\omega\omega}}{\Gamma_\omega-\mu G_\omega}+\frac{\Gamma_\omega}{1-\Gamma}+\frac{\Gamma_\omega-\mu G_\omega}{\Gamma-\mu G}}{\frac{\Gamma_\omega-\mu G_\omega}{\Gamma-\mu G}}\mathbb{E}_t\hat{\triangle}_{t+1}$$
$$-\frac{-\frac{\Gamma_\sigma-\mu G_\sigma}{\Gamma-\mu G}\left(\frac{\Gamma_{\omega\omega}}{\Gamma_\omega}-\frac{\Gamma_{\omega\omega}-\mu G_{\omega\omega}}{\Gamma_\omega-\mu G_\omega}+\frac{\Gamma_\omega}{1-\Gamma}\right)+\frac{\Gamma_\omega-\mu G_\omega}{\Gamma-\mu G}\left(\frac{\Gamma_{\omega\sigma}}{\Gamma_\omega}-\frac{\Gamma_{\omega\sigma}-\mu G_{\omega\sigma}}{\Gamma_\omega-\mu G_\omega}+\frac{\Gamma_\sigma}{1-\Gamma}\right)}{\frac{\Gamma_\omega-\mu G_\omega}{\Gamma-\mu G}}\sigma_\omega\hat{\sigma}_{\omega,t} \tag{C.14}$$

We can simplify this expression to obtain:

$$-\sigma_e(R_{k,t+1}-\mathbb{E}_tR_{k,t+1})+\hat{\Lambda}_{t,t+1}-\mathbb{E}_t\hat{\Lambda}_{t,t+1}-(\hat{\Psi}_{t+1}-\mathbb{E}_t\hat{\Psi}_{t+1})=$$
$$\frac{\frac{\Gamma_{\omega\omega}}{\Gamma_\omega}-\frac{\Gamma_{\omega\omega}-\mu G_{\omega\omega}}{\Gamma_\omega-\mu G_\omega}+\frac{\sigma_e\Gamma_\omega}{1-\Gamma}}{\frac{\Gamma_\omega-\mu G_\omega}{\Gamma-\mu G}}(\hat{R}_{t+1}-\hat{R}_{k,t+1})+\frac{\frac{\Gamma_{\omega\omega}}{\Gamma_\omega}-\frac{\Gamma_{\omega\omega}-\mu G_{\omega\omega}}{\Gamma_\omega-\mu G_\omega}+\frac{\sigma_e\Gamma_\omega}{1-\Gamma}}{\frac{\Gamma_\omega-\mu G_\omega}{\Gamma-\mu G}}\mathbb{E}_t\hat{\triangle}_{t+1} \tag{C.15}$$

We set $\tilde{\alpha}=\frac{\frac{\Gamma_\omega-\mu G_\omega}{\Gamma-\mu G}}{\frac{\Gamma_{\omega\omega}}{\Gamma_\omega}-\frac{\Gamma_{\omega\omega}-\mu G_{\omega\omega}}{\Gamma_\omega-\mu G_\omega}+\frac{\sigma_e\Gamma_\omega}{1-\Gamma}}$, and simplify the above expression to the following:

$$R_{t+1}-\mathbb{E}_tR_{t+1}=R_{k,t+1}-\mathbb{E}_tR_{k,t+1}-\tilde{\alpha}\left[\sigma_e(R_{k,t+1}-\mathbb{E}_tR_{k,t+1})-(\hat{\Lambda}_{t,t+1}-\mathbb{E}_t\hat{\Lambda}_{t,t+1})+\hat{\Psi}_{t+1}-\mathbb{E}_t\hat{\Psi}_{t+1}\right] \tag{C.16}$$

Finally we solve for the dynamics of $\hat{\Psi}_t$:

$$\frac{\Psi}{\Psi-1}\hat{\Psi}_t=(1-\sigma_e)(\hat{\kappa}_t-\frac{\Gamma_\omega}{1-\Gamma}\omega\mathbb{E}_t\hat{\omega}_{t+1}-\frac{\Gamma_\sigma}{1-\Gamma}\sigma_\omega\hat{\sigma}_{\omega,t}+\mathbb{E}_t\hat{R}_{k,t+1})+\mathbb{E}_t\hat{\Psi}_{t+1}. \tag{C.17}$$

Using the participation constraint we eliminate ω in the expression for $\hat{\Psi}_t$:

$$\frac{\Psi}{\Psi-1}\hat{\Psi}_t=(1-\sigma_e)\left[\hat{\kappa}_t-\frac{\frac{\Gamma_\omega}{1-\Gamma}}{\frac{\Gamma_\omega-\mu G_\omega}{\Gamma-\mu G}}\left(\frac{1}{\kappa-1}\hat{\kappa}_t-\mathbb{E}_t\hat{R}_{k,t+1}+\mathbb{E}_t\hat{R}_{t+1}-\frac{\Gamma_\sigma-\mu G_\sigma}{\Gamma-\mu G}\sigma_\omega\hat{\sigma}_{\omega,t}\right)\right.$$
$$\left.+\frac{\Gamma_\sigma-\mu G_\sigma}{\Gamma-\mu G}\sigma_\omega\hat{\sigma}_{\omega,t}+\mathbb{E}_t\hat{R}_{k,t+1}\right]+\mathbb{E}_t\hat{\Psi}_{t+1}. \tag{C.18}$$

We can rearrange this expression to match Corollary 3 in the text:

$$\hat{\Psi}_t=\epsilon_N\mathbb{E}_t\left\{(1-\sigma_e)[(\kappa-1)(\hat{R}_{k,t+1}-\hat{R}_{t+1})+\hat{R}_{k,t+1}+\nu_\Psi\hat{\sigma}_{\omega,t}]+\hat{\Psi}_{t+1}\right\}, \tag{C.19}$$

where $\nu_\Psi=\frac{(\Gamma_\sigma-\mu G_\sigma)\frac{\Gamma_\omega}{\Gamma_\omega-\mu G_\omega}-\Gamma_\sigma}{1-\Gamma}\sigma_\omega$.

Appendix D. The complete log-linearized model

In this section we review the whole model in its log-linearized form.

D.1. New Keynesian components

We begin with the set of equations characterizing the standard New Keynesian components of the model. Equation (D.1) gives the Euler equation for state-contingent assets from the FOC for deposits, while (D.2) is the Euler equation for nominal bonds. The labor market clearing condition is given by (D.3), (D.4) is the New Keynesian Phillips curve, (D.5) is the production function, (D.6) gives the evolution of capital, (D.7) refers to the dynamics of the price of capital, (D.8) gives returns to capital, and (D.9) refers to goods market clearing. Shocks to technology, monetary policy, government spending and idiosyncratic risk are defined by (D.10), (D.11), (D.12) and (D.13).

$$-\sigma\left(\mathbb{E}_t\hat{C}_{t+1}-\hat{C}_t\right)+\mathbb{E}_t\hat{R}_{t+1}=0, \tag{D.1}$$

$$\hat{R}^n_t=\mathbb{E}_t\hat{R}_{t+1}+\mathbb{E}_t\hat{\pi}_{t+1} \tag{D.2}$$

$$\hat{Y}_t-\hat{H}_t-\hat{\mathcal{X}}_t-\sigma\hat{C}_t=\eta\hat{H}_t, \tag{D.3}$$

$$\hat{\pi}_t=-\frac{(1-\theta)(1-\theta\beta)}{\theta}\hat{\mathcal{X}}_t+\beta\mathbb{E}_t\hat{\pi}_{t+1}. \tag{D.4}$$

$$\hat{Y}_t=\hat{A}_t+\alpha\hat{K}_{t-1}+(1-\alpha)(1-\Omega)\hat{H}_t. \tag{D.5}$$

$$\hat{K}_t=\delta\hat{I}_t+(1-\delta)\hat{K}_{t-1}, \tag{D.6}$$

$$\hat{Q}_t=\delta\phi_K(\hat{I}_t-\hat{K}_{t-1}), \tag{D.7}$$

$$\hat{R}_{k,t+1}=(1-\epsilon)(\hat{Y}_{t+1}-\hat{K}_t-\hat{\mathcal{X}}_{t+1})+\epsilon\hat{Q}_{t+1}-\hat{Q}_t \tag{D.8}$$

$$Y\hat{Y}_t=C\hat{C}_t+I\hat{I}_t+G\hat{G}_t+C^e\hat{C}^e_t+\phi_\mu\hat{\phi}_{\mu,t}, \tag{D.9}$$

$$\hat{A}=\rho^A\hat{A}_{t-1}+\epsilon^A_t \tag{D.10}$$

$$\hat{R}^n_t=\rho^{R^n}\hat{R}^n_{t-1}+\xi\hat{\pi}_t+\rho^Y\hat{Y}_t+\epsilon^{R^n}_t \tag{D.11}$$

$$\hat{G}_t=\rho^G\hat{G}_{t-1}+\epsilon^G_t \tag{D.12}$$

$$\hat{\sigma}_{\omega,t}=\rho^{\sigma_\omega}\hat{\sigma}_{\omega,t-1}+\epsilon^{\sigma_\omega}_t \tag{D.13}$$

D.2. Entrepreneurial consumption and net worth

The evolution of entrepreneurial net worth is given by (D.14), where (D.15) defines leverage. Entrepreneurial consumption is defined by (D.16) and the financial premium is given by (D.17).

$$\hat{N}_{t+1}=\epsilon_N(\hat{N}_t+\hat{R}_{t+1}+\kappa(\hat{R}_{k,t+1}-\hat{R}_{t+1})+\nu_\Psi\hat{\sigma}_{\omega,t})+(1-\epsilon_N)(\hat{Y}_t-\hat{\mathcal{X}}_t), \tag{D.14}$$

$$\hat{\kappa}_t=\hat{K}_t+\hat{Q}_t-\hat{N}_t \tag{D.15}$$

$$\hat{C}^e_{t+1}=\hat{N}_t+\hat{R}_{t+1}+\kappa(\hat{R}_{k,t+1}-\hat{R}_{t+1})+\nu_\Psi\hat{\sigma}_{\omega,t} \tag{D.16}$$

$$\mathbb{E}_t\hat{R}_{k,t+1}-\mathbb{E}_t\hat{R}_{t+1}=\nu_\kappa\hat{\kappa}_t+\nu_\sigma\hat{\sigma}_{\omega,t} \tag{D.17}$$

D.3. Dynamics of the lending rate

The BGG lending rate is defined as (D.18a), the MCC lending rate is defined as (D.18b) and the optimal lending rate is defined as (D.18c). These log-linear expressions are derived in Appendix C.

$$\hat{R}_{t+1}-\mathbb{E}_t\hat{R}_{t+1}=\begin{cases}0 & \text{(a)}\\ \hat{R}_{k,t+1}-\mathbb{E}_tR_{k,t+1}-\tilde{\alpha}\sigma(\hat{C}_{t+1}-\mathbb{E}_t\hat{C}_{t+1}) & \text{(b)}\\ \hat{R}_{k,t+1}-\mathbb{E}_tR_{k,t+1}-\tilde{\alpha}\left[\sigma(\hat{C}_{t+1}-\mathbb{E}_t\hat{C}_{t+1})+\hat{\Psi}_{t+1}-\mathbb{E}_t\hat{\Psi}_{t+1}\right] & \text{(c)}\end{cases} \tag{D.18}$$

$$\hat{\Psi}_{t+1}=\epsilon_N\mathbb{E}_{t+1}\left\{(\kappa-1)(\hat{R}_{k,t+2}-\hat{R}_{t+2})+\hat{R}_{k,t+2}+\nu_\Psi\hat{\sigma}_{\omega,t+1}+\hat{\Psi}_{t+2}\right\} \tag{D.19}$$

D.4. Monitoring costs

$$\hat{\phi_\mu}_t=\hat{C}^e_{t+1}+\nu_\mu\left(\frac{1}{\kappa-1}\hat{\kappa}_t-(\hat{R}_{k,t+1}-\hat{R}_{t+1})\right)+\nu_{\sigma,\mu}\hat{\sigma}_{\omega,t} \tag{D.20}$$

Appendix E. Supplementary material

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.red.2016.12.003.

References

Bernanke, B., Gertler, M., 1989. Agency costs, net worth, and business fluctuations. The American Economic Review 79 (1), 14–31. http://ideas.repec.org/a/aea/aecrev/v79y1989i1p14-31.html.

Bernanke, B.S., Gertler, M., Gilchrist, S., 1999. The financial accelerator in a quantitative business cycle framework. In: Taylor, J.B., Woodford, M. (Eds.), Handbook of Macroeconomics. In: Handbook of Macroeconomics, vol. 1. Elsevier, pp. 1341–1393. Ch. 21, http://ideas.repec.org/h/eee/macchp/1-21.html.

Calvo, G., 1983. Staggered prices in a utility-maximizing framework. Journal of Monetary Economics 12 (3), 383–398. https://ideas.repec.org/a/eee/moneco/v12y1983i3p383-398.html.

Candian, G., Dmitriev, M., 2016. Risk aversion, uninsurable idiosyncratic risk, and the financial accelerator. Available at SSRN, https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2460257.

Cao, D., Nie, G., 2016. Amplification and asymmetric effects without collateral constraints. Manuscript.

Carlstrom, C.T., Fuerst, T.S., 1997. Agency costs, net worth, and business fluctuations: a computable general equilibrium analysis. The American Economic Review 87 (5), 893–910. http://ideas.repec.org/a/aea/aecrev/v87y1997i5p893-910.html.

Carlstrom, C., Fuerst, T., Paustian, M., 2016. Optimal contracts, aggregate risk, and the financial accelerator. American Economic Journal: Macroeconomics 8 (1), 119–147. https://www.aeaweb.org/articles?id=10.1257/mac.20120024.

Chari, V., 2003. Discussion presented at conference on "Monetary Stability, Financial Stability and the Business Cycle", organized by the Bank for International Settlements.

Christensen, I., Dib, A., 2008. The financial accelerator in an estimated New Keynesian model. Review of Economic Dynamics 11 (1), 155–178. http://ideas.repec.org/a/red/issued/06-87.html.

Christiano, L., Motto, R., Rostagno, M., 2013. Risk shocks. NBER Working Papers 18682. National Bureau of Economic Research, Inc. http://ideas.repec.org/p/nbr/nberwo/18682.html.

Chugh, S.K., 2013. Firm risk and leverage based business cycles. Boston College Working Papers in Economics 844. Boston College Department of Economics. http://ideas.repec.org/p/boc/bocoec/844.html.

Cole, H.L., 2013. Self-enforcing stochastic monitoring and the separation of claims. Journal of Monetary Economics 60 (6), 632–649. http://dx.doi.org/10.1016/j.jmoneco.2013.05.003. http://www.sciencedirect.com/science/article/pii/S0304393213000792.

Di Tella, S. Uncertainty shocks and balance sheet recessions. Unpublished Manuscript.

Dmitriev, M., Hoddenbagh, J., 2014. Collateral constraints and state-contingent contracts. Available at SSRN, https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2443522.

Fernandez-Villaverde, J., 2010. Fiscal policy in a model with financial frictions. The American Economic Review 100 (2), 35–40. http://ideas.repec.org/a/aea/aecrev/v100y2010i2p35-40.html.

Fernandez-Villaverde, J., Ohanian, L., 2010. The Spanish crisis from a global perspective. Working Papers 2010-03. FEDEA. http://ideas.repec.org/p/fda/fdaddt/2010-03.html.

Gertler, M., Karadi, P., 2011. A model of unconventional monetary policy. Journal of Monetary Economics 58 (1), 17–34. http://ideas.repec.org/a/eee/moneco/v58y2011i1p17-34.html.

Gertler, M., Kiyotaki, N., 2010. Financial intermediation and credit policy in business cycle analysis. In: Friedman, B.M., Woodford, M. (Eds.), Handbook of Monetary Economics. In: Handbook of Monetary Economics, vol. 3. Elsevier, pp. 547–599. Ch. 11, http://ideas.repec.org/h/eee/monchp/3-11.html.

House, C.L., 2006. Adverse selection and the financial accelerator. Journal of Monetary Economics 53 (6), 1117–1134.

Jermann, U., Quadrini, V., 2012. Macroeconomic effects of financial shocks. The American Economic Review 102 (1), 238–271. http://dx.doi.org/10.1257/aer.102.1.238. http://www.aeaweb.org/articles.php?doi=10.1257/aer.102.1.238.

Kiyotaki, N., Moore, J., 1997. Credit cycles. Journal of Political Economy 105 (2), 211–248. http://ideas.repec.org/a/ucp/jpolec/v105y1997i2p211-48.html.

Monnet, C., Quintin, E., 2005. Optimal contracts in a dynamic costly state verification model. Economic Theory 26 (4), 867–885. http://ideas.repec.org/a/spr/joecth/v26y2005i4p867-885.html.

Ottonello, P., 2015. Capital unemployment, financial shocks, and investment slumps. Mimeo. University of Michigan. https://sites.google.com/site/ottonellopablo/research.

Popov, L., 2014. Enforcement frictions and optimal lending contracts. Economic Theory 57 (1), 195–222.

Popov, L., 2016. Stochastic costly state verification and dynamic contracts. Journal of Economic Dynamics and Control 64 (C), 1–22. http://www.sciencedirect.com/science/article/pii/S0165188915002213.

Stiglitz, J.E., Weiss, A., 1981. Credit rationing in markets with imperfect information. The American Economic Review 71 (3), 393–410. http://ideas.repec.org/a/aea/aecrev/v71y1981i3p393-410.html.

Townsend, R., 1979. Optimal contracts and competitive markets with costly state verification. Journal of Economic Theory 21 (2), 265–293. https://ideas.repec.org/a/eee/jetheo/v21y1979i2p265-293.html.

Wang, C., 2005. Dynamic costly state verification. Economic Theory 25 (4), 887–916. http://ideas.repec.org/a/spr/joecth/v25y2005i4p887-916.html.