Problem Set 4 - Causal Effects

Due date: December 9, 2022

1 Question 1 - Differences-in-differences

In this section we we analyze the weights given to individual treatment effects in the standard two-way fixed effects regression, under a parallel trends assumption. Here we will assume a staggered adoption design so that

$$Y_{it} = Y_{it}(0) + \tau_{it} D_{it} = Y_{it}(0) + \tau_{it} 1\{t \geq g_i\}$$

where D_{it} is a treatment dummy variable, τ_{it} is the treatment effect for i in period t, and g_i is the first period in which unit i is treated. We assume that no units are treated in the first period, so that $Y_{i1} = Y_{i1}(0)$ for all i, and that there exist never-treated units in our sample.The parallel trends assumption is

$$E\big[Y_{it}(0) - Y_{is}(0) | g_i = g\big] = E\big[Y_{it}(0) - Y_{is}(0) | g_i = g'\big]$$

for any two periods (t, s) and any two cohorts (g, g').

(a) Show that the parallel trends assumption implies that $Y_{it}(0)$ follows a two-way fixed effects model

$$Y_{it}(0) = \alpha_i + \delta_t + u_{it}$$

in which $E[u_{it}|g_i] = 0$.

Suppose that we estimate a static two-way fixed effects regression, i.e. a regression on individual fixed effects, time fixed effects, and the treatment dummay variable D_{it}.

(b) Show that the OLS estimate $\widehat{\beta}$ is equal to a weighted average of the individual-time treatment

effects τ_{it}, i.e.

$$\widehat{\beta} = \sum_i \sum_t \omega_{it} \tau_{it}$$

for some set of weights ω_{it}.

(Hint: the simplest way to do this is to use FWL theorem. You may use the fact that the fitted value in the regression of D_{it} on the individual and time fixed effects is given by $\widehat{D}_{it} = \bar{D}_i + \bar{D}_t - \bar{D}$, where $\bar{D}_i = \sum_t D_{it}/T$, $\bar{D}_t = \sum_i D_{it}/N$ and $\bar{D} = \sum_{i,t} D_{it}/NT$)

(c) Interpret the weights that you derived. Do they define a sensible estimand? Under what conditions are the weights convex? Under what conditions do the weights give the average treatment effect on the treated?

2 Question 2 - Regression discontinuity

Here we will analyze an empirical setting using regression discontinuity techniques. The data is taken from Lindo et. al (2010) and includes observations on academic performance of students at a Canadian university. At the university, students with a cumulative GPA below 1.5 are placed on academic probation. Students who subsequently improve their grades are returned to good standing, while students who remain below the cutoff may be suspended for an academic year. The goal here is to study the effect of academic probation (essentially the threat of possible suspension) on various academic outcomes.

The running variable in this analysis is *dist_from_cut = GPA-1.5* so that the discontinuity occurs at 0. We will study two outcome variables: *left_school*, an indicator of whether the student dropped out of college following their first year, and *nextGPA*, the student's GPA is the following semester.

(a) Estimate the effect of probation on a student subsequently dropping out of college. Interpret the result. (You should compare estimates for different choices of bandwidth, polynomial degree etc. and discuss which is your preferred estimate).

(b) Present your result from part (a) visually. The figure should be able to go in a published paper, i.e. clearly labelled, easy to see and interpret results etc.

(c) Estimate the effect of probation on a student's subsequent GPA. Interpret the result.

(d) Discuss how the use of nextGPA *as an outcome variable could lead to biases in the RD estimate as a measure of the causal effect of probation on grades (hint: think about the sample of*

individuals included in you estimation). Suggest at least one method of addressing this bias, either directly (by estimating an unbiased effect) or indirectly (by bounding the size of the potential bias).

(e) Perform a series of robustness checks to argue for/against the validity of the regression discontinuity in this setting. You should consider the distribution of the running variable, as well as that of a chosen set of covariates (you only need to choose two or three covariates, but discuss why you choose the covariates you did, and whether other covariates would have been valid/invalid for use in similar robustness checks).

Table 1: Variables

Variable	Description
dist_from_cut	GPA-cutoff in first year
GPA_year'x'	GPA in year x
hsgrade_pct	high school grade percentile
totcredits_year1	credits attempted in first year
age_at_entry	age at entry
probation_year1	on probation after first year
probation_ever	ever on probation
male	male
english	english is first language
left_school	dropped out after first year
nextGPA	GPA-cutoff at second evaluation
bpl_north_america	born in North America
bpl_asia	born in Asia
bpl_other	born in other
suspended_ever	ever supsended
gradin'x'	graduated in x years

References

[1] Lindo, J. M., Sanders, N. J., Oreopoulos, P.(2010), "Ability Gender, and Performance Standards: Evidence from Academic Probation," *American Economics Journal: Applied Economics 2* **2**, 95-117.