Problem Set 3 - Double Machine Learning for Causal Analysis

Due date: November 18, 2022

1 Question 1 - Orthogonal scores

In this section we will construct orthogonal scores for two treatment effects setups: differences-in-differences estimation, and a setting where treatment effects are continuous.

1.1 Differences-in-differences

Suppose we have data on two groups of individuals, $G = \{0, 1\}$, in two time periods, $T = \{0, 1\}$. We are interested in the effect of a treatment D on outcome Y, where the treatment is given to group 1 in the second time period, i.e. $D = 1\{G = 1\} \times 1\{T = 1\}$. We additionally observe a set of control variables X. We do not believe that treatment is assigned independently of potential outcomes, even conditional on X, but we are willing to make the following conditional parallel trends assumption:

$$\begin{aligned} &E\big[Y(0)|G = 1, T = 1, X\big] - E\big[Y(0)|G = 1, T = 0, X\big] \\ &= E\big[Y(0)|G = 0, T = 1, X\big] - E\big[Y(0)|G = 0, T = 0, X\big] \end{aligned} \tag{1}$$

(a) Show that this assumption is sufficient to identify a causal treatment effect. Is the identified effect specific to any sub-group/time period?

(b) How would you estimate the identified effect using standard regression techniques?

Now we imagine that the set of controls is potentially high-dimensional so that we would like to use an estimator that has the Neyman-orthogonality property.

(c) Construct a Neyman-orthogonal score for estimating the treatment effect identified above. What are the nuisance functions? Show that the orthogonality property holds for each nuisance function. (Hint: you should first rewrite your problem as a standard treatment effects estimator by considering the time-differences $\Delta Y_i = Y_{i1} - Y_{i0}$ as your outcomes. Then you can apply the relevant score from Chapter 9 of the notes.)

1.2 Continuous treatment variables

Next we consider a setting in which our treatment D takes on p different values, $D \in \{1, \dots, p\}$. We have the identifying condition

$$Y(d) \perp D|X, \qquad \text{for all } d$$

(a) Write down a Neyman-orthogonal score for the quantity $\theta(d) = E[Y(d)]$. (Hint: this is no different to estimating $E[Y(1)]$ or $E[Y(0)]$ in the binary case.)

(b) Explain in detail how you would estimate the effect of an intervention that shifted treatment from d to d' using the orthogonal score function. Include all steps in the process.

We now want to extend the above to a setting in which the treatment D is continuous.

(c) Write down a Neyman-orthogonal score for the quantity $\theta(d) = E[Y(d)]$ in the continuous case. You should construct the score by analogy with the multi-valued case in part(a); you do not need to prove the validity of the estimator, but explain clearly how you would use the score to estimate $\theta(d)$. Think carefully about how to handle probabilities with continuous variables (hint: a kernel function may be useful).

2 Question 2 - Heterogeneous treatment effects

For this part you will estimate a treatment effect using ML techniques and then analyze heterogeneity in the treatment effect across some dimension/ subgroup. You may choose any data set to work with, although it should contain an outcome of interest, a treatment (ideally binary but discrete is also fine) and a set of control variables (ideally high-dimensional, but you can always add interactions, power terms etc.). An interesting choice would be to reanalyze an existing paper using the techniques we have learnt in class, but any data set is fine (if you are really struggling feel free to use the data set from Pset 1 and estimate the effect of gender on log wages (controlling for education, etc.), but it would be more fun to find a setting that you think is interesting).

(a) Describe the setting of your data - what is the treatment, what covariates are available, is causality plausible here?

(b) Estimate the average treatment effect using standard regression methods. Include specifications with and without interaction of the treatment variable with some set of covariates and comment on whether you find evidence of heterogeneous treatment effects. You should estimate something like

$$
\begin{aligned}
Y_i &= \beta_0 + \beta_1 D_i + \beta_3' X_i + \epsilon_i \\
Y_i &= \beta_0 + \beta_1 D_i + \beta_3' X_i + \beta_4' D_i \times (X_i - \bar{X}) + \epsilon_i
\end{aligned}
$$

(c) Estimate the ATE using the Neyman-orthogonal score covered in class, combined with some choices of ML methods for the nuisance parameters. Describe how you constructed your estimate. Compare the estimate of the ATE to the standard regression based estimates.

(d) Using the score you estimated in (c), perform some analysis of the heterogeneity in treatment effects. You may use any method you like, e.g. choose some set of sub-groups, do a non-parametric regression using the scores as outcomes, or estimate a regression tree. Comment on the evidence of heterogeneity you find and compare it to the results from the standard regression in (b) (i.e. the regression with interacted effects $D_i \times (X_i - \bar{X})$ *). Are there particular groups that have very different treatment effects?*

References

[1] Chernozhukov, V., Chetverikov, D., Demirer, M., Duflo, E., Hansen, C., Newey, W. and Robins, J.(2017), "Double/Debiased Machine Learning for Treatment Effects and Structural Parameters," *The Econometrics Journal* **21**, C1-C68.