Course project

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December 18, 2017

1 Problem statement

1.1 Parameters

TODO: Describe params...

1.2 Decision variables

For the problem, 3 matrices are needed as decision variables. The last one is auxiliary.

Symbol	Type and size
P	Binary matrix of size $N \times H$.
W	Binary matrix of size $N \times H$.
T	Binary matrix of size $N \times H$.

The matrix P has the element $P_{n,h} = 1$ if the nurse n is at the hospital at hour n, also if is working, $W_{n,h} = 1$, otherwise 0. The matrix T is an auxiliary matrix, with the element $T_{n,h} = 1$ if the nurse n is travelling to the hospital at the hour h, otherwise 0.

1.3 Constraints

Constraint 1 At least demand$_h$ nurses should be working at the hour h.

$$\forall h \in H, \sum_{n \in N} W_{n,h} \geq \text{demand}_h$$

Constraint 2 Each working nurse should work at least minHours.

$$\forall n \in N, \sum_{h \in H} W_{n,h} \geq \text{minHours} * \sum_{j \in H} T_{n,j}$$

Constraint 3 Each nurse should work at most maxHours.

$$\forall n \in N, \sum_{h \in H} W_{n,h} \leq \text{maxHours}$$

Constraint 4a Each nurse should work at most maxConsec consecutive hours.

$$\forall n \in N, \forall h \in [1, \text{nHours} - \text{maxConsec}],$$

$$\sum_{k \in [0, maxConsec]} W_{n,h+k} \leq \text{maxConsec}$$

Constraint 4b Wrap around case for maxConsecutive hours.

$$\forall n \in N, \forall h \in [1, \text{maxConsec}],$$

$$\sum_{j \in [1,h]} W_{n,j} + \sum_{j \in [nHours - maxConsec + h, nHours]} W_{n,j} \leq \text{maxConsec}$$

Constraint 5 No nurse can stay at the hospital for more than maxPresence hours.

$$\forall n \in N, \sum_{h \in H} P_{n,h} \leq \text{maxPresence}$$

Constraint 6 No nurse can rest for more than one consecutive hour.

$$\forall n \in N, \forall h \in [1, \text{nHours-1}],$$

$$W_{n,h} + W_{n,h+1} \geq P_{n,h+1}$$

Constraint 7 Working nurses can travel to hospital at most once.

$$\forall n \in N,$$

$$\sum_{h \in H} T_{n,h} \leq 1$$

Constraint 8a If a nurse is present at current hour and wasn't at previous hour, then they traveled to hospital at current hour.

$$\forall nN,\ \forall h \in [2, \text{nHours}],$$

$$T_{n,h} \geq 1 - P_{n,h-1} + P_{n,h} - 1$$

Constraint 8b Add case for wrap-around

$$\forall n \in N,$$

$$T_{n,1} \geq 1 - P_{n,nHours} + P_{n,1} - 1$$

Constraint 9 If nurse travels at current hour, then they were not present at previous hour.

$$\forall n \in N,\ \forall h \in [2, \text{nHours}],$$

$$T_{n,h} \leq 1 - P_{n,h-1}$$

Constraint 10 If nurse travels at current hour, then they are present at current hour.

$$\forall n \in N,\ \forall h \in H,$$

$$T_{n,h} \leq P_{n,h}$$

2 GRASP

The GRASP method is a metaheuristic in which each iteration consists of two phases: construction and local search.

First, we need to specify the problem in terms of part that can be selected. The ground set E contains elements that can be selected in order to build a solution. Let S be a posible solution by selecting parts of E, so $S \in 2^E$. We can build F as the set of solutions that are feasible $F \subseteq 2^E$

In our problem, we need to select the behavior of N nurses. A posible solution can be defined by the selection of elements that describe what a nurse n is doing at the time h. A tuple $t = (n, h, s)$ can code that the nurse n should be at state s in the time h. A feasible solution S should consist of a selection of tuples t such that, they define the behavior of every nurse at every time, and also be consistent with the previous constraints.

Furthermore, in order to build a solution, a notion of cost is needed to select among all the posible tuples that can be selected. Lets ignore the constraints in the cost function, and check the feasibility at the end.

Iteratively we can select a naive solution by a simple procedure. We begin with an empty solution, at time $h = 1$. At that time, we have a demand d given by demand_h. A good candidate will be try to meet the demand by adding working nurses, the RCL will have the tuples with working nurses in such a way that they are selected with high probability. Then, one of the posible combination will be chosen, hopefully assigning one nurse to work in that time. Then we continue adding nurses in a working state. Once the demand is met, then we now will prefer adding to the solution tuples that contain nurses in a non-working status. The probability of such state should now be added with high probability.

The selection of the cost function is the core of the GRASP algorithm, so we need to take some care to guarantee a good behavior.