Course project

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1   Problem statement

1.1   Parameters

TODO: Describe params...

1.2   Decision variables

For the problem, 3 matrices are needed as decision variables. The last one is auxiliary.

Symbol	Type and size
P	Binary matrix of size $N \times H$.
W	Binary matrix of size $N \times H$.
T	Binary matrix of size $N \times H$.

The matrix P has the element $P_{n,h} = 1$ if the nurse n is at the hospital at hour n, also if is working, $W_{n,h} = 1$, otherwise 0. The matrix T is an auxiliary matrix, with the element $T_{n,h} = 1$ if the nurse n is travelling to the hospital at the hour h, otherwise 0.

1.3   Constraints

Constraint 1   At least demand$_h$ nurses should be working at the hour h.

$$\forall h \in H, \sum_{n \in N} W_{n,h} \geq \text{demand}_h$$

Constraint 2 Each working nurse should work at least minHours.

$$\forall n \in N, \sum_{h \in H} W_{n,h} \geq \text{minHours} * \sum_{j \in H} T_{n,j}$$

Constraint 3 Each nurse should work at most maxHours.

$$\forall n \in N, \sum_{h \in H} W_{n,h} \leq \text{maxHours}$$

Constraint 4a Each nurse should work at most maxConsec consecutive hours.

$$\forall n \in N, \forall h \in [1, \text{nHours} - \text{maxConsec}],$$

$$\sum_{k \in [0, maxConsec]} W_{n,h+k} \leq \text{maxConsec}$$

Constraint 4b Wrap around case for maxConsecutive hours.

$$\forall n \in N, \forall h \in [1, \text{maxConsec}],$$

$$\sum_{j \in [1,h]} W_{n,j} + \sum_{j \in [nHours-maxConsec+h, nHours]} W_{n,j} \leq \text{maxConsec}$$

Constraint 5 No nurse can stay at the hospital for more than maxPresence hours.

$$\forall n \in N, \sum_{h \in H} P_{n,h} \leq \text{maxPresence}$$

Constraint 6 No nurse can rest for more than one consecutive hour.

$$\forall n \in N, \forall h \in [1, \text{nHours-1}],$$

$$W_{n,h} + W_{n,h+1} \geq P_{n,h+1}$$

Constraint 7 Working nurses can travel to hospital at most once.

$$\forall n \in N,$$

$$\sum_{h \in H} T_{n,h} \leq 1$$

Constraint 8a If a nurse is present at current hour and wasnt at previous hour, then they traveled to hospital at current hour.

$$\forall n N, \; \forall h \in [2, \text{nHours}],$$

$$T_{n,h} \geq 1 - P_{n,h-1} + P_{n,h} - 1$$

Constraint 8b Add case for wrap-around

$$\forall n \in N,$$

$$T_{n,1} \geq 1 - P_{n,nHours} + P_{n,1} - 1$$

Constraint 9 If nurse travels at current hour, then they were not present at previous hour.

$$\forall n \in N, \; \forall h \in [2, \text{nHours}],$$

$$T_{n,h} \leq 1 - P_{n,h-1}$$

Constraint 10 If nurse travels at current hour, then they are present at current hour.

$$\forall n \in N, \; \forall h \in H,$$

$$T_{n,h} \leq P_{n,h}$$