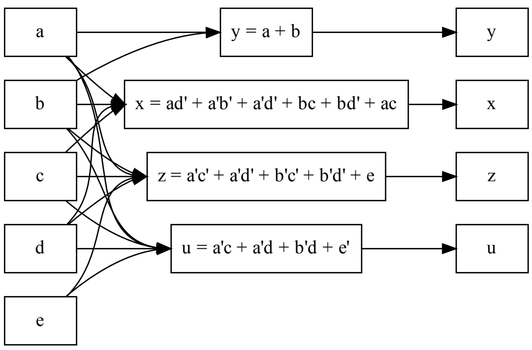
Exercises on physical design

Rodrigo Arias Mallo

December 29, 2017

1 Exercise 5

1.1 Draw the initial network

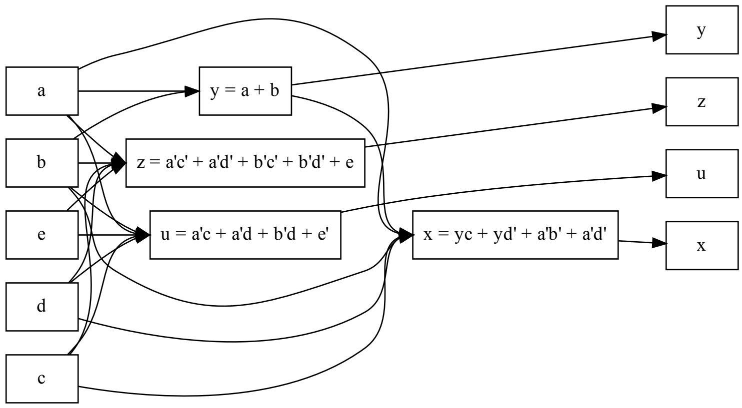


1.2 Algebraic division

In order to perform the algebraic division f_x/f_y we will apply the algorithm WEAK_DIV described in the figure 10.9 of [1]. The functions are defined as

$$f_x = ad' + a'b' + a'd' + bc + bd' + ac \quad f_y = a + b$$

With $F = f_x$ and $P = f_y$ we first compute V^a as the set of cubes of F that contain the literal a, with a removed, giving $V^a = \{d', c\}$. The same for b, giving $V^b = \{d', c\}$. Then we compute Q as the intersection and we get the same $Q = V^a \cap V^b = \{d', c\} = d' + c$. Finally with $PQ = (a + b)(c + d') = ac + ad' + bc + bd'$ we compute the rest as $R = F - PQ = a'b' + a'd'$. So we can confirm that $F = PQ + R$. After substitution we have the following graph.



1.3 Kernels and co-kernels

After trying both algorithms R_KERNELS and KERNELS (8.3.3 and 8.3.4) in [2], I found it very complicated to work by hand. So I switched to the method of the cube insertion table, described in section 10.5.1 of [1]. For $f_z = a'c' + a'd' + b'c' + b'd' + e$, let n be the number of cubes, we first compute the co-kernels of level 0 by building the following table, placing the first $n-1$ cubes in columns, and the last $n-1$ cubes in columns.

	$a'c'$	$a'd'$	$b'c'$	$b'd'$
$a'd'$	a'			
$b'c'$	c'	0		
$b'd'$	0	d'	b'	
e	0	0	0	0

Then, we compute the intersection of the cubes pairwise. Cells above the diagonal are not needed. As we didn't discover co-kernels with more than one literal, we know that there are no co-kernels in the next levels. So we finally get $C(f_z) = \{a', b', c', d', 1\}$. Now, to compute the kernels, we can iterate for each co-kernel, and divide f_z by each one, and the quotient would be the respective kernel. So we easily see that $K(f_z) = \{a' + b', c' + d', f_z\}$

For the function $f_u = a'c + a'd + b'd + e'$ we proceed in a similar way building the table.

	$a'c'$	$a'd'$	$b'd$
$a'd'$	a'		
$b'd$	0	d	
e'	0	0	0

And as before, we finish the procedure at level 0 as we didn't find co-kernels of more than one literal. We have $C(f_u) = \{a', d, 1\}$, and from there we build the associate kernels $K(f_u) = \{a' + b', c + d, f_u\}$

1.4 Multi-cube extraction

Once we have the kernels of both functions, we can extract a multi-cube common subexpression, in order to simplify the logic network. After factoring z and u we have

$$z = (a' + b')(c' + d') + e \quad u = d(a' + b') + a'c + e'$$

Which is decomposed into $w = a' + b'$, $v = c' + d'$ and $z = wv + e$. And for u we have $p = a' + b'$ and $u = dp + a'c + e'$. We see that $w = p$ so we can replace p by w in u, giving $u = dw + a'c + e'$.

References

[1] G. D. Hachtel and F. Somenzi, *Logic Synthesis and Verification Algorithms.* Springer US, 1 ed., 2002.

[2] G. D. Micheli, *Synthesis and Optimization of Digital Circuits.* McGraw-Hill Science/Engineering/Math, 1 ed., 1994.