The Effects of Student Loans on the Provision and Demand for Higher Education*

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Abstract

We characterize the outcomes of the tertiary education market in a context where borrowing constraints bind, there is a two-tier college system operating under monopolistic competition in which colleges differ by the quality offered and returns to education depend on the quality of the school attended. College quality, tuition prices, acceptance cut-offs and education demand are all determined in a general equilibrium model and depend on the borrowing constraints faced by households. Our main finding shows that subsidized student loan policies can lead to a widening gap in the quality of services provided by higher education institutions. This happens because the demand for elite institutions unambiguously increases when individuals can borrow. This does not happen in non-elite institutions, since relaxing borrowing constraints makes some individuals move from non-elite to elite institutions. The higher increase in demand for elite institutions allows them to increase prices and investment per student. As investment and average student ability are complementary inputs in the quality production function, elite universities also increase their acceptance cut-offs. In this new equilibrium, the differentiation of the product offered by colleges increases, where elite universities provide higher quality to high-ability students and non-elite universities offer lower quality to less-able students. We calibrate the model to Colombia, which implemented massive student loan policies during the last decade and experienced an increase in the gap of quality of education provided by elite and non-elite universities. We show that the increase in the quality gap can be a by-product of the subsidized loan policies. Such results show that, when analyzed in a general equilibrium setting, subsidized loan policies can have regressive effects on the income distribution.

JEL Classification: E24, I23, I24, I25, I28,L130

Keywords: Higher education, Student Loans, Education policy, Returns to Education, Labor productivity, Monopolistic Competition

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1 Introduction

The market for higher education has received significant attention in the economics literature. In particular, the effects that subsidized loan policies have on the demand side of the market have been widely studied, given the dramatic increase in student debt during the last two decades in the U.S. Overall, there seems to be a consensus in the literature on the fact that credit constraints explain only a small fraction of enrollment decisions in higher education in the U.S. However, this is not necessarily the case in developing countries, where student financial aid systems are weak and evidence suggests that college enrollment is highly determined by family wealth (World Bank, 2003, 2012). In this context, the implementation of subsidized student loan policies can potentially affect the demand for education, generating equilibrium effects such as increases in tuition prices and changes in the quality of services offered by colleges.

Understanding the effects of these policies is of central importance given the massive investments that have been made in student credit programs during the last two decades in the developing world, in Latin America and some African countries[1]. The demand side effects of these policies in a context where borrowing constraints determine enrollment decisions have been studied by the literature and the conclusions are certainly appealing: an expansion in student loans will lead to an increase in the demand for higher education among the most able students (Canton and Blom, 2004). This will reduce the inefficiency that exists when very high-ability individuals with low initial wealth cannot access tertiary education. A partial equilibrium perspective would unambiguously show that such policies have welfare improving effects on it's beneficiaries. As a consequence, these programs have often received the support of multilateral institutions, such as the World Bank and the Inter-American Development Bank[2].

However, the equilibrium effects of such credit programs have not been widely studied. In particular, not much has been said about the effect of subsidized student loans on tuition prices and on the quality of services provided by educational institutions. Such policies can have negative effects on a subset of the agents in the economy that might offset the overall benefits from introducing such policies[3].

This paper contributes to the literature studying the consequences of subsidized loan policies, by analyzing the general equilibrium effects that such programs have on the quality of education provided by different

[1] See Salmi (1999) and World Bank (2009) for a review on student loan programs in Latin American and African countries.

[2] These institutions have contributed to different student loan projects in the developing world. For example, the World Bank has been financing the Colombian ACCES program since 2002 and committed in 2014 to lend \$200 million during the period 2014-2019. Recently, the IDB provided a \$10 million dollar loan to the Higher Education Finance Fund in 2012, to finance student loan programs in 4 Latin American countries.

[3] Obiols-Homs (2011) argues that in an incomplete markets setting, although increasing borrowing limits increases the welfare of borrowing constrained individuals, in equilibrium this also leads to an increase in the interest rate paid by the borrowers. The two effects oppose each other, so the effect of loosening borrowing limits on welfare is ambiguous and follows a U-shape. Although we do not take into account the effect of borrowing constraints on the interest rate and assume government student loans are subject to an exogenous interest rate, his findings strengthen our theory that student loan policies might have negative effects on welfare, in equilibrium.

tiers of colleges, which we will denote henceforth as low and high-quality, or elite and non-elite colleges. We develop a general equilibrium model where families decide whether or not to go to college, and if they go they need to decide which college they will attend. The market of higher education operates under monopolistic competition, as is common in the literature (Epple et al., 2006), in a two-tier system where students who graduate from elite colleges have higher rewards in the labor market than students who go to the non-elite system. Colleges maximize the quality of the services offered [4] where such quality depends on the skills of their student body and on the investment per student made by the university. Their objective is to chose skills thresholds for admission, tuition, and investment per students, while maintaining a balanced budget in order to maximize the quality of the services offered. We characterize the demand for higher education, the incentives to invest and admit students by each tier-college, as well as the consequences of loan policies in the equilibria arising from such setting.

We find a set of equilibria such that once the subsidized loan policy is implemented, colleges change the quality of services provided in a way such that the gap in quality provided by elite and non-elite institutions becomes wider. Elite universities will face a higher demand for services as a consequence of the subsidized loan policy. They will be able to increase the minimum level of skills for acceptance while maintaining budget balancedness. Increasing the quality of their student body will make elite-universities invest more in their students so long as these two are complement inputs in the provision of quality provided by colleges.. This will lead to a higher segmentation of the market for higher education as high-ability students will attend elite colleges and low-ability students will be concentrated in the non-elite system. In such case, least able students will not be exposed to high-ability students, limiting the extent to which they benefit from their peer effects. Additionally, the marginal productivity of investments per students will decrease in non-elite colleges as the quality of their student body deteriorates, decreasing the incentives by non-elite colleges to invest in their students. This ends up decreasing the gains from higher education for students attending non-elite universities, making subsidized-loan policies a regressive policy in the overall economy.

The rationale behind the widening gap in quality comes from the fact that the market for higher education operates under monopolistic competition and each tier will serve a different segment of the market once the subsidized loan policy have been implemented. Under such circumstances, universities have incentives to differentiate their product -quality offered-, given the differential effect that such policy has on the segments of the market that each university serves. On one hand, the size of the segment of the market that attends the low-quality college is ambiguous after the policy is implemented. It can expand, as people that could not study before can now afford to attend the low-quality college, but can also contract, given that some individuals that prefer the high-quality college can now afford to pay for better education. On the other

[4] This is a common assumption in the literature (Epple et al., 2006)

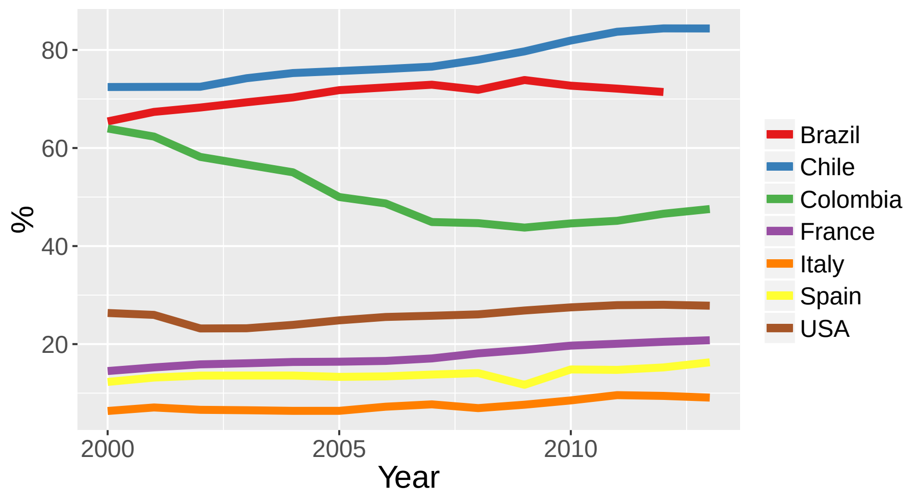

Figure 1: % of enrollment in private institutions by country.

hand, the demand for the elite college unambiguously expands, since now some individuals can afford its services and no individuals will stop attending it because of the policy. The larger increase in the demand for the elite college, as opposed to the non-elite college, allows the former to increase prices per student more than the latter. With more resources, the high-quality college will find it optimal to increase investment per student. Moreover, if the peer effects and the investment per student are complements in the quality production function, the elite institution will increase the cut-offs, so that the average student ability is higher in equilibrium. In this way, the quality gap supplied in equilibrium widens.

Our analysis is also novel, given our focus on developing countries and the fact that the structural literature has only explored the context of the U.S., as far as our knowledge goes. The educational sector in developing economies is particularly different from that of developed economies, for three main reasons. First, in developing countries there is evidence that credit constraints play a role in determining college enrollment decisions among households (Melguizo et al., 2015), as opposed to the case of developed countries, which is reviewed in Section 2. Second, in many developing countries the private institutions own a larger share of the market for higher education, as compared to European countries or even the U.S. (see Figure 1). This is important because public institutions may not be as responsive to market incentives, but rather follow the social planner's objectives. In contrast, private institutions are potentially more responsive to market signals, so any change in demand will generate stronger equilibrium effects in developing economies. Third, enrollment rates in developing countries are very low, when compared to enrollment in developed countries. As documented by Mestieri (2016), there is an existing positive correlation between enrollment rates and income per capita at a cross-country level.

We use the results of the equilibria to rationalize what we observe happened in Colombia after the introduction of a massive subsidized loan policy. Colombia is a developing country that undertook massive expansions of publicly supplied student loan availability during the last decade.

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After the introduction of such policy, not only the number of students increased but we also observe facts consistent with a widening gap in the quality of elite and non-elite universities. The main purpose of this paper is to rationalize how this expansion in the gap can arise as a consequence of the subsidized loan policy previously implemented. This would imply that student loan programs also have downsides when studied in general equilibrium, and all of this should be taken into account for future policy design.

Finally, our analysis will give us tools to discuss the design of the optimal student loan policy in a context where the government has outside funds that has to allocate within the existing population. From a partial equilibrium perspective, we find that the student loan policy that maximizes the average utility of society (and enrollment) is one that gives priority to the lowest-ability individuals that are borrowing constrained and would like to study. The reasoning behind this is the following: high-ability individuals will receive higher incomes over their lifetime, regardless of their education level. Since marginal utility is decreasing, the benefits of studying to have additional income are relatively small. In addition, those that are borrowing constrained and study will not be able to smooth consumption. In this sense, they face a higher opportunity cost of education. Therefore, relaxing the constraint for high ability individuals will change the study decision of fewer individuals, than if the constraint were relaxed among lower ability individuals.

In a general equilibrium setting, there is an opposing force in action. Individuals choose whether and where to study according to the quality offered by each college, since their future earnings will depend on it. The quality offered by colleges, in turn, is a composite of the average ability of the student body (to resemble peer effects) and investments per student made by the college. As assumed by the literature, these two inputs are complements in the quality production function. In this regard, the best quality-enhancing student loan policy would maximize the average student ability, which would increase returns to investments per student, and would lead to higher quality offered. That is, loosening credit constraints to the most able students would lead to higher education quality.

In this regard, there are two opposing forces shaping the optimal student loan policy. Relaxing the borrowing constraints of lower ability households will have the highest impact on school enrollment, but will reduce the returns to investments per student done by schools and the quality of education. In contrast, relaxing the borrowing limits of high ability households maximizes the education quality offered, but enrollment is not as large. The issue becomes even more complex once we incorporate a two-tier education system, in which colleges might respond differently in their pricing, admissions and investment policies when faced to a demand shock of this nature.

The rest of the paper is organized as follows. Section 2 describes the literature relevant to our paper. Section 3 describes an model of the market for higher education, characterizes the demand for a two-tiered education system and explains the mechanism through which borrowing constraints affect equilibrium quality supplied. In this section we illustrate the main theoretical results of the paper. That is, a policy leading to subsidized loans can increase the gap of quality of education. Section 4 describes the case of Colombia and illustrates how this case is consistent with what we predict in the theoretical model. Section 5 estimates the parameters of the quality function, using data on average test scores, professors per student and average wages of students from each university in Colombia. Section 6 describes the computation of the model and the parameters used for calibration. Section 7 shows the main results of the estimations and some counterfactual experiments. Section 8 concludes.

2 Literature Review

Different strands of the literature are related to the present research. First, our paper is related to the literature that studies the importance of borrowing constraints in the schooling decisions. Given that we are studying the welfare effects of government loan policies in developing countries, knowing whether borrowing constraints matter is of central importance. Although there is evidence suggesting that borrowing constraints do not determine school attendance of students in developed countries, the opposite is the case for countries in the developing world. Second, our paper is related to the literature that studies general equilibria in the market for education. This literature has mostly studied what is known as the *"Bennett Hypothesis"*[5]. Our paper adds to this literature because we study equilibrium welfare effects that go beyond prices in the context of binding borrowing constraints, whereas this literature has been concerned only with studying equilibria in the United States, where the inefficiency caused by the existence of borrowing constraints in education is, at best, small. Finally, our paper models the educational sector following the treatment that the industrial organization literature has given to universities, as monopolistically competitive firms that maximize the quality provided, subject to a budget balance constraint.

[5]The *"Bennett Hypothesis"* states that an expansion in the number of grants provided to students are almost totally appropriated by colleges through increases in tuition prices.

2.1 The Role of Borrowing Constraints

Using data from the United States, there is a wide literature that argues that the effects of borrowing constraints on the post-secondary decisions of youngsters are negligible. Using the 1979 of the National Longitudinal Survey of Youth (NLSY79), Carneiro and Heckman (2002) find evidence that borrowing constraints and family income account for a very small fraction of post-secondary school attendance decisions, while early childhood differences are determinant. Once the estimations control for family background and ability, current family wealth becomes unimportant in the schooling decision. According to their estimates, only between 0% and 8% of high school graduates are actually borrowing constrained. Similarly, Keane and Wolpin (2001) find that, although borrowing constraints are tight in the U.S. and individuals cannot even borrow the amount to cover one year of schooling, their existence does not determine the decision to study. In counterfactual experiments, when the authors remove the borrowing constraints, the educational attainment does not change significantly. Borrowing constraints only affect labor supply and savings decisions of students.

Dinarsky (2003) measures the impact of the exogenous elimination of the Social Security Student Benefit Program in 1982 on school attendance in the U.S. This program provided students that came from families with deceased, disabled, or retired Social Security beneficiaries with monthly payments while enrolled in college. The paper finds that the exogenous reduction in aid led to a decrease in the probability of being enrolled by students at the margin. However, the author argues that this cannot be interpreted as existence of borrowing constraints, since grants do not only relax the borrowing constraints of households, but change also the relative price of education.

More recent studies argue that, although credit constraints did not seem to affect the schooling decision some decades ago, during the last two decades they might be playing an important role in post-secondary schooling in the United States. Using data from the 1979 and 1997 National Longitudinal Survey of Youth (NLSY79 and NLSY97), Belley and Lochner (2007) find a dramatic increase in the importance of family income on school attainment, after controlling for family background and ability as in the previous studies. Similarly, Lochner and Monge-Naranjo (2011) estimate a structural model that suggests that, although American households were not borrowing constrained during the 1980s, during the last decade family income has been determinant in schooling decisions. They argue that in the last two decades there have been rising costs and returns to education, while government student loan programs have not grown at the same pace, so people have become borrowing constrained.

Although there is not much research that studies the role of credit constraints in educational choices in developing countries, the existing evidence seems to unambiguously point towards the importance of borrowing constraints in the educational decisions. As Attanasio and Kaufmann (2009) state, *"one important difference between Mexico and the U.S., for instance, might be the wider availability of scholarships and student loans in the U.S., cannot be found in Mexico for higher education."* Attanasio and Kaufmann (2009) and Kaufmann (2014) provide evidence suggesting that liquidity constraints do determine the post-secondary schooling decision in Mexico. They use data that characterizes the expected returns of education for every household in their sample. If credit constraints were not binding, there should exist a positive gradient between subjective expected returns from schooling and school attendance. Their results show that this gradient breaks for the lowest income households in their sample. Under their interpretation, this is evidence of existing borrowing constraints. Solis (2013) studies the existence of borrowing constraints in Chile. Using administrative data on the entire sample of individuals that participate in the college admissions' process, he uses a regression discontinuity approach to study the impact of providing educational loans. After controlling for socio-economic covariates, individuals right above the eligibility threshold for receiving educational loans have a significantly higher probability of enrolling in college than those right below the threshold. The author finds evidence suggesting a positive gradient between income and enrolment among those households that have no access to the government loans. This gradient disappears for individuals that access the program. Also using a regression discontinuity approach, Gurgand et al. (2011) find evidence that the enrolment to college of households without access to student loans is 20 percentage points lower in South Africa. Regarding the Colombian case, Melguizo et al. (2015) find evidence that the implementation of a massive government loan program in the past decade, which is the topic of this paper, did increase student enrolment.

2.2 General Equilibrium Effects and the Bennett Hypothesis

During the last decade, the literature that has tried to explain what has become to be known as the *Bennett Hypothesis*: expansions of government-supplied student aid for education have been almost almost entirely appropriated by colleges through an increase in tuition prices. As the former U.S. Secretary of Education stated in 1987, *"If anything, increases in financial aid in recent years have enabled colleges and universities blithely to raise their tuitions, confident that Federal loan subsidies would help cushion the increase."*[6] Singell and Stone (2007) study the effect that Pell Grants have had on tuition prices of public and private schools. They study the Pell Program, which has been the biggest post-secondary educational loan program in the United States. In 1999, the Pell Grants were awarded to 3 million students across more than 6000 colleges, out of a total of 9 million students. The authors estimate the impact of Pell Grants per student on tuition charged by universities, using a panel of 1554 colleges from 1989 to 1996. They find that the increase in Pell Grants caused an almost one-to-one increase in the price of tuition charged by private and public out-of-state

[6] William Bennett to the New York Times, 1987.

colleges. However, they find no such a causality on the in-state tuition charged by public schools. In contrast, Rizzo and Ehrenberg (2002) find evidence that private and public out-of-state tuition prices were not affected by government loans, while in-state tuition by public colleges were. Finally, Lucca et al. (2016) use exogenous variation in the legislation that rules Pell Grants, to study the relationship between student aid and tuition.

Gordon and Hedlund (2015) study the increase in tuition prices by estimating a structural model in which universities provide human capital and households decide their investments in education. They study the rise in college tuition over the last decades, as a reaction to cuts in state appropriations, an increase in the costs of skilled labor in other industries, and an increase in government supplied loans. The authors find out that the increase in government loans explains 102% of the tuition increase, as opposed to only 16% of the other two hypotheses. This result provides evidence in favor of the Bennett Hypothesis. Our paper differs from theirs in the sense that we want to study the equilibrium effects on quality provided and welfare effects of relaxing borrowing constraints in a context in which they matter. The authors study increases in the borrowing limits in the context of the U.S. As has been already argued, there is evidence that these constraints are of secondary importance on the decision to attend school. Therefore, relaxing these limits does not improve efficiency. In contrast, in countries in which the borrowing constraints are binding, relaxing them does generate efficiency improvements.

2.3 The Education firms

Our paper makes part of the literature that models universities as firms in the educational sector. Universities produce human capital and use households both as inputs and costumers. This approach has been used to study different questions regarding post-secondary education. For instance, Chade et al. (2014) model the universities as an oligopoly with a fixed number of universities (firms), in which the goods produced by universities (education) are ranked exogenously by all households in the same way. Universities only choose admission standards, so as to fill a fixed capacity of students and maximize the ability of the student body. The paper studies the role of frictions in the application process on the student sorting between universities. Namely, the model has information frictions and fixed costs of application. The authors, as Caucutt (2001), treat the utility that households receive of attending each of the universities as exogenous and independent from the product offered by each university. We endogenize the valuation of households as a function of the equilibrium quality offered. The authors do not include tuition prices as a policy of universities, assume an exogenous valuation for the universities and take the size of universities as fixed. We depart from all of these assumptions, but assume there are no frictions in the application process. The reason is that our purpose is not to study the outcome of the application process but, instead, model the strategic interactions in the post-secondary education sector between universities and households.

The educational sector in our model closely mimics Epple et al. (2006). In their paper, the authors model the supply side of the educational sector as an oligopoly sector in which a fixed amount of colleges interact to attract students and maximize the quality of the education they offer, subject to a balanced budget constraint. Quality by universities is a composite of average student ability, to resemble peer effects in schooling, and the average investments per student. This treatment of quality has been standard in the literature that models schools (Caucutt, 2001). Households value quality as an input on their utility function. In their model, households play a passive role in their model, since their purpose is not to estimate equilibrium interactions between households and firms. Rather, they concentrate in studying thoroughly the supply side. Furthermore, they estimate their model by using a "club goods" approach, instead of explicitly solving the Nash equilibrium of the monopolistically competitive market. We depart from this approach, since we consider that the strategic interactions between colleges are of first order importance to explain the different reaction of elite and non-elite institutions to subsidized student loan policies.

Finally, we treat wages of college graduates as a function of the quality supplied by the school attended. To the best of our knowledge, this approach has not been used in structural estimations in the past, but there is empirical evidence that relates future wages to the quality of the education. Black and Smith (2006) estimate a latent model in which quality is a latent variable, and there are "signals" of quality. They find out that SAT scores, faculty-student ratio, rejection rate, freshmen retention rate, and faculty salaries are significant signals of quality. Furthermore, the latent variable of quality significantly affects post-college wages of individuals. Similarly, Black et al. (2005) find evidence that quality increases post college earnings, driven by higher wages. Leaving quality aside, there is extensive evidence that estimates positive returns to college attendance in terms of higher future wages (Zimmerman, 2014; Patrinos et al., 2006). OECD and World Bank (2012) estimates that average starting earnings for individuals with a bachelor's degree were 4 times higher than those of individuals with high-school degree. Although these estimates do not control for unobservable household characteristics, other estimates find that people with post-secondary degrees earn significantly higher wages in Latin America (Gasparini et al., 2011).

3 A Model of the Market for Higher Education with Credit Constraints

There are two types of agents in the economy: households and universities. There is a government that offers educational credits to high-ability individuals that decide to attend college, at an exogenous interest rate $R \geq r$, where r is the risk free interest rate. In addition, the government subsidizes the interest paid by the poorest households that access the credit, at a subsidy rate s. In order to finance these subsidies, the government levies a marginal tax, τ, to every household in the economy. The government policies are exogenous, fixed before the economy starts and satisfy budget balance. Given these policies, the market of higher education operates under monopolistic competition. Universities supply human capital in the market for education, by choosing a tuition price, a minimum ability level for admission and a level of investment per student. Given government and university policies, the households decide if they want to study in any university at the prevailing market prices.

3.1 Households

Households are born with innate ability and wealth (θ, b), according to a bivariate distribution $F(\theta, b)$ over the space $[0,1] \times [\underline{b}, \bar{b}]$. Individuals live for two periods, after which they die with probability equal to one. In period 1, individuals choose either to study at the university or work in the non-skilled labor market at a wage w per efficiency unit of labor. Individuals that do not study receive a wage θw, do not have access to credit markets and can save at the risk-free rate r. There are two universities in the economy denoted by h and l. Each university sets a threshold $\underline{\theta}^j$ for $j = h, l$ such that only students that have ability $\theta \geq \underline{\theta}^j$ are admitted to university j, and we assume that this information is public[7]. Therefore, individuals with $\theta < \min\{\underline{\theta}^h, \underline{\theta}^l\}$ cannot study and have to work. Individuals who decide to study at university j cannot work, and have to pay a tuition, P^j, set by the university.

In order to finance education, the government offers student loans of up to the price of the tuition, P^j, at the interest rate R to people that decide to study and have an ability level $\theta \geq \theta_{min}$. In addition, students with low wealth, $b \leq b_{max}$, that decide to study and have access to the loan will receive a subsidy on the interest rate, s. Loans are given conditional on studying, and individuals that study and are eligible for the loan choose whether to borrow from the government or not. In order to finance these subsidies, the government levies a proportional tax, τ, to every individual in the economy. Individuals for which $\theta < \theta_{min}$ are borrowing constrained and can only finance education with their initial wealth. Therefore, in the first period the household decides its level of consumption, c, whether to study or not in any university, h, l, and the level of savings, a, which can be potentially negative for households that study and satisfy the government conditions for the educational loans.

In the second period, the households are either non-, low- or high-skilled, depending on whether they decided to study in the first period and which college they attended. Those who decided to study in period

[7]We assume that $\underline{\theta}^j, j = h, l$ is a public threshold, since our purpose is not to study the frictions in the college application process, as opposed to some papers in the literature that model explicitly these information frictions (Chade et al., 2014; Fu, 2014).

1, will enter the j-skilled labor market in period 2, and receive a wage equal to $w\theta(1+z^j)$, where z^j is a skill premium that is university specific. This quality is an equilibrium object that depends on the quality of the student body and investments per student, and is fully characterized in the next section. We assume that individuals have perfect foresight of the value of z^j for $j = h, l$ when they optimize. Individuals who do not study will become part of the non-skilled labor force at a wage $w\theta$. We exclude the possibility of default in the model by assuming that repayment is fully enforced, so in the second period individuals that have government debt will repay their student loan. Given prices R, r, w, government policies τ, s, university policies $\{\underline{\theta}^j, P^j\}_{j=h,l}$, and perfect foresight about education quality $\{z^l, z^h\}$, a household that is eligible for studying at the university j, $\theta \geq \underline{\theta}^j$, and decides to study gets a utility equal to:

$$V^j(\theta, b) = \max_{c,a} \quad u(c) + \beta u(c'), \quad \text{s.t.} \tag{1}$$

$$c + a + P^j = b \cdot (1-\tau) \tag{2}$$

$$c' = a(1+r) \cdot \mathbb{1}_{\{a \geq 0\}} + a(1+\tilde{R}) \cdot \mathbb{1}_{\{a<0\}} + w\theta(1+z^j) \tag{3}$$

$$\tilde{R} = \begin{cases} R(1-s) & \text{if } b \leq b_{max} \\ R & \text{if } b > b_{max} \end{cases} \tag{4}$$

$$a \geq -\mathbb{1}_{\{\theta \geq \theta_{min}\}} \cdot P^j, \quad c \geq 0, \quad c' \geq 0 \tag{5}$$

Individuals that decide not to study, get the following utility:

$$V^N(\theta, b) = \max_{c,a} \quad u(c) + \beta u(c'), \quad \text{s.t.} \tag{6}$$

$$c + a = b \cdot (1-\tau) + w\theta \tag{7}$$

$$c' = a(1+r) + w\theta \tag{8}$$

$$a \geq 0, \quad c \geq 0, \quad c' \geq 0 \tag{9}$$

The individual with ability and wealth (θ, b) decides to study at university j whenever $\theta \geq \underline{\theta}^j$ and $V^j(\theta, b) \geq V^N(\theta, b)$, and $V^j(\theta, b) \geq V^{-j}(\theta, b)$ if they can attend to the other university $-j$, i.e. $\theta \geq \underline{\theta}^{-j}$. Otherwise, the individual decides not to study. Therefore, the household's value function is given by:

$$V(\theta, b) = \begin{cases} \max\{V^h(\theta, b), V^l(\theta, b), V^N(\theta, b)\} \text{ if } \theta \geq \max\{\underline{\theta}^h, \underline{\theta}^l\} \\ \max\{V^j(\theta, b), V^N(\theta, b)\} \text{ if } \underline{\theta}^{-j} > \theta \geq \underline{\theta}^j \\ V^N(\theta, b) \text{ if } \theta < \min\{\underline{\theta}^h, \underline{\theta}^l\} \end{cases}$$

The following section gives a detailed characterization of the demand for both tiers of schools in the state space. This characterization will allow us to give insights on the optimal student loan policy on a monopolistically competitive market.

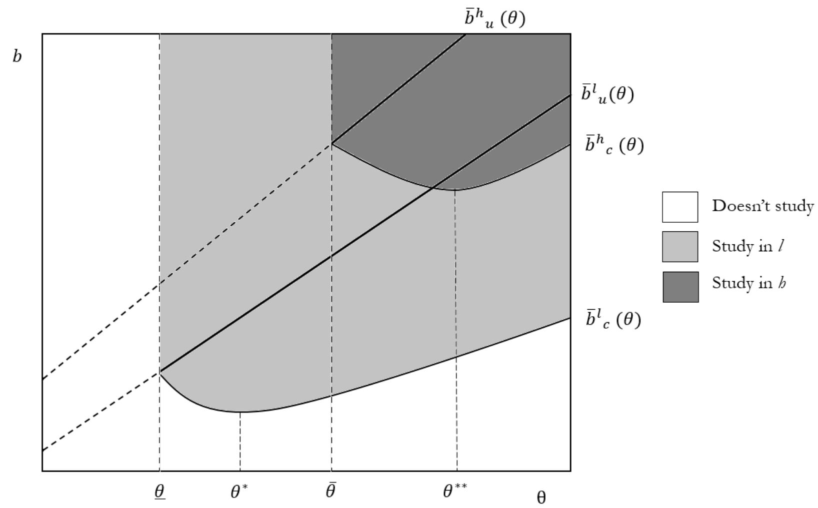


Figure 2: Representation of the education decisions on the state space.

3.1.1 Characterization of the Demand

For a given set of initial parameters, the shaded region in Figure 2 illustrates the individuals that choose to study in the state space when both universities set their acceptance threshold to 0 and there are no government-supplied student loans. The following sequence of theorems characterize the demand for college education on the state space, and its close relationship with borrowing constraints. This will let us derive some results about the socially optimal student loan policy. First, we describe the college decision for households that are unconstrained.

Theorem 1. *Among the unconstrained households, the decision of whether and where to study is independent of initial wealth, b, and follows a cut-off rule on θ. That is, there exist $\underline{\theta}$ and $\bar{\theta}$ such that:*

- *If $\theta \leq \underline{\theta}$, the individual will not study.*
- *If $\underline{\theta} \leq \theta \leq \bar{\theta}$, the individual will attend the low-quality college.*
- *If $\bar{\theta} \leq \theta$, the individual will attend the high-quality college.*

where:

$$\bar{\theta}_l = \frac{1+r}{w}\left(\frac{P_l}{z_l-(1+r)}\right), \quad \bar{\theta}_h = \frac{1+r}{w}\left(\frac{P_h-P_l}{z_h-z_l}\right)$$

Proof. See Proof A.1 □

Theorem 1 is a result of the fact that ability θ, unskilled labor w and quality of the school attended z_j are complements. In particular, this complementarity implies: a) among the unconstrained individuals, those with higher ability face higher marginal returns of education, so will choose, *ceteris paribus*, a higher quality school for a given wealth, b) as the wages of unskilled labor w increase, the marginal returns to education rise for every θ, so marginal individuals will shift to higher levels of education, c) if college j, for $j \in \{l, h\}$, increases its price P_j or reduces its quality z_j, marginal individuals will change their schooling decision in the expected direction. That is, if P_j increases or z_j decreases, marginal individuals will change their decision of attending school j. Finally, d) if the interest rate r increases, present consumption becomes more valuable than future consumption, so marginal individuals will reduce their present expenditures in education. Theorem 2 characterizes the individuals that, given their decision to attend college j, are borrowing constrained.

Theorem 2. *Given an ability θ, there exist cut-offs, $\bar{b}_u^j(\theta)$, $j \in \{N, l, h\}$, on the initial wealth, such that individuals with $b \geq \bar{b}_u^j(\theta)$ that attend college j will not be borrowing constrained. Individuals that attend college j and have $b < \bar{b}^j(\theta)$ will be borrowing constrained and will not be able to smooth consumption over time. The cut-offs are linear, increasing in θ and take the form:*

$$b_u^{\bar{N}}(\theta) = -\bar{A}(1 + (\beta(1+r))^{-1/\sigma}(1+r)) - w\theta(1 - (\beta(1+r))^{-1/\sigma})$$

$$b_u^{\bar{l}}(\theta) = P_l + (\beta(1+r))^{-1/\sigma} w\theta(1+z_l) - \bar{A}(1 + (\beta(1+r))^{-1/\sigma}(1+r))$$

$$b_u^{\bar{h}}(\theta) = P_h + (\beta(1+r))^{-1/\sigma} w\theta(1+z_h) - \bar{A}(1 + (\beta(1+r))^{-1/\sigma}(1+r))$$

Proof. See Proof A.2 □

Given a level of education and initial wealth, individuals with a higher θ have higher lifetime income and in an unconstrained world would consume more in every period of their lives. Given the existence of a borrowing limit $\bar{A}$, for a sufficiently high θ individuals will be borrowing constrained. As a consequence, the initial wealth that individuals must have not to be borrowing constrained is increasing in ability. Figure 2 illustrates the cut-off functions $\bar{b}_u^j(\theta)$ on the state space. As illustrated, individuals above the $\bar{b}_u^j(\theta)$ function, will decide to study in college j whenever her θ falls inside the corresponding interval in the cut-offs defined in Theorem 1. Note also that individuals that are borrowing constrained when studying at college l will also be borrowing constrained when studying in h, assuming a higher price of education in the high-quality college (which, of course, is an equilibrium object). Moreover, the functions $\bar{b}_u^j(\theta)$ are steeper when the quality z_j increases, since quality of schooling and ability are complements. Finally, we do not consider the case in which individuals are borrowing constrained when they do not study. Since in our context, individuals that do not study earn the same wage in every period, they will only be borrowing constrained when the interest rate $\beta(1+r) << 1$. However, for a reasonable calibration, individuals will be able to smooth consumption. The following two theorems illustrate the study decision of individuals that are borrowing constrained.

Theorem 3. *Given ability θ, the decision to study in the low-quality college, l, or not study at all, follows a cut-off strategy on b, such that individuals with $b \geq \bar{b}_c^l(\theta)$ will attend college l, and those with $b < \bar{b}_c^l(\theta)$ will* *not study. The cut-off is characterized implicitly by equation (16) in the proof. Moreover, if the intertemporal elasiticity of substitution is lower than 1 the cutoff is U-shaped and there exists a θ^* such that $\bar{b}_c^l(\theta)$ is:*

- *decreasing in θ for $\theta \leq \theta^*$*
- *increasing in θ for $\theta \geq \theta^*$*

where θ^* solves:

$$\left(\frac{1}{1-\sigma}\right)(b(\theta^*) - P_l + \bar{A})^{1-\sigma} + \left(\frac{\beta}{1-\sigma}\right)(w\theta^*(1+z_l) - (1+r)\bar{A})^{1-\sigma} -$$

$$\Phi(w\theta^*(2+r) + b(\theta^*)(1+r))^{1-\sigma} = 0$$

$$b(\theta) = \theta\left[\frac{wX(1+z_l) - w(2+r)}{1+r}\right] - X\bar{A}$$

$$X = \left[\frac{\Phi(1-\sigma)(2+r)}{\beta(1+z_l)}\right]^{1/\sigma}$$

Proof. See Proof A.3. □

The cut-off $\bar{b}_c^l(\theta)$ is illustrated in Figure 2, where we assume that the utility function is CRRA with $\sigma = 2$, as is common in the literature, so the intertemporal elasticity is lower than 1. The individuals who are constrained (below $\bar{b}_u^l(\theta)$) will choose either to study at l or not, if their initial wealth exceeds $\bar{b}_c^l(\theta)$. The cut-off is U-shaped because two effects are in action. First, the "complementarity" effect means that, given a b, individuals with higher θ will have higher marginal returns from studying, so are willing to study even though they will not be able to smooth consumption. Therefore, the cut-off is initially decreasing. However, the "constrainedness" effect dominates after some point: given an initial wealth b, individuals with higher θ will face a larger wedge in their Euler equation, meaning that they will be able to smooth consumption to a lower extent. When the wedge is large enough, individuals will prefer not to study and smooth consumption by deciding not to study. Of course, this results strongly depends on the value of σ chosen, and continues to hold for any $\sigma > 1$. For the sake of exposition, in Appendix A.7 we characterize the demand for education with a linear utility function (that is, when $\sigma = 0$ and there is an infinite elasticity of substitution). Figure 10 illustrates the decision of individuals in the state space. As can be expected, in the linear case individuals derive no utility from consumption smoothing, so there does not exist such a "constrainedness" effect. In this case, the threshold is never increasing. The next theorem characterizes the cut-off for individuals that are constrained when studying at h. The results are parallel to Theorem 3.

Theorem 4. *Given ability θ, the decision to study in h or l, follows a cut-off strategy on b, such that individuals with $b \geq \bar{b}_c^h(\theta)$ will attend college h, and those with $b < \bar{b}_c^h(\theta)$ will attend l. The cut-off is characterized implicitly by equation (17) in the proof. Moreover, if the intertemporal elasiticity of substitution is lower than 1 the cutoff is U-shaped and there exists a θ^{**} such that $\bar{b}_c^h(\theta)$ is:*

- *decreasing in θ for $\theta \leq \theta^{**}$*
- *increasing in θ for $\theta \geq \theta^{**}$*

where θ^{**} solves:

$$\left(\frac{1}{1-\sigma}\right)\left(b^*(\theta^{**}) - P_h + \bar{A}\right)^{(1-\sigma)} + \left(\frac{\beta}{1-\sigma}\right)\left(w\theta^{**}(1+z^h) - (1+r)\bar{A}\right) -$$

$$\Phi \times \left(w\theta^{**}(1+z_l) + b(1+r) - P_l(1+r)\right) = 0$$

$$b^*(\theta) = \theta w\left(X^*(1+z^h) - (1+z_l)\right) - X^* + P_l$$

$$X^* = \left(\frac{\Phi \times (1-\sigma)(1+z_l)}{\beta(1+z_h)}\right)^{1/\sigma}$$

Proof. See Proof A.4. □

Having characterized the demand for education in the state space, we can say a couple of things about the relationship between borrowing constraints and the demand. The following result describes the differential effect of relaxing the borrowing limits to households, $\bar{A}$.

Theorem 5 (Borrowing constraints)**.** *If the intertemporal elasticity of substitution is lower than 1, for any given θ the cut-offs $\bar{b}_c^l(\theta)$ and $\bar{b}_c^h(\theta)$ are decreasing on $\bar{A}$. Moreover, the elasticities of $\bar{b}_c^l(\theta)$ and $\bar{b}_c^h(\theta)$ with respect to the borrowing limit $\bar{A}$ are decreasing on θ, meaning that a relaxation of the borrowing constraint has a higher impact on enrollment among the marginal individuals that have lower θ.*

Proof. See Proof A.5. □

Theorem 5 states that among the constrained individuals, those with lower θ are more sensitive to relaxing the borrowing constraints. That is, if the borrowing constraints were relaxed by the same amount to all the individuals, more low-θ individuals would change their study decision. This result is a consequence of the decreasing marginal utility. Individuals with high θ and sufficiently low initial wealth have a trade-off between earning relatively high wages in every period and smoothing consumption it they do not study, or studying to earn large wages in the second period at the expense of a very low consumption in the first period. However, because of decreasing marginal utility, the utility of a very large wage in the second period is not as large as for lower θ individuals, so individuals will optimally decide to study only when there is a large increase in the borrowing limits of the first period.

This result has very important implications on the design of an optimal student loan policy in a partial equilibrium setting. If the objective of the government is to maximize enrollment, the policy should target the lower ability individuals. As a matter of illustration of Theorem 5, Figure 3 illustrates the number of individuals of ability θ that change their study decision as the borrowing constraint is relaxed from $\bar{A} = 0$. As stated in Theorem 5, the individuals in the state space with low ability that would study in the unconstrained world (those with $\theta \in [\bar{\theta}_l, \bar{\theta}_h]$) are more sensitive to relaxing borrowing constraints. Therefore, increasing the borrowing capacity increases enrollment more among the low ability individuals.

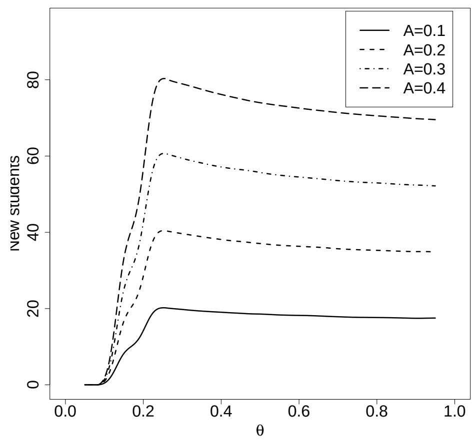


Figure 3: Number of students that change their study decision when borrowing constraints change from $\bar{A} = 0$ to $\bar{A}$, by ability θ.

3.2 Universities

Universities act as firms that maximize an objective function. Given that university systems in most countries are non-profit firms, we follow the literature on education and industrial organization and assume that universities maximize a composite of the quality they offer to students, denoted by z, and the economic diversity of their student body, subject to a budget constraint. Quality offered by universities is an abstract concept. The literature has argued that the quality offered by a school is determined both by the quality of the student body and the investments per student done by the school. Epple et al. (2006), for instance, model the objective function of the university as a composite of the average ability of the student body, the investment per student and the inverse of the mean income. They argue that there is empirical and anecdotal evidence that shows that colleges engage in policies to attract low income students. Universities take as given the values of τ, s, R, w and the distribution F. Additionally, we assume that universities set their policies simultaneously and so, the pricing and admission policies set in equilibrium should satisfy the no profitable one shot deviation principle.

University j takes as given $(\tau, r, s, R, w, P^{-j}, \underline{\theta^{-j}})$ and will set the pricing and admission policies $(P^j, \underline{\theta^j})$ in order to solve the following problem:

$$\max_{\{P^j, \underline{\theta^j}, I^j\}} \left(z^j\right)^{\alpha} \left(\sigma_b^j\right)^{1-\alpha} \tag{10}$$

subject to:

$$z^j = \tilde{\theta^j}^{\alpha_1} (I^j)^{\alpha_2} \tag{11}$$

$$\tilde{\theta^j} = \int_{\Theta \times B} \theta \cdot e^j(\theta, b) dF(\theta, b) \tag{12}$$

$$I^j \cdot N^j + V^j(N^j) + C^j = P^j \cdot N^j + E^j \tag{13}$$

$$N^j = \int_{\Theta \times B} e^j(\theta, b) dF(\theta, b) \tag{14}$$

where $\tilde{\theta^j}$ is the average ability of the individuals that attend school j. σ_b^j is the inverse of the average income of the student body and reflects the fact that universities care about the diversity in their student body. $e^j(b, \theta)$ indicates with values zero or one if a student with ability θ and wealth b decides to study or not. I^j is the monetary amount that the university invests per student, V^j is a convex cost function, N^j is the size of the student body, C^j is a fixed cost and E^j the university's endowment. Note that the policy P^j does not depend on student's characteristics such as wealth or skills. This is not only a simplifying assumption but also follows closely the case of Colombia where private universities do not price-discriminate students based on ability or wealth. As will be discussed in the relevant section, the extent of financial aid provided by such institutions is very limited in the period of analysis.

3.3 Discussion

Although in principle the solution to the problem of the university might seem simple given that there are only two variables of choice, there are several elements of the model that increase the complexity of such decision. First of all, both policies are interdependent. When a university changes one decision variable -either the price or the admission threshold- this will distort the incentives faced when setting the other policy. For instance, a change in tuition price will not only change the revenue of the university but will change the demand in a way that we expect to see a change in the average ability of the student body. Such a change in the average ability of the student body will affect the marginal productivity of investments made by the university, which in turn will affect its pricing decisions.

Moreover, we need to deal with the fact that in equilibrium no university should have incentives to deviate. Given that both universities make the decision simultaneously and that there are no elements of incomplete information in the model, the relevant equilibrium concept is Nash Equilibrium: no university will have incentives to deviate given the decisions made by the other university. Note that given the nature of the problem we cannot be sure of the existence of such equilibrium -university payoffs are not continuous- and moreover, uniqueness cannot be guaranteed.

The aforementioned elements make it clear why analyzing the consequences that subsidized loan policies will have in the market of higher education is a complex problem. Let's suppose that the government imposes such policy by subsidizing the interest rate of student loans. The first effect such policy will have is an increase in the number of students going to college. Note, however, that it is also not unreasonable to assume that the quality of the student body will change. This is because people who changed their decision to go to college are either those who were credit constrained or those having low ability levels that now decide to go to college given the decrease in the opportunity cost.

We can expect that after imposing such a policy, households will react by changing their decision of studying and universities should expect a change not only in the size of their student body but also in their quality. Given such changes, universities might want to change the prices charged to their students. This is due to the fact that as the quality of the student body changes, the productivity of investment will also be affected. Additionally, the willingness to pay for educational services is affected by such policy and universities will react to that. Moreover, universities might want to change the admission threshold either to improve the quality of their student body or to attract less able students that are willing to pay more for education. The overall effect depends on how sensitive is the demand for education with respect to the quality of services being provided.

Finally, note that -as said previously- the decisions of universities need to be analyzed in equilibrium. When deciding what is optimal, each college needs to take into account what their competitor is doing in the market and there should be no room for profitable deviations. After imposing a subsidized loan policy we might end up in an equilibrium where one college serves a specific part of the population. For instance one college serves a large demand for students with relatively low levels of ability whereas the other one specializes in providing high quality education for a reduced number of high ability students. Additionally, we can have a symmetric equilibrium where both firms are indistinguishable from one another or one in which only one firm operates in the market.

3.4 Government

We do not model the government as a welfare maximizing agent in the economy. We abstract from this fact and simply analyse the impact of the change in the government policies on the higher education market. However, we do interpret the student loan policy implementation as a way of the government to reduce the existent inefficiency in the educational market.

In a social planner's solution, the efficient outcome would be one in which the high ability individuals decide to study, independent of their wealth. Thus, the role of the student loan policy can be interpreted as a way to reduce the existing inefficiency in the educational sector, although we do not model it as an optimal decision. We assume that the government has a borrowing constraint in the international borrowing markets, so is only able to finance the education of some fraction of the individuals in the economy. For now, we assume that the government finances individuals that have $\theta \geq \theta_{min}$, and of those that can access the loans, subsidizes the interest rate on the loan for those individuals that have $b \leq b_{max}$. The government sets thresholds $\bar{b}$ and θ_0, such that

$$s \cdot (R - 1 - r) \cdot \int_{\Theta_2 \times B_2} \left(e^l(b,\theta) + e^h(b,\theta)\right) \times dF(\theta, b) = \tau \int_{\Theta \times B} b dF(\theta, b) \tag{15}$$

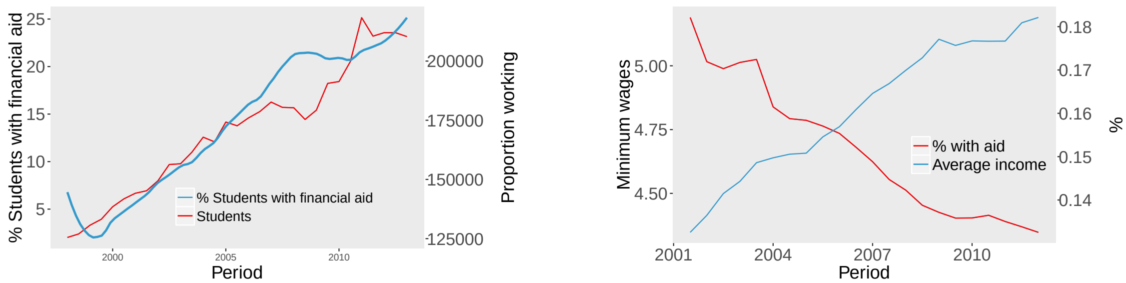
where $\Theta_2 \times B_2 = (\Theta_1 \times B_1) \cap ([\theta_0, 1] \times [0, \bar{b}])$ is the set of households who study and decide to take the subsidy.

Definition 1 (Competitive Equilibrium)**.** *Given a set of government policies, $\tau, s, b_{max}, \theta_{min}$, and prices R, r, w, a competitive equilibrium is a set of university policies $(P^j, \underline{\theta}^j, I^j)_{j=h,l}$ and household's value function $V(\theta, b)$ and policy functions $c(\theta, b), a(\theta, b), e^h(\theta, b), e^l(\theta, b)$, such that:*

1. *Given $\tau, s, b_{max}, \theta_{min}$, R, r, w, and university policies $\{P^j, \underline{\theta^j}, I^j\}_{j=h,l}$, the value function $V(\theta, b)$ solves the household's problem, with $c(\theta, b), a(\theta, b)$, $e^h(\theta, b)$ and $e^l(\theta, b)$ being the corresponding policy functions.*
2. *For each university $j = h, l$, it should hold that given $\tau, s, b_{max}, \theta_{min}$; prices, R, r, w; policy functions $c(\theta, b), a(\theta, b), e^h(\theta, b), e^l(\theta, b)$; and policies from university $-j$, $(P^{-j}, \theta^{-j}, I^{-j})$, university j chooses policies $(P^j, \underline{\theta^j}, I^j)$ that solve the university's problem described in 10-14.*
3. *The government's budget is balanced (equation 15 holds).*

The nature of the problem makes it hard not only to compute the competitive equilibrium but also to show its existence. Note that, by only analyzing the supply side of the market, we cannot be sure that such an equilibrium will exist in this economy. In order to compute the Nash equilibrium of the supply side of the market, we need to find pricing and admission policies that are profit-maximizing given what the policies of the other university.

The computation of such equilibrium is more involved when we note that there is an additional fixed-point problem in the computation of the equilibrium. Universities offer their students a given level of quality that needs to be self-fulfilled: the quality offered by universities will attract certain students to the market but the quality of students going to universities determines the quality offered by universities. It is not possible to use any fixed-point theorem to show existence of a fixed point in this quality self-fulfilling problem given that the necessary assumptions are not satisfied. In particular, note that the fixed-point quality problem is not continuous as whenever the low-quality university offers the same quality as the high one, all students who are beyond the ability threshold will go to the cheapest one, generating a massive exit from one university to the other one, generating a discontinuous jump in the quality being offered.



(a) Enrollment and % of students with financial aid. (b) Average income and % of students with financial aid.

Figure 4: Enrollment, income and financial aid.

In order to illustrate this point extensively, we show in appendix A.7 the failure to prove existence of the equilibrium in the case of a linear utility function.

4 The Case of Colombia

In the present research, we will use Colombia as a natural experiment of a country that implemented a rapid credit expansion program to alleviate credit constraints. Colombia is a developing country which by the beginning of last decade had low enrollment rates in post-secondary education, and significant differences in enrollment by quintiles of income. As will be argued, the majority of students came from high-income families, and the existence of financial constraints kept high-ability individuals from the lowest quintiles out of the education market. During the last decade, the government engaged into the strategy *Revolucion Educativa*, aimed at increasing the education coverage at all levels. During the decade, there were substantial increases in enrollment and educational credit access (see Figure 4).

4.1 Enrollment and inequality

At the beginning of last decade, college enrollment in Colombia was among the lowest in Latin America and a student financial aid system was almost non-existent. In 2000, 23.2% of the people between 18 and 23 years old enrolled in tertiary education, below the enrollment rates of Bolivia, Peru, Brazil, Chile and Venezuela, and very close to the enrollment rates of Mexico. Because of a lack of a well-functioning financial aid system, less than 5%[8] of the entering cohorts had any kind of public or private financial support (World Bank, 2003, 2012). By the end of the decade, the enrollment rates grew to 37%, and reached 50% in 2015. The fraction of students with some type of credit increased to almost 25% of the entering cohorts (see Figure 4(a)).

Access to education has always been unequal and, despite the fast growth of enrollment, many disparities

[8] Extracted from the dataset of indicators for tertiary education, SPADIES, from the Ministry of Education.

persist. In 2013, only 45% of the low-income students graduated from high school, and only 25% of them enrolled in tertiary education. Of the high-income households, 60% graduated from high school and 54% of them enrolled in a post-secondary institution (Melguizo et al., 2015). According to World Bank (2003, 2012), the enrollment gap between the lowest and the highest quintiles of wealth widened throughout the decade: in 2001, the enrollement rates were of 8% in the lowest quintile and 41% in the highest, while in 2010 these numbers grew to 10% and 52%, respectively. If quality is taken into account, disparities are even larger as a larger proportion of the low-income students attend non-professional institutions, which have less resources and offer lower expected income in the future. Many theories have been used to explain the low enrollment of low-income students, such as disparities in the quality of public and private high school education, the high costs of tertiary education and the lack of a well-functioning financial aid system (Melguizo et al., 2015).

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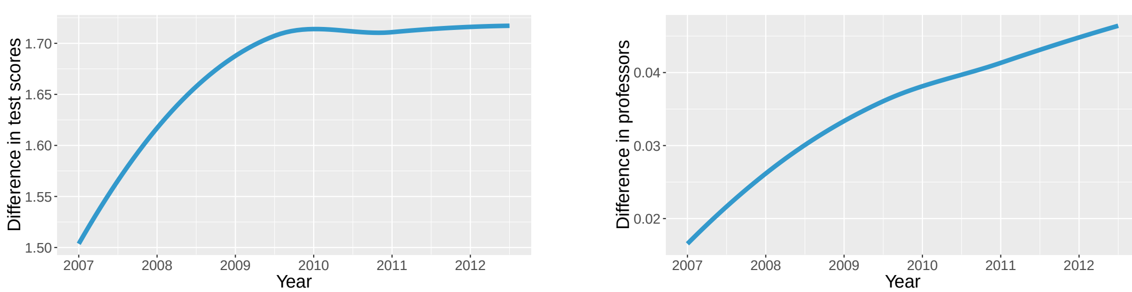
Figure 5 illustrates the difference between tiers of the average decile of test scores of the entering cohorts, and the difference in the number of professors per student. Clearly, there has been a widening gap in student abilities (as measured by test scores) and investments per student[9].



(a) Difference in average test scores of entering cohort.

(b) Difference in professors per student.

Figure 5: Differences in quality indicators between tier 1 and tier 2.

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4.2 Education institutions

The university system in Colombia functions as a monopolistically competitive market in which there are significant institutional barriers to entry, and universities do not have fixed "production capacities", as assumed by Chade et al. (2014) (Figure 6). There are approximately 300 tertiary education institutions, of which

[9]The widening gap can also be observed when looking at other proxies, such as the number of publications per student in peer-reviewed journals and qualifications of faculty. We do not include them, since our quality index will only be a composite of average student ability and investments per student.

around 190 are universities, and the rest offer non-professional degrees (mainly technical and technological). Despite the growing size of the entering cohorts throughout the decade, the number of institutions remained almost constant, while the average size of each institution doubled, on average. It is important to note that around $45-50\%$ of the total student body is enrolled in private tertiary education institutions (OECD and World Bank, 2012). Private institutions do not have any regulations regarding the price or investment per student they offer, although they have to satisfy a minimum quality requirement in terms of the programs and degrees offered. Therefore, the education market in Colombia can be studied as a monopolistically competitive market with barriers to entry and not subject to much government regulation.

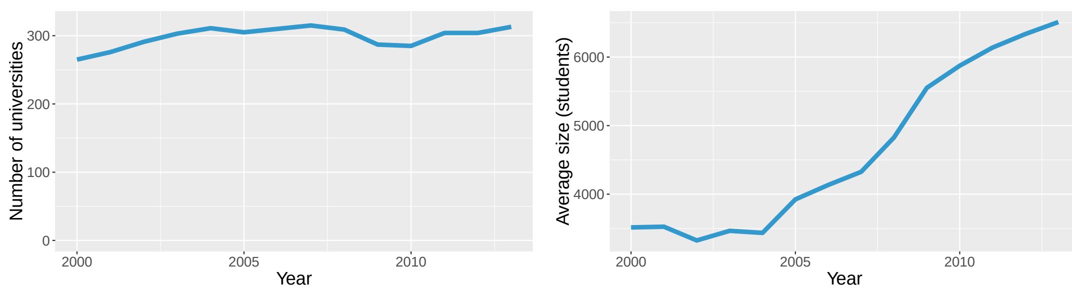


Figure 6: Left: number of universities. Right: average size of universities

Figure 7 illustrates the behavior of the average real price of tuition during the decade, in terms of 2004 pesos. As can be observed, there has been a steady increase in the real price of education throughout the decade for all universities in Colombia. Additionally, the price of the high-quality colleges seems to have peaked at a higher pace for Law, Engineering and Medicine schools. This increasing trend suggests that the Bennett Hypothesis might also be taking place in the Colombian context, given the fast increase in the government provided loans to education.

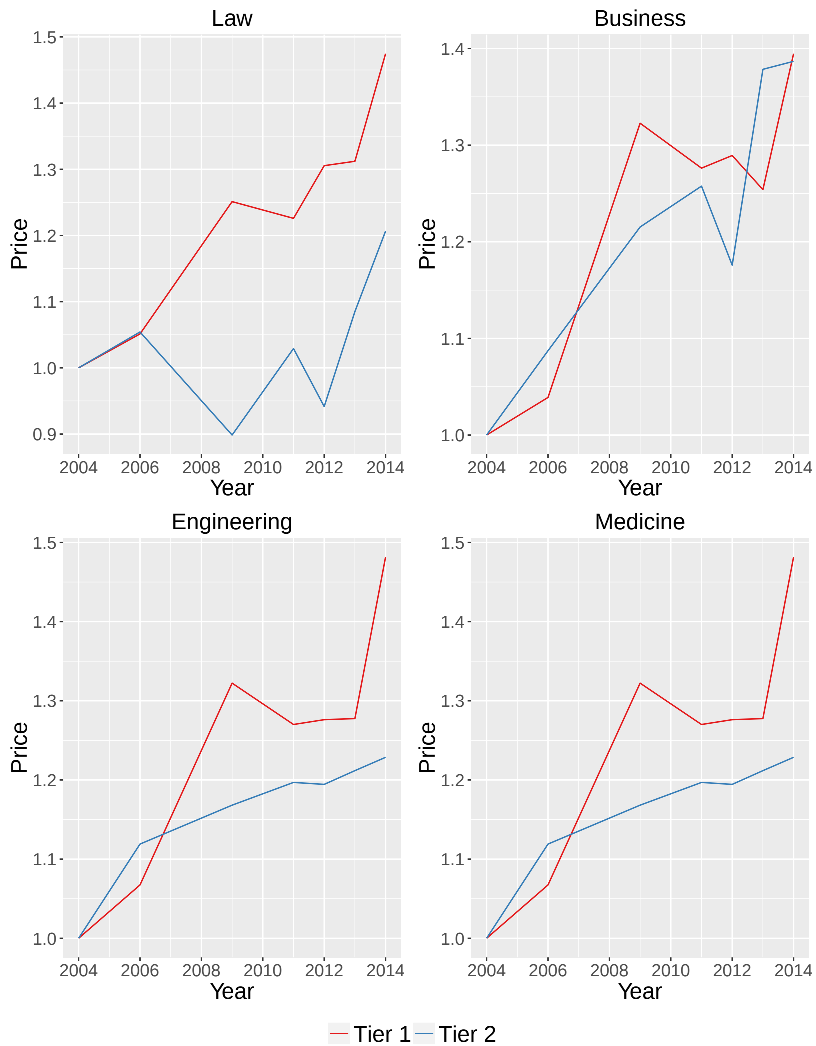


Figure 7: Evolution of average tuition prices over the decade

In Colombia, every student that wants to graduate from high school has to present an exam called SABER11 set by the Colombian Institution for Education Evaluation (ICFES), similar to the SAT test in the U.S. Although not every tertiary education institution takes into account the results of the SABER11 in their admission decision, 78% use it as a criterion for admission (OECD and World Bank, 2012). As SABER11 has no pass-mark, each institution sets its own minimum threshold for admission. In contrast to what happens in Chile and some European countries, in Colombia there is not any institution that clears the market for admissions, so individuals apply to as many institutions as they like and universities choose their admission standards independently (Melguizo et al., 2015). Although not perfect, the results in the SABER11 exam can be used as a proxy for the quality of the student body at universities. Figure 8 illustrates the average decile of the SABER11 scores of the entering cohorts to tertiary education institutions. Throughout the decade,

Figure 8: Average decile of ability of entering student body, measured by test scores

universities seem to have adjusted their admissions standards in such a way that led to a reduction in the ability of the student body, as measured by relative position in the test scores.

4.3 The ACCES Program

To alleviate the low access, in 2002 the government implemented the credit program *Access with Quality to Higher Education*, ACCES, with the support of the World Bank, that massively increased the available credit to students. The credit is awarded to students that have test scores above a threshold set by the government, and covers up to 75% of the tuition for the lowest income students, and up to 50% for the rest. The credit has a subsidized zero-real interest rate for the poorest households, and a real interest rate of 8% for the high-income students. Students that graduate from their programs have twice the time of their study period to repay the loan. The ACCES program has full coverage, in the sense that any student that has test scores on the highest deciles of their region can access this credit line. The test score cut-offs vary by region, to account for disparities in the quality of secondary education across regions with different infrastructure and economic development. Given that the credit is awarded according to regional cutoffs, the disparities in the ability of people accessing the credit are large. The best students from the poorest regions might not have high ability and preparation when compared to the best students of the principal cities, so the credit is not awarded to the highest ability individuals in absolute terms.

Using a regression discontinuity approach, Melguizo et al. (2015) find evidence that the ACCES program had a positive impact on the enrollment rates, especially for individuals that come from poor households. Although the growth in the number of students enrolled in college may have been a consequence of other factors, such as better economic activity, the massive increase in financing seems to be a driving factor of such a trend.

5 Quality Evolution Estimation

According to the specifications assumed in the model, we are able to identify the parameters of the wage equation. For this, we will use data on the average wages of graduates from each university in Colombia through 2007-2012, and the minimum wage, as a measure of w, to estimate the parameters of the quality production function of universities. Per-efficiency unit wages are given by:

$$w_h = w \cdot (1 + z_h), \quad w_l = w \cdot (1 + z_l)$$

Where w_h and w_l are the wages of high- and low-quality college graduates, given equilibrium qualities of education z_h and z_l, respectively, and w is the wage of non-skilled labor per-efficiency unit. The quality of education, z, is given by equation (11) in the universities' problem. We have a panel of data for 50 universities in Colombia from 2007 to 2012. We have the average ability of students in the entering cohorts, number of professors per student and average wages during the first year after graduation. For every university i in our sample, the following equation holds:

$$w_i = w \cdot (1 + \kappa \bar{\theta}_i^{\alpha_1} I_i^{\alpha_2})$$

Rearranging and taking logarithms:

$$\log\left(\frac{w_i}{w} - 1\right) = \log\kappa + \alpha_1 \bar{\theta}_i + \alpha_2 I_i$$

Assuming that there is measurement error in the wages of each of the universities, and assuming an exclusion restriction that the measurement error is uncorrelated with the explanatory variables, we can estimate the following equation:

$$\log\left(\frac{w_{i,t}}{w_t} - 1\right) = \log\kappa + \alpha_1 \bar{\theta}_{i,t} + \alpha_2 I_{i,t} + \eta T_{i,t} + \phi_t + \psi_i + \epsilon_{i,t}$$

where T_i is an indicator function that takes the value of one when the university i is a low-quality institution, and zero otherwise. Under this specification, we can estimate possible differences in the technology parameter, κ, between top and second tier schools. In order to isolate possible omitted variable bias, we

Parameter	OLS	OLS	OLS
$\hat{\alpha_1}$	0.211	0.228	0.168
	(0.026)	(0.026)	(0.026)
$\hat{\alpha_2}$	.358	0.478	0.414
	(0.361)	(0.357)	(0.403)
$\hat{\eta}$	-0.029	0.008	-0.046
	(0.047)	(0.043)	(0.046)
$\widehat{\log(\kappa)}$	-0.84	-0.957	-0.163
	(0.232)	(0.228)	(0.198)
Time fixed effects		Yes	Yes
City fixed effects			Yes
N	382	382	382
R-squared	0.353	0.444	0.567
Prob > F	0.000	0.000	0.000

Robust standard errors in parenthesis

$***p < 0.01, **p < 0.05, *p < 0.1$

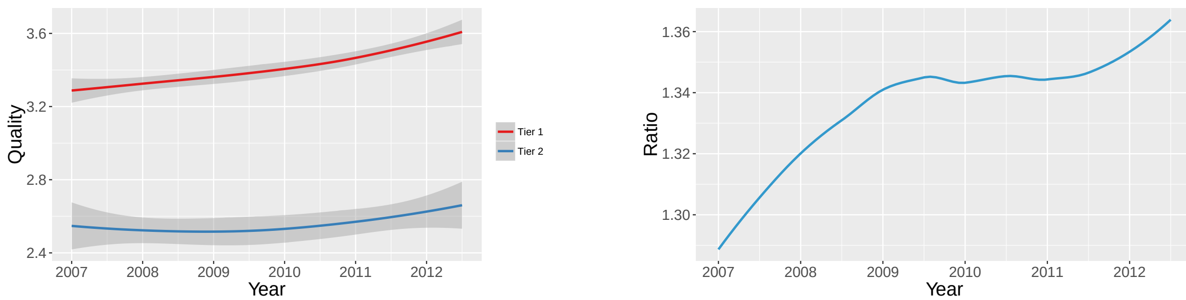
Table 1: Estimates for the quality production function.

estimate the above model under three different specifications, with and without time and geographic fixed effects, ϕ_t and ψ_i, respectively.

For the estimation, we constructed a panel of the top 50 universities in Colombia, according to a quality ranking published by the Ministry of Education in 2014[10]. This panel includes data on average wages during the first year after graduation for graduates of every school, as a measure of $w_{i,t}$, the average test scores for the entering cohorts, as a measure of $\theta_{i,t}$, and the number of professors per student, as a measure for $I_{i,t}$. We also have data on total operational expenditures by each school for 2014. However, with only one year we are not able to construct the evolution of quality of universities over time. Since the number of professors per student are a good indicator of the total expenditures per student, we will use that variable, instead. For the non-skilled labor wages, w_t, we will use the values of the real minimum wage (in 2007 pesos). The average wages of college graduates are strictly above the minimum wage during the period, so the dependent variable is well defined for every college in every period. In addition, we have information about the municipality of the school, to control for regional differences. The results of the estimation are displayed in Table 1.

The estimates show that the elasticities α_1 and α_2 are fairly robust to different specifications and do not change dramatically when including control variables. Moreover, the parameter η is negative in two of the specifications, although non statistically significative. This means that, on average, tier 2 universities have a lower technology parameter, κ, on their quality production function. This will be one of the main differences between tier 1 and tier 2 universities in our calibration of the model.

[10]The ranking is published in the website of the Ministry of Education, and can be found in the following link: http://www.mineducacion.gov.co/cvn/1665/w3-article-351855.html



(a) Estimated quality of tier 1 and tier 2 universities.

(b) Quality ratio of tier 1 versus tier 2 universities.

Figure 9: Quality supplied by colleges.

Figure 9 shows the estimated education qualities offered by both institutions, on average. Two things are worth noting about these results. First, the division of tier 1 and tier 2 universities seems to be robust, as shown by Panel A: the average quality offered by both tiers is significantly different to a 99% confidence level (see Table **??** in the Appendix). Second, during the period 2007-2012, tier 1 universities increased their quality at a roughly constant rate, while tier 2 universities only started increasing it by the end of the period. As Panel B shows, the gap in quality offered by both universities widened over the period.

6 Calibration of the Model

In order to draw conclusions about the relevance of our model, we calibrate the parameters to values that are relevant to the Colombian context. To achieve this, we will map a life-cycle model to a two period model, so the conclusions of Section 3 hold. We follow an approach similar to the one used by Lochner and Monge-Naranjo (2011), but in a discrete time economy. The environment is as follows.

Individuals live for T periods, after which they die with certainty. Individuals start their adult life at $t = 0$, when they must choose whether to attend the low- or high-quality school, or not study at all. Studying lasts for S periods, so those individuals that attend college will not receive any income during $t \in \{0, \dots, S-1\}$ and have to pay a per-period price of P_j for attending school j. Moreover, during the first S periods individuals are borrowing constrained. Those that decide not to study cannot borrow at all. Those that decide to study, can borrow up to the exogenous limit set by the government student loan policy, $\bar{A}$. After period $S-1$, the individual enters the labor market and earns a per-period wage $\theta w(1 + z_j)$, that depends on the quality of the school attended. During periods $S, \dots, T$ individuals only consume and save. We assume that from period S onwards, individuals enter into perfect financial markets where debt repayments are fully enforced. In this context, individuals can borrow any amount they want.

Clearly, individuals that are not borrowing constrained during their study period will perfectly smooth consumption along the life-cycle. However, those individuals that are constrained during the first S years of life will exhibit a jump in their consumption once they graduate from college. This setting can be easily embedded into the two-period model described in last section, by setting the discount factors and budget constraints appropriately. Namely, the problem for the household becomes:

$$\max_{c,c'} \frac{c^{1-\sigma}}{1-\sigma} + \tilde{\beta}\frac{(c')^{1-\sigma}}{1-\sigma}, \quad s.t.$$

$$c + c'\left(\frac{\Phi_S}{\Phi_0(1+r)^S}\right) + (P_H h + P_l l)\left(\frac{\Phi_r^y}{\Phi_0}\right) =$$
$$w\theta(1-h)(1-l)\left(\frac{\Phi_r^y}{\Phi_0}\right) + w\theta(1+z_j)\left(\frac{\Phi_r^0}{\Phi_0(1+r)^S}\right) j + \frac{b}{\Phi_0}$$
$$a \geq \bar{A}$$

The derivation of the parameters $\tilde{\beta}, \Phi_0, \Phi_S, \Phi_r^o, \Phi_r^y$ is explained in detail in the Appendix A.8. In this environment, all the results from Section 3 hold.

6.1 Parameterization

In our calibration, we set one period to be exactly one year. We will set some parameter values to match the Colombian educational market. All parameter values are reported in Table ??.

We set $S = 5$, so that the individuals that choose to attend a college study during 5 periods, since most professional degrees in Colombia take exactly 5 years. In Colombia, life expectancy at birth is 73.95 years of life[11]. Although the National Statistics Department of Colombia (DANE) does not publish the life expectancy by age, we estimate the life expectancy at 18 years to be 55 more years of life[12]. That is, we set $T = 55$ to match the life expectancy in Colombia for high-school graduates.

We set $\sigma = 2$, which a standard parameter in the literature (Fernandez-Villaverde and Krueger, 2011; Lochner and Monge-Naranjo, 2011). For the real interest rate, we choose $r = 8.9\%$, which is the value for Colombia in 2014 published by the World Bank[13]. We do not claim that this value is representative of developing countries, since the real interest rate for most Latin American countries has a huge variation, ranging from negative values in Argentina (-4.1%) and Venezuela (-14.5% in 2013), to very high values like

[11] See life expectancy tables here.

[12] For instance, in the U.S. life expectancy at 18 is only 0.79 more years than life expectancy at birth. Therefore, we will set life expectancy at 18 in Colombia to be 1.05 years above life expectancy at birth, as a conservative estimate.

[13] See the real interest rates for all the countries in this link.

Brazil (23.5%). We choose $\beta = 0.92$ such that $\beta = 1/(1+r)$. With these parameter values, the discount factor in our two-period model becomes $\tilde{\beta} = 1.89$. This reflects the fact that the post-college period is much longer than the study period, even though individuals discount time at a high rate.

As for the university parameters, we use the estimations of Section 5. In particular, we choose $\alpha_1 = 0.211$, $\alpha_2 = 0.358$, $\kappa_l = 0.8$ and $\kappa_h = 0.85$, obtained from the wage regressions displayed in Table 1.

Parameter	**Value**	**Source**
Utility and discount		
β	0.97	Literature
σ	2	Literature
r	2%	Colombia
w	2	Normalization
Time parameters		
T	78	Colombia
S	5	Colombia
University parameters		
α_1	0.211	Estimation
α_2	0.358	Estimation
κ_l	1.4	Estimation
κ_h	1.2	Estimation
$E^h - C^h$	-12	Estimation
$E^l - C^l$	-7	Estimation

Table 2: Parameter values

7 Results

In this section we show the results of the numerical computation of the equilibria without the subsidized-loan policy being implemented and once it was implemented. In order to mimic as closely as possible the post-reform equilibrium, we set up a tax rate of 10% used to fund a subsidized loan policy offering credits for higher education for people whose income is below the median income in the economy. The policy implemented in Colombia is designed as a subsidy to the interest rate paid by students. In the model we set up the subsidy in such a way that students that have access to it only have to pay 50% of the interests accumulated in students debts. In addition to having an income below the median, a student who wants to qualify for the policy must have an ability level in the top 30%[14]. Table 3 illustrates the results before and after the implementation of the student loan policies. As can be observed, after the reform there is a widening gap in the quality offered by each university. Elite universities offer a higher quality, while non-elite universities reduce it. There is

[14]The institutional details of the policy implemented in Colombia are fully described in Melguizo et al. (2015)

also a market segmentation, where better students attend the elite institution, and the ability of the students attending the low-quality institutions falls.

Table 3: Equilibria computations

		Pre-reform	Post-reform
Elite institutions	Students attending	5,863	9,431
	Average ability of student body	0.48	0.64
	Quality offered	1.01	1.19
Non-elite institutions	Students attending	6,971	6,753
	Average ability of student body	0.41	0.38
	Quality offered	0.53	0.42

8 Conclusion

Subsidized loan policies have been used widely in both developing and developed economies, as a policy tool to increase college attendance. Such policies are particularly relevant in a context where credit constraints explain a significant amount in the college non-attendance rate. However, when implementing such policies, it is important to observe not only the distortion it imposes in the demand side of the market but also the way it affects the incentives of the providers of higher education.

We show that subsidized loan policies can distort the incentives of colleges providing services of higher education in a way that can be harmful for a group of households in an economy. Taking into account that the market for higher education operates under a monopolistic competition setting, granting subsidized loans does not translate into an expansion of the providers of higher education services but rather on the same colleges facing a new set of incentives. As elite institutions unambiguously observe an increase in their demand, they can use their pricing and admission policies to be more selective in their admission process and to spend more per student, which translates into providing better services for their student body. On the contrary, the universities in the low-quality tier will observe a migration to the high-quality group when such policies are implemented. The result is a new equilibrium in the market for higher education where the quality gap between elite and non-elite institutions is widened as a result of the implementation of subsidized loan policies.

Our model is consistent with what we observe in the market for higher education in Colombia: an expansion of the gap in the quality offered by different institutions. In such scenario, subsidized policy loans can make some households worse off as, although the attendance to higher education institutions becomes easier, the gains from attending low-quality universities is not offset by the amount households have to pay in taxes in order to pay for the policy implemented.

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A Appendix A

The problem of the households is:

$$\max_{c,l,h,a} \frac{c^{1-\sigma}}{1-\sigma} + \beta \frac{c'^{1-\sigma}}{1-\sigma}, \quad s.t.$$

$$a + c + hP_h + lP_l = w\theta(1-h)(1-l) + b$$

$$c' = w\theta + w\theta z_h h + w\theta z_l l + (1+r)a$$

A.1 Solution of the unconstrained households:

Proof of Theorem 1. The unconstrained consumptions are:

$$c^N = \frac{(\beta(1+r))^{-1/\sigma}(w\theta(2+r)+(1+r)b)}{1+(\beta(1+r))^{-1/\sigma}(1+r)},$$

$$c'^N = \frac{(w\theta(2+r)+(1+r)b)}{1+(\beta(1+r))^{-1/\sigma}(1+r)}$$

$$c^l = \frac{(\beta(1+r))^{-1/\sigma}(w\theta(1+z_l)+(1+r)b-P_l(1+r))}{1+(\beta(1+r))^{-1/\sigma}(1+r)},$$

$$c'^l = \frac{(w\theta(1+z_l)+(1+r)b-P_l(1+r))}{1+(\beta(1+r))^{-1/\sigma}(1+r)}$$

$$c^h = \frac{(\beta(1+r))^{-1/\sigma}(w\theta(1+z_h)+(1+r)b-P_h(1+r))}{1+(\beta(1+r))^{-1/\sigma}(1+r)},$$

$$c'^h = \frac{(w\theta(1+z_h)+(1+r)b-P_h(1+r))}{1+(\beta(1+r))^{-1/\sigma}(1+r)}$$

The utilities of each of the options are:

$$u^N = \Phi \times (w\theta(2+r)+b(1+r))^{1-\sigma}$$

$$u^l = \Phi \times (w\theta(1+z_l)+b(1+r)-P_l(1+r))^{1-\sigma}$$

$$u^h = \Phi \times (w\theta(1+z_h)+b(1+r)-P_h(1+r))^{1-\sigma}$$

where

$$\Phi = \left(\frac{1}{1-\sigma}\right)\left(\frac{1}{1+(\beta(1+r))^{-1/\sigma}(1+r)}\right)^{1-\sigma}\left((\beta(1+r))^{(\sigma-1)/\sigma}+\beta\right)$$

The household's decision of whether and where to study follows a *cut-off* rule on θ, and the decision is independent of initial wealth, b. The cut-offs are:

$$\bar{\theta}_l = \frac{1+r}{w}\left(\frac{P_l}{z_l-(1+r)}\right), \quad \bar{\theta}_h = \frac{1+r}{w}\left(\frac{P_h-P_l}{z_h-z_l}\right)$$

□

A.2 Wealth cutoff rules for households:

Proof of Theorem 2. The debt levels of the unconstrained households are:

$$a^N = \frac{w\theta(1-(\beta(1+r))^{-1/\sigma})+b}{1+(\beta(1+r))^{-1/\sigma}(1+r)}$$

$$a^l = \frac{b-P_l-(\beta(1+r))^{-1/\sigma}w\theta(1+z_l)}{1+(\beta(1+r))^{-1/\sigma}(1+r)}$$

$$a^h = \frac{b-P_h-(\beta(1+r))^{-1/\sigma}w\theta(1+z_h)}{1+(\beta(1+r))^{-1/\sigma}(1+r)}$$

Given the exogenous borrowing constraint $\bar{A}$, for a given θ we can construct a cut-off $\bar{b}(\theta)$ on the initial wealth such that individuals with $b < \bar{b}(\theta)$ are constrained and $b \geq \bar{b}(\theta)$ are unconstrained. These are given by:

$$a^N \geq \bar{A} \quad \Leftrightarrow \quad b \geq -\bar{A}(1+(\beta(1+r))^{-1/\sigma}(1+r)) - w\theta(1-(\beta(1+r))^{-1/\sigma})$$

$$a^l \geq \bar{A} \quad \Leftrightarrow \quad b \geq P_l + (\beta(1+r))^{-1/\sigma}w\theta(1+z_l) - \bar{A}(1+(\beta(1+r))^{-1/\sigma}(1+r))$$

$$a^h \geq \bar{A} \quad \Leftrightarrow \quad b \geq P_h + (\beta(1+r))^{-1/\sigma}w\theta(1+z_h) - \bar{A}(1+(\beta(1+r))^{-1/\sigma}(1+r))$$

That is, the cut-offs are:

$$\bar{b}_u^N(\theta) = -\bar{A}(1+(\beta(1+r))^{-1/\sigma}(1+r)) - w\theta(1-(\beta(1+r))^{-1/\sigma})$$

$$\bar{b}_u^l(\theta) = P_l + (\beta(1+r))^{-1/\sigma}w\theta(1+z_l) - \bar{A}(1+(\beta(1+r))^{-1/\sigma}(1+r))$$

$$\bar{b}_u^h(\theta) = P_h + (\beta(1+r))^{-1/\sigma}w\theta(1+z_h) - \bar{A}(1+(\beta(1+r))^{-1/\sigma}(1+r))$$

□

This subdivides the state space in three subregions, as shown in the following Figure 2.

A.3 Solution of the constrained households:

Next, we have to consider the decision of studying of those households that are constrained. Note that, although if an individual is borrowing constrained when he decides to study, he might prefer to study and not smooth consumption, than not studying and being able to smooth consumption. Therefore, we must compare the utility of studying while being constrained, with the utility of not studying and being unconstrained. The constrained consumptions are given by:

$$c_c^N = w\theta + b + \bar{A}, \quad c_c^{\prime N} = w\theta - (1+r)\bar{A}$$

$$c_c^l = b - P_l + \bar{A}, \quad c_c^{\prime l} = w\theta(1+z_l) - (1+r)\bar{A}$$

$$c_c^h = b - P_h + \bar{A}, \quad c_c^{\prime h} = w\theta(1+z_h) - (1+r)\bar{A}$$

There are three decisions to characterize:

1. Whether to study in l or not study, for individuals that are constrained when studying in l. These individuals will study in l whenever:

$$\left(\frac{1}{1-\sigma}\right)(b - P_l + \bar{A})^{1-\sigma} + \left(\frac{\beta}{1-\sigma}\right)(w\theta(1+z_l) - (1+r)\bar{A})^{1-\sigma} -$$
$$\Phi \times (w\theta(2+r) + b(1+r))^{1-\sigma} \geq 0$$

2. Whether to study in l or in h, for individuals that are constrained when studying in h but not constrained when studying in l. These individuals will study in h whenever:

$$\left(\frac{1}{1-\sigma}\right)\left(b - P_h + \bar{A}\right)^{1-\sigma} + \left(\frac{\beta}{1-\sigma}\right)\left(w\theta(1+z_h) - (1+r)\bar{A}\right)^{1-\sigma} -$$
$$\Phi \times (w\theta(1+z_l) + b(1+r) - P_l(1+r))^{1-\sigma} \geq 0$$

3. Whether to study in l or in h, for individuals that are constrained when they decide to study in h or l. These individuals will study in h whenever:

$$\left(\frac{1}{1-\sigma}\right)\left(b - P_h + \bar{A}\right)^{1-\sigma} + \left(\frac{\beta}{1-\sigma}\right)\left(w\theta(1+z_h) - (1+r)\bar{A}\right)^{1-\sigma}$$
$$\left(\frac{1}{1-\sigma}\right)\left(b - P_l + \bar{A}\right)^{1-\sigma} - \left(\frac{\beta}{1-\sigma}\right)\left(w\theta(1+z_l) - (1+r)\bar{A}\right)^{1-\sigma} \geq 0$$

The cut-offs that define the college decision for constrained individuals are defined in the following theorem proofs:

Proof of Theorem 3. Define the following function:

$$G(\theta, b) = \left(\frac{1}{1-\sigma}\right)(b - P_l + \bar{A})^{1-\sigma} + \left(\frac{\beta}{1-\sigma}\right)(w\theta(1+z_l) - (1+r)\bar{A})^{1-\sigma} -$$
$$\Phi \times (w\theta^*(2+r) + b(1+r))^{1-\sigma} \tag{16}$$

Let the function $\bar{b}_c^l(\theta)$ be implictly defined by the equality $G(\theta, \bar{b}_c^l(\theta)) = 0$. By the implicit function theorem,

$$\frac{\partial \bar{b}_c^l(\theta)}{\partial \theta} = -\frac{\partial G/\partial \theta}{\partial G/\partial b}$$

Setting $\partial G/\partial \theta = 0$ gives the result in Theorem 3. □

A.4 Proof of Theorem 4

Proof of Theorem 4. The proof is similar to Proof A.3. Define:

$$G^*(\theta, b) = \left(\frac{1}{1-\sigma}\right)(b - P_h + \bar{A})^{1-\sigma} + \left(\frac{\beta}{1-\sigma}\right)(w\theta(1+z_h) - (1+r)\bar{A})^{1-\sigma} - \\ \Phi(w\theta^*(1+z_l) + b(1+r) - P_l(1+r))^{1-\sigma} \quad (17)$$

and setting $\partial G/\partial \theta = 0$ gives the result in Theorem 4.

□

A.5 Proof of Theorem 5

Proof of Theorem 5. By implicit function theorem, $\partial b/\partial \bar{A} = -\frac{\partial G/\partial \bar{A}}{\partial G/\partial b}$.

$$\frac{\partial G}{\partial \bar{A}} = (b - P_l + \bar{A})^{-\sigma} + \beta(1+r)(w\theta(1+z_l) - (1+r)\bar{A})^{-\sigma} \geq 0$$

Since $\partial G/\partial b > 0$, the first result follows.

For the second result, note that:

$$\begin{aligned}
\frac{\partial b}{\partial \bar{A}\partial \theta} = {} & \frac{1}{(\cdot)^2}\Big[\Big(\sigma\beta(1+r)w(1+z_l)(w\theta(1+z_l) - (1+r)\bar{A})^{-(1+\sigma)}\Big) \\
& \cdot \big((1+r)((b - P_l + \bar{A})^{-\sigma} - \Phi(1-\sigma)(1+r)(w\theta(2+r) + b(1+r))^{-\sigma})\big)\Big] \\
& + \frac{1}{(\cdot)^2}\big[\big((b - P_l + \bar{A})^{-\sigma} + \beta(1+r)(w\theta(1+z_l) - \bar{A}(1+r))^{-\sigma}\big) \\
& \cdot \big(\sigma\Phi(1-\sigma)w(1+r)(2+r)(w\theta(2+r) + b(1+r))^{-\sigma}\big)\big] \\
\geq {} & 0
\end{aligned} \quad (18)$$

This proves Theorem 5.

□

A.6 Computation of Nash Equillibrium

In this section we will describe the algorithm used to compute the Nash Equilibrium between elite and non-elite universities. The Nash Equilibrium is composed by a tuple $(P_h^*, \underline{\theta_h}^*, P_l^*, \underline{\theta_l}^*)$ such that:

$$(P_i^*, \underline{\theta_i}^*) \in \arg \max_{(P_i,\underline{\theta_i}) \in \mathcal{R}^+ \times [0,1]} \left(z_i(P_i, \underline{\theta_i}, P_{-i}^*, \underline{\theta_{-i}}^*)\right)^{\alpha} \left(\sigma_{b,i}(P_i, \underline{\theta_i}, P_{-i}^*, \underline{\theta_{-i}}^*)\right)^{1-\alpha} \tag{19}$$

Note that the problem defined in 19 involves solving for a fixed point nested within another fixed point problem. In particular, the universities will offer a given level of z_l, z_h to the households and, conditional on such offer households will demand education services that need to fulfill the promised levels of z_l, z_h. This implies that when solving for the optimal of the universities we need to take into account that the offered level of productivities need to be satisfied by the demand of educational services. The full procedure to find the Equilibrium is described below:

Computation of the Nash Equilibrium

1. Start algorithm with some initial guess $\langle P_h^g, \underline{\theta_h^g}, P_l^g, \underline{\theta_l^g} \rangle$. Set $E = 10$.
2. Find $\langle P_h^T, \underline{\theta_h^T} \rangle \in \arg\max_{(P_h,\underline{\theta_h}) \in \mathcal{R}^+ \times [0,1]} \left(z_h(P_h, \underline{\theta_h}, P_l^g, \underline{\theta_l}^g)\right)^{\alpha} \left(\sigma_{b,h}(P_h, \underline{\theta_h}, P_l^g, \underline{\theta_l}^g)\right)^{1-\alpha}$
   (a) Set $\langle P_h^r, \underline{\theta_h^r} \rangle = \langle P_h^g, \underline{\theta_h^g} \rangle$
   (b) Given $\langle P_h^r, \underline{\theta_h^r}, P_l^g, \underline{\theta_l^g} \rangle$, go to 5. to compute $\langle z_h, z_l \rangle$
   (c) Given $S1 = \langle P_h^r, \underline{\theta_h^r}, P_l^g, \underline{\theta_l^g}, z_h, z_l \rangle$ compute the objective function of the university $H(S1)$.
   (d) Update for a new guess of the optimal $\langle P_h^r, \underline{\theta_h^r} \rangle = \langle P_h^{new}, \underline{\theta_h^{new}} \rangle$ according to some rule.
   (e) Repeat $(b) - (d)$ until optimal $\langle P_h^T, \underline{\theta_h^T} \rangle$ is found
3. Find $\langle P_l^T, \underline{\theta_l^T} \rangle \in \arg\max_{(P_l,\underline{\theta_l}) \in \mathcal{R}^+ \times [0,1]} \left(z_l(P_h^g, \underline{\theta_h^g}, P_l, \underline{\theta_l})\right)^{\alpha} \left(\sigma_{b,l}(P_h^g, \underline{\theta_h^g}, P_l, \underline{\theta_l})\right)^{1-\alpha}$
   (a) Set $\langle P_l^r, \underline{\theta_h^l} \rangle = \langle P_l^g, \underline{\theta_l^g} \rangle$
   (b) Given $\langle P_h^g, \underline{\theta_h^g}, P_l^r, \underline{\theta_l^r} \rangle$, go to 5. to compute $\langle z_h, z_l \rangle$
   (c) Given $S1 = \langle P_h^g, \underline{\theta_h^g}, P_l^r, \underline{\theta_l^r}, z_h, z_l \rangle$ compute the objective function of the university $L(S1)$.
   (d) Update for a new guess of the optimal $\langle P_l^r, \underline{\theta_l^r} \rangle = \langle P_l^{new}, \underline{\theta_l^{new}} \rangle$
   (e) Repeat $(b) - (d)$ until optimal $\langle P_l^T, \underline{\theta_l^T} \rangle$ is found
4. Set $E = ||\langle P_h^g, \underline{\theta_h^g}, P_l^g, \underline{\theta_l^g} \rangle - \langle P_h^T, \underline{\theta_h^T}, P_l^T, \underline{\theta_l^T} \rangle||$. If E is smaller than a tolerance level, stop the algorithm, the NE is given by the tuple $\langle P_h^T, \underline{\theta_h^T}, P_l^T, \underline{\theta_l^T} \rangle$. Otherwise, set $\langle P_h^g, \underline{\theta_h^g}, P_l^g, \underline{\theta_l^g} \rangle = \langle P_h^T, \underline{\theta_h^T}, P_l^T, \underline{\theta_l^T} \rangle$ and go to 2.
5. Computation of $\langle z_h, z_l \rangle$ given $\langle P_h, \underline{\theta_h}, P_l, \underline{\theta_l} \rangle$
   (a) Start algorithm with some initial guess $\langle z_h^g, z_l^g \rangle$ and set $\varepsilon = 10$
   (b) Given $\langle P_h, \underline{\theta_h}, P_l, \underline{\theta_l} \rangle$, the guess $\langle z_h^g, z_l^g \rangle$ and the policy functions of the households, compute the realized values of $\langle z_h^r, z_l^r \rangle$
   (c) set $\varepsilon = (z_h^r - z_h^g)^2 + (z_l^r - z_l^g)^2$.
   (d) If ε is smaller to a tolerance level, the algorithm is complete. Otherwise, set $\langle z_h^g, z_l^g \rangle = \langle z_h^r, z_l^r \rangle$ and go to (b).

A.7 Analysis in the linear case

In order to get a clear idea of how credit constraints affect the market for higher education, we illustrate the linear case where $\sigma = 1$. Furthermore, we need to distinguish scenarios where households would like to substitute future for current consumption and the other way around. This is given by the inequality $\beta(1+r) < 1$. Whenever this inequality is satisfied, households would prefer to get as much debt during the first period. The opposite case, when $\beta(1+r) \geq 1$ will motivate households to save as much as possible given that the returns to savings, in terms of utility, are more than one to one.

Case 1.$\beta(1+r) \geq 1$

In this case, households will prefer to save as much as they want and then the value functions for each case (not study, study in low quality university or study in high quality university) are given by:

$$V^N(b,\theta) = \beta\left[b(1-\tau)(1+r) + w\theta(2+r)\right] \tag{20}$$

$$\tag{21}$$

The value function for households going to the low quality university is only defined whenever they can afford it. That is, whenever $P_l - b(1-\tau) \leq \min\{\bar{A}, \frac{w\theta(1+z^l)}{1+r}\}$. In particular, consider the case where $P_l - b(1-\tau) \leq 0$. If this holds, then households are able to afford the price of education with their income after taxes and thus we have no concerns about they not getting enough debt to fund their education.

However, when students should get positive debt in order to attend the low quality university, the amount of debt should satisfy two constraints:

$$P_l - b(1-\tau) \leq \bar{A} \tag{22}$$

$$P_l - b(1-\tau) \leq \frac{w\theta(1+z^l)}{1+r} \tag{23}$$

The constraint given in 22 states that the amount of debt students get should not exceed the upper limit given exogenously in the economy. The inequality given in 23 guarantees that students have enough funds to get the necessary debt to attend college. The two aforementioned inequalities give bounds in b and θ for students to being able to pay the tuition in the low quality college:

$$b \geq b_{p_l} = \frac{\bar{A} - P_l}{1-\tau} \tag{24}$$

$$b \geq L(\theta) = \frac{P_l}{1-\tau} - \frac{w\theta(1+z^l)}{(1-\tau)(1+r)} \tag{25}$$

Now, for households with state variables (b,θ) such that low quality education is affordable, we can define the value of going to the low university as:

$$V^L(b,\theta) = \beta\left[(b(1-\tau) - Pl)(1+r) + w\theta(1+z^l)\right] \tag{26}$$

Similarly, in order to be able to go to the high quality institutions, it should be the case that:

$$b \geq b_{p_h} = \frac{\bar{A} - P_h}{1-\tau} \tag{27}$$

$$b \geq H(\theta) = \frac{P_h}{1-\tau} - \frac{w\theta(1+z^h)}{(1-\tau)(1+r)} \tag{28}$$

For those households, we can define the value of going to the high quality college as:

$$V^H(b,\theta) = \beta\left[(b(1-\tau) - P_h)(1+r) + w\theta(1+z^h)\right] \tag{29}$$

Consider the case of a person who is deciding whether to go to the low quality college or not study. In such case, granted that he could afford to pay tuition, he will decide to attend whenever $V^L(b,\theta) \geq V^N(b,\theta)$. This implies that the decision will be to go to the low quality college whenever:

$$\theta_l \geq \theta_L = \frac{P_l(1+r)}{w[z^l - r - 1]} \tag{30}$$

Similarly, when a person is deciding whether to go to the high quality college or to the low quality one, and granted he could afford both, the relevant decision rule will be to go to the high quality college whenever $V^H(b,\theta) \geq V^L(b,\theta)$. This inequality generates the decision rule of going to college whenever:

$$\theta \geq \theta_H = \frac{(P_h - P_l)(1+r)}{w(z^h - z^l)} \tag{31}$$

The decision rules can be represented in the state space according to the following graph:

Note that we can express N^H in terms of elements that we have found previously:

$$N^H = \int_{\theta^H}^{\theta^{Ih}} \int_{H(\theta)}^{\bar{b}} dF(b,\theta) + \int_{\theta^{Ih}}^{1} \int_{b_{Ph}}^{\bar{b}} dF(b,\theta) \tag{32}$$

where $\bar{b}$ is the maximum level of bequests in the state space and

$$\theta^{Ih} = \frac{(1+r)\bar{A}}{(1+z^h)w} \tag{33}$$

For the sake of simplicity, we will assume a uniform distribution for (b,θ). As long as $P^h > P^l$ and $z^h > z^l$ we can express the measure of people going to the high quality university as:

$$N^H = \frac{1}{\bar{b}}\left[\left(b - \frac{P_h}{1-\tau}\right)\left(\frac{(1+r)\bar{A}}{(1+z^h)w} - \frac{(P_h - P_l)(1+r)}{w(z^h - z^l)}\right)\right. \tag{34}$$

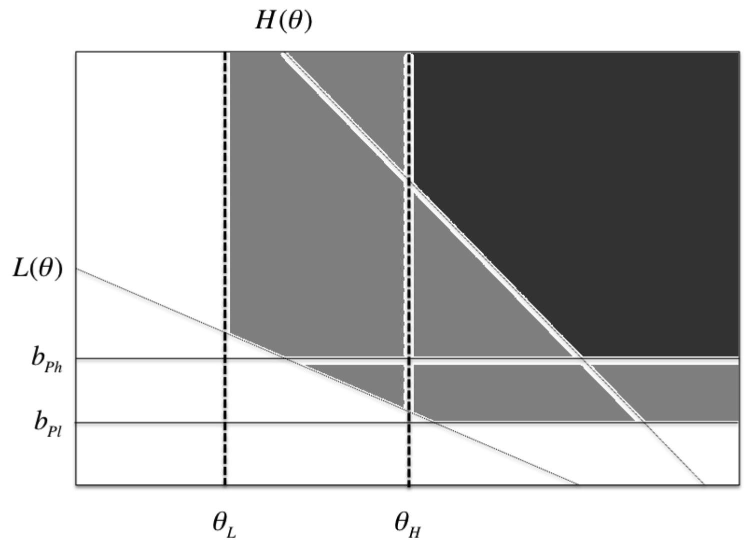

Figure 10: Representation of the education decisions on the state space.

$$+\frac{w(1+z^h)}{(1-\tau)(1+r)}\left[\left(\frac{(1+r)\bar{A}}{w(1+z^h)}\right)^2-\left(\frac{(P_h-P_l)(1+r)}{w(z^h-z^l)}\right)^2\right]\Bigg]$$

Similarly, the average level of skills of people attending such college is given by:

$$\tilde{\theta}^H = \frac{1}{\bar{b}}\Bigg[\left(\left(\frac{(1+r)\bar{A}}{(1+z^h)w}\right)^2-\left(\frac{(P_h-P_l)(1+r)}{w(z^h-z^l)}\right)^2\right)\left(\frac{\bar{b}}{2}-\frac{P_h}{2(1-\tau)}\right)+$$

$$\frac{w(1+z^h)}{3(1-\tau)(1+r)}\left[\left(\frac{(1+r)\bar{A}}{w(1+z^h)}\right)^3-\left(\frac{(P_h-P_l)(1+r)}{w(z^h-z^l)}\right)^3\right]+$$

$$\frac{1}{2}\left[\bar{b}-\frac{\bar{A}}{1-\tau}+\frac{P_h}{1-\tau}\left(1-\left(\frac{(1+r)\bar{A}}{1(1+z^h)}\right)^2\right)\right]\Bigg] \tag{35}$$

We can express the relevant variables for low quality college, granted $P_h > P_l$ and $z_h > z_l$, as:

$$N^L = \int_{\theta_L}^{\theta_H}\int_{L(\theta)}^{1} dF(b,\theta) + \int_{\theta_H}^{\theta^{Il}}\int_{L(\theta)}^{H(\theta)} dF(b,\theta)+$$

$$\int_{\theta^{Il}}^{\theta^{Ih}}\int_{b_{Pl}}^{H(\theta)} dF(b,\theta) + \int_{\theta^{Ih}}^{1}\int_{b_{Pl}}^{b_{Ph}} dF(b,\theta) \tag{36}$$

$$\tilde{\theta}^L = \int_{\theta_L}^{\theta_H} \int_{L(\theta)}^{1} \theta dF(b,\theta) + \int_{\theta_H}^{\theta^{Il}} \int_{L(\theta)}^{H(\theta)} \theta dF(b,\theta) +$$

$$\int_{\theta^{Il}}^{\theta^{Ih}} \int_{b_{Pl}}^{H(\theta)} \theta dF(b,\theta) + \int_{\theta^{Ih}}^{1} \int_{b_{Pl}}^{b_{Ph}} \theta dF(b,\theta) \tag{37}$$

$$\mu_{b^L} = \int_{\theta_L}^{\theta_H} \int_{L(\theta)}^{1} b dF(b,\theta) + \int_{\theta_H}^{\theta^{Il}} \int_{L(\theta)}^{H(\theta)} b dF(b,\theta) +$$

$$\int_{\theta^{Il}}^{\theta^{Ih}} \int_{b_{Pl}}^{H(\theta)} b dF(b,\theta) + \int_{\theta^{Ih}}^{1} \int_{b_{Pl}}^{b_{Ph}} b dF(b,\theta) \tag{38}$$

It is important to note that throughout this analysis we have not implemented the fact that both colleges are able to set a threshold rule such that people with a level of skills below such threshold will not be admitted. In such a case, we will simply modify the regions of integration to consider that only people with ability beyond the threshold will be able to attend.

Existence of equilibrium

The expressions found in 34, 35, 36 and 37 can be used to express the necessary conditions that the offered qualities need to satisfy in equilibrium. In particular, we need to find z^h, z^l such that:

$$\begin{bmatrix} z^h \\ z^l \end{bmatrix} = \begin{bmatrix} \kappa^h \left(\tilde{\theta}^h(\underline{\theta^h}, \underline{\theta^l}, P_h, P_l, z^h, z^l)\right)^{\alpha_1} \left(I(\underline{\theta^h}, \underline{\theta^l}, P_h, P_l, z^h, z^l)\right)^{\alpha_2} \\ \kappa^l \left(\tilde{\theta}^h(\underline{\theta^l}, \underline{\theta^l}, P_h, P_l, z^h, z^l)\right)^{\alpha_1} \left(I(\underline{\theta^h}, \underline{\theta^l}, P_h, P_l, z^h, z^l)\right)^{\alpha_2} \end{bmatrix} \tag{39}$$

We need to prove existence of a fixed point in the qualities offered by universities before proving the existence of the Nash Equilibrium. Note, however, that difficulty arises in this point given the fact that there is no natural way to bound the set of qualities offered by the universities. Additionally, note that equations 34, 35, 36 are not continuous in $z^h = z^l$. The inability of proving the existence of a fixed point in the qualities offered by universities shows that it is not possible to prove existence of the Nash Equilibrium. We rely purely on the computational analysis to find a Nash Equilibrium in this case that might not be unique.
Case 2.$\beta(1+r) < 1$

This case is more involved as households value more current consumption than future and will try to get as much debt as possible. The difficulty arises as even when students can afford to pay college, they might be constrained given that they want to substitute future by current consumption. Additionally, we need to establish which is the relevant constraint that households face when getting the desired level of debt, either the exogenously given level of credit constraint or they reach a point where they can't fund the debt with their resources.

We start analyzing the case of a person who is not going to university. In this case, the person will get as much debt as possible and he will be constrained whenever $\frac{w\theta}{1+r} > \bar{A}$. If this is the case, the person will get the maximum level of debt $\bar{A}$. Taking into account this case when computing the value of not going to college, we see that:

$$V^N(b,\theta) = \begin{cases} b(1-\tau) + w\theta\frac{2+r}{1+r} \text{ if } \theta \leq \frac{\bar{A}(1+r)}{w} \\ b(1-\tau) + w(\theta)(1+\beta) + \bar{A}[1-\beta(1+r)] \text{ if } \theta > \bar{A}\frac{1+r}{w} \end{cases} \tag{40}$$

Now, let's consider a household that goes to the low-quality university. Evidently, the value function will only be defined for the case when it is possible to pay tuition price via endowment or debt. For people whose income is below the tuition price ($b(1-\tau) < P_l$) and who are constrained either by the exogenous level $\bar{A}$ or by their earning capacity $\frac{w\theta(1+z^l)}{1+r}$, the value of going to the low quality college will not be defined.

An individual who is not constrained and takes as much debt as he can, will derive utility given by $b(1-\tau) - P_l + \frac{w\theta(1+z^l)}{1+r}$. The first term, $b(1-\tau) - P_l$ corresponds to net income after tuition and the remaining part $\frac{w\theta(1+z^l)}{1+r}$ is simply the amount they will make in the second period taken to the present value of the first period.

If the net income after tuition is negative, an individual will not be credit constrained so long as:

$$P_l - b(1-\tau) \leq \min\{\bar{A}, \frac{w\theta(1+z^l)}{1+r}\} \tag{41}$$

However, it is possible to have individuals who are borrowing constrained even if the net income after tuition is positive. These individuals are those who would like to borrow against their future income, given that current consumption is more valuable than future consumption, but they are not able to borrow as much as they want given the exogenous limit $\bar{A}$. Those are individuals such that:

$$\frac{w\theta(1+z^l)}{(1+r)} < \bar{A} \tag{42}$$

and they are forced to borrow no more than $\bar{A}$. This implies that we can define the value of going to low-quality college as:

$$V^L(b,\theta) = \begin{cases} b(1-\tau) - P_l + \frac{w\theta(1+z^l)}{1+r} \text{ if } \begin{cases} b(1-\tau) - P_l \geq 0 \quad \theta \leq \frac{\bar{A}(1+r)}{w(1+z^l)} \\ \text{or} \\ b(1-\tau) - P_l < 0 \quad P_l - b(1-\tau) \leq \min\{\bar{A}, \frac{w(\theta)(1+z^l)}{1+r}\} \end{cases} \\ \\ b(1-\tau) - P_l + \bar{A}[1-\beta(1+r)] + w\beta(1+z^l) \text{ if } b(1-\tau) - P_l > 0 \text{ and } \theta > \frac{\bar{A}(1+r)}{w(1+z^l)} \end{cases} \tag{43}$$

Finally, doing the same analysis but with P_h and z^h we can find the value of going to the high quality college:

$$V^H(b,\theta) = \begin{cases} b(1-\tau) - P_h + \frac{w\theta(1+z^h)}{1+r} \text{ if } \begin{cases} b(1-\tau) - P_h \geq 0 \quad \theta \leq \frac{\bar{A}(1+r)}{w(1+z^h)} \\ \text{or} \\ b(1-\tau) - P_h < 0 \quad P_h - b(1-\tau) \leq \min\{\bar{A}, \frac{w(\theta)(1+z^h)}{1+r}\} \end{cases} \\ \\ b(1-\tau) - P_h + \bar{A}[1-\beta(1+r)] + w\beta(1+z^h) \text{ if } b(1-\tau) - P_h > 0 \text{ and } \theta > \frac{\bar{A}(1+r)}{w(1+z^h)} \end{cases} \tag{44}$$

A.8 Life-cycle Model

In this section we embed a life-cycle model into a two-period model, so our calibration of Section 6 is realistic. We solve the household's problem in two parts: 1) during the study periods, $t = 0, \ldots, S-1$, and 2) after college age, $S, \ldots, T$, and leave the problem expressed as a two-period maximization problem in which households decide how much to save for post-college periods. First, we start by solving the post-college optimization problem. We assume that after college graduation, individuals enter perfect financial markets, so there is perfect consumption smoothing. The problem of the households is:

$$\max_{c_t} \sum_{t=S}^{T} \beta^{t-S} \frac{c_t^{1-\sigma}}{1-\sigma}, \quad s.t.$$

$$c_S = b + a_{S+1} + w(1+z_j)\theta$$

$$c_t + a_t(1+r) = a_{t+1} + w(1+z_j)\theta, \quad t \in \{S, \ldots, T\}$$

where a_{t+1} is the debt at period t to be repaid next period, and b are the savings that the individual carries from the college years. In here, we assume that there are no borrowing constraints, since households enter perfect financial markets. Solving this problem, yields the present value budget constraint in period S:

$$\sum_{t=S}^{T} \frac{c_t}{(1+r)^{t-S}} = b + \sum_{t=S}^{T} \frac{w\theta(1+z_j)}{(1+r)^{t-S}}$$

Combining this with the Euler equation, the optimal consumption path is given by:

$$c_S = \frac{1}{\Phi_S}\left(b + w(1+z_j)\theta\Phi_r^o\right)$$

$$c_t = ((1+r)\beta)^{\frac{t-S}{\sigma}} c_S, \quad t \in \{S, \ldots, T\}$$

where Φ_S and Φ_r^o are given by the following expressions:

$$\boxed{\Phi_S = \frac{1 - \left(\frac{\beta}{(1+r)^{\sigma-1}}\right)^{\frac{T-S+1}{\sigma}}}{1 - \left(\frac{\beta}{(1+r)^{\sigma-1}}\right)^{\frac{1}{\sigma}}}} \qquad \boxed{\Phi_r^o = \frac{1 - \left(\frac{1}{1+r}\right)^{T-S+1}}{1 - \left(\frac{1}{1+r}\right)}}$$

The present value utility at time S of this consumption path is given by:

$$\sum_{t=S}^{T} \beta^{t-S} u(c_t) = \Phi_S u(c_S)$$

Note that c_S is determined for every given savings b carried from the college period, so without solving the problem for periods $\{0, \ldots, S-1\}$, it will not be completely pinned down. Now, we solve for the households' problem during periods $0, \ldots, S-1$. Given that during college, there exist exogenous borrowing constraints given by $\bar{A}$, there are two cases: *a*) individuals are unconstrained, and *b*) individuals are constrained. The unconstrained solution of the problem in periods $\{0, \ldots, S-1\}$ yields:

$$c_0\Phi_0 + (P_h h + P_l l)\Phi_r^y + \frac{a}{(1+r)^S} = w\theta\Phi_r^y(1-l)(1-h) + b$$

$$c_t = ((1+r)\beta)^{\frac{t}{\sigma}} c_0, \quad t \in \{1, \ldots, S-1\}$$

where b is the initial wealth of individuals, and Φ_0, Φ_r^y are given by:

$$\boxed{\Phi_0 = \frac{1 - \left(\frac{\beta}{(1+r)^{\sigma-1}}\right)^{\frac{S}{\sigma}}}{1 - \left(\frac{\beta}{(1+r)^{\sigma-1}}\right)^{\frac{1}{\sigma}}}} \qquad \boxed{\Phi_r^y = \frac{1 - \left(\frac{1}{1+r}\right)^{S}}{1 - \left(\frac{1}{1+r}\right)}}$$

Utility in period 0 is given by

$$\sum_{t=0}^{S} \beta^t u(c_t) = \Phi_0 u(c_0)$$

Note that now, the problem can be perfectly embedded in the two-period model described in Section 3. Households solve the following two-period problem:

$$\max_{c_0, c_S} \quad u(c_0) + \tilde{\beta} u(c_S), \quad s.t.$$

$$c_s\Phi_S = a + w\theta(1+z_j)\Phi_r^o$$

$$c_0\Phi_0 + (P_h h + P_l l)\Phi_r^y + \frac{a}{(1+r)^S} = w\theta\Phi_r^y(1-l)(1-h) + b$$

$$a \geq -\bar{A}$$

where:

$$\boxed{\tilde{\beta} = \frac{\beta^S \Phi_S}{\Phi_0}}$$

These two budget constraints can be rewritten as a single lifetime budget constraint:

$$c_0\Phi_0 + (P_h h + P_l l)\Phi_r^y + \frac{c_s\Phi_s}{(1+r)^S} = w\theta\Phi_r^y(1-l)(1-h) + \frac{w\theta(1+z_j)\Phi_r^o}{(1+r)^S} + b$$

The unconstrained consumptions are given by:

$$c_n = \frac{(\beta(1+r))^{(-S/\sigma)}\left[w\theta\left(\frac{\Phi_r^o+(1+r)^S\Phi_r^y}{\Phi_S}\right) + \frac{b(1+r)^S}{\Phi_S}\right]}{1+\frac{(\beta(1+r))^{(-S/\sigma)}\Phi_0(1+r)^S}{\Phi_S}}$$

$$c_h = \frac{(\beta(1+r))^{(-S/\sigma)}\left[w\theta(1+z_h)\frac{\Phi_r^o}{\Phi_S} + \frac{b(1+r)^S}{\Phi_S} - \frac{P_h\Phi_r^y(1+r)^S}{\Phi_S}\right]}{1+\frac{(\beta(1+r))^{(-S/\sigma)}\Phi_0(1+r)^S}{\Phi_S}}$$

$$c_l = \frac{(\beta(1+r))^{(-S/\sigma)}\left[w\theta(1+z_l)\frac{\Phi_r^o}{\Phi_S} + \frac{b(1+r)^S}{\Phi_S} - \frac{P_l\Phi_r^y(1+r)^S}{\Phi_S}\right]}{1+\frac{(\beta(1+r))^{(-S/\sigma)}\Phi_0(1+r)^S}{\Phi_S}}$$