The effects of student loans on the provision and demand for higher education*

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Abstract

We characterize the demand side for tertiary education in a context where borrowing constraints bind, there is a two-tier college system in which colleges differ by the quality offered and returns to education depend on the school attended. This setup is relevant for most developing countries and the U.S. during the last two decades, where there has been evidence of binding borrowing constraints. College quality, tuition prices, acceptance cut-offs and education demand are all determined in a general equilibrium model and depend on the borrowing constraints that agents face. We study the evolution of quality and school enrollment after the implementation of student loan policies. We calibrate the model to Colombia, which implemented massive student loan policies during the last decade based on ability and wealth. We match the moments of the data, to rationalize the quality response to relaxing borrowing constraints.

JEL Classification: E24, I23, I24, I25, I28,L130

Keywords:Higher education, Student Loans, Education policy,Returns to Education,Labor productivity, Monopolistic Competition

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1 Introduction

The market for higher education has received significant attention in the economics literature. In particular, the effects that subsidized loan policies have on the demand side of the market for higher education have been widely studied, given the dramatic increase in student debt over the last two decades in the U.S. Overall, there seems to be a consensus in the literature on the fact that credit constraints explain only a small fraction of enrollment decisions in higher education in the U.S. However, this is not necessarily the case in developing countries, where student financial aid systems are weak and evidence suggests that college enrollment is highly determined by family wealth[1] (Bank, 2003, 2012). In this context, the implementation of student loan policies can potentially affect the demand for education, but can also have equilibrium effects such as an increase in tuition prices and a change in the equilibrium quality offered by colleges.

Understanding the effects of these policies is of central importance, given the massive investments that have been made in student credit programs during the last two decades in the developing world, as part of the democratization processes in Latin America and some African countries. The demand side effects of these policies in a context where borrowing constraints determine enrollment decisions have been studied by the literature and the conclusions are certainly appealing: an expansion in student loans will lead to an increase in the demand for higher education among the most able students. As a consequence, these programs have often received the support of multilateral institutions, such as the World Bank and the Inter-American Development Bank[2]. However, the equilibrium effects of such credit programs have been understudied. Recent evidence suggests that an increase in the availability of student loans is reflected *vis-a-vis* on tuition prices. We argue that colleges also change their quality offered in equilibrium, as a consequence of the massive increase in demand. Whether student loan programs are unambiguously welfare improving or not, is a question still to be answered, since most of the research has drawn only partial equilibrium conclusions.

In this paper we analyze the general equilibrium effects of the implementation of educational loan policies in the context of a developing country. This environment is particularly different from the case of developed economies, for three main reasons. First, in developing countries there is evidence that credit constraints play a role in determining college enrollment decisions among households (Tatiana Melguizo and Velasco, 2015). As will be discussed further in the next section, evidence almost unambiguously suggests that this is not the case for developed countries. Second, in many developing countries the private institutions own a larger share of the market for higher education, as compared to European countries or even the U.S. (see

[1]See for instance Tatiana Melguizo and Velasco (2015)

[2]These institutions have contributed to different student loan projects in the develping world. For example, the World Bank has been financing the Colombian ACCES program since 2002 and committed in 2014 to lend $200 million during the period 2014-2019. Recently, the IDB provided a $10 million dollar loan to the Higher Education Finance Fund in 2012, to finance student loan programs in 4 Latin American countries.

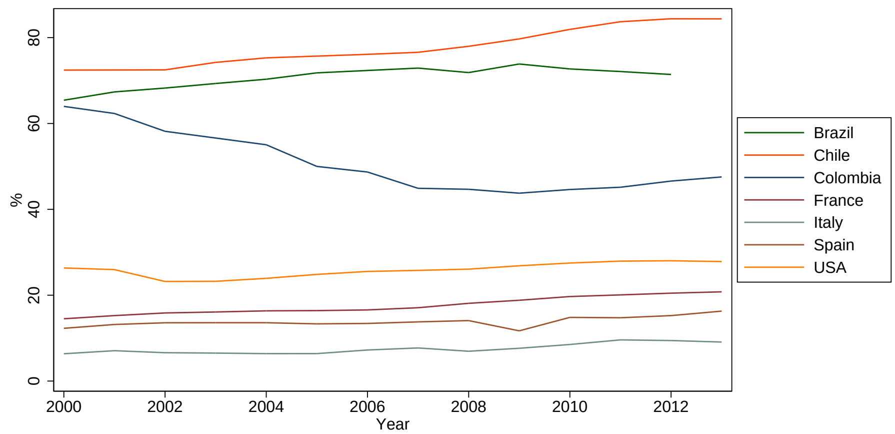


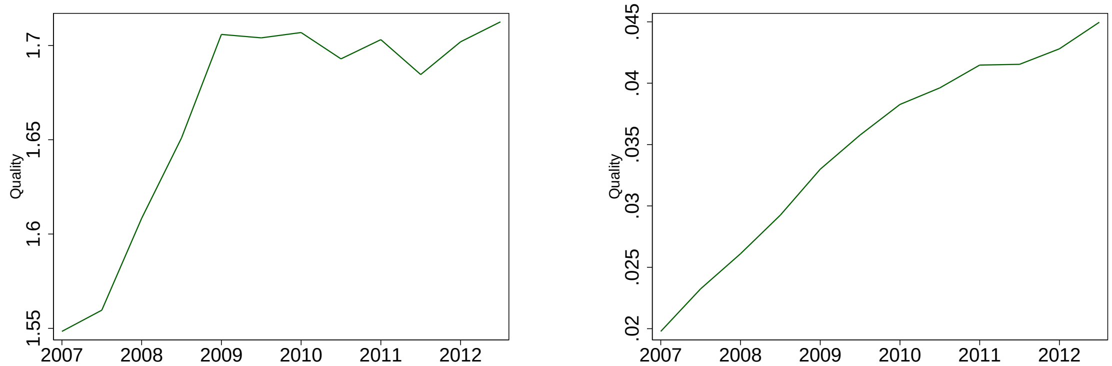
Figure 1: % of enrollment in private institutions by country.

Figure 1). This is important because public institutions may not be as responsive to market incentives, but rather follow the social planner's objetives. In contrast, private institutions are potentially more responsive to market signals, so any change in demand will generate stronger equilibrium effects in developing countries. Third, enrollment rates in developing countries are very low, when compared to enrollment in developed countries. As documented by Mestieri (2016), there is an existing positive correlation between enrollment rates and income per capita at a cross-country level.

The purpose of this paper is to analyze the general equilibrium effects of implementing student loan policies. In particular, we want to study the effect of these policies on the equilibrium quality supplied by different tiers of colleges, which we will denote henceforth as low- and high-quality colleges. We will closely study the case of Colombia, which is a developing country that undertook massive expansions of publicly supplied student loan availability during the last decade and has very good available data. We follow the same approach as the literature on higher education and industrial organization, and model quality of education as a composite of peer effects and investments per student made by the college. Figure 2 illustrates the difference between tiers of the average decile of test scores in each tier, and the difference in the number of professors per student. Clearly, there has been a widening gap in peer effects and investments per student[3]. The main purpose of this paper is to rationalize this widening gap of the quality offered by the two tiers of schools, to argue that the expansion of student loans might accentuate existing inequalities within the educational sector. This would imply that student loan programs also have downsides when studied in general equilibrium, and all of this should be taken into account for future policy design.

Moreover, our analysis will give us tools to discuss the design of the optimal student loan policy in a

[3]The widening gap can also be observed when looking at the number of publications per student in peer-reviewed journals and qualifications of faculty



(a) Difference in average decile of entering cohort.

(b) Difference in professors per student.

Figure 2: Differences in quality indicators between tier 1 and tier 2.

context where the government has outside funds that has to allocate within the existing population. This analysis, of course, has to be done in a general equilibrium setting, which is not a trivial matter. The main elements of the optimal policy are the following. From a partial equilibrium perspective, there are two main forces driving the optimal policy. On one hand, offering loans to people with low income and the highest ability, who can not afford to study due to the credit constraints, will reduce the inefficiencies present by the existence of the constraints. On the other hand, very high ability individuals will receive higher incomes over their lifetime, regardless of their education level, so they face a higher opportunity cost of studying. In this sense, relaxing the constraint for high ability individuals will change the study decision of fewer individuals, than if the constraint were relaxed among lower ability individuals. This implies that there exists a trade-off between reducing the highest inefficiency for fewer individuals, or reducing a smaller inefficiency for more.

When embedding this into a general equilibrium setting, matters become even more involved. Individuals choose which school to attend according to the quality offered by each college, since their future earnings will depend on it. The quality offered by colleges, in turn, is a composite of peer effects in education and investments per student made by the college. As previous literature does, we model these two inputs of quality are complements. In this regard, the best quality-enhancing student loan policy would maximize the peer effects, which would increase returns to investments per student, and would lead to higher quality offered. Therefore, there are two opposing forces in action. Relaxing the borrowing constraints of lower ability households will have the highest impact on school enrollment, but will reduce the returns to investments per student done by schools and the quality of education. The issue becomes even more complex once we incorporate a two-tier education system, in which colleges might respond differently in their pricing, admissions and investment policies when faced to a demand shock of this nature.

With this objective in mind, we analyze the effects that a policy change introducing subsidized loans for college enrollment in Colombia had in the market for higher education. We combine various sources of data such as examination scores for entering cohorts altogether with their socioeconomic background. We collected data on publication records for different universities as well as qualifications of their faculty members, expenditures per student, tuition prices and wages of graduating cohorts. The availability of this information will allow us to get a better description of the overall effect of such loans on the market for higher education.

The results of this paper will help us assess the effect of different counterfactual policies designed to increase college enrollment in the context of a developing country.

The paper is organized as follows. Section 2 describes the literature relevant to our paper. Section 3 describes the case of Colombia and explains why this is an appropriate natural experiment to study massive expansions in student loan programs in the context of developing economies. Section 4 describes the model, characterizes the demand for a two-tiered education system and explains the mechanism through which borrowing constraints affect equilibrium quality supplied. Section 5 estimates the parameters of the quality function, using data on average test scores, professors per student and average wages of students from each university in Colombia. Section 6 describes the computation of the model and some counterfactual experiments. Section 7 concludes.

2 Literature Review

Different strands of the literature are related to the present research. First, our paper is related to the literature that studies the importance of borrowing constraints in the schooling decisions. Given that we are studying the welfare effects of government loan policies in developing countries, knowing whether borrowing constraints matter is of central importance. Although there is evidence suggesting that borrowing constraints do not determine school attendance of students in developed countries, the opposite is the case for countries in the developing world. Second, our paper is related to the literature that studies general equilibria in the market for education. This literature has mostly studied what is known as the *"Bennett Hypothesis"*[4]. Our paper adds to this literature because we study equilibrium welfare effects that go beyond prices in the context of binding borrowing constraints. This literature has been concerned with studying equilibria in the United States, where the inefficiency caused by the existence of borrowing constraints in education is, at best, small. Finally, our paper models the educational sector following the treatment that the industrial organization has

[4]The *"Bennett Hypothesis"* states that an expansion in the number of grants provided to students are almost totally appropriated by colleges through increases in tuition prices.

given to universities, as monopolistically competitive firms that maximize the quality provided, subject to a budget balanced constraint.

2.1 The Role of Borrowing Constraints

Using data from the United States, there is a wide literature that argues that the effects of borrowing constraints on the post-secondary decisions of youngsters are negligible. Using the 1979 of the National Longitudinal Survey of Youth (NLSY79), Carneiro and Heckman (2002) find evidence that borrowing constraints and family income account for a very small fraction of post-secondary school attendance, while early childhood differences are determinant. Once the estimations control for family background and ability, current family wealth becomes unimportant in the schooling decision. According to their estimates, between 0% and 8% of high school graduates are actually borrowing constrained. Similarly, Keane and Wolpin (2001) find that, although borrowing constraints are tight and individuals cannot even borrow the amount to cover one year of schooling, their existence does not determine the decision to study. In counterfactual experiments, when the authors remove the borrowing constraints, the educational attainment does not change significantly. Borrowing constraints only affect labor supply and savings decisions of students.

Dinarsky (2003) measures the impact of the exogenous elimination of the Social Security Student Benefit Program in 1982 on school attendance in the US. This program provided students that came from families with deceased, disabled, or retired Social Security beneficiaries with monthly payments while enrolled in college. The paper finds that the exogenous reduction in aid lead to a decrease in the probability of being enrolled by students at the margin. However, the author argues that this cannot be interpreted as existence of borrowing constraints, since grants do not only relax the borrowing constraints of households, but change also the relative price of education.

More recent studies argue that, although credit constraints did not seem to affect the schooling decision some decades ago, during the last two decades they might be playing an important role in post-secondary schooling in the United States. Using data from the 1979 and 1997 National Longitudinal Survey of Youth (NLSY79 and NLSY97), Belley and Lochner (2007) find a dramatic increase in the importance of family income on school attainment, after controlling for family background and ability as in the previous studies. Similarly, Lochner and Monge-Naranjo (2011) estimate a structural model that suggests that, although American households were not borrowing constrained during the 1980s, during the last decade family income has been determinant in schooling decisions. They argue that in the last two decades there have been rising costs and returns to education, while government student loan programs have not grown at the same pace, so people have become borrowing constrained.

Although there is not much research that studies the role of credit constraints in educational choices in developing countries, the existing evidence seems to unambiguously point towards the importance of borrowing constraints in the educational decisions. As Attanasio and Kaufmann (2009) state, *"one important difference between Mexico and the US, for instance, might be the wider availability of scholarships and student loans in the US, cannot be found in Mexico for higher education."* Attanasio and Kaufmann (2009); Kaufmann (2014) provide evidence suggesting that liquidity constraints do determine the post-secondary schooling decision in Mexico. They use data that characterizes the expected returns of education for every household in their sample. If credit constraints were not binding, there should exist a positive gradient between subjective expected returns from schooling and school attendance. Their results show that this gradient breaks for the lowest income households in their sample. Under their interpretation, this is evidence of existing borrowing constraints. Solis (2013) studies the existence of borrowing constraints in Chile. Using administrative data on the entire sample of individuals that participate in the college admissions' process, he uses a regression discontinuity approach to study the impact of providing educational loans. After controlling for socio-economic covariates, individuals right above the eligibility threshold for receiving educational loans have a significantly higher probability of enrolling in college than those right below the threshold. The author finds evidence suggesting a positive gradient between income and enrolment among those households that have no access to the government loans. This gradient disappears for individuals that access the program. Also using a regression discontinuity approach, Marc Gurgand and Melonio (2011) find evidence that the enrolment to college of households without access to student loans is 20 percentage points lower in South Africa. Regarding the Colombian case, Tatiana Melguizo and Velasco (2015) find evidence that the implementation of a massive government loan program in the past decade, which is the topic of this paper, did increase student enrolment.

2.2 General Equilibrium Effects and the Bennett Hypothesis

During the last decade, the literature that has tried to explain what has become to be known as the *Bennett Hypothesis*: expansions of government-supplied student aid for education have been almost almost entirely appropriated by colleges through an increase in tuition prices. As the former US Secretary of Education stated in 1987, *"If anything, increases in financial aid in recent years have enabled colleges and universities blithely to raise their tuitions, confident that Federal loan subsidies would help cushion the increase."*[5] Singell and Stone (2007) study the effect that Pell Grants have had on tuition prices of public and private schools. They study the Pell Program, which has been the biggest post-secondary educational loan program in the United States. In 1999, the Pell Grants were awarded to 3 million students across more than 6000 colleges,

[5] William Bennett to the New York Times, 1987.

out of a total of 9 million students. The authors estimate the impact of Pell Grants per student on tuition charged by universities, using a panel of 1554 colleges from 1989 to 1996. They find that the increase in Pell Grants caused an almost one-to-one increase in the price of tuition charged by private and public out-of-state colleges. However, they find no such a causality on the in-state tuition charged by public schools. In contrast, Rizzo and Ehrenberg (2002) find evidence that private and public out-of-state tuition prices were not affected by government loans, while in-state tuition by public colleges were. Finally, David O. Lucca and Shen (2016) use exogenous variation in the legislation that rules Pell Grants, to study the relationship between student aid and tuition.

Gordon and Hedlund (2015) study the increase in tuition prices by estimating a structural model in which universities provide human capital and households decide their investments in education. They study the rise in college tuition over the last decades, as a reaction to cuts in state appropriations, an increase in the costs of skilled labor in other industries, and an increase in government supplied loans. The authors find out that the increase in government loans explains 102% of the tuition increase, as opposed to only 16% of the other two hypotheses. This result provides evidence in favor of the Bennett Hypothesis. Our paper differs from theirs in the sense that we want to study the equilibrium effects on quality provided and welfare effects of relaxing borrowing constraints in a context in which they matter. The authors study increases in the borrowing limits in the context of the U.S. As has been already argued, there is evidence that these constraints are of secondary importance on the decision to attend school. Therefore, relaxing these limits does not improve efficiency. In contrast, in countries in which the borrowing constraints are binding, relaxing them does generate efficiency improvements.

2.3 The Education firms

Our paper makes part of the literature that models universities as firms in the educational sector. Universities produce human capital and use households both as inputs and costumers. This approach has been used to study different questions regarding post-secondary education. For instance, Hector Chade and Smith (2014) model the universities as an oligopoly with a fixed number of universities (firms), in which the goods produced by universities (education) are ranked by all households in the same way. Universities only choose admission standards, so as to fill a fixed capacity of students and maximize the ability of the student body. The paper studies the role of frictions in the application process on the student sorting between universities. Namely, the model has information frictions and fixed costs of application. The authors, as Caucutt (2001), treat the utility that households receive of attending each of the universities as exogenous and independent from the product offered by each university. We endogenize the valuation of households as a function of the equilibrium quality offered. The authors do not include tuition prices as a policy of universities, assume an exogenous valuation for the universities and take the size of universities as fixed. We depart from all of these assumptions, but assume there are no frictions in the application process. The reason is that our purpose is not to study the outcome of the application process but, instead, model the strategic interactions in the post-secondary education sector between universities and households.

The educational sector in our model closely mimics Dennis Epple and Sieg (2006). In their paper, the authors model the supply side of the educational sector as an oligopoly sector in which a fixed amount of colleges interact to attract students and maximize the quality of the education they offer, subject to a balanced budget constraint. Quality by universities is a composite of average student ability, to resemble peer effects in schooling, and the average investments per student. This treatment of quality has been standard in the literature that models schools (Caucutt, 2001). Households value quality as an input on their utility function. In their model, households play a passive role in their model, since their purpose is not to estimate equilibrium interactions between households and firms. Rather, they concentrate in studying thoroughly the supply side.

Finally, we treat wages of college graduates as a function of the quality supplied by the school attended. To the best of our knowledge, this approach has not been used in structural estimations in the past, but there is empirical evidence that relates future wages to the quality of the education. Black and Smith (2006) estimate a latent model in which quality is a latent variable, and there are "signals" of quality. They find out that SAT scores, faculty-student ratio, rejection rate, freshmen retention rate, and faculty salaries are significant signals of quality. Furthermore, the latent variable of quality significantly affects post-college wages of individuals. Similarly, Dan Black and Daniel (2005) find evidence that quality increases post college earnings, driven by higher wages. Leaving quality aside, there is extensive evidence that estimates positive returns to college attendance in terms of higher future wages (Zimmerman, 2014; Harry Anthony Patrinos and Sakellariou, 2006). OECD and Bank (2012) estimates that average starting earnings for individuals with a bachelor's degree were 4 times higher than those of individuals with high-school degree. Although these estimates do not control for unobservable household characteristics, other estimates find that people with post-secondary degrees earn significantly higher wages in Latin America (L. Gasparini and Acosta, 2011).

3 The Case of Colombia

In the present research, we will use Colombia as a natural experiment of a country that implemented a rapid credit expansion program to alleviate credit constraints. Colombia is a developing country which by the

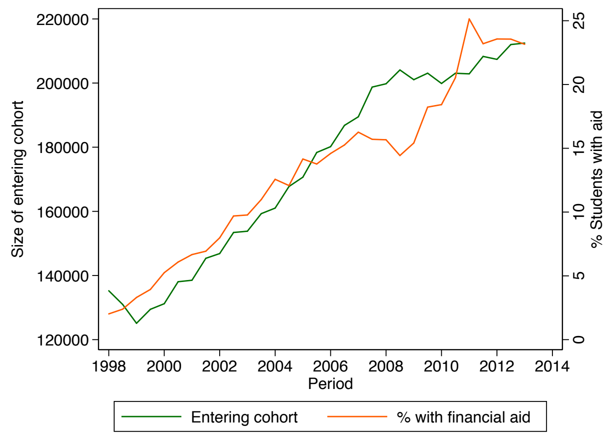


Figure 3: Quality, enrollment and financial aid.

beginning of last decade had low enrollment rates in post-secondary education, and significant differences in enrollment by quintile of income. As will be argued, the majority of students came from high-income families, and the existence of financial constraints kept high-ability individuals from the lowest quintiles out of the education market. During the last decade, the government engaged into the strategy *Revolucion Educativa*, aimed at increasing the education coverage at all levels. During the decade, there were substantial increases in enrollment and educational credit access (see Figure 3), but equilibria effects caused tuition prices to rise and the quality of education offered to change.

3.1 Enrollment and inequality

At the beginning of the last decade, college enrollment in Colombia was among the lowest in Latin America and a financial aid system was almost non-existent for students. In 2000, 23.2% of the people between 18 and 23 years old enrolled in tertiary education, below the enrollment rates of Bolivia, Peru, Brazil, Chile and Venezuela, and very close to the enrollment rates of Mexico. Because of a lack of a well-functioning financial aid system, less than 5%[6] of the entering cohorts had any kind of public or private financial support (Bank, 2003, 2012). By the end of the decade, the enrollment rate grew to 37%, and reached 50% in 2015. Similarly, over the decade the fraction of students with some type of credit increased to almost 25% of the entering cohorts (see Figure 3).

Access to education has always been unequal and, despite the fast growth of enrollment, many disparities persist. In 2013, only 45% of the low-income students graduated from high school, and only 25% of them enrolled in tertiary education. Of the high-income households, 60% graduated from high school and 54%

[6]Extracted from the dataset of indicators for tertiary education, SPADIES, from the Ministry of Education.

of them enrolled in a post-secondary institution (Tatiana Melguizo and Velasco, 2015). According to Bank (2003, 2012), the enrollment gap between the lowest and the highest quintiles of wealth widened throughout the decade: in 2001, the enrollement rates were of 8% in the lowest quintile and 41% in the highest, while in 2010 these numbers grew to 10% and 52%, respectively. If quality is taken into account, disparities are even larger as a bigger proportion of the low-income students attend non-professional institutions, which have less resources and offer lower expected income in the future. Many theories have been used to explain the low enrollment of low-income students, such as disparities in the quality of public and private high school education, the high costs of tertiary education and the lack of a well-functioning financial aid system (Tatiana Melguizo and Velasco, 2015).

3.2 Education institutions

There are approximately 300 tertiary education institutions, of which around 190 are universities, and the rest offer non-professional degrees (mainly technical and technological degrees). Despite the growing size of the entering cohorts throughout the decade, the number of institutions remained almost constant, while the average size of each institution doubled, on average (Figure 4). The university system in Colombia functions as a monopolistically competitive market in which there are significant barriers to entry, and universities do not have fixed "production capacities" (as assumed by Hector Chade and Smith (2014)). It is important to note that around $45-50\%$ of the total student body is enrolled in private tertiary education institutions (OECD and Bank, 2012). Private institutions do not have any regulations regarding the price or investment per student they offer, although they have to satisfy a minimum quality requirement in terms of the programs and degrees offered. Therefore, the education market in Colombia can be studied as a monopolistically competitive market with barriers to entry and not subject to much government regulation.

Figure 5 illustrates the behavior of the average real price of tuition during the decade, in terms of 2004 pesos. As can be observed, there has been a steady increase in the real price of education throughout the decade, and dispersion of prices has nor changed significantly. This increasing trend suggests that the Bennett Hypothesis might also be taking place in the Colombian context, given the fast increase in the government provided loans to education.

In Colombia, every student that wants to graduate from high school has to present an exam called SABER11 set by the Colombian Institution for Education Evaluation (ICFES), similar to the SAT test in the U.S. Although not every tertiary education institution takes into account the results of the SABER11 in their admission decision, 78% use it as a criterion for admission (OECD and Bank, 2012). As SABER11 has no pass-mark, each institution sets its own minimum threshold for admission. In contrast to what happens

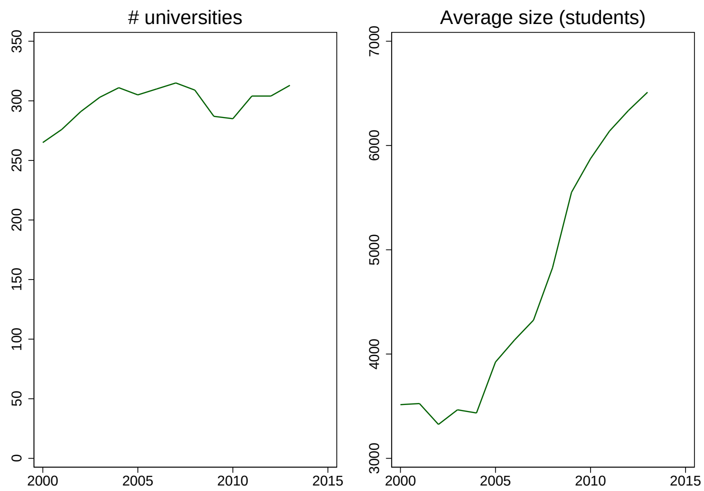


Figure 4: Left: number of universities. Right: average size of universities

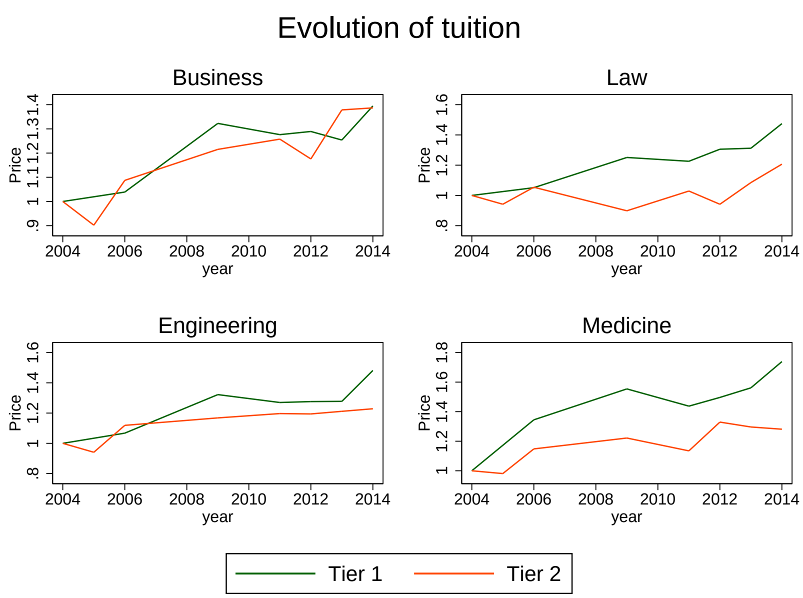


Figure 5: Evolution of average tuition prices over the decade

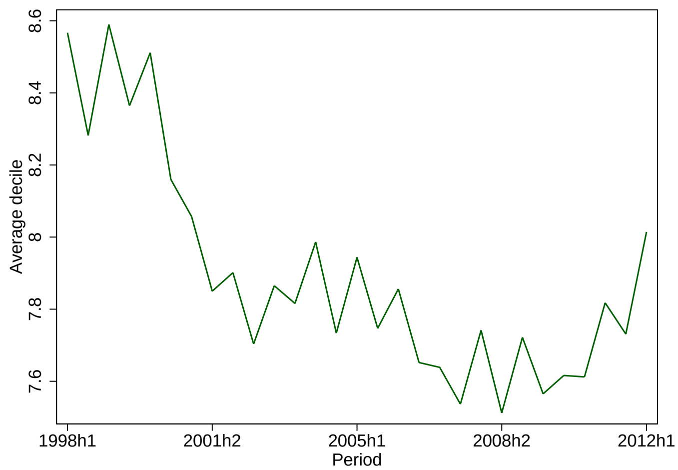


Figure 6: Average decile of ability of entering student body, measured by test scores

in Chile and some European countries, in Colombia there is not any institution that clears the market for admissions, so individuals apply to as many institutions as they like and universities choose their admission standards independently. Although not perfect, the results in the SABER11 exam can be used as a proxy for the quality of the student body at universities. Figure 6 illustrates the average decile of the SABER11 scores of the entering cohorts to tertiary education institutions. Throughout the decade, universities seem to have adjusted their admissions standards in such a way that led to a reduction in the ability of the student body, as measured by relative position in the test scores.

3.3 The ACCES Program

To alleviate the low access, in 2002 the government implemented the credit program *Access with Quality to Higher Education, ACCES*, with the support of the World Bank, that massively increased the available credit to students. The credit is awarded to students that have test scores above a threshold set by the government, and covers up to 75% of the tuition for the lowest income students, and up to 50% for the rest. The credit has a subsidized zero-real interest rate for the poorest households, and a real interest rate of 8% for the high-income students. Students that graduate from their programs have twice the time of their study period to repay the loan. The ACCES program has full coverage, in the sense that any student that has test scores on the highest deciles of their region can access this credit line. The test score cut-offs vary by region, to account for disparities in the quality of secondary education across regions with different infrastructure and economic development. Given that the credit is awarded according to regional cutoffs, the disparities in the ability of people accessing the credit are large. The best students from the poorest regions might not have high ability and preparation when compared to the best students of the principal cities, so the credit is not awarded to the best students in absolute terms.

Tatiana Melguizo and Velasco (2015) find evidence that the ACCES program had a positive impact on the enrollment rates, especially for individuals that come from poor households. Although the growth in the number of students enrolled in college may have been a consequence of other factors, such as better economic activity, the massive increase in financing seems to be a driving factor of such a trend.

4 The Model

There are two types of agents in the economy: households and universities. There is a government that offers educational credits to high-ability individuals that decide to attend college, at an exogenous interest rate $R > r$ with r being the risk free interest rate. In addition, the government subsidizes the interest paid by the poorest households that access the credit, at a subsidy rate s. In order to finance these subsidies, the government levies a marginal tax, τ, to every household in the economy. The government policies are exogenous, fixed before the economy starts and should satisfy budget balance. Given these policies, the market of higher education operates under monopolistic competition. Universities supply human capital in the market for education, by choosing a tuition price, a minimum ability level for admission and a level of investment per student. Given government and university policies, the households decide if they want to study in any university at the prevailing market prices.

4.1 Households

Households are born with innate ability and wealth (θ, b), according to a bivariate distribution $F(\theta, b)$ over the space $[0,1] \times [\underline{b}, \bar{b}]$. Individuals live for two periods, after which they die with probability equal to one. In period 1, individuals choose either to study at the university or work in the low-skilled labor market at a wage w. Individuals that do not study receive a wage w, do not have access to credit markets and can save at the risk-free rate r. There are two universities in the economy denoted by h and l. Each university sets a threshold $\underline{\theta^j}$ for $j = h, l$ such that only students that have ability $\theta \geq \underline{\theta^j}$ are admitted to university j, and we assume that this information is public[7]. Therefore, individuals with $\theta < \min\{\underline{\theta^h}, \underline{\theta^l}\}$ cannot study and have to work. Individuals who decide to study at university j cannot work, and have to pay a tuition, P^j, set by the university.

[7] We assume that $\underline{\theta^j}_{j=h,l}$ is a public threshold, since our purpose is not to study the frictions in the college application process, as opposed to some papers in the literature that model explicitly these information frictions (Hector Chade and Smith, 2014; Fu, 2014).

In order to finance education, the government offers student loans at the interest rate R to people that decide to study and have an ability level $\theta \geq \theta_{min}$. In addition, students with low wealth, $b \leq b_{max}$, that decide to study and have access to the loan will receive a subsidy on the interest rate, s. Loans are given conditional on studying, and individuals that study choose whether to borrow from the government or not. In order to finance these subsidies, the government levies a proportional tax, τ, to every individual in the economy. Therefore, in the first period the household decides its level of consumption, c, whether to study or not in any university, h, l, and the level of savings, a, which can be potentially negative for households that study and satisfy the government conditions for the educational loans.

In the second period, the households are either low- or high-skilled, depending on whether they decided to study in the first period. Those who decided to study in period 1, will enter the high-skilled labor market in period 2, and receive a wage equal to $w\theta(1+z^j)$, where z^j is a skill premium that is university specific. This quality is an equilibrium object that depends on the quality of the student body and investments per student, and is fully characterized in the next section. We assume that individuals have perfect foresight of the value of (z^j) for $j = h, l$ when they optimize. Individuals who do not study will become part of the low-skilled labor force at a wage $w(\theta)$. We exclude the possibility of default in the model by assuming that repayment is fully enforced, so in the second period individuals that have government debt will repay their student loan. Given prices R, r, w, government policies τ, s, university policies $\{\underline{\theta^j}, P^j\}_{j=h,l}$, and perfect foresight about education quality $\{z^l, z^h\}$, a household that is eligible for studying at the university j, $\theta \geq \underline{\theta^j}$, and decides to study gets a utility equal to:

$$V^j(\theta, b) = \max_{c,a} \quad u(c) + \beta u(c'), \quad \text{s.t.} \tag{1}$$

$$c + a + P^j = b \cdot (1-\tau) \tag{2}$$

$$c' = a(1+r) \cdot \mathbb{1}_{\{a \geq 0\}} + a(1+\tilde{R}) \cdot \mathbb{1}_{\{a<0\}} + w\theta(1+z^j) \tag{3}$$

$$\tilde{R} = \begin{cases} R(1-s) & \text{if } b \leq b_{max} \\ R & \text{if } b > b_{max} \end{cases} \tag{4}$$

$$a \geq -\mathbb{1}_{\{\theta \geq \theta_{min}\}} \cdot P, \quad c \geq 0, \quad c' \geq 0 \tag{5}$$

Individuals that decide not to study, get the following utility:

$$
\begin{aligned}
V^N(\theta,b) = \max_{c,a} \qquad & u(c) + \beta u(c'), \quad \text{s.t.} && (6)\\
& c + a = b \cdot (1-\tau) + w\theta && (7)\\
& c' = a(1+r) + w\Theta && (8)\\
& a \geq 0, \quad c \geq 0, \quad c' \geq 0 && (9)
\end{aligned}
$$

The individual with ability and wealth (θ, b) decides to study at university j whenever $\theta \geq \underline{\theta^j}$ and $V^j(\theta,b) \geq V^N(\theta,b)$, and $V^j(\theta,b) \geq V^{-j}(\theta,b)$ if they can attend to the other university $-j$, i.e. $\theta \geq \underline{\theta^{-j}}$. Otherwise, the individual decides not to study. Therefore, the household's value function is given by:

$$
V(\theta,b) = \begin{cases} \max\{V^h(\theta,b), V^l(\theta,b), V^N(\theta,b)\} \text{ if } \theta \geq \max\{\underline{\theta^h}, \underline{\theta^l}\} \\ \max\{V^j(\theta,b), V^N(\theta,b)\} \text{ if } \underline{\theta^{-j}} > \theta \geq \underline{\theta^j} \\ V^N(\theta,b)\} \text{ if } \theta < \min\{\underline{\theta^h}, \underline{\theta^l}\} \end{cases}
$$

For a given set of initial parameters, the shaded region in Figure 7 illustrates the individuals that choose to study, when there are borrowing constraints and there are no government-supplied student loans. Three things are worth noting[8]. First, for a given level of wealth b, the decision to study is a cut-off rule on θ. Individuals that have sufficiently high ability θ will decide to study. This is a direct consequence of the fact that the cost of studying is decreasing in the level of ability. Second, for a given level of ability θ, the decision to study is does not follow a cut-off rule in b. This is a consequence of two facts: the decreasing marginal utility of consumption, and the borrowing constraints.

The following sequence of theorems characterize the demand for college education on the state space, and its close relationship with borrowing constraints. This will let us derive some results about the socially optimal student loan policy. First, we describe the college decision for households that are unconstrained.

Theorem 1. *Among the unconstrained households, the decision of whether and where to study is independent of initial wealth, b, and follows a cut-off rule on θ:*

- *If $\theta \leq \underline{\theta}$, the individual will not study.*
- *If $\underline{\theta} \leq \theta \leq \bar{\theta}$, the individual will attend the low-quality college.*
- *If $\bar{\theta} \leq \theta$, the individual will attend the high-quality college.*

[8] A couple of these should be propositions later.

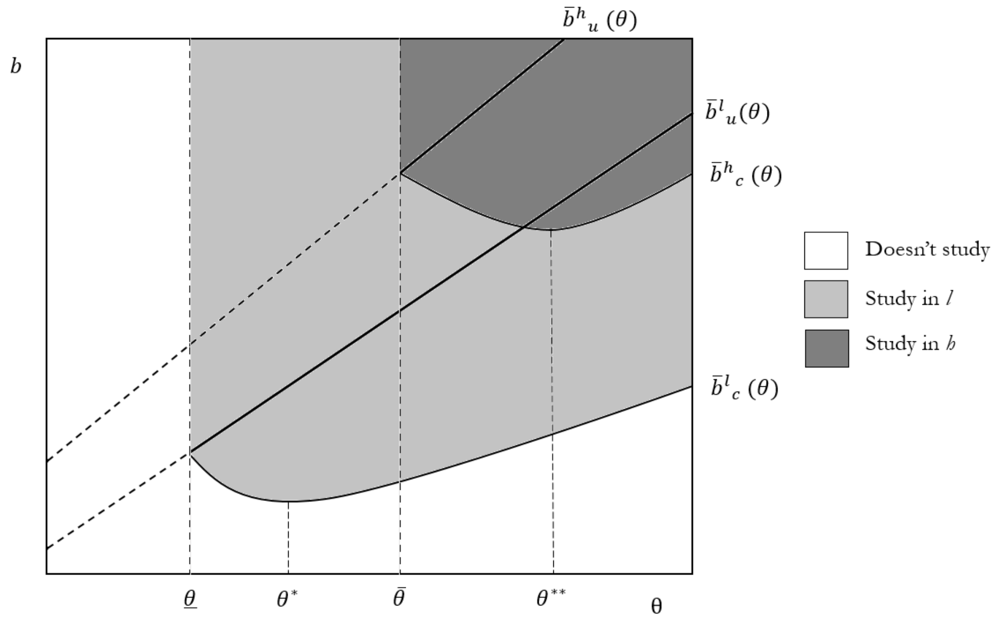


Figure 7: Representation of the education decisions on the state space.

where:

$$\bar{\theta}_l = \frac{1+r}{w}\left(\frac{P_l}{q_l-(1+r)}\right), \quad \bar{\theta}_h = \frac{1+r}{w}\left(\frac{P_h-P_l}{q_h-q_l}\right)$$

Proof. See Proof A.1 □

Theorem 1 is a result of the fact that ability θ, unskilled labor w and quality of the school attended q are complements. In particular, this complementarity implies: a) among the unconstrained individuals, those with higher ability face higher marginal returns of education, so will choose, *ceteris paribus*, a higher quality school for a given wealth, b) as the wages of unskilled labor increase, the marginal returns to education fall for every θ, so marginal individuasl will shift to lower levels of education, c) if college j, for $j \in \{l, h\}$, increases its price P_j or reduces its quality q_j, marginal individuals will change their schooling decision in the expected direction. Theorem 2 characterizes the individuals that, given their decision to attend college j, are borrowing constrained.

Theorem 2. *Given an ability θ, there exist cut-offs, $\bar{b}^j_u(\theta)$, $j \in \{N, l, h\}$, on the initial wealth, such that individuals with $b \geq \bar{b}^j_u(\theta)$ that attend college j will not be borrowing constrained. Individuals that attend college j and have $b < \bar{b}^j(\theta)$ will be borrowing constrained and will not be able to smooth consumption over time. The cut-offs are linear, increasing in θ and take the form:*

$$b^{\bar{N}}_u(\theta) = -\bar{A}(1 + (\beta(1+r))^{-1/\sigma}(1+r)) - w\theta(1 - (\beta(1+r))^{-1/\sigma})$$

$$\bar{b_u^l}(\theta) = P_l + (\beta(1+r))^{-1/\sigma} w\theta(1+q_l) - \bar{A}(1 + (\beta(1+r))^{-1/\sigma}(1+r))$$

$$\bar{b_u^h}(\theta) = P_h + (\beta(1+r))^{-1/\sigma} w\theta(1+q_h) - \bar{A}(1 + (\beta(1+r))^{-1/\sigma}(1+r))$$

Proof. See Proof A.1 □

Given a level of education and initial wealth, individuals with a higher θ have higher lifetime income and in an unconstrained world consume more in every period of their lives. Given the existence of a borrowing limit $\bar{A}$, for a sufficiently high θ individuals will be borrowing constrained. As a consequence, the initial wealth that individuals must have not to be borrowing constrained is increasing in ability. Figure 7 illustrates the cut-off functions $\bar{b}_u^j(\theta)$ on the state space. As illustrated, individuals above the $\bar{b}_u^j(\theta)$ function, will decide to study in college j whenever her θ falls inside the corresponding interval in the cut-offs defined in Theorem 1. Note also that individuals that are borrowing constrained when studying at college l will also be borrowing constrained when studying in h, assuming a higher price of education in the high-quality college. Moreover, the functions $\bar{b}_u^j(\theta)$ are steeper when the quality q_j increases, since quality of schooling and ability are complements. Finally, we do not consider the case in which individuals are borrowing constrained when they do not study. Since in our context, individuals that do not study earn the same wage in every period, they will only be borrowing constrained when the interest rate $\beta(1+r) << 1$. However, for a reasonable calibration, individuals will be able to smooth consumption. The following two theorems illustrate the study decision of individuals that are borrowing constrained.

Theorem 3. *Given ability θ, the decision to study in the low-quality college, l, or not study at all, follows a cut-off strategy on b, such that individuals with $b \geq \bar{b}_c^l(\theta)$ will attend college l, and those with $b < \bar{b}_c^l(\theta)$ will not study. The cut-off is described by Equation 16 in the proof. Moreover, the cutoff is:*

- *Decreasing in θ for $\theta \leq \theta^*$*
- *Increasing in θ for $\theta \geq \theta^*$*

where θ^* solves:

$$\left(\frac{1}{1-\sigma}\right)(b(\theta^*) - P_l + \bar{A})^{1-\sigma} + \left(\frac{\beta}{1-\sigma}\right)(w\theta^*(1+q_l) - (1+r)\bar{A})^{1-\sigma} - \Phi(w\theta^*(2+r) + b(\theta^*)(1+r))^{1-\sigma} = 0$$

$$b(\theta) = \theta\left[\frac{wX(1+q_l) - w(2+r)}{1+r}\right] - X\bar{A}$$

$$X = \left[\frac{\Phi(1-\sigma)(2+r)}{\beta(1+q_l)}\right]^{1/\sigma}$$

Proof. See Proof A.2. □

The cut-off $\bar{b}_c^l(\theta)$ is illustrated in Figure 7. The individuals who are constrained (below $\bar{b}_u^l(\theta)$) will choose either to study at l or not, if their initial wealth exceeds $\bar{b}_c^l(\theta)$. The cut-off is U-shaped because two effects are in action. First, the "complementarity" effect means that, given a b, individuals with higher θ will have higher marginal returns from studying, so are willing to study even though they will not be able to smooth consumption. Therefore, the cut-off is initially decreasing. However, the "constrainedness" effect dominates after some point: given an initial wealth b, individuals with higher θ will face a larger wedge in their Euler equation, meaning that they will be able to smooth consumption to a lower extent. When the wedge is large enough, individuals will prefer not to study and smooth consumption. The following result describes the differential effect of relaxing the borrowing limits to households, $\bar{A}$.

Theorem 4 (Borrowing constraints)**.** *Given θ, the cut-offs $\bar{b}_c^l(\theta)$ and $\bar{b}_c^h(\theta)$ are decreasing on $\bar{A}$. Moreover, the elasticity of $\bar{b}_c^l(\theta)$ and $\bar{b}_c^h(\theta)$ are decreasing on θ, meaning that a relaxation of the borrowing constraint has a higher impact on enrollment among the marginal individuals that have lower θ.*

Proof. See Proof A.2. □

Theorem 4 states that among the constrained individuals, those with lower θ are more sensitive to relaxing the borrowing constraints. That is, if the borrowing constraints were relaxed by the same amount to all the individuals, more low-θ individuals would change their study decision. This result is a consequence of the decreasing marginal utility. Individuals with high θ and sufficiently low initial wealth have a trade-off between earning relatively high wages in every period and smoothing consumption it they do not study, or studying to earn large wages in the second period at the expense of a very low consumption in the first period. However, because of decreasing marginal utility, the utility of a very large wage in the second period is not as large as for lower θ individuals, so individuals will optimally decide to study only when there is a large increase in the borrowing limits of the first period.

This result has very important implications on the design of an optimal student loan policy in a partial equilibrium setting. If the objective of the government is to maximize enrollment, the policy should target the lower ability individuals. Figure 8 illustrates the number of individuals of ability θ that change their study decision as the borrowing constraint is relaxed from $\bar{A} = 0$. As stated by Theorem 4, the individuals in the state space with low ability that would study in the unconstrained world (those with $\theta \in [\bar{\theta}_l, \bar{\theta}_h]$) are more sensitive to relaxing borrowing constraints. Therefore, increasing the borrowing capacity increases enrollment more among the low ability individuals.

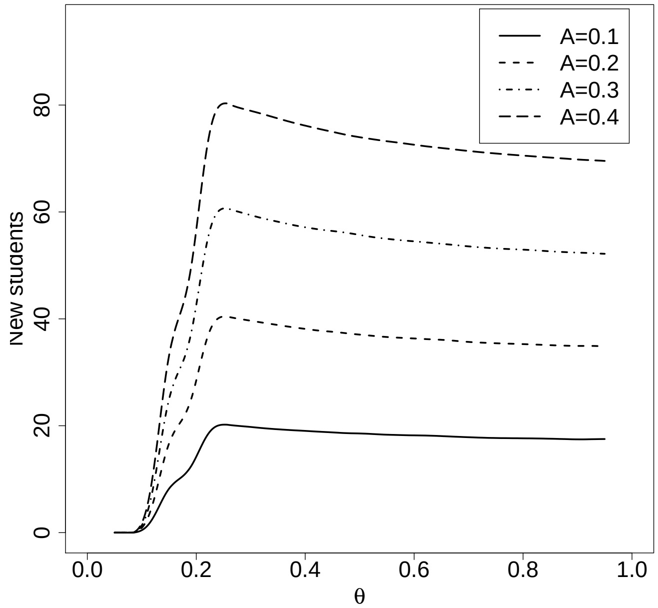


Figure 8: Number of students that change their study decision when borrowing constraints change from $\bar{A} = 0$ to $\bar{A}$, by ability θ.

4.2 Universities

Universities act as firms that maximize an objective function. Given that university systems in most countries are non-profit firms, we follow the literature on education and industrial organization and assume that universities maximize a composite of the quality they offer to students, denoted by q, and the economic diversity of their student body, subject to a budget constraint. Quality offered by universities is an abstract concept. The literature has argued that the quality offered by a school is determined both by the quality of the student body and the investments per student done by the school. Dennis Epple and Sieg (2006), for instance, model the objective function of the university as a composite of the average ability of the student body, the investment per student and the inverse of the mean income. They argue that there is empirical and anecdotal evidence that shows that colleges engage in policies to attract low income students. Universities take as given the values of τ, s, R, w and the distribution F. Additionally, we assume that universities set their policies simultaneously and so, the pricing and admission policies set in equilibrium should satisfy the no profitable one shot deviation principle.

University j takes as given $(\tau, s, R, w, P^{-j}, \underline{\theta^{-j}})$ and will set the pricing and admission policies $(P^j, \underline{\theta^j})$

in order to solve the following problem:

$$\max_{P^j,\underline{\theta^j}} \left(z^j\right)^{\alpha} \left(\sigma_b^j\right)^{1-\alpha} \tag{10}$$

subject to:

$$z^j = \tilde{\theta^j}^{\alpha_1} (I^j)^{\alpha_2} \tag{11}$$

$$\tilde{\theta^j} = \int_{\Theta\times B} \theta \cdot e^j(\theta,b) dF(\theta,b) \tag{12}$$

$$I^j \cdot N^j + V^j(N^j) + C^j = P^j \cdot N^j + E^j \tag{13}$$

$$N^j = \int_{\Theta\times B} s^j(\theta,b) dF(\theta,b) \tag{14}$$

where $\tilde{\theta^j}$ is the average ability of the individuals that attend school j. σ_b^j is the inverse of the average income of the student body and reflects the fact that universities care about the diversity in their student body. I^j is investment per student, V^j is a convex cost function, N^j is the size of the student body, C^j is a fixed cost and E^j is the endowment that the university has.

4.3 Discussion

Although in principle the solution to the problem of the university might seem simple given that there are only two variables of choice, there are several elements of the model that increase the complexity of such decision. First of all, both policies are interdependent. When a university changes one decision variable -either the price or the admission threshold- this will distort the incentives faced when setting the other policy. For instance, a change in tuition price will not only change the revenue of the university but will change the demand in a way that we expect to see a change in the average ability of the student body. Such a change in the average ability of the student body will affect the marginal productivity of investments made by the university, which in turn will affect its pricing decisions.

Moreover, we need to deal with the fact that in equilibrium no university should have incentives to deviate. Given that both universities make the decision simultaneously and that there are no elements of incomplete information in the model, the equilibrium concept we will use at this point is Nash Equilibrium: no university will have incentives to deviate given the decisions made by the other university. Note that given the nature of the problem we cannot be sure of the existence of such equilibrium -university payoffs are not continuous- and in case we find one we cannot be sure it is unique.

The aforementioned elements make it clear why analyzing the consequences that subsidized loan policies will have in the market of higher education is a complex problem. Let's suppose that the government imposes such policy by subsidizing the interest rate of student loans. The first effect such policy will have is an increase in the number of students going to college. Note, however, that it is also not unreasonable to assume that the quality of the student body will change. This is because people who changed their decision to go to college are either those who were credit constrained or those having low ability levels that now decide to go to college given the decrease in the opportunity cost.

We can expect that after imposing such a policy, households will react by changing their decision of studying and universities should expect a change not only in the size of their student body but also in their quality. Given such changes, universities might want to change the prices charged to their students. This is due to the fact that as the quality of the student body changes, the productivity of investment will also be affected. Additionally, the willingness to pay for educational services is affected by such policy and universities will react to that. Moreover, universities might want to change the admission threshold either to improve the quality of their student body or to attract less able students that are willing to pay more for education. The overall effect depends on how sensitive is the demand for education with respect to the quality of services being provided.

Finally, note that -as said previously- the decisions of universities need to be analyzed in equilibrium. When deciding what is optimal, each college needs to take into account what their competitor is doing in the market and there should be no room for profitable deviations. After imposing a subsidized loan policy we might end up in an equilibrium where one college serves a specific part of the population. For instance one college serves a large demand for students with relatively low levels of ability whereas the other one specializes in providing high quality education for a reduced number of high ability students. Additionally, we can have a symmetric equilibrium where both firms are indistinguishable from one another or one in which only one firm operates in the market.

4.4 Government

We do not model the government as a welfare maximizing agent in the economy. We abstract from this fact and simply analyse the impact of the change in the government policies on the higher education market.

The objective of the government is to achieve efficiency in the educational sector. Given that individuals are borrowing constrained, the competitive equilibrium without the government intervention is inefficient, since there are individuals who cannot afford studying because of their initial wealth, but have high ability. In the social planner's solution, the efficient outcome would be one in which the high ability individuals decide to study, independent of their wealth. Thus, the role of the government is to reduce the existing inefficiency in the educational sector. We assume that the government has a borrowing constraint in the international borrowing markets, so is only able to finance the education of some fraction of the individuals in the economy. Further ahead, we will micro found the behavior of the government. For now, we assume that the government finances individuals that have $\theta \geq \theta_{min}$, and of those that can access the loans, subsidizes the interest rate on the loan for those individuals that have $b \leq b_{max}$.

The objective of the government is to relax the borrowing constraints that bind for individuals that cannot attend university but would do it if they had financing. We assume that the government can borrow in private or international markets at a net interest rate R, above the risk-free interest rate r. The government can borrow up to some level exogenous debt limit in order to lend to households that want to study, in such a way that it does not have enough resources to finance the education of every household. That is, the government financing policy is limited to some subset of the population. Moreover, the government has a redistribution motive, so it offers loans at the risk-free interest rates to a subset of the households that get financing. In this way, the government runs in a deficit that must balance by raising tax income. The government sets thresholds $\bar{b}$ and θ_0, such that

$$s \cdot (R - 1 - r) \cdot \int_{\Theta_2 \times B_2} dF(\theta, b) = \tau \int_{\Theta \times B} b dF(\theta, b) \tag{15}$$

where $\Theta_1 \times B_2 = (\Theta_1 \times B_1) \cap ([\theta_0, 1] \times [0, \bar{b}])$ is the set of households who study and decide to take the subsidy.

Definition 1 (Competitive Equilibrium)**.** *Given a set of government policies, $\tau, s, b_{max}, \theta_{min}$, and prices R, r, w, a competitive equilibrium is a set of university policies $(P^j, \underline{\theta}^j, I^j)_{j=h,l}$ and household's value function $V(\theta, b)$ and policy functions $c(\theta, b), a(\theta, b), e^h(\theta, b), e^l(\theta, b)$, such that:*

1. *Given $\tau, s, b_{max}, \theta_{min}$, prices, R, r, w, and university policies $\{P^j, \underline{\theta^j}, I^j\}_{j=h,l}$, the value function $V(\theta, b)$ solves the household's problem, with $c(\theta, b), a(\theta, b)$, $e^h(\theta, b)$ and $e^l(\theta, b)$ being the corresponding policy functions.*

2. *For each university $j = h, l$, it should hold that given $\tau, s, b_{max}, \theta_{min}$; prices, R, r, w; policy functions $(\theta, b), a(\theta, b), e^h(\theta, b), e^l(\theta, b)$; and policies from university $-j$, $(P^{-j}, \theta^{-j}, I^{-j})$, university j chooses policies $(P^j, \underline{\theta^j}, I^j)$ that solve the university's problem described in 10-14.*

3. *The government's budget is balanced (equation 15 holds).*

5 Quality Evolution Estimation

According to the specifications assumed in the model, we are able to identify the parameters of the wage equation:

$$w_h = w \cdot (1 + z_h), \quad w_l = w \cdot (1 + z_l)$$

Where w_h and w_l are the wages of high- and low-quality college graduates, given equilibrium qualities of education z_h and z_l, respectively, and w is the wage of non-skilled labor. The quality of education, z, is given by equation 11 in the universities' problem. We have a panel of data for 50 universities in Colombia from 2007 to 2012. We have the average ability of students in the entering cohorts, number of professors per student and average wages during the first year after graduation. For every university i in our sample, the following equation holds:

$$w_i = w \cdot (1 + \kappa \bar{\theta}_i^{\alpha_1} I_i^{\alpha_2})$$

Rearranging and taking logarithms:

$$\log \left(\frac{w_i}{w} - 1 \right) = \log \kappa + \alpha_1 \bar{\theta}_i + \alpha_2 I_i$$

Assuming that there is measurement error in the wages of each of the universities, and assuming an exclusion restriction that the measurement error is uncorrelated with the explanatory variables, we can estimate the following equation:

$$\log \left(\frac{w_{i,t}}{w_t} - 1 \right) = \log \kappa + \alpha_1 \bar{\theta}_{i,t} + \alpha_2 I_{i,t} + \eta T_{i,t} + \phi_t + \psi_i + \epsilon_{i,t}$$

where T_i is an indicator function that takes the value of one when the university i is a low-quality institution, and zero otherwise. Under this specification, we can estimate possible differences in the technology parameter, κ, between top and second tier schools. In order to isolate possible omitted variable bias, we estimate the above model under three different specifications, with and without time and geographic fixed effects, ϕ_t and ψ_i, respectively.

For the estimation, we constructed a panel of the top 50 universities in Colombia, according to a quality ranking published by the Ministry of Education in 2014. This panel includes data on average wages during

Parameter	OLS	OLS	OLS
$\hat{\alpha_1}$	0.211	0.228	0.168
	(0.026)	(0.026)	(0.026)
$\hat{\alpha_2}$	.358	0.478	0.414
	(0.361)	(0.357)	(0.403)
$\hat{\eta}$	-0.029	0.008	-0.046
	(0.047)	(0.043)	(0.046)
$\widehat{\log(\kappa)}$	-0.84	-0.957	-0.163
	(0.232)	(0.228)	(0.198)
Time fixed effects		Yes	Yes
City fixed effects			Yes
N	382	382	382
R-squared	0.353	0.444	0.567
Prob > F	0.000	0.000	0.000

Robust standard errors in parenthesis
$***p < 0.01, **p < 0.05, *p < 0.1$

Table 1: Estimates for the quality production function.

the first year after graduation for graduates of every school, as a measure of $w_{i,t}$, the average test scores for the entering cohorts, as a measure of $\theta_{i,t}$, and the number of professors per student, as a measure for $_{i,t}$. We also have data on total operational expenditures by each school for 2014. However, with only one year we are not able to construct the evolution of quality of universities over time. Since the number of professors per student are a good indicator of the total expenditures per student, we will use that variable, instead. For the non-skilled labor wages, w_t, we will use the values of the real minimum wage (in 2007 pesos). The average wages of college graduates are strictly above the minimum wage during the period, so the dependent variable is well defined for every college in every period. In addition, we have information about the municipality of the school, to control for regional differences. The results of the estimation are displayed in Table 1.

The estimates show that the elasticities α_1 and α_2 are fairly robust to different specifications and do not change dramatically when including control variables. Moreover, the parameter η is negative in two of the specifications, although non statistically significative. This means that, on average, tier 2 universities have a lower technology parameter, κ, on their quality production function. This will be one of the main differences between tier 1 and tier 2 universities in our calibration of the model.

Figure 2 shows the estimated education qualities offered by both institutions, on average. Two things are worth noting about these results. First, the division of tier 1 and tier 2 universities seems to be robust, as shown by Panel A: the average quality offered by both tiers is significantly different to a 99% confidence level (see Table **??** in the Appendix). Second, during the period 2007-2012, tier 1 universities increased their quality at a roughly constant rate, while tier 2 universities only started increasing it by the end of the period.

As Panel B shows, the gap in quality offered by both universities widened over the period.

6 Computation of the equilibrium

Given that the model presented above is composed by a government, who chooses its policies before the economy starts, and by a monopolist, that perfectly anticipates the demand for education it faces, the model is solved by backwards induction in three stages: first we solve for the household's problem, then the university maximizes, and then we solve for the government policies that satisfy budget balance. To do this, we need to use the following subroutines:

6.1 Subroutine to find equilibrium z - fixed point:

1. Set $n = 0$, and start with a guess z^*.
2. Assume that the quality offered by the university is $z_n = z^*$.
3. Given z_n, government policies, $\tau, s, b_{max}, \theta_{min}$, prices, R, r, w, university policies $P, \underline{\theta}, I$, solve the household's problem for every pair (θ, b) in the state space.
4. Given the households' decisions, compute $\bar{\theta}$ and the size of the student body, N, given by (12) and (14).
5. Find the investment per student, I, residually, using (13).
6. Compute quality offered by the university, z^*, using (11).
7. If $|z^* - z_n| < \epsilon$, end here. Otherwise, go to 2.

6.2 Model solution:

1. Given the government policies and prices $\tau, s, b_{max}, \theta_{min}, R, r, w$, the university maximizes the quality offered, z, subject to a balanced budget constraint. For every set of policies, $P, \underline{\theta}, I$, the university finds the corresponding quality z offered, using the subroutine described in last section. The university then iterates over $P, \underline{\theta}, I$ until it finds the maximum z.
2. The government chooses policies $\tau, s, b_{max}, \theta_{min}$ that satisfy a balanced budget in equilibrium, given by equation (**??**).

7 Conclusion

References

Attanasio, O. P. and Kaufmann, K. M. (2009). Educational choices, subjective expectations and credit constraints. *NBER Working Papers*.

Bank, W. (2003). Tertiaty education in colombia: Paving the way for reform. *World Bank*.

Bank, W. (2012). Revies of national policies for education tertiaty education in colombia. *World Bank*.

Belley, P. and Lochner, L. (2007). The changing role of family income and ability in determining educational achievement. *Journal of Human Capital*.

Black, D. A. and Smith, J. A. (2006). Estimating the returns to college quality with multiple proxies for quality. *Journal of Labor Economics*.

Carneiro, P. and Heckman, J. J. (2002). The evidence on credit constraints in post-secondary schooling. *The Economic Journal*.

Caucutt, E. M. (2001). Peer group effects in applied general equilibrium. *Economic Theory*.

Dan Black, J. S. and Daniel, K. (2005). College quality and wages in the united states. *German Economic Review*.

David O. Lucca, T. N. and Shen, K. (2016). Credit supply and the rise in college tuition: Evidence from the expansion in federal student aid programs. *Federal Reserve Bank of New York Staff Reports*.

Dennis Epple, R. R. and Sieg, H. (2006). Admission, tuition and financial aid policies in the market for higher education. *Econometrica*.

Dinarsky, S. M. (2003). Does aid matter? measuring the effect of student aid on college attendance and completion. *American Economic Review*.

Fu, C. (2014). Equilibrium tuition, applications, admissions and enrollment in the college market. *Journal of Political Economy*.

Gordon, G. and Hedlund, A. (2015). Accounting for the rise in college tuition. *NBER Chapters*.

Harry Anthony Patrinos, C. R.-C. and Sakellariou, C. (2006). Estimating the returns to education: Accounting for heterogeneity in ability. *World Bank Policy Research Working Paper*.

Hector Chade, G. L. and Smith, L. (2014). Student portfolios and the college admissions problem. *Review of Economic Studies*.

Kaufmann, K. M. (2014). Understanding the income gradient in college attendance in mexico: the role of heterogeneity in expected returns. *Quantitative Economics*.

Keane, M. P. and Wolpin, K. (2001). The effect of parental transfers and borrowing constraints on educational attainment. *International Economic Review*.

L. Gasparini, S. Galiani, G. C. and Acosta, P. (2011). Educational upgrading and returns to skills in latin america: Evidence from a demand-supply framework, 1990-2010. *World Bank Policy Research Working Paper Series*.

Lochner, L. J. and Monge-Naranjo, A. (2011). The nature of credit constraints and human capital. *American Economic Review*.

Marc Gurgand, A. L. and Melonio, T. (2011). Student loans: liquidity constraint and higher education in south africa. *Paris School of Economics Working Papers*.

Mestieri, M. (2016). Wealth distribution and human capital: How do borrowing constraints shape schooling systems? *Working Paper*.

OECD and Bank, W. (2012). Tertiary education in colombia. *Reviews of National Policies for Education*.

Rizzo, M. J. and Ehrenberg, R. G. (2002). Resident and nonresident tuition and enrollment at flagship state universities. *NBER*.

Singell, L. D. and Stone, J. A. (2007). For whom the pell tolls: The response of university tuition to federal grants-in-aid. *Economics of Education Review*.

Solis, A. (2013). Credit access and college enrollment. *Uppsala University Working Papers*.

Tatiana Melguizo, F. S. and Velasco, T. (2015). Credit for low-income students, access to and academic performance in higher education in colombia: A regression discontinuity approach. *World Development*.

Zimmerman, S. D. (2014). The returns to college admission for academically marginal students. *Journal of Labor Economics*.

A Appendix A

The problem of the households is:

$$\max_{c,l,h,a} \frac{c^{1-\sigma}}{1-\sigma} + \beta \frac{c'^{1-\sigma}}{1-\sigma}, \quad s.t.$$

$$a + c + hP_h + lP_l = w\theta(1-h)(1-l) + b$$

$$c' = w\theta + w\theta q_h h + w\theta q_l l + (1+r)a$$

A.1 Solution of the unconstrained households:

Proof of Theorem 1. The unconstrained consumptions are:

$$c^N = \frac{(\beta(1+r))^{-1/\sigma}\left(w\theta(2+r)+(1+r)b\right)}{1+(\beta(1+r))^{-1/\sigma}(1+r)}, \qquad c'^N = \frac{\left(w\theta(2+r)+(1+r)b\right)}{1+(\beta(1+r))^{-1/\sigma}(1+r)}$$

$$c^l = \frac{(\beta(1+r))^{-1/\sigma}\left(w\theta(1+q_l)+(1+r)b-P_l(1+r)\right)}{1+(\beta(1+r))^{-1/\sigma}(1+r)}, \qquad c'^l = \frac{\left(w\theta(1+q_l)+(1+r)b-P_l(1+r)\right)}{1+(\beta(1+r))^{-1/\sigma}(1+r)}$$

$$c^h = \frac{(\beta(1+r))^{-1/\sigma}\left(w\theta(1+q_h)+(1+r)b-P_h(1+r)\right)}{1+(\beta(1+r))^{-1/\sigma}(1+r)}, \qquad c'^h = \frac{\left(w\theta(1+q_h)+(1+r)b-P_h(1+r)\right)}{1+(\beta(1+r))^{-1/\sigma}(1+r)}$$

The utilities of each of the options are:

$$u^N = \Phi\left(w\theta(2+r)+b(1+r)\right)^{1-\sigma}$$

$$u^l = \Phi\left(w\theta(1+q_l)+b(1+r)-P_l(1+r)\right)^{1-\sigma}$$

$$u^h = \Phi\left(w\theta(1+q_h)+b(1+r)-P_h(1+r)\right)^{1-\sigma}$$

where

$$\Phi = \left(\frac{1}{1-\sigma}\right)\left(\frac{1}{1+(\beta(1+r))^{-1/\sigma}(1+r)}\right)^{1-\sigma}\left((\beta(1+r))^{(\sigma-1)/\sigma}+\beta\right)$$

The household's decision of whether and where to study follows a *cut-off* rule on θ, and the decision is independent of initial wealth, b. The cut-offs are:

$$\bar{\theta}_l = \frac{1+r}{w}\left(\frac{P_l}{q_l-(1+r)}\right), \quad \bar{\theta}_h = \frac{1+r}{w}\left(\frac{P_h-P_l}{q_h-q_l}\right)$$

□

Proof of Theorem 2. The debt levels of the unconstrained households are:

$$a^N = \frac{w\theta(1-(\beta(1+r))^{-1/\sigma})+b}{1+(\beta(1+r))^{-1/\sigma}(1+r)}$$

$$a^l = \frac{b-P_l-(\beta(1+r))^{-1/\sigma}w\theta(1+q_l)}{1+(\beta(1+r))^{-1/\sigma}(1+r)}$$

$$a^h = \frac{b-P_h-(\beta(1+r))^{-1/\sigma}w\theta(1+q_h)}{1+(\beta(1+r))^{-1/\sigma}(1+r)}$$

Given the exogenous borrowing constraint $\bar{A}$, for a given θ we can construct a cut-off $\bar{b}(\theta)$ on the initial wealth such that individuals with $b < \bar{b}(\theta)$ are constrained and $b \geq \bar{b}(\theta)$ are unconstrained. These are given by:

$$a^N \geq \bar{A} \quad \Leftrightarrow \quad b \geq -\bar{A}(1+(\beta(1+r))^{-1/\sigma}(1+r)) - w\theta(1-(\beta(1+r))^{-1/\sigma})$$

$$a^l \geq \bar{A} \quad \Leftrightarrow \quad b \geq P_l + (\beta(1+r))^{-1/\sigma}w\theta(1+q_l) - \bar{A}(1+(\beta(1+r))^{-1/\sigma}(1+r))$$

$$a^h \geq \bar{A} \quad \Leftrightarrow \quad b \geq P_h + (\beta(1+r))^{-1/\sigma}w\theta(1+q_h) - \bar{A}(1+(\beta(1+r))^{-1/\sigma}(1+r))$$

That is, the cut-offs are:

$$\bar{b_u^N}(\theta) = -\bar{A}(1+(\beta(1+r))^{-1/\sigma}(1+r)) - w\theta(1-(\beta(1+r))^{-1/\sigma})$$

$$\bar{b_u^l}(\theta) = P_l + (\beta(1+r))^{-1/\sigma}w\theta(1+q_l) - \bar{A}(1+(\beta(1+r))^{-1/\sigma}(1+r))$$

$$\bar{b_u^h}(\theta) = P_h + (\beta(1+r))^{-1/\sigma}w\theta(1+q_h) - \bar{A}(1+(\beta(1+r))^{-1/\sigma}(1+r))$$

□

This subdivides the state space in three subregions, as shown in the following Figure 7.

A.2 Solution of the constrained households:

Next, we have to consider the decision of studying of those guys that are constrained. Note that, although if an individual is borrowing constrained when he decides to study, he might prefer to study and not smooth consumption, than not studying and being able to smooth consumption. Therefore, we must compare the utility of studying while being constrained, with the utility of not studying and being unconstrained. The constrained consumptions are given by:

$$c_c^N = w\theta + b + \bar{A}, \quad c'^N_c = w\theta - (1+r)\bar{A}$$

$$c_c^l = b - P_l + \bar{A}, \quad c'^l_c = w\theta(1+q_l) - (1+r)\bar{A}$$

$$c_c^h = b - P_h + \bar{A}, \quad c'^h_c = w\theta(1+q_h) - (1+r)\bar{A}$$

Individuals that are constrained when studying in l, will choose to study whenever (b, θ) are such that:

$$\left(\frac{1}{1-\sigma}\right)(b - P_l + \bar{A})^{1-\sigma} + \left(\frac{\beta}{1-\sigma}\right)(w\theta(1+q_l) - (1+r)\bar{A})^{1-\sigma} - \Phi(w\theta(2+r) + b(1+r))^{1-\sigma} \geq 0$$

Here, b is an implicit function of θ. There is no analytical solution for the cutoff that divides the state space. However, using the implicit function theorem we can derive the following result:

Proof of Theorem 3. Among constrained households there are two cases:

1. Individuals that are constrained when studying at h, and constrained when studying at l.
2. Individuals that are constrained when studying at h, but not constrained when studying at l.

The decision between studying at l, h or not studying at all depends on whether the individual is constrained under l, h or both. The following three equations will implicitly define the cut-offs

:

$$G(\theta, b) = \left(\frac{1}{1-\sigma}\right)(b(\theta^*) - P_l + \bar{A})^{1-\sigma} + \left(\frac{\beta}{1-\sigma}\right)(w\theta^*(1+q_l) - (1+r)\bar{A})^{1-\sigma} - \Phi(w\theta^*(2+r) + b(\theta^*)(1+r))^{1-\sigma} \tag{16}$$

and $b(\theta)$ implictly defined by the equality $G(\theta, b(\theta)) = 0$. By the implicit function theorem,

$$\partial b / \partial \theta = -\frac{\partial G / \partial \theta}{\partial G / \partial b}$$

Setting $\partial G / \partial \theta = 0$ gives the result in Theorem 3. □

Individuals that are constrained when studying in both l and h, will decide to study in l when:

$$\left(\frac{1}{1-\sigma}\right)(b - P_l + \bar{A})^{1-\sigma} + \left(\frac{\beta}{1-\sigma}\right)(w\theta(1+q_l) - (1+r)\bar{A})^{1-\sigma} \geq$$
$$\left(\frac{1}{1-\sigma}\right)(b - P_h + \bar{A})^{1-\sigma} + \left(\frac{\beta}{1-\sigma}\right)(w\theta(1+q_h) - (1+r)\bar{A})^{1-\sigma}$$

Proof. By implicit function theorem, $\partial b/\partial \bar{A} = -\frac{\partial G/\partial \bar{A}}{\partial G/\partial b}$.

$$\frac{\partial G}{\partial \bar{A}} = (b - P_l + \bar{A})^{-\sigma} + \beta(1+r)(w\theta(1+q_l) - (1+r)\bar{A})^{-\sigma} \geq 0$$

Since $\partial G/\partial b > 0$, the first result follows.

For the second result, note that:

$$\begin{aligned}
\frac{\partial b}{\partial \bar{A}\partial \theta} = & \frac{1}{(\cdot)^2}\Big[\Big(\sigma\beta(1+r)w(1+q_l)(w\theta(1+q_l) - (1+r)\bar{A})^{-(1+\sigma)}\Big) \\
& \qquad \cdot \big((1+r)((b - P_l + \bar{A})^{-\sigma} - \Phi(1-\sigma)(1+r)(w\theta(2+r) + b(1+r))^{-\sigma})\big)\Big] \\
& + \frac{1}{(\cdot)^2}\big[\big((b - P_l + \bar{A})^{-\sigma} + \beta(1+r)(w\theta(1+q_l) - \bar{A}(1+r))^{-\sigma}\big) \\
& \qquad \cdot \big(\sigma\Phi(1-\sigma)w(1+r)(2+r)(w\theta(2+r) + b(1+r))^{-\sigma}\big)\big] \\
& \geq 0
\end{aligned} \tag{17}$$

This proves Theorem 4.

□

The above theorem is important for the following reason. The inefficiency in the education sector is given mainly by the highest θ individuals that, because of the borrowing constraints, decide not to study. Implementing a policy that relaxes borrowing constraints affects to a higher extent the lower θ's. Therefore, the most cost-effective government policy should be targeted at relaxing the borrowing constraints of the highest θ's.

A.3 Computation of Nash Equillibrium

In this section we will describe the algorithm used to compute the Nash Equilibrium between elite and non-elite universities. The Nash Equilibrium is composed by a tuple $(P_h^*, \underline{\theta_h}^*, P_l^*, \underline{\theta_l}^*)$ such that:

$$(P_i^*, \underline{\theta_i}^*) \in \arg \max_{(P_i, \underline{\theta_i}) \in \mathcal{R}^+ \times [0,1]} \Big(z_i(P_i, \underline{\theta_i}, P_{-i}^*, \underline{\theta_{-i}}^*) \Big)^{\alpha} \Big(\sigma_{b,i}(P_i, \underline{\theta_i}, P_{-i}^*, \underline{\theta_{-i}}^*) \Big)^{1-\alpha} \tag{18}$$

Note that the problem defined in 18 involves solving for a fixed point nested within another fixed point problem. In particular, the universities will offer a given level of z_l, z_h to the households and, conditional on such offer households will demand education services that need to fulfill the promised levels of z_l, z_h. This implies that when solving for the optimal of the universities we need to take into account that the offered level of productivities need to be satisfied by the demand of educational services. The full procedure to find the Equilibrium is described below:

Computation of the Nash Equilibrium

1. Start algorithm with some initial guess $\langle P_h^g, \underline{\theta_h^g}, P_l^g, \underline{\theta_l^g}\rangle$. Set $E = 10$.
2. Find $\langle P_h^T, \underline{\theta_h^T}\rangle \in \arg\max_{(P_h,\underline{\theta_h})\in\mathcal{R}+\times[0,1]} \left(z_h(P_h, \underline{\theta_h}, P_l^g, \underline{\theta_l}^g)\right)^{\alpha} \left(\sigma_{b,h}(P_h, \underline{\theta_h}, P_l^g, \underline{\theta_l}^g)\right)^{1-\alpha}$
   (a) Set $\langle P_h^r, \underline{\theta_h^r}\rangle = \langle P_h^g, \underline{\theta_h^g}\rangle$
   (b) Given $\langle P_h^r, \underline{\theta_h^r}, P_l^g, \underline{\theta_l^g}\rangle$, go to 5. to compute $\langle z_h, z_l\rangle$
   (c) Given $S1 = \langle P_h^r, \underline{\theta_h^r}, P_l^g, \underline{\theta_l^g}, z_h, z_l\rangle$ compute the objective function of the university $H(S1)$.
   (d) Update for a new guess of the optimal $\langle P_h^r, \underline{\theta_h^r}\rangle = \langle P_h^{new}, \underline{\theta_h^{new}}\rangle$ according to some rule.
   (e) Repeat $(b)-(d)$ until optimal $\langle P_h^T, \underline{\theta_h^T}\rangle$ is found
3. Find $\langle P_l^T, \underline{\theta_l^T}\rangle \in \arg\max_{(P_l,\underline{\theta_l})\in\mathcal{R}+\times[0,1]} \left(z_l(P_h^g, \underline{\theta_h^g}, P_l, \underline{\theta_l})\right)^{\alpha} \left(\sigma_{b,l}(P_h^g, \underline{\theta_h^g}, P_l, \underline{\theta_l})\right)^{1-\alpha}$
   (a) Set $\langle P_l^r, \underline{\theta_h^l}\rangle = \langle P_l^g, \underline{\theta_l^g}\rangle$
   (b) Given $\langle P_h^g, \underline{\theta_h^g}, P_l^r, \underline{\theta_l^r}\rangle$, go to 5. to compute $\langle z_h, z_l\rangle$
   (c) Given $S1 = \langle P_h^g, \underline{\theta_h^g}, P_l^r, \underline{\theta_l^r}, z_h, z_l\rangle$ compute the objective function of the university $L(S1)$.
   (d) Update for a new guess of the optimal $\langle P_l^r, \underline{\theta_l^r}\rangle = \langle P_l^{new}, \underline{\theta_l^{new}}\rangle$
   (e) Repeat $(b)-(d)$ until optimal $\langle P_l^T, \underline{\theta_l^T}\rangle$ is found
4. Set $E = ||\langle P_h^g, \underline{\theta_h^g}, P_l^g, \underline{\theta_l^g}\rangle - \langle P_h^T, \underline{\theta_h^T}, P_l^T, \underline{\theta_l^T}\rangle||$. If E is smaller than a tolerance level, stop the algorithm, the NE is given by the tuple $\langle P_h^T, \underline{\theta_h^T}, P_l^T, \underline{\theta_l^T}\rangle$. Otherwise, set $\langle P_h^g, \underline{\theta_h^g}, P_l^g, \underline{\theta_l^g}\rangle = \langle P_h^T, \underline{\theta_h^T}, P_l^T, \underline{\theta_l^T}\rangle$ and go to 2.
5. Computation of $\langle z_h, z_l\rangle$ given $\langle P_h, \underline{\theta_h}, P_l, \underline{\theta_l}\rangle$
   (a) Start algorithm with some initial guess $\langle z_h^g, z_l^g\rangle$ and set $\varepsilon = 10$
   (b) Given $\langle P_h, \underline{\theta_h}, P_l, \underline{\theta_l}\rangle$, the guess $\langle z_h^g, z_l^g\rangle$ and the policy functions of the households, compute the realized values of $\langle z_h^r, z_l^r\rangle$
   (c) set $\varepsilon = (z_h^r - z_h^g)^2 + (z_l^r - z_l^g)^2$.
   (d) If ε is smaller to a tolerance level, the algorithm is complete. Otherwise, set $\langle z_h^g, z_l^g\rangle = \langle z_h^r, z_l^r\rangle$ and go to (b).

A.4 Analysis in the linear case

In order to get a clear idea of how credit constraints affect the market for higher education, we illustrate the linear case where $\sigma = 1$. Furthermore, we need to distinguish scenarios where households would like to substitute future for current consumption and the other way around. This is given by the inequality $\beta(1+r) < 1$. Whenever this inequality is satisfied, households would prefer to get as much debt during the first period. The opposite case, when $\beta(1+r) \geq 1$ will motivate households to save as much as possible given that the returns to savings, in terms of utility, are more than one to one.

Case 1.$\beta(1+r) \geq 1$

In this case, households will prefer to save as much as they want and then the value functions for each case (not study, study in low quality university or study in high quality university) are given by:

$$V^N(b,\theta) = \beta\left[b(1-\tau)(1+r) + w\theta(2+r)\right] \tag{19}$$

$$\tag{20}$$

The value function for households going to the low quality university is only defined whenever they can afford it. That is, whenever $P_l - b(1-\tau) \leq \min\{\bar{A}, \frac{w\theta(1+z^l)}{1+r}\}$. In particular, consider the case where $P_l - b(1-\tau) \leq 0$. If this holds, then households are able to afford the price of education with their income after taxes and thus we have no concerns about they not getting enough debt to fund their education.

However, when students should get positive debt in order to attend the low quality university, the amount of debt should satisfy two constraints:

$$P_l - b(1-\tau) \leq \bar{A} \tag{21}$$

$$P_l - b(1-\tau) \leq \frac{w\theta(1+z^l)}{1+r} \tag{22}$$

The constraint given in 21 states that the amount of debt students get should not exceed the upper limit given exogenously in the economy. The inequality given in 22 guarantees that students have enough funds to get the necessary debt to attend college. The two aforementioned inequalities give bounds in b and θ for students to being able to pay the tuition in the low quality college:

$$b \geq b_{p_l} = \frac{\bar{A} - P_l}{1-\tau} \tag{23}$$

$$b \geq L(\theta) = \frac{P_l}{1-\tau} - \frac{w\theta(1+z^l)}{(1-\tau)(1+r)} \tag{24}$$

Now, for households with state variables (b,θ) such that low quality education is affordable, we can define the value of going to the low university as:

$$V^L(b,\theta) = \beta\left[(b(1-\tau) - Pl)(1+r) + w\theta(1+z^l)\right] \tag{25}$$

Similarly, in order to be able to go to the high quality institutions, it should be the case that:

$$b \geq b_{p_h} = \frac{\bar{A} - P_h}{1-\tau} \tag{26}$$

$$b \geq H(\theta) = \frac{P_h}{1-\tau} - \frac{w\theta(1+z^h)}{(1-\tau)(1+r)} \tag{27}$$

For those households, we can define the value of going to the high quality college as:

$$V^H(b,\theta) = \beta\left[(b(1-\tau) - P_h)(1+r) + w\theta(1+z^h)\right] \tag{28}$$

Consider the case of a person who is deciding whether to go to the low quality college or not study. In such case, granted that he could afford to pay tuition, he will decide to attend whenever $V^L(b,\theta) \geq V^N(b,\theta)$. This implies that the decision will be to go to the low quality college whenever:

$$\theta_l \geq \theta_L = \frac{P_l(1+r)}{w[z^l - r - 1]} \tag{29}$$

Similarly, when a person is deciding whether to go to the high quality college or to the low quality one, and granted he could afford both, the relevant decision rule will be to go to the high quality college whenever $V^H(b,\theta) \geq V^L(b,\theta)$. This inequality generates the decision rule of going to college whenever:

$$\theta \geq \theta_H = \frac{(P_h - P_l)(1+r)}{w(z^h - z^l)} \tag{30}$$

The decision rules can be represented in the state space according to the following graph:

Note that we can express N^H in terms of elements that we have found previously:

$$N^H = \int_{\theta^H}^{\theta^{Ih}} \int_{H(\theta)}^{\bar{b}} dF(b,\theta) + \int_{\theta^{Ih}}^{1} \int_{b_{Ph}}^{\bar{b}} dF(b,\theta) \tag{31}$$

where $\bar{b}$ is the maximum level of bequests in the state space and

$$\theta^{Ih} = \frac{(1+r)\bar{A}}{(1+z^h)w} \tag{32}$$

For the sake of simplicity, we will assume a uniform distribution for (b,θ). As long as $P^h > P^l$ and $z^h > z^l$ we can express the measure of people going to the high quality university as:

$$N^H = \frac{1}{\bar{b}}\left[\left(b - \frac{P_h}{1-\tau}\right)\left(\frac{(1+r)\bar{A}}{(1+z^h)w} - \frac{(P_h - P_l)(1+r)}{w(z^h - z^l)}\right) + \frac{w(1+z^h)}{(1-\tau)(1+r)}\left[\left(\frac{(1+r)\bar{A}}{w(1+z^h)}\right)^2 - \left(\frac{(P_h - P_l)(1+r)}{w(z^h - z^l)}\right)^2\right]\right] \tag{33}$$

Similarly, the average level of skills of people attending such college is given by:

$$\tilde{\theta}^H = \frac{1}{\bar{b}} \left[\left(\left(\frac{(1+r)\bar{A}}{(1+z^h)w} \right)^2 - \left(\frac{(P_h - P_l)(1+r)}{w(z^h - z^l)} \right)^2 \right) \left(\frac{\bar{b}}{2} - \frac{P_h}{2(1-\tau)} \right) + \frac{w(1+z^h)}{3(1-\tau)(1+r)} \left[\left(\frac{(1+r)\bar{A}}{w(1+z^h)} \right)^3 - \left(\frac{(P_h - P_l)(1+r)}{w(z^h - z^l)} \right)^3 \right] + \right.$$

$$\left. \frac{1}{2} \left[\bar{b} - \frac{\bar{A}}{1-\tau} + \frac{P_h}{1-\tau} \left(1 - \left(\frac{(1+r)\bar{A}}{1(1+z^h)} \right)^2 \right) \right] \right] \tag{34}$$

And finally, the average level of income of students entering the high-quality college will be given by:

$$\mu_b^H = \frac{1}{2\bar{b}} \left[\left((\bar{b})^2 - \left(\frac{\bar{A} - P_h}{1-\tau} \right)^2 \right) \left(1 - \frac{(1+r)\bar{A}}{w(1+z^h)} \right) + \left((\bar{b})^2 - \left(\frac{P_h}{1-\tau} \right)^2 \right) \left[\frac{(1+r)\bar{A}}{w(1+z^h)} - \frac{(P_h - P_l)(1+r)}{w(z^h - z^l)} \right] + \right.$$

$$\left. \frac{P_h w(1+z^h)}{(1-\tau)(1+r)} \left[\left(\frac{(1+r)\bar{A}}{(1+z^h)w} \right)^2 - \left(\frac{(P_h - P_l)(1+r)}{w(z^h - z^l)} \right)^2 \right] - \frac{w(1+z^h)}{3(1-\tau)(1+r)} \left[\left(\frac{(1+r)\bar{A}}{w(1+z^h)} \right)^3 - \left(\frac{(P_h - P_l)(1+r)}{w(z^h - z^l)} \right)^3 \right] \right] \tag{35}$$

We can express the relevant variables for low quality college, granted $P_h > P_l$ and $z_h > z_l$, as:

$$N^L = \int_{\theta_L}^{\theta_H} \int_{L(\theta)}^{1} dF(b,\theta) + \int_{\theta_H}^{\theta^{Il}} \int_{L(\theta)}^{H(\theta)} dF(b,\theta) + \int_{\theta^{Il}}^{\theta^{Ih}} \int_{b_{Pl}}^{H(\theta)} dF(b,\theta) + \int_{\theta^{Ih}}^{1} \int_{b_{Pl}}^{b_{Ph}} dF(b,\theta) \tag{36}$$

$$\tilde{\theta}^L = \int_{\theta_L}^{\theta_H} \int_{L(\theta)}^{1} \theta dF(b,\theta) + \int_{\theta_H}^{\theta^{Il}} \int_{L(\theta)}^{H(\theta)} \theta dF(b,\theta) + \int_{\theta^{Il}}^{\theta^{Ih}} \int_{b_{Pl}}^{H(\theta)} \theta dF(b,\theta) + \int_{\theta^{Ih}}^{1} \int_{b_{Pl}}^{b_{Ph}} \theta dF(b,\theta) \tag{37}$$

$$\mu_{b^L} = \int_{\theta_L}^{\theta_H} \int_{L(\theta)}^{1} b dF(b,\theta) + \int_{\theta_H}^{\theta^{Il}} \int_{L(\theta)}^{H(\theta)} b dF(b,\theta) + \int_{\theta^{Il}}^{\theta^{Ih}} \int_{b_{Pl}}^{H(\theta)} b dF(b,\theta) + \int_{\theta^{Ih}}^{1} \int_{b_{Pl}}^{b_{Ph}} b dF(b,\theta) \tag{38}$$

It is important to note that throughout this analysis we have not implemented the fact that both colleges are able to set a threshold rule such that people with a level of skills below such threshold will not be admitted. In such a case, we will simply modify the regions of integration to consider that only people with ability beyond the threshold will be able to attend.

Existence of equilibrium

The expressions found in 33, 34, 36 and 37 can be used to express the necessary conditions that the offered qualities need to satisfy in equilibrium. In particular, we need to find z^h, z^l such that:

$$\begin{bmatrix} z^h \\ z^l \end{bmatrix} = \begin{bmatrix} \kappa^h \left(\tilde{\theta}^h(\underline{\theta}^h, \underline{\theta}^l, P_h, P_l, z^h, z^l) \right)^{\alpha_1} \left(I(\underline{\theta}^h, \underline{\theta}^l, P_h, P_l, z^h, z^l) \right)^{\alpha_2} \\ \kappa^l \left(\tilde{\theta}^h(\underline{\theta}^l, \underline{\theta}^l, P_h, P_l, z^h, z^l) \right)^{\alpha_1} \left(I(\underline{\theta}^h, \underline{\theta}^l, P_h, P_l, z^h, z^l) \right)^{\alpha_2} \end{bmatrix} \tag{39}$$

We need to prove existence of a fixed point in the qualities offered by universities before proving the existence of the Nash Equilibrium. Note, however, that difficulty arises in this point given the fact that there is no natural way to bound the set of qualities offered by the universities. Additionally, note that equations 33, 34, 36 are not continuous in $z^h = z^l$. The inability of proving the existence of a fixed point in the qualities offered by universities shows that it is not possible to prove existence of the Nash Equilibrium. We rely purely on the computational analysis to find a Nash Equilibrium in this case that might not be unique.

Case 2.$\beta(1+r) < 1$

This case is more involved as households value more current consumption than future and will try to get as much debt as possible. The difficulty arises as even when students can afford to pay college, they might be constrained given that they want to substitute future by current consumption. Additionally, we need to establish which is the relevant constraint that households face when getting the desired level of debt, either the exogenously given level of credit constraint or they reach a point where they can't fund the debt with their resources.

We start analyzing the case of a person who is not going to university. In this case, the person will get as much debt as possible and he will be constrained whenever $\frac{w\theta}{1+r} > \bar{A}$. If this is the case, the person will get the maximum level of debt $\bar{A}$. Taking into account this case when computing the value of not going to college, we see that:

$$V^N(b,\theta) = \begin{cases} b(1-\tau) + w\theta\frac{2+r}{1+r} \text{ if } \theta \leq \frac{\bar{A}(1+r)}{w} \\ b(1-\tau) + w(\theta)(1+\beta) + \bar{A}[1-\beta(1+r)] \text{ if } \theta > \bar{A}\frac{1+r}{w} \end{cases} \tag{40}$$

Now, let's consider a household that goes to the low-quality university. Evidently, the value function will only be defined for the case when it is possible to pay tuition price via endowment or debt. For people whose income is below the tuition price ($b(1-\tau) < P_l$) and who are constrained either by the exogenous level $\bar{A}$ or by their earning capacity $\frac{w\theta(1+z^l)}{1+r}$, the value of going to the low quality college will not be defined.

An individual who is not constrained and takes as much debt as he can, will derive utility given by $b(1-\tau) - P_l + \frac{w\theta(1+z^l)}{1+r}$. The first term, $b(1-\tau) - P_l$ corresponds to net income after tuition and the remaining part $\frac{w\theta(1+z^l)}{1+r}$ is simply the amount they will make in the second period taken to the present value of the first period.

If the net income after tuition is negative, an individual will not be credit constrained so long as:

$$P_l - b(1-\tau) \leq \min\{\bar{A}, \frac{w\theta(1+z^l)}{1+r}\} \tag{41}$$

However, it is possible to have individuals who are borrowing constrained even if the net income after tuition is positive. These individuals are those who would like to borrow against their future income, given that current consumption is more valuable than future consumption, but they are not able to borrow as much as they want given the exogenous limit $\bar{A}$. Those are individuals such that:

$$\frac{w\theta(1+z^l)}{(1+r)} < \bar{A} \tag{42}$$

and they are forced to borrow no more than $\bar{A}$. This implies that we can define the value of going to low-quality college as:

$$V^L(b,\theta) = \begin{cases} b(1-\tau) - P_l + \frac{w\theta(1+z^l)}{1+r} \text{ if } \begin{cases} b(1-\tau) - P_l \geq 0 \quad \theta \leq \frac{\bar{A}(1+r)}{w(1+z^l)} \\ \text{or} \\ b(1-\tau) - P_l < 0 \quad P_l - b(1-\tau) \leq \min\{\bar{A}, \frac{w(\theta)(1+z^l)}{1+r}\} \end{cases} \\ \\ b(1-\tau) - P_l + \bar{A}[1-\beta(1+r)] + w\beta(1+z^l) \text{ if } b(1-\tau) - P_l > 0 \text{ and } \theta > \frac{\bar{A}(1+r)}{w(1+z^l)} \end{cases} \tag{43}$$

Finally, doing the same analysis but with P_h and z^h we can find the value of going to the high quality college:

$$V^H(b,\theta) = \begin{cases} b(1-\tau) - P_h + \frac{w\theta(1+z^h)}{1+r} \text{ if } \begin{cases} b(1-\tau) - P_h \geq 0 \quad \theta \leq \frac{\bar{A}(1+r)}{w(1+z^h)} \\ \text{or} \\ b(1-\tau) - P_h < 0 \quad P_h - b(1-\tau) \leq \min\{\bar{A}, \frac{w(\theta)(1+z^h)}{1+r}\} \end{cases} \\ \\ b(1-\tau) - P_h + \bar{A}[1-\beta(1+r)] + w\beta(1+z^h) \text{ if } b(1-\tau) - P_h > 0 \text{ and } \theta > \frac{\bar{A}(1+r)}{w(1+z^h)} \end{cases} \tag{44}$$

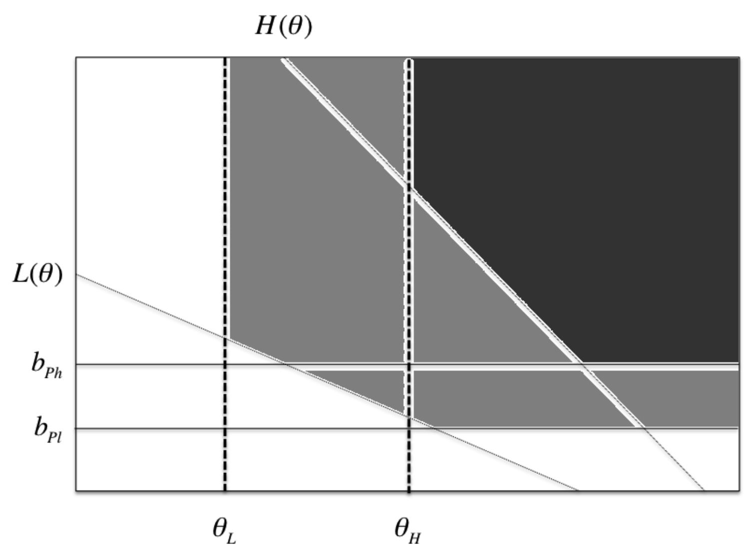


Figure 9: Representation of the education decisions on the state space.