Parents and Children: a Collective Model of Household Behavior and Child Development

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August 7, 2016

Abstract

There is extensive evidence suggesting that skills developed early in life have consequences on adult life outcomes. Such findings have motivated a large body of literature analyzing the production of skills in young children. Nonetheless, very little is known about how families make decisions of investments in their children. In this article we estimate a collective model of labor supply and investment in children. The parameters estimated are used to simulate the effects of various policies aimed at increasing skills of children in disadvantaged households. The results show that policies subsidizing monetary investments in children are the most efficient in closing the gap of skills between rich and poor households. However, the overall effect of such policies is very small, evidencing that developing policies to increase skills in young children is a huge challenge for policymakers.

1 Introduction

Research in medicine, psychology and economics, shows that the way skills are formed during the first years of life has significant consequences on adult life outcomes[1]. This fact has motivated a large number of studies aimed at identifying the determinants of having an adequate early childhood development. Some of the results of such studies have allowed us to get a better understanding of which are the key inputs to promote skills in young children[2]. For instance, we know that parenting and general family environment are amongst the most relevant inputs in the production of skills (Heckman & Mosso, 2014; Schoellman, 2014).

*University of Pennsylvania. This research is partially supported by the Eunice Kennedy Shriver National Institute of Child Health and Human Development (NICHD R01HD065436) grant on "Early Child Development Programs: Effective Interventions for Human Development". I thank Petra Todd, Andrew Shephard and Jere Behrman for their advice and support. Aureo de Paula, Chris Flinn, Holger Sieg and Hanming Fang as well as participants at the University of Pennsylvania applied micro club, ECONCON-2015 and LACEA-2015 made comments that significantly improve the quality of this paper. I am grateful to the Centro Nacional de Microdatos for their support with the dataset as well as Daniela Marshall and Alejandra Abufhele for their guidance and help with the data.

[1]For a review see Conti and Heckman (2012)

[2]See, for example, Cunha, Heckman, and Schennach (2010)

However, having a good characterization of the production function is not enough to assess the effectiveness of policies aimed at improving skills of children. Families administer the resources and make the relevant decisions that determine the allocation of inputs for young children. For that reason, in order to fully understand the consequences of public policies on the skills of young children, we need to get a better understanding of how families decide on the allocation of resources for their children. In this article, I fill this gap in the literature by estimating a production function of skills in young children nested within a collective model of household behavior.

In order to identify how families allocate resources to their children, we need to take into account some basic facts about this process. First of all, the decisions in the family are rarely made by one individual but by multiple ones with different resources and preferences. Parents might have different preferences about what is the optimal amount of time and money that they should spend in their children and somehow they conciliate these differences in order to reach an agreement. Considering households as a single economic entity with a unique utility function have several empirical implications that have been widely rejected in various contexts[3]. In this paper I acknowledge such possibility by allowing parents to have different preferences and capabilities in various ways.

An important feature regarding the process of skills formation in young children is that mothers tend to invest significantly more time in their children than fathers. One possible explanation for this is that mother's time is more productive than father's time. Additionally, it can be due to differences in preferences or to a relative disempowerment of women with respect to men. We cannot asses the extent to which each factor contribute to the overall difference between fathers and mother by using a unitary model of household decision-making. By estimating a collective model of household behavior I am able to disentangle the contribution of each factor to the total difference in time invested between parents. I find that 14% of such difference is due to women being relatively less empowered than men in the average household and the remainder is explained by differences in preferences. I find no evidence of mother's time being more productive than father's time in the production of skills.

When estimating a production of skills in young children, it is important to acknowledge the possibility of measurement error in both inputs and outputs. The results of test scores and questionnaires are never a perfect measure of skills and investments in young children. Ignoring the possibility of measurement error can lead to radically different conclusions about the efficiency of policy interventions (Cunha et al., 2010). The data used for this article contains multiple measures

[3] See Browning, Chiappori, and Weiss (2014) for a review.

of investments and skills that allow me to treat such information as noisy measures of underlying factors representing the true elements of analysis.

This article also makes methodological contributions to the literature on Family Economics and Labor Economics. First of all, there are very few empirical applications of the collective model of household behavior and this article proposes a novel estimation strategy for such models. The few empirical applications of the collective model use information on the consumption of private goods in order to asses the relative importance of each household member in the process of decision-making. The crucial assumption is that there is reliable information available on the consumption of purely private goods, which is arguable[4]. By having information about measures of empowerment and gender roles within the household, I do not need to impose such assumptions and I am able to use direct information on the process of decision-making -allowing for measurement error- in order to identify the bargaining power of each member of the household.

Finally, particle filtering techniques are often used in macroeconomics and macro econometrics in order to estimate non-linear non-gaussian processes Fernández-Villaverde and Rubio-Ramírez (2007). Although such techniques are computationally expensive, they allow better empirical estimates by imposing fewer assumptions compared to alternatives such as the Extended Kalman Filter. This article is, to the best of my knowledge, the first one to use such techniques in the context of family economics and one of the few applications in applied microeconomics. As such, it brings to the table a new methodology for the estimation of such models.

The estimation results are used to asses the effects of different policy counterfactuals on the skills of children coming from disadvantaged households. Initially, we observe that children in the lowest quintile of the income distribution are, on average, 1.1 standard deviations below the mean of stock of skills. Implementing a policy that doubles the amount of monetary transfers that these households receive from the government reduces such gap only in 2% and decreases female employment in three percentage points. Using the same amount of money to subsidize childcare increases female employment in 0.4 percentage points but the gap in skills is only reduced in about 1.5%.

I find that the most effective policy is to subsidize monetary investments in children. By halving the price of goods that are productive for children skills such as books for children, toys and food, the top-bottom gap in skills is decreased in about 4.5% while not discouraging women from participating in the labor market. In addition to being more effective closing the gap in skills and in discouraging women from the labor market, this policy is much cheaper than the other

[4]Perhaps the most common private good used for this purpose is clothing. However, the assumption that each spouse is indifferent about the clothing of the other seems problematic.

alternatives.

One of the most relevant policies aimed at improving the skills of young children is giving direct monetary transfers to poor households. Such policies give a specified amount of money to a household member that might be conditional on making specific investments in the human capital of their children and in some countries they represent the largest social assistance program from the central government[5]. Monetary transfers increase the resources of the household but also distort the behavior of its members in ways that could have consequences on the production of skills of their children. Increasing a household's non-labor income might modify the labor force participation decisions of its members, thus altering the amount of time that parents can spend with their children. Additionally, such transfers make monetary investments in children -such as taking them childcare or improving the quality of their food or toys- cheaper relative to time investments. By having detailed information on time and monetary investments I am able to use exogenous policy variations in order to identify the effect of such policies on the investment parents make on their children.

An important feature of cash transfers programs is that it is almost always the mother who is the recipient of the money. There is overwhelming evidence supporting the fact that making a transfer to the mother, rather than the father, translates into better child outcomes[6]. A common interpretation given by the literature to this fact is that mothers have stronger preferences for children and thus monetary transfers should be made to mothers rather than fathers. Nonetheless, having stronger preferences for children is not a necessary or sufficient condition guaranteeing that transferring the dollar to a mother will be better in terms of child development than doing so to the father. The reason why it is hard to isolate the mechanism is because transfers modify household behavior in a wide variety of aspects. To identify what mechanism is generating such differences in outcomes, it is necessary to estimate an economic model of household behavior. This allows identification of the mechanisms generating the asymmetry of results of monetary transfers and also to generate policy counterfactuals taking into account that the identity of the beneficiary matters. The results of this article show that mothers and fathers care the same for their children, which implies that the mechanism generating the better outcomes when transfers are done to the mother should be related to several other features of the behavior of the household.

In order to rationalize the fact that the identity of the recipient of monetary transfer matters it is not possible to model the household as a single entity with a utility function, the so called

[5]Such is the case of Brazil and Mexico (Fiszbein, Schady, & Ferreira, 2009)

[6]See Thomas (1990), Thomas (1994), Browning, Bourguignon, Chiappori, and Lechene (1994),

unitary approach of household behavior. The collective model of household behavior has been the main alternative to the unitary approach. The assumptions of the collective model are much less restrictive and its empirical predictions have been shown to be more accurate than the unitary model. This paper is, to my knowledge, the first attempt to estimate a collective model of household behavior and child outcomes.

Although this paper uses some of the identification results from existing literature, the estimation strategy is novel. The empirical implementations of the collective model rely on observing the consumption of a private good -usually clothing- by each household member which can reveal information about the relative importance of each family member [7]. Such approach assumes that one spouse does not care at all about the consumption of the other and that there is a one to one mapping between the consumption level and the Pareto weight or bargaining power within the household, without allowing for measurement error. In this paper I use a set of questions regarding the process of decision making within the household in order to back up the relative weight of each household member while allowing for measurement error.

We will use data from Chile, which is a country that has been historically characterized by low levels of female labor force participation for its income level. The central government has created a wide variety of policies intended to increase the participation of women in the labor market such as extension of childcare services or the provision of health services to women participating in the labor market. The results of this paper can be used to provide counterfactual analysis in the design of such policies taking into account not only the labor market response of adult members but also the consequences on the production of skills in children.

The estimates of the economic model are used to asses the consequences of increasing the amount of monetary transfers that poor households receive. The results of the counterfactual analysis show that, while the monetary transfer itself increases the amount of resources for the household, an important feature of such policies is the distortion generated in the behavior of the household. As women get more monetary transfers they become relatively more empowered and will have more control of the resources of the household. Balancing the power within the household reduces the marginal utility of private consumption of both household members, leaving monetary investments to children as the most profitable alternative to invest the additional resources received by the household.

The remaining of this article is structured as follows: In Section 2 we do a brief review of the literature in order to identify what is the main contribution of this article. We will describe the

[7] See for instance Cherchye, De Rock, and Vermeulen (2012)

data used in Section 3. In Section 4 we present some preliminary evidence motivating the economic model, which will be described in Section 5. The estimation procedure, altogether with the relevant identification arguments are introduced in Section 8. The main results of the paper will be included in Section 9 and we will summarize the main points of the article in Section 11.

2 Review of the literature

This article is related to four areas of the literature and, to different extents, it makes a contribution in each of them. First of all, this paper is related to the literature analyzing how household behavior affects the production of skills in children. One of the most important decisions families need to make that will have consequences on the production of skills in children is that of labor supply. As household members increase the participation in the labor market this will bring more monetary resources to the household but the amount of time parents interact with their children is reduced. For this reason, it is not evident at first hand what is the impact of labor force participation on children's skills.

The question of how labor supply decisions affect the production of skills in young children has been somewhat explored in the literature. Bernal (2008) estimates a structural model of female labor force participation taking into account that skills depend not on income and on the total amount of time that mothers interact with their children. The author estimates the model in a sample of single mothers and thus, when such mothers decide to participate full time in the labor market they will have to use childcare services. Taking into account the overall effect of an increase in income but a decrease in the amount of time that mothers interact with their children, the author finds that one year of full employment has an impact of decreasing skills in children by approximately 0.13 standard deviations.

Del Boca, Flinn, and Wiswall (2014) extend the results of Bernal (2008) in order to take into account families with two members besides the child: father and mother. The authors estimate a unitary dynamic model of household behavior where each parent allocates time to labor market, leisure or interacting with their children. Additionally, the authors incorporate the decision of how much money to allocate to monetary investments in their children or to consumption. The authors find that when mothers increase the amount of labor being supplied, the negative effect this might have is not only alleviated by the increase in the amount of resources brought from wages but also by the fact that the father starts to spend more time with the children at home. One of the main conclusions of the authors is that time of both, fathers and mothers, are relatively more important than monetary investments into the production of skills in children.

Although Del Boca et al. (2014) brings to the literature the consideration of fathers and mothers altogether in the production of cognitive skills in children, it does so in a unitary approach. Their modeling approach fails to incorporate the fact that parents might have differences in preferences. It is important to take into account this fact as cash transfers are an important policy tool to alleviate poverty in developing countries. When making a transfer to a mother, not only the resources of the household will increase but the decisions of the household will be more aligned with preferences of the mothers. The unitary approach fails to take into account this second effect which the literature has acknowledged to be significant [8]. Additionally, one limitation of Del Boca et al. (2014) is that they don't observe investments in children but identification comes exclusively from assuming a particular functional form of the utility and the production functions. By considering household behavior using a unitary framework, several features of the policies intended to increase female labor force participation cannot be included in the model. For instance, wage subsidies are a main component of the set of policies for promoting female employment in Chile. One important part of such subsidies is that women become more empowered by bringing more resources to the household. Such effects cannot be considered in a unitary framework. Finally, Del Boca et al. (2014) do not incorporate the possibility of childcare in their model. This limitation will be addressed in the current paper by allowing parents to decide whether or not to use such services.

One of the main contributions of this article is in incorporating a collective model to analyze how household behavior might affect the production of skills in children. By doing so we take a more realistic approach to the fact that parents are allowed to have different preferences and also take into account features of policy tools that will have an impact in the process of intrahousehold bargaining. Additionally, the data used for this analysis allows to distinguish not only the amount of time but also the quality that parents spent with their children. This is an important feature that the literature has acknowledged to be important when taking into account parental interactions and child development (Hanushek, 1992).

As opposed to Bernal (2008) and Del Boca et al. (2014), I will not focus on child cognitive development by using a score in a unique test but rather I will proxy the level of child development given by the scores on a variety of widely used development tests for children. Moreover, the identification strategy will rely to a less extent on functional form assumptions rather than on exclusion restrictions and exogenous variations given in the dataset. Finally, the model of Del Boca et al. (2014) is only identified when both parents devote positive amount of time with

[8] See for instance Attanasio and Lechene (2014) and Bobonis (2009)

their children. In the dataset being used, approximately 20% of fathers report to spend no time in productive activities with their children. The model derived in the present article is defined in both types of households and thus the conclusions extend to this non-random sample of households where there are limited interactions with children.

The second area of the literature to which this article is related is to the empirical implementation of collective models of household behavior. The income pooling assumption has been rejected in contexts as diverse as Sweden (Cesarini, Lindqvist, Notowidigdo, & Ostling, 2013), South Africa (Duflo, 2000), Mexico (Attanasio & Lechene, 2014) Brazil, the US and Ghana (Thomas, 1994). This has motivated a significant amount of research towards exploring alternatives such as the collective model of household behavior. In the collective model of household behavior agents are allowed to have different preferences. However, often invoking folk theorem arguments as the nature of interactions within the family is repeated, it is assumed that the results of the decisions within families are Pareto Optimal. Some of the main theoretical characteristics of the collective model of household behavior are developed in Blundell, Chiappori, and Meghir (2005).

Although some of the main properties of the collective model of household behavior have been explored, there are still very few empirical implementations of such model, one exception being Cherchye et al. (2012). In their model, the authors assume that each parent has its own preferences and each parent derives utility from the time they spent with their children. By following such approach, they ignore the fact that the amount of time parents spend on their children might have an impact on their skills. This present article contributes to the literature related to the collective model of the household by being the first one to rationalize such model of household behavior taking into account the production of skills in children.

Additionally, this article provides a new framework to estimating collective models of household behavior. The usual identification strategy of such models relies on observing the consumption of a given number of private goods, being common the case of clothing. Once the decisions of consumption of such private goods are observed, there is a one to one mapping from these decisions into the Pareto weight given to each agent. However, such arguments deny the fact that every good consumed within the household has a public component, as it is reasonable to assume that couples care about each other's clothing, for example. In this article I will improve upon such estimation strategies in two points by using a novel component of the dataset which includes elements of women empowerment within the household.

First of all, the estimation strategy takes into account the fact that the point of the Pareto frontier might be contaminated with measurement error. Additionally, in order to get an idea of where in the Pareto frontier a family is located I will use a set of questions related to empowerment within the household rather than private consumption. The mapping from such questions to the Pareto frontier, and taking into account measurement error, becomes more transparent that getting identification from the consumption of private goods.

This article also contributes to the literature that addresses how to design optimal policies to disadvantaged households. Currently, Conditional Cash Transfers (CCT) are one of the most important policies to alleviate inequality in most developing countries. Every country in Latin America has a CCT program and in some cases, such as in Brazil and Mexico, this single program accounts for the largest social assistance program executed by the central government (Fiszbein et al., 2009). In most countries, the design of such program establishes that the mother of the child should be the one receiving the monetary transfers. This is supported by findings such as in Bobonis (2009) and Duflo (2000) where the authors explore whether or not the gender of the recipient of a monetary transfer matters in terms of child development. In both cases it is found that transfers made to women translate into better child outcomes than those made to men. The common interpretation of this fact is that preferences of women are more aligned with that of child outcomes and making the transfers to them is more efficient. However, in order to establish what mechanism is generating such outcome it is necessary to estimate an economic model able to identify all possible channels.

The finding that transfers made to women translate into better child outcomes still deserves some analysis from the literature. Although one valid interpretation is that women are expected to spend their own income on public goods within the household, as explained by Bobonis (2009), or to the fact that they simply have stronger preferences for child outcomes than men, there are multiple possible explanations. Blundell et al. (2005) show that as long as the marginal willingness to pay for child outcomes is higher for women than for men we will have such result. However, having women with stronger preferences for child outcomes is not a necessary condition for such statement. Similarly, Basu (2006) provides an example where even in the case that women care more for their children, there might be an inverted-U relationship between the bargaining power of the women and the welfare of children as once women become relatively more powerful they can devote all the resources derived from child labor into their own private consumption. All this shows that it is important for the design of policies to understand which is the mechanism generating the positive relationship between women empowerment and child outcomes. In this article I explicitly allow parents to have different preferences for children and by estimating the structural parameters of the model I can analyze which mechanisms generate such relationship.

Finally, this article is related to the literature exploring the production of skills in children. Todd and Wolpin (2007) present different alternatives to estimating the production function of cognitive skills in children depending on the type of data available to the researcher. Cunha et al. (2010) estimate a production function of skills in children taking into account that the productivity of inputs might vary with age. As both inputs and outputs are observed with error the authors estimate such production function via a dynamic latent factor structure. In this article I use the estimation methods presented in Todd and Wolpin (2007) taking into account that the availability of data allows me to use a value-added specification and for the econometric implementation I will use a latent factor structure as in Cunha et al. (2010). However, in order to solve for the endogeneity of inputs I will use the economic model of household behavior. Although their article is considered as a seminal contribution to the literature of production of skills, there is little scope to counterfactual analysis as the inputs are hard to interpret: the measures of investments into the production function do not map to any possible effort level or monetary investment in the family. In this article, by linking the literature of household behavior and skills production within a latent factor framework, counterfactual analysis can be performed with easy interpretation of findings.

Additionally, this will be one of the few articles that have attempted to estimate a production function of skills in a developing country. Much attention has been focused on the United States and Europe due to the availability of data. For the purpose of this article I will use a unique dataset from Chile that will bring the estimation of production of skills into a new context.

3 Data

I will use a rich longitudinal dataset from Chile. Chile is the country of Latin America with the highest GDP per capita -$US 20,000 PPP- and is often considered a case of economic success in the region due to the good economic performance during the last twenty years. Two of the most distinctive facts about Chilean economy are it's high level of inequality and the low levels of female labor force participation. Women's participation in the labor market has been historically low not only when the comparison is made with developed economies but also with similar countries in terms of income and geographic location.

The dataset used for this project comes from the Early Childhood Longitudinal Survey from Chile (ECLS). The first wave of this survey was collected in 2010 and includes a nationally representative sample of all households in Chile with a child under 3 years of age, which accounts for 15,000 households. The second wave was implemented in 2012 and included 85% of the households in the original sample and 3,135 new households with children younger than 2 years of age. In each wave information about labor force participation for every member older than 15 was collected, together with income, educational background, knowledge about the process of early childhood development and productive routines performed with the child such as reading books, teaching letters and taking children to the park.

Given that I want to identify how families make decisions of investments in young children, I restrict the sample to children living with both biological parents. I do this because the main goal of the article is to be able to identify how parents reach such decisions in a context where there are multiple members with plausibly different preferences.

Additionally, I limit the sample to families with only one children. The reason for doing this is that allowing for multiple children in the economic model would imply solving additional questions that are not the main goal of this article. For instance, we would need to identify take a stance on whether parents have the same preferences for boys and girls, or if they have preferences for equality of skills among children or rather they would devote more resources to the most promising child. Moreover, we also would need to understand to what extent there is a quality-quantity tradeoff in the fertility decisions. Do parents prefer to have more children and devote fewer resources to each of them or terminate early their fertility and devote most resources to a limited number of children.

I drop from the sample those families that do not satisfy these criteria and those that did not complete all the tests performed in order to assess the skills level of their children. After doing this, I end up with a sample of 1,035 families. Some descriptive statistics of the sample used, for the 2012 wave, are included in Table 1 and some details about the age distribution of the children included, for the 2012 wave, are included in Table 2.

We see that fathers, whose average age is 37, are on average three years older than mothers, whose average age is 34. There is not much difference in terms of schooling as both fathers and mothers attain on average 11 years of education. We do observe significant differences between fathers and mothers in labor market variables. Fathers participate on average 42.86 hours a week, which is almost twice the average of mothers at 24 hours. As will be mentioned in the preliminary evidence section, unemployment does not explain a great deal of the low levels of hours that mothers participate in the labor market. It is rather due to women being actively out of the labor force, not looking for a job but rather arguing that they don't work because they have to take care of their children.

When comparing fathers who work and mothers who work, we observe a dramatic difference in the wages earned by both. Fathers earn \$84,700 Chilean Pesos on average whereas the weekly wage for mothers is \$54,000 Chilean Pesos. In terms of age of children, we see that the sample includes a somewhat homogeneous group as the average age is five years old, the oldest one being six and the youngest one being four.

The dataset includes multiple test scores solved by the children and questionnaires answered by the primary caregiver of the child in order to assess the skills level of children, for different domains such as socio-emotional development, behavioral problems and development of vocabulary. Unfortunately, not every test was answered by all the children as all of them include different age specifications[9]. The description of the tests included in the sample is included in Tables 3 and 4. These test scores will be considered a noisy signal of the true level of skills for children.

The survey also reports the frequency with which parents perform different types of activities with their children. The description of each of these activities is presented in Tables **??** and **??**. In Figure 1 I present the average frequency for each activity that parents report to perform with the child. As can be seen, in every activity fathers report a lower frequency than mothers. The most common activities that parents perform with their children is sharing a meal, talking to them and teaching them the numbers. The most unusual activities are taking the children to cultural activities, parks or reading to them.

A novel feature of this dataset is the inclusion of questions regarding female empowerment and gender roles within the household. For instance, there is information on whether it is the mother or the father who administers the income and whether the mother considers that it is better to have a bad marriage than to remain single. These variables allow us to identify to what extent the woman has a saying in the household and if she has some power at all when making the decisions of economic relevance. The variables used to asses the degree of women empowerment in the household are presented in Table **??**. Tables 11 and 12 include summary statistics of the answers provided about the empowerment questionnaires. It is interesting to see, for instance, that 65% of men think that women should devote all their time to taking care of children and should work only in the case there is remaining time. However, as noted in Table 12 women also consider that they should be more in charge of children than working, as for instance the question related to "A woman in charge of chores should not work" receives an average score of 2.61 out of 4. These facts show that female empowerment should be an important concern for policymakers in this subpopulation.

The dataset also contains information about other important inputs into the production of skills in children. For instance, there is significant information about issues for the child during preg-

[9]For instance, the Batelle Index of Development, a questionnaire included in the 2010 survey to be answered by the primary caregiver of the child, is designed for children between 6 and 24 months of age. Given that most children are older than 24 months in the 2010 survey, I do not include this test when performing the analysis of skills in young children.

nancy and the health conditions at birth. This information will be used in order to take into account the value-added specification in the production of skills are all of these are variables that can have consequences on the cognitive and non-cognitive development of children. The indicators of health at birth and conditions during pregnancy are reported in Table **??**.

A relevant input into the production of skills is the amount of monetary investments that parents exert into their children. These type of investments can be considered as any type of materials that can improve the living conditions of children or that can stimulate the learning experiences of children such as toys, food investments, physical space exclusively used by the child, and so on. Previous studies such as Del Boca et al. (2014) and Bernal (2008) take into account such factors into the production of skills in children but do not observe such measures of investments. The identification of how monetary investments affect the production of skills in children relies then on functional forms assumption. Contrary to previous cases in the literature, I will use some indicators of parental investments in children that will give some idea of how parents invest in their children. Some of these measures are exactly the same as used in Cunha et al. (2010), which come from the HOME inventory test score. The details of the measures used to asses the level of monetary investment in the children can be found in Tables **??** and **??**.

4 Preliminary Evidence

In this section I will present four facts found in the dataset that motivates the economic model developed in the next section.

4.1 Gaps in skills emerge early in life

We do not observe inequalities in health at birth related to income for those children included in the sample. When analyzing height at birth, weight at birth and the incidence of pre-term births[10], for different income groups, we do not observe huge differences between poor and rich children, as can be seen in Figure 2. However, when we do observe differences in various dimensions of development such as vocabulary, communication skills, motor skills and cognitive achievement, when children are five years olds. This can be seen in Figure 3. The case is most dramatic for the case of vocabulary, where children in the lowest income quintile score 50% of a standard deviation below children located in the the richest income quintile. This early emergence of gaps in the development of children is consistent with the literature (Schady et al., 2015; Cunha et al.,

[10]These are variables that have often be used as a measure of health at birth (Sørensen et al., 1999).

2010) and because of this interventions aimed at improving skills of people should focus on early childhood.

4.2 Low levels of female participation in the labor market are not explained by female unemployment

As mentioned before, mothers participate on average 24 hours a week on the labor market whereas fathers do so on 43 hours a week. One plausible explanation can be due to unemployment: it is harder for women to find a job and because of that they do not actively participate in the labor market. However, it turns out to be the case that female unemployment in the population analyzed is low, at levels below 5%. The main reason for observing these low levels of female participation in the labor market is due to voluntary unemployment: women decide not to participate in the labor market. As can be seen in Figure 4, this is characteristic of women across all age groups. Most of them are not working or looking for a job and 83% of them argue that the main reason they do not do it is because they are taking care of children.

The fact that unemployment plays a small role in explaining the low levels of female activity in the labor market should guide the economic model as to how to approach the problem of deciding whether or not to work. Including frictions in the model, as is usually done in the literature in order to explain unemployment and variation in earnings for observationally equivalent agents, would complicate the model and the gains from doing so might not be significant. Because of this, I will simplify the usual decision of labor force participation as is usually done in the neoclassical model of household behavior, where people decide whether or not to work at a given wage recognized by the market.

4.3 Mothers spend more time with children than fathers

As shown previously in Figure 1, mothers spend more time with their children, in every activity, than fathers do. One possible explanation for this factor is given by the labor supply differences. Fathers specialize in remunerated activities in the labor market whereas mothers do so taking care of children. Indeed there is a negative correlation between time investments and labor supply decisions for both, fathers and mothers, in the two waves of the dataset being used, as can be seen in Tables 13 and 14.

Additionally, we observe a positive correlation between each parent's own effort and the labor supply of his/her spouse. This might be evidence of a compensating behavior arising from parents as when one parent increases his/her labor supply, they decrease the amount of time spent with their children and thus the other parent might react by increasing the amount of time interacting with their child. This compensating behavior might diminish the plausible negative impact on child development of an increase in female's labor force participation.

Although labor market behavior might explain part of the differences in the time investments between mothers and fathers, there are more stories consistent with such result. The differences might be due to preferences, as mothers find it less costly in to invest time with their children, or due to productivities, as the amount of time that mothers spend with their children might be more efficient in enhancing children's skills than that of fathers. Moreover, there is a possible explanation related to the fact that the utility derived from children's skills is a public good but the time investments are privately exerted. As women are relatively less empowered than men, the cost of effort exerted by women is less than the cost of effort exerted by men. This implies that even with the same preferences and resources, women would spend more time taking care of children. In the economic model I allow all these aforementioned factors to be a possible explanation of the differences in time investment between fathers and mothers.

4.4 There is a positive relationship between female empowerment and child outcomes

The last point to be mentioned in the preliminary evidence section is the correlation between female empowerment and child outcomes. There is evidence in the literature pointing at the fact that women empowerment is associated to better child outcomes in various contexts (Attanasio & Lechene, 2014; Thomas, Contreras, & Frankenberg, 2002).

We do observe evidence of a positive relationship between female empowerment and child outcomes. Table 15 presents the results of various regressions showing positive correlations between child outcomes and the share of income earned by women. Even after controlling for variables such as IQ level of primary caregiver, total household income, years of schooling of both parents and their ages, we do observe a positive relationship between the share of the total household income earned by mothers and children's outcomes.

When analyzing the responses to the female empowerment questionnaires, we also observe a positive relationship between female empowerment and child outcomes. In Table 15 some regressions of child investments and female empowerment are presented. We show again, that even after controlling for the same variables as mentioned before, those households where women are relatively less empowered make fewer investments in their children: they are less likely to have literary and musical instruments in the 2010 survey, the household seems to be less adequate for a child in 2012, and the consumption of rice for these children is done less frequently.

The results of these regressions cannot be interpreted as incorruptible evidence of a causal relationship between female empowerment and child outcomes. Nonetheless, it suggests that there is either some unobservables that are not captured in the regressions, that are also correlated with female empowerment, and that affect positively child outcomes, or that it is indeed female empowerment that improves the conditions of children in the households. In order to incorporate such findings in the economic model, I allow parents to have different preferences regarding leisure, consumption, skills in children, among others, so that we can rationalize the relationship between female empowerment and child outcomes arising from such patterns or either due to unobserved heterogeneity.

5 Economic Model

Each household (h) is composed by two agents (j); namely the father (f) and the mother (m). In each household there is also a child with a level of skills denoted by (s) who is not a decision maker. In each period t, parents make decisions of time investments (e_t^j) and monetary investments for the child (I_t), private consumption (c_t^j), preschool attendance of the child (a_t) and whether or not to participate in labor market (h_t^j). There is a preference shock $\boldsymbol{\epsilon_t}$ associated to each decision of labor supply and preschool service. As there are two decisions of labor supply and two possible of preschool services this shock is four-dimensional. In particular, the choice set for labor supply and childcare decisions is given by $D_t = \{(h_t, a_t) : h_t \in \{0,1\}, a_t \in \{0,1\}\}$. $q_t^{j,d}$ is an indicator function for individual j in period t taking the value of 1 if decision $d \in D_t$ is taken and 0 otherwise. I assume the preference shock follows a multivariate normal distribution with mean zero and variance Ω. The flow utility derived for each parent j in time t is given by the following utility function:

$$
\begin{aligned}
u_t^j(c_t^j, h_t^j, e_t^j, d_t^j, s_t) =& \alpha_{1,t}^j \ln(c_t^j) + \alpha_{2,t}^j \ln(s_t) - \alpha_{3,t}^j(h_t^j) - (1+h_t^j)\alpha_{4,t}^j e_t^j - \\
& \alpha_{5,t}^j h_t^j(1-a_t) + \epsilon_{d,t}^j q_t^{j,d}
\end{aligned}
\tag{1}
$$

where $\epsilon_{d,t}^j$ is the d-th element of the vector $\boldsymbol{\epsilon_t}$

At period t the skills of the child are produced by monetary investments I_t, time investments from both parents (e_t^j), preschool attendance (a_t), the ability of the primary caregiver of the child (PG) which is constant over time, the previous level of skills s_{t-1} and the age of the child in months (τ_t). I allow for unobserved heterogeneity in the production of skills denoted by $(\eta_{s,t})$. The production of skills is specified in the following equation:

$$s_t = r_t s_{t-1}^{\theta_0} \tilde{I}_t^{\theta_1} \tilde{e}_t^{\theta_2} \tag{2}$$

where r_t denotes the total factor productivity specified as:

$$r_t = exp\left(\delta_0 + \delta_1 \tau_t + \delta_2 a_t + \delta_3 PG + \eta_{s_t}\right) \tag{3}$$

$\tilde{e}_t$ is the total time effort invested in the child given by the production function:

$$\tilde{e}_t = \left[\gamma_0 \left(\tilde{e}_t^f\right)^\phi + \gamma_1 \left(\tilde{e}_t^m\right)^\phi\right]^{1/\phi} \tag{4}$$

where

$$\tilde{e}_t^j = e_t^j \exp\left(\eta_{e_t^j}\right) \tag{5}$$

and

$$\tilde{I}_t = I_t \exp\left(\eta_{I_t}\right) \tag{6}$$

The terms $\eta_{e_t^j}$ and η_{I_t} are unobserved heterogeneity. It allows for parents to differ in how productive they are in terms of the time effort and monetary investments in their children in unobserved ways. That is, even with the same amount of effort and monetary investment, the productivity of these inputs might be different across households. The terms $\eta_{I_t^j}$, $\eta_{e_t^j}$ and $\eta_{s_t^j}$ are complete information in the sense that parents make decisions knowing the productivity of their own inputs.

5.1 Dynamic problem

I assume that parents need to make investment decisions for two periods. Each period lasts for two years and the first period starts when children are on average three years old. After the two periods, children enter a different stage in which parents and children face different set of incentives in the process of skills production. Parents face a different set of incentives given that children start the formal schooling years and children start behaving more as an agent making their own decisions that might have consequences in their own skills. For this reason, I only model childhood lasting for three periods: birth, age 3 and age 5. This is exactly the same assumption made as in Del Boca et al. (2014). The problem of the household in the second period is given by:

$$V_2(\Psi_2) = \max_{\{I_2, \{c_2^j, e_2^j, c_2^j, h_2^j\}_{j=m,f}\}} \mu_2 u_2^f(c_2^f, h_2^f, e_2^f, d_2^f, s_2) + (1-\mu_2) u_2^m(c_2^m, h_2^m, e_2^m, d_2^m, s_2) \quad (7)$$

Ψ_2, which will be defined below, includes the state variables relevant to the decisions made in the second period, $\mu \in [\underline{\mu}, \bar{\mu}] \subseteq [0,1]$ represents the Pareto weight or bargaining power of the father subject to the technological constraint given in 2, to the time constraint for each agent:

$$h_2^j \in \{0,1\}, \text{ for } j = m, f \quad (8)$$

and to the budget constraint

$$c_2^f + c_2^m + I_2 = Y_2^f + Y_2^m + w_2^m h_2^f + w_2^f h_2^f + \Xi_2 \quad (9)$$

where w_2^j represents the wage offer for individual j, Y^j is the corresponding non-labor income and Ξ_2 is the total non-labor income that cannot be attributed to any specific household member. Examples of elements included in the Ξ_2 term are subsidies for water consumption for the household. Note that in the second period parents don't make decisions regarding childcare attendance as virtually every child in the sample goes to any kind of preschool service during the second period.

The problem of the household during the first period is given by:

$$V_1(\Psi_1) = \max_{\{I_1, \{c_1^j, e_1^j, c_1^j, h_1^j\}_{j=m,f}\}} \mu_1 u_1^f(c_1^f, h_1^f, e_1^f, d_1^f, s_1) + (1-\mu_1) u_1^m(c_1^m, h_1^m, e_1^m, d_1^m, s_1) +$$

$$\beta \mathbb{E}\left[V_2(\Psi_2)\right] \quad (10)$$

I assume that wages are following a mincer equation:

$$\ln(w_t^j) = \beta_0^j + \beta_1^j yrschool_t^j + \beta_2^j age_t^j + \beta_3^j (age_t^j)^2 + \varepsilon_{t,w^j} \quad (11)$$

where $\varepsilon_{t,w^j} \sim N(0, \varepsilon_{w_j})$ is measurement error[11]. Additionally, the relative importance of each household member will depend on characteristics of the household. In particular, I assume the following parametrization of μ_t:

$$\mu_t(X_t) = \underline{\mu} + \bar{\mu}\left(\frac{\exp(\Lambda' X_t + \eta_{\mu_t})}{1+\exp(\Lambda' X_t + \eta_{\mu_t})}\right) \tag{12}$$

where $\Lambda \in \mathbb{R}^L$ is a vector of coefficients; X are variables affecting the the relative bargaining power of each member in the household and $\eta_{\mu,t}$ is unobserved heterogeneity. $\underline{\mu}$ and $\bar{\mu}$ are the lower and upper bounds for the Pareto weight. In the X variables I includethe ratio of offered wages, the difference of ages between spouses, the difference in years of schooling, the father's share in non labor income and the relationship between male and female unemployment and the sex ratio in the region of residence of the household. Similar specifications to this one have been used previously in the literature [12].

$$X = \left[\frac{w^f}{w^m}, \frac{Y^f}{Y^f + Y^m}, age^f - age^m, yrschool^f - yrschool^m, \frac{\bar{U}^{male}}{U^{f\bar{e}male}}, \frac{M\bar{a}le}{Fe\bar{m}ale}\right] \tag{13}$$

where $\bar{U}$ denotes the unemployment rate for each gender $\frac{M\bar{a}le}{Fe\bar{m}ale}$ is the sex ratio in the region of residence for the household. These variables are what the literature refers to as distribution factors, variables that affect the behavior of the household only through its direct impact on the bargaining power.

The state variables are given by:

$$\Psi_t = \{r_t, s_{t-1}, \boldsymbol{\eta}_t, \Xi_t, X_t, \{\epsilon^j_{d,t}, Y^j_t, w^j_t\}_{j=m,f}\} \tag{14}$$

where the vector $\boldsymbol{\eta}_t$ collects the unobserved heterogeneity:

$$\boldsymbol{\eta}_t = \{\eta_{I_t}, \eta_{e^f_t}, \eta_{e^m_t}, \eta_{\mu_t}, \eta_{s_t}\} \tag{15}$$

[11] Note that I am imposing a separate distribution for men and women. We could assume that all the correlation is yet given by assortative mating and is no necessary to assume a bivariate distribution in their wages. The only difference will be to estimate an additional parameter which will be the correlation between wage offers

[12] Again, this determinants of the bargaining power have been previously used in the literature (Cherchye et al., 2012), Bruins (2015) and Browning, Chiappori, and Lewbel (2013)

6 Model solution

Note that the model involves a set of discrete choices -childcare and labor supply- altogether with continuous decisions such as investment, effort and consumption. The way I solve this is to first find the optimal decisions of investment, consumption and effort, for each labor supply-childcare decision, and then chose the discrete alternatives that derives the highest utility. Given the dynamic nature of the problem, I first solve for the second period problem. The solution is given by:

$$e_2^{m,*} = \frac{\kappa_2^2(\mu_2)\theta_2\gamma_1}{(1-\mu)\alpha_{4,2}^m(1+h_2^m)}\xi_2(m)\exp\left(-\eta_{e_2^m}\right) \tag{16}$$

$$e_2^{f,*} = \frac{\kappa_2^2(\mu_2)\theta_2\gamma_0}{\mu\alpha_{4,2}^f(1+h_2^f)}\xi_2(f)\exp\left(-\eta_{e_2^f}\right) \tag{17}$$

$$I_2^* = \frac{\kappa_2^2(\mu_2)\theta_1\left(h_2^f w_2^f + h_2^m w_2^m + Y_2^f + Y_2^m + \Xi\right)}{\kappa_2^1(\mu_2) + \kappa_2^2(\mu_2)\theta_1 P_I}\exp\left(-\eta_{I_2}\right) \tag{18}$$

$$c_2^{f,*} = \max\{\frac{\alpha_{1,2}^f\mu_2 I_2}{\theta_1\kappa_2^2(\mu)}, \zeta\} \tag{19}$$

$$c_2^{m,*} = \max\{\frac{\alpha_{1,2}^f\mu_2 I_2}{\theta_1\kappa_2^2(\mu)}, \zeta\} \tag{20}$$

$$e_1^{m,*} = \frac{\left[\kappa_2^2(\mu_2)\theta_2 + \beta\kappa_2^2(\mu_2)\theta_2\theta_0\right]\gamma_1}{(1-\mu)\alpha_{4,2}^m(1+h_2^m)}\xi_1(m)\exp\left(-\eta_{e_1^m}\right) \tag{21}$$

$$e_1^{f,*} = \frac{\left[\kappa_1^2(\mu_1)\theta_2 + \beta\kappa_2^2(\mu_2)\theta_2\theta_0\right]\gamma_0}{\mu\alpha_{4,2}^f(1+h_2^f)}\xi_1(f)\exp\left(-\eta_{e_1^f}\right) \tag{22}$$

$$I_1^* = \frac{\left[\kappa_1^2(\mu_1)\theta_1 + \kappa_2^2(\mu_2)\theta_0\theta_1\beta\right]\left(h_2^f w_2^f + h_2^m w_2^m + Y_2^f + Y_2^m + \Xi - P_a a\right)}{\kappa_1^1(\mu_1) + \kappa_1^2(\mu_1)\theta_1 + \beta\theta_0\theta_1\kappa_2^1(\mu_2)}\exp\left(-\eta_{I_1}\right) \tag{23}$$

$$c_1^{f,*} = \max\{\frac{\alpha_{1,2}^f\mu_2 I_2}{\theta_1\kappa_1^2(\mu_1) + \beta\theta_0\theta_1\kappa_2^2(\mu_2)}, \zeta\} \tag{24}$$

$$c_1^{m,*} = \max\{\frac{\alpha_{1,2}^f\mu_2 I_2}{\theta_1\kappa_1^2(\mu_1) + \beta\theta_0\theta_1\kappa_2^2(\mu_2)}, \zeta\} \tag{25}$$

where

$$\xi_t(j) = \frac{\left(\gamma_j \mu \alpha^f_{4,t}(1+h^f_t)\right)^{\frac{\phi}{1-\phi}}}{\gamma_0\left[\gamma_0(1-\mu)\alpha^m_{4,t}(1+h^m_t)\right]^{\frac{\phi}{1-\phi}} + \gamma_1\left[\gamma_1\mu\alpha^f_{4,t}(1+h^f_t)\right]^{\frac{\phi}{1-\phi}}} \tag{26}$$

$$\kappa^i_t(\mu) = \mu\alpha^f_{i,t} + (1-\mu)\alpha^m_{i,t} \tag{27}$$

$$\zeta = 1.0e-5 \tag{28}$$

and

$$\gamma_j = \begin{cases} \gamma_0 \text{ if } j = f \\ \gamma_1 \text{ if } j = m \end{cases} \tag{29}$$

The optimal decisions of labor supply and childcare are given by:

$$\begin{aligned}(h^{f,*}_2, h^{m,*}_2) = \max_{\{h^f_2, h^m_2\}} &\mu_2 u^f_2(c^{f,*}_2(h^f_2,h^m_2), h^f_2, e^{f,*}_2(h^f_2,h^m_2), d^f_2(h^f_2,h^m_2), s_2(h^f_2,h^m_2))+ \\ &(1-\mu_2)u^m_2(c^m_2(h^f_2,h^m_2), h^m_2(h^f_2,h^m_2), e^{m,*}_2(h^f_2,h^m_2), d^m_2(h^f_2,h^m_2), s_2(h^f_2,h^m_2))\end{aligned} \tag{30}$$

$$\begin{aligned}(h^{f,*}_1, h^{m,*}_1, a) = \max_{\{h^f_1, h^m_1, a\}} &\mu_1 u^f_1(c^{f,*}_1(h^f_1,h^m_1,a), h^f_1, e^{f,*}_1(h^f_1,h^m_1,a)), d^f_1(h^f_1,h^m_1,a), s_1(h^f_1,h^m_1,a))+ \\ &(1-\mu_1)u^m_1 c^m_1(h^f_1,h^m_1,a), h^m_1(h^f_1,h^m_1,a), e^{m,*}_1(h^f_1,h^m_1,a), d^m_1(h^f_1,h^m_1,a), s_1(h^f_1,h^m_1,a)) \\ &+ \beta\left[V_2(\Psi_2(h^f_1,h^m_1,a)\right]\end{aligned} \tag{31}$$

7 Inefficiency in Child Investments and Female Empowerment

As shown in the preliminary evidence, women spend more time with their children even when controlling for the labor supply. This, altogether with the evidence that cash in the hands of women translates into better child outcomes than cash in the hands of men, is often used as evidence that women have stronger preferences for children and thus monetary transfers should be given to women. Nonetheless, there are different explanations to why women spend more time with their children than men.

First of all, women's time might be more productive in enhancing skills of children than men's time. If it were the case, the optimal allocation of time in the household would be for women to spend more time with their children without invoking any preferences-based argument. However, in addition to this argument, the relative empowerment of each member might come as a plausible explanation. Given that both parents are making investments in a public good (skills of children) and that effort is costly and privately exerted, the fact that women spend more time with children might be a consequence of their relative disempowerment in the household rather than having different preferences in terms of child development[13].

The allocation of time investments is a result of maximizing the skills of children taking into account the cost of exerting these efforts. However, the time cost of each member is not equally weighed, it depends on the relative empowerment of each household member. If the mother is relatively less empowered, the cost of its time is lower than that of the father. This differences in empowerment levels distort the cost of providing effort and implies inefficiencies in the allocation of resources for children. Put it differently, with the same amount of total effort being provided, we can find an alternative allocation of time investments that would make children better off.

Consider the centralized problem of choosing the effort levels for the second period in order to maximize the skills of children -taking all other inputs as fixed- subject to the fact that the total amount of effort exerted should not exceed the total amount of effort found in the problem of the household described in 16-17. We are basically asking whether or not it is possible to find an alternative allocation of time that would make children better off, whiteout modifying the total amount of effort exerted by both parents. The problem is formally defined as:

$$\max_{e^f, e^m} s_2(e^f, e^m, .) \text{ subject to } e^f + e^m = e^{f,*} + e^{m,*} \tag{32}$$

Define the solution to the problem in 32 as $\left(e^{f,c_1}, e^{m,c_1}\right)$.

Similarly, we can define an alternate centralized problem where we maximize skills subject to

[13] Doepke and Tertilt (2014) develop a non-cooperative model of household behavior to answer the question of how female empowerment might promote economic development. The authors argue that the reason to develop a non-cooperative model of household behavior lies in the fact that the only mechanism capable of generating differences in investments in children in a collective approach would be that of preferences. However, in this paper I present a collective model of household behavior where differences in investment can arise for a variety of reasons other than preferences.

the fact that the total time-cost exerted into the production of skills should not exceed that found in the household's problem defined in 1-15. Formally:

$$\max_{e^f,e^m} s_2(e^f, e^m, .) \text{ subject to } c\left(e^f\right) + c\left(e^m\right) = c\left(e^{f,*}\right) + c\left(e^{m,*}\right) \tag{33}$$

Where the cost of effort is given by $c^j(e^j) = \alpha^j_{4,2} e^j(1 + h^j)$. We will call the solution to 33 as $\left(e^{f,c_2}, e^{m,c_2}\right)$. In both cases, for $l = 1, 2$, we do find that:

$$\frac{\left(\frac{e^{f,c_l}}{e^{m,c_l}}\right)}{\left(\frac{e^{f,*}}{e^{m,*}}\right)} \propto \left[\frac{(1-\mu)}{\mu}\right]^{\phi/(1-\phi)} \tag{34}$$

The difference of ratios of effort in the centralized solutions and in the household problem originally defined in 1-15 depends on the Pareto weight and the degree of substitutability between parental efforts. When the Pareto weight is heavily leaned towards one member, and if there is some degree of substitutability between parental effort, there would be an inefficient allocation in time investments given that we can find an alternative allocation with the same amount of cost, or the same amount of total effort, that will yield better child outcomes.

It is often argued in the literature that, in a collective model of household behavior, observing different child outcomes when there is a shift in the bargaining power can only be explained by differences in preferences or productivities between parents (Doepke & Tertilt, 2014). Nonetheless, if we take into account that child skills are a public good produced with effort whose cost is privately exerted, shifts in bargaining power can translate in changes in child skills even when parents are identical in terms of preferences and productivities.

This result can be interpreted as an additional argument for female empowerment within households. Not invoking an argument of equality but one of efficiency: disparities in bargaining power lead to inefficient allocations within the household. Taking this into account, and with the estimates of the economic model, we can quantify to what extent the differences observed in time spent with children are due to productivity, preferences or empowerment differences.

8 Estimation

The unobserved latent variables in the model are given by:

$$K = \{\{\ln(s_t), \ln(\hat{e}^f_t), \ln(\hat{e}^m_t), \ln(\hat{I}_t), \mu\}_{t=1,2}, \ln(PG), \ln(s_0)\} \tag{35}$$

and are not observed perfectly. Rather than observing them directly, we have a set of measures that give some information about the true latent level of each variable. Such relationship between the measures and the latent factors can be described in the following system:

$$Z_m^k = \iota_{m,0}^k + \iota_{m,1}^k k + \varepsilon_m^k \text{ for } m = 1...N_k \tag{36}$$

where Z_m^k denotes the measure m for the latent variable k and N_k denotes the number of measures available for the latent factor k. The variables used as measurements for each factor are described in Tables 5 - 10. We assume the ε_m^k are uncorrelated across observations and follow a distribution $\mathcal{N}(0, \sigma_{km})$

Given the structure of the model, there is a well defined likelihood function denoted by:

$$P(O|X;\Theta) = \mathcal{L}\left(\Theta|O;X\right) \tag{37}$$

where (O) denotes the observed outcomes in the three periods: $O = \{O_0, O_1, O_2\}$ and X is the set of exogenous characteristics in the model. The set of outcomes for the period 0 are composed exclusively of the measures of primary caregiver's skills and birth outcomes. In the first and second period the the measures corresponding to the specified factors in addition to the labor supply decision and the wages for the cases where labor supply is positive. Formally:

$$O_0 = \{\{z_m^{PG}\}_{m=1}^{N_{PG}}, \{z_m^{S_0}\}_{m=1}^{N_{S_0}}\}$$

for t=1,2:

$$O_t = \{h_t^f, h_t^m, a_t, \mathcal{Z}_t\} \cup \underbrace{\{w_t^f\}}_{\text{if } h_t^f > 0} \cup \underbrace{\{w_t^m\}}_{\text{if } h_t^m > 0}$$

$$\mathcal{Z}_1 = \{\ln(s_1), \ln(\hat{e}_1^f), \ln(\hat{e}_1^m), \ln(\hat{I}_1)\}$$

$$\mathcal{Z}_2 = \{\ln(s_2), \ln(\hat{e}_2^f), \ln(\hat{e}_2^m), \ln(\hat{I}_2), \mu_2\} \tag{38}$$

Note that we only have measures of μ_2 available for the second period. We thus need to integrate over the distribution of the bargaining power in the first period.

The exogenous characteristics are given by the age, years of schooling, age of parents and the distribution factors:

$$X = \{\{y_t^j, age_t^j, yrschool_t^j\}\}_{j=m,f}, \Xi_t, DF_t, Age_t\}_{t=1,2} \tag{39}$$

Given that we need to integrate over the the distribution of the unobserved factors as they are not observed, the expression of the likelihood function becomes a high-dimensional integral with no closed form solution. The natural approach to estimate such likelihood is to approximate the integral via Monte-Carlo methods. That is, drawing shocks from the distribution of the unobserved factors, estimating the likelihood and averaging over these draws. However, note that the time-dependency arising in the production of skills generates an additional difficulty for this approach as for each draw in period 0 we would have to generate multiple draws in the first period and for each draw in the first period we would have to draw multiple draws in the second period. The curse of dimensionality makes it infeasible to estimate this likelihood with the usual simulation techniques.

A pure simulation strategy to estimate the model would be computationally infeasible. We use particle filtering techniques in order to be able to estimate the model via simulated methods. The full description of the estimation technique altogether with the derivation of the likelihood function are described in Appendix 13.2

8.1 Identification

The identification argument is divided in three parts. First, I will show how to the parameters of the measurement system described in 36 are identified. Secondly, I show what variation in the data allows us to recover the distribution of the latent factors. Finally, I will show how the parameters of the economic model are recovered.

8.1.1 Measurement System

The general measurement system in a factor model can be written as:

$$Z = \iota_0 + \iota_1 K + \boldsymbol{\varepsilon} \tag{40}$$

where $Z \in \mathbb{R}^M$ contains all the measures available, M is the total number of measurements for all the factors, $K \in \mathbb{R}^{11}$ is the vector of 11 factors and $\boldsymbol{\varepsilon} \in \mathbb{R}^M$ is measurement error. $\iota_1 \in \mathbb{R}^{M\times 11}$ is the matrix of factor loadings. We normalize $E[k] = 0$ for each factor. This normalization is irrelevant given that we can re-define new measures $Z - \iota_0$ and the analysis will remain unchanged. From the observed measures Z we can obtain the covariances by noting that:

$$\Sigma_Z = \iota_1 \Sigma_K \iota_1' + \Sigma_\varepsilon \tag{41}$$

where σ_x is the variance covariance matrix of x. Note that we have $M \times (M+1)/2$ moments in order to identify $M \times 11$ factor loadings, $11 \times (11+1)/2$ elements in Σ_k and $M \times (M+1)/2$ elements in Σ_ε. As is often the case in factor analysis, it is necessary to make further assumptions in order to identify the relevant parameters of the model.

The fact that we have a dedicated measurement system reduces the parameters to estimate in the ι_1 matrix to only M. Additionally, we can impose the normalization of one factor loading into each factor as is usual in factor models. That is, the first measure used for each factor will have a factor loading normalized to one. This implies that the number of factor loadings to estimate becomes $M - 11$.

Note that even with these assumptions, we still need to make further assumptions to recover all the relevant parameters. By making the assumption that the measurement error of the skills at birth is independent to the measurement error of the measures corresponding to the remaining factors, we have enough moments to identify all the parameters. Formally, the assumption is given by $\varepsilon_m^{\ln(s_0)} \perp\!\!\!\perp \varepsilon_{m'}^{k'}$ for $m = 1 ... N_{ln(s_0)}$, $k \neq \ln(s_0)$, $m' = 1 ... N_k$. The details of why this is enough to identify the parameters in the measurement system are described in Appendix 13.1.

We can recover ι_m^k for $k \neq \ln(s_0)$ by noting that:

$$\frac{Cov(Z_m^k, Z_1^{\ln(s_0)})}{Cov(Z_1^k, Z_1^{\ln(s_0)})} = \iota_{m,1}^k \tag{42}$$

and the factor loadings of $\ln(s_0)$ are obtained simply by changing the roles of k by $\ln(s_0)$:

$$\frac{Cov(Z_m^{\ln(s_0)}, Z_1^k)}{Cov(Z_1^{\ln(s_0)}, Z_1^k)} = \iota_{m,1}^{\ln(s_0)} \tag{43}$$

8.1.2 Distribution of unobserved factors

Once the identification of the factor loadings is secured, we can non-parametrically estimate the distribution of the latent factors using a version of Kotlarsky Theorem. Define:

$$ME_j =\{\frac{Z_j^k}{\iota_{j,1}^k}\}_{k\in K} \tag{44}$$

$$me_i =\{\frac{\varepsilon_j^k}{\iota_{j,1}^k}\}_{k\in K} \tag{45}$$

So long as for at least two measures $j = 1, 2$ the following holds:

$$E\left[me_1|K, me_2\right] = 0 \tag{46}$$

$$me_2 \perp\!\!\!\perp \theta \tag{47}$$

we can use Theorem 1 in Schennach (2004) in order to non-parametrically identify the distribution of factors. The theorem notes that the distribution of factors can be expressed as a function of the Fourier transformation of the distribution of measures under the aforementioned assumptions:

$$p(K) = \frac{\int_{-\infty}^{\infty} e^{-i\chi K} e^{\left(\int_0^{\chi} \frac{E\left[iME_1 e^{i\psi ME_2}\right]}{\left[e^{i\psi ME_2}\right]} d\psi\right)} d\chi}{2\pi} \tag{48}$$

once the distribution $p(K)$ has been identified, we can recover the second-order moments $Cov(k, k')$ for any $k, k' \in K$. And once we recover the second order moments, we can identify the remaining elements of Σ_ε from the system of equations:

$$Cov(Z_m^l, Z_{m'}^{k'}) = \iota_{m,1}^k \iota_{m',1}^{k'} Cov(k, k') + Cov(\varepsilon_m^k, \varepsilon_{m'}^{k'}) \tag{49}$$

8.1.3 Identifying the Production of skills

Since we have secured identification of $p(K)$, we can recover the conditional distribution:

$$p\left(\ln(s_{t+1})|\ln(s_t), \ln(\hat{e}_{t+1}^f), \ln(\hat{e}_{t+1}^m), \ln(\hat{I}_{t+1}), \mu, \ln(PG)\right) \tag{50}$$

from $p(K)$ for $t = 0, 1$. We can define the following function:

$$s_{t+1} = f_s\left(s_t, \hat{e}_t^f, \hat{e}_t^m, \hat{I}_t^m\right) =$$

$$E\left[\exp\left(\ln(s_{t+1})|\ln(s_t), \ln(\hat{e}_{t+1}^f), \ln(\hat{e}_{t+1}^m), \ln(\hat{I}_{t+1}), \mu, \ln(PG)\right)\right] \tag{51}$$

where the expectation is taken with respect to the distribution in 50. However, note that we are interested in a function s_{t+1} that has as additional argument the term η_{s_t} corresponding to heterogeneity. Matzkin (2007) has negative identification results in this case and shows that in order to be able to non-parametrically identify the function we are interested in, we need to impose some restrictions. In particular, if we assume that the term η_{s_t} enters additively in 51 we can trivially identify the production of skills. Additionally, the distribution of η_s is identified as:

$$F_{\left(s_{t+1}|\ln(s_t),\ln(\hat{e}_t^f),\ln(\hat{e}_t^m),\ln(\hat{I}_t^m)\right)}\left(\tilde{s}_{t+1}|\ln(s_t), \ln(\hat{e}_t^f), \ln(\hat{e}_t^m), \ln(\hat{I}_t^m)\right) =$$

$$P\left(s_{t+1} \leq \tilde{s}_{t+1}|\ln(s_t), \ln(\hat{e}_t^f), \ln(\hat{e}_t^m), \ln(\hat{I}_t^m)\right) =$$

$$P\left(f_s\left(s_t, \hat{e}_t^f, \hat{e}_t^m, \hat{I}_t^m\right) + \eta_{s,t} \leq \tilde{s}_{t+1}|\ln(s_t), \ln(\hat{e}_t^f), \ln(\hat{e}_t^m), \ln(\hat{I}_t^m)\right) =$$

$$P\left(\eta_{s,t} \leq \tilde{s}_{t+1} - f_s\left(s_t, \hat{e}_t^f, \hat{e}_t^m, \hat{I}_t^m\right)|\ln(s_t), \ln(\hat{e}_t^f), \ln(\hat{e}_t^m), \ln(\hat{I}_t^m)\right) \tag{52}$$

and thus we can identify the cdf of $\eta_{s,t}$ conditional on factors other than s_{t+1}. With similar arguments we can identify the distribution of the remaining factors.

8.1.4 Parameters of the economic model

The parameters of the economic model are identified by a combination of exclusion restrictions, exogenous sources of variations and functional form specifications using some of the parameters already recovered from the distribution of the underlying factors. For instance, it is not intuitive how to separately identify preferences for skills of child and price of childcare. I use as exclusion restriction the distance to the nearest childcare service provider in order to be able to separately identify both components. The idea is that we can write the price of childcare as:

$$P_a = P_{a,0} + P_{a,1}\text{DChildcare} \tag{53}$$

where DChildcare is the distance from the household to the nearest childcare service provider. The key assumption is that the distance to childcare services affects the cost of such services but is not related to preferences for child development. I use the same exclusion restriction to identify the price of monetary investments in children. The idea is that the distance from households to the nearest childcare center is correlated with the supply of goods available for children in that neighborhood but is independent, conditional on the remaining factors, to skills of children.

In order to separately identify the parameters of the utility function of fathers and mothers we use common arguments used in the literature of collective models of household behavior (Chiappori & Donni, 2009). The use of distribution factors -variables that affect the behavior of the household but do not modify household behavior in any other way- allows us identify preferences for mothers and fathers. The main idea is that variation in such instruments will generate a movement along the Pareto frontier exclusively generated by the change in the bargaining power. The distribution factors used in this article have been previously used in the literature (Cherchye et al., 2012; Attanasio & Lechene, 2014; Blundell et al., 2005).

One additional identification issue to be mentioned is that of separately identifying preferences for skills in children and cost of exerting effort. We can rationalize a given amount of effort exerted from parents to children by a locus combinations of $(\alpha^j_{t,2}, \alpha^j_{t,4})$. Rather than relying exclusively on functional form identification, I use information about additional family members contributing to household chores in order to obtain information about the cost of exerting effort into the children. The idea is that having an additional family member contributing to tasks such as cleaning the house shifts the cost of investing time in children.

9 Results

The results of the parameters estimated, altogether with the corresponding Standard Errors are presented in Tables 17 - 33. As we see, fathers have stronger preferences for consumption and lower disutility from working. An interesting result is related to preferences for child skills: we see that preferences are the same for fathers and mothers. This is an important result as it shows that it is not necessarily through preferences for children that households where women are more empowered see better outcomes in their children not as a result of them having stronger preferences but due to a different mechanism which we will describe below.

Interestingly, we see also that in families where there is nobody able to help in the household chores the marginal cost of providing an additional unit of effort $\alpha^j_{4,1}$ is the same for both fathers and mothers. Nonetheless, the fact that the parameter estimate of $\alpha^f_{4,1} = 0$ and $\alpha^m_{4,1} = 0.01$ shows that when there is someone helping in household chores the effort cost of investing time in children is reduced for mothers but not for fathers. This is due probably to the fact that in the absence of any help in the household chores it is the mother who is responsible of them.

Regarding the estimates of the production of skills, we see that there is no statistical difference in the productivity of time investments of mothers and fathers. It seems to be equally efficient for a child to spend one hour of productive time investment with her father or mother. It is not possible to make comparisons between the productivities of different inputs as they are measured in different units (except father's and mother's effort). Nonetheless, we see that monetary investments, childcare attendance, skills of primary caretaker and having adequate birth conditions all seem to have positive effects on the quality of a child. As expected, the only input with a negative effect on the acquisition of skills is having a harsher type of discipline.

Looking at the estimates of the determinants of the Pareto weight, we see there is an effect of the wage ratio on the Pareto weight. This is important as the relationship holds even when we control for differences in education, age and in non labor income. All the coefficients have the expected sign and we see that as long as the man has higher income (labor or non-labor) is relatively older or more educated his Pareto weight increases. We see then that subsidizing wages or non-labor income might be a way to increase the women's say within the household and the decisions will be more aligned towards her preferences.

The results of the estimation exercise are also useful to have an idea of which measures are more informative about the underlying factors. Every measure is contaminated by measurement error and with the estimation results we are able to extract the proportion of the variance due to true signal and the proportion due to noise.

$$\text{Signal-noise ratio}_{m,k} = \frac{\iota^2_{m,1} Var(k)}{\iota^2_{m,1} Var(k) + Var(\varepsilon^k_m)} \tag{54}$$

In Figures **??** and **??** I present the signal to noise ratio of parental investments. We find that cultural activities are the most informative about time investment in children and sharing a meal or performing household chores are within the group of less informative activities. We should be careful with the interpretation of these results: it does not mean that cultural activities are the most productive ones but rather they are the most informative ones. It can certainly be the case that there is an underlying activity that is not reported in the dataset that is performed more often by those parents who perform cultural activities and it is the one that is really productive, not performing cultural activities by itself. Making inferences about which activities are more productive requires more analysis in this point.

Similarly, in Figure 16 I present the signal to noise ratio of the measures of investment in the second wave of the sample. We observe that frequency of consumption of different items is not as informative as availability of toys and books for children. In Figure ?? I report the ratio for the test scores. We observe that the test scores related to executive function (BDST,HTKS) are not very informative. The Batelle total test score is the most informative with a signal to noise ratio of 45%.

9.1 Model fit

In Figures ?? to ?? we see the performance of the model in terms of predicting the labor force participation. In Figure ?? we can compare the implied labor force participation for mothers and fathers depending on their level of education. We see that the model fits well the participation profile as men seem to participate at all educational groups and women increase their participation as they have more education. The model not only fits well the labor force participation by educational category but also depending on the age of people. In Figure ?? we see that participation profile of women matches across all age groups. Young women (younger than 20 years) tend not to participate in the labor market. This fact changes from the years 25 until 40 and we see a rapid decline at this point. Regarding the labor force participation profile of men across age groups there is nothing surprising as fathers seem to participate regardless of their age group in both the observed data and the implied observations by the model, which can be seen in Figure ??.

As said previously during the description of the dataset used by this study, Chilean economy is characterized by its high levels of inequality. Such levels of inequality are not only reflected in the monetary income of households but also in the skills attained by their children. In Figure ??, in the lower panel, we see the gradient of skills according to household income. The model fits well the skills gradient as those children coming from families who are at the lowest decile of income have a level of skills corresponding approximately to the percentile 45 whereas those who are at the top decile will be on average at the 55-percentile. In Panel A of Figure ?? we see that such gradients are much more pronounced when we use monetary investments from parents rather than household income. The model predicts well the gradient generated by inequalities due to investments in children. The level of skills in children goes from the 40th percentile, for families with investment in the first decile, to approximately the 60th percentile for those at the top decile. When making policy recommendations it is important to take into account the potential of monetary investment. We might see that direct monetary transfers to poor households not only account directly to the reduction of inequality but also to future inequality as it might enhance the level of skills in the children.

10 Policy Counterfactuals

In this section I describe the effects that different policy experiments have not only on the skills of young children but also on the female labor force participation, childcare attendance and on female empowerment. The policies considered are: 1. doubling the amount of monetary transfers from the government to poor households; 2. same as 1. but instead rather than having the mother as the recipient of such transfers, it will be the father; 3. using the same amount of money necessary to implement policies 1 or 2 to subsidize childcare services; 4. subsidize the price of monetary investments in young children.

10.1 Cash transfers to the mother

Cash transfers are a widely used program used by developing countries in their fight to eradicate poverty. Every country in Latin America has a form of Cash transfer that varies by the amount given to the households and the type of conditions established to the families in order to be considered as eligible recipients. Policymakers often invoke the effect of such programs on the promotion of skills of young children as one of its many benefits. Moreover, the vast majority of these programs establish as an explicit condition that the transfer should be done to the woman whenever the benefactor is a household with children and multiple adult members. The main argument for this being the evidence that cash in the hands of women translates into better child outcomes than cash in the hands of men.

Given the high use of cash transfer as a policy tool in developing countries, and given the explicit condition that it should be done to mothers rather than fathers or other adult members, the first counterfactual policy that I consider is to double the amount of cash transfers given to mothers of young children. Although in principle doubling the amount of monetary transfers to poor households might seem as an expensive change in policy, such a policy is not actually that radical in the context analyzed. The size of the basic monetary transfer to poor families in Chile has increased approximately 30% from 2012 to 2014. Additionally, it does not represent a large fraction of the median income. In the case of Mexico, the OPORTUNIDADES program gives a transfer equivalent to 20% of mean household income whereas in Chile such transfer only represents 1.6%.

I analyze the consequences of doubling cash transfers to poor households located in the lowest

As of 2012, approximately 50% of the households in Chile received some type of direct monetary transfer from the government. For households in the poorest decile of the income distribution, such subsidy represent 40% of their total labor income. The highest paying subsidy is the Subsidy to Intellectual disabilities, which is given to adults responsible of a child under 18 years of age with a condition of learning disability and represents $53,000 Chilean Pesos (CLP) of 2012. The most important monetary transfer in terms of the number of beneficiaries is are the Family Allowance and the Unique Family Subsidy[14].

The monetary amount that families receive by each of these subsidies corresponds to $7,179 CLP in 2012. Families with a child under the age of 18 who depends economically from a worker earning less than $187,515 CLP who is making contributions to social security can get the Family Allowance benefit. A family in a similar condition, but where the child depends on someone who is not making contributions to social security and where economic urge can be proved is eligible to receive the Family Allowance. If a family is eligible to receive any of these subsidies the transfer should be done to the mother of the child causing the benefit. In case the mother doesn't live with the child, such transfer is made to the father or the person responsible of the child.

Consider implementing a policy that doubles the amount of money that families get from the aforementioned subsidies. As the requirements to be considered eligible for the Family Subsidy cannot be mapped directly to the family income, let's consider the case where families located in the poorest four deciles of the income distribution qualify for such benefits. Given that we estimated the structural parameters of the model, we can consider the consequences of implementing such policies in the production of skills in children taking into account not only the consequences on labor force participation but also on the process of intra-household bargaining. This is an important fact as such monetary transfers that are done to the mother not only increases the resources poor households but serves the purpose of empowering women within households.

In Figure **??** we can see the consequences of imposing such policy on the labor force participation of mothers. I consider three scenarios to analyze the consequences of such policy: one where

[14]In Spanish "Asignación Familiar" y "Subsidio Único Familiar"

the consequences of such policy on the Pareto weight are ignored and an additional one where we incorporate the fact that giving additional resources to women will increase their Pareto weight. We can see that increasing the monetary resources to women decreases slightly their participation in the labor market: the total participation rate goes from 46% to 40% in both cases, when we take into account the impact on the bargaining power and when it is ignored. The result is unsurprising and can be seen as an argument to limiting the amount of subsidies that are given unconditionally to families, as people will have less incentives to go to the labor market.

The consequences of such policy on the production of skills in children can be analyzed in Figure **??**. As can be seen in Panel A of the figure, this policy reduces the existent inequality in terms of skills. Before implementing the policy, we see that those children living in families that are located in the lowest decile of monetary investments into children are located in between between the 40th and 45th percentile while those in the richest decile are located slightly above the 60th. When the increase in the subsidy is implemented, note that the inequalities in skills are significantly reduced. Note that the effect is stronger when we take into account the consequences on the bargaining power within the household: families in the lowest decile in the distribution of monetary investments have children whose skills are located slightly below the median and those children whose families are in the top deciles are just slightly above the 55th percentile. In Panel B of Figure **??**.

The important point to note in the analysis of the counterfactuals is not only that monetary transfers can alleviate early inequalities in skills but that if we consider the role that such transfers have in distorting the process of intra-household bargaining the reduction in the early inequalities is more dramatic. Note that these results hold not only due to the fact that mothers have stronger for their children. First of all, the estimation results show that mothers have smaller marginal utility of consumption than fathers. For this reason, households where resources are more controlled by women will derive more utility investing them in the production of skills rather than on private consumption. Additionally, those households that were the Pareto weight of women was smaller to that of men will se a balance of power and with such, the marginal utility to both types of private consumption decreases. The only additional alternative to spend such resources is in monetary investments for children, which will enhance their level of abilities.

We see that the results of the model can be taken to analyze the results of policy changes into household behavior and taking into account the consequences on the skills of children. Other policies that have been implemented throughout the world with the purpose of empowering women and also improving the skills of children are reduction in the price of childcare and wage subsidies to women. The case of wage subsidies for women is particularly important for the case of Chile given the low levels of female labor force participation. Right now, a program has been implemented in order to give a subsidy equivalent to up to 20% of the wage to those women who earn less than approximately $430,00 CLP. The estimates of this current model will help to analyze the consequences of this policy on labor force participation, on women empowerment and on the production of skills in children. However, this is left for future work.

11 Conclusions

The fact that skills produced during the first years of life have consequences on adult life outcomes has motivated a significant amount of research directed to analyzing the determinants of the production of cognitive and non cognitive skills in children. Previous work from Heckman, Todd, Wolpin and coauthors, previously mentioned in this article, have helped us to characterize the way skills are produced during the first years of life. Nonetheless, it is still unclear to see how families decide to invest resources in the corresponding inputs of such production function.

Del Boca et al. (2014) analyze simultaneously the problem of household behavior and skills production, they do so from a unitary perspective ignoring the possible implications of policies on the process of intra-household bargaining. As seen from the results of the counterfactual experiments, ignoring the consequences that monetary transfers have on the acquisition of skills in children might be problematic as this is an additional channel through which these policies will reduce inequalities in skills.

The article also makes a contribution in the estimation of collective models of household behavior. The few empirical implementations of these models rely on observing private consumption to fully identify the model. In this case I take into account information about household decision making in order to have an idea of the balance of power within the household. Additionally, I take into account the fact that such observation contains measurement error and thus include a factor analysis framework into the estimation of the economic model.

12 Figures and Tables

Table 1: Summary statistics

Variable	Mean	(Std. Dev.)
Mother's age	34	(6.64)
Father's age	36.94	(7.56)
Mother's years of schooling	11.44	(3.09)
Father's years of schooling	11	(3.21)
Mother's hours of work (week)	23.79	(21.26)
Mother's hours of work (week)	42.86	(16.16)
Mother's wage (Weekly-Chilean Pesos thousands)	53.99	(86.75)
Father's wage (Weekly-Chilean pesos thousands)	84.7	(112.47)
Household's total Income (Weekly-Chilean pesos thousands)	137.73	(138)
Age of child (months)	67.03	(7)
N	1035	

Table 2: **Age distribution (2012)**

Item	Number	Per cent
4	186	17.97
5	528	51.01
6	321	31.01
Total	1,035	100.00

Table 3: 2010 Tests-Measures of child skills

Test	Description	Scoring Interpretation	Ages (in months)	Abbreviation
TEPSI	Psychomotor development test. Three areas of psychomotor development are included: coordination, language and gross motor development. A score including all these areas is also computed.	Higher score indicates a higher level of psychomotor development.	24-60	$MS_{1,10}$-$MS_{3,10}$
TVIP	Peabody Picture Vocabulary Test. A raw score as well as a standardized score is computed.	Higher scores indicate higher levels of verbal intelligence for children	30-60	$MS_{4,10}$
CBCL	Child Behavior Checklist. This tool gives a general diagnosis of the socioemotional development of children in seven dimensions: Emotional reactivity, Anxiety-depression, Somatic complaints, Isolation, sleeping disorders, aggressive behaviors and attention deficit.	A higher score indicates more persistence of behavioral problems.	18-60	$MS_{5,10}$-$MS_{11,10}$

Table 4: 2012 Tests-Measures of child skills

Test	Description	Scoring Interpretation	Ages (in months)	Abbreviation
TADI	Test of Early Childhood Learning. 4 dimensions including cognition, motor skills, language and socio-emotional development. For each one, two scores are computed: raw and total.	Higher scores indicate higher levels of childhood development	6-84	$MS_{1,12}$-$MS_{4,12}$
BATELLE	Batelle Instrument for Child Develpoment. Five dimensions of child development in addition to a total-comprehensive child development score	Higher score indicates a higher level of child development.	6-84	$MS_{5,12}$-$MS_{10,12}$
HTKS	Head Toes Knees and Shoulders. Executive function test. It evaluates the performance of the three sub-areas of executive function. Children should do a different action as instructed. For instance, they should touch their head after the word ”Toe” etc..	The test is divided in stages that will be scored according to the enumerator observation. It consists of four stages and the score is the stage to which the child reached.	36-84	$MS_{11,12}$
BDST	Backward Digit Span Task. Executive function test where children repeat a set of digits backwards..	The test is cmposed by different stages. Stage 2 consists of sets of two digits and so on. The score is the final stage reached.	36-84	$MS_{12,12}$
TVIP	Peabody Picture Vocabulary Test. A raw score as well as a standardized score is computed.	Higher scores indicate higher levels of verbal intelligence for children	30-84	$MS_{13,12}$

Table 5: Measures used for parental effort in 2012

Abbreviation	Activity
$MS1_{EF,12}$	Reads Children's storybooks or drawing books
$MS2_{EF,12}$	Tells her stories
$MS3_{EF,12}$	Sings to child
$MS4_{EF,12}$	Takes her to parks
$MS5_{EF,12}$	Takes her to museums, zoos, libraries or other cultural activities
$MS6_{EF,12}$	Spends time with her chatting or drawing
$MS7_{EF,12}$	Invites her to participate in household chores
$MS8_{EF,12}$	Takes her to the supermarket
$MS9_{EF,12}$	Shares a meal with her
$MS10_{EF,12}$	Teaches the animals and their sounds
$MS11_{EF,12}$	Teaches her the colors
$MS12_{EF,12}$	Goes with her to visit friends or family members
$MS13_{EF,12}$	Teaches her the numbers and how to count
$MS14_{EF,12}$	Teaches her words

For each question parents reply how often, during the last seven days, they perform each activity. The possible answers are: Never, 1-3 times, 4-6 times.

Table 6: Measures used for parental effort in 2010

Abbreviation	Activity
$MS1_{EF,10}$	Reads Childre's storybooks or drawing books
$MS2_{EF,10}$	Tells her stories
$MS3_{EF,10}$	Sings to her
$MS4_{EF,10}$	Takes her to parks
$MS5_{EF,10}$	Takes her to museums, zoos, libraries or other cultural activities
$MS6_{EF,10}$	Plays with her
$MS7_{EF,10}$	Spends time with her talking or drawing

Table 7: Measures used for Pareto weight

Abbreviation	Activity
$\mathrm{MS}1_{BARG}$	A woman who is in charge of most part of tasks of the household has no time to work*
$\mathrm{MS}2_{BARG}$	Both spouses should contribute to household income*
$\mathrm{MS}3_{BARG}$	It is better for everyone if the man goes to work and the woman takes care of the household and the family*
$\mathrm{MS}4_{BARG}$	Men should assume a more active role in the household chores and childcare than what they actually do*
$\mathrm{MS}5_{BARG}$	If my spouse earned enough there is no reason for me to work*
$\mathrm{MS}6_{BARG}$	After having children, the best for a woman is to develop her carreer*
$\mathrm{MS}7_{BARG}$	Taking into account the pros and cons, it is very important for me to have a paying job*
$\mathrm{MS}8_{BARG}$	Having a payed job is the best way for a woman to become independent*
$\mathrm{MS}9_{BARG}$	Father's and mother's time is equally important for the children*
$\mathrm{MS}10_{BARG}$	It is better to have a bad marriage than to remain single*
$\mathrm{MS}11_{BARG}$	Woman participates in the process of administering income (yes-no)
$\mathrm{MS}12_{BARG}$	Man participates in the process of administering income (yes-no)
$\mathrm{MS}13_{BARG}$	Both, father and mother participate in the process of administering income (yes-no)
$\mathrm{MS}14_{BARG}$	(Mother) Who should take care of children (Father-Mother-Both-Other)
$\mathrm{MS}15_{BARG}$	(Man) Women should only be in charge of taking care of children (yes-no)
$\mathrm{MS}16_{BARG}$	(Man) Women should take care of children and work part time (yes-no)
$\mathrm{MS}17_{BARG}$	(Man) Women should work full-time and delegate childcare to someone else (yes-no)
$\mathrm{MS}18_{BARG}$	(Man) Men are better at childcare than women (yes-no)

*: For each question the woman provides an answer between 1 to 5 with the following scale:
Disagrees very much; disagrees; doesn't know; agrees; agrees very much.

Table 8: Measures used for Skills at birth

Abbreviation	Activity
$MS1_{BIRTH}$	Mother diagnosed with Preeclampsia during pregnancy
$MS2_{BIRTH}$	Mother diagnosed with Cholestasis during pregnancy
$MS3_{BIRTH}$	Mother diagnosed with Urinary infections during pregnancy
$MS4_{BIRTH}$	Mother diagnosed with Hemorrages during pregnancy
$MS5_{BIRTH}$	Mother diagnosed with Hipertension during pregnancy
$MS6_{BIRTH}$	Mother diagnosed with Placenta Previa during pregnancy
$MS7_{BIRTH}$	Mother diagnosed with Diabetes G during pregnancy
$MS8_{BIRTH}$	Mother diagnosed with Anemia during pregnancy
$MS9_{BIRTH}$	Mother diagnosed with Toxoplasmosis during pregnancy
$MS10_{BIRTH}$	Mother diagnosed with Depression during pregnancy
$MS11_{BIRTH}$	Mother diagnosed with Bipolar D. during pregnancy
$MS12_{BIRTH}$	Mother diagnosed with Anxiety D. during pregnancy
$MS13_{BIRTH}$	Mother diagnosed with Obsesive compulsive D. during pregnancy
$MS14_{BIRTH}$	Mother diagnosed with Fobia during pregnancy
$MS15_{BIRTH}$	Mother diagnosed with Panic D. during pregnancy
$MS16_{BIRTH}$	Mother diagnosed with PTSD during pregnancy
$MS17_{BIRTH}$	Cigarrettes consumed during pregnancy
$MS18_{BIRTH}$	Cigarrettes consumed during the first six months of life of child
$MS19_{BIRTH}$	Alcohol consumption during pregnancy*
$MS20_{BIRTH}$	Substance abuse during pregnancy*
$MS21_{BIRTH}$	Child was born pre-term
$MS22_{BIRTH}$	Weight at birth (grams)
$MS23_{BIRTH}$	Height at birth (cm)

*Possible answers are never (0), rarely (1) and often (2).

Table 9: Measures used for Investments in 2010

Abbreviation	Activity
$\mathrm{MS1}_{INV,10}$	Child has a special place where to store toys and belongings
$\mathrm{MS2}_{INV,10}$	Child has at least one toy that involves muscular activity
$\mathrm{MS3}_{INV,10}$	Child has toys to pull and push
$\mathrm{MS4}_{INV,10}$	Child has at least one toy with wheels
$\mathrm{MS5}_{INV,10}$	Availability of plush toys-stuffed animals
$\mathrm{MS6}_{INV,10}$	Availability of mobiles for child
$\mathrm{MS7}_{INV,10}$	Availability of musical or literary toys
$\mathrm{MS8}_{INV,10}$	Child has three or more books of his own

Table 10: Measures used for Investment in 2012

Abbreviation	Activity
$MS1_{INV,12}$	Consumption of hamburger-pizza-fries*
$MS2_{INV,12}$	Consumption of Fish-Beef-Chicken*
$MS3_{INV,12}$	Consumption of bread-rice-pasta
$MS4_{INV,12}$	Consumption of legumes*
$MS5_{INV,12}$	Consumption of Chocolate-Candy*
$MS6_{INV,12}$	Consumption of juice*
$MS7_{INV,12}$	Consumption of snacks in bags*
$MS8_{INV,12}$	Consumption of milk*
$MS9_{INV,12}$	Consumption of water*
$MS10_{INV,12}$	Consumption of cookies*
$MS11_{INV,12}$	Consumption of fruits and vegetables*
$MS12_{INV,12}$	There are two or more toys in the household where child can learn colors, sizes and shapes
$MS13_{INV,12}$	Child has three or more puzzles
$MS14_{INV,12}$	There is a music device where child can listen children's music
$MS15_{INV,12}$	There are two or more toys for free expression or impersonations such as tools and customs
$MS16_{INV,12}$	There are two or more toys in the household that can help with learning numbers
$MS17_{INV,12}$	There are at least ten children's books available in the house
$MS18_{INV,12}$	There are at least ten books for adults
$MS19_{INV,12}$	At first sight, there is very little evidence that there is a child living in the household
$MS20_{INV,12}$	Number of people with whom child shares bed
$MS21_{INV,12}$	Number of people with whom child shares room

*: The possible answers are 1: never, 2: one to two times a month; 3: one to three times a week;
4: four to six times a week; 5: once a day; 6: two or more times a day.

Table 11: **Father's opinion on gender roles**

Item	Number	Per cent
Women should only spend time taking care of chidlren	313	30
Women should take care of children and work if there is remaining time	669	65
Women should work full time	47	5
Men take care better of children than women	6	1
Total	1,035	100

Table 12: Summary statistics-Measures of bargaining power

Variable	**Mean**	**(Std. Dev.)**
A woman in charge of chores should not work	2.61	(0.84)
Both parents should contribute equally to household income	1.76	(0.63)
It is better if the man goes to work and the woman stays at home	2.54	(0.84)
Men should be more involved in household chores	1.76	(0.66)
If husband earned enough there is no reason for woman to work	2.19	(0.88)
It is better if woman has children after having a succesful carreer	2.38	(0.84)
It is very important for a woman to have a job	1.8	(0.65)
Having a job is the best way for a woman to achieve independence	1.77	(0.64)
Father's time is as important as mother's time for children	1.48	(0.6)
It is better to have a bad marriage than being single	3.3	(0.75)
N	1035	

All questions are answered by the mother of the child. The possible answers are 1: strongly agrees; 2: agrees; 3: disagrees; 4: strongly disagrees.

Table 13: Time investments and labor supply (2010)

$$\text{Effort}_i = \beta_0 + \beta_1 \text{Hours worked mother}_i + \beta_2 \text{Hours worked father}_i + \beta_3 X_i + \varepsilon_i$$

VARIABLES	(1) Mother's effort (2010)	(2) Father's effort (2010)
Mother: hours worked weekly	-0.01**	0.01*
	(0.01)	(0.01)
Father: hours worked weekly	0.03***	-0.00
	(0.01)	(0.01)
Observations	1,035	1,035
Adjusted R-squared	0.05	0.05

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1
Additional controls include age of child, race, age of both parents and test scores of primary caregiver

The measure of effort is constructed via Principal component analysis, extracting one factor for the variables used as measures of time investments by parents.

Table 14: Time investments and labor supply (2012)

$$\text{Effort}_i = \beta_0 + \beta_1 \text{Hours worked mother}_i + \beta_2 \text{Hours worked father}_i + \beta_3 X_i + \varepsilon_i$$

VARIABLES	(1) Mother's effort (2012)	(2) Father's effort (2012)
Mother: hours worked weekly	0.00	0.01***
	(0.00)	(0.00)
Father: hours worked weekly	-0.00	-0.01**
	(0.00)	(0.00)
Observations	1,035	1,035
Adjusted R-squared	0.04	0.05

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1
Additional controls include age of child, race, age of both parents and test scores of primary caregiver

The measure of effort is constructed via Principal component analysis, extracting one factor for the variables used as measures of time investments by parents.

Table 15: Child outcomes and share of income earned by women

VARIABLES	(1) Batelle (2012)	(2) Tadi cognitive (2012)	(3) Tadi motor skills (2012)	(4) Behavioral problems (2010)	(5) Vocabulary (2010)
Mother's income share	0.17**	0.10*	0.18***	-0.25*	0.22**
	(0.07)	(0.06)	(0.06)	(0.13)	(0.11)
Total household income	0.00	0.00	-0.00	0.00	0.00
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Mother's years of schooling	0.01	0.02***	0.01	0.01	0.02**
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Father's years of schooling	0.00	0.00	0.01	-0.03***	0.04***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Observations	1,035	1,035	1,035	1,035	1,035
Adjusted R-squared	0.23	0.36	0.25	0.24	0.48

Robust standard errors in parentheses

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Additional controlsi include age of child, race, age of both parents and test scores of primary caregiver

Table 16: Female empowerment and Child outcomes

VARIABLES	(1) Literary and musical instruments (2010)	(2) Adequacy of household for a child (2012)	(3) Consumption of rice (2012)
Man's bargaining power	-0.31**	-0.32*	-0.93***
	(0.13)	(0.17)	(0.33)
Mother's years of schooling	0.01	0.00	0.01
	(0.01)	(0.01)	(0.01)
Father's years of schooling	0.02***	0.00	-0.01
	(0.00)	(0.01)	(0.01)
Observations	1,035	1,035	1,035
Adjusted R-squared	0.09	0.00	0.00

Robust standard errors in parentheses

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Additional controlsi include age of child, race, age of both parents and test scores of primary caregiver

Man's bargaining power is constructed using all the responses from the female empowerment questionnaire reported in Table **??** and extracting the first factor after doing a Principal Component Analysis. Literary and Musical instruments (2010) is a yes (1)/ no (0) question in the 2010 survey. Adequacy of household for a child is a yes (1)/ no (0) question answered by the enumerator based on the observation of the household. Consumption of rice is related to the frequency of consumption of this food on a weekly basis. More details can be found in Table **??**.

Table 17: Estimates: Utility function. Mother's preferences

Parameter	Estimate	Standard Error
$\alpha^m_{1,12}$	0.647	0.000
$\alpha^m_{2,12}$	0.053	0.000
$\alpha^m_{2,12}$	0.291	0.001
$\alpha^m_{4,0,12}$	0.010	0.000
$\alpha^m_{4,1,12}$	0.002	0.000
$\alpha^m_{1,10}$	0.164	0.000
$\alpha^m_{2,10}$	0.008	0.000
$\alpha^m_{3,10}$	0.090	0.000
$\alpha^m_{4,0,10}$	0.002	0.000
$\alpha^m_{4,1,10}$	0.002	0.000
$\alpha^m_{5,10}$	0.736	0.001

Table 18: Estimates: Utility function. Father's preferences

Parameter	Estimate	Standard Error
$\alpha^f_{1,12}$	0.030	0.000
$\alpha^f_{2,12}$	0.002	0.000
$\alpha^f_{3,12}$	0.968	0.001
$\alpha^f_{4,0,12}$	0.001	0.000
$\alpha^f_{4,1,12}$	0.000	0.000
$\alpha^f_{1,10}$	0.004	0.000
$\alpha^f_{2,10}$	0.910	0.000
$\alpha^f_{3,10}$	0.968	0.000
$\alpha^f_{4,0,10}$	0.073	0.000
$\alpha^f_{4,1,10}$	0.073	0.000
$\alpha^f_{5,10}$	0.010	0.000

Table 19: Estimates: Preference shock

Parameter	Estimate	Standard Error
$\sigma^m_{W,A}$	3.393	0.000
$\sigma^m_{NW,A}$	1.231	0.000
$\sigma^m_{W,NA}$	0.030	0.000
$\sigma^m_{NW,NA}$	1.600	0.000
$\sigma^f_{W,A}$	0.502	0.000
$\sigma^f_{NW,A}$	0.013	0.000
$\sigma^f_{W,NA}$	0.629	0.000
$\sigma^f_{NW,NA}$	1.012	0.000

Table 20: Estimates: Mothers wages

Parameter	Estimate	Standard Error
β^m_0	5.787	0.007
β^m_1	0.276	0.001
β^m_2	0.071	0.001
β^m_3	-0.001	0.000
σ_{w_m}	0.828	0.055

Table 21: Estimates: Fathers wages

Parameter	Estimate	Standard Error
β^f_0	5.810	0.005
β^f_1	0.126	0.000
β^f_2	0.187	0.000
β^f_3	-0.002	0.000
σ_{w_f}	0.690	0.011

Table 22: Estimates: Production of Skills

Parameter	Estimate	Standard Error
θ_0	0.213	0.000
θ_1	0.267	0.000
θ_2	0.520	0.000
ϕ	0.469	0.000
γ_f	0.498	0.000
γ_m	0.502	0.000
δ_0	-0.800	0.000
δ_1	-0.000	0.000
δ_2	0.001	0.000
$\delta_{3,10}$	4.505	0.000
$\delta_{3,12}$	5.300	0.001
δ_4	0.013	0.000
σ_s	1.575	0.000

Table 23: Estimates: Pareto weight

Parameter	Estimate	Standard Error
λ_0	-1.477	0.000
λ_1	0.001	0.000
λ_2	0.017	0.000
λ_3	-0.042	0.000
λ_4	0.002	0.000
λ_5	-2.532	0.000
λ_6	-0.000	0.000
λ_7	0.932	0.000
λ_8	0.003	0.000
σ_μ	0.213	0.000

Table 24: Estimates: Prices

Parameter	Estimate	Standard Error
Price_{I_0}	966.238	0.233
Price_{I_1}	0.238	0.000
Pchildcare_0	1808.042	0.000
Pchildcare_1	622.610	0.000

Table 25: Estimates: Distribution of latent factors

Parameter	Estimate	Standard Error
σ_{ef}^{m}	3.187	0.001
σ_{ef}^{f}	3.470	0.004
σ_{inv}	1.943	0.012

Table 26: Estimates: Measurement system -Skills in 2010

Parameter	Estimate	Standard Error
$\mathrm{MS}_{1,10}$	0.066	0.000
$\mathrm{VS}_{1,10}$	0.954	0.000
$\mathrm{MS}_{2,10}$	0.060	0.000
$\mathrm{VS}_{2,10}$	0.939	0.000
$\mathrm{MS}_{3,10}$	0.040	0.000
$\mathrm{VS}_{3,10}$	0.935	0.000
$\mathrm{MS}_{4,10}$	0.080	0.000
$\mathrm{VS}_{4,10}$	1.023	0.000
$\mathrm{MS}_{5,10}$	-0.232	0.000
$\mathrm{VS}_{5,10}$	0.790	0.000
$\mathrm{MS}_{6,10}$	-0.193	0.000
$\mathrm{VS}_{6,10}$	0.870	0.000
$\mathrm{MS}_{7,10}$	-0.123	0.000
$\mathrm{VS}_{7,10}$	0.974	0.000
$\mathrm{MS}_{8,10}$	-0.153	0.000
$\mathrm{VS}_{8,10}$	0.906	0.000
$\mathrm{MS}_{9,10}$	-0.127	0.000
$\mathrm{VS}_{9,10}$	0.911	0.000
MS_{10_10}	-0.204	0.000
VS_{10_10}	0.860	0.000
MS_{11_10}	-0.381	0.000
VS_{11_10}	0.002	0.000

Table 27: Estimates: Measurement system -Skills in 2012

Parameter	Estimate	Standard Error
$MS_{1_1 2}$	0.150	0.000
$VS_{1_1 2}$	0.378	0.000
$MS_{2_1 2}$	0.137	0.000
$VS_{2_1 2}$	0.445	0.000
$MS_{3_1 2}$	0.160	0.000
$VS_{3_1 2}$	0.402	0.000
$MS_{4_1 2}$	0.151	0.000
$VS_{4_1 2}$	0.425	0.000
$MS_{5_1 2}$	0.142	0.000
$VS_{5_1 2}$	0.493	0.000
$MS_{6_1 2}$	0.156	0.000
$VS_{6_1 2}$	0.329	0.000
$MS_{7_1 2}$	0.155	0.000
$VS_{7_1 2}$	0.369	0.000
$MS_{8_1 2}$	0.152	0.000
$VS_{8_1 2}$	0.352	0.000
$MS_{9_1 2}$	0.137	0.000
$VS_{9_1 2}$	0.439	0.000
$MS_{10_1 2}$	0.174	0.000
$VS_{10_1 2}$	0.001	0.000
$MS_{11_1 2}$	0.008	0.000
$VS_{11_1 2}$	1.154	0.000
$MS_{12_1 2}$	0.081	0.000
$VS_{12_1 2}$	1.133	0.000
$MS_{13_1 2}$	0.166	0.000
$VS_{13_1 2}$	0.675	0.000

Table 28: Estimates: Measurement system -Skills at birth

Parameter	Estimate	Standard Error
$MS1_{BIRTH}$	-2.473	0.000
$VS1_{BIRTH}$	0.380	0.000
$MS2_{BIRTH}$	-1.978	0.000
$VS2_{BIRTH}$	0.275	0.000
$MS3_{BIRTH}$	-1.190	0.000
$VS3_{BIRTH}$	0.307	0.000
$MS4_{BIRTH}$	-3.304	0.000
$VS4_{BIRTH}$	0.549	0.000
$MS5_{BIRTH}$	-0.703	0.000
$VS5_{BIRTH}$	0.149	0.000
$MS6_{BIRTH}$	-2.759	0.000
$VS6_{BIRTH}$	0.491	0.000
$MS7_{BIRTH}$	-0.507	0.000
$VS7_{BIRTH}$	0.104	0.000
$MS8_{BIRTH}$	-1.758	0.000
$VS8_{BIRTH}$	0.436	0.000
$MS9_{BIRTH}$	-1.794	0.000
$VS9_{BIRTH}$	0.198	0.000
$MS10_{BIRTH}$	-15.424	0.000
$VS10_{BIRTH}$	3.171	0.000
$MS11_{BIRTH}$	-0.295	0.000
$VS11_{BIRTH}$	0.037	0.000
$MS12_{BIRTH}$	-0.831	0.000
$VS12_{BIRTH}$	0.113	0.000
$MS13_{BIRTH}$	-4.158	0.000
$VS13_{BIRTH}$	0.393	0.000
$MS14_{BIRTH}$	-3.563	0.000
$VS14_{BIRTH}$	0.379	0.000
$MS15_{BIRTH}$	-0.645	0.000
$VS15_{BIRTH}$	0.086	0.000
$MS16_{BIRTH}$	-0.575	0.000
$VS16_{BIRTH}$	0.071	0.000
$MS17_{BIRTH}$	-0.536	0.000
$VS17_{BIRTH}$	0.000	0.000
$MS18_{BIRTH}$	-0.627	0.000
$VS18_{BIRTH}$	0.000	0.000
$MS19_{BIRTH}$	-0.932	0.000
$VS19_{BIRTH}$	0.000	0.000
$MS20_{BIRTH}$	-0.296	0.000
$VS20_{BIRTH}$	0.000	0.000
$MS21_{BIRTH}$	-1.962	0.000
$VS21_{BIRTH}$	0.392	0.000
$MS22_{BIRTH}$	0.400	0.000
$VS22_{BIRTH}$	1.000	0.000
$MS23_{BIRTH}$	0.234	0.000
$VS23_{BIRTH}$	1.022	0.000

Table 29: Estimates: Measurement system -Pareto weight

Parameter	Estimate	Standard Error
$\mathrm{MS}1_{BARG}$	-0.337	0.000
$\mathrm{VS}1_{BARG}$	0.985	0.000
$\mathrm{MS}2_{BARG}$	0.137	0.000
$\mathrm{VS}2_{BARG}$	0.997	0.000
$\mathrm{MS}3_{BARG}$	-0.229	0.000
$\mathrm{VS}3_{BARG}$	1.001	0.000
$\mathrm{MS}4_{BARG}$	0.045	0.000
$\mathrm{VS}4_{BARG}$	0.959	0.000
$\mathrm{MS}5_{BARG}$	0.221	0.000
$\mathrm{VS}5_{BARG}$	1.004	0.000
$\mathrm{MS}6_{BARG}$	-0.337	0.000
$\mathrm{VS}6_{BARG}$	1.014	0.000
$\mathrm{MS}7_{BARG}$	0.231	0.000
$\mathrm{VS}7_{BARG}$	0.946	0.000
$\mathrm{MS}8_{BARG}$	0.108	0.000
$\mathrm{VS}8_{BARG}$	0.950	0.000
$\mathrm{MS}9_{BARG}$	0.232	0.000
$\mathrm{VS}9_{BARG}$	0.970	0.000
$\mathrm{MS}10_{BARG}$	-0.044	0.000
$\mathrm{VS}10_{BARG}$	0.983	0.000
$\mathrm{MS}11_{BARG}$	0.026	0.000
$\mathrm{VS}11_{BARG}$	0.046	0.000
$\mathrm{MS}12_{BARG}$	0.070	0.000
$\mathrm{VS}12_{BARG}$	0.018	0.000
$\mathrm{MS}13_{BARG}$	1.018	0.000
$\mathrm{VS}13_{BARG}$	0.911	0.000
$\mathrm{MS}14_{BARG}$	-0.991	0.000
$\mathrm{VS}14_{BARG}$	0.761	0.000
$\mathrm{MS}15_{BARG}$	0.450	0.000
$\mathrm{VS}15_{BARG}$	0.233	0.000
$\mathrm{MS}16_{BARG}$	-1.173	0.000
$\mathrm{VS}16_{BARG}$	0.836	0.000
$\mathrm{MS}17_{BARG}$	1.549	0.000
$\mathrm{VS}17_{BARG}$	0.246	0.000
$\mathrm{MS}18_{BARG}$	0.951	0.000
$\mathrm{VS}18_{BARG}$	0.100	0.000

Table 30: Estimates: Measurement system -Investments 2010

Parameter	Estimate	Standard Error
$MS1_{INV,10}$	0.638	0.000
$VS1_{INV,10}$	0.955	0.000
$MS2_{INV,10}$	4.771	0.000
$VS2_{INV,10}$	10.334	0.000
$MS3_{INV,10}$	1.984	0.000
$VS3_{INV,10}$	3.196	0.000
$MS4_{INV,10}$	1.700	0.000
$VS4_{INV,10}$	2.481	0.000
$MS5_{INV,10}$	0.261	0.000
$VS5_{INV,10}$	0.315	0.000
$MS6_{INV,10}$	0.214	0.000
$VS6_{INV,10}$	0.885	0.000
$MS7_{INV,10}$	0.536	0.000
$VS7_{INV,10}$	1.102	0.000
$MS8_{INV,10}$	0.438	0.000
$VS8_{INV,10}$	1.175	0.000

Table 31: Estimates: Measurement system -Investments 2012

Parameter	Estimate	Standard Error
$MS1_{INV,12}$	0.024	0.000
$VS1_{INV,12}$	0.868	0.000
$MS2_{INV,12}$	0.016	0.000
$VS2_{INV,12}$	0.993	0.000
$MS3_{INV,12}$	0.002	0.000
$VS3_{INV,12}$	0.911	0.000
$MS4_{INV,12}$	0.033	0.000
$VS4_{INV,12}$	0.944	0.000
$MS5_{INV,12}$	0.034	0.000
$VS5_{INV,12}$	1.004	0.000
$MS6_{INV,12}$	0.063	0.000
$VS6_{INV,12}$	0.966	0.000
$MS7_{INV,12}$	-0.001	0.000
$VS7_{INV,12}$	0.986	0.000
$MS8_{INV,12}$	0.030	0.000
$VS8_{INV,12}$	0.546	0.000
$MS9_{INV,12}$	0.063	0.000
$VS9_{INV,12}$	0.883	0.000
$MS10_{INV,12}$	0.029	0.000
$VS10_{INV,12}$	0.932	0.000
$MS11_{INV,12}$	0.042	0.000
$VS11_{INV,12}$	1.009	0.000
$MS12_{INV,12}$	0.233	0.000
$VS12_{INV,12}$	0.403	0.000
$MS13_{INV,12}$	0.440	0.000
$VS13_{INV,12}$	0.938	0.000
$MS14_{INV,12}$	0.224	0.000
$VS14_{INV,12}$	0.652	0.000
$MS15_{INV,12}$	0.000	0.000
$VS15_{INV,12}$	0.002	0.000
$MS16_{INV,12}$	0.397	0.000
$VS16_{INV,12}$	0.937	0.000
$MS17_{INV,12}$	0.102	0.000
$VS17_{INV,12}$	0.195	0.000
$MS18_{INV,12}$	0.394	0.000
$VS18_{INV,12}$	1.292	0.000
$MS19_{INV,12}$	0.966	0.000
$VS19_{INV,12}$	4.359	0.000
$MS20_{INV,12}$	-0.083	0.000
$VS20_{INV,12}$	1.070	0.000
$MS21_{INV,12}$	-0.084	0.000
$VS21_{INV,12}$	0.947	0.000

Table 32: Estimates: Measurement system -Parental effort 2010

Parameter	Estimate	Standard Error
$\mathrm{MS}1_{EF,10}$	0.209	0.000
$\mathrm{VS}1_{EF,10}$	0.833	0.000
$\mathrm{MS}2_{EF,10}$	0.175	0.000
$\mathrm{VS}2_{EF,10}$	0.660	0.000
$\mathrm{MS}3_{EF,10}$	0.161	0.000
$\mathrm{VS}3_{EF,10}$	0.288	0.000
$\mathrm{MS}4_{EF,10}$	0.212	0.000
$\mathrm{VS}4_{EF,10}$	0.789	0.000
$\mathrm{MS}5_{EF,10}$	0.232	0.000
$\mathrm{VS}5_{EF,10}$	0.327	0.000
$\mathrm{MS}6_{EF,10}$	0.757	0.000
$\mathrm{VS}6_{EF,10}$	0.810	0.000

Table 33: Estimates: Measurement system -Parental effort 2012

Parameter	Estimate	Standard Error
$\text{MS}1_{EF,12}$	0.123	0.000
$\text{VS}1_{EF,12}$	0.642	0.000
$\text{MS}2_{EF,12}$	0.131	0.000
$\text{VS}2_{EF,12}$	0.629	0.000
$\text{MS}3_{EF,12}$	0.183	0.000
$\text{VS}3_{EF,12}$	0.689	0.000
$\text{MS}4_{EF,12}$	0.088	0.000
$\text{VS}4_{EF,12}$	0.633	0.000
$\text{MS}5_{EF,12}$	0.080	0.000
$\text{VS}5_{EF,12}$	0.607	0.000
$\text{MS}6_{EF,12}$	0.195	0.000
$\text{VS}6_{EF,12}$	0.685	0.000
$\text{MS}7_{EF,12}$	0.191	0.000
$\text{VS}7_{EF,12}$	0.659	0.000
$\text{MS}8_{EF,12}$	0.146	0.000
$\text{VS}8_{EF,12}$	0.663	0.000
$\text{MS}9_{EF,12}$	0.156	0.000
$\text{VS}9_{EF,12}$	0.714	0.000
$\text{MS}10_{EF,12}$	0.308	0.000
$\text{VS}10_{EF,12}$	0.510	0.000
$\text{MS}11_{EF,12}$	0.327	0.000
$\text{VS}11_{EF,12}$	0.438	0.000
$\text{MS}12_{EF,12}$	0.154	0.000
$\text{VS}12_{EF,12}$	0.603	0.000
$\text{MS}13_{EF,12}$	0.368	0.000
$\text{VS}13_{EF,12}$	0.019	0.000
$\text{MS}14_{EF,12}$	0.365	0.000
$\text{VS}14_{EF,12}$	0.006	0.000

Table 34: Model Fit - I

Childcare Attendance	Predicted	Data
Working Mothers	78.3%	74.04%
Not-working Mothers	53.7%	52.1%
Total	68.5%	65.34%

Table 35: Model Fit - II

Male Labor Force Participation	Predicted	Data
2010	91.8%	91.6%
2012	93.1%	91.0%

Table 36: Model Fit - II

Male Labor Force Participation	Predicted	Data
2010	63.38%	60.28%
2012	63.7%	61.6%

Table 37: **Beneficiaries of monetary transfers**

Item	Number	Per cent
Non-Beneficiary	450	43
Beneficiary	585	57
Total	1,035	100

Table 38: Effects of Policy counterfactuals. Change in Female employment (percentage points)

Counterfactual	Effect on Female employment
1	-3.29
2	-3.29
3	0.39
4	0.00

Table 39: Effects of Policy counterfactuals. Change in Male employment (percentage points)

Counterfactual	Effect on Male employment
1	-1.55
2	-1.55
3	0.00
4	0.00

Table 40: Effects of Policy counterfactuals. Change in Mother's effort (standard deviations)

Counterfactual	Change in Mother's Effort (Standard Deviations)
1	0.14
2	0.14
3	-0.13
4	0.00

Table 41: Effects of Policy counterfactuals. Change in Father's effort (standard deviations)

Counterfactual	Change in Father's Effort (Standard Deviations)
1	0.15
2	0.15
3	-0.09
4	0.00

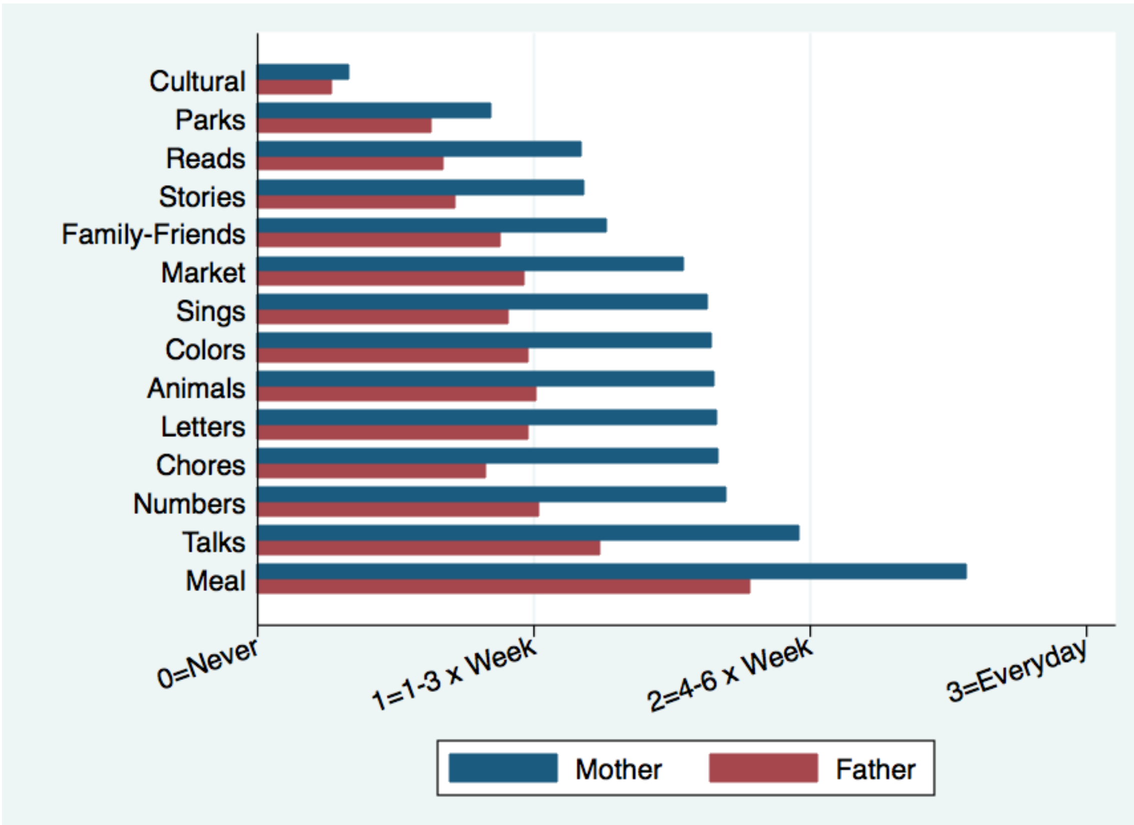
Table 42: Effects of Policy counterfactuals. Change in Father's effort (standard deviations)

Counterfactual	Change in Money Invested (Chilean Pesos)
1	37.13
2	37.13
3	18.99
4	3471.50

Table 43: Effects of Policy counterfactuals. Gaps in skills between the lowest and the richest income quintiles. Measured in Standard Deviations.

Counterfactual	20-20 gap in Skills
Initial situation	-1.11389
1	-1.09046
2	-1.09047
3	-1.09556
4	-1.06267

Figure 1: Frequency of activities between parents and children



For each activity there are possible answers: 0: never, 1: one to three times a week; 2: four to six times a week; 3: everyday.

Figure 2: Gaps in health at birth (%)

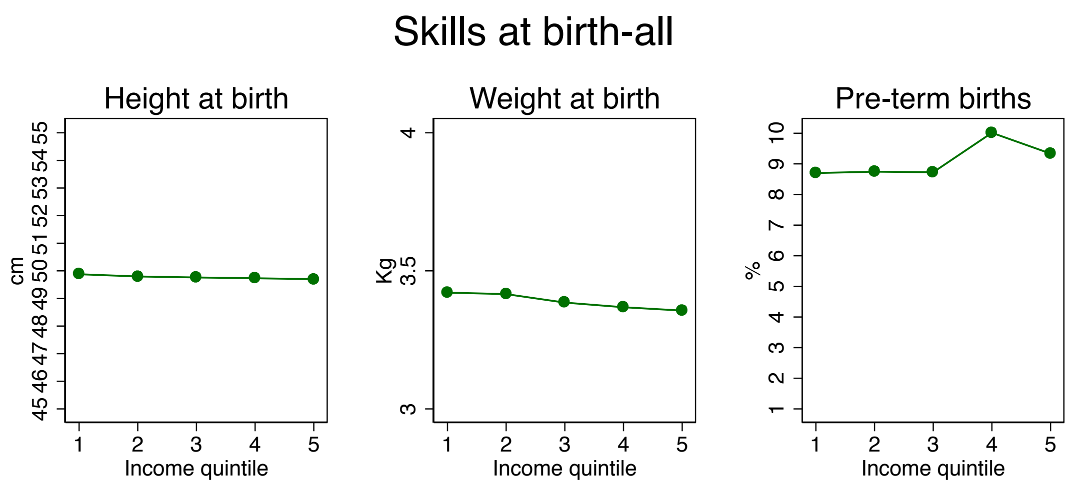


Figure 3: Gaps in skills at age 5

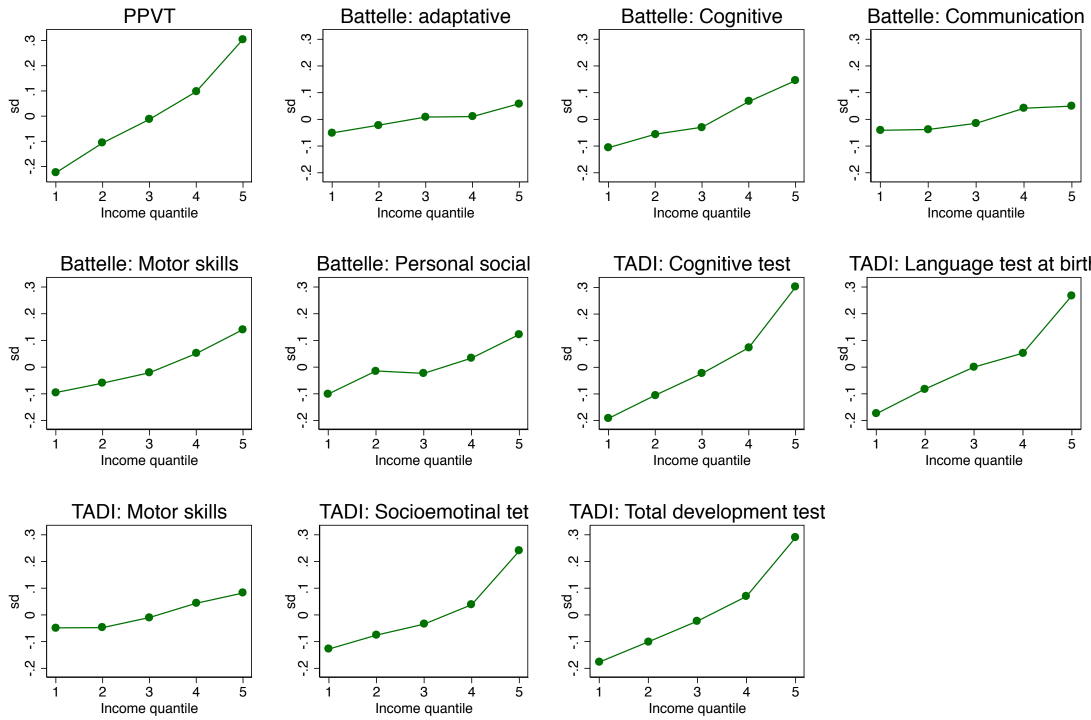


Figure 4: Mother's labor force participation (%)

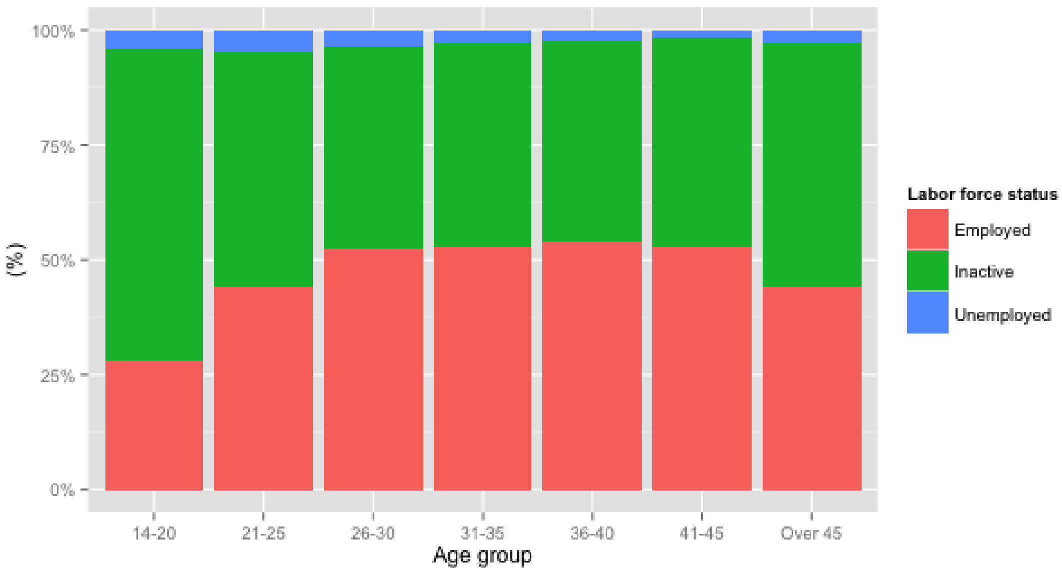


Figure 5: Model fit: Female labor force participation (%)

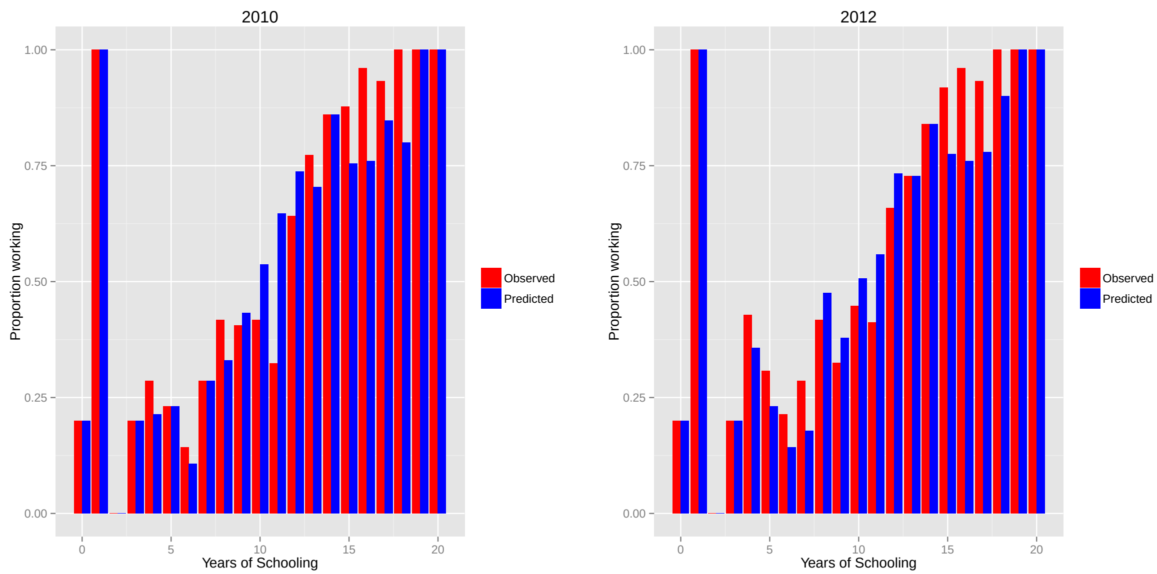


Figure 6: Model fit: Female labor force participation (%)

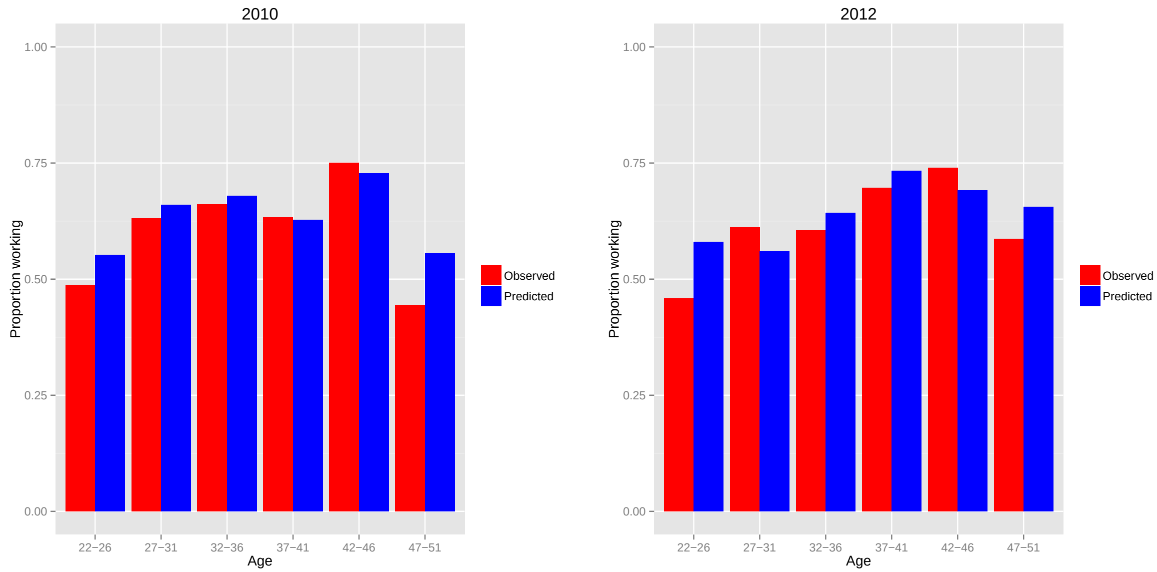


Figure 7: Model fit: Female labor force participation (%)

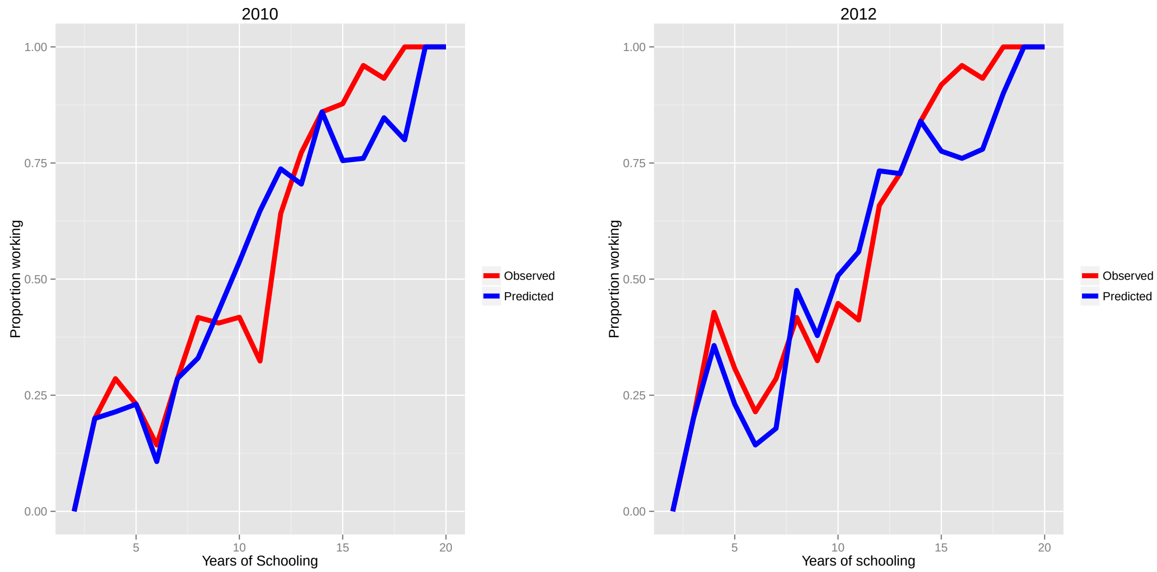


Figure 8: Model fit: Female labor force participation (%)

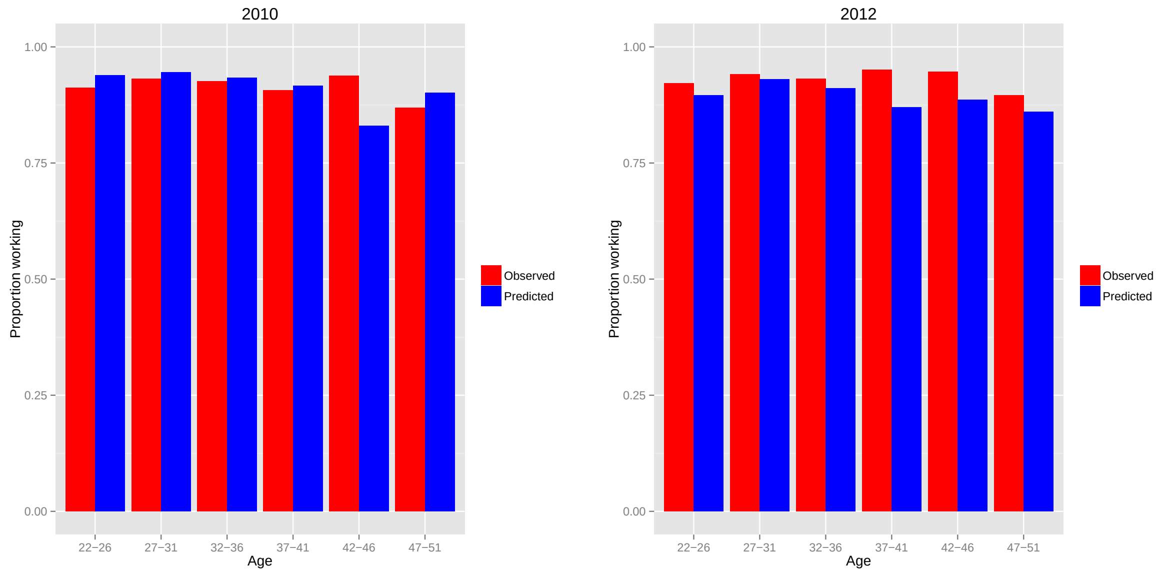


Figure 9: Model fit: Female labor force participation (%)

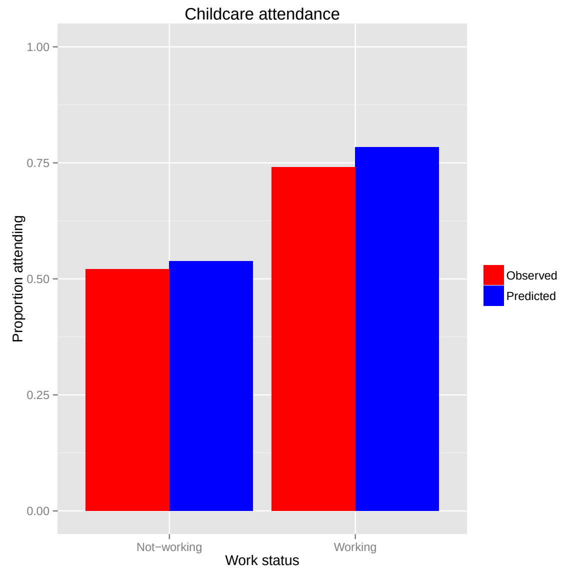


Figure 10: Distribution of skills. Smoothing distribution

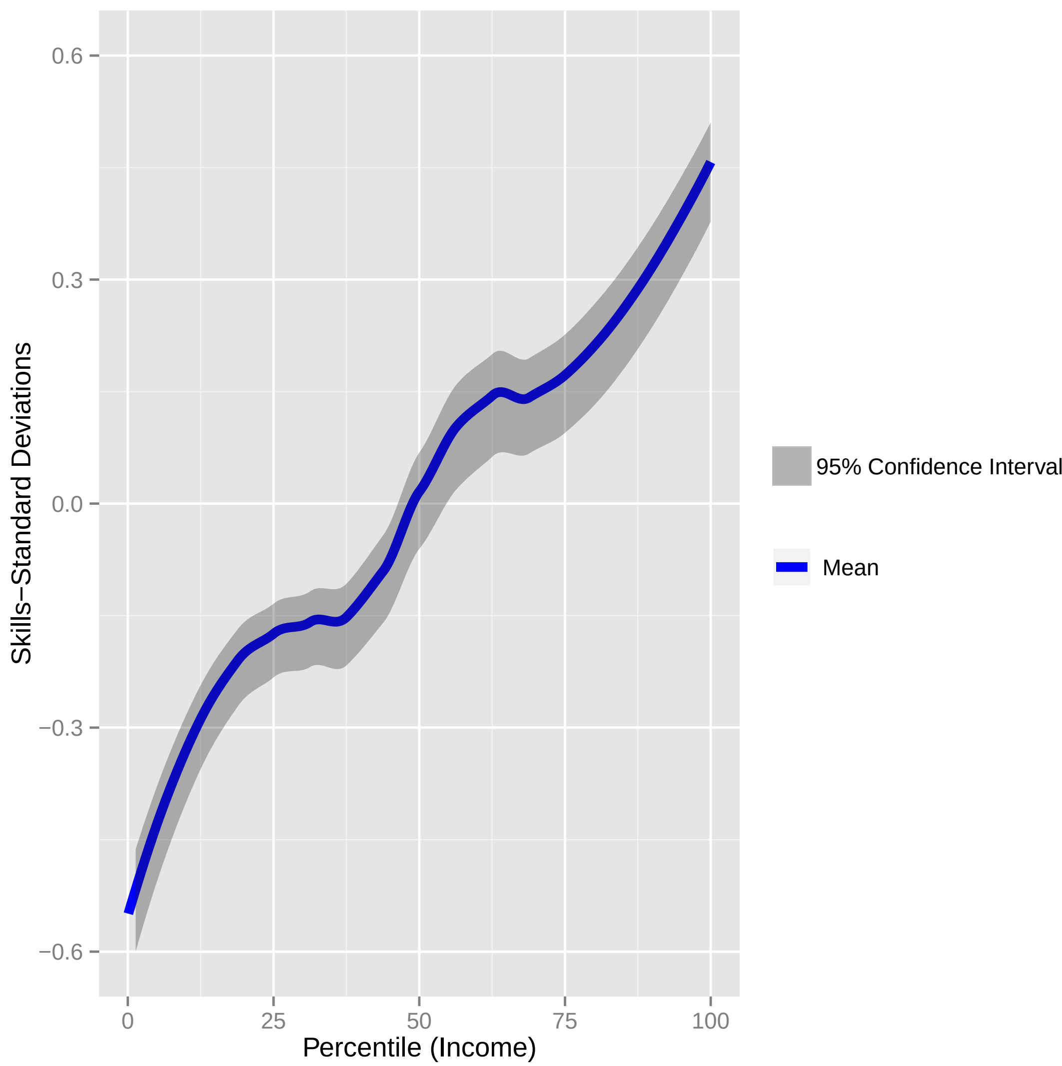


Figure 11: Signal to noise ratio. Mother's effort (2012)

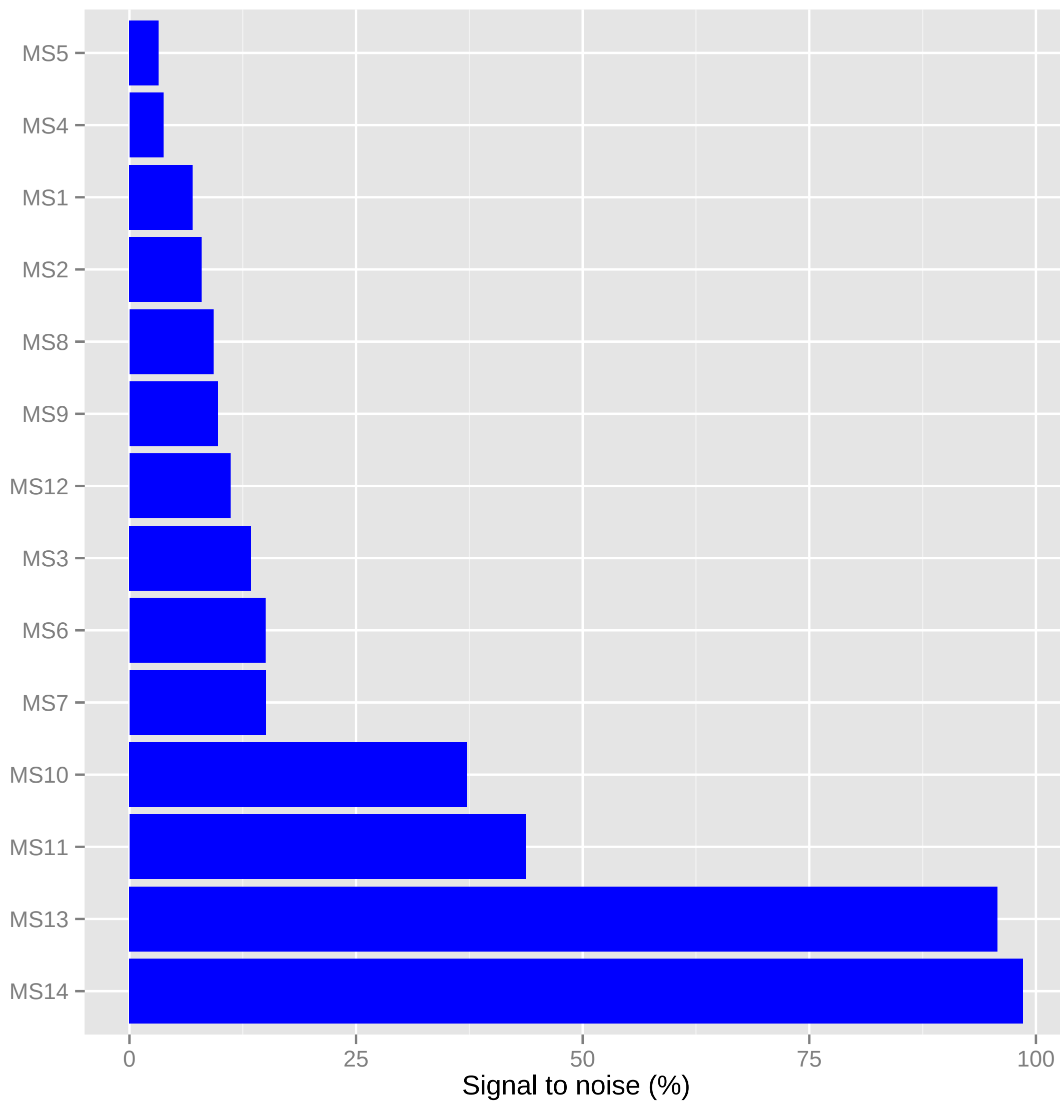


Figure 12: Signal to noise ratio. Father's effort (2012)

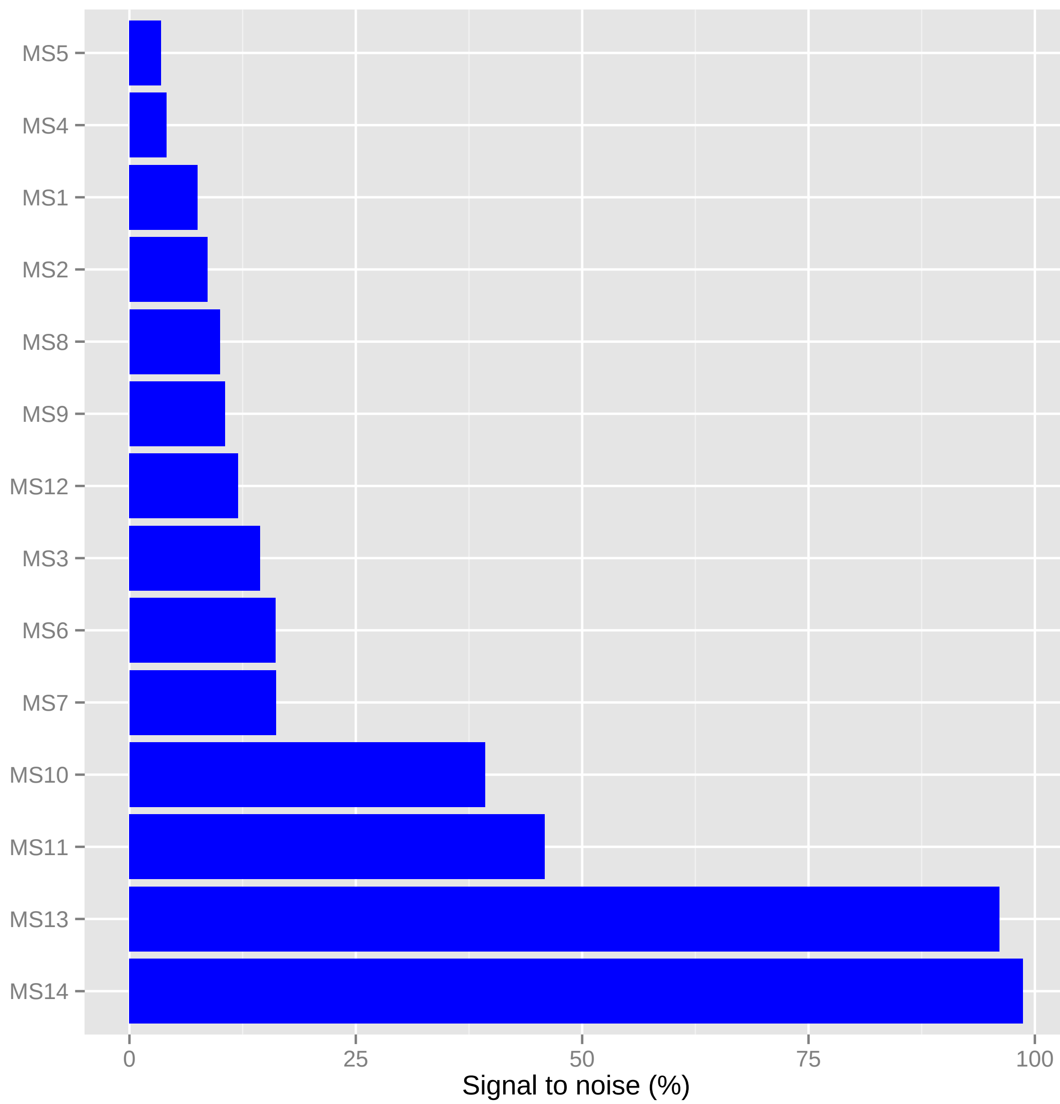


Figure 13: Signal to noise ratio. Mother's effort (2010)

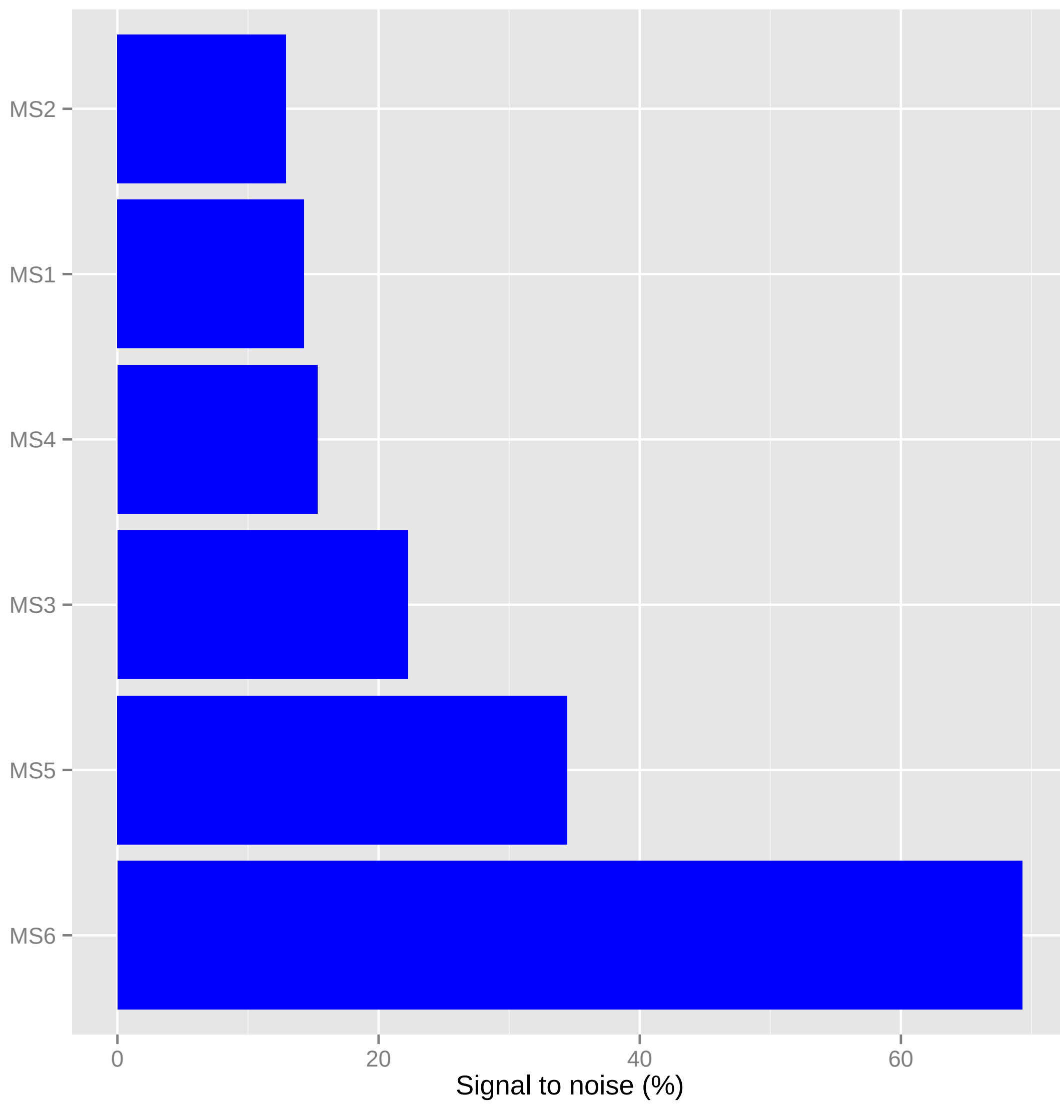


Figure 14: Signal to noise ratio. Father's effort (2010)

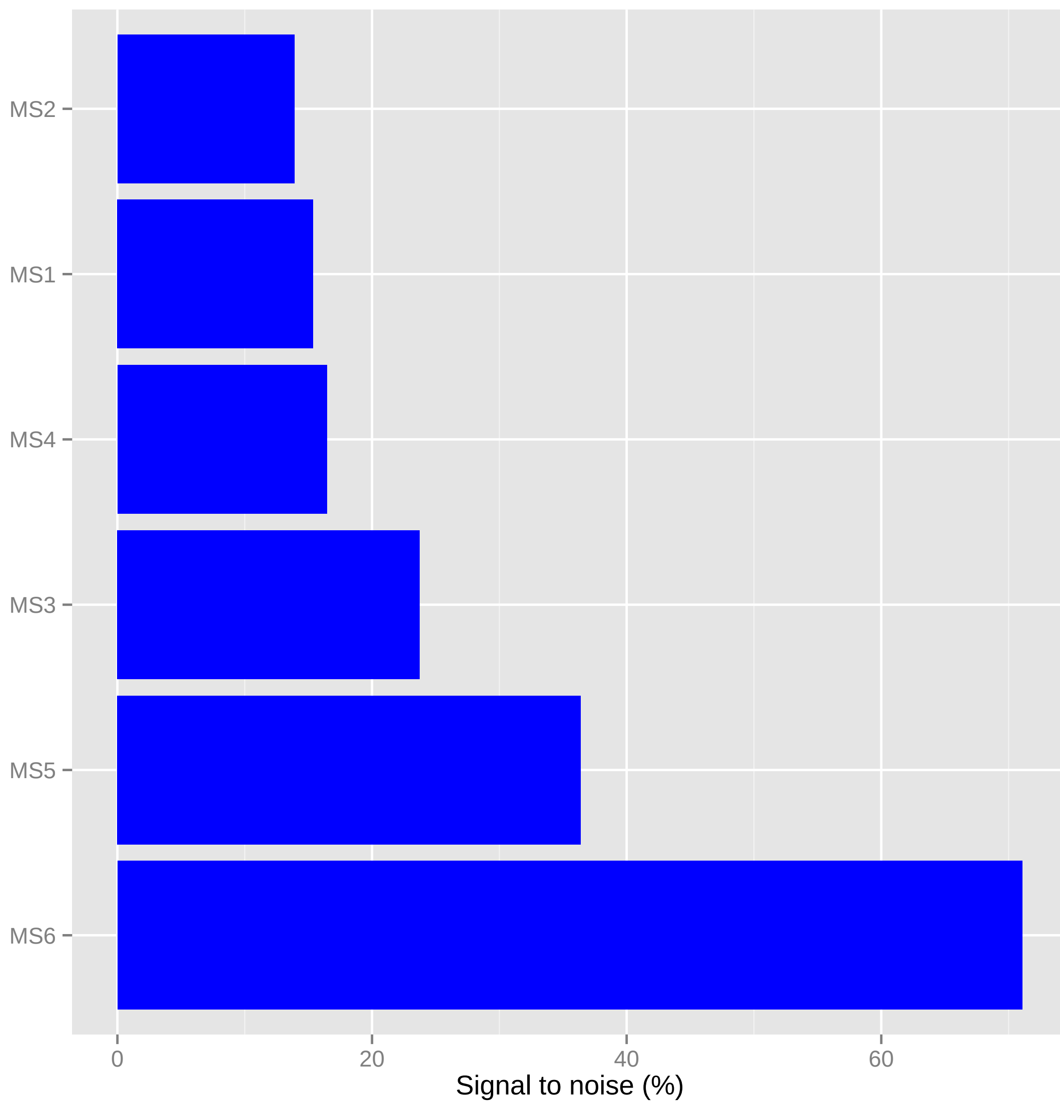


Figure 15: Signal to noise ratio. Monetary Investment (2010)

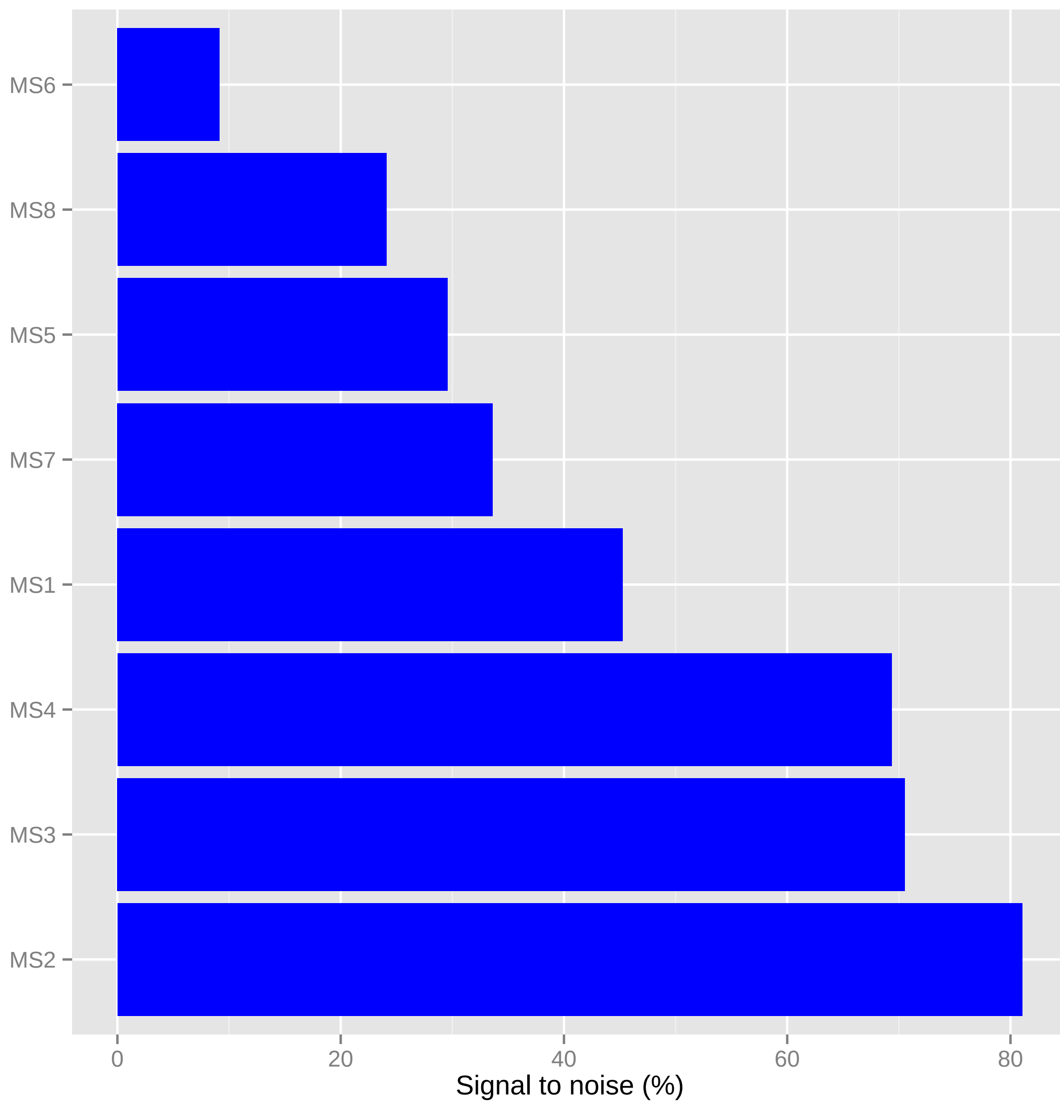


Figure 16: Signal to noise ratio. Monetary Investment (2012)

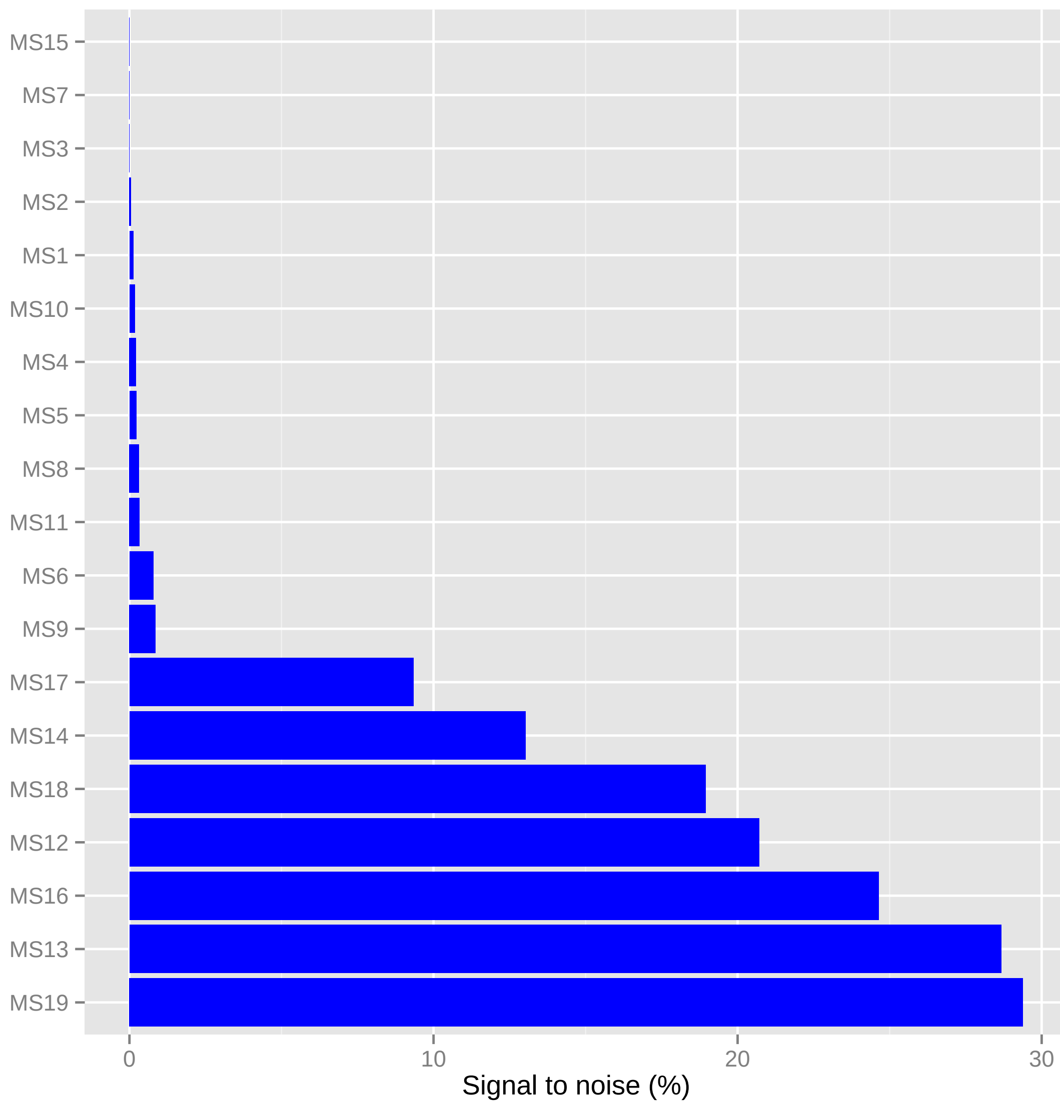


Figure 17: Signal to noise ratio. Birth Outcomes

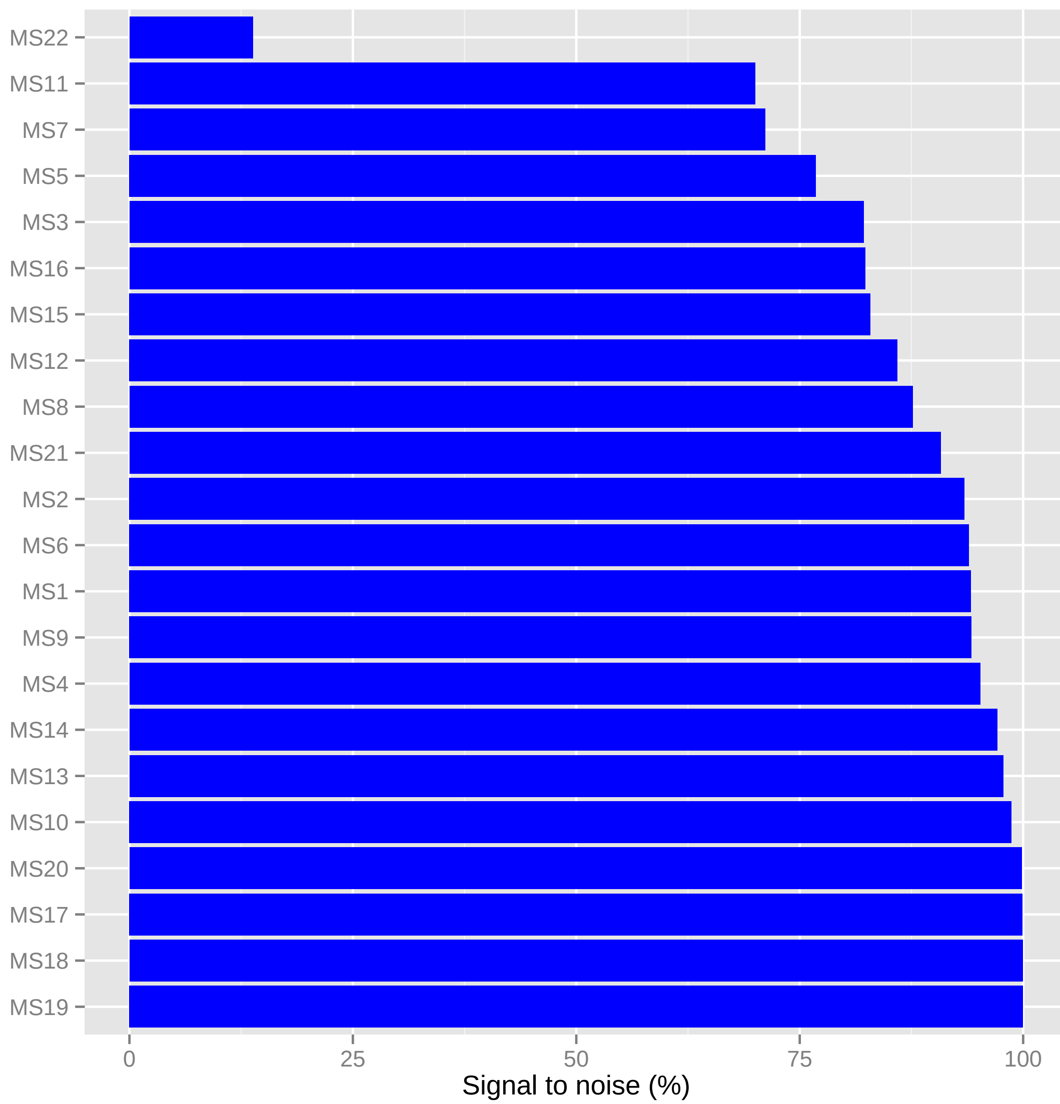


Figure 18: Signal to noise ratio. Skills (2010)

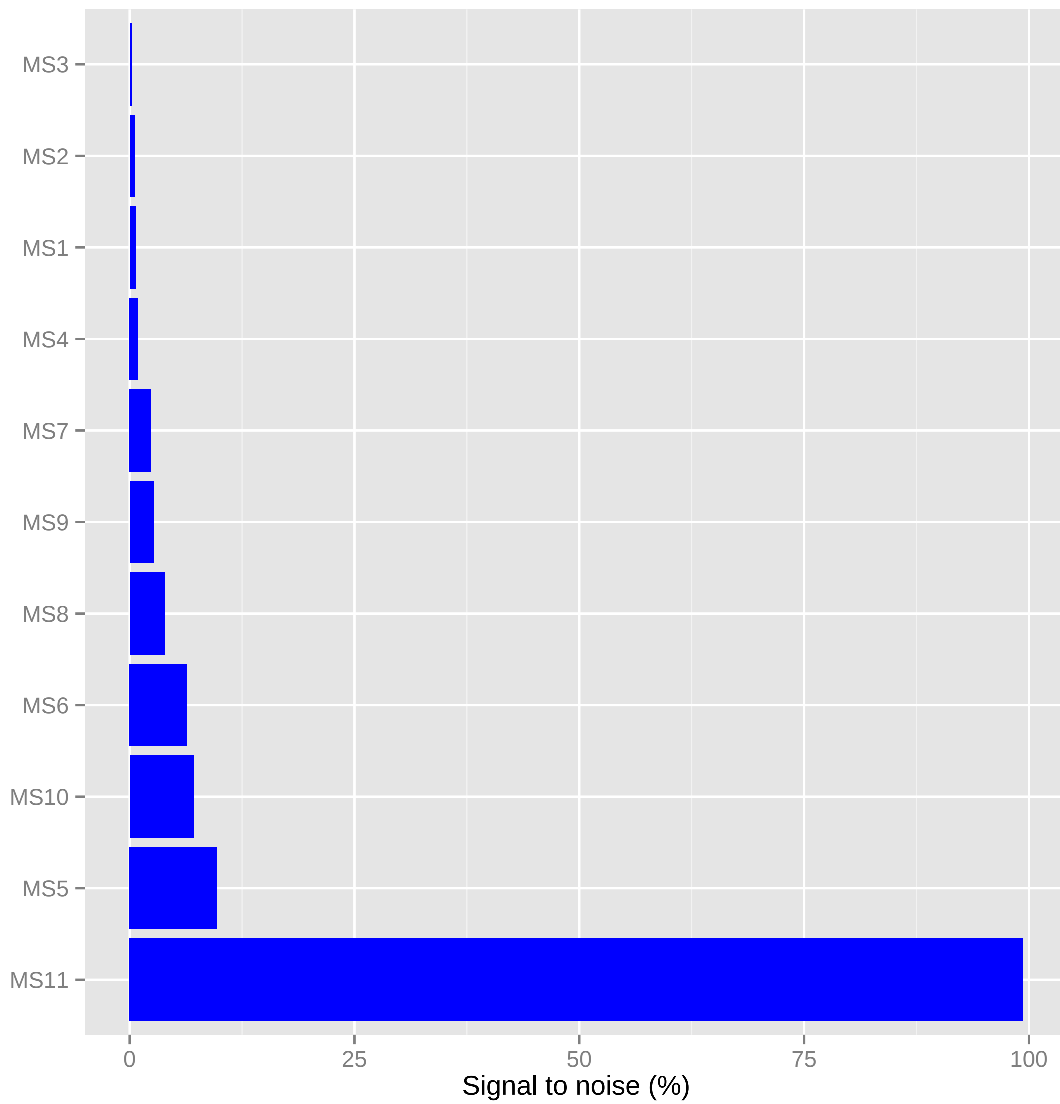


Figure 19: Signal to noise ratio. Skills (2012)

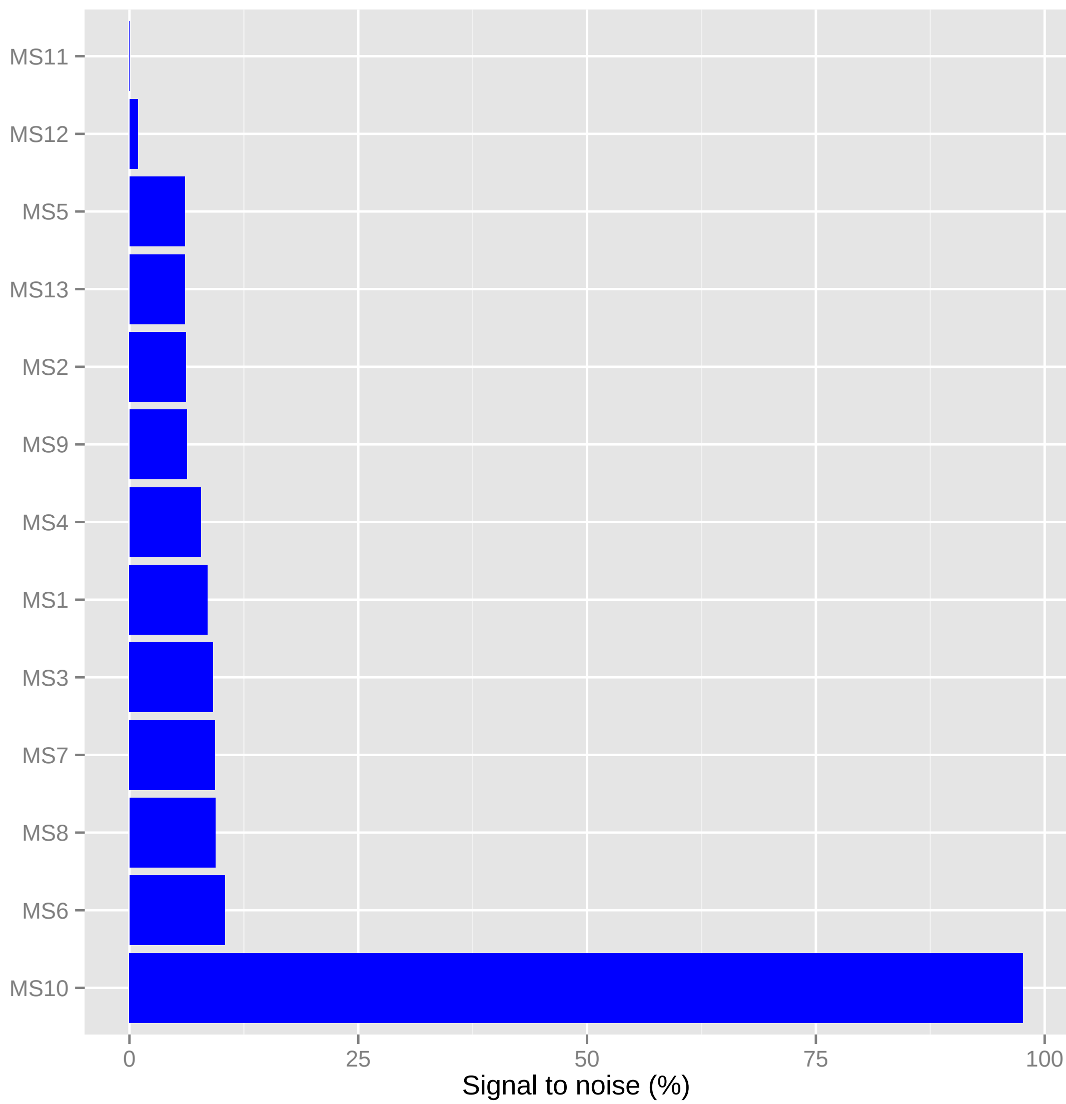


Figure 20: Signal to noise ratio. Bargaining power

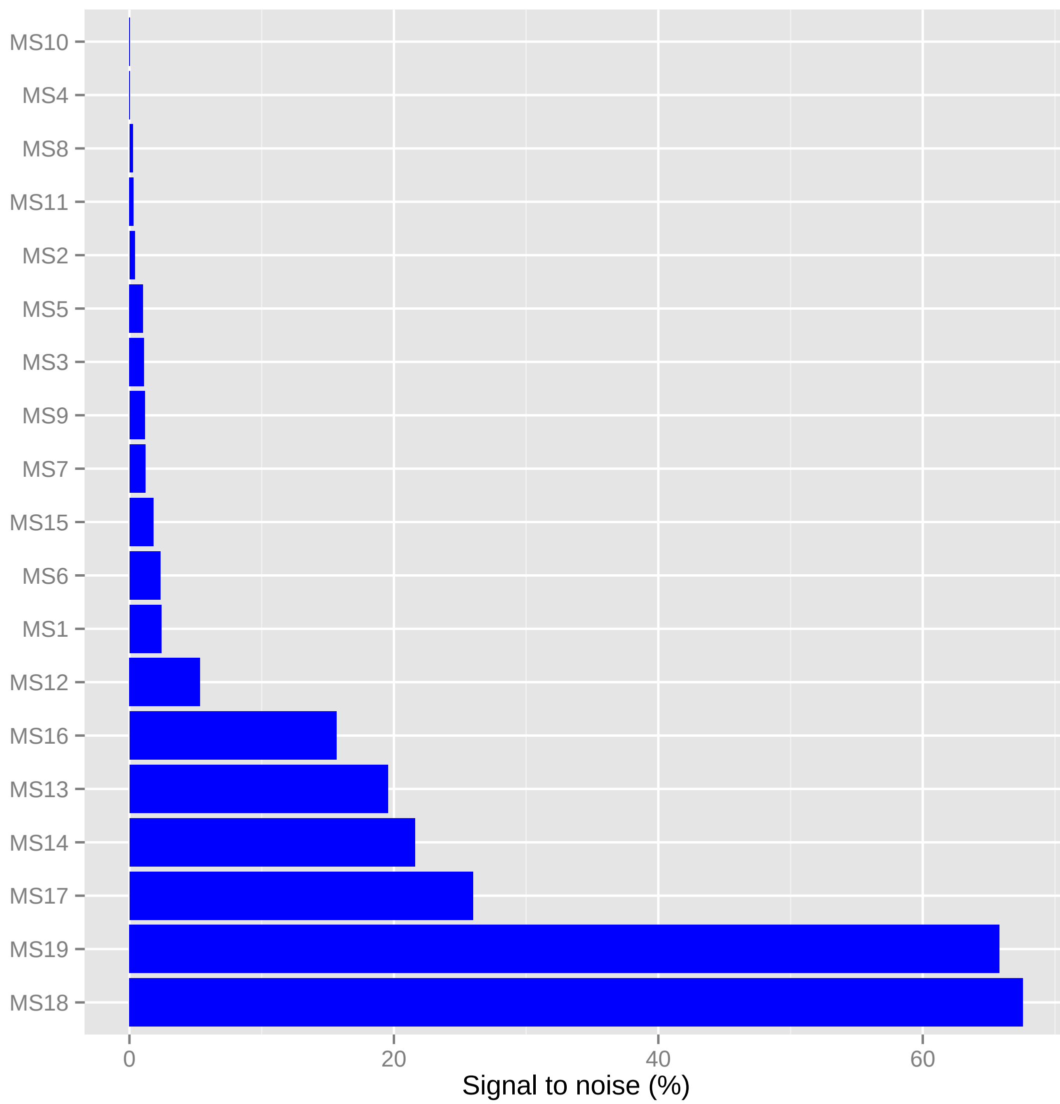


Figure 21: Signal to noise ratio. Primary Caregiver Skills

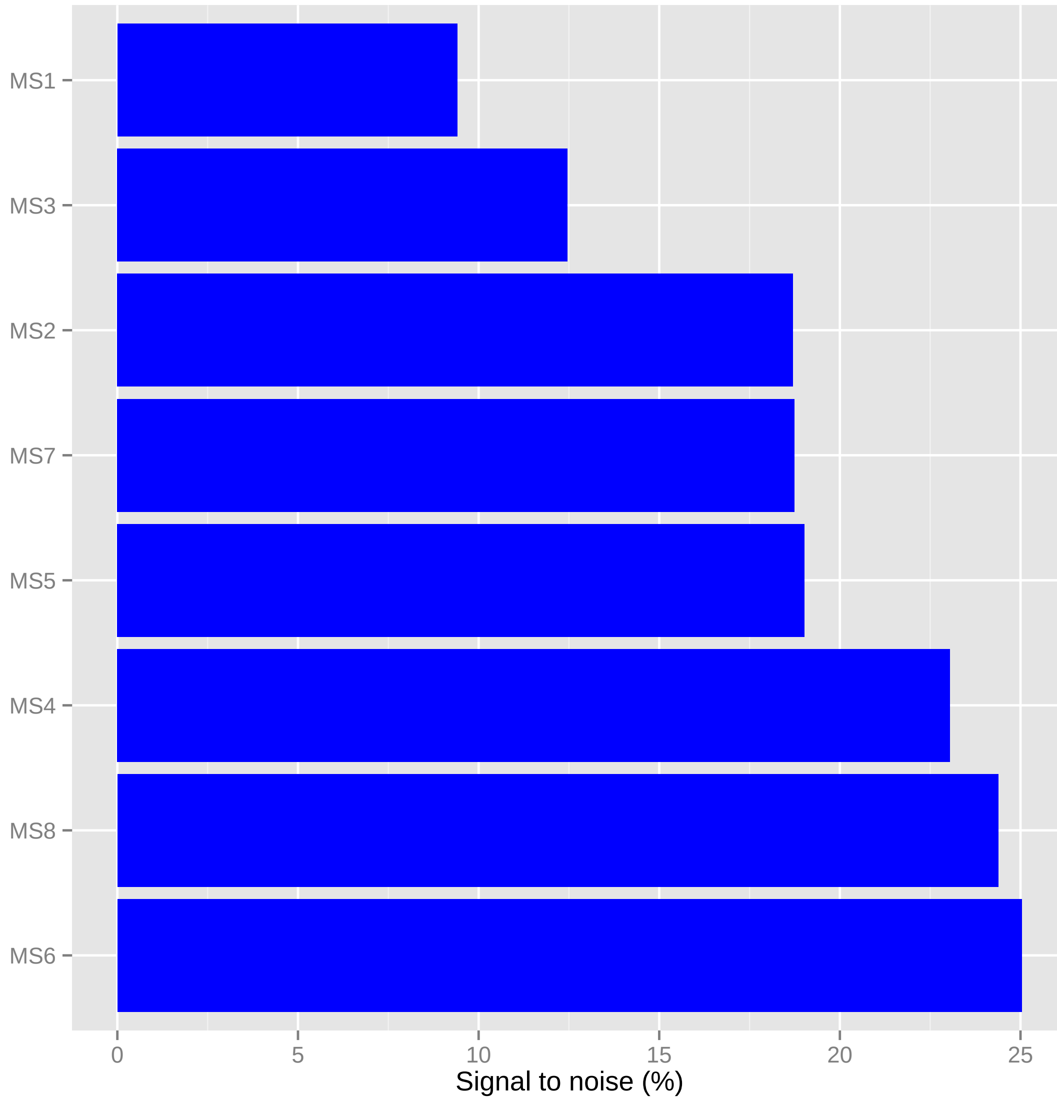


Figure 22: Distribution of monetary transfers

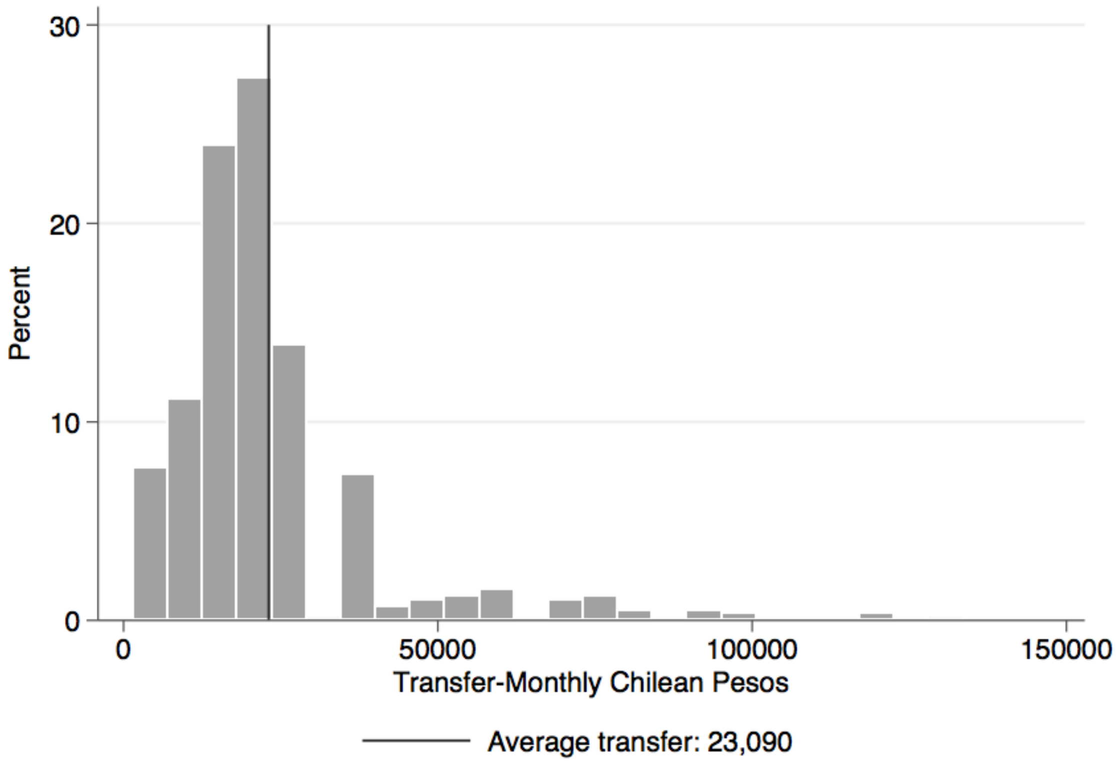


Figure 23: Effects of Policy Experiments

Percentage change on female employment as a result of policy counterfactuals

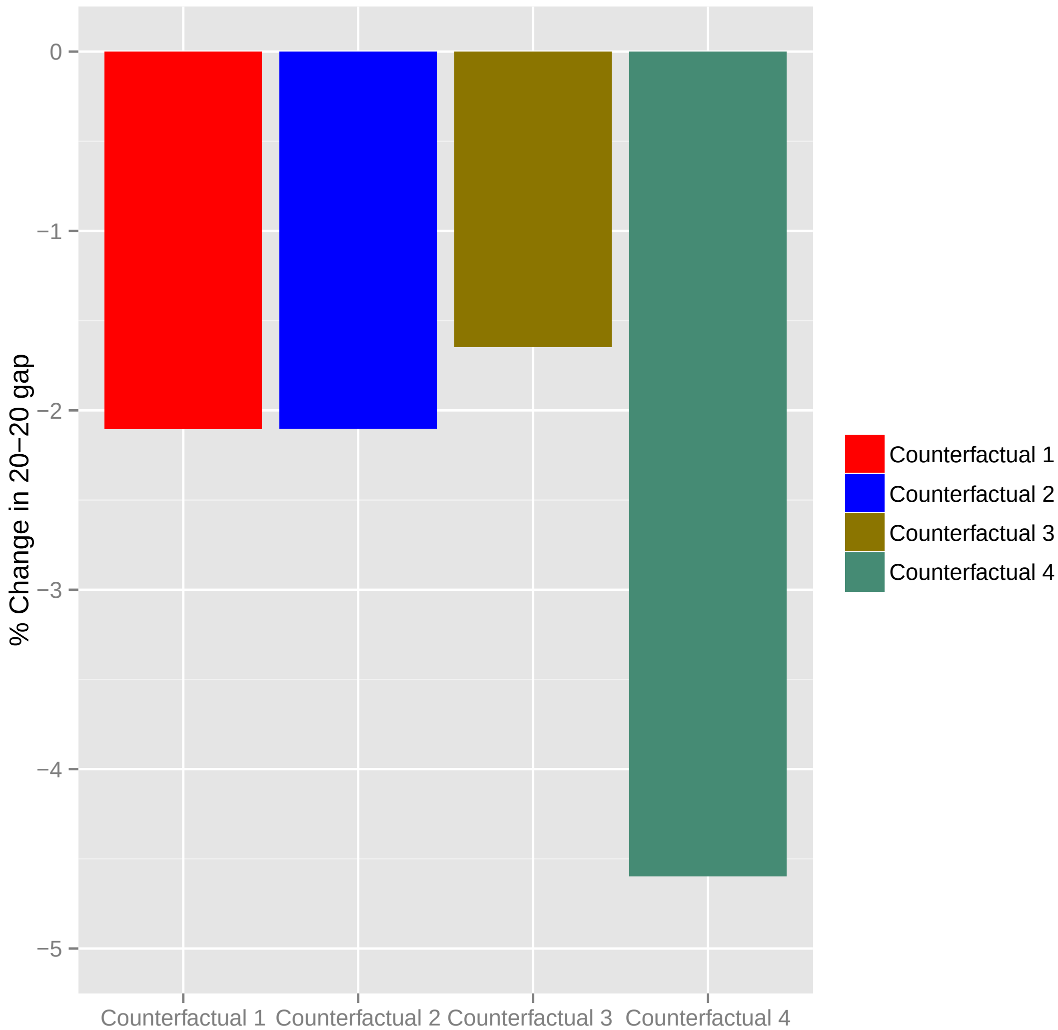
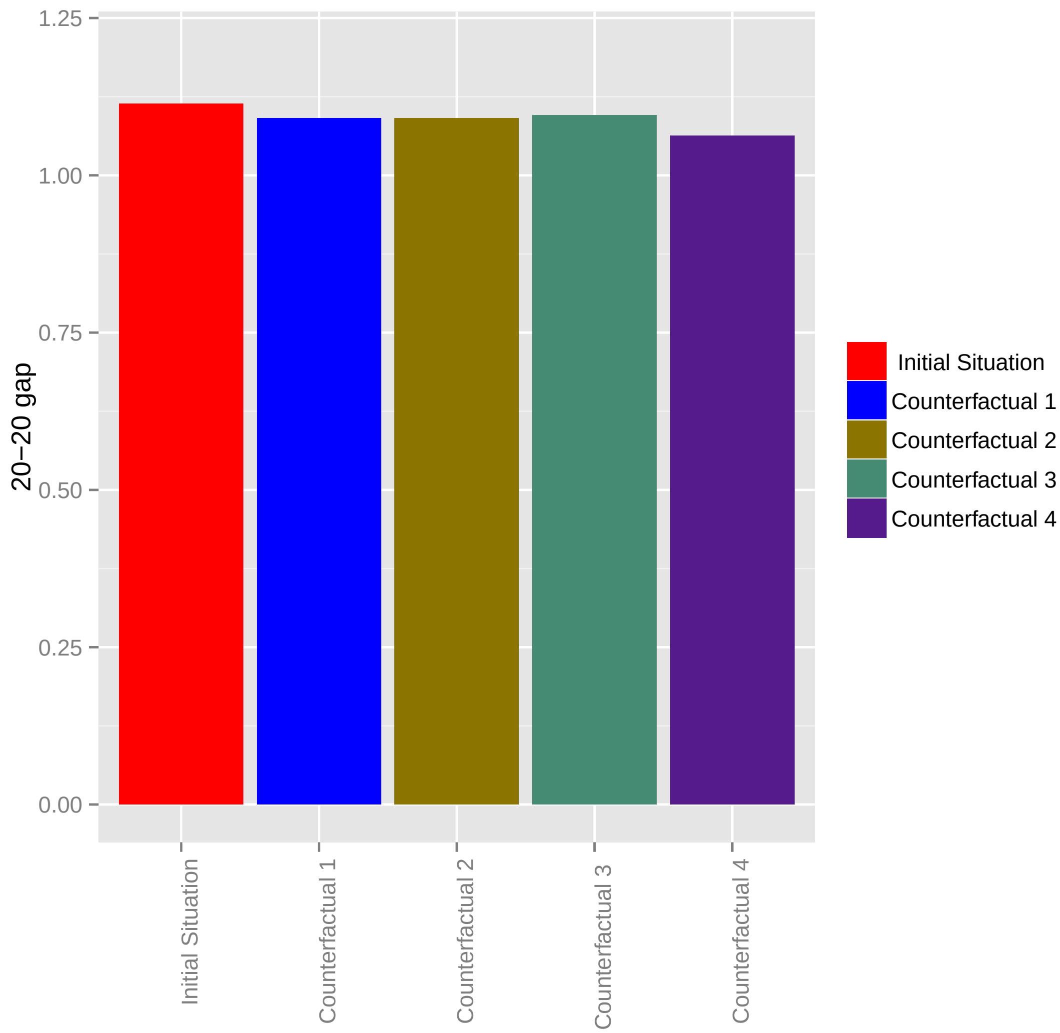


Figure 24: Effects of Policy Experiments

Percentage change on female employment as a result of policy counterfactuals



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13 Appendix

13.1 Identification of Measurement System

The measurement system is described by:

$$Z = \iota_0 + \iota_1 K + \boldsymbol{\varepsilon} \tag{55}$$

We normalize $E[k] = 0$ for every factor. The variance-covariance matrix of the measurement system is given by:

$$\Sigma_Z = \iota_1 \Sigma_K \iota_1' + \Sigma_\varepsilon \tag{56}$$

The matrix of moments Σ_Z contains $M(M+1)/2$ moments in order to identify the necessary parameters of the models. M is the total number of measures available and is equal to the sum of measures for each factor:

$$M = \sum_{k \in K} N_k = 151 \tag{57}$$

as we have

$$N_{ln(S_0)} = 23$$
$$N_{ln(S_1)} = 11$$
$$N_{ln(S_2)} = 13$$
$$N_{ln(PG)} = 8$$
$$N_{\mu} = 19$$
$$N_{ln(I_1)} = 8$$
$$N_{ln(I_2)} = 21$$
$$N_{ln(\hat{e}_1^f)} = 10$$
$$N_{ln(\hat{e}_2^f)} = 14$$
$$N_{ln(\hat{e}_1^m)} = 10$$
$$N_{ln(\hat{e}_2^m)} = 14$$

The dedicated factor structure assumed imposes that each measure loads exclusively to one factor. This implies that rather than $11 \times M$ factor loadings to obtain we only have to estimate M elements in ι_1 to be estimated. Given that the scale of the factor is irrelevant for the analysis, we can normalize one factor loading for each factor to be 1. In total, we have $M - 11 = 140$ factor loadings to be estimated.

The matrix Σ_K contains $(11 \times (11+1)/2)$ covariances to be estimated and Σ_ε has $M \times (M + 1)/2$. We see that it is necessary to make some assumptions about the correlation structure of the factors or of the measurement error system in order to be able to identify the system. If we assume that the measurement error in the system for skills at birth is independent of measurement error in the remaining systems $\varepsilon_m^{\ln(s_0)} \perp\!\!\!\perp \varepsilon_m'^k$ for $m = 1...N_{ln(s_0)}$, $k \in K, k \neq \ln(s_0)$, $m' = 1...N_k$ we have enough moments to identify the system. By doing this assumption, we are assuming that the elements in Σ_ε that correspond corresponding to $ln(s_0)$ and other factors are zero. With this, we have enough moments to identify the system.

13.2 Estimation

In this section I will derive the full likelihood function of the model as well as the filtering procedure to estimate it.

13.2.1 Likelihood function

The likelihood of the model is:

$$\mathcal{L}(\Theta|O;X) = P(O|X;\Theta) = P(O_1, O_2, O_3|X;\Theta)$$

$$p_0(O_0|\Theta, X)p_1(O_1|O_0, \Theta, X)p_2(O_2|O_1, \Theta, X) \tag{58}$$

Now, inspecting every element. The first term is composed by the observed outcomes in period zero. Given that the only one observed in this case is the first period of skills, this is composed then by that.

$$p_0(O_0|\Theta, X) = \int p_0(O_0, K_0|\Theta, X)dK_0 =$$

$$\int p_0(O_0|K_0, \Theta, X)p(K_0|\Theta, X)dK_0 =$$

$$E_{p(K_0|\Theta,X)}\left[P_0(O_0|K_0, \Theta, X)\right] \approx$$

$$\sum_{rr=1}^{RR} P_0(O_0|K_0^{\{rr\}}, \Theta, X) \tag{59}$$

for RR large, and for the $\{K_0^{\{rr\}}\}_{rr=1}^{RR}$ being drawn from the distribution $p(K_0|\Theta, X)$. K_0 is the set of unobserved factors relevant for period zero given by

$$K_0 = \{\ln(s_0), \ln(PG)\} \tag{60}$$

Note that in the model the distribution $p(K_0|\Theta, X)$ is not specified. I will assume that both factors are independent and each follow a normal distribution with mean zero and variance $\sigma^2_{s_0}$ and σ^2_{PG} respectively. This way, evaluating the likelihood for period 0 ends up being a process of drawing shocks from the distribution $p(K_0|\Theta, X)$, computing the likelihood of each shock given by the measurement system of the unobserved latent factors and averaging such likelihoods over the RR shocks.

For the first period the set of relevant factors is given by:

$$K_1 = \{\ln(s_1), \ln(\hat{e}_1^f), \ln(\hat{e}_1^m), \ln(\hat{I}_1)\} \tag{61}$$

and the likelihood can be expressed as:

$$p_1(O_1|O_0, \Theta, X) = \int p_1(O_1, K_1|O_0, \Theta, X) dK_1 =$$

$$\int \int p_1(O_1, K_1, K_0|O_0, \Theta, X) dK_1 dK_0 =$$

$$\int \int p_1(O_1|K_1, K_0, O_0, \Theta, X) p(K_1|O_0, K_0, \Theta, X) p(K_0|O_0, \Theta, X) dK_1 dK_0 \tag{62}$$

Note that

$$p(K_1|O_0, K_0, \Theta, X) = p(K_1|K_0, \theta, X) \tag{63}$$

as O_0 would not carry more information beyond that in K_0 that is relevant for K_1. Also, note that

$$p_1(O_1|K_1, K_0, O_0, \Theta, X) = p_1(O_1|K_1, \Theta, X) \tag{64}$$

Taking into account the facts presented in Equations 63 and 64 we can express 62 as:

$$\int \int p_1(O_1|K_1, \Theta, X) p(K_1|K_0, \Theta, X) p(K_0|O_0, \Theta, X) dK_1 dK_0 =$$

$$\int p(K_0|O_0, \Theta, X) \left[\int p(O_1|K_1, \Theta, X) p(K_1|K_0, \Theta, X) dK_1 \right] dK_0 =$$

$$E_{p(K_0|O_0,\Theta,X)} \left[\int p(O_1|K_1, \Theta, X) p(K_1|K_0, \Theta, X) dK_1 \right] = \tag{65}$$

in Equation 65 $p(O_1|K_1, \Theta, X)$ is given by the measurement system of factors, the likelihood of wages (for those that are observed) and the preference shocks cdf. We can re-write such expression as:

$$p(O_1|K_1,\Theta,X) =$$

$$p(\mathcal{Z}_1|K_1,\Theta,X) \times p(w^f|K_1,\Theta,X)^{(1-h^{f,*})} \times p(w^m|K_1,\Theta,X)^{1-h^{m,*}}$$

$$\times\, p(h^{f,*},h^{m,*},a|w^f,w^m,K_1,\Theta,X) \tag{66}$$

As specified previously, $p(\mathcal{Z}_1|K_1,\Theta,X)$ is given by the measurement system. $p(w^f|K_1,\Theta,X)$ is given by the measurement error associated to the observed wages:

$$w^j = \beta_0^j + \beta_1^j yrschool^j + \beta_2 Age^j + \beta + 3(Age^j)^2 + \varepsilon_{w^j} \tag{67}$$

where ε_{w^j} is measurement error following a distribution $\varepsilon_{w^j} \sim N(0,\sigma_{\varepsilon^j})$.
Finally, $p(h_f^*,h_m^*,a^*|w^f,w^m,K_1,\Theta,X)$ is given by the probability of having the observed decisions as the optimal ones:

$$p(h_f^*,h_m^*,a^*|w^f,w^m,K_1,\Theta,X) =$$

$$p_{(\varepsilon_d^f,\varepsilon_d^m)}\left(W(u^f(h^{f*},h^{m,*},a^*),u^m(h^{f,*},h^{m,*},a^*)) \in \arg\max_{\{h^f,h^m,a\}} W(u^f(h^f,h^m,a),u^m(h^f,h^m,a))|K_1,\Theta,X\right) \tag{68}$$

where $p_{(\varepsilon_d^f,\varepsilon_d^m)}$ is the distribution of the preference shocks $\boldsymbol{\varepsilon_d^f,\varepsilon_d^m}$.

$p(K_1|K_0,\Theta,X)$ is given by the transition equation. Note, however, that the dynamics of the system are only given through the skills of the child, the remaining factors do not have any dynamics carried from the previous period. This implies that such expression will be given by the skills production function and the distribution of heterogeneity in the remaining factors. Being explicit:

$$p(K_1|K_0,\Theta,X) = p(\ln(s_1),\ln(\hat{e}_1^f),\ln(\hat{e}_1^m),\ln(\hat{I}_1)|\ln(PG),\ln(s_0),\Theta,X)$$

$$\int p(\ln(s_1),\ln(\hat{e}_1^f),\ln(\hat{e}_1^m),\ln(\hat{I}_1),\mu_1|\ln(PG),\ln(s_0),\Theta,X)d\mu =$$

$$\int p(\ln(s_1)|\ln(\hat{e}_1^f),\ln(\hat{e}_1^m),\ln(\hat{I}_1),\mu_1,\ln(PG),\ln(s_0),\Theta,X)d\mu \tag{69}$$

$$\times \int p(\ln(\hat{e}_1^f)|\mu_1\Theta,X)d\mu \tag{70}$$

$$\times \int p(\ln(\hat{e}_1^m)|\mu_1,\Theta,X)d\mu \tag{71}$$

$$\times \int p(\ln(\hat{I}_1)|\mu_1,\Theta,X)d\mu \tag{72}$$

We integrate over the factor μ given that we have no measures for it during the first period. The term 69 is given by the production of skills and the remaining 70-72 are given by the distribution of heterogeneity in each factor: η_{e^f}, η_{e^m} and η_I. Note that we can also use Monte-Carlo techniques to approximate the expression in 65 by:

$$\sum_{rr=1}^{RR} \hat{w}_0^{\{rr\}} \left[\int p(O_1|K_1,\Theta,X)p(K_1|K_0^{\{rr\}},\Theta,X)dK_1 \right] \tag{73}$$

where $\{K_0^{\{rr\}}\}_{rr=1}^{RR}$ are drawn from an importance distribution $g_0(K_0|\mathcal{Z}_0,\Theta,X)$ and the weights are given by:

$$\hat{w}_0^{rr} = \frac{w_0^{rr}}{\sum_{rr=1}^{RR} w_0^{rr}} \tag{74}$$

and the individual weights are defined:

$$w_0^{rr} \propto \frac{p(K_0|O_0,\Theta,X)}{g_0(K_0|\mathcal{Z}_0,\theta_0,\Theta,X)} \tag{75}$$

Note that after some algebra, we can define:

$$\tilde{w}_1 = \frac{p(O_1|K_1,\Theta,X)p(K_1|K_0,\Theta,X)}{g_t(K_0|,O_0,O_1,\Theta,X)} \tag{76}$$

where $g_t(K_0|,O_0,O_1,\Theta,X)$ is the proposal -importance- distribution from which the particles are going to be drawn. We will explain below what this distribution is. Note that replacing 76 into 65 we obtain:

$$\sum_{rr=1}^{RR} \hat{w}_0^{\{rr\}} \left[\int p(O_1|K_1,\Theta,X)p(K_1|K_0^{\{rr\}},\Theta,X)dK_1 \right] =$$

$$\sum_{rr=1}^{RR} \hat{w}_0^{\{rr\}} \left[\sum_{rr'=1}^{RR} \tilde{w}_1^{rr'}(rr) \right] \tag{77}$$

And finally note that the dependence given between rr and rr' generates a $dirac$ measure in dependence (all that follow from rr different in the dependence path go to zero in rr'). Then, we can write the expression of the likelihood in the first period as:

$$p_1(O_1|K_1,K_0,O_0,\Theta,X) = \sum_{rr=1}^{RR} \hat{w}_0^{rr}\tilde{w}_1^{rr} \tag{78}$$

The computation of the likelihood for the second period is identical to that of the first period with the exception that we do not need to integrate over the factor μ given that we have a set of measures for it. In particular:

$$
\begin{aligned}
p(K_2|K_1,\Theta,X) = & p(\ln(s_2),\ln(\hat{e}_2^f),\ln(\hat{e}_2^m),\ln(\hat{I}_2),\mu_2|\ln(PG),\ln(s_1),\Theta,X) = \\
& p(\ln(s_2)|\ln(\hat{e}_2^f),\ln(\hat{e}_2^m),\ln(\hat{I}_2),\mu_2,\ln(PG),\ln(s_1),\Theta,X) = && (79) \\
& \times p(\ln(\hat{e}_2^f)|\mu_2\Theta,X) = && (80) \\
& \times p(\ln(\hat{e}_2^m)|\mu_2,\Theta,X) = && (81) \\
& \times p(\ln(\hat{I}_2)|\mu_2\Theta,X) = && (82) \\
& \times p(\mu_2|\Theta,X) && (83)
\end{aligned}
$$

In this case we will use as importance distribution the same transition equation. The literature refers to this type of filtering as the bootstrap filter Creal (2012). The distribution $p(\mu_2|\Theta,X)$ is given by the corresponding transformation from from Equation 12

13.2.2 Filtering

Now that we have an expression for the likelihood function in a way that can be computed via simulation, I will present the algorithm used to evaluate the likelihood function at a given point:

13.3 Filtering Algorithm

1. Set t=0.

   (a) For rr=1....RR:

      i. draw $K_0^{\{rr\}}$ from proposal distribution $g(K_0|\Theta, X)$

      ii. Compute the weights $\hat{w}_0^{\{rr\}} = \frac{1}{RR}$

   (b) Compute likelihood for measurement system in $t = 0$: $\frac{1}{RR}\sum_{rr=1}^{RR} P_0(O_0|K_0^{\{rr\}}, \Theta, X)$

2. Set t=t+1

   (a) For rr=1....RR:

      i. Draw θ_t from proposal distribution (transition equation): $p(K_t^{\{rr\}}|K_{t-1}^{\{rr\}}, \Theta, X)$

      ii. Compute the weights $\tilde{w}_t^{\{rr\}} = p(O_t|K_t^{\{rr\}}, \Theta, X)$

      iii. Define $w_t^{\{rr\}} = \hat{w}_{t-1}^{\{rr\}}\tilde{w}_t^{\{rr\}}$

   (b) For rr=1...RR

      i. Define $\hat{w}_t^{\{rr\}} = \frac{w_t^{\{rr\}}}{\sum_{rr=1}^{RR} w_t^{rr}}$

   (c) Compute the likelihood for period t: $\sum_{rr=1}^{RR} \tilde{w}_t^{rr}\hat{w}_{t-1}^{rr}$

   (d) For rr=1....RR

      i. Re-sample RR particles $\theta_t^{\{rr\}}$ from step $(2.i)$ with probabilities $\hat{w}_t^{\{rr\}}$

      ii. Set $w_t^{rr} = \frac{1}{RR}$

It is usually assumed that it is costly to sample from the original distribution $p(K_t|K_{t-1}, \psi, X)$. Such is not the case of this article and then as importance distribution we will use the transition system as the importance distribution. When such distribution is used, the algorithm implemented receives the name of the bootstrap filter.

13.4 Smoothing distribution

The smoothing distribution is useful if we are interested in making inference about the state of the unobserved factors. In this case, it is particularly interesting to make inference about the skills of children. The following procedure describes how to use the information provided in the model and in the data in order to derive the smoothing distribution of the unobserved latent factors. This procedure is adapted from Klaas et al. (2006):

I will use as main input for this file the article "Fast Particle Smoothing: If I had a Million Particles". I translate the notation in the one used in the paper. Define $O_{0:t} = \{O_0, O_1..., O_t\}$. The smoothed density is:

$$p(K_t|O_{0:2}) \tag{84}$$

where we basically condition on all the measures we have. Note that we can write Equation 84 as:

$$p(K_t|O_{0:2}) = p(K_t|O_{0:t}) \int \left(\frac{p(K_{t+1}|O_{0:2})p(K_{t+1}|\theta_t)}{\int p(K_{t+1}|\theta_t)p(K_1|O_{0:t})dK_t} \right) dK_{t+1} \tag{85}$$

And then we can approximate this distribution by $\hat{p}(\theta_t|O_{0:2})$ with:

$$\hat{p}(K_t|O_{0:2}) = \sum_{rr=1}^{RR} w_{t|T}^{(rr)} \delta_{K_t^{(rr)}}(K_t) \tag{86}$$

where $\delta_{K_t^{(rr)}}(K_t)$ is the Dirac distribution and

$$w_{t|T}^{(mm)} = w_t^{(mm)} \left[\sum_{rr=1}^{RR} w_{t+1|T}^{(rr)} \left(\frac{p\left(K_{t+1}^{(rr)}|K_t^{(mm)}\right)}{\sum_{kk=1}^{KK} w_t^{(kk)} p\left(K_{t+1}^{(rr)}|K_t^{(kk)}\right)} \right) \right] \tag{87}$$

where $w_{T|T} = w_T$

13.5 Smoothing algorithm

1. For t=0,1,2 perform the particle filtering to obtain $\{K_t^{rr}, w_t^{rr}\}_{rr=1}^{RR}$
2. Set $w_{2|2}^{rr} = w_2^{rr}$ for $rr = 1...RR$
3. For t=1,0 define $w_{t|2}^{(mm)} = w_t^{(mm)} \left[\sum_{rr=1}^{RR} w_{t+1|2}^{(rr)} \left(\frac{p\left(x_{t+1}^{(rr)}|x_t^{(mm)}\right)}{\sum_{kk=1}^{KK} w_t^{(kk)} p\left(x_{t+1}^{(rr)}|x_t^{(kk)}\right)} \right) \right]$