Linear Programming MILP FICO Xpress Mosel Operations Research

Airline Hub Location Optimisation

What happens to a hub-and-spoke network when budget, emissions, and profit maximisation enter the cost function simultaneously?

MIS41160 Optimisation in Business · 2025

$199.1M	12 Cities	3 Hubs	6
Optimal Profit (160M budget)	US + France Network	Selected Optimally	Sensitivity Scenarios

BACKGRO The Classical Hub Model Is Strategically Incomplete

The starting point for this project is the classical airline hub location model from Guéret, Prins and Sevaux (2002). The original formulation covers six cities, selects two hubs, and minimises total transportation cost. Freight between any origin–destination pair is routed through two hubs following the pattern: **origin → hub 1 → hub 2 → destination**.

The limitation is that this model assumes a cost-neutral business environment. It does not account for hub opening investment, budget constraints, environmental penalties, direct-flight alternatives, or profit potential. All of these are now material considerations for a real-world airline expanding its network — not externalities to be ignored.

French Air Lines (FAL) approached us with a plan to expand their freight network and needed help identifying which airports to designate as major hubs. With a fixed investment budget of $150 million and specific operational requirements, the objective shifted from pure cost minimisation to **profit maximisation under real-world constraints**.

DEL EXTE Five Extensions to the Base Case

The extensions are interdependent. Expanding the network creates new routing options. Adding hub costs introduces a budget dimension. Allowing direct flights creates a three-way routing choice for every origin–destination pair. The model resolves all of these simultaneously.

EXT 01

Network Expansion
The original 6-city test instance was expanded to a 12-city network covering six US airports (Atlanta, Boston, Chicago, Dallas, Los Angeles, Denver) and six French airports (Marseille, Nice, Paris, Lille, Lyon, Nantes).

EXT 02

Hub Opening Costs & Budget Constraint
Each candidate hub carries a realistic opening cost. Total investment across selected hubs must not exceed the $150M budget ceiling. This transforms hub selection from an unconstrained decision into a capital allocation problem.

EXT 03

Environmental Emissions Penalty
An emissions penalty of $250 per flight leg per unit of flow is applied to total transport cost, capturing carbon liability and incentivising route efficiency.

EXT 04

Routing Flexibility
For each origin–destination pair, the optimiser chooses between: direct flight (no discount), one-hub routing (10% distance discount), or two-hub routing (20% hub-to-hub discount).

EXT 05

Profit Maximisation Objective
The objective shifts from minimising cost to maximising total profit: Profit = Revenue – Transport Cost – Environmental Cost – Hub Opening Cost.

FORMULAT Decision Variables and Objective Function

Decision Variables

Variable	Domain	Description
`open_i`	`{0, 1}`	Airport i is active in the network
`hub_i`	`{0, 1}`	Airport i is selected as a hub
`direct_ij`	`{0, 1}`	Freight between i and j transported directly

`flow_ij^kl`	`{0, 1}`	Freight from i to j routed via hubs k and l

Objective Function

Rather than minimising transportation cost, the model maximises total profit:

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max Profit = Revenue − TransportCost − EnvironmentalCost − OpeningCost
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Key Constraints

Constraint	Formula	Meaning
Hub consistency	`hub_i ≤ open_i`	Hubs can only be selected at open airports
Hub count	`Σ hub_i = 3`	Exactly three hubs must be selected
Network expansion	`Σ open_i ≥ 2  (i ∈ NEW)`	At least two new airports must open
Budget ceiling	`Σ OPEN_i · open_i ≤ $160M`	Total hub investment within budget
Unique routing	`direct_ij + Σ flow_ij^kl = open_i + open_j − 1`	One routing per city pair
Flow consistency	`flow_ij^kl ≤ hub_k  and  flow_ij^kl ≤ hub_l`	Flows only through chosen hubs

/ RESULT Two Budgets, Two Network Strategies

Both budget scenarios produce profitable three-hub networks, but they reflect fundamentally different strategic postures. The model's hub selection shifts completely when the investment ceiling moves by just $10 million.

Metric	Budget = $150M (Denver Scenario)	Budget = $160M (Los Angeles Scenario)	Change
Total Profit	$184.9M	$199.1M	**+$14.2M (+7.7%)**
Total Revenue	$440.6M	$493.7M	**+$53.1M (+12.1%)**
Transport + Env. Cost	$120.7M	$134.6M	**+$13.9M (+11.5%)**
Hub Opening Cost	$135.0M	$160.0M	**+$25.0M**
US Hub Selected	Denver	Los Angeles	**Network pivot**
Full Hub Set	ATL · DEN · PAR	ATL · LAX · PAR	**DEN → LAX**
BOS & CHI Routing	Direct transatlantic	Direct transatlantic	**Unchanged**

The most significant finding is that LAX, despite higher opening costs, generates substantially more revenue by capturing West Coast–Europe corridors that Denver cannot reach as efficiently. The extra $10M in budget yields $14.2M in additional profit — a return of 142% on the incremental investment.

SENSITIV The Network Is Robust to Moderate Policy Change

Budget Sensitivity

Budget	Profit ($M)	Revenue ($M)	Cost ($M)	Selected Hubs
$150M	184.9	440.6	120.7	Atlanta · Denver · Paris
$160M	199.1	493.7	134.6	Atlanta · Los Angeles · Paris
$170M	207.0	519.7	142.7	Atlanta · Denver · Paris

Emission Penalty Sensitivity

Emissions Penalty	Profit ($M)	Revenue ($M)	Cost ($M)	Selected Hubs
$200/unit	188.9	440.6	116.7	Boston · Nantes · Denver

$250/unit	184.9	440.6	120.7	Atlanta · Denver · Paris
$300/unit	181.5	440.6	124.0	Atlanta · Denver · Paris

Demand Quantity Sensitivity

Adjustment	Profit ($M)	Revenue ($M)	Cost ($M)	Hub Response
Dallas +10%	No change	—	—	Dallas remains unselected
Denver +10%	197.1	457.0	124.9	Denver hub confirmed

Emission penalties between $200–$300 per unit produce only marginal profit changes, confirming the network is robust to moderate regulatory shifts. Dallas consistently fails to justify hub investment because its demand is insufficient relative to its opening cost — a finding that holds across all scenarios tested.

OMMEND Strategy Depends on Risk Appetite

Conservative ($150M — ATL · DEN · PAR)
Lower capital expenditure, operationally robust, solid transatlantic coverage. Denver provides a strong mid-continent connection point while Boston–Europe and Chicago–Europe flows remain direct, limiting flight legs and environmental impact. The trade-off: some revenue potential on West Coast corridors is left uncaptured.

Growth-Oriented ($160M — ATL · LAX · PAR)
Opening LAX captures high-yield West Coast–Europe traffic and channels flows such as BOS–LAX through an efficient one-hub pattern. LAX becomes a key global gateway alongside Paris. The trade-off: higher fixed investment and greater operational dependence on Los Angeles — any underperformance at LAX directly affects the primary US–France flow pattern.

Overall recommendation: if the priority is strict cost control and lower capital risk, the $150M ATL–DEN–PAR scenario is defensible. If the objective is to maximise profit and strengthen long-haul and West Coast positioning, the $160M ATL–LAX–PAR scenario is preferable — the incremental $10M generates $14.2M in additional profit, representing a compelling return on investment.

CHNICAL Implementation Notes

The model was implemented in **FICO Xpress Mosel**, selected for its native MILP capability and modular constraint structure. Binary variables for airport opening, hub selection, and flow routing are resolved using branch-and-bound / branch-and-cut algorithms. Data parameters (distances, demand quantities, opening costs, emission rates) are loaded from an external **.dat** file, making scenario analysis a parameter change rather than a structural code edit.

Component	Detail
Solver	`FICO Xpress Mosel (MILP)`
Problem class	`Mixed Integer Linear Programme (MILP)`
Decision vars	`open_i, hub_i, direct_ij, flow_ij^kl — all binary`
Revenue model	`REV_DOM = $3.0/mile/unit · REV_INT = $4.0/mile/unit`
Cost factors	`ONE_HUB_FACTOR = 0.90 · TWO_HUB_FACTOR = 0.80`
Emissions	`EMISS = $250 per flight leg per unit of flow`
Cities	`12 total: 6 US (ATL, BOS, ORD, DAL, LAX, DEN) + 6 France`
Hubs required	`Exactly 3, selected from any open city`

References

- Guéret, C., Prins, C. and Sevaux, M. (2002). *Applications of Optimization with Xpress-MP*. Dash Optimization.
- Alumur, S. and Kara, B.Y. (2008). Network Hub Location Problems: The State of the Art. *European Journal of Operational Research*, 190(1), pp. 1–21.
- O'Kelly, M. et al. (2025). Hub Location Problems: A Meta Review and Ten Disruptive Research Challenges. *Journal of the Air Transport Research Society*, 4, p. 100073.