School of Physics & Astronomy

THE UNIVERSITY of EDINBURGH

Modelling and Visualisation in Physics

PHYS10035 (SCQF Level 10)

Thursday 30th April, 2020 13:00-16:00 BST
(May Diet)

Please read full instructions before commencing writing.

Examination Paper Information

Answer all questions.
Completed codes should be uploaded via Learn as a single .tar, .tar.gz, or .zip file immediately after you have finished, together with graphs and datafiles.

You may use any resources available on the internet at the beginning of the examination, or present in your CPlab home directory but you may not communicate with any other person electronically or otherwise.

Special Instructions

- This is an open-book examination.
- Electronic Calculators may be used during this examination.
- A sheet of physical constants is supplied for use in this examination.

Special Items

- School supplied Constant Sheets

Chairman of Examiners: Prof J Dunlop
External Examiner: Prof I Ford

The contact process is a simple model for epidemic spreading, which can be formulated in terms of a cellular automaton, defined on a 2-dimensional square $L \times L$ lattice with periodic boundary conditions, as follows.

Each cell in the lattice represents an individual (or agent), which can be in one of two states: "active" (or infected), and "inactive" (or healthy). The update rule defining the contact process is the following. First, a cell (lattice site) is chosen randomly. If the selected cell is inactive, nothing happens. If the cell is active, this becomes inactive with probability $1 - p$, whereas with probability p it infects one of its four nearest neighbours (the neighbour to be infected is chosen randomly, and if the chosen neighbour is already infected nothing happens).

a. Write a Python code to simulate the dynamics of the contact process model. You should take $L = 50$ (i.e., a 50×50 lattice). Your code should allow you to set the value of p as an argument, and it should also display the state of the system in real time as it is running. To initialise the system you can start with a random state with equal probability of having an active or inactive state at each of the lattice points. You should measure time in units of sweeps, where a sweep consists of L^2 updates. **[20]**

b. Plot or sketch the fraction of active/infected sites over time for $p = 0.6$ and $p = 0.7$. Explain why these two plots show that the two points correspond to two different phases. **[3]**

c. Call A the total number of active/infected sites in a configuration, and $N = L^2$ the total number of sites. Compute the average fraction of active sites, $f \equiv \frac{\langle A \rangle}{N}$, where $\langle \cdot \rangle$ denotes averaging, between $p = 0.55$ and $p = 0.7$ (use a resolution of 0.005 in p if at all possible). Plot the resulting graph. **[6]**

d. Compute $\Delta \equiv \frac{\langle A^2 \rangle - \langle A \rangle^2}{N}$ as a function of p, again between $p = 0.55$ and $p = 0.7$ (with the same resolution as in part c.). Your plot should have appropriately computed error bars. Discuss the resulting graph in conjunction with that found in part c. Identify the point at which a phase transition occurs, justifying your reasoning. **[6]**

e. The "survival probability" is the probability that a simulation contains a non-zero fraction of active/infected sites at time t (measured, recall, in sweeps). Modify your code, or write another version of it, which computes the survival probability versus time. As an initial condition, you should place a single active cell in a random position in the lattice. The survival probability should be computed by running many simulations and counting how many have active sites left in them as a function of time t. **[5]**

f. Compute the survival probability as a function of time for $p = 0.6$, $p = 0.625$ and $p = 0.65$, up to $t = 300$ sweeps. Plot the resulting curves in a log-log plot, and comment on the results. **[10]**