Practice problems week 4

September 18, 2023

Problem 1 Verify that the following set of vectors is an orthonormal basis for $\mathbb{R}^4$

$$\mathcal{B} = \left\{ \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}, 0 \right), \left(0, \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right) \left(\frac{-1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{-1}{2} \right), \left(\frac{1}{2}, \frac{1}{2}, \frac{-1}{2}, \frac{-1}{2} \right) \right\}$$

Problem 2 Let,

$$W = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \middle| 3x + 4y = 0 \right\}$$

find $W^\perp$ as a set then give a basis for $W^\perp$.

Problem 3 Let $W \subset \mathbb{R}^4$ be a subspace with basis vectors

$$\vec{x}_1 = \begin{bmatrix} 2 \\ -1 \\ 1 \\ 2 \end{bmatrix}, \vec{x}_2 = \begin{bmatrix} 3 \\ -1 \\ 0 \\ 4 \end{bmatrix}, \vec{x}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

use the Gram-Schmidt process to obtain an orthonormal basis for W

Problem 4 Let $W \subset \mathbb{R}^n$ be any subspace. Prove that $W \cap W^\perp = \{\vec{0}\}$

Hint: This is done via a method called a 'double inclusion' argument. Meaning it requires 2 steps, first show that $W \cap W^\perp \subset \{\vec{0}\}$ then $\{\vec{0}\} \subset W \cap W^\perp$. In order to do this each time you take an arbitrary element in the left set and show it must be in the right. (note this is not intended to be overly difficult and only requires a few lines of work when thought of properly. If you are struggling please see the TA's or myself for a more complete explanation.

Odd numbered problems from Poole (the ones with answers) that are relevant to this week's discussion can be found on pages 387-388 and 394-395