Math 225 lecture 7 Sept 20^{th} 2023

Goal: grand students understanding of abstract vector spaces from last day with examples & non-examples.

Class Q: How many axioms are there to identify a space as a vector space? and how many is a space allowed not to obey before we cannot call it a vector space?

section 6.1

Recall; last time we defined a v. space that was

↳ a set V together with two operations $+, \cdot$ is a v. space if it satisfies

1. $\vec{u}+\vec{v} \in V$
2. $\vec{u}+\vec{v} = \vec{v}+\vec{u}$
3. $(\vec{u}+\vec{v})+\vec{w} = \vec{u}+(\vec{v}+\vec{w})$
4. $\exists\ \vec{0} \in V$ st $\vec{u}+\vec{0}=\vec{u}$
5. $\forall \vec{u} \in V\ \exists -\vec{u} \in V$ st $\vec{u}+(-\vec{u}) = \vec{0}$
6. $c\vec{u} \in V$
7. $c(\vec{u}+\vec{v}) = c\vec{u}+c\vec{V}$
8. $(\vec{u}+\vec{v})c = c\vec{u}+c\vec{v}$
9. $c(d\vec{u}) = (cd)\vec{u}$
10. $1\vec{u} = \vec{u}$

lets repeat the verification process but with two slightly stranger vector spaces, $\{0\}$, $\mathbb{R}$ where $x \oplus y = xy$ $c \odot x = x^c$

	$\{0\} = Z$	$\{\mathbb{R}, \oplus, \odot\} = \mathbb{R}_w$
1	$0+0 = 0 \in Z$ ✓	$x \oplus y = xy \in \mathbb{R}_w$
2	$0+0 = 0 = 0$ ✓	$x \oplus y = xy = yx = y \oplus x$ ✓

3	$(0+0)+0 = 0+0 = 0 = 0+0 = 0+(0+0)$	$(X \oplus Y) \oplus Z = XY \oplus Z = XYZ = X \oplus YZ = X \oplus (Y \oplus Z)$
4	$0 \in \mathbb{Z}$ st $0+0=0$	$1 \in \mathbb{R}_w$ st $1 \oplus X = 1X = X$ $1 = \vec{0}$
5	let $0 \in \mathbb{Z}$ $-0 \in \mathbb{Z}$ $0+(-0)=0$	for $X \in \mathbb{R}_w$ $\frac{1}{X} \in \mathbb{R}_w$ st $X \oplus \frac{1}{X} = \frac{X}{X} = 1 = \vec{0}$
6	$c0 = 0 \in \mathbb{Z}$	$c \odot X = X^c \in \mathbb{R}_w$
7	$c(0+0) = c(0) = 0 = 0+0 = c0+c0$	$c \odot (X \oplus Y) = c \odot (XY) = X^c Y^c = X^c \oplus Y^c = c \odot X \oplus c \odot Y$
8	$(0+0)c = 0c = 0 = 0+0 = c(0)+c(0)$	$(X \oplus Y) \odot c = (XY) \odot c = X^c Y^c = X^c \oplus Y^c = c \odot X \oplus c \odot Y$
9	$c(d0) = c(0) = 0 = (cd)0$	$c \odot (d \odot X) = c \odot X^d = (X^d)^c = X^{dc} = X^{cd} = (cd) \odot X$
10	$1(0) = 0$	$1 \odot X = X^1 = X$

Non examples! lets identify at least one axiom each of the following spaces fail to satisfy.

1. let $V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 \,\middle|\, x \geq 0,\ y \geq 0 \right\}$

   fails 5, 6

2 let $V = \{ \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 \mid xy \geq 0 \}$ (quadrant 1 & 3)

fails 1

3 let $V = \{ \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 \}$ w/ usual scalar mult but $u + v = \begin{bmatrix} u_1 + v_1 + 1 \\ u_2 + v_2 + 1 \end{bmatrix}$

Fails 7 & 8

working on these collectively.

Discuss the importance of vetor spaces & emphasize that we learn this to be combined with a similar notion later & tie everything together

Tell a story!