Practice problems week 3

September 12, 2023

Problem 1 Let $V = \mathbb{R}^2$ with the following operations,

$$(u_1, u_2) + (v_1, v_2) = (u_1 + v_1 + 1, u_2 + v_2 - 1)$$

$$k(u_1, u_2) = (ku_1 + k - 1, ku_2 - k + 1)$$

- Find the 0-vector for these operations (i.e. the vector for which $\vec{v} + \vec{0} = \vec{v}$, HINT, its not $(0,0)$.)

- find the vector $-(2,5)$ in this space (i.e the vector that you add to $(2,5)$ such that you get the 0-vector you found in part one)

- Prove that $\mathbb{R}^2$ with these operations is a vector space (show all 10 axioms)

Problem 2 Show that $\mathbb{R}^2$ with the following operations

$$(u_1, u_2) + (v_1, v_2) = (2u_1 + 2v_1, u_2 + v_2)$$

$$k(u_1, u_2) = (ku_1, ku_2)$$

is NOT a vector space, which axioms does it fail to satisfy?

Problem 3 Verify weather or not each of the following matrices are orthogonal. You may use any of the properties given in the class notes.

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}, B = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Odd numbered problems from Poole (the ones with answers) that are relevant to this week's discussion can be found on pages 441-442, 376-378