MATH 225, Written Assignment # 3,
Due: **Thursday Nov. 16** by 11:59pm.

Instructor: Brandon Gill

1. (25 points) In this question, you will compare the QR and SVD decomposition of a matrix. Let
$$A = \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ -\sqrt{2} & \sqrt{2} \end{bmatrix}$$

   (a) Find the QR decomposition for the above matrix

   (b) Find an SVD decomposition for the matrix A above, i.e. find orthogonal matrices U_A, V_A and a "diagonal" matrix (in the sense of SVD) Σ_A such that
$$A = U_A \Sigma_A {}^t V_A$$

   (c) For the matrix R which you found in part (a), find an SVD decomposition, i.e. matrices U_R, Σ_R, V_R such that
$$R = U_R \Sigma_R {}^t V_R$$

   (d) In general, suppose A is an $n \times n$ matrix with linearly independent columns, and so has a QR decomposition with Q an orthogonal matrix. If we compare the SVD decomposition for R and A, what changes? In other words: what is the relation between the singular values of R and those of A? what is the relation between U_R and U_A? what is the relation between V_R and V_A?

2. (25 points) For this problem we will be looking at the following
$$A = \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ -1 & 1 \end{bmatrix}, \vec{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

   and solving the least squares problem for the system $A\vec{x} = \vec{b}$ in each of the three ways described in class.

   (a) Solve $A\vec{x} = \vec{b}$ by solving the normal system. This method is the most direct and should be what you compare your other answers with.

   (b) Solve $A\vec{x} = \vec{b}$ via the direct method by projecting onto column space. (Remember, what do you need to project onto a subspace)

   (c) Lastly solve $A\vec{x} = \vec{b}$ using the QR factorization of A. (Note, you should already have done the hardest part of finding the QR decomposition above)

3. (20 points) Let $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be given by a reflection about the y-axis. Let

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \end{bmatrix} \right\},\ \mathcal{D} = \left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ -2 \end{bmatrix} \right\}$$

be bases for $\mathbb{R}^2$. Find $[T]_{\mathcal{D}\leftarrow\mathcal{B}}$ and confirm that it works as desired (like we did in class on November 1st) by applying it to $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$. (**Important**: don't forget that the vector you are testing it on is written with respect to the standard basis).