

Q1 - 15 marks

Consider the following matrix

$$A = \begin{pmatrix} 2 & -4 & 0 & 2 & 1 \\ -1 & 2 & 1 & 2 & 3 \\ 1 & -2 & 1 & 4 & 4 \end{pmatrix}$$

a) Write down the RREF of A

$$\begin{aligned} A &= \begin{pmatrix} 2 & -4 & 0 & 2 & 1 \\ -1 & 2 & 1 & 2 & 3 \\ 1 & -2 & 1 & 4 & 4 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \begin{pmatrix} 1 & -2 & 1 & 4 & 4 \\ -1 & 2 & 1 & 2 & 3 \\ 2 & -4 & 0 & 2 & 1 \end{pmatrix} \\ &\xrightarrow{R_2 + R_1} \begin{pmatrix} 1 & -2 & 1 & 4 & 4 \\ 0 & 0 & 2 & 6 & 7 \\ 2 & -4 & 0 & 2 & 1 \end{pmatrix} \xrightarrow{R_3 - 2R_1} \begin{pmatrix} 1 & -2 & 1 & 4 & 4 \\ 0 & 0 & 2 & 6 & 7 \\ 0 & 0 & -2 & -6 & -7 \end{pmatrix} \\ &\xrightarrow{R_3 + R_2} \begin{pmatrix} 1 & -2 & 1 & 4 & 4 \\ 0 & 0 & 2 & 6 & 7 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{\frac{1}{2}R_2} \begin{pmatrix} 1 & -2 & 1 & 4 & 4 \\ 0 & 0 & 1 & 3 & \frac{7}{2} \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \\ &\xrightarrow{R_1 - R_2} \begin{pmatrix} 1 & -2 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 & 3 & \frac{7}{2} \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{aligned}$$

b) What is the dimension of the row space of A

$$\dim(A) = 2$$

c) Write down a basis of the column space of A

$$\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$$

d) What is the nullity of A . Write down a basis of the nullspace A ?

$$\text{nullity}(A) = 3$$

From a):

$$\left(\begin{array}{ccccc|c} 1 & -2 & 0 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & 3 & \frac{7}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$$\begin{aligned} x_1 - 2x_2 + x_4 + \frac{1}{2}x_5 &= 0 \\ x_3 + 3x_4 + \frac{7}{2}x_5 &= 0 \end{aligned}$$

$$x_1 = 2s - t - \frac{1}{2}u$$

$$x_2 = s$$

$$x_3 = -3t + \frac{7}{2}u$$

$$x_4 = t$$

$$x_5 = u$$

$$\vec{x} = s \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ -3 \\ 1 \\ 0 \end{bmatrix} + u \begin{bmatrix} -\frac{1}{2} \\ 0 \\ \frac{7}{2} \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore \mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{1}{2} \\ 0 \\ \frac{7}{2} \\ 0 \\ 1 \end{bmatrix} \right\}$$