MATH 225, Written Assignment # 3,
Due: **Saturday Dec 2nd** by 11:59pm.

Instructor: Brandon Gill

1. (20 points) Let $V = \mathcal{P}_2(\mathbb{R})$ that is the the polynomials of degree less than or equal to two with real coefficients. Let the map $T : \mathcal{P}_2(\mathbb{R}) \longrightarrow \mathcal{P}_2(\mathbb{R})$ be given by $p(x) \mapsto p'(x)$ i.e the first derivative.

   - Verify that T is a linear map. (remember to check all three conditions)
   - Find the kernel of T and describe it as a set.
   - Find the range of T and describe it as a set.
   - is T onto? is it one-to-one? If yes, prove it, if not find a counter example.

2. (15 points) Let

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\} \mathcal{C} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\} \vec{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

   Calculate $[\vec{v}]_{\mathcal{D}}$ specifically by computing computing each of $\mathcal{P}_{\mathcal{B}\leftarrow\mathcal{S}}$ and $\mathcal{P}_{\mathcal{D}\leftarrow\mathcal{B}}$ (Note: $\mathcal{S}$ is the standard basis for $\mathbb{R}^3$) and verify your answer by showing the linear combination defined by $[\vec{v}]_{\mathcal{D}}$ does indeed get you back to $\vec{v}$.

3. (15 points) Let $\mathcal{P}_2(\mathbb{R})$ be as it was in problem 1 and define the following,

$$\langle f(x), g(x) \rangle = f_0 g_0 + f_1 g_1 + f_2 g_2$$

   where $f(x) = f_0 + f_1 x + f_2 x^2$ (similarly for g).

   - Verify that $\mathcal{P}_2(\mathbb{R})$ with this product is an inner product space. (note you may use that $\mathcal{P}_2(\mathbb{R})$ is a vector space and do not need to verify that part)
   - In this space what is the length/norm of the "vector" $f(x) = 3 + 2x + 4x^2$
   - In this space what is the distance between $f(x) = 4 - 5x + 2x^2$ and $g(x) = -1 + 8x + 7x^2$
   - In this space find TWO different "vectors" which are orthogonal to $f(x) = 1 + x + x^2$