Math 225 lecture 30 Nov 24^{th} 2023

goal: help students appreciate the scope and significance of the topics covered in the course. Paint a big picture perspective to show applicability.

Class Q: do you trust me to take you through this wild ride?

What have we done since the midterm?
- linear transformations, & matricies there of
- basis, dimension, linear independance, kernel, range
- change of basis
- inner products & inner prod. spaces

Why?! (discussion)

Every topic we covered before this ~~relies~~ relies on these concepts. As such everything we've done is applicable in a far broader context than the one we have been working in i.e.

what could we do! (discussion)

- orthogonalize ~~polynomials~~ matricies (even non square.)
- compute diagonalizations in a space where the vectors are matricies
- compute our decompositions (SVD, QR, LSS) in more exotic vector spaces!

- find an orthogonal complement to a subspace of a polynomial space

and any others arrived at in class

Now that we have all these tools in such generality where might they point?

- inside math, DE's, polynomials, higher algebra & geometry, quadratic forms, constrained optimization and more! essentially it is the springboard for most higher math.
- outside math, coding over finite fields, statistical analysis economic modles, compter graphics, chemistry, engineering medical imaging and so on!

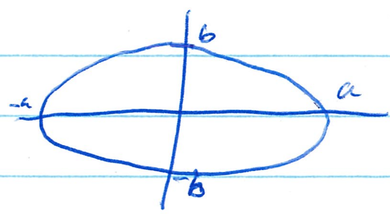
lets see some of these werder examples! inner prod spaces.

$\mathrm{Mat}_{m,n}(\mathbb{R})$ with $\langle A, B \rangle = \mathrm{tr}(AB^T)$

$\mathbb{R}^n$ with $\langle \vec{u}, \vec{v} \rangle = \vec{u}^t A \vec{v}$ for A a positive definite matrix (that is, one A hermetian and with all positive real e. vals $A = A^* = \overline{A}^T$)

$\mathbb{R}^2$ with $\langle \vec{u}, \vec{v} \rangle = \dfrac{u_1 v_1}{a^2} + \dfrac{u_2 v_2}{b^2}$ for some positive real numbers a & b

(note the "unit vectors" in this space form an ellips



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V a v. space & $\langle \vec{u}, \vec{v} \rangle = \langle \vec{u}, \vec{v} \rangle_1 + \langle \vec{u}, \vec{v} \rangle_2$ where $\langle , \rangle_1$ & $\langle , \rangle_2$ are any two other inner products on V

Vector spaces

$\mathbb{C}$ as a v. space over $\mathbb{R}$

$\mathbb{R}$ with $x \oplus y := (x^n + y^n)^{1/n}$ and $a \odot x := (a x^n)^{1/n} = a^{1/n} x$ for any odd positive integer n

$(-1, 1) \subset \mathbb{R}$ (open interval) with $x \oplus y := \dfrac{x+y}{1+xy}$ &

$$a \odot x := \frac{(1+x)^a - (1-x)^a}{(1+x)^a + (1-x)^a}$$

$\mathbb{R}^3$ with $\vec{u} \oplus \vec{v} := \begin{pmatrix} u_1 v_1 \\ (u_2^3 + v_2^3)^{1/3} \\ u_3 + v_3 + 1 \end{pmatrix}$ and $a \odot \vec{u} = \begin{pmatrix} u_1^a \\ a^{1/3} u_2 \\ a u_3 + a - 1 \end{pmatrix}$

the point being that no matter how scary weird, unusual, or exotic a space looks if its a v. space our entire discussion about linear maps applies, change of basis etc. & if you have an inner prod. space everything we talked about where orthogonality is involved can be done!

its a fun wild world and you have more tools than you know to take it on!