Math 225 lecture 1 sept 6^{th} 2023

Goal: Give introduction to the course. Begin reviewing content from MATH 125, in particular systems of linear equations and Gaussian Elimination

Class Q: How do you identify weather or not a system is consistent or not?

- Intro and housekeeping.
  - My office is CAB 546, OH T/R: 1:00-2:30, W: 11:00-12:30 or by appointment
  - email btgill@ualberta.ca (include MATH 225)
  - go over syllabus including grade scheme
  - explain class question.
  - Note text book, not required but will give references.
  - Weekly Assignments w/ labs, graded HW, Midterm, final
  - Final tentatively 9:00am Dec 21 (check this!)
  - Midterm on Oct 27 (in class)
  - graded HW due: saturdays sept 23, oct 14, Nov 11, Dec 2
  - notes posted for each class (bare bones/hilights) will check in at the 2 week mark to see if this is working
  - optional MATH 125 bootcamp/review Thursday sept 7, 14, 21 4:30-6:00pm CCIS 1-140
  - Questions, comments, concerns, etc.
  - at the 2ish week ~~mark~~ mark I will ask for feedback

section 3.1

- Basics of matrices

let A be an $m \times n$ matrix & B be an $n \times l$ matrix

$$A = \underbrace{\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}}_{n\text{-columns}} \Bigg\} m\text{-rows} \qquad B = \underbrace{\begin{bmatrix} b_{11} & \cdots & b_{1l} \\ \vdots & \ddots & \vdots \\ b_{n1} & \cdots & b_{nl} \end{bmatrix}}_{l\text{-columns}} \Bigg\} n\text{-rows}$$

basic operations add and multiply

to add matrices they must be of the same size and addition is done entry wise

Ex: let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$

then $A+B$ is defined and

$$A+B = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{bmatrix}$$

can only multiply matrices if there sizes are compatable
☆ (recall matrix mult is non-commutative $AB \neq BA$ in general)

if A is $m \times n$ and B is $n \times l$ then

AB exists but BA does not

$m \times n \quad n \times l$ ✓ result

$n \times l \quad m \times n$ ✗ NOPE

Note: $A = \begin{bmatrix} - \vec{a}_1 - \\ \vdots \\ - \vec{a}_m - \end{bmatrix}$ or $B = \begin{bmatrix} | & & | \\ \vec{b}_1 & \cdots & \vec{b}_n \\ | & & | \end{bmatrix}$, are matricies written as rows or colums

To multiply compatible matricies you take dot products

let A & B be as above then

$$AB = \begin{bmatrix} \vec{a}_1 \cdot \vec{b}_1 & \cdots & \vec{a}_1 \cdot \vec{b}_n \\ \vdots & \ddots & \vdots \\ \vec{a}_m \cdot \vec{b}_1 & \cdots & \vec{a}_m \cdot \vec{b}_n \end{bmatrix}$$ where " $\cdot$ " denotes dot product.

Recall: distributivity $A \cdot (B+C) = AB + AC$

$I_n = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$ the $n \times n$ identity matrix

$A I_n = A = I_n A$ provided sizes are compatible

$0 = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix}$ the zero matrix

$0 + A = A + 0 = A$ for appropriate sizes

Important: $A + B = B + A$ $\qquad AB \neq BA$

Section 2.2

Systems of linear equations

Ex:

$$
\begin{aligned}
2y + 3z &= 8 \\
2x + 3y + z &= 5 \\
x - y - 2z &= -5
\end{aligned}
$$

create Augmented Mat

$$\left[\begin{array}{ccc|c} 0 & 2 & 3 & 8 \\ 2 & 3 & 1 & 5 \\ 1 & -1 & -2 & -5 \end{array}\right]$$

Bring to (R)REF (reduced row echelon form) via interchanging rows & adding multipuls of one to another

$$\xrightarrow{R_2 - 2R_3} \left[\begin{array}{ccc|c} 0 & 2 & 3 & 8 \\ 0 & 5 & 5 & 15 \\ 1 & -1 & -2 & -5 \end{array}\right]$$

$$\xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & -1 & -2 & -5 \\ 0 & 5 & 5 & 15 \\ 0 & 2 & 3 & 8 \end{array}\right]$$

$$\xrightarrow{R_2 - 2R_3} \left[\begin{array}{ccc|c} 1 & -1 & -2 & -5 \\ 0 & 1 & -1 & -1 \\ 0 & 2 & 3 & 8 \end{array}\right]$$

$$\xrightarrow{R_3 - 2R_2} \left[\begin{array}{ccc|c} 1 & -1 & -2 & -5 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 5 & 10 \end{array}\right] \xrightarrow{\frac{1}{5}R_3} \left[\begin{array}{ccc|c} 1 & -1 & -2 & -5 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

can stop here or go to RREF

$$
\begin{aligned}
x - y - 2z &= -5 & & \longrightarrow & x - 1 - 2(2) &= -5 \\
y - z &= -1 \rightarrow & y - 2 &= -1 & x - 5 &= -5 \\
z &= 2 & y &= 1 & x &= 0
\end{aligned}
$$

meaning the system is consistent and has a unique solution

Hilroy

<u>Recall</u>: a system of the form $A\vec{x} = \vec{b}$ (i.e. from $[A \,\vdots\, \vec{b}]$)

can have $\underbrace{\text{i) no sol'n}}_{\text{inconsistent}}$ $\underbrace{\text{ii) a unique sol'n iii) } \infty\text{-many sol'n}}_{\text{consistent}}$

moving to REF via Gaussian Elimination (what we just did) makes

① all zero rows at the bottom
② first non-zero entry in each row is a 1 with zeros to its left & below

from here have info

$$\left[\begin{array}{ccccc|c} 1 & * & * & \cdots & & * \\ 0 & 1 & * & \cdots & & \vdots \\ 0 & 0 & 0 & 0 & 1 & \vdots \\ 0 & \cdots & & 0 & & * \end{array}\right]$$

① if we have a row $[0 \cdots 0 \mid *]$
inconsistent, no sol'n

② # of non-zero rows is the rank of the system
vars − rank = # of free vars

if # free vars ≥ 1 the system is consistent and has ∞-many sol'n's

③ if rank = # of vars (and is consistent) then there is a unique sol'n