CMPUT 428: Tracking

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Contents

1 Lecture - Feb 1

lec06ProjG2D.pdf

1.1 Homogenous Coordinates

Homog rep of 2D points and lines:

$$ax + by + c = 0 \Rightarrow (a, b, c)^\top (x, y, 1) = 0$$

point $\mathbf{x}$ lies on line $\mathbf{l}$ iff

$$\mathbf{l}^\top \mathbf{x} = \mathbf{x}^\top \mathbf{l} = 0$$

Scale not inportant for incidence relation; equivalence class of vectors (any vector representative)

$$(a, b, c)^\top \sim k(a, b, c)^\top, \forall k \neq 0 \qquad (x, y, 1)^\top \sim k(x, y, 1)^\top, \forall k \neq 0$$

Set of all equivalence classes in $\mathbb{R}^3$ is $(0, 0, 0)^\top$, forms $\mathbb{P}^2$
2 homogenous points are equivalent if they are collinear.
Homogenous coords - $(x_1, x_2, x_3)^\top$ (2DOF)
Inhomogenous coords - $(x, y)^\top$

$\begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$ - point at infinity (ideal/vanishing point)

Intersection of two lines, $\mathbf{l}$ and $\mathbf{l}'$:

$$\mathbf{x} = \mathbf{l} \times \mathbf{l}'$$

Line connecting two points, $\mathbf{x}$ and $\mathbf{x}'$:

$$\mathbf{l} = \mathbf{x} \times \mathbf{x}'$$

Expln 1.1.1: Cross Product

$$\mathbf{x} \times \mathbf{x}' = [\mathbf{x}]_\times \mathbf{x}'$$

Easier to represent cross product with:

$$[\mathbf{x}]_\times = \begin{bmatrix} 0 & z & -y \\ -z & 0 & x \\ y & -x & 0 \end{bmatrix}$$

Intersection of Parallel Lines, $\mathbf{l} = (a, b, c)^\top$ and $\mathbf{l}' = (a, b, c')^\top$
(parallel lines meet at ideal points in projective geometry; don't meet in Euclidean)

$$\mathbf{l} \times \mathbf{l}' = (b, -a, 0)^\top$$

$(b, -a)$ - tangent vector; (a, b) - normal direction

Ideal Point:

$$\mathbf{x}_\infty = \begin{bmatrix} x_1 & x_2 & 0 \end{bmatrix}^\top$$

Line at Infinity:

$$\mathbf{l}_\infty = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^\top$$

$\mathbb{P}^2 = \mathbb{R}^2 \cup \mathbf{l}_\infty$ ($\mathbb{P}^2$ is $\mathbb{R}^2$ plus a "line at infinity")

1.2 2D Projective Plane

Perspective imaging models 2d projective space. Each 3D ray is a point in $\mathbb{P}^2$ (homogenous coords) Homog coords:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = s \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \forall s \neq 0$$

Inhomog Coords:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \frac{1}{z} \begin{bmatrix} x \\ y \end{bmatrix}$$

principal axis - center ray

1.3 Lines

Plane through origin projects to mathetmatical line when it is intersected w/ img plane
Projective Line - a plane through the origin

$$\mathbf{l}^\top \mathbf{x} = \mathbf{x}^\top \mathbf{l} = 0$$

Ideal Line - plane parallel to img

$$\mathbf{l}_\infty = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Expln 1.3.1: Duality

For any 2D projective property, dual property holds when the role of points and lines are interchanged
(symmetrical)

1.4 Conics

Curve described by 2nd degree equation in plane:

$$ax^2 + bxy + cy^2 + dx + ey + f = 0$$

or homogenized: $x \mapsto \frac{x_1}{x_3}, y \mapsto \frac{x_2}{x_3}$

$$ax_1^2 + bx_1x_2 + cx_2^2 + dx_1x_3 + ex_2x_3 + fx_3^2 = 0$$

or in matrix form:

$$\mathbf{x}^\top \mathbf{C} \mathbf{x} = 0 \text{ with } \mathbf{C} = \begin{bmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{bmatrix}$$

5DOF (ratios b/w distinct elements)

1.4.1 Find Conics

5 points define a conic;

$$[x_i^2, x_i, y_i, y_i^2, x_i, y_i, 1] \cdot \mathbf{c} \qquad \mathbf{c} = [a, b, c, d, e, f]^\top$$

Stacking contraints yields:

$$\begin{bmatrix} x_1^2 & x_1y_1 & y_1^2 & x_1 & y_1 & 1 \\ x_2^2 & x_2y_2 & y_2^2 & x_2 & y_2 & 1 \\ x_3^2 & x_3y_3 & y_3^2 & x_3 & y_3 & 1 \\ x_4^2 & x_4y_4 & y_4^2 & x_4 & y_4 & 1 \\ x_5^2 & x_5y_5 & y_5^2 & x_5 & y_5 & 1 \end{bmatrix} \mathbf{c} = 0$$

1.4.2 Tangent Lines to Conics

The line $\mathbf{l}$ tangent to $\mathbf{C}$ at point $\mathbf{x}$ on $\mathbf{C}$ is given by $\mathbf{l} = \mathbf{C}\mathbf{x}$

$$\mathbf{x}_0^\top \mathbf{C} \mathbf{x} = 0$$

1.4.3 Dual Conics

Line tangent to conic $\mathbf{C}$ statisfies $\mathbf{l}^\top \mathbf{C}^* \mathbf{l} = 0$
In general ($\mathbf{C}$ full rank) - $\mathbf{C}^* = C^{-1}$
Dual conics = line conics = convic envelope

1.4.4 Degneerate Conics

Conic **degenerate** iff matrix $\mathbf{C}$ is not full rank
Degenerate line conics: 2 points (rank 2), double point (rank 1)
For degen conics: $(\mathbf{C}^*)^* \neq \mathbf{C}$

2 Lecture - Feb 6

2.1 Lines

Line b/w 2 points is cross product of the points; point intersecting 2 lines is cross product of lines.
Parallel lines (distinct at small scale) become indistinguishable at infinity.

2.2 Projective Transformation

Definition 2.2.1: Projectivity

Projectivity is an invertible mapping h from $\mathbb{P}^2$ to itself s.t. three points $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3$ lie on the same line iff $h(x_1), h(x_2), h(x_3)$ do

Thm 2.2.1

Mapping $h : P^2 \to P^2$ is a projectivity iff there exists a nonsingular 3×3 matrix H s.t. for any point in P^2 represented by a vector $\mathbf{x}$, it is true that $h(\mathbf{x}) = H\mathbf{x}$

Transforming Lines:

$$\mathbf{l}' = H^{-\top}\mathbf{l}$$

affine - parallelism
metric - orthogonalism
euclidian - absolute scale

2.2.1 Affinity and Projectivity on line at inf

Affine preserves 0 (stays at infinity)
Projectivity changes 0, line at infinity potentionally becomes finite

Thm 2.2.2

Line at infinity is a fixed line under a projective transformation H iff H is affinity.
Affine transformation is one that preserves line at infinity

Rectification/post-processing to get line at infinity back (fix parallel lines) after projective.

2.3 Affine Rectification

Take img, get parallel lines, construct v_1, v_2 as intersections for 2 sets of parallel lines, intersescted points on $\mathbf{l}_\infty$

$$\mathbf{l}_\infty = [l_1, l_2, l_3]$$

Fix 2 DOF relating to projective geometry, undo projective part of transformation; just get affine part

$$H_{PA} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l_1 & l_2 & l_3 \end{bmatrix} H_A$$

2.3.1

$$x'_i = Hx_i$$

Want to show that the points are collinear

$$\lambda \begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

2 independent eq / point; 8DOF for projective $\rightarrow$ 4 points needed

2.4 DLT

Estimate H; Hx_i involves homogenous vectors vectors, $H_i x$ and x_i need only be in the same direction and not strictly equal

$$\begin{bmatrix} x \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \lambda x \\ \lambda \end{bmatrix}$$

Specify "same directionality" via cross product formulation

$$x'_i \times Hx_i = 0$$

Conditioning of A isn't good in general
Minimizes

$$\|Ah\|$$

$e = Ah$ (residual)

2.4.1 Importance of Normalization

Order of magnitudes different, close to singular (poorly conditioned).
Normalize so centroid is origin, rescale points to be w/in 1.
Improve numerical problems.
Denormalize solution to get pixel coordinates.

2.4.2 Degen. Config

Avoid having more than 3 collinear points.
Typically want 4 points clustered

2.5 Parameter Estimation

trifocal tensor - relate point w/ 3 imgs (not covered in class)

2.5.1 Solving LS

if $m = n$ then

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$$

if $m > n$, overconstrained, cant invert; use LSS (via QR or pseudoinverse)

2.5.2 Homogenous Sys. of Eq

Solve $A\mathbf{x} = 0$, trivial sol $\mathbf{x} = \vec{0}$ but don't want to use this. Find other vals.
Solve via SVD ($A = UDV^\top$), with $\mathbf{x}$ as the **last column** of V

2.5.3 Inhomogenous sol

assumes $h_9 = 1$, wont work if we want to have horizon in center