CMPUT 428: 3D Modelling

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Contents

1 Lecture - Feb 29

lec07modeling18EmbMedia.pdf

3D from video is:

- inexpensive
- quick and convenient for the user
- integrates with existing software (ie. maya, blender, c4d, ue4, etc.)

1.1 Prelims; Macro Geometry

Shape from silhouettes:

more = better

provides good starting approx. for object

Structure from motion:

uses triangulations to get 3d points

can be done with 'unstructured' imgs/scene (ie. from phone camera)

1.2 Fundamental Problems

1. **Resection** of 3D camera matrices
2. **Intersection**
3. **Structure From Motion (SFM)**

1.2.1 Resection

Projection Equation:

$$x_i = P_i X$$

Resection:

$$x_i, X \rightarrow P$$

Imagine you have x (ie. 3d model) and image x_i (projection of points on img). Finding camera = finding projection of rays that map 3d point to image point. Can have many potential camera planes that all map (?)

1.2.2 Intersection

Projection Equation:

$$x_i = P_i X$$

Intersection:

$$x_i, P_i \rightarrow X$$

Have ≥ 2 cameras; projections in ≥ 2 images. Intersect rays to get 3d points. Need to 'know' the camera properties.

1.2.3 SFM

Projection Equation:

$$x_i = P_i X$$

Structure From Motion (SFM):

$$x_i \rightarrow P_i, X$$

Get image predictions (x_i) from point tracking.
Get image points in $\geq$ 2 views, calculate 3d points (structure) and camera projection matrices (motion)
Estimate projective structure
Rectify the reconstruction to metric (autocalibration)
(dont know the cameras; figure out constraints)

1.3 Seq. 3D Structure From Motion

Use 2 and 3 view geometry

1. Initialize structure and motion from 2 views
2. for each additional view:
   (a) determine pose
   (b) refine and extend structure
3. determine correspondences robustly by jointly estimating matchies and epipolar constraints

1.4 2 View Geometry

2 points in views x_i, comes from 3d point X. If you have camera center, know that ray from center to point extends to infinity; X lies somewhere on ray.
line connecting camera centers
epipole - intersecting point b/w line and image plane
epipolar line - line b/w imaged x_i and epipole

1.4.1 Epipolar Geometry

C, C' x, x' and X are coplanar
Epipolar geometry exists independent of 3d point, only related to camera and 2d point projections.
There is a family of epipolar planes
Epipolar lines go towards parallel lines
Family of planes and lines intersect infinitely far apart.
Epipoles (e, e') can be finite or infinite

intersection of baseline with img plane

projection of projection center in other img

vanishing point of camera motion direction

Epipolar plane - plane containing baseline (1D family) Epipolar line - intersection of epipolar plane with image

Example 1.4.1: Parallel Motion with img plane

Epipoles at infinity

1.5 Fundamental Matrix

Algebraic representation of epipolar geometry.

$$\mathbf{x} \mapsto \mathbf{l}'$$

This mapping is a singular correlation; represented by fundamental matrix F
3×3 matrix with 7 dof (8 independent ratios)

1.5.1 Algebraic Derivation

Let x be PX

$$P^{+}x = P^{+}PX$$

Say that pseudoinverse represents line from center to point

$$\begin{aligned}
\mathbf{X}(\lambda) &= \mathbf{P}^{+}\mathbf{x} + \lambda\mathbf{C} \\
\mathbf{l}' &= \mathbf{P}'\mathbf{C} \times \mathbf{P}'\mathbf{P}^{+}\mathbf{x} \\
F &= [e']_{\times} P' P^{+}
\end{aligned}$$

λ is a scalar
$[e']_{\times}$ skew symmetric cross product matrix (rank 2)
doesnt work for $C = C' \Rightarrow F = 0$ (must have distinct camera centers)

Alternatively can write:

$$\begin{aligned}
F &= [e']_{\times} H_{\infty} \\
H_{\infty} &= K^{-1} R K
\end{aligned}$$

1.5.2 Geometric Derivation

Step 1: X on a plane π

$$x' = H_{\pi} x$$

Step 2: Epipolar line l'

$$l' = e' \times x' = [e']_{\times} H_{\pi} x = F x$$

Map from 2D to 1D family (rank 2)
(plane π can be different for each point, **unrelated** to epipolar plane)

1.5.3 Correspondence Condition

Exists 2D matrix that ouputs epipolar line.
If $\mathbf{x}'$ lies on $\mathbf{l}'$, $\mathbf{x}'\mathbf{l}' = 0$

$$\mathbf{x}' \cdot \mathbf{F}\mathbf{x} = 0$$

$\mathbf{F}$ is the unique 3×3 rank 2 matrix that satisfies

$$\mathbf{x}'^{\top}\mathbf{F}\mathbf{x} = 0 \quad \forall \mathbf{x} \leftrightarrow \mathbf{x}'$$

1. **Transpose**: if $\mathbf{F}$ is a fundamental matrix for (P, P'), $\mathbf{F}^{\top}$ is fundamental matrix for (P', P)
2. Epipolar Lines: $\mathbf{l}' = \mathbf{F}\mathbf{x}$ and $\mathbf{l} = \mathbf{F}^{\top}\mathbf{x}'$
3. Epipoles: on all epipolar lines, $\mathbf{e}'^{\top}\mathbf{F}\mathbf{x} = \mathbf{0}\ \forall \mathbf{x}, \Rightarrow\ e'^{\top}F = 0$, similarly $\mathbf{F}\mathbf{e} = 0$
4. $\mathbf{F}$ has 7dof (3×3 - 1 (homog), - 1 (rank 2))
5. $\mathbf{F}$ is a correlation, projective mapping from a point $\mathbf{x}$ to a line $\mathbf{l}' = \mathbf{F}\mathbf{x}$ (not a proper corr., ie. not invertible)

1.5.4 Summary

Algebraic rep. of epipolar geom.
Step 1: $\mathbf{X}$ on plane π

$$\mathbf{x}' = \mathbf{H}\mathbf{x}$$

Step 2: epipolar line $\mathbf{l}'$

$$\begin{aligned} \mathbf{l}' &= \mathbf{e}' \times \mathbf{x}' = [\mathbf{e}']_{\times}\mathbf{x}' \\ &= [\mathbf{e}']_{\times}\mathbf{H}\mathbf{x} = \mathbf{F}\mathbf{x} \end{aligned}$$

$$\mathbf{x}'^{\top}\mathbf{F}\mathbf{x} = \mathbf{0}$$

1.5.5 Computing F

Use 8 points

$$\mathbf{x}'^{\top}\mathbf{F}\mathbf{x} = 0$$

(SLIDE 43-44)

2 Lecture - Mar 5

2.1 Fundamental Matrix (Summary)

get ray from point, real point could be any point that lies on ray, take multiple imgs and get intersection of rays to find point.
find mathematical nature of point b/w 2 views (?)

Definition 2.1.1: F

3×3 Rank 2, $\det(F) = 0$
Linear solution - 8 corr. points (unique)
Nonlinear sol - 7 corr. points (3 sol)
Very sensitive to **noise** and **outliers** (need to know x and x' very precisely)

Epipolar Lines:

$$\mathbf{l}' = F\mathbf{x},\ \mathbf{l} = F^\top \mathbf{x}'$$

Epipoles:

$$F\mathbf{e} = 0, F^\top \mathbf{e}' = 0$$

Projection Matrices:

$$P = \begin{bmatrix} I & | & \mathbf{0} \end{bmatrix}$$

$$P' = \begin{bmatrix} [\mathbf{e}']_\times F + \mathbf{e}'\mathbf{v}^\top & | & \lambda \mathbf{e}' \end{bmatrix}$$

2.2 3D geom and 2D images w/ F

F relates to 3 questions:

1. **Correspondence geometry** - given img point x in first view, how does this constrain the position of the corresponding point x' in the second image?
2. **Camera geometry (motion)** - given a set of corresponding img points $\{x_i \leftrightarrow x'_i\}, i = 1, \ldots, n$ what are the cameras P and P' for the two views
3. **Scene geometry (structure)** - given corresponding image points $x_i \leftrightarrow x'_i$ and cameras P, P', what is the position of (their pre-image) X in space

2.3 Compute F

(slide 43-44)
should normalize pixel values $[0, \sqrt{2}]$
Trifocal tensor - 3 view geom
Quadrifocal tensor - 4 view geom
Multiview tensors
(no additional constraint b/w > 4 imgs, all constraints can be expressed using F, triliear tensor or quadrifocal tensor)

2.4 Canonical Cameras given F

F matrix corresponds to P, P' iff $P'^\top F P$ is skew symmetric

$$X^\top P'^\top F P X = 0, \forall X$$

F matrix, S skew symmetric matrix

$$P = \begin{bmatrix} I & | & 0 \end{bmatrix}, P' = \begin{bmatrix} SF & | & e' \end{bmatrix}$$

Possible Choice:

$$P = \begin{bmatrix} I & | & 0 \end{bmatrix} \quad P' = \begin{bmatrix} [e']_\times F & | & e' \end{bmatrix}$$

Canonical Rep:

$$P = \begin{bmatrix} I & | & 0 \end{bmatrix} \quad P' = \begin{bmatrix} [e']_\times F + e' v^\top & | & \lambda e' \end{bmatrix}$$

2.5 Struct from imgs

compute points, triangulate position.

2.6 Linear Triangulation

(slide 49-51)
multiplicative and additive relationships in A
not best for geometric error, good for algebraic error; sufficient to show theres a computational solution for computing 3d

2.7 Ambiguities

Reconstruction uncertainty - angles/rays close to colinear → less specific solution space

2.8 Refining truct and motion

Minimize reprojection error

$$\min_{\hat{P}_k, \hat{M}_i} \sum_{k=1}^{m} \sum_{j=1}^{n} D(m_{kj}, \hat{P}_k \hat{M}_i)^2$$

2.9 Proj. ambiguity + self calibration

autocalibration (self cal.) - find proj T that upgrades proj. rec. to metric

2.10 Orthographic Factorization

first 2 rows = im1, next 2 = im2, . . .
trackers on points, move camera, etc.
3 first rows in M - 3d structure

2.11 weak perspective

M changes by scale, do same thing as orthog but constraints involve scale

3 Lecture - Mar 7

3.1 Factorization Method Derivation

2 Steps:

1. Show Existence/Mathematical Structure of Prblm
2. How do we apply to data

(see notebook)

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} r_1^\top & t_1 \\ r_2^\top & t_2 \\ 0^\top & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} r_1^\top \\ r_2^\top \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$$

Zero mean data $\rightarrow$ remove translation; centered coord:

$$\begin{bmatrix} \vec{u}_{ij} \\ \vec{v}_{ij} \end{bmatrix} = \begin{bmatrix} \vec{u}_{ij} - \text{mean } u \\ \vec{v}_{ij} - \text{mean } v \end{bmatrix}$$

Const. equation

Let

$$\vec{m}_{ij} = \begin{bmatrix} u_{ij} \\ v_{ij} \end{bmatrix} = \begin{bmatrix} r_1^\top \\ r_2^\top \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = [P][M_j]$$

Many pts, Im_1

$$\begin{bmatrix} u_{11} & u_{12} & \cdots & u_{1n} \\ v_{11} & v_{12} & \cdots & v_{1n} \end{bmatrix} = P_1 \begin{bmatrix} M_1 & M_2 & \cdots & M_n \end{bmatrix}$$

Im_2

$$\begin{bmatrix} u_{21} & u_{22} & \cdots & u_{2n} \\ v_{21} & v_{22} & \cdots & v_{2n} \end{bmatrix} = P_2 \begin{bmatrix} M_1 & M_2 & \cdots & M_n \end{bmatrix}$$

$$\vdots$$

Im_m

$$\begin{bmatrix} u_{m1} & u_{m2} & \cdots & u_{mn} \\ v_{m1} & v_{m2} & \cdots & v_{mn} \end{bmatrix} = P_m \begin{bmatrix} M_1 & M_2 & \cdots & M_n \end{bmatrix}$$

Put Together

$$\overline{m} = \begin{bmatrix} u_{11} & \cdots \\ v_{11} & \cdots \\ u_{21} & \cdots \\ v_{21} & \cdots \end{bmatrix}_{2m}^{n} = \begin{bmatrix} P_1 \\ P_2 \\ | \\ P_m \end{bmatrix} \begin{bmatrix} M_1 & M_2 & \cdots & M_n \end{bmatrix}^n$$

Showed in rank 3
Perform SVD on $\overline{m}$

$$\begin{aligned} \overline{m} &= USV^\top \\ &= \tilde{P}\tilde{M} \text{ where } \tilde{P} = U \text{ and } \tilde{M} = SV^\top \\ &= (PQ)(Q^{-1}M) = \overline{PM} \ (\tilde{P} \neq \overline{P},\ \tilde{M} \neq \overline{M}\text{; diff coords}) \end{aligned}$$

We compute 3D structure and cameras up to an affine coordinate transformation
Rectify Affine to Euclidean:

Selected

$$[\tilde{m}] = Q \underset{\text{(eucl.)}}{\overline{M}} = Q \begin{bmatrix} 0 & 0 & \cdots \\ 0 & 2 & \cdots \\ 0 & 0 & \cdots \end{bmatrix}$$

Solve for Q

$$\begin{aligned} \tilde{M} &= Q\overline{M} \\ ()^\top &= (\ )^\top \\ \tilde{M}^\top &= \overline{M}^\top Q^\top \end{aligned}$$

Solve LU fact. for $\overline{M}$

With no 3D ground truth, what do we know $\overline{P} \leftarrow$ desired

$$\overline{P} = Q\tilde{P}$$

rot matrix

$$\overline{P} = \begin{bmatrix} \cdots & r_1 & \cdots \\ \cdots & r_2 & \cdots \\ & \vdots & \end{bmatrix}$$

Get cholesky decomp.

$$
\begin{aligned}
0 = r_1^\top r_2 &= P_1 Q^\top Q P_2 \\
&= P_1 C P_2 \\
s^2 = r_1^\top r_1 &= C \\
s^2 = r_2^\top r_2 &= C \\
&\downarrow \\
&\dots
\end{aligned}
$$