

CMPUT 428: 3D Projective Geometry

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1 Lecture - Feb 13

lec06Proj3D.pdf

Points:

$$\mathbf{X} = (x_1, x_2, x_3, x_4); x_4 \neq 0$$

Point at inf:

$$(x_1, x_2, x_3, 0)$$

Planes:

$$\pi = (\pi_1, \pi_2, \pi_3, \pi_4)$$

Plane at inf:

$$\pi^\top \mathbf{X} = 0$$

proj transform $\rightarrow 4 \times 4$ nonsingular matrix H

Point Transformation: $X' = HX$

Plane Transformation: $\pi' = H^{-\top} \pi$

3D point:

$(X, Y, Z)^\top$ in \mathbb{R}^3

$X = (X_1, X_2, X_3, X_4)$ in P^3

Euclidean Rep:

$$n^\top \tilde{X} + d = 0$$

$n = (\pi_1, \pi_2, \pi_3)^\top$, $\tilde{X} = (X, Y, Z)^\top$

$\pi_4 = d$, $X_4 = 1$

d is distance

1.1 Planes from Points

Solve π for:

$$\mathbf{X}_1^\top \pi = 0, \mathbf{X}_2^\top \pi = 0, \text{ and } \mathbf{X}_3^\top \pi = 0$$

(use SVD)

or implicitly **Coplanarity Condition** (check slides)

Canonical Form:

If we have a plane, we can get nullspace span $\mathbf{M} = \begin{bmatrix} p \\ I \end{bmatrix}$

$$p = \left[-\frac{b}{a}, -\frac{c}{a}, -\frac{d}{a}\right]^\top$$

1.2 Points from Planes

Solve \mathbf{X} from

$$\pi_1^\top \mathbf{X} = 0, \pi_2^\top \mathbf{X} = 0, \text{ and } \pi_3^\top \mathbf{X} = 0$$

1.3 Lines

Join of two points (A, B)

$$W = \begin{bmatrix} A^\top \\ B^\top \end{bmatrix}, \lambda A + \mu B$$

Intersection of two planes (P, Q)

$$W^* = \begin{bmatrix} P^\top \\ Q^\top \end{bmatrix}, \lambda P + \mu Q \text{ "pencil of planes"}$$

pencil of planes - all the planes that could generate an intersection of this line

$$W^* W^\top = W W^{*\top} = 0_{2 \times 2}$$

Define basis/direction vectors as pure direction ($x_4 = 0$) rather than a finite point

W is 2×4

1.4 Points, lines, planes

Join of point X and line W is plane π

$$\mathbf{M} = \begin{bmatrix} W \\ X^\top \end{bmatrix} \quad \mathbf{M}\pi = 0$$

Intersection of line W with plane π is point X

$$\mathbf{M} = \begin{bmatrix} W^* \\ \pi^\top \end{bmatrix} \quad \mathbf{M}X = 0$$

1.5 Affine space

Difficulties with a projective space:

- Nonintuitive notion of direction
 - Parallelism not represented
 - Restore parallelism \rightarrow need to know/fix some affine property
- Infinity not distinguished
- No notion of "inbetweenness"
 - Projective lines are topologically circular
- Only cross-ratios are available

- Ratios are required for many practical tasks

Solution:

Find the point at infinity

$$\pi_{\infty} = (0, 0, 0, 1)$$

Transform model to give π_{∞} canonical coords

2D analogy: fix horizontal line $\mathbf{l}_{\infty} = (0, 0, 1)$

1.5.1 Proj to Affine

Get "parallel" lines in img, find projective transformation that would make lines parallel

Affine transformation:

$$H_A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

12 DOF, π_{∞} unchanged

Invariants:

Ratio of lengths on a line

Ratios of angles

Parallelism

1.6 Metric Space

Metric transformation (similarity)

$$H_S = \begin{bmatrix} sR & \mathbf{t} \\ \mathbf{0}^t & 1 \end{bmatrix}$$

7DOF, maps absolute conic to itself

Invariants:

Length ratios

Angles

Absolute conic

Highest level of geometric structure that can be retrieved from images (without yard stick/ruler)

Overview 1.6.1: Main Takeaway of 428

"Stratified Geometry" vs Euclidean Geometry

Euclidean geometry is not the only repr for:

1. building models from imgs
2. building imgs from models

When 3D realism is the goal; how can we effectively build and use projective, affine, metric representations.

2 Lecture - Feb 15

lec08Proj, lec06Proj3D; **READ SLIDES!!!!!!!!!!**

Projections - use similar triangles

Camera matrix

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ W \end{bmatrix}$$

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} \approx \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ W \end{bmatrix}$$

$$(U, V, W) \rightarrow \left(\frac{U}{W}, \frac{V}{W} \right) = (u, v)$$

when coord sys not aligned, P becomes arbitrary singular or nonsingular matrix

$$x = PX$$

2.1 Intrinsic Parameters

Conversion from metric to pixel coords (and reverse)

$$x_{\text{mm}} = -(x_{\text{pix}} - o_x)s_x$$

$$y_{\text{mm}} = -(y_{\text{pix}} - o_y)s_y$$

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix}_{\text{pix}} = \begin{bmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}_{\text{mm}} = \mathbf{M}_{\text{int}} \mathbf{p}$$

Expln 2.1.1

f is focal length (mm); s is scale factor (aspect ratios), o center coords

2.2 Relative Location (cam/laser)

create homog transforms that represent the alignments;

camera internal params from manufacturers

first matrix via calibration eq.

Internal params = known information; need to find ext. params (unknown)

2.3 Diff Cam Models

Orthographic \rightarrow pure parallel projection

Weak Perspective \rightarrow parallel projection onto imaginary plane (obj plane) and then perspective transformation. (single scaling factor)

Paraperspective \rightarrow

Perspective \rightarrow

2.4 Weak Perspective

Uses a scaling factor T

$$u = Tx, v = Ty, T = f/Z$$

Z is the **average** depth

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & f/Z^* \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ T \end{bmatrix}$$

Z^* fixed val, usually mean/centroid distance

2.5 Full Affine Linear Camera

factor into rotation part and scale part (?)

2.6 Camera Models