Formalizing Modal Embeddings of Call-by-Name and Call-by-Value

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Research Question

How can the unification of call-by-name and call-by-value evaluation strategies using modal logic be formalised in Agda?

Introduction

Call-by-name

Let f be defined as

$f(x) = x * x$

Call-by-name

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Call-by-name evaluation of f (3 + 3)

$$f(3+3) \rightarrow (3+3) * (3+3)$$

Call-by-name

Let f be defined as

$f(x) = x * x$

Call-by-name evaluation of f (3 + 3)

$$\begin{aligned} f(3+3) &\rightarrow (3+3) * (3+3) \\ &\rightarrow 6 * (3+3) \end{aligned}$$

Call-by-name

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Call-by-name evaluation of f (3 + 3)

$$\begin{aligned} f(3+3) &\rightarrow (3+3) * (3+3) \\ &\rightarrow 6 * (3+3) \\ &\rightarrow 6 * 6 \end{aligned}$$

Call-by-name

Let f be defined as

$f(x) = x * x$

Call-by-name evaluation of f (3 + 3)

$$\begin{aligned} f(3+3) &\rightarrow (3+3) * (3+3) \\ &\rightarrow 6 * (3+3) \\ &\rightarrow 6 * 6 \\ &\rightarrow 36 \end{aligned}$$

Call-by-value

Let f be defined as

$f(x) = x * x$

Call-by-value

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$f(x) = x * x$

Call-by-value evaluation of f (3 + 3)

$$f(3 + 3) \rightarrow f(6)$$

Call-by-value

Let f be defined as

$f(x) = x * x$

Call-by-value evaluation of f (3 + 3)

$$\begin{aligned} f(3+3) &\rightarrow f(6) \\ &\rightarrow 6 * 6 \end{aligned}$$

Call-by-value

Let f be defined as

$f(x) = x * x$

Call-by-value evaluation of f (3 + 3)

$$\begin{aligned} f(3+3) &\rightarrow f(6) \\ &\rightarrow 6 * 6 \\ &\rightarrow 36 \end{aligned}$$

Unifying cbn and cbv

- Why unify cbn and cbv?
- Some approaches to unification:
  - Modal logic
  - Linear logic
  - Thunks

Background

Grammar Lambda Calculus (λ-calculus)

$$M, N, P, Q \quad ::= \quad x \mid \lambda x.M \mid MN$$

Call-by-name λ-calculus

Lorem ipsum

Call-by-value λ-calculus

Lorem ipsum

Evaluation Relations

Lorem ipsum

Call-by-box λ-calculus

Lorem ipsum

Evaluation Relations of λ_b

Lorem ipsum

Girard's Translation

Lorem ipsum

Gödel's Translation

Lorem ipsum

Formalisation in Agda

λ-terms in Agda

Lorem ipsum

Propositions

Girard's Translation

Lorem ipsum

Gödel's Translation

Lorem ipsum

Conclusion

Conclusion

Lorem ipsum