线性代数 矩阵及其运算

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矩阵就是两个变量空间的线性对应关系
线性关系表示 变量之间没有乘除关系，只有加减关系和倍数关系

矩阵实际意义：

$$\begin{pmatrix} y_1 = a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ y_2 = a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \cdots\cdots\cdots\cdots \\ y_m = a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \end{pmatrix}$$

R_n空间

$$\begin{pmatrix} \boxed{\begin{matrix} y_1 \\ y_2 \\ \\ y_m \end{matrix}} & \begin{matrix} = \\ = \\ \\ = \end{matrix} & \boxed{\begin{matrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \cdots\cdots\cdots\cdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \end{matrix}} \end{pmatrix}$$

R_m空间

其中有意义的部分只有参数a
其他 x y 变量都可以省略，产生最后的矩阵形式

$$R_n \longrightarrow \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \longrightarrow R_m$$

矩阵加减法：

实际就是 两个变量空间的线性参数的 线性叠加
注意：矩阵加减法前后两个矩阵大小必须一致

矩阵乘法：

矩阵乘法的实际意义是两个线性空间的嵌套叠加
比如一个 m个参数的空间 Ry

$$\begin{pmatrix} y_{a1} = a_{11}x_1 + a_{12}x_2 \\ y_{a2} = a_{21}x_1 + a_{22}x_2 \\ \cdots\cdots\cdots\cdots \\ y_{am} = a_{m1}x_1 + a_{m2}x_2 \end{pmatrix}$$

点乘

$$\begin{pmatrix} y_{b1} = b_{11}x_1 + b_{12}x_2 + \cdots + b_{1n}x_n \\ y_{b2} = b_{21}x_1 + b_{22}x_2 + \cdots + b_{2n}x_n \end{pmatrix}$$

就是将每个y_b 分别带入前式的 x
计算方式就不用标准公式展开，可参考手写推导：

所以 前式的列数必须与后式的行数相同，
最终产生的新矩阵，行数为前式的行数，列数为后式的列数

满足结合律： (AB)C = A(BC)
不满足交换律：AB ≠ BA

$$\begin{pmatrix} y_{a1} = a_{11}y_{b1} + a_{12}y_{b2} \\ y_{a2} = a_{21}y_{b1} + a_{22}y_{b2} \\ \vdots \\ y_{am} = a_{m1}y_{b1} + a_{m2}y_{b2} \end{pmatrix}$$

$$\begin{pmatrix} y_{a1} = a_{11}(b_{11}x_1 + b_{12}x_2 + \cdots + b_{1n}x_n) + a_{12}(b_{21}x_1 + b_{22}x_2 + \cdots b_{2n}x_n) \\ y_{a2} = a_{21}(b_{11}x_1 + b_{12}x_2 + \cdots + b_{1n}x_n) + a_{22}(b_{21}x_1 + b_{22}x_2 + \cdots + b_{2n}x_n) \\ \vdots \\ y_{am} = a_{m1}(b_{11}x_1 + b_{12}x_2 + \cdots + b_{1n}x_n) + a_{m2}(b_{21}x_1 + b_{22}x_2 + \cdots + b_{2n}x_n) \end{pmatrix}$$

$$\begin{pmatrix} y_{a1} = (a_{11}b_{11} + a_{12}b_{21})x_1 + (a_{11}b_{12} + a_{12}b_{22})x_2 + \cdots + (a_{11}b_{1n} + a_{12}b_{2n})x_n \\ y_{a2} = (a_{21}b_{11} + a_{22}b_{21})x_1 + (a_{21}b_{12} + a_{22}b_{22})x_2 + \cdots + (a_{21}b_{1n} + a_{22}b_{2n})x_n \\ \vdots \\ y_{am} = (a_{m1}b_{11} + a_{m2}b_{21})x_1 + (a_{m1}b_{12} + a_{m2}b_{22})x_2 + \cdots + (a_{m1}b_{1n} + a_{m2}b_{2n})x_n \end{pmatrix}$$

$$\begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} & \cdots & a_{11}b_{1n} + a_{12}b_{2n} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} & \cdots & a_{21}b_{1n} + a_{22}b_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1}b_{11} + a_{m2}b_{21} & a_{m1}b_{12} + a_{m2}b_{22} & \cdots & a_{m1}b_{1n} + a_{m2}b_{2m} \end{pmatrix}$$

即 $A_{mk} \cdot b_{kn}$

$$c_{ij} = \sum_{s=1}^{k} a_{is} b_{sj}$$