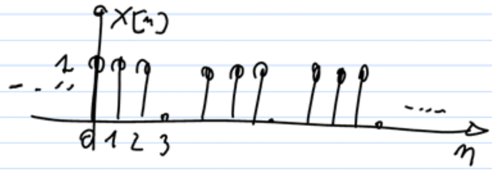
29/11/2017 CLASE PROBLEMAS Nº 10, miércoles

Título de la nota 29/11/2017

TEMA 4

DTFT de señales periódicas (antes se realiza el D.S.F. del tema 1)

Dada la señal x[n]



a) Calcula el DSF y los c_k (TEMA 1)

b) Calcula y representa la DTFT de x[n] (TEMA 4)

a) TEMA 1 $\rightarrow$ $N_0 = 4$, $c_k = \frac{1}{N_0}\sum_{n=0}^{N_0-1} x[n]\, e^{-j\frac{2\pi k n}{N_0}}$

$c_0 = 3/4 \qquad c_1 = \frac{1}{4} e^{-j\frac{\pi}{2}} \qquad c_2 = \frac{1}{4} \qquad c_3 = \frac{1}{4} e^{+j\frac{\pi}{2}}$

para obtener x[n]

$$\boxed{x[n] = \frac{3}{4} + \frac{1}{4} e^{-j\frac{\pi}{2}} e^{j\frac{\pi}{2}n} + \frac{1}{4} e^{j\pi n} + \frac{1}{4} e^{j\frac{\pi}{2}} e^{+j\frac{3}{2}\pi n}}$$

el D.S.F.

b) Tema 4

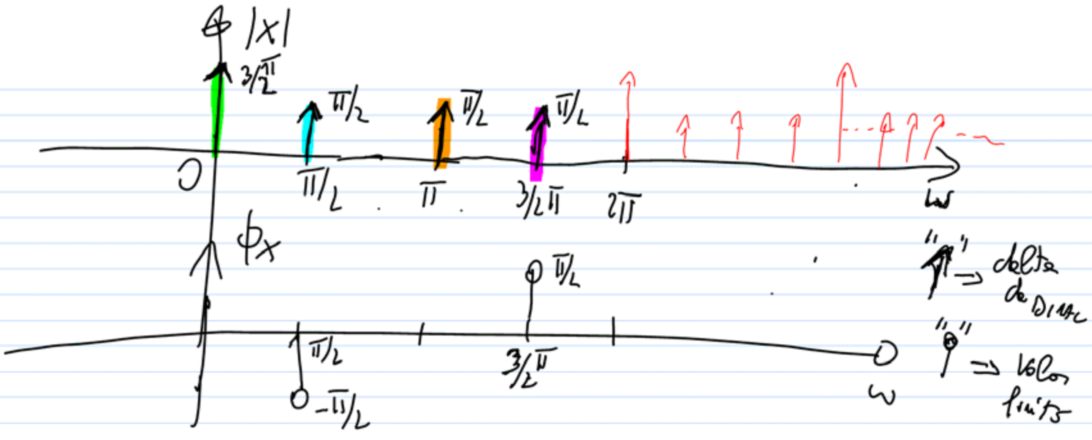
$$x[n] = A \xleftrightarrow{\text{DTFT}} X(e^{j\omega}) = 2\pi A \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$$

$$x[n] = A e^{j\phi} e^{j\omega_k n} \xleftrightarrow{\text{DTFT}} X(e^{j\omega}) = 2\pi A e^{j\phi_0} \sum_{k=-\infty}^{\infty} \delta(\omega - \omega_k - 2\pi k)$$

$$X(e^{j\omega}) = \cancel{2}\pi \frac{3}{\cancel{4}} \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k) + \cancel{2}\pi \cdot \frac{1}{\cancel{4}_{2}} e^{-j\frac{\pi}{2}} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{\pi}{2} - 2\pi k\right) +$$

$$+ \frac{1}{4} 2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - \pi - 2\pi k) + \cancel{2}\pi \frac{1}{\cancel{4}_{2}} e^{+j\frac{\pi}{2}} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{3}{2}\pi - 2\pi k\right)$$

$$= \sum_{k=0}^{N_0-1} 2\pi c_k e^{j\phi_{c_k}} \sum_{\ell=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{N_0} - 2\pi \ell\right)$$



1.4.3 Calcula la DTFT^{-1} de

$$X(e^{j\omega}) = 1 - 2e^{-j\omega} + 4e^{j2\omega} + 3e^{-j6\omega}$$

Sol.

$$A\,\delta[n] \xleftrightarrow{DTFT^{-1}} A$$

$$A\,\delta[n+n_0] \longleftrightarrow A\,e^{j\omega n_0}$$

$$A\,\delta[n-n_0] \longleftrightarrow A\,e^{-j\omega n_0}$$

$$x[n] = \delta[n] - 2\delta[n-1] + 4\delta[n+2] + 3\delta[n-6]$$

$$X(e^{j\omega}) = \cos^2(\omega) = \frac{1+\cos(2\omega)}{2} = \frac{1}{2} + \frac{1}{2}\cos(2\omega) =$$

$$= 1/2 + \frac{1}{2}\cdot\frac{1}{2}e^{j2\omega} + \frac{1}{2}\cdot\frac{1}{2}e^{-j2\omega}$$

$$\boxed{\cos(2\omega) = \frac{e^{j2\omega}+e^{-j2\omega}}{2}}$$

$$= \frac{1}{2} + \frac{1}{4}e^{j2\omega} + \frac{1}{4}e^{-j2\omega}$$

$\downarrow$ DTFT^{-1}

$$x[n] = \frac{1}{2}\delta[n] + \frac{1}{4}\delta[n+2] + \frac{1}{4}\delta[n-2]$$

1.4.7 Dada la ec. en diferencias

$$y[n] + \frac{1}{2}y[n-1] = x[n]$$

a) Calcula $H(e^{j\omega})$ c) Calcula $y[n]$ para

b) Calcula $h[n]$ $\qquad$ $x_1[n] = \left(-\frac{1}{2}\right)^n u[n]$

$x_2[n] = \delta[n] + \frac{1}{2}\delta[n-1]$

Sol.

a) es ec. en diferencias de tipo IIR y orden $N=1$

$$Y(e^{j\omega}) + \frac{1}{2}e^{-j\omega}Y(e^{j\omega}) = X(e^{j\omega})$$

$$Y(e^{j\omega})\left(1 + \frac{1}{2}e^{-j\omega}\right) = X(e^{j\omega})$$

$$Y(e^{j\omega}) = X(e^{j\omega})\left(\frac{1}{1+\frac{1}{2}e^{-j\omega}}\right) = X(e^{j\omega})\,H(e^{j\omega})$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1}{1+\frac{1}{2}e^{-j\omega}}$$

b) $a^n u[n] \Longleftrightarrow \frac{1}{1-ae^{-j\omega}} \Rightarrow$ si $a = -\frac{1}{2}$

$|a| < 1$

$$h[n] = DTFT^{-1}\left\{H(e^{j\omega})\right\} = \left(-\frac{1}{2}\right)^n u[n]$$

c) $y_1[n]$? cuando $x_1[n] = \left(-\frac{1}{2}\right)^n u[n]$

$y_1[n] \to y_1[n] = x_1[n] * h_1[n]$

$y_1[n] \to Y_1(e^{j\omega}) = X_1(e^{j\omega})\,H(e^{j\omega}) \xrightarrow{DTFT^{-1}} y[n]$

$$H(e^{j\omega}) = \frac{1}{1+\frac{1}{2}e^{-j\omega}} \qquad X_1(e^{j\omega}) = \frac{1}{1+\frac{1}{2}e^{-j\omega}}$$

$$Y_1(e^{j\omega}) = X_1(e^{j\omega})\,H(e^{j\omega}) = \underbrace{\frac{1}{\left(1+\frac{1}{2}e^{-j\omega}\right)}}_{X_1} \cdot \underbrace{\frac{1}{\left(1+\frac{1}{2}e^{-j\omega}\right)}}_{H} = \frac{1}{\left(1+\frac{1}{2}e^{-j\omega}\right)^2}$$

$$\boxed{\begin{array}{c} n\,a^n u[n] \Longleftrightarrow \dfrac{a e^{-jw}}{(1-a e^{-jw})^2} \\ |a|<1 \end{array}} \qquad a = \left(-\tfrac{1}{2}\right)$$

$$Y_1(e^{jw}) = \frac{1}{(1+\frac{1}{2}e^{-jw})^2} = \frac{\left(-\frac{1}{2}e^{-jw}\right)\cdot\left(-2\right)e^{jw}}{\left(1+\frac{1}{2}e^{-jw}\right)^2}$$

DTFT^{-1}

$$y_1[n] = -2\,(n+1)\left(-\tfrac{1}{2}\right)^{n+1} u[n+1]$$

$y_2[n]$? cuando $x_2[n] = \delta[n] + \frac{1}{2}\delta[n-1]$

$$h[n] = \left(-\tfrac{1}{2}\right)^n u[n] \longrightarrow \boxed{H(e^{jw}) = \frac{1}{1+\frac{1}{2}e^{-jw}}}$$

$$y_2[n] = \begin{cases} \to x_2[n] * h[n] \\ \to X_2(e^{jw})\cdot H(e^{jw}) \xRightarrow{DTFT^{-1}} y_2[n] \end{cases}$$

CAMINO POR EL TIEMPO

$$y_2[n] = \left(-\tfrac{1}{2}\right)^n u[n] * \left(\delta[n] + \tfrac{1}{2}\delta[n-1]\right) = \left(-\tfrac{1}{2}\right)^n u[n] + \tfrac{1}{2}\left(-\tfrac{1}{2}\right)^{n-1} u[n-1] = \delta[n]$$

CAMINO POR LA FREC.

$$X_2(e^{jw}) = 1 + \frac{1}{2}e^{-jw}$$

$$Y_2(e^{jw}) = X_2(e^{jw})\,H(e^{jw}) = \cancel{\left(1+\tfrac{1}{2}e^{-jw}\right)}\cancel{\left(\frac{1}{1+\frac{1}{2}e^{-jw}}\right)} = 1$$

DTFT^{-1}

$$\boxed{y_2[n] = \delta[n]}$$