RODRIGO ALMEIDA - LISTA 06

01) $Y(s) = (R(s) - Y(s)) \dfrac{K \cdot 5}{(s+2)(s+3)(s+4)}$

$((s^2 + 5s + 6)(s+4) + 5k) Y(s) = 5k \; R(s)$

$$\frac{Y(s)}{R(s)} = \frac{5k}{s^3 + 9s^2 + 26s + (24 + 5k)}$$

$s^3: \; 1 \quad 26$

$s^2: \; 9 \quad 24+5k$

$s^1: \; 26 - \dfrac{(24+5k)}{9}$

$s^0: \; 24+5k$

$b_1 = \dfrac{9 \cdot 26 - 24 - 5k}{9}$

$c_1 = 24 + 5k$

$\begin{cases} 24 + 5k < 26 \cdot 9 \\ 24 + 5k > 0 \end{cases}$

$\{ \; 42 > k > -4{,}8$

02) $Y(s) = G_R(s) R(s) + G_D(s) D(s) = \dfrac{k_p s + k_i}{ms^2 + (b + k_p)s + k_i} R(s) + \dfrac{1}{s(ms^2 + (b+k_p)s + k_i)}$

$-E(s) = Y(s) - R(s) = \dfrac{-ms - b}{ms^2 + (b+k_p)s + k_i} + \dfrac{1}{s(ms^2 + (b+k_p)s + k_i)}$

$E(s) = \dfrac{ms^2 + bs - 1}{s(ms^2 + (b+k_p)s + k_i)}$

$e_\infty = \lim_{s \to 0} sE(s) = \boxed{-\dfrac{1}{k_i}}$

(03)

$$\frac{d}{dt}\vec{x} = F(\vec{x},u) \quad , \quad \vec{x} = \begin{bmatrix} y \\ v \end{bmatrix}$$

$$\frac{d}{dt}\begin{bmatrix} y \\ v \end{bmatrix} = \begin{bmatrix} v \\ \frac{k_f}{m}\frac{u^2}{(y_{max}-y)^2} - \frac{b}{m}v - g \end{bmatrix} \rightarrow \text{não linear}$$

$$F(\vec{x},u) \approx F(x_0,u_0) + \left.\frac{\partial F}{\partial \vec{x}}\right|_{\substack{x=x_0\\u=u_0}}(\vec{x}-\vec{x}_0) + \left.\frac{\partial F}{\partial u}\right|_{\substack{x=x_0\\u=u_0}}(u-u_0)$$

$$F(\vec{x},u) \approx \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{2g}{y_{max}-y_0} \end{bmatrix}(y-y_0) + \begin{bmatrix} 1 \\ -b/m \end{bmatrix}(v-v_0) + \begin{bmatrix} 0 \\ \frac{2g}{u_0} \end{bmatrix}(u-u_0)$$

$$\frac{d}{dt}\begin{bmatrix} \delta y \\ \delta v \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{2g}{y_{max}-y_0} & -\frac{b}{m} \end{bmatrix}\begin{bmatrix} \delta y \\ \delta v \end{bmatrix} + \begin{bmatrix} 0 \\ 2g/u_0 \end{bmatrix}\delta u$$

(04) polos :

$$\begin{cases} s=-10 & |s|=10 \\ s=-20 & |s|=20 \\ s=-1+\sqrt{3}i & |s|=2 \\ s=-1-\sqrt{3}i & |s|=2 \end{cases} \Big\} \text{ polos dominantes}$$

$$G_f(s) = \frac{1}{10}\cdot\cancel{\frac{1}{s/10+1}}^{\,1}\cdot\frac{1}{20}\;\frac{1}{s/20+1}^{\,1}\cdot\frac{1}{s^2+2s+4} \approx \frac{1}{200}\;\frac{1}{s^2+2s+4}$$

$$G_f(s) = \frac{1}{800}\cdot\frac{4}{s^2+2s+4} \quad , \quad w_n = 2 \quad \zeta = 1/2$$

$$t_s|_{2\%} \approx \frac{3,9}{\zeta w_n} \approx 3,9 \qquad t_r|_{0\%}^{100\%} \approx \frac{2,16\zeta+0,60}{w_n} \approx 0,84$$

$$E(s) = \frac{1}{200\,s(s^2+2s+4)} - \frac{1}{s} = \frac{1-200s^2-400s-800}{200s(s^2+2s+4)}$$

$$e_0 = \lim_{s\to 0} sE(s) = \lim_{s\to 0}\frac{-200s^2-400s-799}{200s^2+400s+800} = -\frac{799}{800}$$

(05) $V(t) = K_p e(t) + K_i \int_0^t e(\tau)\,d\tau$, $e = q_r\left(1 - e^{-\frac{K_i}{K_p}t}\right) - q$

$$V = R\dot{q} + \frac{q}{C}$$

$$E(s) = q_r\left(\frac{1}{s} - \frac{1}{s+\frac{K_i}{K_p}}\right) - Q(s) =$$

$$= \frac{q_r K_i}{s(K_p s + K_i)} - Q(s)$$

$$\frac{K_p K_i q_r}{s(K_p s+K_i)} - K_p Q(s) + \frac{q_r K_i^2}{s^2(K_p s+K_i)} - \frac{K_i}{s} Q(s) = R s\, Q(s) + \frac{Q(s)}{C}$$

$$\frac{K_i q_r \cancel{(K_p s + K_i)}}{s^{\cancel{2}} \cancel{(K_p s+K_i)}} = Q(s)\left(\frac{K_p s + K_i + R s^2 + s/C}{\cancel{s}}\right)$$

$$Q(s) = \frac{K_i q_r}{s\left(R s^2 + s(K_p + 1/C) + K_i\right)}$$

$\zeta = 0{,}7$

$w_n = 0{,}3285$

$$\left\{\begin{array}{l} w_n = \sqrt{\frac{K_i}{R}} \\ \frac{K_p}{R} + \frac{1}{CR} = 2\zeta w_n \end{array}\right.$$

$K_i = 1079{,}3$

$K_p = -995401$