RODRIGO ALVES DE ALMEIDA

P2

(01) $(R-Y)\dfrac{(s+10)K}{(s+1)(s+2)(s+3)} = Y$

$R(s+10)K = Y\left((s^2+3s+2)(s+3) + sk + 10K\right)$

$G(s) = \dfrac{Y(s)}{R(s)} = \dfrac{(s+10)K}{s^3+6s^2+(11+k)s+(6+10K)}$

tabela de Routh:

$s^3: \quad 1 \qquad 11+K$

$s^2: \quad 6 \qquad 6+10K$

$s^1: \quad 10-\frac{2}{3}K \qquad 0$

$s^0: \quad 6+10K$

$b_1 = 11+K - \dfrac{6+10K}{6} = 10 - \dfrac{2}{3}K$

$b_2 = 0$

$c_1 = 6+10K$

$\begin{cases} 6+10K > 0 \\ K > -0.6 \ (1) \end{cases} \quad \begin{cases} 10-\frac{2}{3}K > 0 \\ K < 15 \ (2) \end{cases}$

$R = 1/s$ (degrau unitário)

$E(s) = \dfrac{1}{s} - Y(s) = \dfrac{1}{s} - \dfrac{(s+10)K}{s(s^3+6s^2+(11+k)s+6+10K)} = \dfrac{s^3+6s^2+11s+6}{s(s^3+6s^2+(11+k)s+6+10K)}$

$e_\infty = \lim_{s\to 0} sE(s) = \dfrac{6}{6+10K}$

Intersecção de (1), (2), (3), (4).

$\begin{cases} -0.1 < \dfrac{6}{6+10K} < 0.1 \\ 5.4 < K \ (3), \quad K < -6.6 \ (4) \end{cases} \rightarrow 5.4 < K < 15$

Q2 a) $\begin{cases} \dot{x}(x_0, \dot{x}_0, f_0) = 0 \\ \ddot{x}(x_0, \dot{x}_0, f_0) = 0 \end{cases} \Rightarrow \begin{cases} \dot{x}_0 = 0 \\ k_1 x_0 + k_2 x_0^3 - mg = f_0 \end{cases}$

Os pontos são: $(x_0, 0, k_1 x_0 + k_2 x_0^3 - mg)$

b)

$$\frac{d}{dt}\begin{bmatrix} x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \frac{f}{m} + g - \frac{b}{m}\dot{x} - \frac{k_1}{m}x - \frac{k_2}{m}x^3 \end{bmatrix} = \begin{bmatrix} f_1(x, \dot{x}, f) \\ f_2(x, \dot{x}, f) \end{bmatrix}$$

linearizando f_2 por taylor:

$$f_2(x, \dot{x}, f) \approx f_2(x_0, \dot{x}_0, f_0) + \frac{\partial f_2}{\partial x}(x_0, \dot{x}_0, f_0)(x - x_0) + \frac{\partial f_2}{\partial \dot{x}}(x_0, \dot{x}_0, f_0)(\dot{x} - \dot{x}_0) +$$

$$+ \frac{\partial f_2}{\partial f}(x_0, \dot{x}_0, f_0)(f - f_0)$$

$$f_2(x, \dot{x}, f) \approx \left(-\frac{k_1}{m} - \frac{3k_2}{m}x_0^2\right)(x - x_0) + \left(-\frac{b}{m}\right)(\dot{x} - \dot{x}_0) + \left(\frac{1}{m}\right)(f - f_0)$$

Reescrevendo a equação

$$\frac{d}{dt}\begin{bmatrix} \delta x \\ \delta \dot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k_1}{m} - \frac{3x_0^2 k_2}{m} & -\frac{b}{m} \end{bmatrix}\begin{bmatrix} \delta x \\ \delta \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix}\delta f$$

03) Primeiro, nota-se que

$$\psi(s)\cdot\frac{v}{s}=H(s)$$

$$\boxed{\psi(s)=\frac{sH(s)}{v}}$$

$$\left(\left(R(s)-H(s)\right)K_p-\frac{sH(s)}{v}\right)\frac{K_\psi\cdot a\cdot v}{(s+a)s^2}=H(s)$$

$$K_pK_\psi a v\,R(s)-K_pK_\psi a v\,H(s)-sK_\psi a\,H(s)=(s^3+as^2)H(s)$$

$$K_pK_\psi a v\,R(s)=\left(s^3+as^2+aK_\psi s+aK_\psi K_p v\right)H(s)$$

$$G_F(s)=\frac{K_pK_\psi a v}{s^3+as^2+aK_\psi s+aK_\psi K_p v}$$

A partir do Polinômio:

$$D(s)=s^3+as^2+aK_\psi s+aK_\psi K_p v$$

sendo r_1 raiz real do polinômio, $b=-r_1$

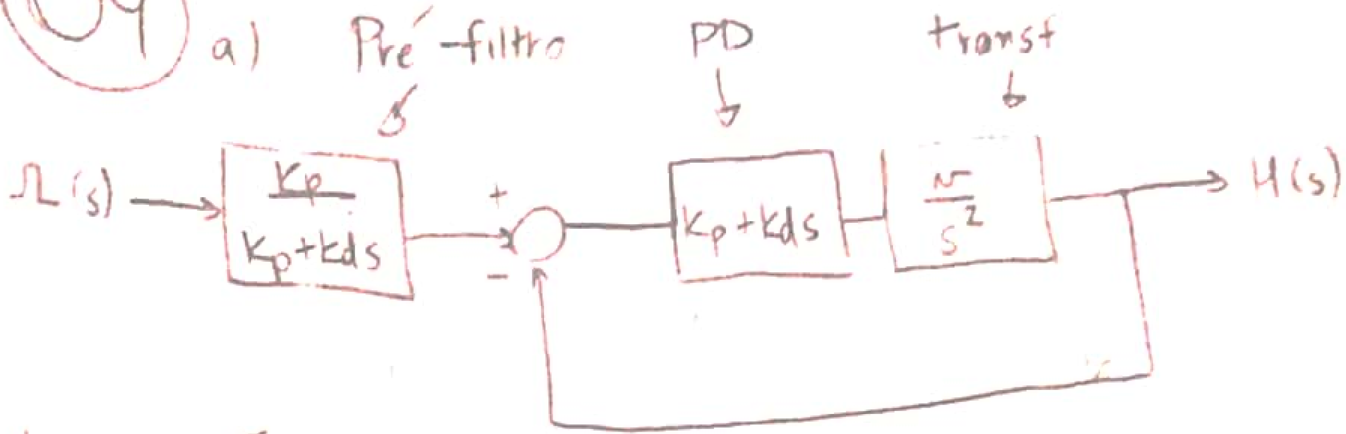
As outras raízes r_2, r_3 podem ser ambas reais ou imaginárias conjugadas:

$$\begin{cases} w_n=\sqrt{r_2\,r_3} \\ \xi=\dfrac{-(r_2+r_3)}{2\sqrt{r_2\cdot r_3}} \end{cases}$$

b)

Scanned with CamScanner

04 a)



$$M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \qquad t_r\Big|_{0\%}^{100\%} = \frac{\pi - a\cos\zeta}{\omega_n\sqrt{1-\zeta^2}}$$

$$\zeta = 0{,}69 \qquad \omega_n = 6{,}44$$

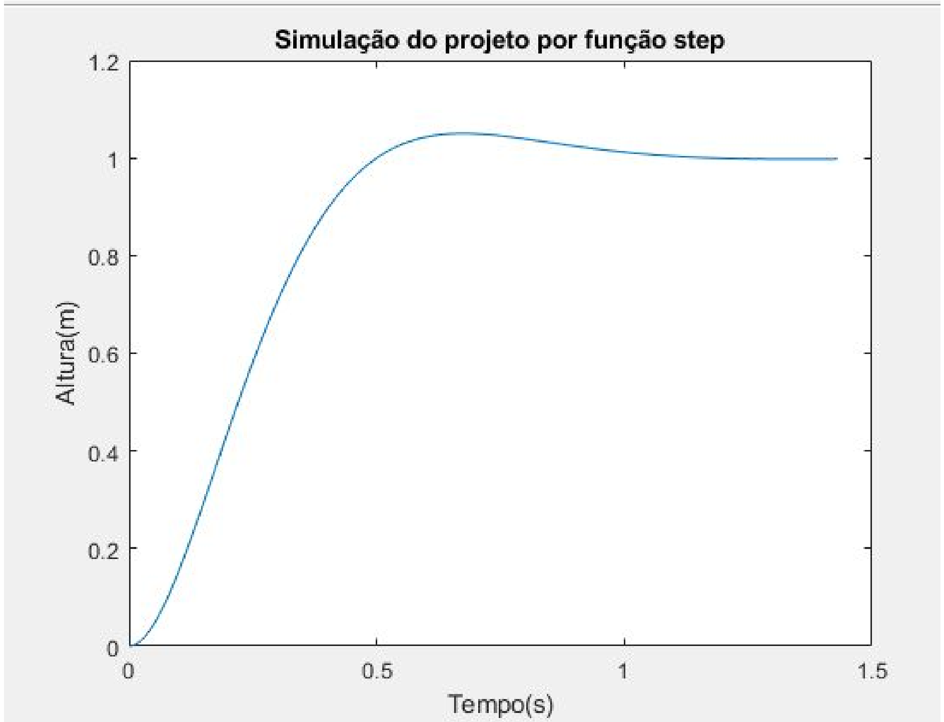
$$\left(\frac{K_p}{K_p+K_d s}\,\Omega(s) - H(s)\right)\frac{(K_p+K_d s)\,\nu}{s^2} = H(s)$$

$$K_p\nu\,\Omega(s) = \left(s^2 + K_d\nu s + K_p\nu\right)H(s)$$

$$\frac{H(s)}{\Omega(s)} = \frac{K_p\nu}{s^2+K_d\nu s+K_p\nu}$$

- $K_p \cdot 0{,}5 = (6{,}44)^2 \rightsquigarrow K_p = 82{,}95$
- $K_d \cdot 0{,}5 = 2 \cdot 0{,}69 \cdot 6{,}44 \rightsquigarrow K_d = 17{,}77$

4b)



Observa-se que o sobressinal e o tempo de subida são consistentes com os previstos no enunciado.