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CMC-12 LISTA 10 COMP-22

$$\textcircled{02} ((R(s) - X(s))K_p - X(s)s)K_r = X(s)(s(ms+b))$$

$$(R(s) - X(s))K_p K_r = X(s)(ms^2 + (b+K_r)s)$$

$$G(s) = \frac{X(s)}{R(s)} = \frac{K_p K_r}{ms^2 + (b+K_r)s + K_p K_r}, \quad G(s) = \frac{K_p K_r}{s(ms+b+K_r)}$$

$$\begin{cases} \omega_n^2 = \frac{K_p K_r}{m} \\ 2\zeta\omega_n = \frac{b+K_r}{m} \end{cases}$$

$$\begin{cases} \left(\frac{2\pi}{\omega_n}\right)^2 = 1 - 2\zeta^2 + \sqrt{4\zeta^4 - 4\zeta^2 + 2} \\ \frac{4\zeta^2}{3} = -2\zeta^2 + \sqrt{4\zeta^4 + 1} \end{cases}$$

$$\frac{10}{3}\zeta^2 = \sqrt{4\zeta^4 + 1}$$

$$\frac{100}{9}\zeta^4 = 4\zeta^4 + 1$$

$$\zeta^2 = \frac{3}{8}$$

$$\omega_n = \frac{2\pi}{\sqrt{1 - 2\zeta^2 + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}}$$

3

$$|G(j\omega)|_{dB} = 0 \Rightarrow \angle(G(j\omega)) = -105^\circ$$

$$PM = 75^\circ$$

$$\angle(G(j\omega)) = -180^\circ \Rightarrow |G(j\omega)|_{dB} = -16$$

$$GM = 16 \text{ dB}$$

4

$$G_a(s) = \frac{9s+6}{0,1s+1} \cdot \frac{1}{s^2} \cdot e^{-Ts}$$

$$|G_a(j\omega_{cp})| = 1 \Rightarrow \left| \frac{6+9j\omega_{cp}}{1+0,1j\omega_{cp}} \right| \cdot \left| \frac{1}{-j\omega_{cp}^2} \right| \cdot |e^{-Tj\omega_{cp}}| = 1$$

$$\frac{\sqrt{36+81\omega^2}}{\sqrt{1+0,01\omega^2}} \cdot \frac{1}{\omega^2} = 1$$

$$36+81\omega^2 = \omega^4(1+0,01\omega^2)$$

$$0,01\omega^6 + \omega^4 - 81\omega^2 - 36 = 0$$

$$\text{roots } \omega_{cp} = 7,3$$

$$\Delta PM = (100-30)ms \cdot 7,3 \frac{\text{rad}}{s} = 0,511 \text{ rad} =$$

$$= 29,3^\circ$$

5)  $\cdot G_R$

$$\left( R(s) \cdot \frac{k_i}{K_p s + K_i} - Y(s) \cdot \frac{a}{s+a} \right) (K_p s + K_i) = Y(s) \cdot s(Ls + R)$$

$$R(s) \cdot K_i = Y(s) \left( Ls^2 + Rs + \frac{K_p s a + K_i a}{s+a} \right)$$

$$G_R(s) = \frac{(s+a) K_i}{Ls^3 + (La + R)s^2 + (R + K_p)a s + K_i a}$$

$\cdot G_N$

$$-(Y(s) + N(s)) \frac{a}{s+a} \cdot \frac{K_p s + K_i}{s(Ls + R)} = Y(s)$$

$$-N(s) \cdot (a(K_p s + K_i)) = Y(s) \left( (s+a)(Ls + R) + a(K_p s + K_i) \right)$$

$$G_N(s) = \frac{-a K_p s - a K_i}{Ls^3 + (R + La)s^2 + (R + K_p)a s + a K_i}$$

$\cdot G_D$

$$D(s) - Y(s) \frac{a}{s+a} \cdot \frac{K_p s + K_i}{s} = Y(s) \cdot (Ls + R)$$

$$G_D(s) = \frac{s(s+a)}{Ls^3 + (R + La)s^2 + (R + K_p)a s + a K_i}$$