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PROVA 3    CMC 12    COMP 22

Q1    $G(s) = \dfrac{10}{(s+1)(s+3)(s+5)}$

Regra 1: ramos partem de $p_1 = -1$  $p_2 = -3$  $p_3 = -5$

Regra 2: há LGR em $(-3,-1)$ e $(-\infty,-5)$

Regra 3:

$n - m = 3$

$\boxed{\phi_{a_1} = 60^\circ}$  $\boxed{\phi_{a_2} = 180^\circ}$  $\boxed{\phi_{a_3} = 300^\circ}$

$\alpha = \frac{1}{3}(-1-3-5) = \boxed{-3}$

Regra 5: $\dfrac{d}{ds}\dfrac{1}{G(s)} = 0 \rightarrow (s+3)(s+5) + (s+1)(s+5) + (s+1)(s+3) = 0$

$3s^2 + 18s + 23 = 0$

$\cancel{s = -4{,}155}$ v $\boxed{s = -1{,}8453}$

não há ramo

eixo imaginário: $s = jw$, $w \in \mathbb{R}^*$

$$\frac{10}{-w^3 j - 9w^2 + 23wj + 15} = -\frac{1}{K}$$

$$23wj - w^3 j = 0$$

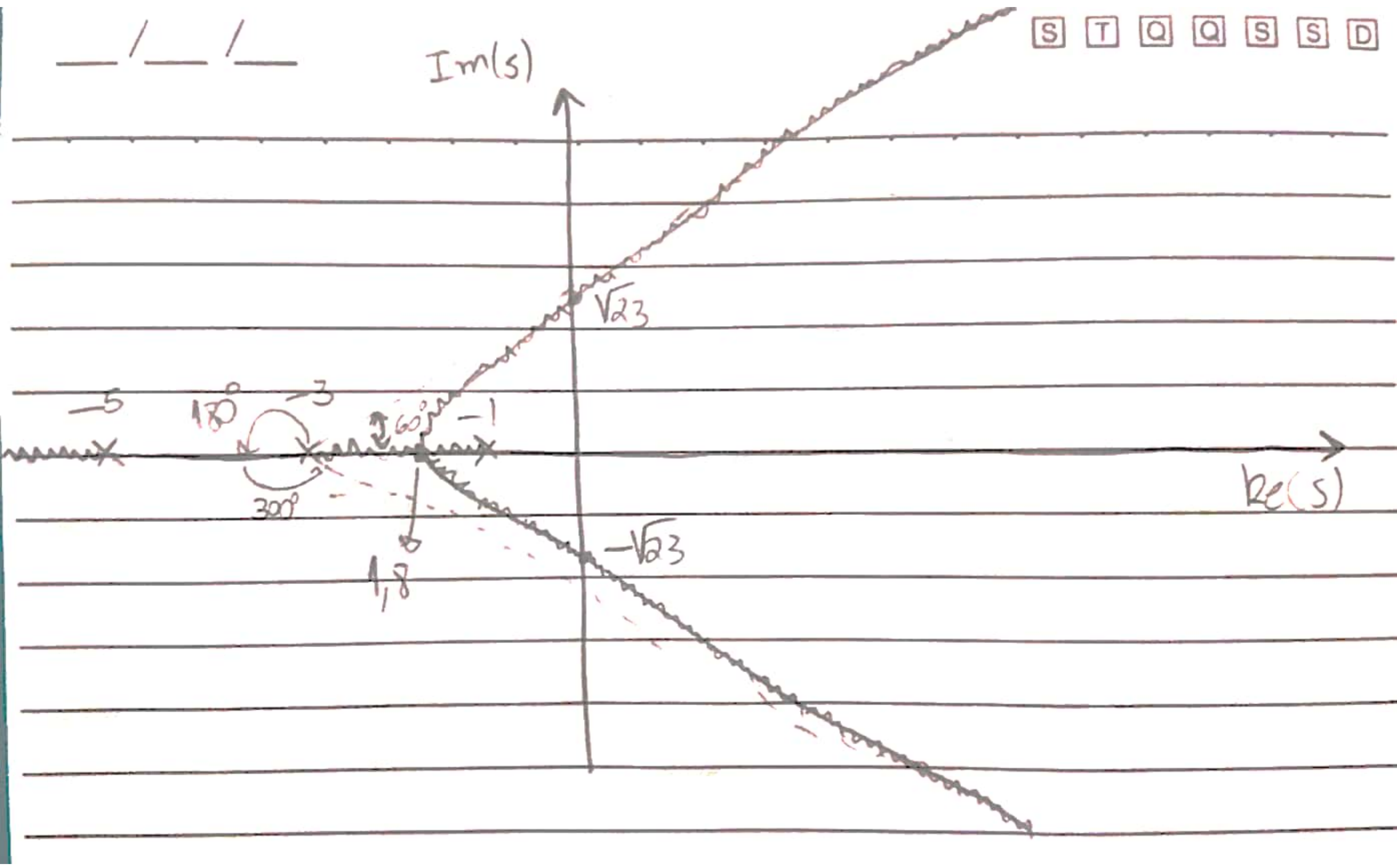
$\cancel{w=0}$ v $w = \sqrt{23}$ v $w = -\sqrt{23}$

$$\boxed{s = \pm\sqrt{23}\,j}$$

Limiar da estabilidade: polos no eixo imaginário

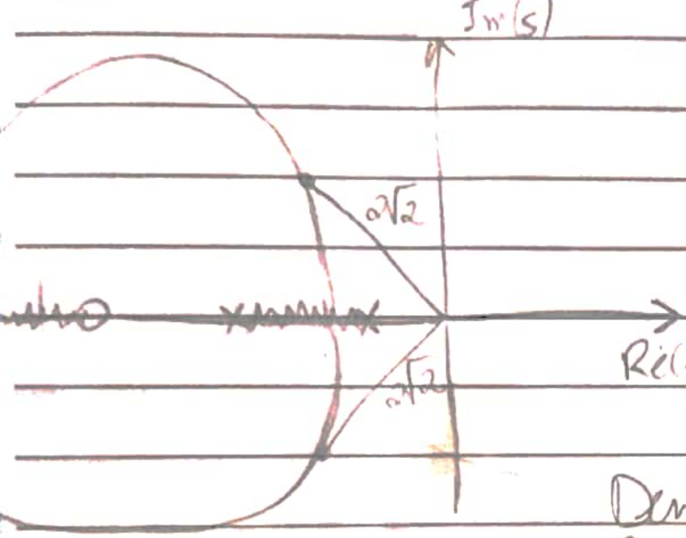
$$K = \left.\frac{9w^2 - 15}{10}\right|_{w = \pm\sqrt{23}}$$

$$\boxed{K = 19{,}2}$$



Im(s)
Re(s)
√23
−√23
−5
−3
−1
180°
300°
60°
1,8

Q2 $\omega_n \to$ módulo dos polos



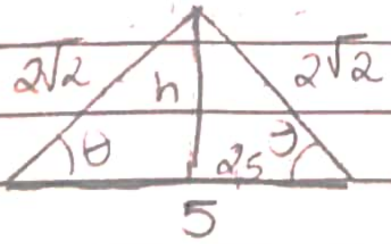

$$\frac{d}{ds}\left(\frac{1}{G(s)}\right) = 0 \rightarrow$$

$$(2s+4)(s+5) = s^2 + 4s + 3$$

$$s^2 + 10s + 17 = 0$$

$$s = -5 \pm \sqrt{8}$$

Deve-se, portanto, encontrar os pontos da circunferência com centro em $-5$ e raio $\sqrt{8}$ com módulo $2\sqrt{2}$.




$$h = \sqrt{8 - 6,25} = \frac{\sqrt{7}}{2}$$

$$s = -2,5 + \frac{\sqrt{7}}{2} j$$

$$G(s) = -0,5$$

$$\boxed{K = 2}$$

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Q3 (i) $|G(jw)|_{w\to 0} = 1$   $|G'(jw)|_{w\to\infty} = 0$

$\angle G(jw)_{w\to\infty} = -180^\circ$

(ii) $|G(jw)|_{w\to 0} = 1$   $|G(jw)|_{w\to\infty} = 0$

$\angle G(jw)_{w\to\infty} = -180^\circ$

(iii) $|G(jw)|_{w\to 0} = 1$   $|G(jw)|_{w\to\infty} = 0$

$\angle G(jw)_{w\to\infty} = -90^\circ$

(iv) $|G(jw)|_{w\to 0} = 1$   $|G'(jw)|_{w\to\infty} = 0$

$\angle G(jw)_{w\to\infty} = -270^\circ$

(v) $|G(jw)|_{w \to 0} = 10$ $|G(jw)|_{w \to \infty} = 0$

(vi) $|G(jw)|_{w \to 0} = \infty$ $|G(jw)|_{w \to \infty} = 0$

a $\to$ ii: variação no gráfico de magnitude ocorre apenas em $w=1$
b $\to$ iii: ângulo tende a $-90°$
c $\to$ iv: ângulo tende a $-270°$
d $\to$ i: variação na magnitude em $w=0{,}1$ e $w=10$
e $\to$ vi: $|G(jw)|$ diverge quando $w \to 0$
F $\to$ v: $20 \log |G(jw)| = 20$ quando $w \to 0$

S T Q Q S S D

Q4 $\left( (R(s)-X(s))K_p - X(s)\cdot s \right) K_v = X(s)\cdot(ms+b)s$

$(R(s)-X(s))K_pK_v = X(s)\left(ms^2+(b+K_v)s\right)$

$R(s)K_pK_v = X(s)\left(ms^2+(b+K_v)s+K_pK_v\right)$

$$G(s)=\frac{K_pK_v}{ms^2+(b+K_v)s+K_pK_v}$$

$$G(jw)=\frac{K_pK_v}{(K_pK_v-mw^2)+(b+K_v)w\,j}$$

$$\begin{cases} w=1 \Rightarrow 20\log|G(j)| = 2{,}7 & (1) \\ \left.\dfrac{d}{dw}|G(jw)|\right|_{w=1} = 0 & (2) \end{cases}$$

de (2):

$$\left.\frac{d}{dw}\left((K_pK_v-mw^2)^2+(b+K_v)^2w^2\right)\right|_{w=1}=0$$

$$\left.\cancel{2}(-2mw)(K_pK_v-mw^2)+\cancel{2}w(b+K_v)^2\right|_{w=1}=0$$

$$(b+K_v)^2=2m(K_pK_v-m) \quad *$$

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de ①:

$$1,86 = \frac{K_p^2 K_v^2}{(K_p K_v - m)^2 + (b+K_v)^2} \rightarrow *$$

$$1,86 = \frac{K_p^2 K_v^2}{(K_p K_v - m)(K_p K_v + m)}$$

$$0,86 (K_p K_v)^2 = 1,86\, m^2$$

$$K_p K_v = \frac{\alpha m}{\sqrt{\alpha^2 - 1}} = 1469,7$$

de *

$$\boxed{\begin{aligned} K_v &= 919,2 \\ K_p &= 1,60 \end{aligned}}$$

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$$G(jw) = \frac{50(jw+1)(jw-5)(jw-10)}{-(w^2+1)(w^2+25)(w^2+100)}$$

$$G(jw) = \frac{50\cdot(-(14w^2+50) + (w^2-35)w j)}{(w^2+1)(w^2+25)(w^2+100)}$$

$C_1 \rightsquigarrow G(jw) \quad w \geq 0$

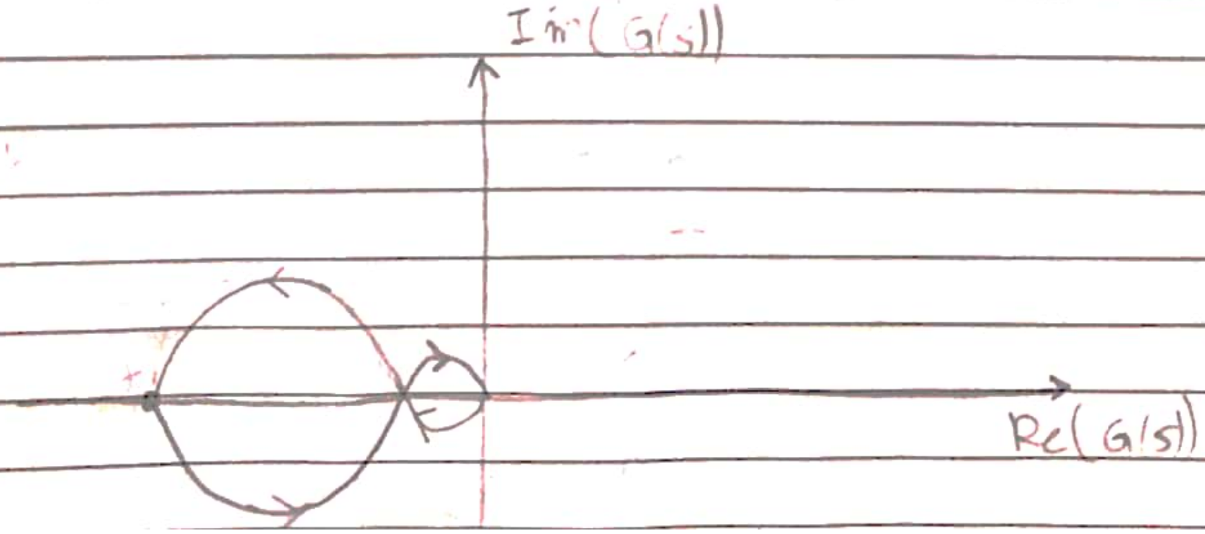
$C_2 \rightsquigarrow G(\infty) = 0$

$C_3 \rightsquigarrow G(-jw) \quad w \leq 0$

$w = 0 \rightsquigarrow G(0) = -1$

$0 < w < \sqrt{35} \rightsquigarrow 3^{\underline{o}}$ quadrante

$\sqrt{35} < w \rightsquigarrow 2^{\underline{o}}$ quadrante

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a parte imaginária de $G(j\omega)$ se anula em:

$(\omega^2 - 35)\,\omega = 0$

$\underline{\omega = 0}$ $\quad$ $\underline{\omega = \sqrt{35}}$ $\quad$ $\omega = -\sqrt{35}$

$\omega \geq 0$

$$\boxed{\begin{aligned} G(0) &= -1 \\ G(\sqrt{35}\,j) &= -0{,}1 \end{aligned}}$$

$0 < K < 1 \;\Rightarrow\; N = 0 \quad P = 1 \quad Z = 1$ instável

$-1 < K < 10 \;\Rightarrow\; N = -1 \quad P = 1 \quad Z = 0$ estável

$10 < K \;\Rightarrow\; N = 1 \quad P = 1 \quad Z = 2$ instável