CMC 12 - LISTA 4

RODRIGO ALVES DE ALMEIDA

1

$$F(s) = \frac{3s+5}{(s+1)^2(s+2)} = \frac{As+B}{(s+1)^2} + \frac{C}{s+2} \quad \Big\{ As^2 + 2As + Bs + 2B + Cs^2 + 2Cs + C = 3s+$$

$$\begin{cases} A + C = 0 \\ 2A + B + 2C = 3 \\ 2B - C = 5 \end{cases} \rightarrow \begin{aligned} A &= 1 \\ B &= 3 \\ C &= -1 \end{aligned} \qquad F(s) = \frac{s+3}{(s+1)^2} - \frac{1}{s+2}$$

$$F(s) \overset{s+1=u}{=} \frac{u+2}{u^2} - \frac{1}{s+2} = \left(\frac{1}{u} + \frac{2}{u^2}\right) - \frac{1}{s+2} =$$

$$= (1+2t)e^{-t} - e^{-2t}$$

Scanned with CamScanner

$= (1+2t)e^{-t} - e^{-2t}$

② $\ddot{x} + 6\dot{x} + 18x = 18 \quad , \quad t>0$

$\mathcal{L}\{x(t)\} = F(s)$

$\mathcal{L}\{\dot{x}(t)\} = sF(s) - x(0) = sF(s) - 1$

$\mathcal{L}\{\ddot{x}(t)\} = s^2F(s) - s - \dot{x}(0) = s^2F(s) - s - 3$

$\mathcal{L}\{\ddot{x} + 6\dot{x} + 18x\} = \mathcal{L}\{18\}$

$s^2F(s) - s - 3 + 6sF(s) - 6 + 18F(s) = \frac{18}{s}$

$(s^2 + 6s + 18)F(s) = \frac{18 + 9s + s^2}{s}$

$F(s) = \frac{s^2 + 9s + 18}{s(s^2+6s+18)} = \frac{1}{s} + \frac{3}{s^2+6s+18} = \frac{1}{s} + \frac{3}{(s+3)^2+9}$

$\mathcal{L}^{-1}\{F(s)\} = x(t) = 1 + e^{-3t}\,\mathrm{sen}(3t)$

Scanned with CamScanner

(3)

$$\begin{cases} V = iR + \dot{i}L + V_b \\ K_t i - b\omega = J\dot{\omega} \end{cases}$$

$$i = \frac{J\dot{\omega}}{K_t} + \frac{b\omega}{K_t}$$

$$\dot{i} = \frac{J}{K_t}\ddot{\omega} + \frac{b\dot{\omega}}{K_t}$$

$$\begin{cases} V = \frac{RJ}{K_t}\dot{\omega} + \frac{bR}{K_t}\omega + \frac{LJ\ddot{\omega}}{K_t} + \frac{Lb}{K_t}\dot{\omega} \\ \quad + K_t\,\omega \end{cases}$$

$$V(t) = \frac{LJ}{K_t}\dddot{\theta} + \left(\frac{RJ + Lb}{K_t}\right)\ddot{\theta} + \left(\frac{bR}{K_t} + K_t\right)\dot{\theta}$$

$$V(s) = \frac{LJ}{K_t}s^3\Theta(s) + \left(\frac{RJ+Lb}{K_t}\right)s^2\Theta(s) + \left(\frac{bR}{K_t} + K_t\right)s\,\Theta(s)$$

$$G(s) = \frac{\Theta(s)}{V(s)} = \frac{K_t}{LJs^3 + (RJ+Lb)s^2 + (Rb+K_t^2)s}$$

1) $m\ddot{x} + (b + K_v)\dot{x} + K_p K_v x = u$ $\quad u(t) = K_p K_v t$

$$ms^2 X(s) + (b + K_v) s X(s) + K_p K_v X(s) = U(s)$$

$$X(s) = \frac{U(s)}{(ms^2 + (b + K_v)s + K_p K_v)}$$