Quantum Eraser

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Abstract

Originally the quantum erasure experiment was developed to establish the validity of the complementarity principle. The principle was given by the Danish physicist Neils Bohr in which he conjectured that objects have certain pairs of complementary properties which cant be determined or observed simultaneously. In the case of the quantum eraser experiment, the complementarity properties are particle and wave nature, and therefore an object cannot behave as a particle and wave simultaneously. If the information providing the objects trajectory can be determined without significantly perturbing it, then the interference should disappear (in accordance with complementarity). But if that information is subsequently erased, then the interference should return.

In this paper, we are trying to figure out whether a quantum eraser can be simulated computationally without invoking the complementarity principle.

1 Introduction

One of the most salient consequences of Bohrs Complementarity Principle is that we can only say something about the behavior of the system (particle or wave) after the measurement has been carried out. Therefore, one needs to choose beforehand which phenomenon one wishes to observe and then devise the experimental settings to observe that phenomenon (particle or wave nature). The most famous experiment to demonstrate this principle is the single-photon Mach Zehnder Interferometer. In this experiment, the two mutually complementary pieces of knowledge are the path which the object took and the existence of interference fringes.

This characteristic led Wheeler to formulate his delayed-choice quantum eraser experiment. Wheeler speculations were directed towards knowing whether the object passing through the interferometer could know before entering the interferometer, which information we choose to obtain and then behave accordingly? To find a definite answer, Wheeler proposed that this choice must be made available to the object only after it had passed through the first beam splitter. Though the experiments confirmed the apparent ability of measurements made in the present to alter events occurring in the past, this is only true if we stick with a non-standard view of quantum mechanics. If the photon in flight is assumed to be in a superposition of states, i.e. it has the potentiality to manifest itself as a particle or wave, then there is no time paradox.

2 Experimental Procedure

The MachZehnder interferometer consists of two (non-polarizing) 50-50 beam splitters (BS) and two mirrors (M). One of the mirrors is mounted on a PZT so that phase difference $\delta\phi$,of the two paths can be varied.we introduce a pre-polariser (PP) between source and BS_1. On the upper path , we put a horizontal polarizer (P_1) between mirror and exitting beam spillter (BS_2), and similarly we put vertical polarizer (P_2) in the other path. At the ends of (BS_2) , we put two lens in the paths of lights between (BS_2) and metal screens to see the interference patterns. Now using a mono-chromatic laser , we irradiate on one beam splitter as shown. Now we observe the two metallic screens. And we will repeat the same but with 45° polarizers (P_3) between the lens and the beam splitter of both paths. Here, we again observe the two metallic screens.

3 Simulation

```
# Importing the libraries

from qutip import *
from math import *
from numpy import sqrt

# Preparing the initial state after applying polarizer P1

v = basis(2,1)
P1 = (Qobj([[1,1],[1,1]]).unit()*(sqrt(2)))
v = P1*v
print(v)
```

Quantum object: dims = [[2], [1]], shape = (2, 1), type = ket

$$\begin{pmatrix} 0.707 \\ 0.707 \end{pmatrix}$$

```
# Beam splitters considered identical, two polarizers P2 = P(pi/2) and P3 = P(theta applied)

angle = 0;
theta = radians(angle)

# Intensity can be varied by changing theta, if we take theta=90 we lose the which way info.
# For theta other than 90 and 270, the polarizers P2 and P3 are different, thus we have the which way path info and interference
# doesn't happen

POL = Qobj([[cos(theta)*cos(theta),cos(theta)*sin(theta)],[cos(theta)*sin(theta),sin(theta)*sin(theta)+1]])
a = sqrt(1+sin(theta)*sin(theta)+2*cos(theta)*sin(theta)) # Normalization factor

# Polarizers act as detectors and destroy the interference giving output state
v=POL*v/a
print(v)
```

Quantum object: dims = [[2], [1]], shape = (2, 1), type = ket

$$\begin{pmatrix} 0.707 \\ 0.707 \end{pmatrix}$$

```
# Inserting another polarizer brings back the interference

angle2 = 0
phi = radians(angle2)
P4 = Qobj([[cos(phi)*cos(phi),cos(phi)*sin(phi)],[cos(phi)*sin(phi),sin(phi)*sin(phi)]]).unit()

b = (cos(theta)*cos(theta)+cos(theta)*sin(theta))/(a*sqrt(2)) # Normalization factor

# Only detector 1 clicks
if(b!=0): v=P4*v/b
print(v)

```

```
# Although Polarizer 2 and 3 determined which way info, and thus forced photon to one path,inserting another polarizer brought
# back its superposition characteristic and thus interference. Photon now interferes with itself and goes only to detector 1
```

Quantum object: dims = [[2], [1]], shape = (2, 1), type = ket
$\begin{pmatrix} 1.0 \\ 0.0 \end{pmatrix}$

4 Classical interpretation

As we have seen that in the first part of the experiment (before introducing P_3) , we see that there is no interference pattern on both the screens. This is due to the reason that according to classical physics orthogonal waves do not interfere. Since we used horizontal and vertical polarizers in the upper and lower paths respectively , the beams passing through BS_2 are orthogonal , so hence we can explain the result. After introducing the P_3 , we can observe the interference pattern only in the lower path. This is because the light coming from two paths are not orthogonal any more as their polarisation changed due to P_3, so now they interfere and form interference pattern in the lower detector.

5 Quantum interpretation

When we try to irradiate the MZI with photon by photon, before introducing P_3 , we can easily find the path through which the photon travelled, as each path has different polarisation. So we have "which way" information. By complementarity principle, in this experiment it behaves like a particle. The mathematics can be shown as follows:

In the second experiment , after the BS_2 , as we have placed 45° polarizers . This makes the two orthogonal polarised to 45° polarization. Now we cannot defferentiate the paths. So the "which way" information is *erased*. (Note that we did no re-write the past ; Our experiment does not imply retrocausality.) As it is a single photon which is travelling and as we observe interference on a detector and no photon detection on another (after sending many photons one-by-one) , we can say that each photon interferes with itself. So by complementarity principle, we can cannot detect particle nature in this experiment. The mathematics for the second experiment is as follows:

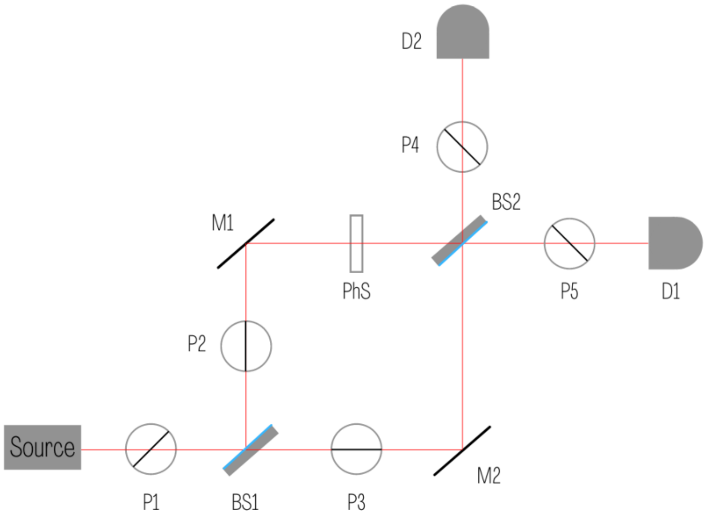
We use Jones Calculus, introduced by Robert Clark Jones in 1941, for the analytical treatment of the problem. The Electric fields are written as column vectors and called Jones vector. A Jones vector represents the amplitude and the phase of the various components of electric field of the electromagnetic radiation.

Jones vectors are represented as:

Horizontally Polarized: $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	Vertically Polarized: $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
Diagonally Polarized: $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	Anti-diagonally Polarized: $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

Various optical instruments such as Polarizers, Mirrors, Phase shifters and Beam splitters are represented by two-dimensional matrices. Below is the schematic of the setup to observe Quantum erasure followed by the matrix representation of each of the components.

Linear Polarizer (transmission axis at an angle θ with the horizontal: $\begin{pmatrix} cos^2(\theta) & cos(\theta)sin(\theta) \\ cos(\theta)sin(\theta) & sin^2(\theta) \end{pmatrix}$



Let us take a vertically polarised light of vector v and pass it through diagonally polarized light. i.e.

$$v = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{1}$$

After passing it through the diagonal polarizer v becomes as follows :

$$v = \begin{pmatrix} cos^2(45°) & cos(45°)sin(45°) \\ cos(45°)sin(45°) & sin^2(45°) \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2}$$

After normalizing , we get v=$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
In terms of intensity of electric field we can write:
$\Psi = \frac{E_0}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
The beam splitter BS_1 splits (reflects and transmits light) into equal intersities.The effect of an ideal beam splitter is described by multiplying the incident spinor by the amplitude reflection and transmission factors :

$$r = \frac{i}{\sqrt{2}} \tag{3}$$

$$t = \frac{1}{\sqrt{2}} \tag{4}$$

The horizontal and vertical polarizers are described by the matrices respectively:

$$\mathrm{M}_H = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \Bigg| \ \mathrm{M}_V = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

The polarizers P_4 and P_5 act as erasing polarisers , let the they are aligned with angle α with respect to horizontal , then the corresponding matrix will be :

$\mathrm{M}_\alpha = \begin{pmatrix} cos^2(\alpha) & cos(\alpha)sin(\alpha) \\ cos(\alpha)sin(\alpha) & sin^2(\alpha) \end{pmatrix}$ (5)

We can write Ψ as follows:

$|\Psi\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)\,|d\rangle$

where $|H\rangle$ and $|V\rangle$ horizontal and vertical polarisation and $|d\rangle$ is the direction of propagation of the photon.The operator of beam spiltter BS_1 is as follows:

$P_{BS1}= r\,|1\rangle\,\langle d| + t\,|2\rangle\,\langle d|$
where $|1\rangle$ and $|2\rangle$ are the directions of reflection and transmission from BS_1. Now, the photon passes through the polarizers P_2 and P_3. So the operator for this polarizer will be:

$P_{V/H}= |V\rangle\,\langle V|1\rangle\,\langle 1| + |H\rangle\,\langle H|2\rangle\,\langle 2|$

Now the photon passes through the phase shifter PS and changes phase by $\delta\phi$, so the operator will be as follows:

$P_{PS}= e^{i\delta\phi}\,|1\rangle\,\langle 1| + |2\rangle\,\langle 2|$

Now finally, we have beam splitter BS_2. It acts as:
$P_{BS2}= r\,|u\rangle\,\langle 1| + t\,|l\rangle\,\langle 1| + r\,|l\rangle\,\langle 2| + r\,|u\rangle\,\langle 2|$

where u and v are the directions of transmission and reflection from BS_2.
If we want to measure the part of wave function acting on the detector D_2, i.e. in the direction of u and that of detector D1. The appropriate operators are

$P_u=|u\rangle\,\langle u|$
$P_l=|l\rangle\,\langle l|$

The operator for the erasure which is at angle α with horizontal is P_{u_α} .
The only states that can pass through the eraser along u is :
$|\psi_{u_\alpha}\rangle= cos(\alpha)\;|H\rangle\,|u\rangle + sin(\alpha)\;|V\rangle\,|u\rangle$

Therefore, the operator such that the wave function passing through the eraser will be :
$P_{u_\alpha} = |\psi_{u_\alpha}\rangle\,\langle\psi_{u_\alpha}|$

If there is no eraser is present, the wave function reaching the detector D_2 will be:

$$|\psi_{u1}\rangle = P_u P_{BS2} P_{PS} P_{V/H} P_{BS1}\,|\Psi\rangle = \frac{1}{2\sqrt{2}}(-\,|V\rangle\,e^{i\delta\phi} + |H\rangle)\,|u\rangle \tag{6}$$

and that of D_1 will be:

$$|\psi_{l1}\rangle = P_l P_{BS2} P_{PS} P_{V/H} P_{BS1}\,|\Psi\rangle = \frac{i}{2\sqrt{2}}(|V\rangle\,e^{i\delta\phi} + |H\rangle)\,|l\rangle \tag{7}$$

The probabilities of photon approaching D_2 and D_1 are w_{u1} and w_{l1}. Therefore,

$$w_{u1} = \langle\psi_{u1}|\psi_{u1}\rangle = \frac{1}{4} \tag{8}$$

$$w_{l1} = \langle\psi_{l1}|\psi_{l1}\rangle = \frac{1}{4} \tag{9}$$

When the polarizers P_4 and P_5 are introduced, the wave functions along D_2 will be

$$|\psi_{u2}\rangle = P_{u_\alpha} P_u P_{BS2} P_{PS} P_{V/H} P_{BS1} |\Psi\rangle = \frac{1}{2\sqrt{2}}[-sin^2(\alpha)|V\rangle e^{i\delta\phi} + cos^2(\alpha)|H\rangle + sin(\alpha)cos(\alpha)(|V\rangle - e^{i\delta\phi}|H\rangle)] \tag{10}$$

The probabilities of photon approaching D_2 is :

$$w_{u2} = \langle\psi_{u2}|\psi_{u2}\rangle = \frac{1}{8}(1 + sin(2\alpha)cos(\delta\phi)) \tag{11}$$

Similarly we get the probability for D_1 that :

$$w_{l2} = \frac{1}{8}(1 - sin(2\alpha)cos(\delta\phi)) \tag{12}$$

We know that the after firing infinite photons , the intensity on the screens will be proportional to the probability of a single photon to reach the corresponding detector.Clearly we can see that when eraser is not introduced, the intensities on the detectors are same , thus the light did not participate in interference. But after we introduced the polarizers we can see the difference in the intensities observed on the two detectors.

6 Conclusions

Now that we have successfully constructed a simulation model for quantum eraser which agrees with actual data generated from physical experiments, we can return to our original question, "whether a quantum eraser can be simulated computationally without using the complementarity principle?" The answer is yes.

The existence of quantum eraser easily becomes apparent when one tries to simulate it computationally, as it naturally springs out of the linear algebra formalism, we have used to simulate it.

7 On complementarity and causality

Complementarity dictates that if a photon manifests itself as though it had come by two indistinguishable paths, then it must have entered the apparatus as a wave. If the experimental apparatus is changed while the photon is in midflight, then the photon should reverse its original "decision" as to whether to be a wave or a particle. To a gullible observer, it might seem that future events are somehow altering events in past, i.e. effect seems to precede cause, but that's obviously not true.

References

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[4] Event-by-event simulation of a quantum eraser experiment

[5] Experimental analysis of the quantum complementarity principle