Dynamics of Uncertain and Conflicting Opinions in Social Networks

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Abstract*— In this paper, we study the evolution of opinions where people are not sure of their own opinions and/or their opinions may be conflicting to others' in social networks. We model two types of agents so-called *informed agents* (IAs) and *uninformed agents* (UIAs). The IAs have a strong opinion agreeing or disagreeing toward a proposition without being influenced by other agents' opinions and have a high confidence (low uncertainty) toward its own opinion. The UIAs have a weak opinion without either agreeing or disagreeing toward a proposition and lack confidence with a high uncertainty. Based on *subjective logic*, we consider a binomial opinion to deal with an opinion with a degree of uncertainty. We develop two types of trust attitudes for agents to update their opinions upon their interactions with other agents: uncertainty-based trust (UT) and similarity-based trust (ST). In the UT, a UIA updates its opinion based on an interacting agent's uncertainty toward a proposition. In the ST, a UIA updates its opinion based on the degree of similarity between its own opinion and the interacting agent's opinion toward the proposition. Our results show that more IAs slow down the convergence of the opinions under the UT while they can quickly lead to opinion convergence under the ST. In addition, the ST leads uncertain opinions to two extremes, either 0 or 1, if consensus exists. On the other hand, the UT can make opinions converge to a certain point between two extreme opinions although the converged point is significantly affected by the dominant agents' opinions. Furthermore, we observe that under the UT, more IAs with a high centrality increase dissonance of opinions, while more IAs with a low centrality offer better chances for opinion consensus in both the UT and the ST.

***Index Terms*— Consensus, opinion dynamics, similarity, subjective logic (SL), uncertainty.**

I. Introduction

HUMANS learn from their social networks where social learning can also come by receiving information from public/social media, observing others' actions, and/or directly interacting with others [1]. A person's attitude affects whether to accept opinions and how to interact with others. For example, stubborn people tend to influence others without accepting others' opinions, while open-minded people are influenced by others' opinions as well as influence them. In addition, people are more likely to interact with those who have like-mindedness or an expertise in an area of their interest.

A proposition, as an opinion or a particular issue, is often discussed with two extremes, "pros versus cons" or "agree versus disagree." For example, many common social problems have been discussed as typical controversial issues, such as abortion, health care, same sex marriage, and so forth. However, humans have cognitive limitations in investigating all uncertain aspects of a proposition based on given evidence. Thus, we cannot have a full sense of confidence in any given proposition; our lack of confidence or a limited level of certainty due to our cognitive limitation naturally leads us to make heuristic decisions in forming our own opinions.

Although humans' sense of confidence can increase as more evidence is available or depending on a person's cognitive capability or personality [27], an individual may not be perfectly sure about a proposition in most situations due to the inherent uncertainty existing in this world. Thus, humans may use heuristic ways to form their opinions, such as biases, attitudes, or logic. In this paper, we are interested in how a different trust attitude in accepting others' opinions can affect propagation, formation, and convergence/divergence of opinions after a sufficient number of interactions among entities. An entity can represent a person, an organization, or a community in social networks. This paper simply assumes that an entity, or an agent, represents a person who interacts with other people to influence others' opinions or update his or her opinion based on the opinions of entities the person interacts with.

Couzin *et al*. [10], [11] and West and Bergstrom [43] have studied how increasing or decreasing the number of unbiased or uninformed agents (UIAs) (i.e., agents who are not given any exact information to perform a task) can promote consensus. They found that consensus can be reached faster when the majority of agents are not informed with the exact information on where to move for finding food sources or migration routes. Similarly, we model informed agents (IAs) and UIAs in order to examine the effect of the number of IAs on the convergence or divergence of opinions in social networks. However, unlike [10], [11], and [43], an opinion in this paper is associated with a degree of uncertainty which can play a critical role in an agent's decision to update its opinion.

In this paper, we devise the following two opinion update models that agents can use upon interactions with other agents: *uncertainty-based trust* (UT) and *similarity-based trust* (ST). In the UT, assuming agents tend to trust more certain opinions, agents update their opinions based on the degree of uncertainty (or certainty) of interacting agents' opinions. In the ST, agents are assumed to trust more similar opinions and thus update their opinions based on the degree of similarity between their own opinions and interacting agents' opinions.

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This paper has the following key contributions.

1) *Subjective logic* (SL) operators [23] are used to formulate uncertain opinions and investigate their evolution upon more interactions over time. By using the formulation of a binomial opinion in the SL, we devised two opinion models based on trust attitudes that an agent can use to update its opinion, which are UT and ST. These two opinion models are studied in order to examine their effect on the convergence or divergence of uncertain opinions and the speed of reaching consensus. To the best of our knowledge, this paper is the first to study the evolution of "uncertain" opinions where an agent's binomial opinion is formulated based on the SL and its attitudes of updating opinions are developed by considering the dimension of uncertainty in the opinions. The preliminary results are published in our prior work in [9]. But in this journal version paper, we significantly revised the ST model in order to investigate the effect of conflicting opinions in updating opinions based on the trust revision operator [26]. This operator is proposed to estimate one's trust based on conflicting opinions from two agents where a third party agent evaluates their opinions as supporting evidence to update its opinions. Unlike the original concept of the trust revision operator [26], we refine the trust revision operator that can offer agents' opinion updates in a dyadic relationship where each agent evaluates the interacting agent's opinion in order to update its own opinion in three-fold: 1) the distance between two agents' opinions; 2) the trust estimation toward each other based on the degree of uncertainty; and 3) the degree of conflicts in two opinions in terms of uncertainty. We discuss more details on the ST model in Section III-B2.
2) By modeling the types of agents in [10] based on the opinions formulated in SL, we study the impact of two different agent types, IAs and UIAs. An IA is an agent who has a high confidence (low uncertainty) in its opinion. Thus, the IA influences others' opinions while not being influenced by them, which is often called a *stubborn* agent. A UIA is an agent who has a low confidence (high uncertainty) and can change its opinion based on certain criteria, which is often called an *open-minded* agent.
3) This paper provides the validation of the proposed opinion update models, UT and ST, via both mathematical convergence analysis (Section IV) and simulation experiments (Section V). In our prior work [9], we only provided the mathematical convergence analysis of UT. In this journal version, using the revised version of the proposed ST, we provided a convergence analysis of the ST model in Section IV-B, which proves that the opinion convergence of ST is slower than that of UT. This mathematical convergence analyses of UT and ST are also validated based on our simulation experiments in Section V.
4) We evaluate the performance of two opinion models in *Erdős–Rényi* (ER), *Barabási–Albert* (BA), and real Facebook networks to validate their effect under various social network environments. In our prior work [9], we only validated our proposed opinion update models based on an ER network [17]. In this paper, we added experimental results under a BA network [3] and a Facebook network [29] to examine the effect of network topologies on the convergence/divergence of uncertain opinions. In addition, we investigate how the influence of IAs based on their network centrality measures (i.e., degree, betweenness, or pagerank) can affect the convergence or divergence of opinions among UIAs.

The rest of this paper is organized as follows. Section II gives an overview of related work in terms of opinion models and SL. Section III describes models of opinion formation, interactions, and agent types. Section IV provides the convergence analysis of the two opinion models, the UT and the ST. Section V describes the experimental setting used for the simulation experiments and shows the results with the discussions of the observed trends. Finally, Section VI summarizes key findings and suggests future work directions.

II. Related Work

We discuss some existing studies related to opinion models and the SL.

A. Opinion Models

Opinion spreading or diffusion has been extensively studied in the literature. A popular information diffusion model is the *random walk model* [16], [21]. *Random walk processes* have been applied in stochastic continuous-time networks [21]. Another popular model is *epidemic spreading*. If an infected entity recovers or infects other neighboring entities, the number of infected entities decreases or increases correspondingly. Two well-known classes of the model using the *disease spreading model* are the *susceptible–infected–removed (or recovered)* (SIR) and the *susceptible–infected–susceptible* (SIS) model [5]. The SIR/SIS model has been variously extended by many studies [34], [41].

Valdez *et al.* [41] use the SIR model to propose their intermittent social distancing strategy and examine their strategy on disseminating epidemics in adaptive complex networks. Mieghem and van de Bovenkamp [34] studied how non-Markovian infection time can dramatically change the SIS epidemic threshold in networks. Zhao *et al*. [44] extended the SIR model by incorporating a "forgetting" factor to investigate the rumor propagation process based on the degree, infection rate, and forgetting rate. Zhao *et al*. [45], [46] developed a susceptible–infected–hibernator–removed model by adding a direct link from ignorants (i.e., uninfected nodes by a rumor) to stiflers (i.e., stifling nodes against a rumor), which represent a new type of people, hibernators.

Rumor spreading or *gossip* has also been extensively examined in the areas of computer science, information theory [6], and sociology [7]. Daley and Kendall [12] proposed a standard rumor model, called the DK model, similar to the epidemic spreading model. The DK model has been extensively varied to study rumor spreading mechanisms [14], [33], [37]. Demers *et al*. [14] have extended the DK model to maintain replicated database. The DK model has been explored such as the Maki–Thompson model [33] as a variant. In addition, sociologists have studied rumor spreading using the DK model [37].

Many researchers have studied the characteristics of agents that lead to opinion consensus or divergence in social networks. Verma *et al*. [42] studied the impact of zealotry in the *naming game model*. Ghaderi and Srikant [18] examined how consensus emerges in terms of agents' type (e.g., stubborn versus open-minded), network structure, and initial opinions. Deffuant *et al.* [13] proposed a simple and intuitive interaction model, called the *Deffuant–Weisbuch* (DW) model, in which interacting agents update their opinions only if the *distance* between opinions is below a threshold. Variants of the DW model [13] have been extensively studied [2], [3], [17], [30], [31].

Alaali *et al*. [2] extended the DW model using an *Euclidean distance* metric to measure the distance in opinions and test their model on two well-known network typologies, the BA network [3] and the ER network [17]. Li *et al.* [30], [31] proposed an interaction model as a variant of the DW model using trust that may exist among agents with similar opinions, called *like-minded* agents, assuming that the link-minded may exchange their opinions with each other more actively.

Diffusion models are the methods by which members of a society adopt new behaviors. *Linear threshold model* [20], [28], [32] considers the influence of collective behavior on a person's adoption behavior. *An independent cascading model* derives from interaction processes in particle systems [15]. Based on this particle interaction process, Kempe *et al.* [28] explained the diffusion process in which a node υ is activated by its neighbor ω based on a success probability, $p_{\omega,\upsilon}$, at a discrete time with an independent activation order of its neighbors ω's. Goldenberg *et al.* [19] used this cascading model in explaining the speed of accepting a new product in the area of marketing. Axelrod [4] proposed an agent-based adaptive model to explain the effect of convergent social influence. Similar people tend to be aggregated by forming a community, and a person is more likely to be influenced by his or her neighbors. In [4], culture is investigated as an individual's attribute.

Moscovici and Zavalloni [35] studied the polarization effect of group interactions on reaching a consensus. Based on their empirical study, the polarization effect is more pronounced when the group commits itself to a given opinion position; and individuals often adopt the group opinions as their personal opinions. This paper is partly aligned with some similar discussions which can be found in our ST attitude that is used for an agent to update its opinion. However, this paper focuses more on investigating the effect of varying IAs (i.e., zealotry) in terms of the consensus of opinions where each individual's opinion considers uncertainty, representing lack of the individual's confidence toward his or her own opinion.

Unlike the existing works discussed above, this paper deals with uncertain opinions which are commonly observed in reality but have not been much addressed adequately in the literature. In addition, while similarity-based opinion consensus among the like-minded is well studied, a little work has addressed how uncertainty (or certainty) plays a role in accepting others' opinions when an agent adjusts its opinion.

B. Subjective Logic

Opinions are basically *subjective* as humans may not have a full sense of confidence in their opinions. A belief theory has been used to represent the aspect of "subjectivity" such as in *Dempster–Shafer Theory* (DST) [39] and *Transferrable Belief Model* [40]. In the same context, Jøsang [22], [23] proposed an opinion update model called *Subjective Logic* (SL) in which a binomial opinion has three dimensions, *belief*, *disbelief*, and *uncertainty*. The SL has been applied in many domains, including trust network analysis, security services, and information fusion [24]. We use the SL as an opinion update model to reflect the aspect of uncertainty in a subjective opinion and its impact on opinion dynamics and/or consensus. We discuss how the SL is applied in Section III.

III. Modeling of Opinions and Agents

In this section, we discuss how uncertain opinions and their opinion update models based on different trust attitudes are formulated using the SL. In addition, we discuss how different types of agents are modeled in this paper.

A. Opinion Formation

The SL has been proposed as a technique to represent opinions and to combine multiple opinions based on belief theory (i.e., DST [39]) to deal with continuous uncertainty and belief parameters. The SL provides specific operators for consensus and recommendation to form and update opinions. In the SL, a binomial *opinion* is represented by three dimensions: *belief* (b), *disbelief* (d), and *uncertainty* (u) [22]. A single binomial opinion on a given proposition is represented as

$$b, d, u \in [0, 1]^3, \quad b + d + u = 1. \tag{1}$$

We consider a binomial opinion representing two strong extremes in [0, 1], such as *agree* or *disagree*. For example, we may agree or disagree on abortion rights. The agent may have degrees of belief and/or disbelief (i.e., agree or disagree) in an opinion with some degree of uncertainty. Thus, agent i's opinion on proposition A, denoted as w_i^A, can be represented as

$$w_i^A = \left(b_i^A, d_i^A, u_i^A\right). \tag{2}$$

Different people can absorb other opinions and/or observations based on different factors. For example, some people are more likely to be persuaded than others. In addition, people tend to absorb others' opinions more readily either when they are like-minded in general, when their opinions are similar, or when the persuading agent has a very strong belief with a high confidence. We are interested in understanding how these attitudes can affect the dynamics of opinion formation and convergence (or divergence).

An agent collects evidence which leads to an opinion. To map evidence to opinion spaces, agent i must map the evidence for proposition A into the three dimensions of the opinion. From now on, we will omit the superscript A for notational simplicity, assuming that we deal with a particular proposition in all the cases we discuss. Thus, $\omega = (b, d, u)$ is given by

$$b = \frac{r}{r+s+W}, \; d = \frac{s}{r+s+W}, \; u = \frac{W}{r+s+W} \quad (3)$$

where r is the amount of positive evidence (i.e., agree) and s is the amount of negative evidence (i.e., disagree) for a particular proposition. For simplicity, we dropped the subscript i denoting the agent in the above equation. When $W = 0$, b is a natural estimate of the fractional evidence in favor of the proposition. W indicates the amount of uncertainty which can be affected by the agent's propensity (e.g., more likely to trust or not) or inherent errors that can be introduced by the environment itself.

B. Opinion Update Models

An agent can form an opinion based on direct interactions with other agents, which is called *communication learning* [1]. In addition, the agent can update its opinion after observing or being exposed to public media or publicly available information (e.g., reading articles, listening to radio/TV discussions, and social media like Facebook or Twitter), which is called *observational learning* [1]. To reflect the influence of others' opinions on the agent's opinion, we use SL's *consensus* operator [22]. The consensus operator offers a solution on how two agents can agree with a certain opinion based on the position of their current opinions. Agent i's opinion after interacting with agent j is updated by considering its own belief weighted by j's uncertainty plus j's belief weighted by its own uncertainty in its opinion. Similarly, i's disbelief is computed based on its own disbelief weighted by j's uncertainty plus j's disbelief weighted by its own uncertainty in its opinion. i's uncertainty after interacting with j reduces to the product of the uncertainty values of the both parties in their opinions. All opinion updates are performed as long as both parties' uncertainty levels do not reach zero, representing a full sense of confidence. The updated opinion of agent i after interacting with agent j, denoted by $\omega_{i\oplus j} = \omega_i \oplus \omega_j = (b_{i\oplus j}, d_{i\oplus j}, u_{i\oplus j})$, is given by

$$b_{i\oplus j} = \frac{b_i u_j + b_j u_i}{\beta}, \; d_{i\oplus j} = \frac{d_i u_j + d_j u_i}{\beta}, \; u_{i\oplus j} = \frac{u_i u_j}{\beta} \quad (4)$$

where $\beta = u_i + u_j - u_i u_j$, and $\beta \neq 0$ is assumed. This condition implies that i and j can update their opinions as long as they are not 100% sure about their opinions. (4) supports the natural phenomenon about how human agents can reduce their perceived uncertainty as they process more information by interacting with others. In the SL, uncertainty refers to ignorance about "the likelihood of possible events" due to the lack of information [25]. Although conflicting evidence is received, either belief or disbelief increases while uncertainty will decrease as far as any evidence is received. Since i and j may be uncertain toward a given proposition due to the lack of the same evidence, β (i.e., overall uncertainty level from both i and j) is computed based on the union of two probabilities where each probability (i.e., uncertainty) is independent to each other. Since $0 \leq u \leq 1$, we see that $\beta = 0$ only if $(u_i, u_j) = (0, 0)$, corresponding to two completely confident agents. In our model, an agent with $u = 0$ no longer updates its own opinion. Note that the left-hand side indicates an opinion with three dimensions at interaction index $(k + 1)$, while the right-hand side refers to the opinion at interaction index k. We omit interaction indices k and $(k+1)$ for notational simplicity in the above.

The above update rule is mechanistic; in reality, we often observe that an agent uses its own opinion to judge another's opinions before accepting them to update its own opinions as shown in (4). To reflect this, we use the *discounting* rule [22] as follows. Agent i's opinion toward j's advice is represented as $\tilde{\omega}_i^j = (\tilde{b}_i^j, \tilde{d}_i^j, \tilde{u}_i^j)$, where $\tilde{b}_i^j$ can be interpreted as trust of agent i in agent j. Agent j's advice on a particular proposition is represented as $\omega_j = (b_j, d_j, u_j)$. Agent i's opinion after this trust-weighted interaction with agent j is given by $\omega_{i\otimes j} = \tilde{\omega}_i^j \otimes \omega_j = (b_{i\otimes j}, d_{i\otimes j}, u_{i\otimes j})$ as

$$\begin{aligned} b_{i\otimes j} &= \tilde{b}_i^j b_j \\ d_{i\otimes j} &= \tilde{b}_i^j d_j \\ u_{i\otimes j} &= 1 - (\tilde{b}_i^j b_j + \tilde{b}_i^j d_j) \\ &= \tilde{d}_i^j + \tilde{u}_i^j + \tilde{b}_i^j u_j \end{aligned} \quad (5)$$

where $u_{i\otimes j}$ can be simply derived as above based on $b + d + u = 1$. Agent j performs the same process to update its opinion. In (5), all of the elements are at interaction index k; we have omitted this index for simplicity. $b_{i\otimes j}$ and $d_{i\otimes j}$ are simply computed by considering i's trust in j's belief or disbelief, respectively. $u_{i\otimes j}$ is computed by subtracting a certain opinion (i.e., belief and disbelief of j weighted by i's trust in j from 1), as shown in the above equation.

Agent i's trust in agent j, $\tilde{\omega}_i^j = (\tilde{b}_i^j, \tilde{d}_i^j, \tilde{u}_i^j)$, can be updated based on two criteria: uncertainty and similarity. We explain the two opinion models as below.

1) Uncertainty-Based Trust: Agent i's trust in agent j may be based on *uncertainty* on a given opinion: the more confident agent j is that the more trusting agent i is toward agent j. The UT of agent i in agent j, $\tilde{\omega}_i^j = (\tilde{b}_i^j, \tilde{d}_i^j, \tilde{u}_i^j)$, is given by

$$\begin{aligned} \tilde{b}_i^j &= (1 - u_i)(1 - u_j) \\ &= 1 - \beta \\ \tilde{d}_i^j &= u_j - u_i u_j \\ &= \beta - u_i \\ \tilde{u}_i^j &= u_i \\ &= \beta - \tilde{d}_i^j. \end{aligned} \quad (6)$$

In the above equation, trust is estimated based on the level of uncertainty that i has toward j. i's belief toward how certain j's opinion, $\tilde{b}_i^j$, is computed based on i's certainty toward j's certainty, leading to $(1 - u_i)(1 - u_j)$. i's disbelief toward how certain j's opinion, $\tilde{d}_i^j$, is captured only based on j's uncertainty that i is not aware of. i's uncertainty toward j's uncertainty, $\tilde{u}_i^j$, remains as i's uncertainty as i is certain about j's opinion based on the sum of its belief and disbelief toward j (i.e., $\tilde{b}_i^j + \tilde{d}_i^j$).

In the UT, the trust relationship between agent i and agent j may be *asymmetric* as agents i and j may hold different (b, d, u) values.

2) Similarity-Based Trust: People update their opinions as the result of interacting with those who have similar opinions to themselves. Based on this assumption, another approach to estimate trust can be based on *dissimilarity* (or similarity) in opinions. In estimating similarity of opinions, how to deal with conflicting opinions is a major problem to solve. In this paper, we leverage Jøsang's trust revision operator [26] to deal with conflicting opinions. In [26], the dissimilarity (i.e., the degree of dissimilarity agent i perceives toward agent j), is based on the following three components: 1) a projected distance between the two opinions of agents i and j (PD^i_j); 2) a trust revision factor agent i considers for agent j (RF^i_j); and 3) a degree of conflict in the two opinions, C_{ij}.

The projected opinion distance between agents i and j, PD^i_j, is based on the distances in both belief and disbelief of the two agents by

$$\mathrm{PD}^i_j = \frac{|b_i - b_j| + |d_i - d_j|}{2}. \tag{7}$$

This value is symmetric in that $\mathrm{PD}^i_j = \mathrm{PD}^j_i$. All of the elements are at interaction index k.

Trust revision factor, RF^i_j, is determined based on the uncertainty difference (UD) [26] as

$$\begin{aligned} \mathrm{RF}^i_j &= \frac{1 - \frac{u_i - u_j}{u_i + u_j}}{2} = \frac{u_j}{u_i + u_j} = 1 - \mathrm{RF}^j_i \\ \mathrm{RF}^j_i &= \frac{1 - \frac{u_j - u_i}{u_i + u_j}}{2} = \frac{u_i}{u_i + u_j} = 1 - \mathrm{RF}^i_j. \end{aligned} \tag{8}$$

The above trust revision, RF^i_j or RF^j_i, is computed based on the UD, $(u_i - u_j)/(u_i + u_j)$ or $(u_j - u_i)/(u_i + u_j)$, where $UD \in [-1, 1]$. $UD = 0$ means that the two opinions have equal uncertainty, while $UD = 1$ or $UD = -1$ implies that one party's opinion is infinitely more uncertain than the other's [26]. RF^i_j or RF^j_i are represented as above in order to make it ranged in $[0, 1]$ where $\mathrm{RF}^i_j + \mathrm{RF}^j_i = 1$. As seen above, the trust revision factor is asymmetric, $\mathrm{RF}^i_j \neq \mathrm{RF}^j_i$. We consider this trust revision factor in order for i to consider j's trust when considering j's opinion while C_{ij} is used for i to consider similarity of their opinions in the level of uncertainty.

C_{ij} is the conjunctive conflict between agents i and j and is given by

$$C_{ij} = (1 - u_i)(1 - u_j) = 1 - \beta. \tag{9}$$

Recall that $\beta = u_i + u_j - u_i u_j$ in (4). C_{ij} implies the degree that both i and j are sure about a given opinion. Also notice that C_{ij} is the same as $\tilde{b}^j_i$ in (6), because C_{ij} represents the confidence level over an opinion an agent holds. The unique feature of an opinion in the SL is to explicitly deal with uncertainty. Thus, we consider uncertainty in estimating dissimilarity (or similarity) of an opinion.

Then, agent i updates its opinion, $\tilde{\omega}^j_i = (\tilde{b}^j_i, \tilde{d}^j_i, \tilde{u}^j_i)$, based on the revised trust of agent j in order to consider agent j's opinion as

$$\begin{aligned} \tilde{b}^j_i &= C_{ij}(1 - \mathrm{PD}^i_j)(1 - \mathrm{RF}^i_j) \\ \tilde{d}^j_i &= C_{ij}\mathrm{PD}^i_j(1 - \mathrm{RF}^i_j) \\ \tilde{u}^j_i &= 1 - C_{ij}(1 - \mathrm{RF}^i_j) \end{aligned} \tag{10}$$

where $\tilde{b}^j_i$ represents the degree that agent i trusts agent j based on the degree of the opinion similarity of the two agents, i and j. Since PD^i_j indicates the difference between agent i's opinion and agent j's opinion, the opinion similarity is $(1 - \mathrm{PD}^i_j)$. The validity of the estimated similarity is ensured by: 1) the degree of certainty based on the uncertainty that two agents, i and j, have toward a given proposition and 2) the degree of the trust revision agent i needs to consider in estimating trust in agent j. We multiply the opinion similarity, $(1 - \mathrm{PD}^i_j)$, by C_{ij} to consider 1) and $(1 - \mathrm{RF}^i_j)$ to consider 2). Note that a lower RF^i_j means a smaller adjustment from the current trust state. If $\mathrm{RF}^i_j = 0$, it means agent i can fully trust what j says. Thus, we use $(1 - \mathrm{RF}^i_j)$ to indicate more trusting in the estimated similarity.

Similarly, in $\tilde{d}^j_i$, where the opinion dissimilarity is PD^i_j, 1) and 2) above are considered by multiplying PD^i_j with C_{ij} and $(1 - \mathrm{PD}^i_j)$. Also the uncertainty for agent i to trust agent j's opinion is computed based on the trust revision factor as above.

The way of formulating (10) as above is motivated based on the rationale that agent i's trust in agent j is based on: 1) how similar i has an opinion with j, reflected by $(1 - \mathrm{PD}^i_j)$; 2) the relative degree of uncertainty that i has compared with that of uncertainty that j has, considered in RF^i_j (i.e., lower RF^i_j is more desirable implying that less trust revision is needed for j because of high certainty); and 3) the overall certainty based on uncertainty perceived by both i and j, considered in C_{ij} (i.e., higher C_{ij} implies that both i and j are certain about their opinions). Based on these factors, (10) provides i's high belief, low disbelief, and low uncertainty in j when i and j have similar opinions with low uncertainty for both opinions.

Based on (4)–(6) and (10), the interaction rule is given by

$$\omega_{i \oplus j} = \omega_i \oplus \omega_{i \otimes j} = \omega_i \oplus \left(\tilde{\omega}^j_i \otimes \omega_j\right). \tag{11}$$

To obtain $\tilde{\omega}^i_j$, we can use either the UT or the ST. Again, $\omega_{i \oplus j}$ means that i's opinion is updated based on j's opinion which is weighted by i's opinion toward it. This implies that i reflects j's opinion in updating its opinion based on how much it trusts j's opinion. In Section V, we compare and analyze their performance. Further, motivation and properties of the UT and the ST are discussed with the convergence analysis in Section IV.

C. Agent Types

The following two agent types are considered in updating opinions.

1) *IA:* This agent holds its own subjective opinion with a high confidence (i.e., minimal uncertainty). This type of agent does not change its opinion and only tries to influence others' opinions. For example, influential

leaders in society show a firm belief and they often are not affected by other opinions.

2) *UIA:* This agent does not have any firm opinion on a given proposition and hence has a high uncertainty. This type of agent will update its opinion based on interactions with others or learning through observations.

We model an IA who has a strong opinion with either belief (i.e., agreeing), denoted as IAB [i.e., $(b, d, u) = (b \to 1, d \to 0, u \to 0)$], or disbelief (i.e., disagreeing), denoted as IAD [i.e., $(b, d, u) = (b \to 0, d \to 1, u \to 0)$]. On the other hand, a UIA is initialized with positive evidence $r = 1$ for belief and negative evidence $s = 1$ for disbelief to derive b, d, and u based on the *mapping* rule discussed in (3) [i.e., $(b, d, u) = (b \to 0, d \to 0, u \to 1)$]. These two types of agents are initialized in the beginning and they update opinions as they interact with others. While IAs will not change their opinions, UIAs will update their opinions upon interactions with other agents. We say that an IA is stubborn while a UIA is open-minded.

IV. Convergence Analysis of UT and ST

In this section, we mathematically analyze the convergence of the two proposed opinion models and discuss the key findings from the analysis.

A. Analysis of UT-Based Opinion Update

Let agents i and j interact at interaction index k. Using (4)–(6), the updated opinion vector for i can be written as

$$b_i[k+1] = \frac{(1-\beta)(b_i u_j + b_j u_i) + \beta b_i}{\beta(2-\beta)}$$
$$d_i[k+1] = \frac{(1-\beta)(d_i u_j + d_j u_i) + \beta d_i}{\beta(2-\beta)}$$
$$u_i[k+1] = \frac{(1-\beta)(u_i u_j) + \beta u_i}{\beta(2-\beta)}$$

where all the terms on the right-hand side are at index k.

Remarks: We have found the following from the analysis of UT-based opinion update.

1) *R1:* The update equation for belief (disbelief) is a nonconvex linear *state-dependent* combination of the beliefs (disbeliefs) of the interacting agents. From the above, $b_i[k+1]$ can be rewritten as

$$\begin{aligned} b_i[k+1] &= b_i \frac{(1-\beta)u_j + \beta}{\beta(2-\beta)} + b_j \frac{(1-\beta)u_i}{\beta(2-\beta)} \\ &= f b_i + g b_j \\ d_i[k+1] &= d_i \frac{(1-\beta)u_j + \beta}{\beta(2-\beta)} + d_j \frac{(1-\beta)u_i}{\beta(2-\beta)} \\ &= f d_i + g d_j \\ u_i[k+1] &= u_i \frac{(1-\beta)u_j + \beta}{\beta(2-\beta)} \\ &= f u_i \end{aligned}$$

where the definitions of f and g are obvious. All the terms on the right-hand side are at index k. We can easily show that $0.5 \le f < 1$, $0 \le g < 0.5$, and $f + g \ge 1$.

2) *R2:* The nonlinearity is entirely in the update of the uncertainty, which evolves independent of (b, d).

3) *R3:* Uncertainty always decreases after an interaction (unless the agent already has $u = 0$). Correspondingly, belief or disbelief or both increase.

4) *R4:* If two agents with the same (b, d, u) interact, u will decrease (as noted earlier) and both b and d will increase by the same factor. Thus, updates cease only when $u = 0$; a state in which all (uninformed) agents have the same (b, d), $u > 0$, is not a stable consensus state.

5) *R5:* Consider the change in the opinion triple of the interacting agents

$$b_i[k+1] - b_j[k+1] = \frac{b_i[k] - b_j[k]}{2-\beta}$$
$$d_i[k+1] - d_j[k+1] = \frac{d_i[k] - d_j[k]}{2-\beta}$$
$$u_i[k+1] - u_j[k+1] = \frac{u_i[k] - u_j[k]}{2-\beta}$$

since $0 < u < 1$, $1 < (2-\beta) < 2$. Thus, if $u < 1$ (agents are not completely ignorant), then the agents move closer in each of the dimensions (b, d, u). Furthermore, the ordering does not change; i.e., $b_i[k] > b_j[k]$ implies $b_i[k+1] > b_j[k+1]$. From R1, we note that even though the distance decreases, the two agents need not move toward each other in b or d.

B. Analysis of ST-Based Opinion Update

Again agents i and j interact with each other at interaction index k. According to (4), (5), and (7)–(10), the opinion vector for agent j based on agent i's trust, $\omega_{i\otimes j} = \omega_i^j \otimes \omega_j$, as shown in (11), is

$$\begin{aligned} b_{i\otimes j} &= b_j \tilde{b}_i^j \\ &= b_j C_{ij}(1 - \mathrm{PD}_j^i)(1 - \mathrm{RF}_j^i) \\ &\le b_j C_{ij} = b_j(1-\beta) \\ d_{i\otimes j} &= d_j \tilde{b}_i^j \\ &= d_j C_{ij}(1 - \mathrm{PD}_j^i)(1 - \mathrm{RF}_j^i) \\ &\le d_j C_{ij} = d_j(1-\beta) \\ u_{i\otimes j} &= \tilde{d}_i^j + \tilde{u}_i^j + \tilde{b}_i^j u_j \\ &\le \tilde{d}_i^j + \tilde{u}_i^j + \tilde{b}_i^j = 1 \end{aligned}$$

since $(1 - \mathrm{PD}_j^i)$ and $(1 - \mathrm{RF}_j^i)$ lie between 0 and 1. A UIA with $(b_i, d_i, u_i) = (\to 0, \to 0, \to 1)$ can update its opinion, while an IA with $(b_i, d_i, u_i) = (\to 1, \to 0, \to 0)$ or $(\to 0, \to 1, \to 0)$ does not update its opinion. Thus, the UIA can update its opinion when it interacts with an agent with either the IA or another UIA. Then, $\omega_i \oplus \omega_{i\otimes j}$ can be computed by applying (4) as

$$\begin{aligned} b_i[k+1] &= \frac{b_i u_{i\otimes j} + b_{i\otimes j} u_i}{\beta_2} \\ &\leq \frac{b_i + b_j(1-\beta)u_i}{\beta_2} \\ d_i[k+1] &= \frac{d_i u_{i\otimes j} + d_{i\otimes j} u_i}{\beta_2} \\ &\leq \frac{d_i + d_j(1-\beta)u_i}{\beta_2} \\ u_i[k+1] &= \frac{u_i u_{i\otimes j}}{\beta_2} \\ &\leq \frac{u_i}{\beta_2} \end{aligned}$$

where $\beta_2 = u_i + u_{i\otimes j} - u_i u_{i\otimes j}$ and all the terms on the right-hand side are at index k.

Remarks: We have found the following from the analysis of ST-based opinion update.

1) *R1:* Similar to the opinion update in the UT, the updated belief is based on the combination of the beliefs (or disbeliefs) of the two interacting agents. The updated belief can be derived from the above equation as

$$\begin{aligned} b_i[k+1] &\leq \frac{b_i}{\beta_2} + b_j \frac{u_i(1-\beta)}{\beta_2} \\ &= f b_i + g b_j \\ d_i[k+1] &\leq \frac{d_i}{\beta_2} + d_j \frac{u_i(1-\beta)}{\beta_2} \\ &= f d_i + g d_j \\ u_i[k+1] &\leq \frac{u_i}{\beta_2} \\ &= f u_i. \end{aligned}$$

Based on (12), $\beta_2 = u_i + u_{i\otimes j} - u_i u_{i\otimes j} = u_i + u_{i\otimes j}(1 - u_i) \geq u_i \leq 1$ where $u_{i\otimes j} \leq 1$. Thus, $u_i \leq \beta_2 \leq 1$. g and f are bounded by

$$\begin{aligned} f &= 1/\beta_2 \geq 1 \\ g &= \frac{u_i(1-\beta)}{\beta_2} \leq u_i \leq 1 \end{aligned}$$

where $0 \leq u_i \leq 1$, $0 \leq (1-\beta) \leq 1$, and $f+g \geq 1$. Thus, when $f = 1$, $u_i[k+1] \leq u_i[k]$; otherwise, $u_i[k+1] \geq u_i[k]$. This implies that in the ST, uncertainty does not always decrease.

2) *R2:* The updates of an opinion is dependent upon the degree of an agent's uncertainty as a belief or a disbelief is updated based on a function of uncertainty, f or g, as above. Recall that $\beta = u_i + u_j - u_i u_j$.

3) *R3:* Similar to the UT, uncertainty decreases as agents interact with other agents upon interactions. This is seen from $u_i[k+1] \leq f u_i[k]$ as discussed above. Since $f \geq 1$, $u_i[k+1]$ may not decrease. Due to the possibility that uncertainty does not decrease, the convergence is very slow or does not occur, because the significant update is performed only when two agents have similar opinions.

4) *R4:* Similar to the UT, if two agents with the same (b, d, u) interact, u will decrease while b or d will increase or decrease to the same extent. The update will stop when u reaches 0.

5) *R5:* Since the updates will stop at $u = 0$, consensus may not be reached during the updates. The changes of two agents' opinions from iteration k to $k+1$ are

$$\begin{aligned} |b_i[k+1] - b_j[k+1]| &\leq |b_i[k] - b_j[k]| \\ |d_i[k+1] - d_j[k+1]| &\leq |d_i[k] - d_j[k]| \\ |u_i[k+1] - u_j[k+1]| &\leq |u_i[k] - u_j[k]|. \end{aligned}$$

The above proves that when $0 < u < 1$, the two agents have a close opinion in (b, d, u) as the difference between two opinions becomes smaller at $k+1$ than at k. The absolute difference between the opinions of two agents decreases over time. However, when u reaches 0, the updates cease, in which two opinions may not reach consensus. In addition, as an agent updates its opinions significantly only when they have very similar opinions, the speed of convergence is very slow.

V. Simulation Results

In this section, we describe the experimental setup used to obtain simulation results, including node selection at each iteration, network environments for both synthetic networks and a real network (i.e., the Facebook network topology data set), and default values used for key design parameters. We conduct a comparative study of the two opinion models, the UT and the ST, under the three network topologies (i.e., ER, BA, and Facebook networks).

A. Experimental Setup

We randomly select an agent and one of its neighbors based on the uniform distribution at every iteration at k. We used this random selection of the link between two agents to investigate how two agents' opinions are being updated based on their opinion models reflecting two different trust attitudes. Unlike epidemic models (e.g., SIR or SIS), this paper is interested in how both agents change their opinions after interacting with each other (i.e., bidirectional opinion updates), rather than how one agent can make another agent's opinion changed (i.e., one directional opinion update). Note that the directional opinion update can happen in the interactions between IAs and UIAs where only UIAs' opinions change. However, even in this case, the UIAs' opinions do not become as the same as the IAs' opinions, as shown in Section IV.

The simulation results are based on the data collected from 10 000 interactions where each interaction may be between two UIAs (opinion updates for both agents) or an IA and a UIA (an opinion update for the UIA). There will be no opinion updates between two interacting IAs. If β reaches 0, there will be no opinion updates of agents, implying zero uncertainty.

We use three network environments where two experiments were conducted with simulated networks using the ER network [17] and the BA network [3], and one real network using a Facebook network [29], respectively. An ER network is generated with a probability $p = 0.25$ that a random pair of agents are connected. A BA network is created where each agent is initially connected with 10 neighbors, and at each time

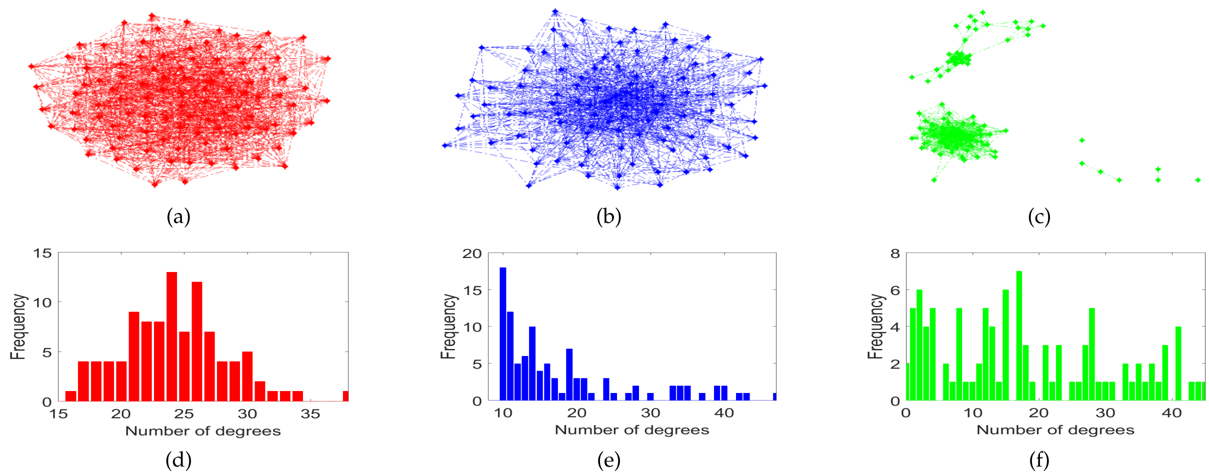

Fig. 1. Network topologies used for the experiments and their degree distributions. (a) ER network with 100 users and 1213 edges with 0.25 probability that selects a random pair connected. (b) BA network with 100 users and 909 edges with a randomly selected 10 initial members. (c) Facebook network with 100 users and 902 edges [29]. (d) Degree distribution of the ER network in (a). (e) Degree distribution of the BA network in (b). (f) Degree distribution of the Facebook network in (c).

step, each agent is further connected with $m = 10$ agents based on its degree [3]. Experiments using a real network is based on a subset of the Facebook data set by randomly selecting 100 users in one of the ego networks [29].

To get a sense of the density of these networks, we show the network topologies in Fig. 1. Fig. 1(a) is an ER network with 1213 edges, which is a connected network. Fig. 1(b) is a BA network with 909 edges and is also a connected network. The Facebook network used in this paper has 902 edges, which does not create a connected network, as shown in Fig. 1(c). We also show the degree distributions of those networks, respectively, in Fig. 1(d) and (e). To define consensus of opinions, we treat that two agents have the same opinion when their opinions (i.e., belief, disbelief, and uncertainty) have less than 0.01 difference.

B. Results

In this section, we analyze the results under three network environments: ER, BA, and Facebook networks.

1) Result Analysis Under an ER Network: Figs. 2 and 3 show how the UT and the ST affect the evolution of opinions with an ER network. We vary the number of IAs with a strong disbelief [i.e., $(b, d, u) = (\rightarrow\ 0, \rightarrow\ 1, \rightarrow\ 0]$ with #IAD (# of IADs) = 10, 30, or 50 under a fixed number of IA agreeing to a strong belief [i.e., $(b, d, u) = (\rightarrow\ 1, \rightarrow\ 0, \rightarrow\ 0]$ with #IAB (# of IABs) = 10 where the total number of agents (N) is 100.

In Fig. 2(a)–(c), we show how agents' opinions evolve as more interactions are performed. Fig. 2(d)–(f) is the histograms based on the opinions converged after 10 000 interactions, corresponding to Fig. 2(a)–(c). When #IAD = #IAB = 10 as shown in Fig. 2(a) and (d), the opinions of UIAs converge quickly and reache consensus with a neutral opinion, $(b, d, u) = (0.5, 0.5, 0)$. The convergence of opinions moves toward the dominance of disbelief when more #IAD are added, such as 30 or 50, as shown in Fig. 2(b) and (c), respectively. However, when more #IADs are added to the network, the speed of consensus is slowed down due to the increased interactions among the IAs themselves. However, we can still observe consensus with $(b, d, u) = (0.25, 0.75, 0)$ or $(b, d, u) = (0.18, 0.82, 0)$ when #IAD is 30 or 50 among all UIAs, respectively. From Fig. 2(b) and (c), we can notice that agents' uncertainty diminishes more quickly with more IADs. From Fig. 2, we can conclude that under the UT, the increasing number of stubborn agents slows down the speed of convergence although it quickly drops uncertainty.

When UIAs use the ST for their opinion updates, the evolution of their opinions are quite different from what we observed in Fig. 2. Fig. 3(a)–(c) shows the evolution of agents' opinions as more interactions are performed under the ST while Fig. 3(d)–(f) shows the histograms of Fig. 3(a)–(c) correspondingly based on the converged opinions after 10 000 interactions. When #IAD = #IAB = 10 in Fig. 3(a) and (d), no consensus is reached and agents' uncertainty decreases quite slowly showing its significant reduction after 3 000 interactions. However, as the number of IADs increases, such as #IAD = 30 or 50 with #IAB = 10, a clearer convergence of opinions among UIAs appears as shown in Fig. 3(b) and (e) or Fig. 3(c) and (f), respectively. Opinions of all UIAs converge toward the opinions of IADs which are more dominant than those of IABs in terms of the number of agents. This is because a UIA updates its opinion when it encounters other agents who have similar opinions to itself. Since agents have more chances to meet IADs than IABs, if their opinions initially lean toward more dominant agents' opinions, the final converged opinions tend to converge toward the dominant opinions which are the opinions of IADs in this case study.

In Fig. 3(c) and (f), the opinions of UIAs clearly converge to $(b, d, u) = (\rightarrow 0, \rightarrow 1, 0)$ with a higher speed of convergence, compared with the cases with a fewer number of IADs in Fig. 3(a) and (d) or Fig. 3(b) and (e). From Fig. 3, under the ST, we conclude that more IAs can expedite the speed of

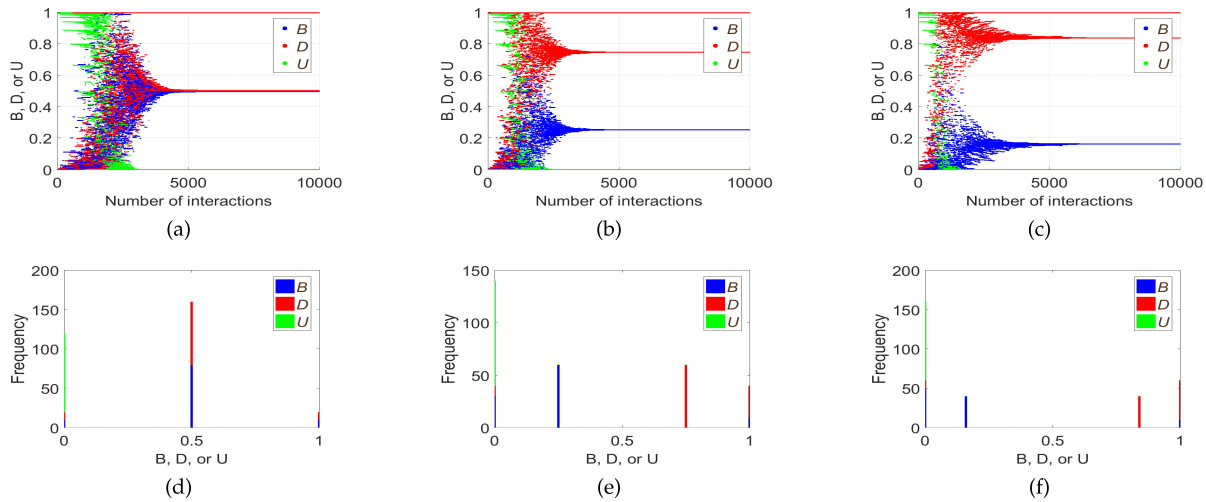

Fig. 2. Under the ER network: temporal evolution of opinion triple (b, d, u) under UT with # IABs = 10. (a) (b, d, u) with # IADs = 10 under UT. (b) (b, d, u) with # IADs = 30 under UT. (c) (b, d, u) with # IADs = 50 under UT. (d) Opinion histogram with # IADs = 10 after 10 000 interactions under UT. (e) Opinion histogram with # IADs = 30 after 10 000 interactions under UT. (f) Opinion histogram with # IADs = 50 after 10 000 interactions under UT.

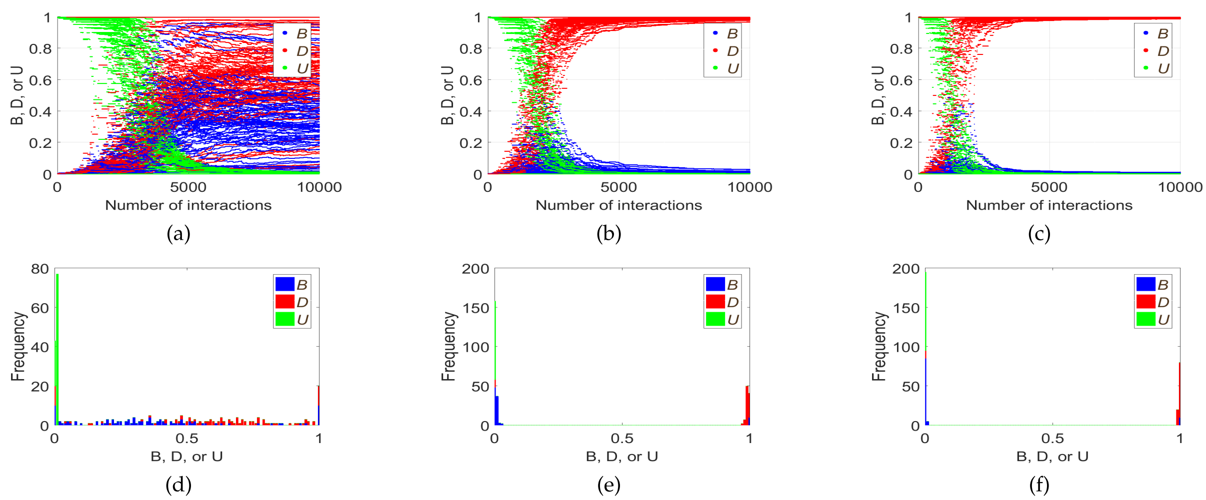

Fig. 3. Under the ER network: temporal evolution of opinion triple (b, d, u) under ST with # IABs = 10. (a) (b, d, u) with # IADs = 10 under ST. (b) (b, d, u) with # IADs = 30 under ST. (c) (b, d, u) with # IADs = 50 under ST. (d) Opinion histogram with # IADs = 10 after 10 000 interactions under ST. (e) Opinion histogram with # IADs = 30 after 10 000 interactions under ST. (f) Opinion histogram with # IADs = 50 after 10 000 interactions under ST.

opinion convergence while leading to consensus in a network. Note that agents' uncertainty quickly reduces with more IAs, implying that reducing uncertainty may not lead to consensus of opinions.

Comparison Discussion Between UT and ST Under the ER Network: In the results under the ER network, we observe the following major trends: 1) opinions with UT converges faster than those with ST; 2) less IAs can help faster convergence in UT while more IAs can lead to better convergence in ST; and 3) in UT, a converged opinion can be between 0 and 1, implying that both agents update their opinions in a certain point based on their current opinions. On the other hand, in the ST, the converged opinion is toward either 0 or 1, agreeing to either extreme opinion where the current study exhibits the convergence to a dominant opinion (i.e., disbelief by IADs).

2) Result Analysis Under a BA Network: We study how these two opinion models, UT and ST, affect the evolution of opinions in a BA network. Recall that the BA network has a significantly different degree distribution, as shown in Fig. 1(e). It is characterized as a scale-free network based on preferential attachment assuming that a node is more likely attached to a high-degree node.

In Figs. 4 and 5, we observe very similar trends as demonstrated in Fig. 2 and 3. Fig. 4 shows the evolution of opinions under the UT; a higher and faster convergence of opinions is observed when #IAD is small. On the other hand, as more IADs are added to the network, we notice a lower and slower convergence of opinions. In particular, when #IAD = 50, a higher divergence of opinions is observed. In Fig. 5, although the overall trends are similar to Fig. 3,

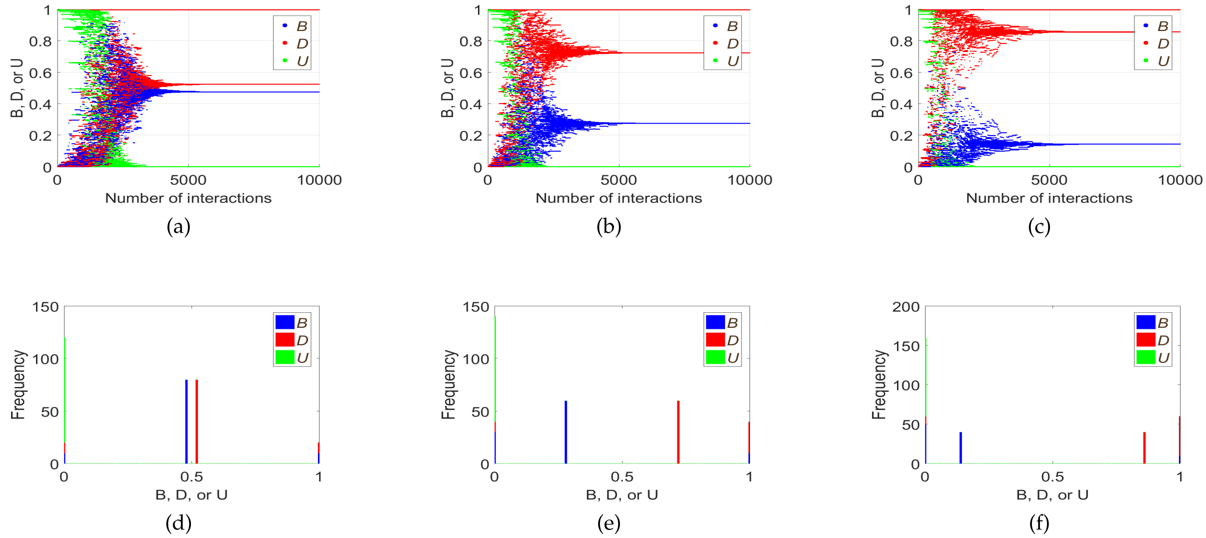

Fig. 4. Under the BA network: temporal evolution of opinion triple (b, d, u) under UT with # IABs = 10. (a) (b, d, u) with # IADs = 10 under UT. (b) (b, d, u) with # IADs = 30 under UT. (c) (b, d, u) with # IADs = 50 under UT. (d) Opinion histogram with # IADs = 10 after 10 000 interactions under UT. (e) Opinion histogram with # IADs = 30 after 10 000 interactions under UT. (f) Opinion histogram with # IADs = 50 after 10 000 interactions under UT.

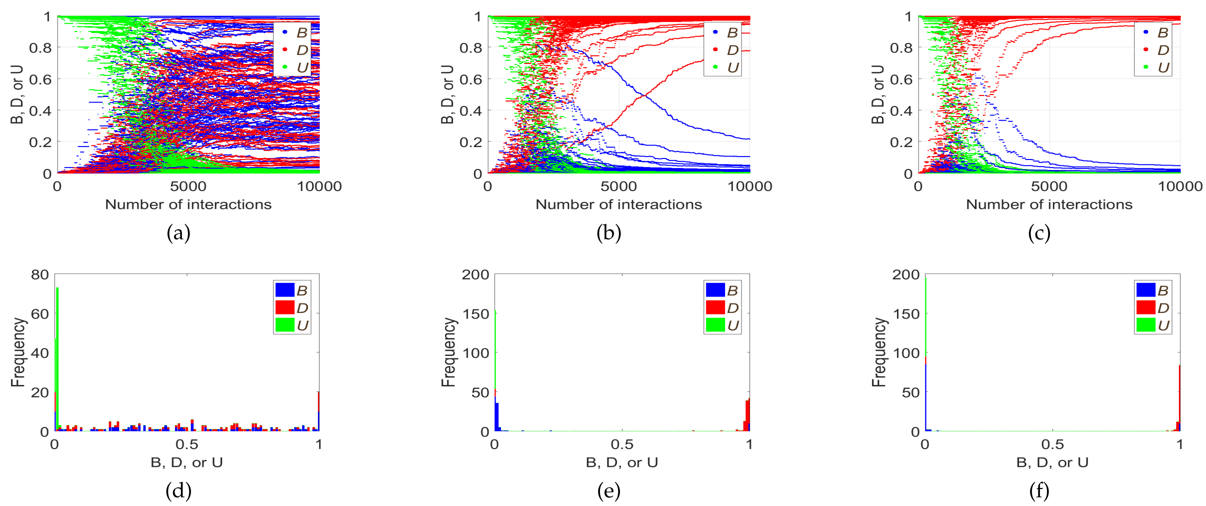

Fig. 5. Under the BA network: temporal evolution of opinion triple (b, d, u) under ST with # IABs = 10. (a) (b, d, u) with # IADs = 10 under ST. (b) (b, d, u) with # IADs = 30 under ST. (c) (b, d, u) with # IADs = 50 under ST. (d) Opinion histogram with # IADs = 10 after 10 000 interactions under ST. (e) Opinion histogram with # IADs = 30 after 10 000 interactions under ST. (f) Opinion histogram with # IADs = 50 after 10 000 interactions under ST

the severity of divergent opinions under small #IAD, such as 10 or 30, is more prominent than that under #IAD = 50.

Comparison Discussion Between UT and ST Under the BA Network: Although the BA network shows higher noises in opinion consensus compared with the results under the ER network, the overall trends still follow the three observations shown in the ER network discussed under Section V-B1. Some distinct observations from the observations in the ER network are as follows. When less IAs exist in the BA network, opinions in the UT quickly converges while opinions in the ST exhibits the total chaos with huge divergences of uncertain opinions. As more IAs are added, the speed of the convergence in UT is slowed down while the opinions in the ST quickly converges to the dominant opinion (i.e., disbelief by IADs). Regardless of a different density of IAs, UIAs' opinions in the UT bring a consensus even if more IAs slow down the speed of the consensus. When agents are willing to accept others' opinions based on their expertise or confidence regardless of whether their opinions are similar to their own or not, they are more likely to reach consensus even if their original opinions are significantly different, as shown in the evolution of opinions under the UT.

3) Result Analysis Under a Facebook Network: In this section, we show the experimental results with exactly the same parameter setting used in Section V-B1 but under a real network. We use the Facebook network as described in Section V-A. Since the network density of Fig. 1(c) is relatively very low compared with those of Fig. 1(a) and (b),

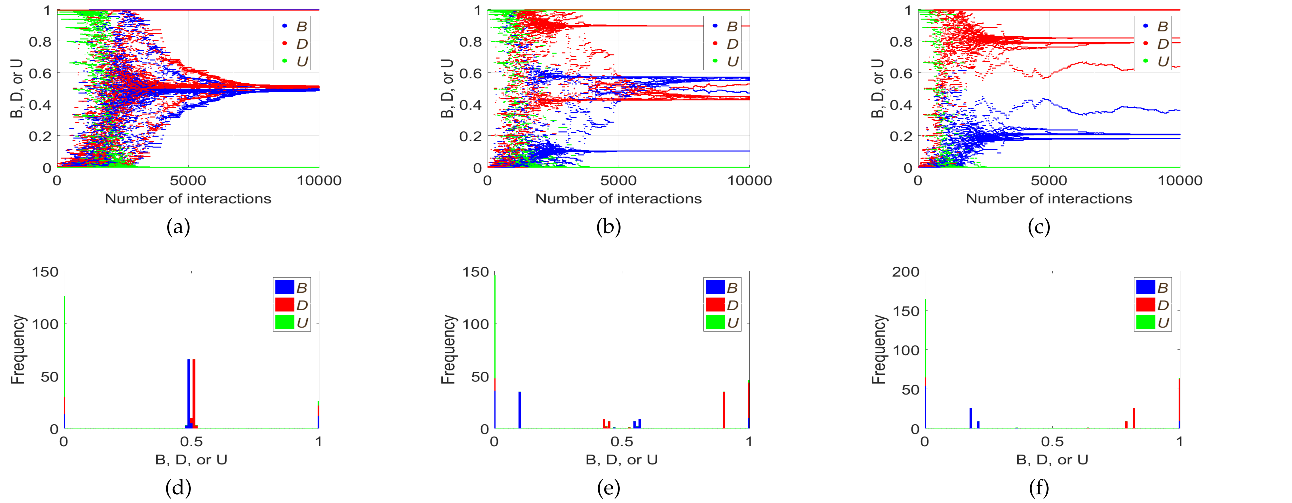

Fig. 6. Under the Facebook network: temporal evolution of opinion triple (b, d, u) under UT with # IABs = 10. (a) (b, d, u) with # IADs = 10 under UT. (b) (b, d, u) with # IADs = 30 under UT. (c) (b, d, u) with # IADs = 50 under UT. (d) Opinion histogram with # IADs = 10 after 10 000 interactions under UT. (e) Opinion histogram with # IADs = 30 after 10 000 interactions under UT. (f) Opinion histogram with # IADs = 50 after 10 000 interactions under UT.

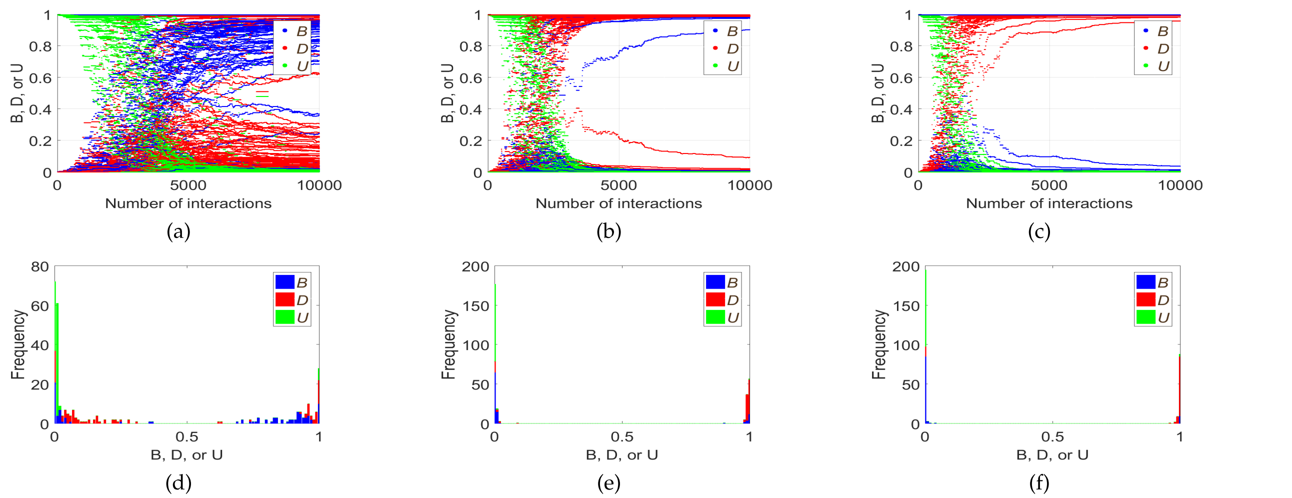

Fig. 7. Under the Facebook network: temporal evolution of opinion triple (b, d, u) under ST with # IABs = 10. (a) (b, d, u) with # IADs = 10 under ST. (b) (b, d, u) with # IADs = 30 under ST. (c) (b, d, u) with # IADs = 50 under ST. (d) Opinion histogram with # IADs = 10 after 10 000 interactions under ST. (e) Opinion histogram with # IADs = 30 after 10 000 interactions under ST. (f) Opinion histogram with # IADs = 50 after 10 000 interactions under ST.

we do not observe a clear matching between results observed in the ER/BA networks and the Facebook network. However, we can still observe the effect of more IAs (i.e., #IADs) under the UT and the ST in that fewer IADs expedite the convergence of opinions under the UT while more IADs facilitate convergence under the ST. This can be observed clearly in Figs. 6(a) and 7(c).

Different from the results observed in the ER and BA networks, even under the UT, if there are more IAs, such as #IADs = 30 or 50, consensus may not be observed, because the network is not fully connected. As shown in Fig. 7(c) and (f), under the ST, although it takes many interactions for an agent to change its opinion from one extreme to the other extreme (e.g., complete belief to complete disbelief or vice-versa), we can see that consensus may appear after a sufficient amount of interactions. Although we cannot say there exists 100% consensus as shown in Fig. 7(f), we can conjecture that there may exist a minimum number of IAs for reaching a consensus under the ST.

Comparison Discussion Between UT and ST Under the Facebook Network: Although we can observe high noises in the convergence or divergence of uncertain opinions in both the UT and the ST, compared with the results observed under the ER and BA networks, similar trends are also observed in terms of the three observations mentioned in Section V-B1. However, due to the irregular patterns of the network degree distributions as shown in Fig. 1(e) and the disconnected network topology, even with a small number of IAs (i.e., # of IADs), under UT, the speed of convergence is significantly slow compared with the cases in the ER and BA networks;

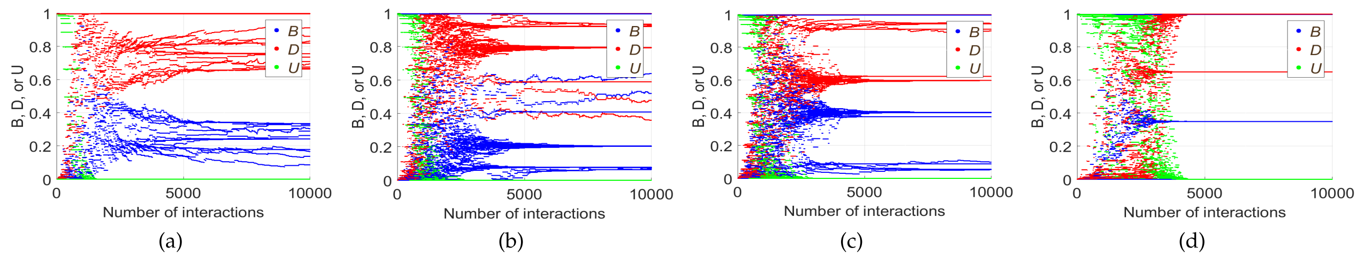

Fig. 8. Under the Facebook network: temporal evolution of opinion triple (b, d, u) under UT with # IABs = 10 and # IADs = 30 when IAs are selected based on various centrality measures. (a) (b, d, u) with high-degree IAs under UT. (b) (b, d, u) with high-betweenness IAs under UT. (c) (b, d, u) with high-pagerank IAs under UT. (d) (b, d, u) with low-degree IAs under UT.

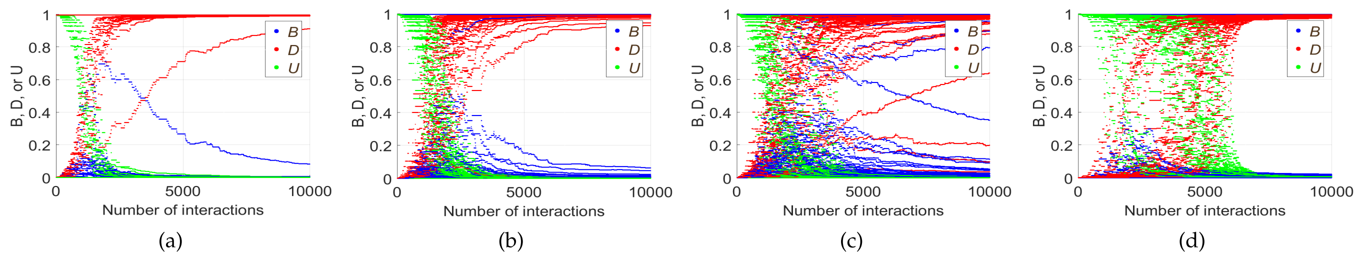

Fig. 9. Under the Facebook network: temporal evolution of opinion triple (b, d, u) under ST with # IABs = 10 and # IADs = 30 when IAs are selected based on various centrality measures. (a) (b, d, u) with high-degree IAs under ST. (b) (b, d, u) with high-betweenness IAs under ST. (c) (b, d, u) with high-pagerank IAs under ST. (d) (b, d, u) with low-degree IAs under ST.

even in the case with more IADs under the UT, opinion convergence is not observed. Under the ST, a small number of IADs generated a complete chaos, while more IADs showed some converging trends toward two extreme opinions, either 0 or 1, over time. However, even by the end of 10 000 interactions, no complete opinion convergence is observed due to highly different degree distributions and the disconnected network topology.

4) Effect of (Non) Influential Informed Agents under a Facebook Network: In this section, we investigate how IAs' characteristics in terms of its network centrality, representing the degree of its influence in the network, impact the evolution of uncertain opinions under a Facebook network [29]. We tested the effect of IAs' centrality (or noncentrality) on the opinion updates of UIAs in terms of the four centrality measures [36], including high degree, high betweenness, high pagerank, and low degree. To select a set of IAs, we rank them based on each centrality criterion, such as the top k agents based on the highest degree, betweenness, and pagerank, and the lowest degree. We first select top 10 IABs, and then select top 30 IADs based on each criterion. Figs. 8 and 9 show how each centrality type of IAs affects the evolution of their opinions over time under either the UT or the ST.

As shown in Fig. 8(a)–(c), in the UT, IAs with high-centrality, including high degree, high betweenness, or high pagerank, do not help converge opinions; rather, they even facilitate divergence of opinions. This is because dominant opinions (i.e., high disbelief by IADs) are more propagated by high-centrality agents with higher chances, leading UIAs to quickly reach their uncertainty zero. This makes their opinion updates stopped and accordingly this increases dissonance of opinions. On the other hand, when the IAs are selected based on the lowest degree, dominant opinions are less propagated over the network where more UIAs have a chance to meet other agents which do not have dominant opinions and can have better chances to converge to a certain opinion, as shown in Fig. 8(d).

Fig. 9(a)–(d) shows how the centrality of IAs can affect the evolution of opinions over time when UIAs use the ST for their opinion updates. Similar to Fig. 8, we set the number of IABs to 10 while the number of IADs is set to 30 based on given centrality criteria. Since in the ST, more IAs lead to better convergence of opinions, we can observe less noisy, chaos divergence when IAs are selected based on high centrality, high betweenness, and high pagerank. In particular, when IAs have high-degree centralities, we can clearly notice that much less noises are shown with much less diverging opinions, as shown in Fig. 9(a). However, particularly when the IAs have high-pagerank, we can observe much more noisy, diverging opinions. This is because the pagerank centrality is measured based on the number of incoming links from an agent's neighbors; the dominant opinions (i.e., IADs' opinions) are more quickly propagated, leading to faster updates of opinions with zero uncertainty. This can make the agents stop updating their opinions and lose the chances to reach consensus over time. However, when the IAs are selected based on the rank of the lowest degrees, their uncertainty slowly reduces because not many agents are connected to them, resulting in low chance to be dominated by the IADs. This allows the agents to keep updating their opinions but slowly until their uncertainty becomes zero.

Comparison Discussion Between UT and ST Under Varying the Centrality of IAs: In the UT, high-degree IAs least help converge opinions of UIAs, while in the ST, high-pagerank

IAs introduce more diverging opinions. In both the UT and the ST, the low-degree IAs can significantly help the convergence of opinions over time. The opinion convergence is more prominently observed under the UT; but the speed of opinion convergence under ST is quite slow with the slow reduction of uncertainty although the opinions ultimately start to converge over time.

VI. Conclusion

This paper studied how uncertain and conflicting opinions can evolve upon interactions of agents which allows their opinion updates. By leveraging a belief model called the SL that has an opinion formulation explicitly dealing with uncertainty, we proposed two types of opinion models that agents can update their opinions based on either the UT or the ST. We investigated how UT- and ST-based opinion updates affect the convergence or divergence of opinions in a social network via both mathematical and simulation analysis. We examined the effect of the number of IAs with strong disbelief (or belief) on whether UIAs' opinions quickly converge or not under ER, BA, or Facebook networks.

From our comprehensive simulation experiments, we summarize the following key findings.

1) More IAs (i.e., stubborn agents only influencing others while not being affected by others) can slow down the speed of opinion convergence of UIAs under the UT, while they can quickly lead to opinion convergence under the ST.
2) More IAs can decrease UIAs' uncertainty quickly under both the ST and the UT.
3) The UT achieves a faster opinion convergence than the ST.
4) Under the UT, an opinion can converge to a certain point between two extremes, somewhere between 0 and 1 although the converged point is affected by the dominant agents' opinions. On the other hand, under the ST, an opinion only converges to one of extremes, either 0 or 1 if consensus exists.
5) Although a real network (i.e., a Facebook network [29]) shows significant noises in opinion consensus, compared with the two synthetic networks (i.e., ER and BA networks), the general trends under the UT and the ST are similar to the trends observed under the ER and BA networks.
6) In the UT, overall high-centrality IAs introduce high diverging opinions; in particular, we observed that high-degree IAs show highly diverging opinions even compared with other high-centrality IAs (i.e., high betweenness or high pagerank). In the ST, high-degree IAs can perform fairly well in generating converging opinions, compared with other high centrality IAs, such as high betweenness or high pagerank.
7) In both the UT and the ST, low-degree IAs show converging opinions although the speed of the convergence is not really fast and the uncertainly slowly reduces.

Overall, similarity-based opinion updates are viewed as more conservative in that agents are more likely to stick to the opinions they have in the past. On the other hand, uncertainty-based opinion updates can allow agents' to update their opinions based on expertise or confidence an information source has, which is observed as more open-minded.

As future research directions, we plan to: 1) validate the UT and ST models using a longitudinal real data set, so we can observe the real dynamics of a network and the effect of the proposed interaction models; 2) investigate whether the order of interactions affects opinion convergence using the concept of "path dependency" [38]; 3) examine if the *Lyapunov equilibrium* exists to identify the converged point of opinions and how long it takes to reach the equilibrium [8]; and 4) consider the effect of different levels of pay-off for decisions and actions taken on the basis of agents' beliefs, influenced by the states of the world.

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References

[1] D. Acemoglu and A. Ozdaglar, "Opinion dynamics and learning in social networks," *Dyn. Games Appl.*, vol. 1, pp. 3–49, Mar. 2011.
[2] A. Alaali, M. A. Purvis, and B. T. R. Savarimuthu, "Vector opinion dynamics: An extended model for consensus in social networks," in *Proc. IEEE/WIC/ACM Int. Conf. Web Intell. Intell. Agent Technol.-Workshops*, Sydney, NSW, Australia, Dec. 2008, pp. 394–397.
[3] R. Albert and A.-L. Barabási, "Statistical mechanics of complex networks," *Rev. Mod. Phys.*, vol. 74, no. 1, pp. 47–97, Jan. 2002.
[4] R. Axelrod, "The dissemination of culture: A model with local convergence and global polarization," *J. Conflict Resolution*, vol. 41, no. 2, pp. 203–226, 1997.
[5] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, and D.-U. Hwang, "Complex networks: Structure and dynamics," *Phys. Rep.*, vol. 424, nos. 4–5, pp. 175–308, 2006.
[6] S. Boyd, A. Ghosh, B. Prabhakar, and D. Shah, "Randomized gossip algorithms," *IEEE Trans. Inf. Theory*, vol. 52, no. 6, pp. 2508–2530, Jun. 2006.
[7] H. T. Buckner, "A theory of rumor transmission," *Public Opinion Quart.*, vol. 29, no. 1, pp. 54–70, 1965.
[8] F. Ceragioli and P. Frasca, "Continuous and discontinuous opinion dynamics with bounded confidence," *Nonlinear Anal. Real World Appl.*, vol. 13, no. 3, pp. 1239–1251, May 2012.
[9] J.-H. Cho and A. Swami, "Dynamics of uncertain opinions in social networks," in *Proc. IEEE MILCOM*, Baltimore, MD, USA, Oct. 2014, pp. 1627–1632.
[10] I. D. Couzin, J. Krause, N. R. Franks, and S. A. Levin, "Effective leadership and decision-making in animal groups on the move," *Nature*, vol. 433, pp. 513–516, Feb. 2005.
[11] I. D. Couzin *et al.*, "Uninformed individuals promote democratic consensus in animal groups," *Science*, vol. 334, pp. 1578–1580, Dec. 2011.
[12] D. J. Daley and D. G. Kendall, "Epidemics and rumours," *Nature*, vol. 204, no. 225, p. 1118, Dec. 1964.
[13] G. Deffuant, D. Neau, F. Amblard, and G. Weisbuch, "Mixing beliefs among interacting agents," *Adv. Complex Syst.*, vol. 3, nos. 1–4, pp. 87–98, 2000.
[14] A. Demers *et al.*, "Epidemic algorithms for replicated database maintenance," in *Proc. 6th Annu. ACM Symp. Principles Distrib. Comput.*, 1987, pp. 1–12.
[15] R. Durrett, *Lecture Notes on Particle Systems and Percolation*. Belmont, CA, USA: Wadsworth Publishing, 1988.
[16] R. Durrett, "Some features of the spread of epidemics and information on a random graph," *Proc. Nat. Acad. Sci. USA*, vol. 107, no. 10, pp. 4491–4498, Mar. 2010.
[17] P. Erdős and A. Rényi, "On random graphs, I," *Publ. Math.*, vol. 6, pp. 290–297, 1959.
[18] J. Ghaderi and R. Srikant, "Opinion dynamics in social networks: A local interaction game with stubborn agents," in *Proc. Amer. Control Conf.*, Jun. 2013, pp. 1982–1987.

[19] J. Goldenberg, B. Libai, and E. Müller, "Talk of the network: A complex systems look at the underlying process of word-of-mouth," *Marketing Lett.*, vol. 12, no. 3, pp. 211–223, 2001.
[20] M. Granovetter, "Threshold models of collective behavior," *Amer. J. Sociol.*, vol. 83, no. 6, pp. 1420–1443, May 1978.
[21] T. Hoffmann, M. A. Porter, and R. Lambiotte, "Random walks on stochastic temporal networks," in *Temporal Networks* (Understanding Complex Systems). Springer, 2013, pp. 295–313.
[22] A. Jøsang, "A logic for uncertain probabilities," *Int. J. Uncertainty, Fuzziness Knowl.-Based Syst.*, vol. 9, no. 3, p. 279, Jun. 2001.
[23] A. Jøsang, "The consensus operator for combining beliefs," *Artif. Intell.*, vol. 141, pp. 157–170, Oct. 2002.
[24] A. Jøsang and R. Hankin, "Interpretation and fusion of hyper opinions in subjective logic," in *Proc. 15th Int. Conf. Inf. Fusion (FUSION)*, Jul. 2012, pp. 1225–1232.
[25] A. Jøsang, *Subjective Logic: A Formalism for Reasoning Under Uncertainty*. Springer, 2016.
[26] A. Jøsang, M. Ivanovska, and T. Müller, "Trust revision for conflicting sources," in *Proc. 18th Int. Conf. Inf. Fusion (FUSION)*, Jul. 2015, pp. 550–557.
[27] P. Juslin, A. Winman, and H. Olsson, "Naive empiricism and dogmatism in confidence research: A critical examination of the hard–easy effect," *Psychol. Rev.*, vol. 107, no. 2, pp. 384–396, 2000.
[28] D. Kempe, J. Kleinberg, and É. Tardos, "Maximizing the spread of influence through a social network," in *Proc. 9th ACM SIGKDD*, New York, NY, USA, 2003, pp. 137–146.
[29] J. Leskovec. (2012). *Social Circles: Facebook*. [Online]. Available: https://snap.stanford.edu/data/egonets-Facebook.html
[30] L. Li, A. Scaglione, A. Swami, and Q. Zhao, "Phase transition in opinion diffusion in social networks," in *Proc. IEEE Int. Conf. Acoust., Speech Signal Process. (ICASSP)*, Mar. 2012, pp. 3073–3076.
[31] L. Li, A. Scaglione, A. Swami, and Q. Zhao, "Consensus, polarization and clustering of opinions in social networks," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 6, pp. 1072–1083, Jun. 2013.
[32] M. W. Macy, "Chains of cooperation: Threshold effects in collective action," *Amer. Sociol. Rev.*, vol. 56, no. 6, pp. 730–747, 1991.
[33] D. P. Maki, *Mathematical Models and Applications, With Emphasis on Social, Life, and Mangement Sciences*. Englewood Cliffs, NJ, USA: Prentice-Hall, 1973.
[34] P. V. Mieghem and R. van de Bovenkamp, "Non-Markovian infection spread dramatically alters the susceptible-infected-susceptible epidemic threshold in networks," *Phys. Rev. Lett.*, vol. 110, pp. 108701-1–108701-5, Mar. 2013.
[35] S. Moscovici and M. Zavalloni, "The group as a polarizer of attitudes," *J. Pers. Social Psychol.*, vol. 12, no. 2, pp. 125–135, 1969.
[36] M. E. J. Newman, *Networks: An Introduction*, 1st ed. London, U.K.: Oxford Univ. Press, 2010.
[37] A. Noymer, "The transmission and persistence of 'urban legends': Sociological application of age-structured epidemic models," *J. Math. Sociol.*, vol. 25, no. 3, pp. 299–323, 2001.
[38] S. E. Page, "Path dependence," *Quart. J. Political Sci.*, vol. 1, no. 1, pp. 87–115, 2006.
[39] G. Shafer, *A Mathematical Theory of Evidence*. Princeton Univ. Press, Princeton, NJ, USA: 1976.
[40] P. Smets and R. Kennes, "The transferable belief model," *Artif. Intell.*, vol. 66, no. 2, pp. 191–234, Apr. 1994.
[41] L. D. Valdez, P. A. Macri, and L. A. Braunstein, "Intermittent social distancing strategy for epidemic control," *Phys. Rev. E, Stat. Phys. Plasmas Fluids Relat. Interdiscip. Top.*, vol. 85, p. 036108, Mar. 2012.
[42] G. Verma, A. Swami, and K. Chan, "The impact of competing zealots on opinion dynamics," *Phys. A, Statist. Mech. Appl.*, vol. 395, pp. 310–331, Feb. 2014.
[43] J. D. West and C. T. Bergstrom, "Can ignorance promote democracy?" *Science*, vol. 334, pp. 1503–1504, Dec. 2011.
[44] L. Zhao, Q. Wang, J. Cheng, Y. Chen, J. Wang, and W. Huang, "Rumor spreading model with consideration of forgetting mechanism: A case of online blogging livejournal," *Phys. A, Statist. Mech. Appl.*, vol. 390, no. 13, pp. 2619–2625, 2011.
[45] L. Zhao, J. Wang, Y. Chen, Q. Wang, J. Cheng, and H. Cui, "SIHR rumor spreading model in social networks," *Phys. A, Statist. Mech. Appl.*, vol. 391, no. 7, pp. 2444–2453, 2012.
[46] L. Zhao, X. Qiu, X. Wang, and J. Wang, "Rumor spreading model considering forgetting and remembering mechanisms in inhomogeneous networks," *Phys. A, Statist. Mech. Appl.*, vol. 392, no. 4, pp. 987–994, 2013.

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