Designing a Supersonic Nuzzle Using Method of Characteristics

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Abstract

For the design of a two-dimensional supersonic nozzle, a method based on the theory of characteristics is presented in this paper. Using MATLAB, the optimum Mach number at the nozzle exit with uniform flow at the diverging section of the nozzle was calculated for the minimum length of the supersonic nozzle. Two-dimensional, steady, viscous, irrotational, and supersonic flows are solved numerically. The diverging section is the focus of the design considerations.

Introduction Theory

The method of characteristics (MOC) is a mathematical technique (numerical procedure) for solving hyperbolic type of partial differential equations and involves reducing the partial differential equation (PDE) to an ordinary differential equation (ODE). The steady state supersonic flows are governed by hyperbolic PDEs therefore the method characteristics can be used. MOC is generally based on discretization along a set of characteristic lines. These lines (in a supersonic flow) are aligned in specific directions along which pressure waves are propagated, therefore by using this technique flow properties such as direction and velocity can be calculated at specific points in a flow field.

Combining the continuity and Euler equation under the assumption of 2D irrotational flow can be written as:

$$\left(1 - \frac{\emptyset_x^2}{a^2}\right)\emptyset_{xx} + \left(1 + \frac{\emptyset_y^2}{a^2}\right)\emptyset_{xx} - 2\frac{\emptyset_x \emptyset_y}{a^2}\emptyset_{xy} = 0 \qquad (1)$$

For a supersonic flow Eqn.1 becomes hyperbolic in type since the square of velocity magnitude ($\emptyset_x^2 + \emptyset_y^2$) divided by the speed of sound (a) is larger than one. A hyperbolic equation presents particular directions in space called characteristics. Along the characteristic lines the flow properties are continuous and the derivatives are both indeterminate and can be discontinuous.

The velocity potential equation can be combined with chain rule for potential derivatives to obtain three linear algebraic equations (Eqn 1,2,3). The set of equations can be solved using Cramer's rule.

$$d\left(\frac{\partial\emptyset}{\partial y}\right) = du = \frac{\partial^2\emptyset}{\partial x^2}dx + \frac{\partial^2\emptyset}{\partial x\partial y}dy \quad (2)$$

$$d\left(\frac{\partial\emptyset}{\partial y}\right) = dv = \frac{\partial^2\emptyset}{\partial y^2}dy + \frac{\partial^2\emptyset}{\partial x\partial y}dx \quad (3)$$

Using Cramer's rule we find that $\frac{\partial^2\emptyset}{\partial x\partial y}$ is indeterminate. (a characteristic line)

The slope of the characteristic lines is then written as:

$$\left(\frac{dy}{dx}\right) = \left(\frac{-\frac{uv}{a^2} \pm \sqrt{\frac{u^2+v^2}{a^2} - 1}}{1 - \frac{u^2}{a^2}}\right) \qquad (4)$$

Following a set of algebraic and trigonometric manipulation, the slope of the characteristic lines is equal to the tangent of the angle between the characteristic line and a horizontal line.

$$\left(\frac{dy}{dx}\right) = \tan(\theta \mp \mu) \qquad (5)$$

Eq 5. Shows that's a characteristic line running through a point have slopes equal to $\tan\,(\theta - \mu)$ denoted as C_ and $\tan\,(\theta + \mu)$ denoted as C_{+}.

Depending on the position in the flow, the fluid could have a different Mach number and velocity vector orientation causing the characteristic lines to have a different orientation.

Along characteristic lines, the governing equation describing the flow reduces to ODEs known as compatibility equations.

$$d\theta = -\sqrt{M^2 - 1}\frac{dV}{V} \qquad along\ C_- \qquad (6)$$

$$d\theta = \sqrt{M^2 - 1}\frac{dV}{V} \qquad along\ C_+ \qquad (7)$$

By integration the equations in terms of Prandtl-Meyer function become:

$$\theta + \upsilon(M) = const\ = K_- \qquad along\ C_- \qquad (8)$$

$$\theta - \upsilon(M) = const\ = K_+ \qquad along\ C_+ \qquad (9)$$

K_ and K_+ are constants along their respective characteristics known as Riemann invariants. The equations can be combined to obtain simple expressions to calculate θ and υ.

$$\theta = \frac{1}{2}[\,K_- + \ K_+\,] \qquad (10)$$

$$\upsilon = \frac{1}{2}[\,K_- - \ K_+\,] \qquad (11)$$

The wall contour for a supersonic nozzle to allow shock free isentropic expansion can then be designed using the equations defined. For a minimum nozzle length, the expansion angle of the wall downstream of the throat is equal to on one half of the Prandtl-Meyer function for the design Mach number.

In order to calculate the static temperature, static pressure, velocity, and density; we assume that the engine is under choke conditions where the stagnation upstream temperatures and pressures are constant. It's also noted that the mass-flow rate is constant throughout the nozzle.

When the stagnation temperature $P_t = 1500\ KPa$ is constant throughout the whole engine, the Mach number at each node will be used to calculate the corresponding static temperature using equation (12)

$$\frac{T_t}{T} = \left(1 + \frac{\gamma - 1}{2} M^2\right) \qquad (12)$$

Similarly, assume that the stagnation pressure is constant at $P_t = 1500\ KPa$ and find the static pressure at each node with respect to its Mach number

$$\frac{P_t}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}} \qquad (13)$$

$$\frac{\rho_t}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{1}{\gamma-1}} \qquad (14)$$

Since that the project does not provide the stagnation density, the dependence on constant mass-flowrate $\dot{m} = 10\ kg/m^3$ throughout the chocked nozzle and equation (15) will be crucial to finding the density at each node.

$$\dot{m} = \rho v A \qquad (15)$$

Where A the cross-sectional area at every respective node and the velocity is v can be computed using the following equation.

$$V = M\sqrt{\gamma RT} \qquad (16)$$

The Mach number is known from the Prandtl-Meyer relationships and the static temperature calculated in Equation (12) is used at its respective point to find its velocity.

Design Input Parameters

- Total temperature in the combustion chamber, T_t = 3000 K
- Total pressure in combustion chamber, P_t =1500 kPa
- Mass flow rate through the nozzle, m= 10 kg/s
- Stagnation density is 1.718 kg/m^3

Assuming sea level conditions,

$$P_o = 101325\ Pa$$

$$T_o = 288.15\ K$$

$$\gamma = 1.4$$

Exit Mach number is calculated using Eqn. 17

$$M = \sqrt{\frac{2}{\gamma - 1} * \frac{\left(\frac{P_t}{P_o}\right)^{\frac{\gamma-1}{\gamma}}}{1 + \frac{\gamma - 1}{2}}} \qquad (17)$$

$$M_e \approx 2.2$$

The throat area calculated using Eqn. 18

$$\dot{m} = \frac{\dot{A^*} P_t}{\sqrt{T_t}} * \sqrt{\frac{\gamma}{R}} * \left(\frac{\gamma + 1}{2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}} \qquad (18)$$

$$A^* = \frac{\dot{m}\sqrt{T_t}}{P_t\sqrt{\frac{\gamma}{R}} * \left(\frac{\gamma+1}{2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}}}$$

$$A^* = 0.009\, m^2$$

Calculating maximum wall angle

$$\theta_{\text{wall,max}} = \frac{v_e}{2} \tag{19}$$

Where v_e is found using the Prandtl-Meyer table that relates v_e to M_e which corresponds to $v_e = 31.7325°$

$$\theta_{\text{wall,max}} = \frac{(31.7325)}{2}$$

$$\Rightarrow \theta_{\text{wall,max}} = 15.86625°$$

For this specific maximum wall angle we chose our initial angle for node 1 to be $\theta_{\text{initial}} = 0.86625°$ and a step size of three degrees was used.

The total required nodes was calculated using the following relationship 2 + (n+1)*n2 where n is the specified characteristic lines. In this case n will be a total of 6 characteristic lines that result in 27 nodal points across the nozzle.

Using these conditions and the theory explained previously; the iterations that calculate the Mach number, angle and characteristic line lengths will be done in MATLAB.

Results and Discussion

1-Minimum Length nozzle

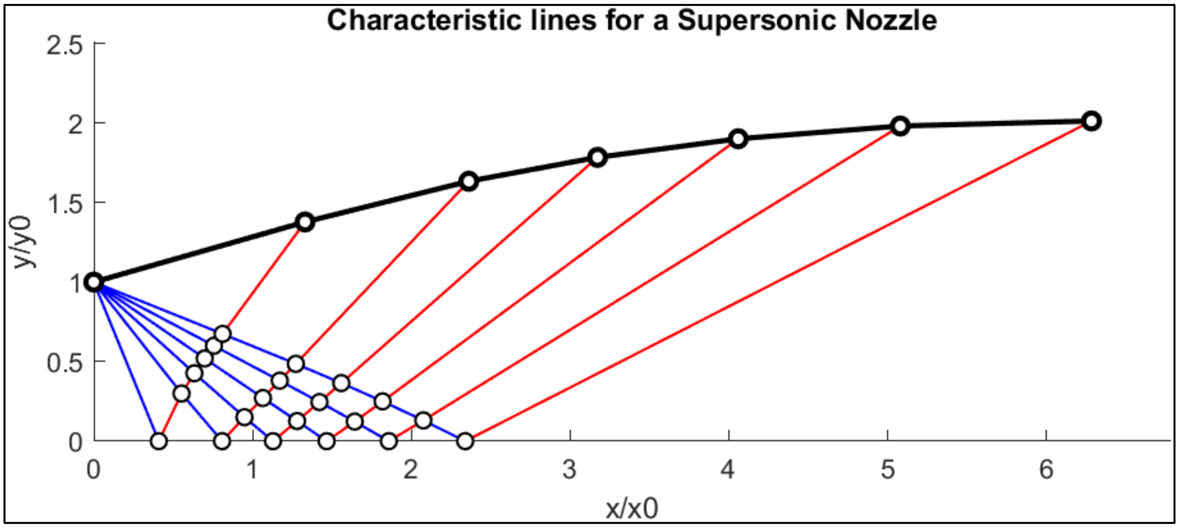

Figure 1

As we can observe in figure (1), the length and radius of the nozzle is non-dimensional. The relationship of y/y0 describes the ratio between the areas of each point from the wall to the throat area. For an example, the ratio at $y_{27} = \frac{A_e}{A^*} = 2.01297$ can be multiplied by the throat area $A^* = 0.009\, m^2$. This will yield the physical area at the exit of the nozzle which is $A_e = 0.0181\, m^2$.

Calculating the ratio $\frac{A_e}{A^*}$ using the following equation

$$\frac{A}{A*} = \frac{1}{M}\left(\frac{\frac{\gamma+1}{2}}{1+\frac{\gamma-1}{2}M^2}\right)^{\frac{\gamma+1}{2-2\gamma}} \qquad (20)$$

$$\frac{A}{A*} = \frac{1}{2.2}\left(\frac{\frac{1.4+1}{2}}{1\ +\frac{1.4-1}{2}*2.2^2}\right)^{\frac{1.4+1}{2-2*1.4}} = 2.00497$$

The real physical area is therefore $\frac{A_e}{A^*} = 2.00497$. Furthermore, comparing the real result to the MOC approximation we can see an error of 0.39% which is remarkably accurate.

Similarly, the relationship x/x0 describes the non-dimensional length to the throat radius. In order to find the physical length, the ratio will be multiplied by the throat radius

$$L_x = \frac{x}{x_o} * Throat\ Radius \quad \Rightarrow \quad L_x = 0.32 \text{ m}$$

2-Temperature

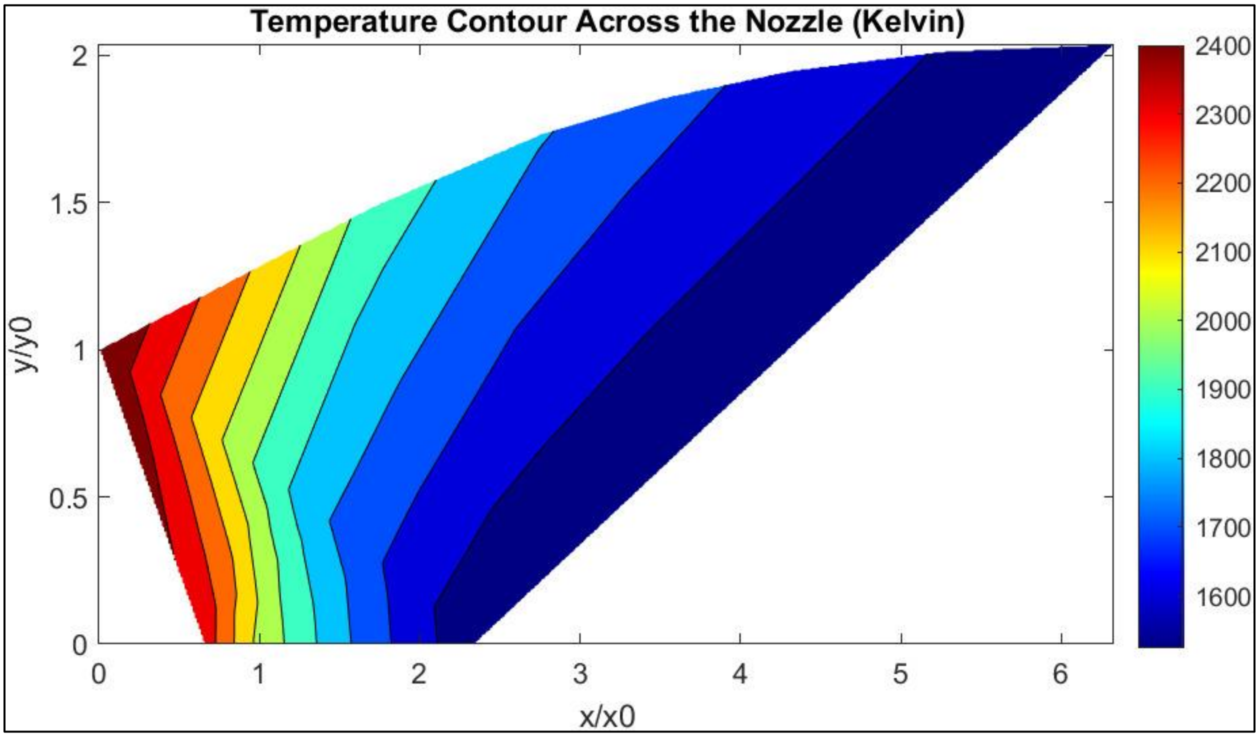

Figure 2

As observed from figure (2). The static temperature reduces as you make your way down to the end of the nozzle, with the highest value of 2362 K at the throat of the nozzle and the lowest value of about 1524 K at the end-tip of the nozzle.

3-Density

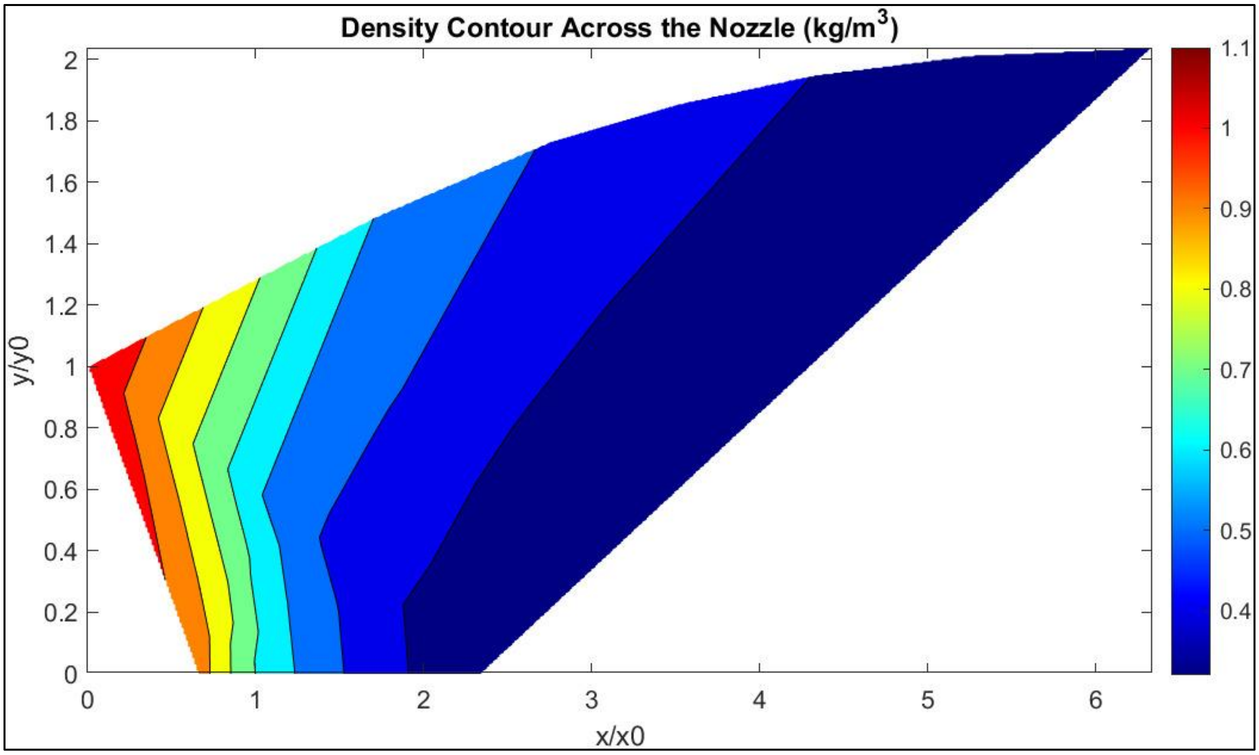

Figure 3

Figure (3) shows the variation of static density across the nozzle. The density decreases as we approach the end of the nozzle. The density value at the throat of 0.9578 kg/m^3, and the value of about 0.32 kg/m^3 at the end of the nozzle.

4-Mach number

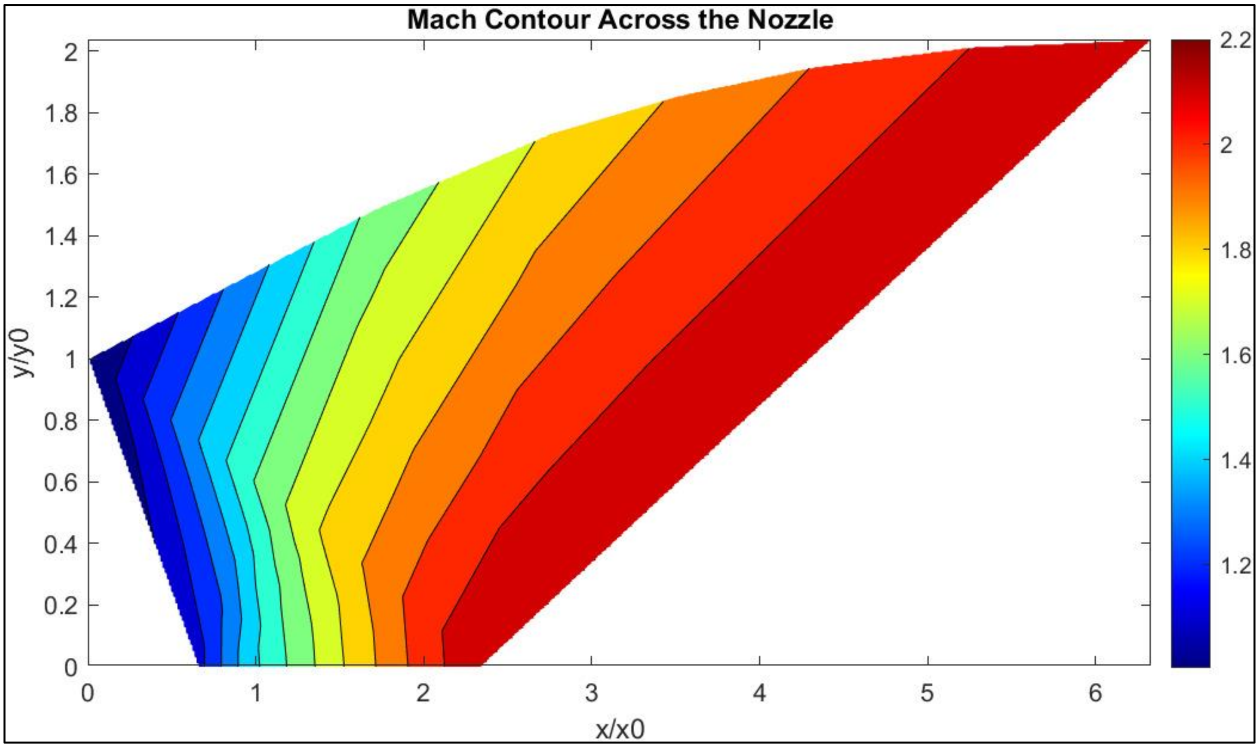

Figure 4

Observing figure (4), the trend of Mach number increases the closer we are to the exit of the nozzle. Both the chocked throat condition of 1 and exit Mach number of 2.2 are satisfied. Additionally, at node 26 we can see that the Mach number is satisfied for 2.2 as well. As a result, the exit conditions around the axis of symmetry are satisfied at 37% of the nozzle length.

5-Velocity

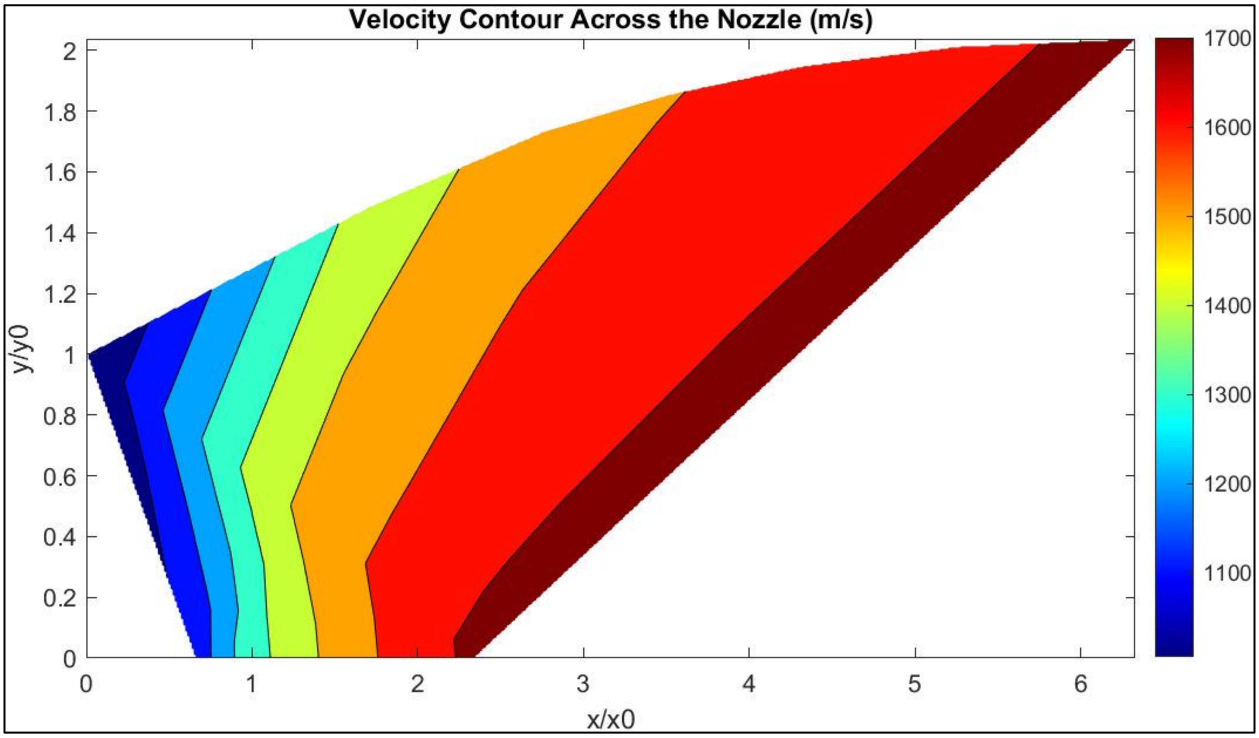

Figure 5

Figure (5) follows the same trend as the Mach number. This confirms that the follow is accelerating towards the exit of the nozzle. The velocity value at the throat is 1131 m/s and at the end of the nozzle a velocity value ıs 1722 m/s.

6-Pressure

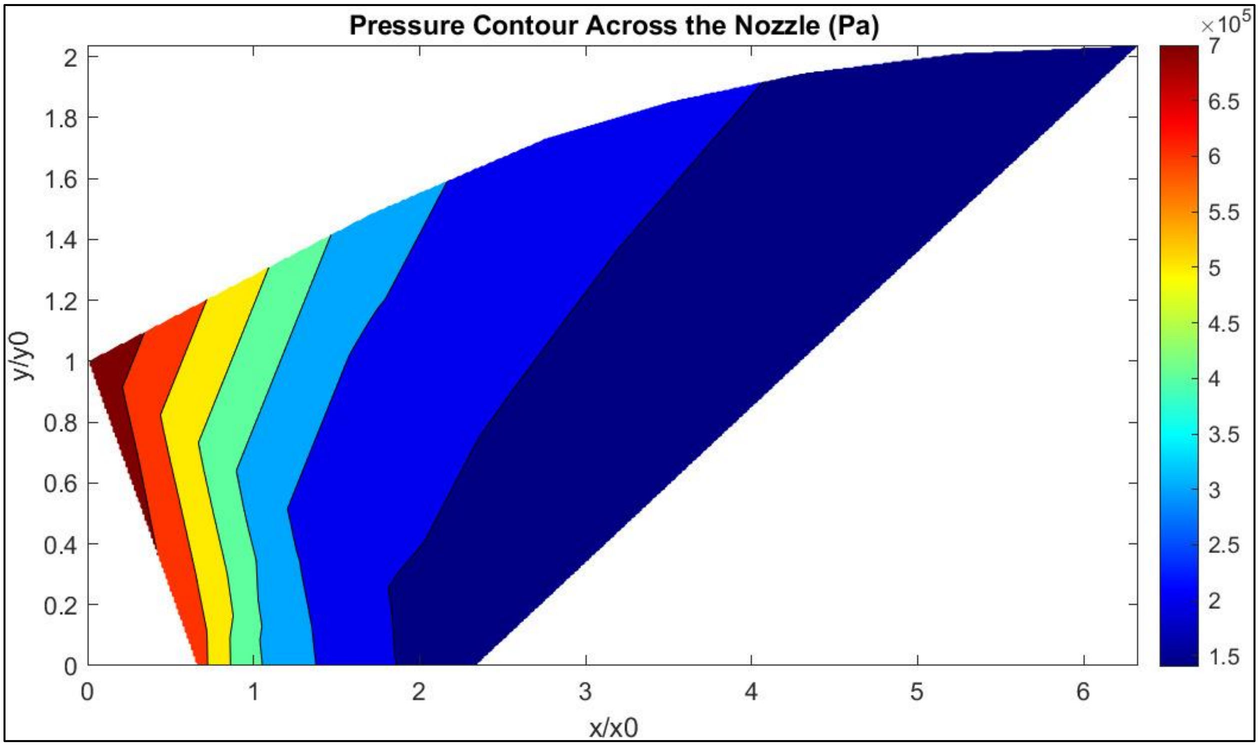

Figure 6

As observed from figure (6), the static pressure reduces as you make your way down to the end of the nozzle. The maximum value at the throat of 650030 Pa and the minimum value of 140240 Pa at the end of the nozzle. This confirms the idea that in order to accelerate the velocity we are losing pressure in the process.

Temperature (K)	Mach number	Density (kg/m^3)	Velocity (m/s)	Pressure (Pa)
1525.42	2.2	0.32	1720	140240

Table 1: Conditions at Nozzle Exit

Temperature (K)	Mach number	Density (kg/m^3)	Velocity (m/s)	Pressure (Pa)
2493	1	1.07	1021	650030

Table 2: Conditions at the throat

Node	M	θ	v	μ	k^-	k^+	$\frac{x}{x_0}$	$\frac{y}{y_0}$
1	1.1616	0.8662	0.8662	68.5924	1.7325	0	0.6551	0
2	1.2674	3.8662	3.8662	55.5686	7.7325	0	0.7639	0.1863
3	1.3632	6.8662	6.8662	48.9898	13.7325	0	0.8458	0.3089
4	1.4548	9.8662	9.8662	44.3557	19.7325	0	0.9165	0.4083
5	1.5446	12.8662	12.8662	40.7291	25.7325	0	0.9816	0.4972
6	1.6341	15.8662	15.8662	37.7318	31.7325	0	1.0436	0.5812
7	1.6341	15.8662	15.8662	37.7318	31.7325	0	1.711	1.4863
8	1.4548	0	7.7325	47.5119	7.7325	-7.7325	0.9495	0
9	1.5446	3	10.7325	43.2271	13.7325	-7.7325	1.0801	0.1226
10	1.6341	6	13.7325	39.8098	19.7325	-7.7325	1.195	0.2299
11	1.724	9	16.7325	36.9533	25.7325	-7.7325	1.3029	0.3312
12	1.815	12	19.7325	34.4924	31.7325	-7.7325	1.4078	0.4314
13	1.815	12	19.7325	34.4924	31.7325	-7.7325	2.7532	1.7312
14	1.6341	0	13.7325	39.8098	13.7325	-13.7325	1.2386	0
15	1.724	3	16.7325	36.9533	19.7325	-13.7325	1.381	0.1109
16	1.815	6	19.7325	34.4924	25.7325	-13.7325	1.5166	0.2184
17	1.9077	9	22.7325	32.3266	31.7325	-13.7325	1.6502	0.3271
18	1.9077	9	22.7325	32.3266	31.7325	-13.7325	3.4975	1.8524
19	1.815	0	19.7325	34.4924	19.7325	-19.7325	1.5509	0
20	1.9077	3	22.7325	32.3266	25.7325	-19.7325	1.7152	0.1102
21	2.0025	6	25.7325	30.39	31.7325	-19.7325	1.8795	0.2242
22	2.0025	6	25.7325	30.39	31.7325	-19.7325	4.3182	1.9476
23	2.0025	0	25.7325	30.39	25.7325	-25.7325	1.9104	0
24	2.0998	3	28.7325	28.6365	31.7325	-25.7325	2.108	0.1165
25	2.0998	3	28.7325	28.6365	31.7325	-25.7325	5.2529	2.0124
26	2.2002	0	31.7325	27.0329	31.7325	-31.7325	2.3426	0
27	2.2002	0	31.7325	27.0329	31.7325	-31.7325	6.3355	2.0374

Table 3: Mach and Prandtl-Meyer relationships for 27 nodes

Node	M	$T_S(kelvin)$	$v\ (m\ /\ s)$	$\rho\ \ (kg\ /\ m^3)$	$P_S * 10^5(Pa)$
1	1.1616	2362	1131	0.9578	6.5003
2	1.2674	2270	1210	0.8674	5.6578
3	1.3632	2187	1278	0.7899	4.963
4	1.4548	2107	1338	0.7202	4.3609
5	1.5446	2030	1395	0.6563	3.8289
6	1.6341	1955	1448	0.5971	3.3548
7	1.6341	1955	1448	0.5971	3.3548
8	1.4548	2107	1338	0.7202	4.3609
9	1.5446	2030	1395	0.6563	3.8289
10	1.6341	1955	1448	0.5971	3.3548
11	1.724	1881	1499	0.5422	2.9308
12	1.815	1808	1547	0.4911	2.5514
13	1.815	1808	1547	0.4911	2.5514
14	1.6341	1955	1448	0.5971	3.3548
15	1.724	1881	1499	0.5422	2.9308
16	1.815	1808	1547	0.4911	2.5514
17	1.9077	1736	1593	0.4435	2.2122
18	1.9077	1736	1593	0.4435	2.2122
19	1.815	1808	1547	0.4911	2.5514
20	1.9077	1736	1593	0.4435	2.2122
21	2.0025	1664	1637	0.3993	1.9097
22	2.0025	1664	1637	0.3993	1.9097
23	2.0025	1664	1637	0.3993	1.9097
24	2.0998	1594	1680	0.3583	1.6407
25	2.0998	1594	1680	0.3583	1.6407
26	2.2002	1524	1722	0.3203	1.4024
27	2.2002	1524	1722	0.3203	1.4024

Table 4: data for all nodes

Conclusion

Using the method of characteristics for 27 nodes and specified Mach exit number of 2.2 at the given conditions; the minimum non-dimensional length was concluded to be $x = 2.01297$ and the corresponding physical length is $x = 0.32\text{m}$. Furthermore, the non-dimensional area of the throat at the exit is $y_{27} = \frac{A_e}{A^*} = 2.01297$ and the real isentropic ratio is 2.00497. The error between them was found to be 0.8% which is very accurate. It was confirmed that increasing the characteristic lines also increases the accuracy of the result at the cost of computational time. The exit Mach number flow conditions were satisfied at the node 26 of the centerline which is 37% of the total nozzle length. For each node the Mach number was computed using the Prandtl-Meyer relation and MOC. The temperature, density, pressure and velocity were then calculated using isentropic flow and choked conditions for each respective nodes Mach number and the results were compared to various supersonic nozzles and seem to be realistic. The contours for the full engine were only calculated along the nodal points which may reduce the accuracy between the nodes.

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Appendix

```
clear all;close all;clc


%% Input parameters %%
R=287.05287;
g=1.4;  % Cp/Cv ratio
Me=2.2; % Mach number at the exit of the nozzle
n=6;
At=9.042073087e-3; % Area of the thoat of the nozzle
theta_max=PrandtlMeyer(Me,g)*180/(2*pi);
theta_0=theta_max-15;
[v,KL,KR,theta]=MOC(theta_max,theta_0,n);
node=((3/2)*n+n^2/2);
M=zeros(1,node);
mu=zeros(1,node);
for i=1:node
M(i)=InvPrandtlMeyer(v(i));
mu(i)=Mu(M(i));
end
%% Plotting the Geometry of the Nozzle
figure(1)
d=1; % Non-Dimensional y co-ordinate of throat wall
i=1;
x=zeros(1,node);y=zeros(1,node);
wall=theta_max;
hold on
while (i<=n+1)
if i==1
x(i)=-d/(tand(theta(i)-mu(i)));
y(i)=0;
plot([0 x(i)],[d 0],'b','LineWidth',1);
else if i==n+1
x(i)=(y(i-1)-d-x(i-1)*tand((theta(i-1)+theta(i)+mu(i-
1)+mu(i))*0.5))/(tand(0.5*(wall+theta(i)))-tand((theta(i-1)+theta(i)+mu(i-
1)+mu(i))*0.5));
y(i)=d+x(i)*tand(0.5*(wall+theta(i)));
plot([x(i-1) x(i)],[y(i-1) y(i)],'r','LineWidth',1);
plot([0 x(i)],[d y(i)],'black','LineWidth',2);
else
x(i)=(d-y(i-1)+x(i-1)*tand(0.5*(mu(i-1)+theta(i-1)+mu(i)+theta(i))))/(tand(0.5*(mu(i-
1)+theta(i-1)+mu(i)+theta(i)))-tand(theta(i)-mu(i)));
y(i)=tand(theta(i)-mu(i))*x(i)+d;
plot([x(i-1) x(i)],[y(i-1) y(i)],'r','LineWidth',1);
plot([0 x(i)],[d y(i)],'b','LineWidth',1);
end
end
i=i+1;
end
```

```
h=i;k=0;i=h;
for j=1:n-1
while (i<=h+n-k-1)
if (i==h)
x(i)=x(i-n+k)-y(i-n+k)/(tand(0.5*(theta(i-n+k)+theta(i)-mu(i-n+k)-mu(i))));
y(i)=0;
plot([x(i-n+k) x(i)],[y(i-n+k) y(i)],'b','LineWidth',1);
else if (i==h+n-k-1)
x(i)=(x(i-n+k)*tand(0.5*(theta(i-n+k)+theta(i)))-y(i-n+k)+y(i-1)-x(i-
1)*tand((theta(i-1)+theta(i)+mu(i-1)+mu(i))*0.5))/(tand(0.5*(theta(i-n+k)+theta(i)))-
tand((theta(i-1)+theta(i)+mu(i-1)+mu(i))*0.5));
y(i)=y(i-n+k)+(x(i)-x(i-n+k))*tand(0.5*(theta(i-n+k)+theta(i)));
plot([x(i-1) x(i)],[y(i-1) y(i)],'r','LineWidth',1);
plot([x(i-n+k) x(i)],[y(i-n+k) y(i)],'black','LineWidth',2);
else
s1= tand(0.5*(theta(i)+theta(i-1)+mu(i)+mu(i-1)));
s2= tand(0.5*(theta(i)+theta(i-n+k)-mu(i)-mu(i-n+k)));
x(i)=(y(i-n+k)-y(i-1)+s1*x(i-1)-s2*x(i-n+k))/(s1-s2);
y(i)=y(i-1)+(x(i)-x(i-1))*s1;
plot([x(i-1) x(i)],[y(i-1) y(i)],'r','LineWidth',1);
plot([x(i-n+k) x(i)],[y(i-n+k) y(i)],'b','LineWidth',1);
end
end
i=i+1;
end
k=k+1;h=i;i=h;
end
title(sprintf('Characteristic lines for a Supersonic Nozzle'))
xlabel('x/x0');ylabel('y/y0');
axis equal
xlim([0 x(node)+0.5])
ylim([0 y(node)+0.5])

%% Mach number including throat mach
M=[1 1.16161275616419 1.26736618883442 1.36316417173638    1.45478567959913
      1.54464203256408    1.63407630119466    1.63407630119466    1.45478567959913
      1.54464203256408    1.63407630119466    1.72397410185001    1.81499361719474
      1.81499361719474    1.63407630119466    1.72397410185001    1.81499361719474
      1.90767120599972    1.90767120599972    1.81499361719474    1.90767120599972
      2.00247704388902    2.00247704388902    2.00247704388902    2.09984784829619
      2.09984784829619    2.20020813210228    2.20020813210228];
%% Calculation of all values
Ts = 3000.*(1+((g-1)/2)*(M).^2).^(-1); % Static temperature
velocity = M.*sqrt(g.*R.*Ts); % Velocity
tt=(((g+1)./2)./(1+((g-1)./2).*(M).^2)).^((g+1)./(2-2*g)); % Static Temperature to
Total Temperature ratio
Area=(At).*(1./M).*(tt); % Area of the Nozzle
Density=(10)./(velocity.*Area); % Density
P_s = 1500000.*(1+((g-1)/2)*(M).^2).^-(g/(g-1));
```

```
%% Contour for temperature
figure(2)
x=[0 0.655123051291342    0.763907141960214   0.845771795458690   0.916536655714000
      0.981619733130140   1.04361457072508    1.71095144849225    0.949482835670749
      1.08005471999580    1.19502884106223    1.30288948115994    1.40776295681694
      2.75320248633135    1.23864321942748    1.38095915961355    1.51656180253649
      1.65023088968390    3.49746294099476    1.55086372976895    1.71524506875150
      1.87951345408413    4.31821135695853    1.91036654109851    2.10801492727385
      5.25285558728972    2.34260622543538    6.33548750403854];
y=[1 0 0.186323853746791   0.308855818895714   0.408288587811591   0.497162471757082
      0.581197627351075   1.48628783895323    0      0.122644653181451
      0.229879929149396   0.331182204015620   0.431398603103537   1.73116542764767
      0      0.110851035756273   0.218368567103062   0.327133265870499
      1.85244676212798    0      0.110235044918529   0.224206034855789
      1.94756935858685    0      0.116522794863841   2.01237794279746    0
      2.03736575772961];
xx = linspace(min(x),max(x),500);
yy = linspace(min(y),max(y),500);
[X,Y] = meshgrid(xx,yy);
Z = griddata(x,y,Ts,X,Y);
contourf(X,Y,Z)
colormap jet
title('Temperature Contour Across the Nozzle');xlabel('x/x0');ylabel('y/y0');

%% Contour for Velocity
figure(3)
x=[0 0.655123051291342    0.763907141960214   0.845771795458690   0.916536655714000
      0.981619733130140   1.04361457072508    1.71095144849225    0.949482835670749
      1.08005471999580    1.19502884106223    1.30288948115994    1.40776295681694
      2.75320248633135    1.23864321942748    1.38095915961355    1.51656180253649
      1.65023088968390    3.49746294099476    1.55086372976895    1.71524506875150
      1.87951345408413    4.31821135695853    1.91036654109851    2.10801492727385
      5.25285558728972    2.34260622543538    6.33548750403854];
y=[1 0 0.186323853746791   0.308855818895714   0.408288587811591   0.497162471757082
      0.581197627351075   1.48628783895323    0      0.122644653181451
      0.229879929149396   0.331182204015620   0.431398603103537   1.73116542764767
      0      0.110851035756273   0.218368567103062   0.327133265870499
      1.85244676212798    0      0.110235044918529   0.224206034855789
      1.94756935858685    0      0.116522794863841   2.01237794279746    0
      2.03736575772961];
xx = linspace(min(x),max(x),500);
yy = linspace(min(y),max(y),500);
[X,Y] = meshgrid(xx,yy);
Z = griddata(x,y,velocity,X,Y);
contourf(X,Y,Z)
colormap jet
title('Velocity Contour Across the Nozzle');xlabel('x/x0');ylabel('y/y0');
```

```
%% Contour for Density
figure(4)
x=[0 0.655123051291342    0.763907141960214   0.845771795458690   0.916536655714000
      0.981619733130140   1.04361457072508    1.71095144849225    0.949482835670749
      1.08005471999580    1.19502884106223    1.30288948115994    1.40776295681694
      2.75320248633135    1.23864321942748    1.38095915961355    1.51656180253649
      1.65023088968390    3.49746294099476    1.55086372976895    1.71524506875150
      1.87951345408413    4.31821135695853    1.91036654109851    2.10801492727385
      5.25285558728972    2.34260622543538    6.33548750403854];
y=[1 0 0.186323853746791   0.308855818895714   0.408288587811591   0.497162471757082
      0.581197627351075   1.48628783895323    0      0.122644653181451
      0.229879929149396   0.331182204015620   0.431398603103537   1.73116542764767
      0      0.110851035756273   0.218368567103062   0.327133265870499
      1.85244676212798    0      0.110235044918529   0.224206034855789
      1.94756935858685    0      0.116522794863841   2.01237794279746    0
      2.03736575772961];
xx = linspace(min(x),max(x),500);
yy = linspace(min(y),max(y),500);
[X,Y] = meshgrid(xx,yy);
Z = griddata(x,y,Density,X,Y);
contourf(X,Y,Z)
colormap jet
title('Density Contour Across the Nozzle');xlabel('x/x0');ylabel('y/y0');

%% Contour for Mach number
figure(5)
x=[0 0.655123051291342    0.763907141960214   0.845771795458690   0.916536655714000
      0.981619733130140   1.04361457072508    1.71095144849225    0.949482835670749
      1.08005471999580    1.19502884106223    1.30288948115994    1.40776295681694
      2.75320248633135    1.23864321942748    1.38095915961355    1.51656180253649
      1.65023088968390    3.49746294099476    1.55086372976895    1.71524506875150
      1.87951345408413    4.31821135695853    1.91036654109851    2.10801492727385
      5.25285558728972    2.34260622543538    6.33548750403854];
y=[1 0 0.186323853746791   0.308855818895714   0.408288587811591   0.497162471757082
      0.581197627351075   1.48628783895323    0      0.122644653181451
      0.229879929149396   0.331182204015620   0.431398603103537   1.73116542764767
      0      0.110851035756273   0.218368567103062   0.327133265870499
      1.85244676212798    0      0.110235044918529   0.224206034855789
      1.94756935858685    0      0.116522794863841   2.01237794279746    0
      2.03736575772961];
xx = linspace(min(x),max(x),500);
yy = linspace(min(y),max(y),500);
[X,Y] = meshgrid(xx,yy);
Z = griddata(x,y,M,X,Y);
contourf(X,Y,Z)
colormap jet
title('Mach Contour Across the Nozzle');xlabel('x/x0');ylabel('y/y0');

vd=v*(180/pi);





%% Contours for Pressure
```

```
figure(6)
x=[0 0.655123051291342    0.763907141960214   0.845771795458690   0.916536655714000
       0.981619733130140   1.04361457072508    1.71095144849225    0.949482835670749
       1.08005471999580    1.19502884106223    1.30288948115994    1.40776295681694
       2.75320248633135    1.23864321942748    1.38095915961355    1.51656180253649
       1.65023088968390    3.49746294099476    1.55086372976895    1.71524506875150
       1.87951345408413    4.31821135695853    1.91036654109851    2.10801492727385
       5.25285558728972    2.34260622543538    6.33548750403854];
y=[1 0 0.186323853746791   0.308855818895714   0.408288587811591   0.497162471757082
       0.581197627351075   1.48628783895323    0      0.122644653181451
       0.229879929149396   0.331182204015620   0.431398603103537   1.73116542764767
       0      0.110851035756273   0.218368567103062   0.327133265870499
       1.85244676212798    0      0.110235044918529   0.224206034855789
       1.94756935858685    0      0.116522794863841   2.01237794279746    0
       2.03736575772961];
xx = linspace(min(x),max(x),500);
yy = linspace(min(y),max(y),500);
[X,Y] = meshgrid(xx,yy);
Z = griddata(x,y,P_s,X,Y);
contourf(X,Y,Z)
colormap jet
title('Pressure Contour Across the Nozzle');xlabel('x/x0');ylabel('y/y0');

%% Prandtl-meyer
function M=InvPrandtlMeyer(v)
v=v*pi/180;
A=1.3604;
B=0.0962;
C=-0.5127;
D=-0.6722;
E=-0.3278;
v_0=0.5*pi*(sqrt(6)-1);
y=(v/v_0)^(2/3);
M=(1 + A*y + B*y^2 + C*y^3)/(1 + D*y + E*y^2);
end
function v=PrandtlMeyer(M,gamma)
v=sqrt((gamma+1)/(gamma-1))*atan(sqrt((gamma-1)/(gamma+1)*(M^2-1)))-atan(sqrt(M^2-
1));
end
function [v,KL,KR,theta]=MOC(theta_max,theta_0,n)
dtheta=(theta_max-theta_0)/(n-1);
node=0.5*n*(4+n-1);
KL=zeros(1,node);
KR=zeros(1,node);
theta=zeros(1,node);
v=zeros(1,node);
for i=1:n
theta(i)=theta_0+(i-1)*dtheta;
v(i)=theta(i);
KL(i)=theta(i)-v(i);
KR(i)=theta(i)+v(i);
end
i=n+1;
theta(i)=theta(i-1);
v(i)=v(i-1);
```

```
KL(i)=KL(i-1);
KR(i)=KR(i-1);
p=2;
q=n+2;
for k=1:n-1
j=p;
h=q;
theta(h)=0;
KR(h)=KR(j);
v(h)=KR(j)-theta(h);
KL(h)=theta(h)-v(h);
j=j+1;
for i=h+1:n-p+q
KR(i)=KR(j);
KL(i)=KL(i-1);
theta(i)=0.5*(KL(i)+KR(i));
v(i)=0.5*(KR(i)-KL(i));
j=j+1;
end
if i==n-p+q
h=i+1;
else
h=h+1;
end
theta(h)=theta(h-1);
v(h)=v(h-1);
KL(h)=KL(h-1);
KR(h)=KR(h-1);
p=p+1;
q=h+1;
end
end
function mu=Mu(M)
mu=asind(1/M);
end
```