Equations for solving functions.

Length of section = L
Height at start of section = Y_0
Height at end of section = Y_L
Slope at start of section = S_0
Slope at end of sextion = S_Y

$$f(x) = Ax^3 + Bx^2 + Cx + D \tag{1}$$

$$f'(x) = 3Ax^2 + 2Bx + C \tag{2}$$

$$f(0) = Y_0 = D \tag{3}$$

$$f(L) = Y_L = AL^3 + BL^2 + CL + D \tag{4}$$

$$f'(0) = S_0 = C \tag{5}$$

$$f'(L) = S_L = 3AL^2 + 2BL^2 + S_0 \tag{6}$$

Subtract $2f(L) - Lf'(L)$ to get A

$$A = \frac{L(S_0 + S_L) + 2(Y_0 - Y_L)}{L^3} \tag{7}$$

Solve for B in $f'(x)'$

$$B = \frac{(-2S_0 - S_L) - 3(Y_0 - Y_L)}{L^2} \tag{8}$$

$$C = S_0 \tag{9}$$

$$D = Y_0 \tag{10}$$

Convert range from $[0 : L]$ to $[0 : 1]$

$$A = L(S_0 + S_L) + 2(Y_0 - Y_L) \tag{11}$$

$$B = (-2S_0 - S_L) - 3(Y_0 - Y_L) \tag{12}$$

$$C = LS_0 \tag{13}$$

$$D = Y_0 \tag{14}$$