Machine Learning and Pattern Recognition
A High Level Overview

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(Main bulk of slides kindly provided by **Prof. Sandra Avila**)
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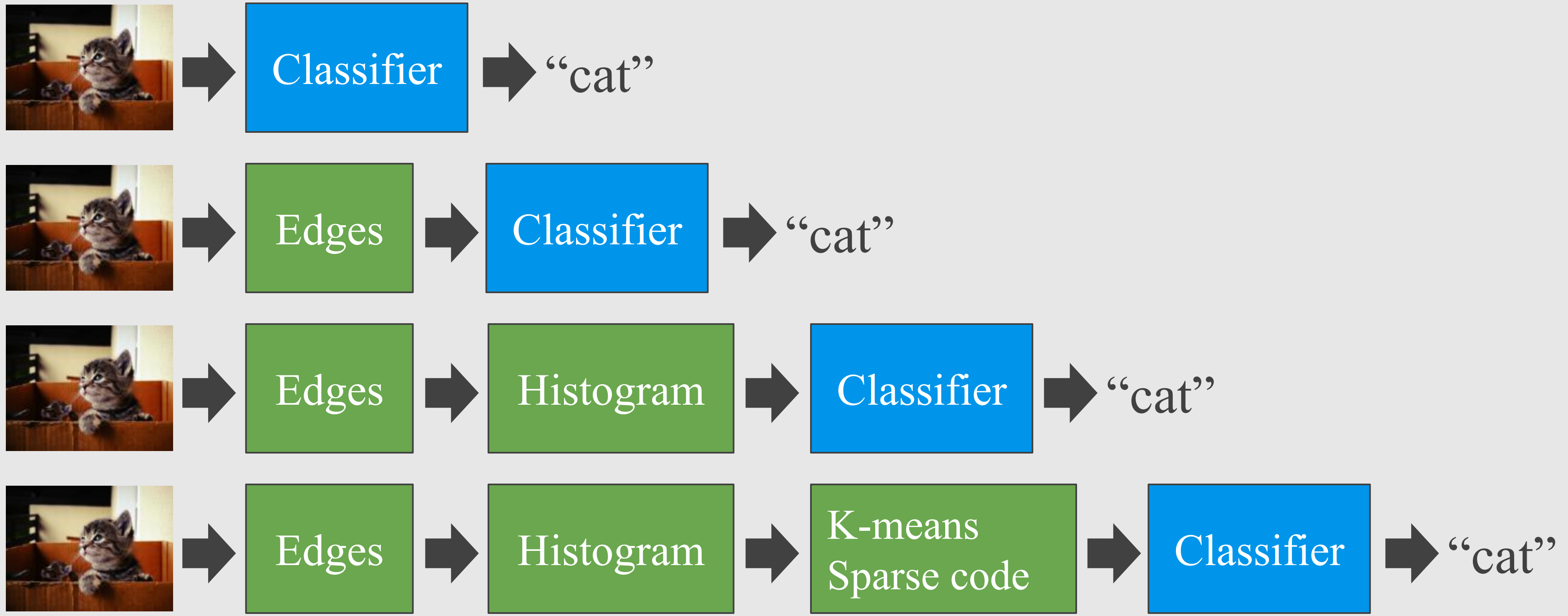
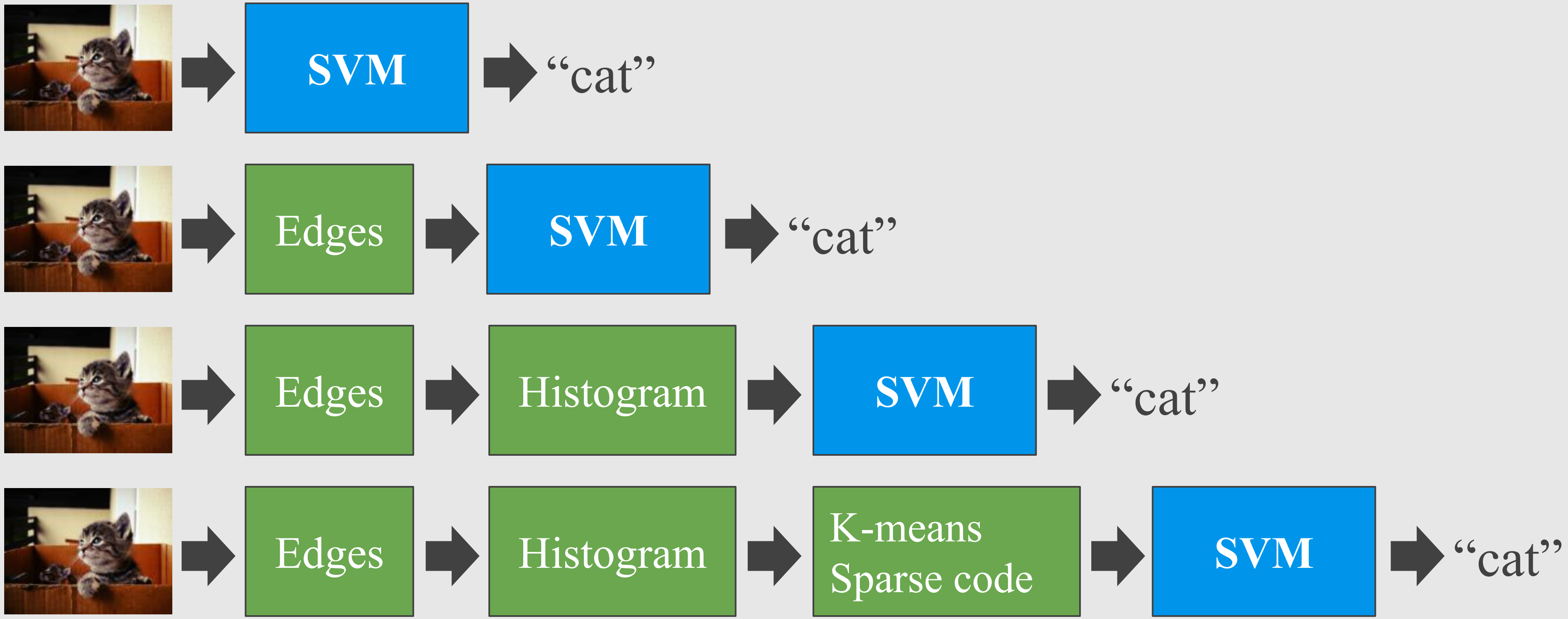
MC886/MO444

SVMs are among the best “off-the-shelf” supervised learning algorithm.

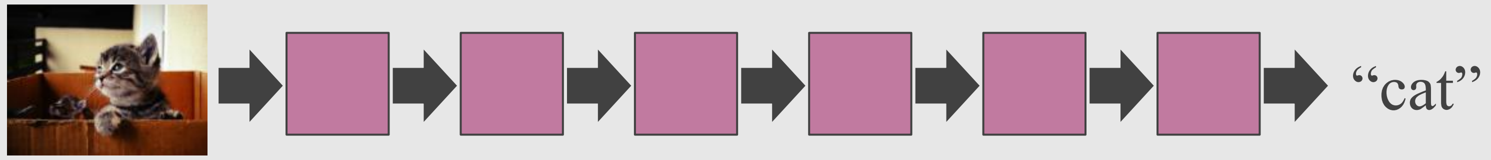
Traditional Recognition



Traditional Recognition



Deep Learning



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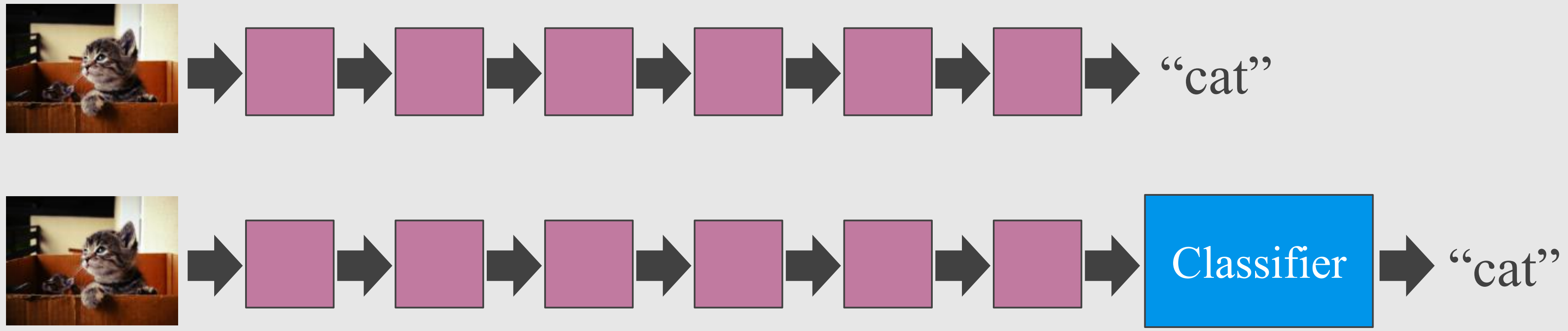

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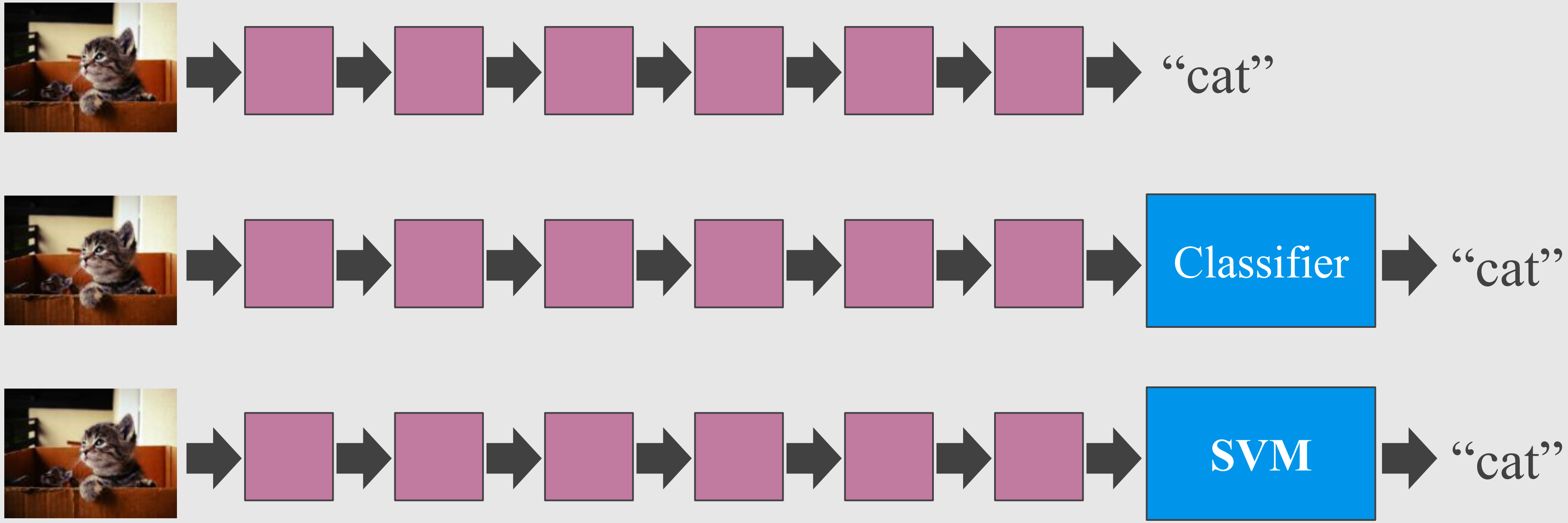
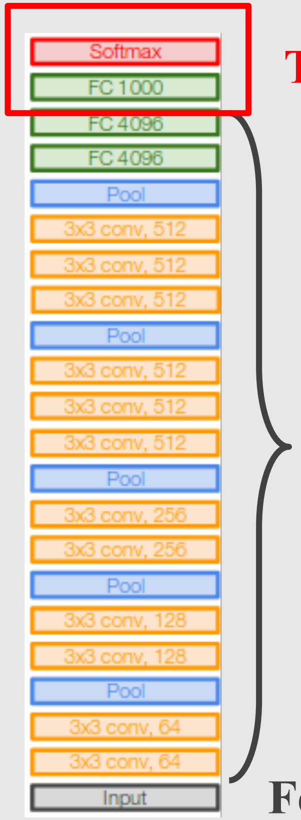


Transfer Learning

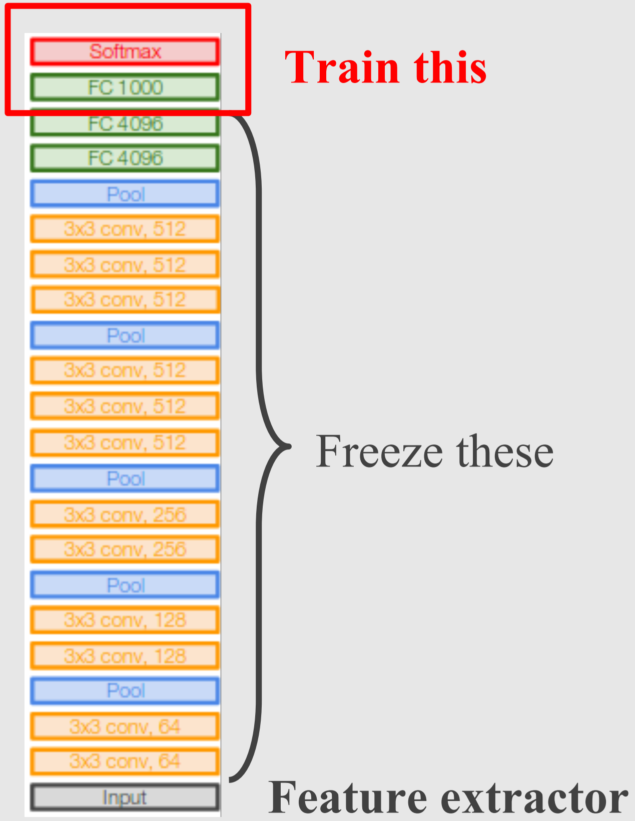


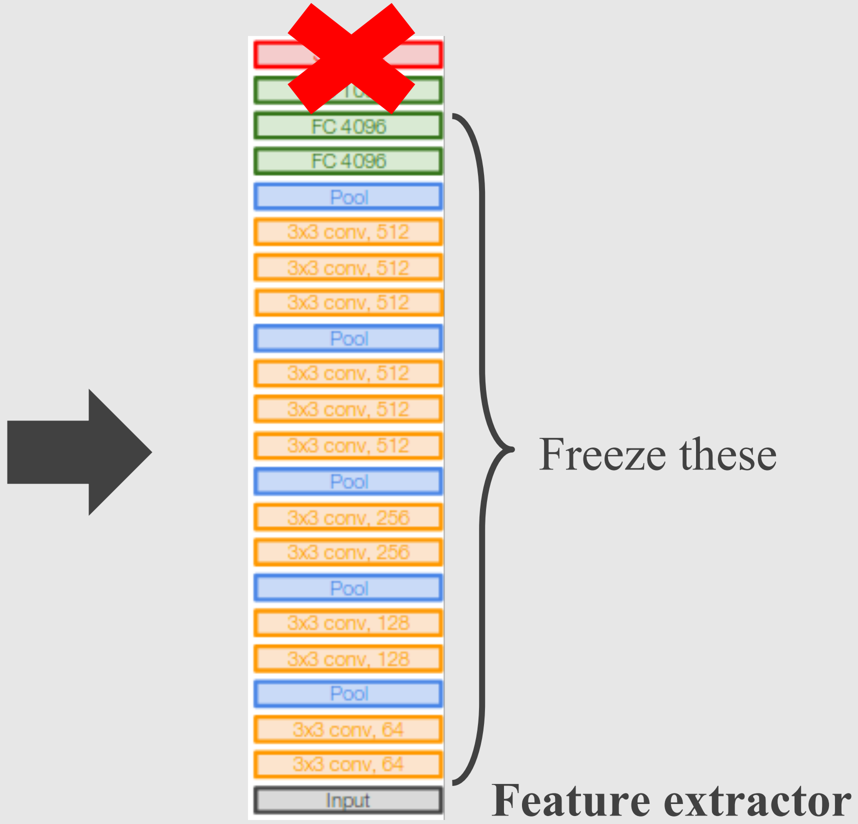
Train this

Freeze these

Feature extractor

Transfer Learning





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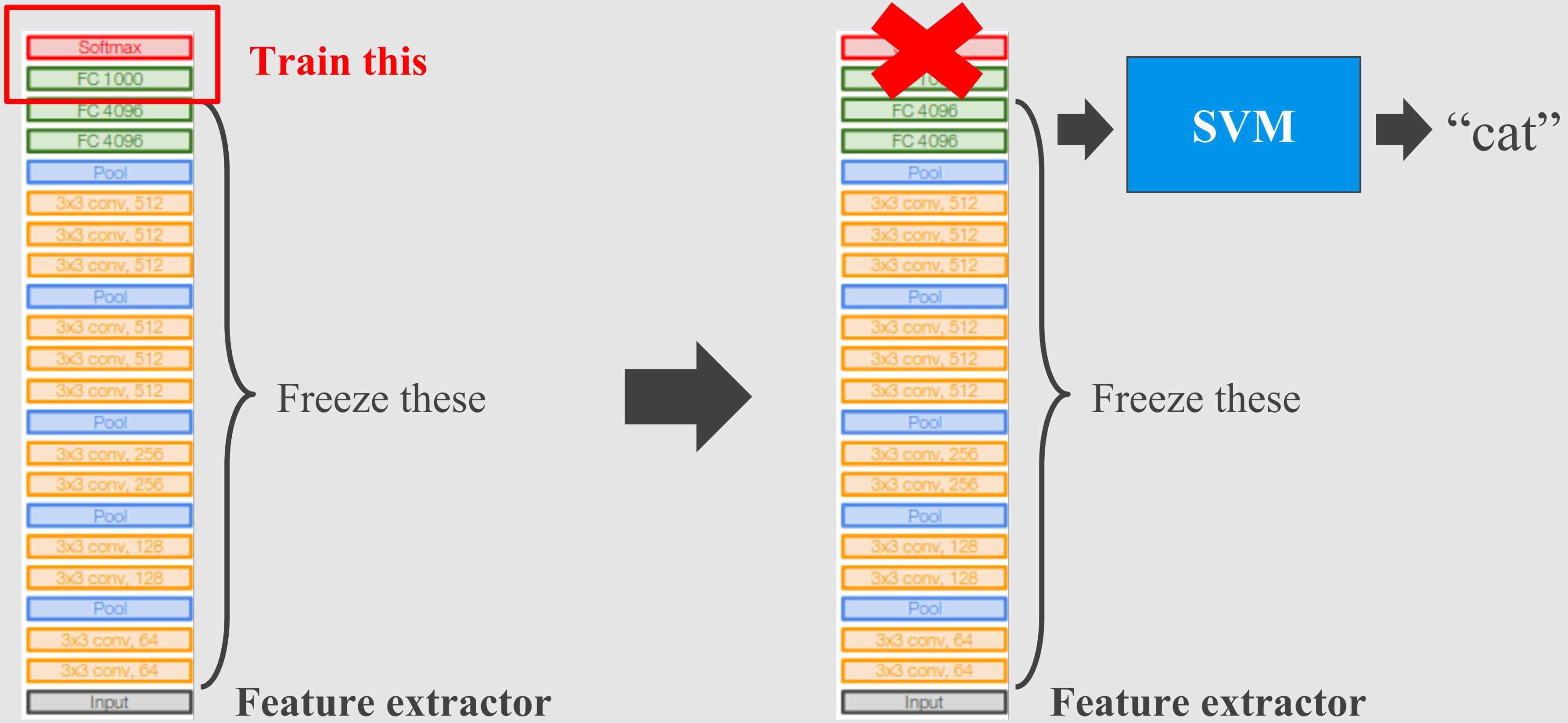
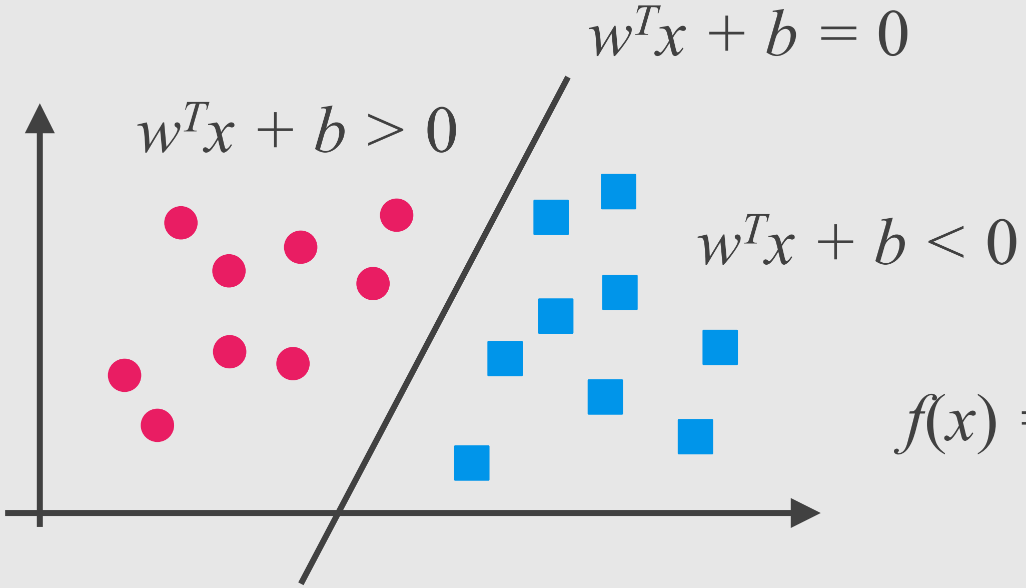


What is Support Vector Machine?

Linear Classifier

Binary classification can be viewed as the task of separating classes in feature space:

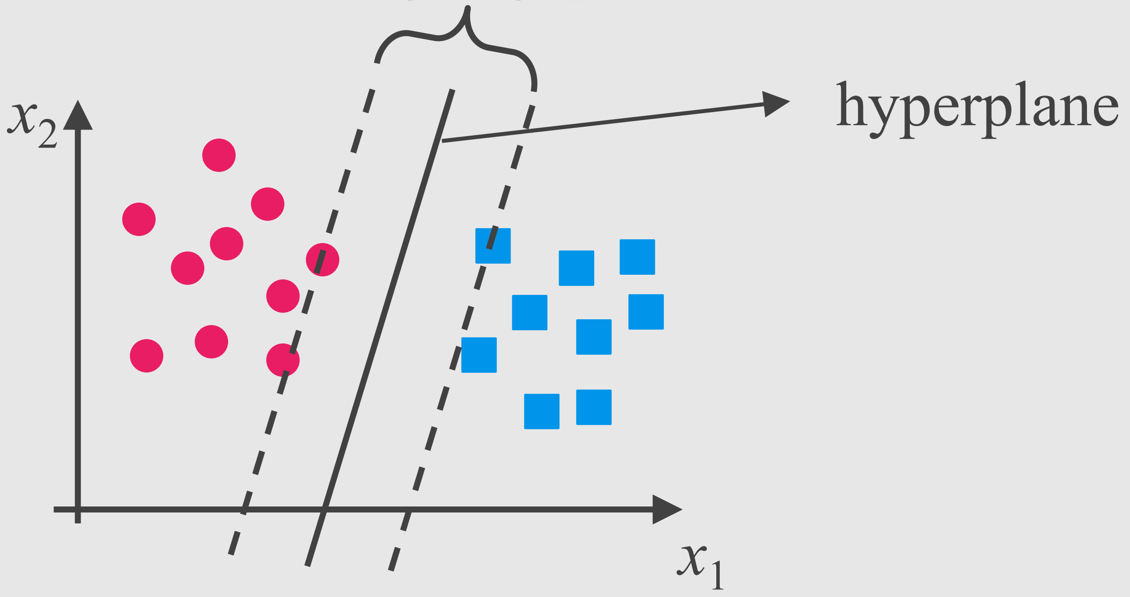


$f(x) = \text{sign}(w^Tx + b)$

Support Vector Machine

[Vapnik and Chervonenkis, 1964; Vapnik, 1982; Vapnik, 1995]

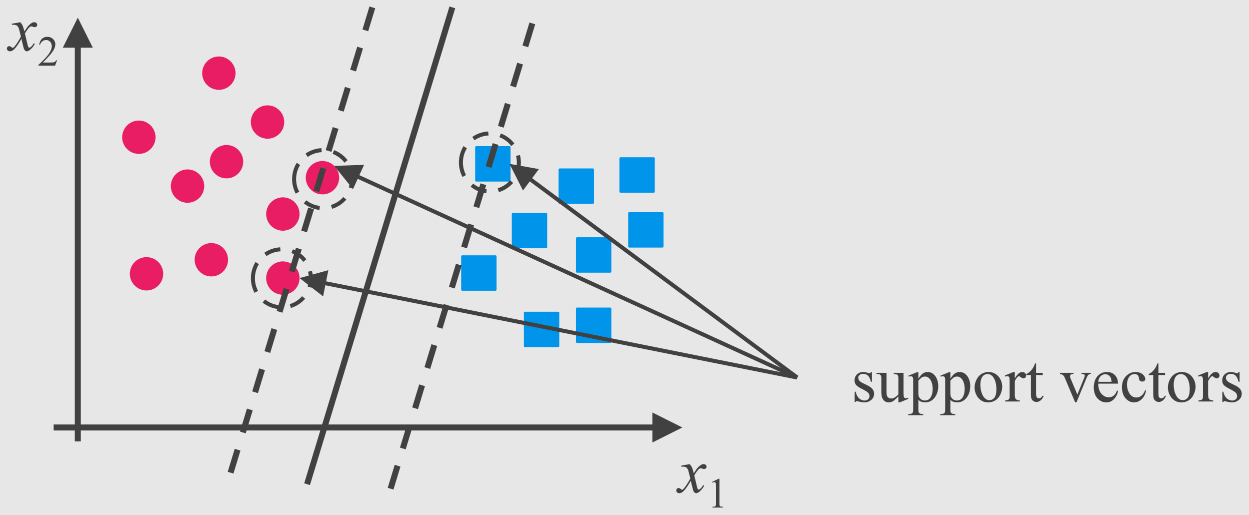
Idea of separating data with **a large "gap"**.



Support Vector Machine

[Vapnik and Chervonenkis, 1964; Vapnik, 1982; Vapnik, 1995]

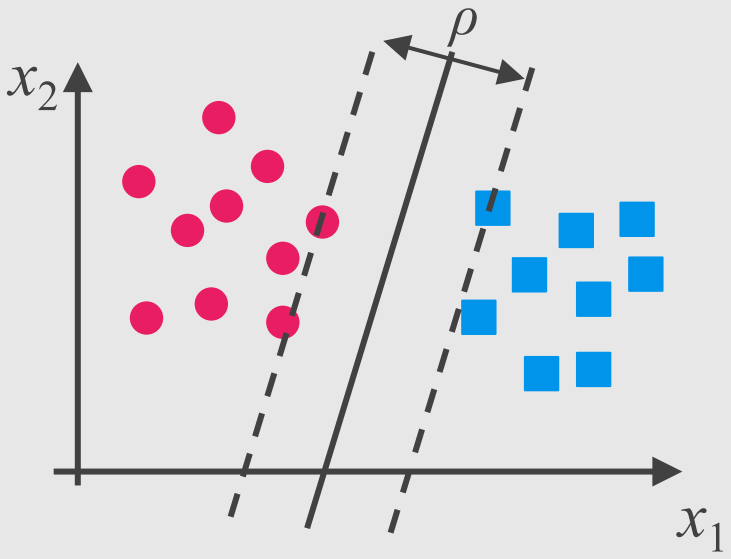
Examples closest to the hyperplane are support vectors.



Support Vector Machine

[Vapnik and Chervonenkis, 1964; Vapnik, 1982; Vapnik, 1995]

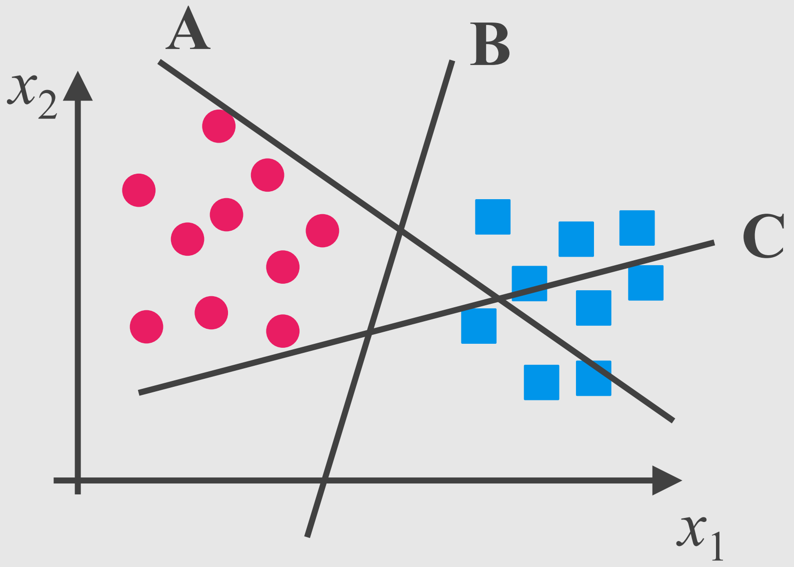
Margin ρ of the separator is the distance between support vectors.



How does SVM work?

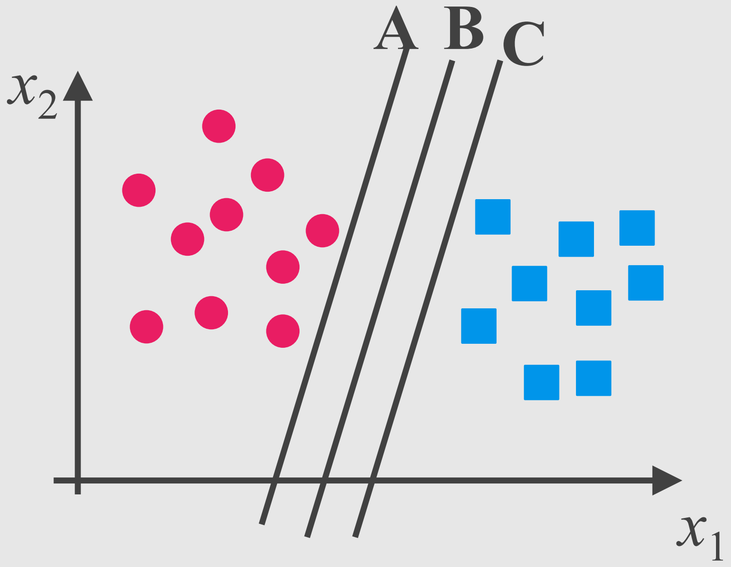
How can we identify the right hyperplane?

Scenario 1



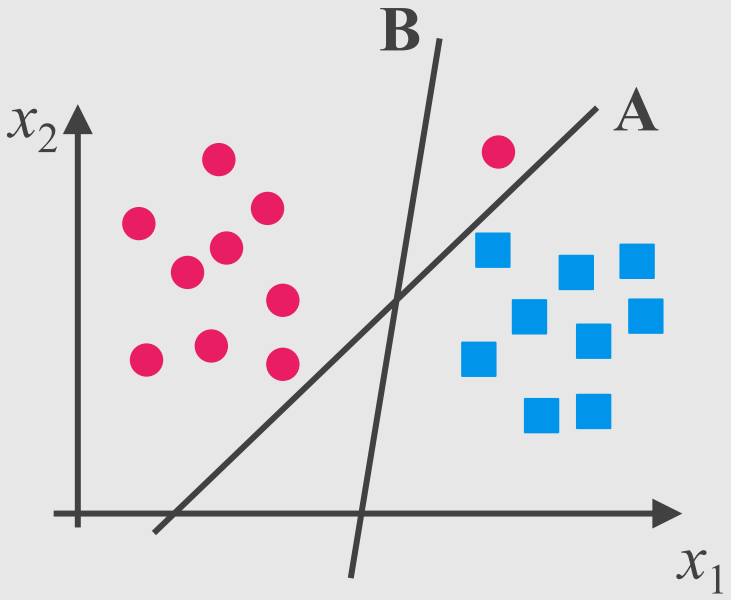
How can we identify the right hyperplane?

Scenario 2



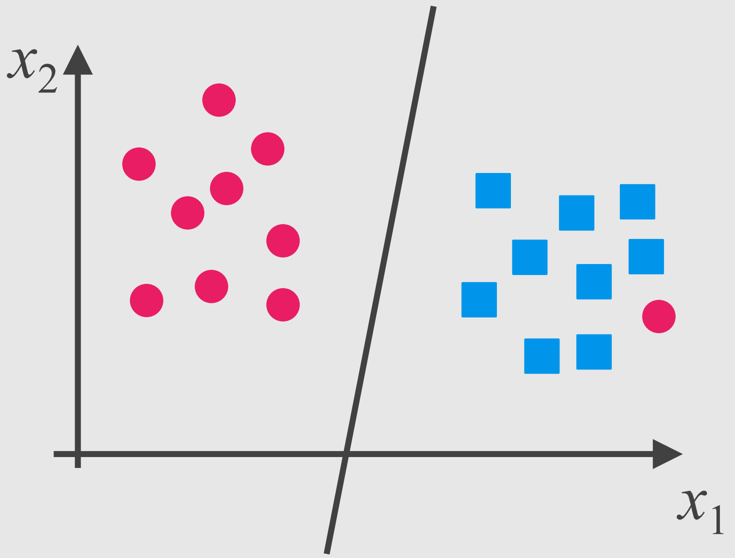
How can we identify the right hyperplane?

Scenario 3



How can we identify the right hyperplane?

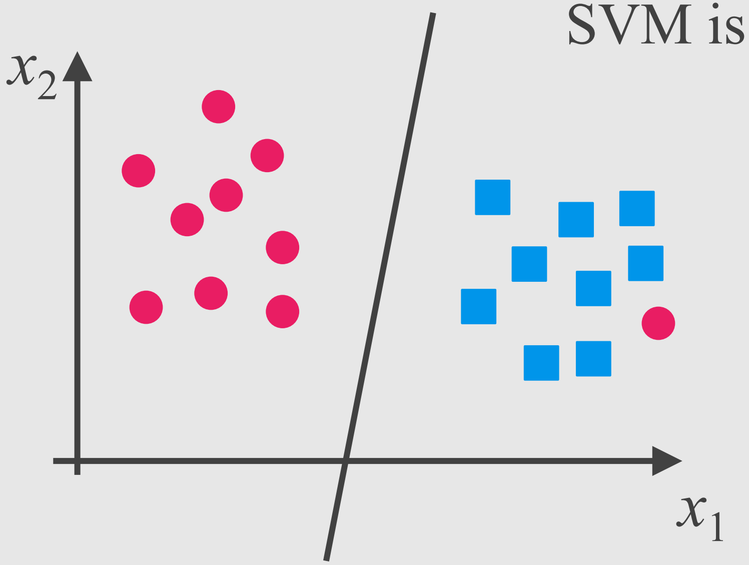
Scenario 4



How can we identify the right hyperplane?

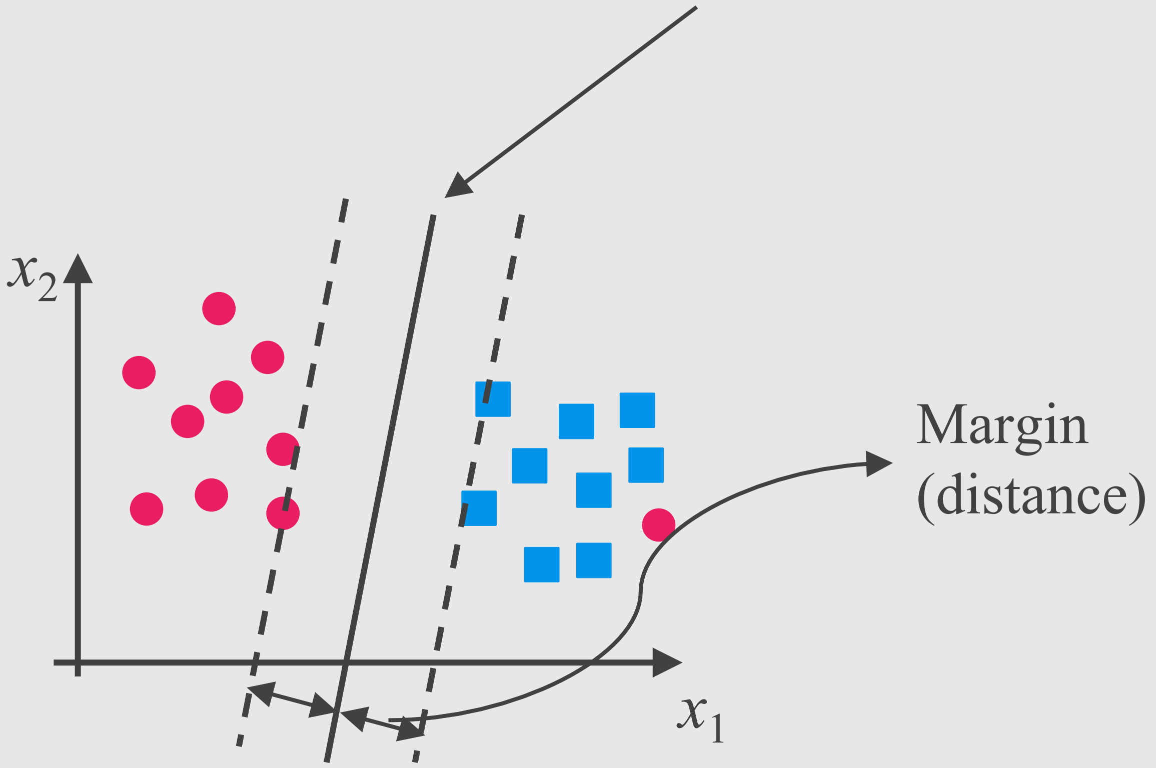
Scenario 4

SVM is robust to outliers



How can we identify the right hyperplane?

Scenario 4



SVM: Notation

We will be considering a **linear classifier for a binary classification** problem with labels y and features x.

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- Parameters: w, b (instead of vector θ)

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- Class labels: $y \in \{-1,1\}$ (instead of $\{0,1\}$)
- Parameters: w, b (instead of vector θ)
- Classifier: $h_{w,b}(x) = g(w^T x + b)$
    - $g(z) = 1$ if $z \geq 0$, and $g(z) = -1$ otherwise

SVM: The Optimal Hyperplane

Let training set $\{(x_i, y_i)\}_{i=1..m}$, $x_i \in \Re^d$, $y_i \in \{-1, 1\}$ be separated by a hyperplane with margin ρ. Then for each training example (x_i, y_i):

$$\begin{array}{l} w^T x + b \leq -\rho/2 \text{ if } y_i = -1 \\ w^T x + b \geq \rho/2 \text{ if } y_i = 1 \end{array} \quad \Longleftrightarrow \quad y_i(w^T x + b) \geq \rho/2$$

SVM: The Optimal Hyperplane

For every support vector x_s the previous inequality is an equality. After rescaling w and b by ρ/2 in the equality, we obtain that distance between each x_s and the hyperplane is

$$\mathrm{r} = \frac{y_s\,(w^T x_s + b)}{||w||}$$

SVM: The Optimal Hyperplane

Then we can formulate the quadratic optimization problem:

$$\min_{w,b} \ \tfrac{1}{2}||w||^2$$
$$\text{s.t. } y_i(w^T x + b) \geq 1,\ i = \{1, \ldots, m\}$$

http://cs229.stanford.edu/notes/cs229-notes3.pdf

Need to optimize a quadratic function subject to linear constraints

SVM: The Optimal Hyperplane

The solution involves constructing a dual problem where a Lagrange multiplier α i is associated with every inequality constraint in the primal (original) problem:

Find $\alpha_i \ldots \alpha_n$ such that

$\mathbf{Q}(\boldsymbol{\alpha}) = \Sigma\, \alpha_i - \frac{1}{2} \Sigma\Sigma\, \alpha_i \alpha_j\, y_i y_j\, \boldsymbol{x_i}^T \boldsymbol{x_j}$ is maximized and

(1) $\Sigma\, \alpha_i y_i = 0$

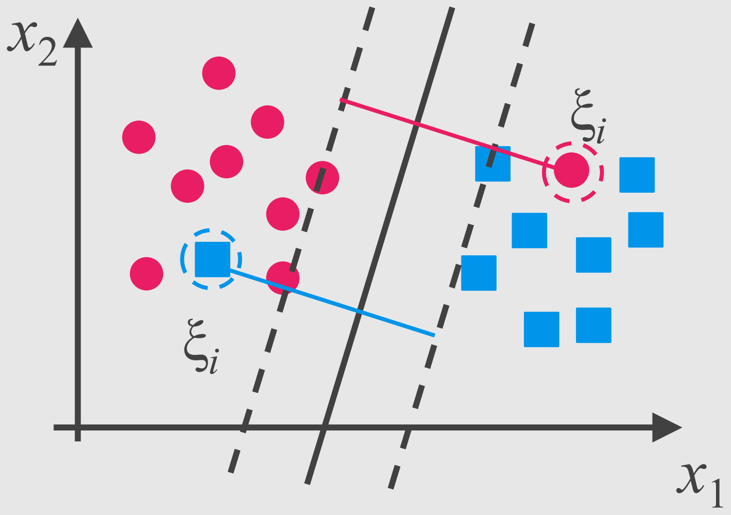
(2) $\alpha_i \geq 0$ for all α_i

Soft Margin Classification

What if the training set is not linearly separable?

Soft Margin Classification

Slack variables ξ_i can be added to allow misclassification of difficult or noisy examples, resulting margin called **soft**.



Soft Margin Classification

Modified formulation incorporates slack variables:

$$\boxed{\begin{gathered}\min_{w,b,\xi}\ \tfrac{1}{2}||w||^2\ + C\Sigma\xi_i \\ \text{s.t. } y_i(w^Tx + b) \geq 1 - \xi_i,\ \xi_i \geq 0,\ i = \{1, \ldots, m\}\end{gathered}}$$

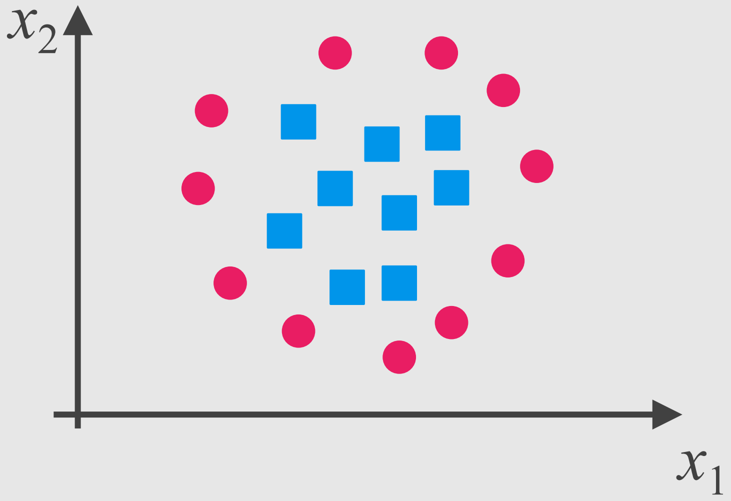
Parameter *C* can be viewed as a way to control overfitting:
it “trades off” the relative importance of maximizing the margin and fitting the training data.

Linear SVMs: Overview

- The classifier is a *separating hyperplane*.
- Most "important" training points are support vectors; they define the hyperplane.
- Quadratic optimization algorithms can identify which training points x_i are support vectors with non-zero Lagrangian multipliers α_i.

How can we identify the right hyperplane?

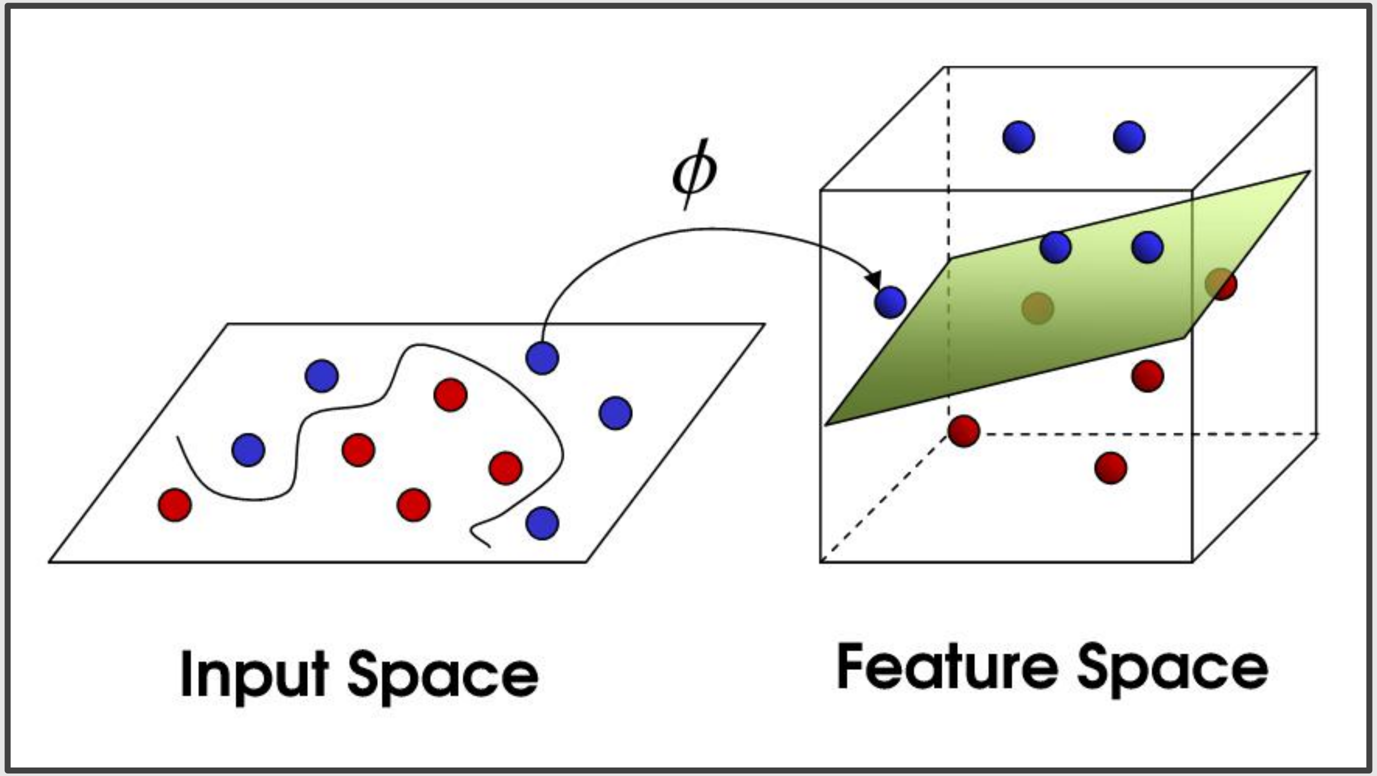
Scenario 5



Non-linear SVMs: Feature spaces

General idea: the original feature space can always be mapped to some higher-dimensional feature space where the training set is separable.

$$\Phi : x \longrightarrow \varphi(x)$$



Kernel Trick

- The linear classifier relies on inner product between vector
  $K(x_i, x_j) = x_i^{\mathrm{T}} x_j$
- If every datapoint is mapped into high-dimensional space via some transformation Φ: $x \rightarrow \varphi(x)$, the inner product becomes:

$$\boldsymbol{K(x_i, x_j) = \varphi(x_i)^{\mathrm{T}} \varphi(x_j)}$$

- A kernel function is a function that is equivalent to an inner product in some feature space.

Kernelized SVM

Suppose you want to apply a 2nd degree polynomial transformation to a 2-dimensional training set, then train a linear SVM classifier on the transformed training set.

$$\varphi(x) = \varphi((x_1\ x_2)) = (x_1^2\ \ \sqrt{2}x_1x_2\ \ x_2^2)$$

The transformed vector is 3-dimensional instead of 2-dimensional.

Kernelized SVM

Let's look at what happens to a couple 2-dimensional vectors **a** and **b** if we apply this 2nd degree polynomial mapping then compute the dot product of the transformed vectors.

$$\varphi(\mathbf{a})^T \varphi(\mathbf{b}) =$$

Kernelized SVM

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$$\varphi(\mathbf{a})^T\varphi(\mathbf{b}) = (a_1^2 \;\; \sqrt{2}a_1a_2 \;\; a_2^2)^T(b_1^2 \;\; \sqrt{2}b_1b_2 \;\; b_2^2) =$$

Kernelized SVM

Let's look at what happens to a couple 2-dimensional vectors **a** and **b** if we apply this 2nd degree polynomial mapping then compute the dot product of the transformed vectors.

$$\begin{aligned}
\varphi(\mathbf{a})^{\mathrm{T}}\varphi(\mathbf{b}) &= (a_1{}^2 \;\; \sqrt{2}a_1a_2 \;\; a_2{}^2)^{\mathrm{T}}(b_1{}^2 \;\; \sqrt{2}b_1b_2 \;\; b_2{}^2) = \\
&= a_1{}^2b_1{}^2 + 2a_1a_2b_1b_2 + a_2{}^2b_2{}^2 = \\
&= (a_1b_1 + a_2b_2)^2 = \\
&= ((a_1 \; a_2)^{\mathrm{T}}(b_1 \; b_2))^2 = \\
&= (\mathbf{a}^{\mathrm{T}} \cdot \mathbf{b})^2
\end{aligned}$$

Kernelized SVM

The dot product of the transformed vectors is equal to the square of the dot product of the original vectors: $\varphi(\mathbf{a})^T\varphi(\mathbf{b}) = (\mathbf{a}^T \cdot \mathbf{b})^2$

Kernelized SVM

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So **you don't actually need to transform the training instances** at all: just replace the dot product by its square.

This is the essence of the kernel trick.

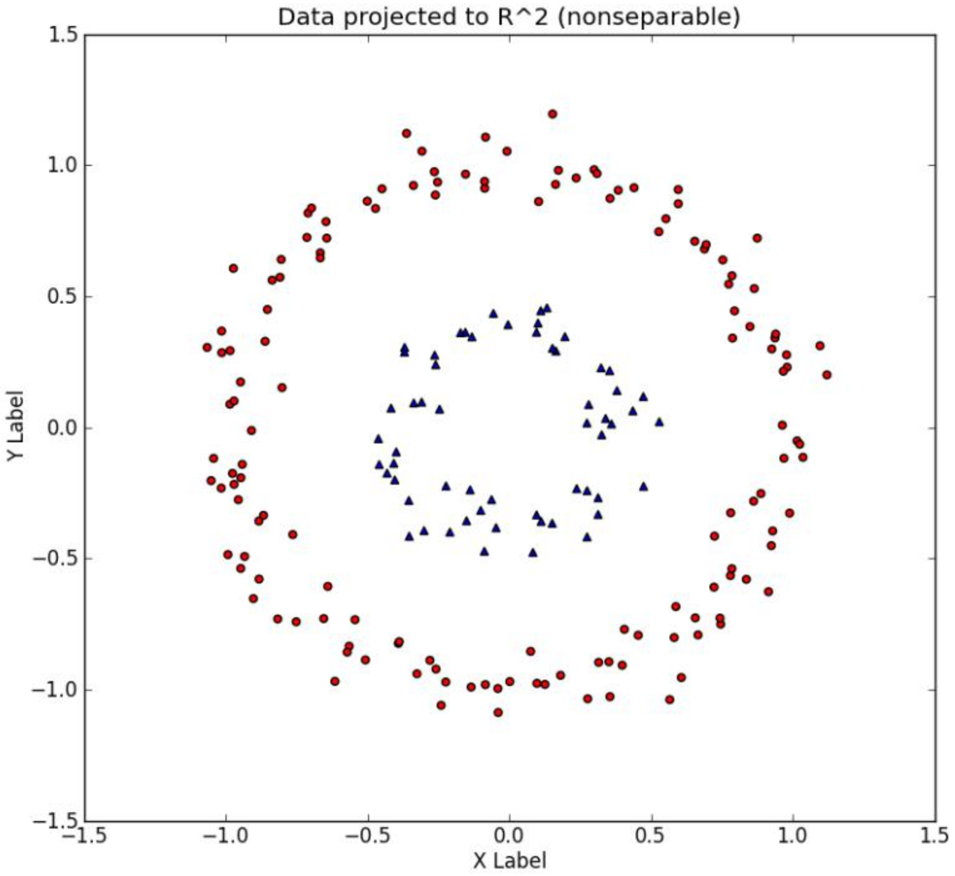
Kernelized SVM

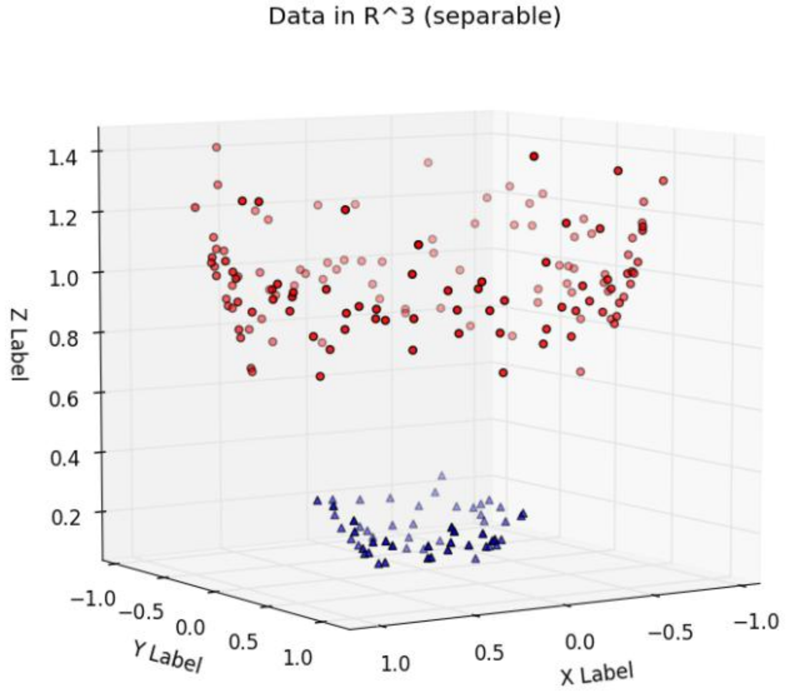
In Machine Learning, a ***kernel*** is a function capable of computing the dot product $\varphi(\mathbf{a})^T\varphi(\mathbf{b})$ based only on the original vectors **a** and **b**, without having to compute (or even to know about) the transformation φ.

Kernel Functions

- Linear SVM = Kernel SVM: $K(x_i, x_j) = x_i^T x_j$
- Nonlinear SVM: $K(x_i, x_j) = \varphi(x_i) \cdot \varphi(x_j)$

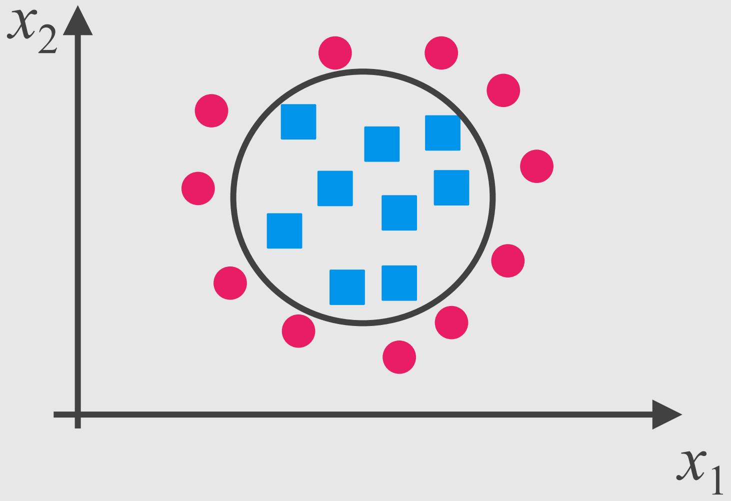
- Gaussian kernel: $K(x_i, x_j) = \exp(-\|x_i - x_j\|^2/(2\sigma^2))$
- Polynomial kernel: $K(x_i, x_j) = (x_i \cdot x_j + 1)^d$, d degree
- Chi-square kernel, histogram intersection kernel, string kernel, ...


Data projected to R^2 (nonseparable)
X Label
Y Label
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5


Data in R^3 (separable)
Z Label
Y Label
X Label
1.4
1.2
1.0
0.8
0.6
0.4
0.2
−1.0
−0.5
0.0
0.5
1.0

How can we identify the right hyperplane?

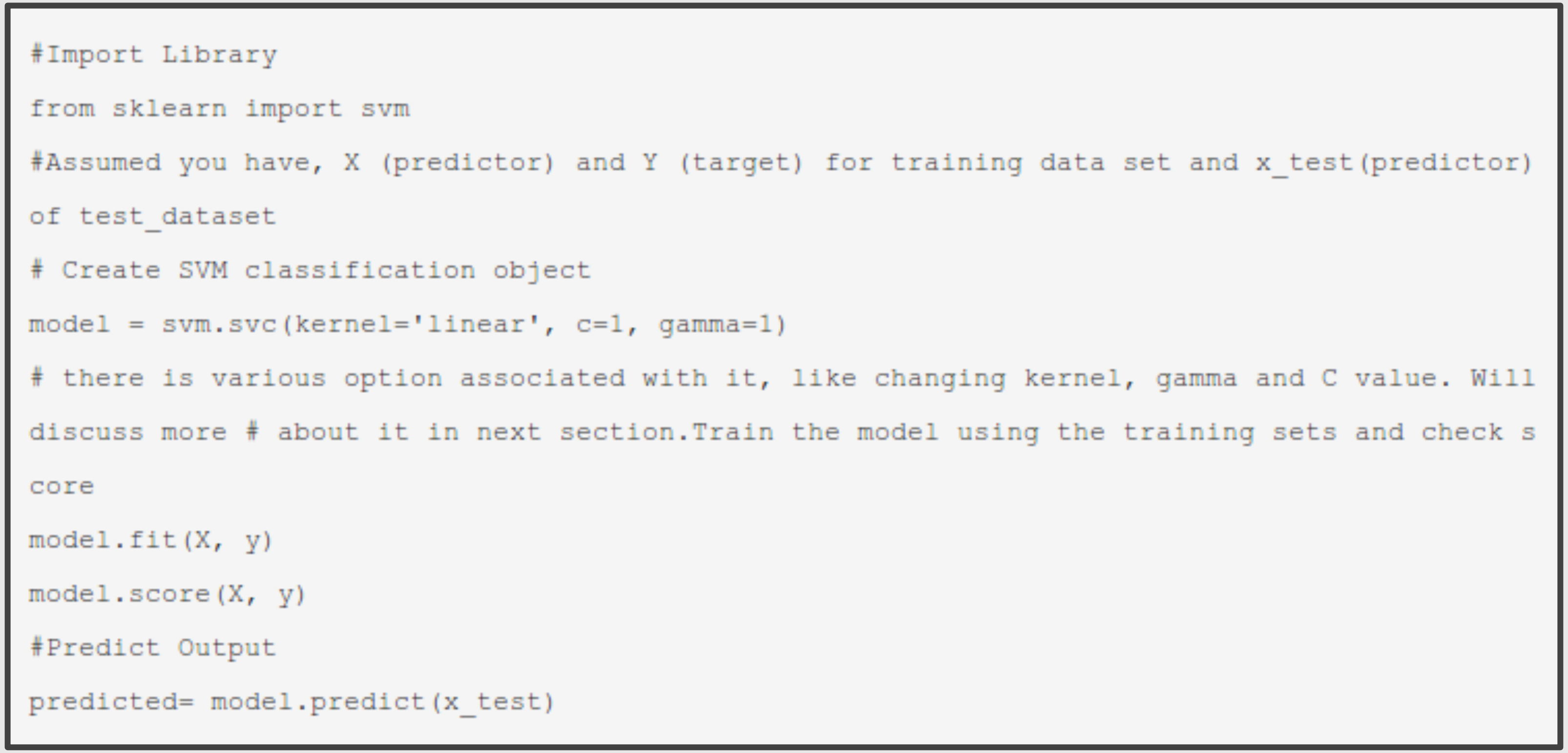
Scenario 5



SVM & scikit-learn

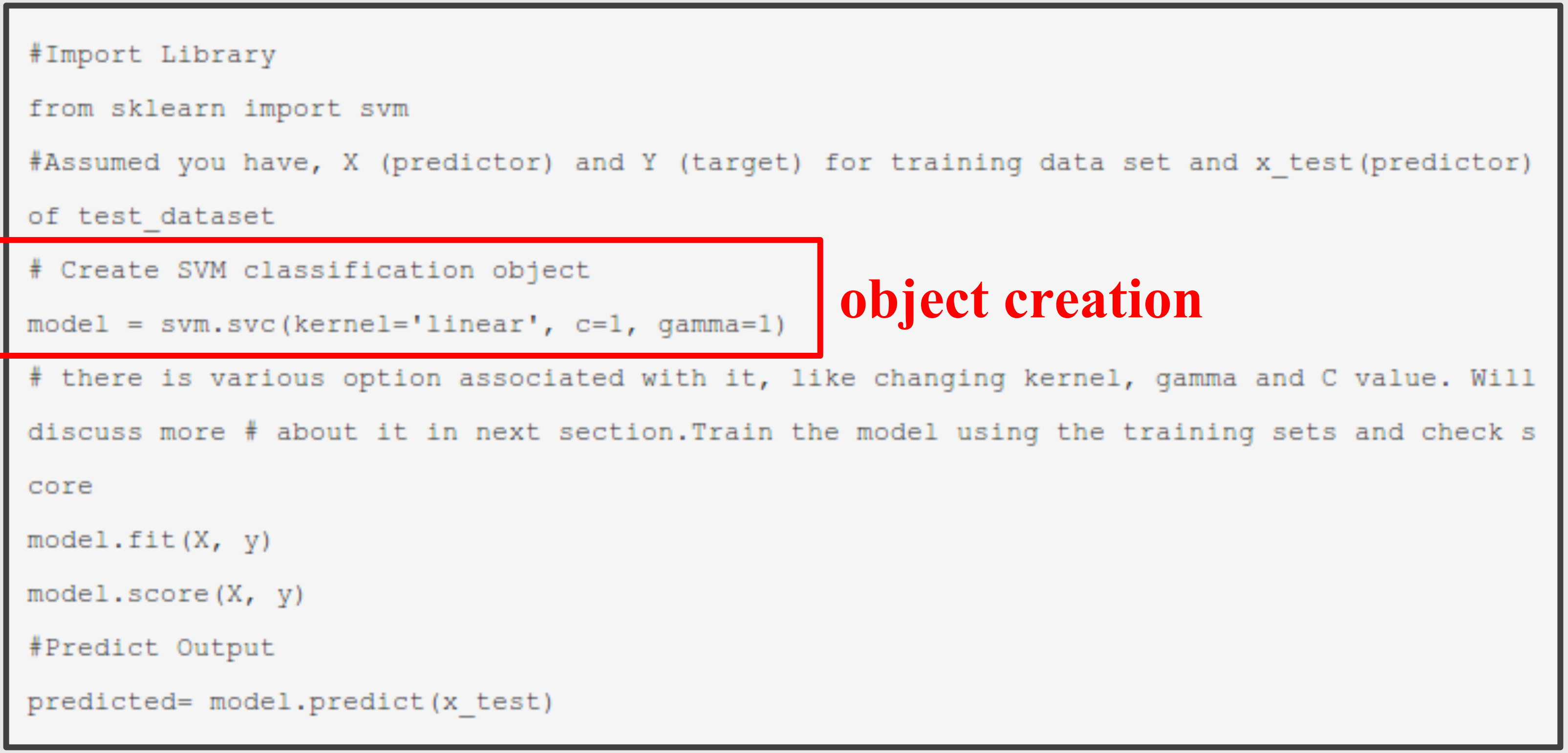
SVM is also available in scikit-learn library and follow the same structure : import library, object creation, fitting model and prediction.

SVM & scikit-learn



```
#Import Library

from sklearn import svm

#Assumed you have, X (predictor) and Y (target) for training data set and x_test(predictor)

of test_dataset

# Create SVM classification object

model = svm.svc(kernel='linear', c=1, gamma=1)

# there is various option associated with it, like changing kernel, gamma and C value. Will

discuss more # about it in next section.Train the model using the training sets and check s

core

model.fit(X, y)

model.score(X, y)

#Predict Output

predicted= model.predict(x_test)
```

SVM & scikit-learn



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object creation

SVM & scikit-learn

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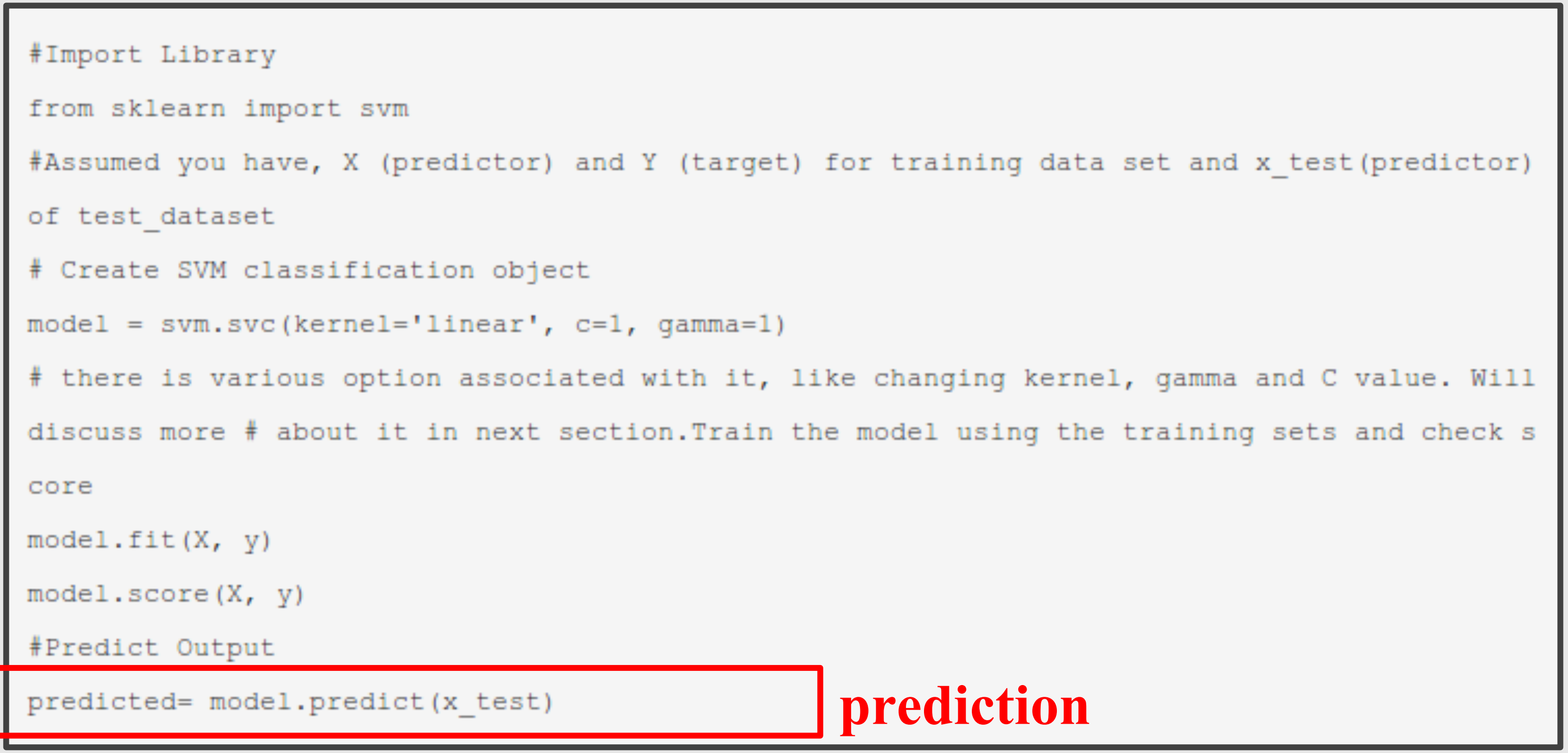
fitting model

SVM & scikit-learn



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prediction

Important Parameters

Important parameters having higher impact on model performance, “kernel”, “gamma” and “C”.

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C: Penalty parameter C of the error term. It also controls the trade off between smooth decision boundary and classifying the training points correctly.

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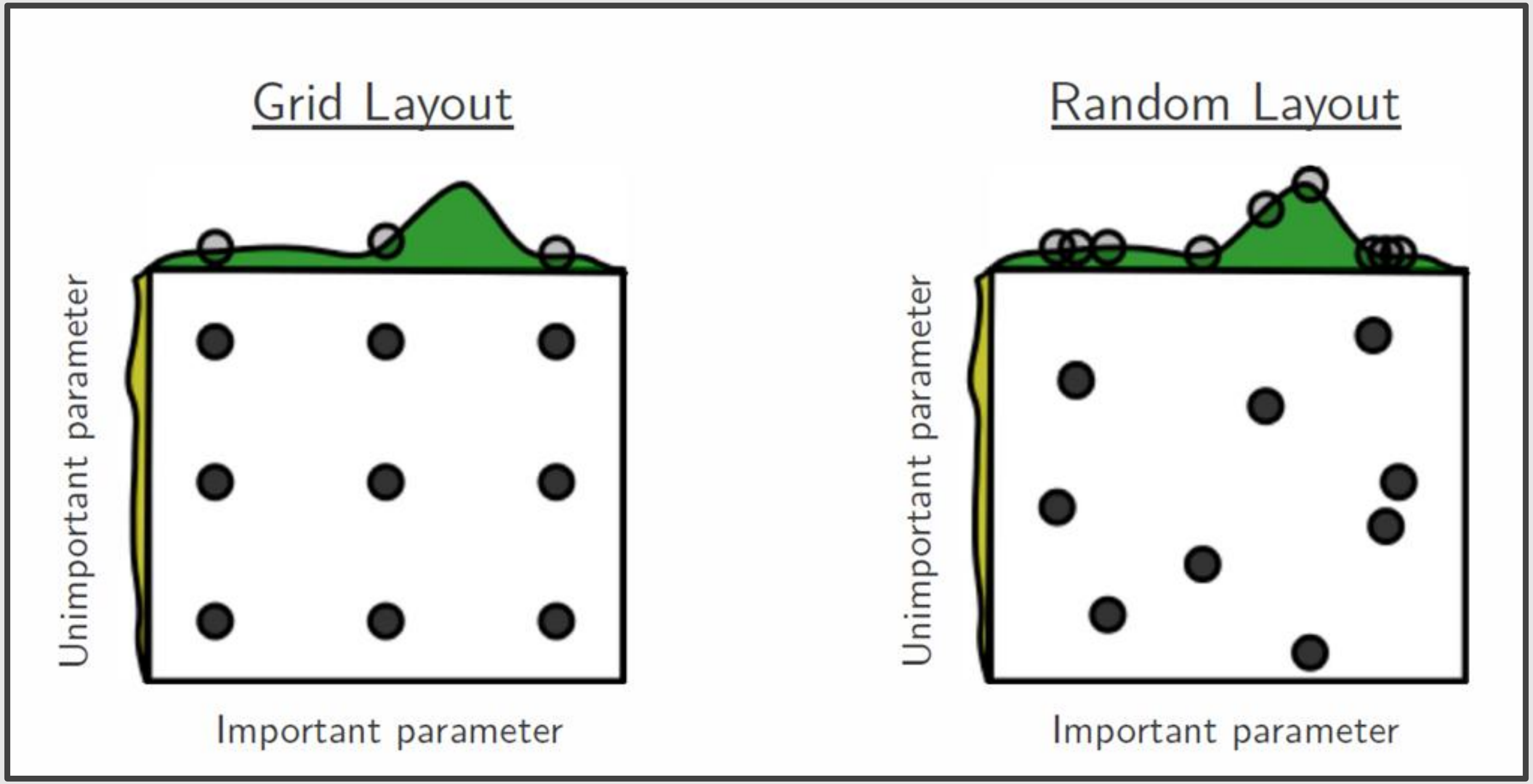
C: Penalty parameter C of the error term. It also controls the trade off between smooth decision boundary and classifying the training points correctly.

The parameters can be tuned using grid-search.

Grid Search


Grid Layout
Random Layout
Unimportant parameter
Important parameter
Unimportant parameter
Important parameter

References

Machine Learning Books

- Hands-On Machine Learning with Scikit-Learn and TensorFlow, Chap. 5
- Pattern Recognition and Machine Learning, Chap. 6 & 7

Machine Learning Courses

- https://www.coursera.org/learn/machine-learning, Week 7
- http://cs229.stanford.edu/syllabus.html, http://cs229.stanford.edu/notes/cs229-notes3.pdf