



The price of gold and the exchange rate

LARRY A SJAASTAD*

*Department of Economics, University of Chicago, 1126 East 59th
Street, Chicago, IL 60637, USA*

AND

FABIO SCACCIAVILLANI

International Monetary Fund, Washington, DC 20431, USA

This paper examines the theoretical relationship between the major exchange rates and the prices of internationally-traded commodities. In the empirical section, the case of gold is analyzed using forecast error data. Among other things, it is found that, since the dissolution of the Bretton Woods International monetary system, floating exchange rates among the major currencies have been a major source of price instability in the world gold market and, as the world gold market is dominated by the European currency bloc, appreciations or depreciations of European currencies have strong effects on the price of gold in other currencies. (JEL D40, F33). Copyright © 1996 Elsevier Science Ltd

The main objective of this paper is to identify the effect of major currency exchange rates on the prices of internationally traded commodities. For commodities that are traded continuously in organized markets such as the Chicago Board of Trade, a change in any exchange rate will result in an immediate adjustment in the prices of those commodities *in at least one currency*, and perhaps in both currencies if both countries are 'large'. For example, when the dollar depreciates against the Deutsche mark, dollar prices of commodities tend to rise (and DM prices fall) even though the fundamentals of the markets—all relevant factors other than exchange rates and price levels—remain unchanged. The power of this effect is suggested by the events surrounding the intense appreciation of the dollar from early 1980 until early 1985 during which the US price level *rose* by 30% but the IMF dollar-based commodity price

*The authors are grateful for comments by participants in seminars at the Australian National University, Curtin University, the Universities of Chicago and Western Australia, and particularly for comments by Kenneth W. Clements, Michael McAleer, and anonymous referees; the usual disclaimer holds. The original version of this paper appeared as Discussion Paper 92.05 of the Economic Research Centre and the Department of Economics of the University of Western Australia.

index *fell* by 30%, and dollar-based unit-value indices for both imports and exports of commodity-exporting countries as a group *declined* by 14%. The explanation for this anomaly may lie in the exchange rate: with respect to the DM, for example, the dollar appreciated by more than 90% in nominal terms, and by 45% in *real* terms.

The potential importance of this phenomenon is not limited to the major currency countries. With more than two-thirds of the minor currencies of the world being directly or indirectly tied to one of the three major currencies (the dollar, the Deutsche mark, and the yen) or to a currency basket, shocks to the major currency exchange rates may be felt not only by producers and consumers of internationally-traded commodities in the major currency countries but also by many of the smaller, commodity-exporting countries in the form of inflationary (or deflationary) shocks transmitted by fluctuations in the international prices of commodities.¹

In the first of the five sections to follow, an international pricing model is developed, which predicts that changes in major currency exchange rates will impact on the prices of many commodities in all currencies—major and minor alike. Section II is concerned with preliminary test of the data and section III—the core of the paper—reports the findings of a pilot study of the international market for gold. In section IV we quantify the contribution of floating exchange rates to the coefficient of variation of the international price of gold since the dissolution of the Bretton Woods system in 1973. A short summary section concludes the paper.

Gold is a prime candidate for a pilot study of the effects on commodity prices of fluctuations in major currency exchange rates. A highly homogeneous commodity, gold is traded almost continuously in well organized spot and future markets. Moreover, as annual production (and consumption) of gold is minuscule compared with the global stock, the gold producing countries, whose currencies typically are not traded in organized markets, are unlikely to dominate the world gold market.

I. Exchange rates and commodity prices: the model

The model developed in this section focuses on the effect of movements in exchange rates on the international price of a homogeneous commodity that is traded in an organized market; it is not the usual asset pricing model as it is not concerned with the rate of return on holding commodity in question.² The model has two basic elements: the law of one price and global market clearing in a world of M countries or currency blocs. Ignoring all barriers to trade and with all variables expressed in natural logarithms, the law of one price for an internationally-traded commodity is simply:

$$\langle 1 \rangle \quad P_1 = P_j + E_{1j}, \quad j = 1, \dots, M,$$

P_j being the commodity price in currency j and E_{1j} price of currency j in terms of the reference currency 1. Feedback from the commodity market to exchange rates is assumed to be negligible.

The *excess demand* (i.e. net imports), Q_j , for that commodity in currency bloc j is a function of its *real* price, $P_j^R \equiv P_j - P_j^*$, where P_j^* is the *price level* in that bloc, and a 1 by N vector $X_j = (X_{j1}, X_{j2}, \dots, X_{jN})$ of (yet to be specified) market 'fundamentals' specific to the commodity in question and currency bloc j :

$$Q_j = Q_j(P_j^R, X_j), \quad \partial Q_j / \partial P_j^R \leq 0, \quad j = 1, \dots, M.$$

Global market clearing requires:

$$\sum_{j=1}^M Q_j(P_j^R, X_j) = 0,$$

and hence a local log-linear approximation can be written as:

$$\langle 2 \rangle \quad \sum_{j=1}^M \left(\partial Q_j / \partial P_j^R \right) \cdot (P_j^R - \bar{P}_j^R) + \sum_{j=1}^M \left(\sum_{i=1}^N (\partial Q_j / \partial X_{ji}) \cdot (X_{ji} - \bar{X}_{ji}) \right) = 0,$$

where \bar{P}_j^R and \bar{X}_{ji} are means of the distributions of P_j^R and X_{ji} , respectively.

From Eq. $\langle 1 \rangle$, $P_j^R = P_1 - E_{1j} - P_j^*$, so Eq. $\langle 2 \rangle$ can be rearranged into a fairly simple expression for P_1 :

$$\langle 3 \rangle \quad P_1 = \text{constant} + \sum_{j=1}^M \theta_j \cdot (E_{1j} + P_j^*) + K(X),$$

where $\theta_j = (\partial Q_j / \partial P_j^R) / \sum_{j=1}^M (\partial Q_j / \partial P_j^R)$; while Q_j may be positive or negative, $\partial Q_j / \partial P_j^R$ is non-positive so the θ_j are non-negative fractions that sum to unity. The *global* fundamentals are captured by $K(X) = -\sum_{j=1}^M \sum_{i=1}^N [(\partial Q_j / \partial X_{ji}) \cdot X_{ji}] / \sum_{j=1}^M (\partial Q_j / \partial P_j^R)$; i.e. X is a vector containing all elements of the country-specific X_j vectors.³ Since $P_k = P_1 - E_{1k}$ and $E_{kj} = E_{1j} - E_{1k}$, Eq. $\langle 3 \rangle$ can be specified in any currency k : $P_k = \text{constant} + \sum_{j=1}^M \theta_j \cdot (E_{kj} + P_j^*) + K(X)$. Changes in the global fundamentals have identical effects on the price of the commodity in question regardless of currency of denomination.

By subtracting P_1^* from both sides of Eq. $\langle 3 \rangle$ we obtain:

$$\langle 3R \rangle \quad P_1^R = \sum_{j=1}^M \theta_j \cdot E_{1j}^R + K(X),$$

where $E_{1j}^R \equiv E_{1j} + P_j^* - P_1^*$ is the common PPP *real* exchange rate between currency blocs 1 and j . For the commodity in question, then, θ_j is simultaneously the elasticity of its *nominal* price in currency bloc 1 with respect to the *nominal* exchange rate (or price level) of bloc j , and the elasticity of its real price in currency bloc 1 with respect to the PPP *real* exchange rate between blocs 1 and j , holding all other variables constant in both cases. While the θ_j can be estimated with either nominal or real variables, the actual estimation will use forecast errors.

I.A. An interpretation of the 'thetas'

The 'thetas' in Eq. $\langle 3 \rangle$ are key to the analysis as they measure the *relative*

market power possessed by each participant in the world market for the commodity in question. Consider a small depreciation of currency 1 against all other currencies (holding all P_j^* constant); the effect of that depreciation on P_1 is $\sum_{j=2}^M \theta_j = 1 - \theta_1$. If currency bloc 1 is a *price-taker* in, say, the world gold market, that depreciation will have no effect on the price of gold in other currencies so the entire impact falls on the price of gold in currency 1 and hence $\theta_1 = 0$. In other words, currency bloc 1 is the classic 'small' economy in the world gold market. On the other hand, if bloc 1 is an absolute *price-maker*, that depreciation will have no effect the price of gold in currency 1 as that bloc totally dominates the world gold market, so $\theta_1 = 1$; all of the effect of the depreciation will appear in the price of gold in other currencies.

To dominate the world market for any commodity, a country must have an extremely elastic excess demand for that commodity. When stocks are small compared with annual production and consumption (as in the case of wheat or copper), a country must be a major producer and/or consumer in order to dominate the price of a commodity. Precious metals are unusual in that *stocks* are very large compared with annual production or consumption and hence a country with a high propensity to hoard gold might dominate the world gold market without being a major producer.

Given the high variability of major currency exchange rates since 1973, the term $\sum_{j=1}^M \theta_j \cdot (E_{ij} + P_j^*)$ of Eq. (3) is a potentially important source of shocks to the price of a commodity such as gold, and hence estimates of the θ_j can be useful.⁴ That information can help identify the sources (exchange rates vs the fundamentals) of the price shocks experienced by consumers and producers of gold, to the extent that exchange rates can be predicted, one can forecast the effects of movements in those rates on the price of gold. Finally, information about the θ_j can be exploited for portfolio management; by denominating their assets and liabilities in foreign currency in accordance with the θ_j , firms involved with gold can reduce the financial impact of exchange rate shocks.

I.B. The forecast error approach

With appropriate time series data, the θ_j coefficients in Eq. (3) can be estimated, but that procedure confronts long standing issues such as the stationarity of exchange rates. However, when the currencies and commodities are traded in both *spot* and *forward* markets, those issues can be finessed by using forecast errors, which involves writing Eq. (3) in terms of those errors extracted from spot and forward price and exchange rate data rather than with actual prices and exchange rates. As forecast error data are usually stationary and, if the relevant markets are 'efficient', serially uncorrelated as well, the econometric analysis is considerably simplified.

To develop this approach, we begin with spot and forward versions of Eq. (3); apart from notation, the former is identical with Eq. (3):

$$(3S) \quad P_{1,t}^S = \text{constant} + \sum_{j=1}^M \theta_j \cdot (E_{1j,t}^S + P_{j,t}^*) + K^S(X_t),$$

and the forward version is written as:

$$\langle 3F \rangle \quad P_{1,t,n}^F = \text{constant} + \sum_{j=1}^M \theta_j \cdot (E_{1j,t,n}^F + P_{j,t,n}^{*F}) + K^F(X_t),$$

where the S and F superscripts denote spot and forward, P_j^{*F} and $K^F(X)$ are unobserved market forecasts of P_j^* and $K^S(X)$, and n is the length of the forward contract. The θ_j are set equal in Eqs. $\langle 3S \rangle$ and $\langle 3F \rangle$ as there is no reason to expect them to differ for short-term (e.g. 90-day) contracts.

The forecast error for the price of the commodity in question, $Z_{1,t,n}$, is the difference between realized and forward prices:

$$Z_{1,t,n} \equiv P_{1,t}^S - P_{1,t-n,n}^F,$$

and for exchange rates, the forecast error is:

$$Z_{E_{1j,t,n}} \equiv E_{1j,t}^S - E_{1j,t-n,n}^F.$$

Neglecting the constant term, the forecast error version of Eq. $\langle 3 \rangle$ is just the difference between Eqs. $\langle 3F \rangle$ and $\langle 3S \rangle$, with an n period lag:

$$\begin{aligned} Z_{1,t,n} &= \sum_{j=1}^M \theta_j \cdot (Z_{E_{1j,t,n}} + [P_{j,t}^* - P_{j,t-n,n}^{*F}]) + [K^S(X_t) - K^F(X_{t-n})] \\ &= \sum_{j=1}^M \theta_j \cdot Z_{E_{1j,t,n}} + [P_{W,t}^* - P_{W,t-n,n}^{*F}] + [K^S(X_t) - K^F(X_{t-n})], \end{aligned}$$

where $P_{W,t}^* = \sum_{j=1}^M \theta_j \cdot P_{j,t}^*$ is the 'world' price level and, as $P_{W,t-n,n}^{*F}$ is the forecast of $P_{W,t}^*$, the terms $P_{W,t}^* - P_{W,t-n,n}^{*F} \equiv Z_{P,t,n}$ and $K^S(X_t) - K^F(X_{t-n}) \equiv Z_{K,t,n}$ also are forecast errors. Since neither $Z_{P,t,n}$ nor $Z_{K,t,n}$ are observable, the forecast error version of Eq. $\langle 3 \rangle$ is written as:

$$\langle 4 \rangle \quad Z_{1,t,n} = \text{constant} + \sum_{j=1}^M \theta_j \cdot Z_{E_{1j,t,n}} + v_{1t},$$

where $v_{1t} \equiv Z_{P,t,n} + Z_{K,t,n}$ also is a forecast error.

If markets are weakly efficient, $K^F(X_{t-n}) = E[K^S(X_t) | I_{t-n}]$ and $P_{W,t-n,n}^{*F} = E(P_{W,t}^* | I_{t-n})$, where $E(\cdot)$ is the conditional expectation operator and I_{t-n} the information set at time $t-n$, and hence $Z_{P,t,n}$, $Z_{K,t,n}$, and v_{1t} are serially uncorrelated. Weak market efficiency implies, then, that all variables in Eq. $\langle 4 \rangle$ are serially uncorrelated.⁵ Given the potentially superior characteristics of forecast error data, Eq. $\langle 4 \rangle$ will be the center piece for the empirical implementation of the pricing model.

II. Preliminary tests on the data

The spot gold price data consist of daily observations from January 1982 through December 1990 and the forward price data refer to 108 90-day contracts let at the beginning of each month during the same period, both in US dollars. The daily spot gold prices are from the London Gold Market

(Reuters), and forward gold prices were computed using closing quotations on 3-month COMEX contracts. Spot and 90-day forward exchange rates between the US dollar and the Deutsche mark, the UK pound sterling, and the Japanese yen were obtained from the International Monetary Fund Data Bank and cover the same period. Because the forecast errors require a 3-month lag on the forward series, the useful data set is reduced to 105 overlapping (and hence serially correlated) observations. Alternatively, the data can be divided into three subsets of 35 nonoverlapping observations for the same period. Finally, preliminary tests indicated that the UK 'theta' is approximately zero, so the pound sterling was designated currency 1 and the price of gold and all exchange rates were denominated in pounds.

As the empirical analysis focuses on the relation between exchange rates and the price of gold, and since the fundamentals are difficult to specify in advance (apart from world inflation, which may influence the appeal of gold as a store of value), we made no attempt to do so; accordingly, in estimating Eq. (4), we assume that the exchange-rate forecast errors, $Z_{E_{ij},t,n}$, and those concerning gold-market fundamentals, v_{1t} , are orthogonal. The period of analysis, 1982–1990, however, was deliberately chosen to exclude the price explosion of 1979–1980 due to international political instability; moreover, as the IMF *International Financial Statistics* indicate that central bank gold reserves remained quite constant at just under one billion fine troy ounces throughout the 1982–1990 period, the world gold market was not influenced by large net sales of gold on the part of central banks during that period.

II.A. Stationarity tests

Two distinct stationarity tests based on all 108 *overlapping* observations on spot and forward gold prices and the original US dollar exchange rate data indicate that all spot and forward series are non-stationary; these results are reported in the Appendix. This finding suggests that those variables may be co-integrated, and hence the relationship described by Eq. (3) might be represented by the Engle–Granger (1987) error correction mechanism. Tests of the no-cointegration null hypothesis were based on a technique proposed by Hamilton (1994, Chapter 19), which consists of estimating the co-integration vector via a regression in which the regressors are exchange rates and the dependent variable is the price of gold, and then test the residuals for unit roots using the augmented Dickey–Fuller (1981) (ADF) test. The results of the unit-root tests on both spot and forward data are reported in Table 1; as the no-cointegration null hypothesis is not rejected in either case, the error correction approach is not appropriate.⁶

As estimation of Eq. (4) requires stationarity of forecast errors rather than prices and exchange rates, the non-overlapping forecast error data were tested for stationarity by ADF unit-root tests with up to five lags. The usual procedure involves testing the ADF statistic corresponding to highest lag with a significant t -lag statistic but, as is indicated in Table 1, none of the t -lag statistics were significant at even the 10% level.⁷ While this inconclusive result may be due to

TABLE 1. Augmented Dickey–Fuller unit root tests on residuals of co-integration equations: gold prices and exchange rates, 1982:07–90:12 Critical values for *t*-ADF statistic: 5% = −3.66, 1% = −4.65^a

LAG	Spot			Forward		
	<i>t</i> -ADF stat	<i>t</i> -lag statistic		<i>t</i> -ADF stat	<i>t</i> -lag statistic	
		Value	<i>P</i> -value		Value	<i>P</i> -value
1	−2.7826	−0.7844	0.4346	−2.7203	−2.1173	0.0369
2	−2.5501	−0.5546	0.5804	−2.1900	−1.7289	0.0872
3	−2.4526	−0.2514	0.8021	−2.0284	−0.3696	0.7125
4	−2.7099	−1.4279	0.1565	−1.5853	−1.4108	0.1617
5	−2.5899	−0.0312	0.9751	−1.5125	−0.0346	0.9725

^aCritical values are for non-zero drift in the explanatory variables.

sample size, augmented Dickey–Fuller tests often fail to provide solid evidence.

More conclusive stationarity tests were obtained by using the fractional differencing approach, which involves a non-integer ‘order of differentiation,’ *d*. For any time series X_t , the Wald representation is:

$$(1 - L)^d X_t = A(L) \varepsilon_t,$$

where ε_t is white noise, *L* is the usual lag operator, and stationarity is determined by the value of *d* (see the Appendix for further details). We used the Sowell (1991) maximum likelihood estimate, which gives more reliable results, particularly with small samples. The parameter *d* was estimated for each of the three subsets of 35 forecast error observations (based on the original data set), and the results appear in Panel A, Table 2.⁸ As none of the estimates of *d* are significantly different from zero, the forecast errors for all four series appear to be stationary.⁹

II.B. Market efficiency tests

As was argued earlier, if the gold and foreign exchange markets are efficient, estimation of Eq. <4> is simplified as both the forecast errors and the residuals of Eq. <4> will be serially uncorrelated. Tests of both weak and semi-strong market efficiency were conducted.

II.B.i. Weak market efficiency

The classic test for weak market efficiency is based on estimating the equation $P_{i,t}^S = \alpha + \beta \cdot P_{i,t-3,3}^F$ and testing the joint restriction $\alpha = 0$ and $\beta = 1$. But as market efficiency also requires serially uncorrelated forecast errors, the test for market efficiency was based on Eq. <5>:

TABLE 2. Stationarity and market efficiency tests on forecast error data: 1982:2–90:4

Panel A. Stationarity tests on forecast error data								
Subset	Maximum likelihood estimates of d (with t -statistics)							
	Gold	Mark	Yen	Pound				
First	–0.07 (–0.32)	0.19 (0.93)	0.03 (0.22)	0.10 (1.12)				
Second	–0.15 (–0.30)	0.15 (0.88)	0.11 (1.53)	0.05 (0.18)				
Third	–0.08 (–0.47)	0.20 (1.07)	0.03 (0.70)	0.18 (1.44)				

Panel B. Market efficiency tests								
Statistic	Weak (Eq. (5)) Forecast error for:				Semi-strong (Eq. (6)) Forecast error for:			
	Gold	DM	Dollar	Yen	Gold	DM	Dollar	Yen
$\chi^2(1)^a$	0.03	0.00	1.13	1.05	2.58	17.22	15.02	10.46
P -Value:	0.87	1.00	0.29	0.31	0.63	0.00	0.00	0.03
$F(1,100)^b$	0.06	0.00	1.50	1.06	0.48	15.25	4.41	8.74
P -Value:	0.81	1.00	0.22	0.30	0.75	0.00	0.00	0.00

^aOLS estimates using White's (1980) Robust Error routine with 2 lags.

^bStandard errors estimated by the Hansen–Hodrick (1980) method.

$$\langle 5 \rangle \quad Z_{\cdot,t,3} = \gamma + \delta \cdot Z_{\cdot,t-3,3},$$

in which estimates of δ should not differ significantly from zero. Four χ^2 statistics on the restriction $\delta = 0$ based on OLS estimates of Eq. (5) for the 105 overlapping observations using White's (1980) robust standard error routine appear in Panel B of Table 2; the restriction is not rejected.¹⁰ The standard errors were re-estimated by the Hansen–Hodrick (H–H) (1980) method and since the significance of F statistics on the $\delta = 0$ restriction, also reported in Panel B of Table 2, are similar to the χ^2 statistics, weak market efficiency cannot be rejected.

II.B.ii. Semi-strong market efficiency

In the context of the model developed in section I, semi-strong market efficiency requires past gold price *and* exchange-rate forecast errors to be orthogonal with both the current gold price *and* exchange-rate forecast errors. The test for the gold market involves estimating the following equation:

$$\langle 6 \rangle \quad Z_{1,t+3,3} = \mu + \vartheta_1 \cdot Z_{1,t,3} + \sum_{i=2}^4 \vartheta_i \cdot Z_{E_{1i},t,3}$$

and, for the j^{th} exchange rate, Z_1 is replaced with $Z_{E_{ij}}$. Semi-strong market efficiency is tested by the joint restriction that estimates of all four ϑ_i are zero. Panel B of Table 2 presents the four χ^2 statistics on that joint restriction based on OLS robust-error estimates of Eq. (6) using the 105 overlapping observations, and the four F statistics based on H-H estimates of standard errors; semi-strong market efficiency is not rejected for gold, but is decisively rejected for all exchange rates. In summary, all forecast error series are stationary and hence no filtering is required; in addition, weak market efficiency cannot be rejected for any case, but semi-strong efficiency can be rejected for all exchange rates.

III. Estimates of the 'thetas' for the world gold market

As the pound sterling was designated currency 1 (the reference currency) and the price of gold and all exchange rates were denominated in that currency, there are but three parameters to estimate: θ_{DM} , $\theta_{US\$}$, and θ_{Yen} . Equation 4 was estimated using all 105 overlapping observations with the standard errors estimated by the H-H method.¹¹ As a t -test on the estimates of the θ_j parameters, reported in Panel A of Table 3, indicates that the unit-sum restriction cannot be rejected, that restriction was imposed and the results (again with standard errors estimated by the H-H method) are summarized in Panel B of Table 3. As that restriction was not binding, the restricted and unrestricted regressions are nearly identical—apart from an increase in the t statistics. Apparently the major gold producers, which includes Australia, South Africa, and the former USSR, have little power in the world gold market. Rather, that market is dominated by the ECU and dollar blocs (with the ECU

TABLE 3. OLS estimate of Eq. (4): gold, 1983:01–90:12 (Hansen–Hodrick standard errors)

Panel A. Unrestricted			
Sum of θ_j coefficients:		0.9756	
Standard error of sum:		0.0852	
t -Statistic (against unity):		–0.2862	
P -Value:		0.7754	
Panel B. Restricted			
Parameter	Estimate	t -Statistic	P -Value
θ_{DM}	0.5339	4.0913	0.0001
$\theta_{US\$}$	0.2759	3.4978	0.0007
θ_{Yen}	0.1902	1.8940	0.0614

$$\bar{R}^2 = 0.3904; \text{SEE} = 0.0596; \text{D-W} = 1.1824 \text{ } Q(24) = 41.7801, \text{ } P\text{-Value} = 0.0137$$

having by far the larger weight), and to a lesser extent by the yen bloc. Note further that, while the estimates of θ_{DM} and $\theta_{US\$}$ are significant at the 0.001% level, that of θ_{Yen} is not significant at the 5% level.¹²

To test whether the θ estimates vary over time, Eq. (4) was re-estimated using two sub samples of equal length, which resulted in the following estimates: 0.5455 and 0.4995 for θ_{DM} , 0.2472 and 0.3045 for $\theta_{US\$}$, and 0.2073 and 0.1960 for θ_{Yen} , respectively, for the subperiods 1983:01–86:12 and 1987:01–90:12. While the importance of the dollar bloc may have increased over time (and that of Europe declined), in no case did the difference between the two estimates exceed the smaller of the two standard errors.

III.A. World inflation and the price of gold

The estimate of Eq. (4) reported in Table 3 does not include any variables for the fundamentals, which are captured in Eqs. (3S) and (3F) by $K^S(X_t)$ and $K^F(X_{t-3})$. A likely candidate is world inflation, changes in which may affect the price of gold (but not exchange rates). The 'world' price level, P_W^* , was defined as the natural logarithm of a weighted average of the European, US, and Japanese price levels, the weights being the theta estimates reported in Table 3.¹³ The quarterly world inflation rate, defined as $\Pi_{t,3} \equiv P_{W,t}^* - P_{W,t-3}^*$, was converted to an annual rate Π_t and, as inflation may have lagged effects, the inflation components of $K^S(X_t)$ and $K^F(X_{t-3})$ were defined as $\gamma(L) \cdot \Pi_t$ and $\gamma(L) \cdot \Pi_{t-3}$. The inflation component of v_{1t} is $\gamma(L) \cdot (\Pi_t - \Pi_{t-3})$, and was parameterized as $\gamma_0(\Pi_{t,3} - \Pi_{t-3,3}) + \gamma_1 \cdot (\Pi_{t-1,3} - \Pi_{t-4,3}) + \gamma_2 \cdot (\Pi_{t-2,3} - \Pi_{t-5,3})$. Moreover, using the property of any polynomial $A(L) = \sum_{i=0}^N a_i \cdot L^i$ and any time series Y_t , that $A(L) \cdot Y_t$ can be reparameterized in error correction form as:

$$A(L) \cdot Y_t = \sum_{i=0}^{N-1} \left(\sum_{j=0}^i a_j \right) \cdot \Delta Y_{t-i} + A(1) \cdot Y_{t-N},$$

the inflation term $\gamma(L) \cdot (\Pi_t - \Pi_{t-3})$ is expressed, in the case two lags, as $\gamma_0 \cdot \Delta(\Pi_t - \Pi_{t-3}) + \gamma_1 \cdot \Delta(\Pi_{t-1} - \Pi_{t-4}) + \gamma(1) \cdot (\Pi_{t-2} - \Pi_{t-5})$ which permits a direct estimate of $\gamma(1)$, the long run impact on the spot price of gold of a permanent change in the rate of world inflation.

When the inflation variable was included, the no-cointegration null hypothesis was not rejected, nor was stationarity of the inflation variable rejected: the estimate of d for $\Pi_{t,3} - \Pi_{t-3,3}$ was 0.082. The results of an OLS estimate of Eq. (4) augmented by the inflation variable with two lags are summarized in Table 4; the estimates of θ_j are similar to those reported in Table 3, although the standard errors are smaller. The results show that world inflation, as a fundamental, is an anemic one: a (permanent) rise in the (annual) rate of world inflation by one percentage point leads to a mere 0.78% rise in the price of gold.

III.B. A more general formulation

The *overlapping* nature of the forecast error data results in strong serial

TABLE 4. OLS estimate of Eq. <4> with inflation variables: gold, 1983:01–90:12 (Hansen–Hodrick standard errors)

Panel A. Unrestricted				
Sum of θ_j coefficients:			0.9718	
Standard error of sum:			0.0837	
t -statistic (against unity):			–0.3365	
P -value:			0.7373	
Panel B. Restricted				
Parameter	Lag	Estimate	t -statistic	P -value
θ_{DM}	—	0.5478	4.3817	0.0000
$\theta_{US\$}$	—	0.2521	3.5118	0.0007
θ_{Ycn}	—	0.2001	2.2683	0.0257
$\gamma(1)$	2	0.7777	3.2081	0.0019

$\bar{R}^2 = 0.4074$; $SEE = 0.0587$; $D-W = 1.2521$ $Q(24) = 40.8280$, P -Value = 0.0174

correlation in the data and in the residuals of OLS estimates based on those data; indeed, the partial auto-correlations for the forecast error data are high for up to seven or eight lags and the Q statistics reported in Tables 3 and 4 are highly significant. This serial correlation suggests that lags may be useful even though weak market efficiency was not rejected. A more general specification of Eq. <4>, which incorporate lags, is the following:

$$\langle 4' \rangle \quad \alpha(L) \cdot Z_{1,t,n} = \text{cons} + \sum_{j=2}^M \Theta_j(L) \cdot Z_{E_{1j},t,n} + \gamma(L)(\Pi_t - \Pi_{t-3}) + v_{1t}.$$

Experimentation indicated that lags on the independent variables became redundant (i.e. $\Theta_j(L) \equiv \Theta_j$) once lags on the dependent variable were introduced; accordingly, the final effect on the spot price of gold of a permanent shock to the j^{th} exchange rate is $\theta_j \equiv \Theta_j/\alpha(1)$, where $\alpha(1) = \sum_{i=0}^I \alpha_i$ and $\alpha_0 = 1$, and the long run reaction of the real spot price of gold to a permanent shock to world inflation is captured by the parameter $\Gamma \equiv \gamma(1)/\alpha(1)$.

With this modification, Eq. <4'> was estimated by OLS with lags being added until the estimate of $\alpha(1)$ stabilized (which occurred after the eight lag) and the standard errors were estimated by the H–H technique; the results appear in Panel A in Table 5.¹⁴ Since the unit-sum restriction on the θ_j was not rejected at the 70% level of significance, Eq. <4> was re-estimated with that restriction imposed; the results for both the Θ_j and θ_j are reported in panel B in Table 5.

Despite first differencing of the dependent variable, the new estimate of Eq. <4'> dominates those reported in Tables 3 and 4. The \bar{R}^2 has increased by one quarter to 0.51 and the standard error of estimate has declined to 0.051 from

TABLE 5. OLS estimate of Eq. (4') with inflation variables and nine lags on dependent variable: gold, 1983:01–90:12(Hansen–Hodrick standard errors)

Panel A. Unrestricted				
Sum of θ_j coefficients			0.9654	
Standard Error of Sum:			0.0933	
t -Statistic (against unity)			–0.3705	
P -Value:			0.7120	
Panel B. Restricted				
Parameter	Lag	Estimate	t -Statistic	P -Value
Θ_{DM}	0	0.4919	5.4858	0.0000
$\Theta_{US\$}$	0	0.1284	2.7900	0.0053
Θ_{Yen}	0	0.1327	2.3992	0.0164
$\alpha(1)$	9	–0.7531	–12.6609	0.0000
θ_{DM}	0	0.6532	5.2959	0.0000
$\theta_{US\$}$	0	0.1705	2.9334	0.0044
θ_{Yen}	0	0.1763	2.3580	0.0208
Γ	2	0.6446	2.8119	0.0062

$\bar{R}^2 = 0.5105$; SEE = 0.0507

0.059. The estimates of all thetas—both short and long-run—are significant at the 2% level, and the point estimate of θ_{DM} , which increased from 0.55 to 0.65, is nearly four times that of both $\theta_{US\$}$ and θ_{Yen} . The estimate of Γ also is highly significant but declined from 0.78 to 0.64: a (permanent) rise by one point in the annual rate of world inflation rate leads to an increase in the *real* spot price of gold of only two-thirds of 1%.

It is clear from Table 5 that the European countries heavily dominate the international market for gold and hence movements in European exchange rates against the US dollar impact heavily on the dollar price of gold. While a 10% appreciation of the Deutsche mark (against all other currencies) increases the dollar price of gold by 6.5% (and vice versa), the same appreciation of the yen increases the dollar price of gold by only 1.7%. A 10% appreciation of the dollar against both currencies depresses the dollar price of gold by about 8%, and vice versa.

Three simulations, based on the restricted estimate of Eq. (4'), depicting the response of the US dollar price of gold to transitory and permanent depreciations of the US dollar *vis à vis* the DM, the yen, and both currencies are reported in Figure 1. The striking effect of the dollar/DM (i.e. US dollar/ECU) exchange rate on the dollar price of gold is readily evident.

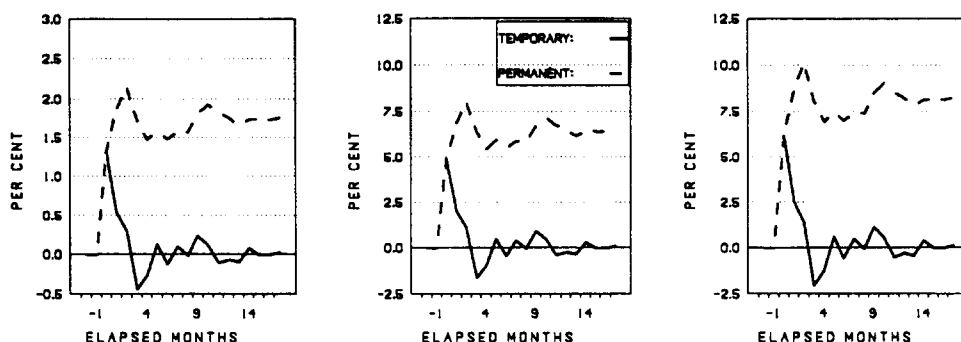


FIGURE 1. Simulated response of US dollar price of gold to a 10% depreciation of the dollar against: the Yen, the DM, both.

IV. Floating exchange rates and the stability of the gold market

There can be little doubt that floating exchange rates among the major currencies have contributed substantially to the variability of the price of gold during the 1980s. There is, of course, no way of divining the behavior of the free market price of gold had the Bretton Woods fixed exchange rate system endured (but without the link between the dollar and gold), but to gain an idea of the degree to which the world gold market has been influenced by floating exchange rates, an experiment was conducted. Equation (4') was parameterized in level form as:

$$Z_{1,t,n} = \text{constant} + \sum_{k=1}^9 \alpha_k \cdot Z_{1,t-j,n} + \sum_{j=2}^4 \theta_j \cdot Z_{E_{1j},t,n} + v_{1t},$$

and OLS estimates of the α_k and θ_j , with the unit-sum restriction (i.e. $\sum_{j=1}^3 \hat{\theta}_j - \sum_{k=1}^9 \hat{\alpha}_k = \hat{\alpha}_0 = \alpha_0 = 1$) imposed on the long run thetas, were used to calculate the residuals of a reparameterized version of Eq. (3S):

$$(3S') \quad \hat{u}_{1t} = P_{1,t}^S - \sum_{k=1}^9 \hat{\alpha}_k \cdot P_{1,t-j}^S - \sum_{j=2}^4 \hat{\theta}_j \cdot E_{1j,t}^S.$$

Those residuals, which reflect all influences on the real spot price of gold *other than major currency exchange rates*, were taken as estimates of the fundamentals (the $K(X)$ term in Eq. (3)).

As all variables in Eq. (3S') are in natural logs, the calculated residuals, \hat{u}_{1t} , were transformed into arithmetic values to be dimensionally identical with the spot price of gold and then converted into both nominal and real 'prices' in all four currencies. The coefficients of variation of the spot gold price and the transformed residuals appear in Panel A, Table 6. For the 1982:11–90:12 period, the coefficients of variation of the actual spot prices of gold—both nominal and real—are roughly 80% larger than those of the transformed residuals and, apart from the case of the pound sterling, the differences according to currency of denomination are very small.¹⁵ Fluctuation in real

TABLE 6. The stability of the gold market and real exchange rates

Panel A. Sources of variation in gold prices: 1982:11–90:12						
Currency of denomination	Coefficient of variation of the					
	Nominal price of gold			Real price of gold		
	Actual Prices	\hat{u}_{it}	Ratio	Actual prices	\hat{u}_{1t}	Ratio
Pound:	5.11	2.43	2.10	5.40	2.53	2.14
Mark:	5.28	2.88	1.83	5.33	2.91	1.83
Dollar:	4.92	2.78	1.77	5.04	2.83	1.78
Yen:	5.68	3.15	1.80	5.63	3.10	1.82

Panel B. Standard deviations of quarterly first differences of logarithms of the U.K. real exchange rates with Germany, the US and Japan: 1960–1970 and 1973–1990, in %				
Real exchange rate defined on:	Years	Germany	US	Japan
Consumer prices:	1960–1970	2.1067	1.7755	2.1934
	1973–1990	4.8402	5.4086	5.6491
Ratio:		2.30	3.05	2.58
Producer prices:	1960–1970	2.0059	1.6974	2.1153
	1973–1990	4.6049	5.3005	5.7939
Ratio:		2.30	3.12	2.74

exchange rates appear to account for nearly half of the variance in gold prices during the sample period.

It cannot be assumed that real exchange rates would have been constant had the Bretton Woods regime been preserved (again without the link between the dollar and gold), nor is it possible to know how they would have behaved under that system since 1973. Mussa (1986) and others, however, have shown that there is ample evidence that real exchange rates have been far less stable with floating exchange rates than under the Bretton Woods regime. To quantify the effect of the regime change, we calculated the standard deviations of quarterly first differences of the natural logarithms of the real exchange rates between the UK and the US, Germany and Japan for 1960–1970 and 1973–1990 periods; price levels and exchange rates were defined as the natural logarithms of quarterly averages. The results, presented in Panel B of Table 6, show that the variability in the UK real exchange rates with the US and Japan have nearly tripled since 1973. Moreover, despite the growing monetary integration between the UK and Germany, that variability of that real exchange rate has more than doubled. While not definitive, these results support the proposition that floating exchange rates among the major currencies have exacerbated the instability of the free market price of gold since 1973.

V. Summary of the main results

While we cannot claim that the empirical results for the case of gold can be generalized to other commodities, the main findings, based on an analysis of the gold and foreign exchange markets for the 1982–1990 period, are:

- The world gold market is dominated by the European currency bloc which possesses about two-thirds of the ‘market power’ enjoyed by all participants in that market. Accordingly, real appreciations or depreciations of the European currencies have profound effects on the price of gold in all other currencies.
- Although gold is usually denominated in US dollars, the dollar bloc has but a small influence on the dollar price of gold. Moreover, the major gold producers of the world (Australia, South Africa, and the former USSR) appear to have no significant influence on the world price of gold.
- Gold continues to be a store of value as ‘world’ inflation increases the demand for gold; it is estimated that the real price of gold rises by between two-thirds and three-quarters of 1% in response to a one point increase in the world inflation rate.
- The evidence strongly supports the market efficiency hypothesis for the international gold market for the 1982–1990 period.
- During the 1982–1990 period, floating exchange rates among the major currencies contributed substantially to the instability of the world price of gold; indeed, fluctuations in the real exchange rates among the major currencies account for nearly half of the observed variance in the spot price of gold during that period.

Appendix

Fractional differencing

In Box–Jenkins (1976) terminology, time series data usually are assumed to be integrated of either degree zero or one (and occasionally of degree two); when a variable X_t is integrated of degree zero [i.e., $X_t \sim I(0)$], its variance is finite and innovations have no lasting effect as the auto-correlation decays at an exponential rate for distant lags. However, if $X_t \sim I(1)$, the variance of X_t is not finite and, as X_t is the sum of all previous innovations, those innovations have a permanent effect.

A limitation of the conventional approach is that it allows only for discrete values for the degree of integration whereas there may exist a range of intermediate values that involve the so-called long memory models. These models stem from Granger (1966) who demonstrated that economic variables have a ‘typical spectral shape’, concluding that ‘...long term fluctuations in economic variables, if decomposed into frequency components, are such that the amplitudes of the components decrease smoothly with decreasing period’. In other words, the spectral density of economic time series is bounded at the origin and the auto-correlation function declines smoothly as the lag between observations increases. This means that variables tend to display long, irregular cycles or, stated differently, shocks are persistent.

The correct method for analyzing economic time series, developed in the early 1980s, is known as fractional differencing. The intuition is rather straightforward. Standard time series methodology considers only processes such as ARIMA (p, d, q) where d , the order of

differentiation, is assumed to be an integer. Granger and Joyeux (1980) and Hosking (1981) argue, however, that d is not necessarily an integer; rather, it is a real number. They suggested a procedure by which d is estimated (which is closely related to unit-root tests) and then a filter based on the estimate of d is applied to preserve the information on persistence. The transformed series then can be analyzed as an ARMA (p, q) process or by traditional time series methods.

The simplest long memory process—the basic building block—is the fractional noise defined as:

$$(1-L)^d X_t = \sum_{j=0}^{\infty} a_j \cdot \varepsilon_{t-j} = A(L) \varepsilon_t,$$

where d is a real number and ε_t is white noise. The process is stationary and invertible if $-0.5 < d < 0.5$ and the binomial expansion of $(1-L)^d$ allows one to express a_j , as:

$$a_j = \frac{\Gamma_{(j-d)}}{\Gamma_{(j+1)} \cdot \Gamma_{(-d)}},$$

which converges in mean square for $-0.5 < d < 0.5$. The fractional noise for $-0.5 < d < 0.5$ has an auto-regressive representation:

$$\sum_{j=1}^{\infty} \left(\frac{\Gamma_{(j-d)}}{\Gamma_{(j+1)} \cdot \Gamma_{(-d)}} \right) L^j X_t = \varepsilon_t,$$

and a moving average representation:

$$X_t = \sum_{j=1}^{\infty} \left(\frac{\Gamma_{(j-d)}}{\Gamma_{(j+1)} \cdot \Gamma_{(d)}} \right) L^j \varepsilon_t.$$

More general processes can be obtained from fractional noise; these are usually referred to as Auto-Regressive Fractionally Integrated Moving Average (ARFIMA) and, in addition to the long memory component, they contain an ARMA component that determines the short term movements:

$$(1-L)^d B(L) X_t = C(L) \varepsilon_t.$$

If d is in the open interval (0,0.5), the series X_t is stationary but displays non-periodic, irregular cycles. The auto-covariance of the series is positive and decays at a geometric rate (compared to the exponential rate of the standard ARIMA models). Alternatively, if $d \in (0.5, 1)$ the series is non-stationary; in either case, if d is significantly different from zero, a filter $(1-L)^d$ is required to obtain a series integrated of degree zero.

Stationary tests on basic data

The first test to determine if the gold price and exchange rate data are random walks or (possibly non-stationary) long memory processes utilizes a procedure designed by Diebold (1989) and based on the variance-time function. With sample of size T and mean μ , the variance of the k^{th} difference is:

$$\sigma_X^2(k) = \sum_{t=k}^T (X_t - X_{t-k} - k \cdot \mu)^2 / (T - k + 1),$$

and if X_t follows a random walk, $\sigma_X^2(k)$ is proportional to k :

$$\sigma_X^2(k) = k \cdot \sigma_X^2(1).$$

Under the null hypothesis that the series follows a random walk with drift, a simple scalar

TABLE A1. Stationarity tests on gold price and exchange rate data

Panel A. Diebold random walk test on gold prices and exchange rates								
Test	Gold		Mark		Yen		Pound	
	Spot	Forward	Spot	Forward	Spot	Forward	Spot	Forward
R2(2)	0.98	0.91	1.17	1.19	1.10	1.11	0.90	0.80
R2(4)	0.86	0.76	0.93	0.95	0.87	0.90	0.96	0.93
R2(8)	1.84	2.30	0.65	0.65	0.58	0.61	1.11	1.14
R2(16)	1.42	1.85	0.47	0.45	0.81	0.82	0.81	0.85
R2(32)	1.95	2.06	0.20	0.20	0.34	0.34	0.33	0.34
J2	0.94	1.77	4.96	5.27	2.08	1.97	1.60	1.48

Panel B. Maximum likelihood estimates of d : gold and the exchange rates				
Prices	Gold	Mark	Yen	Pound
Spot	0.83	1.02	1.07	1.05
Forward	0.86	1.08	1.07	1.06

asymptotic test statistic, $R2(k)$, is calculated as:

$$R2(k) = k \cdot \sigma_X^2(1) / \sigma_X^2(k).$$

The fractiles of the $R2(k)$ statistic for $k = 1, 2, 4, 8, 16, 32$ corresponding to the (random-walk) null hypothesis that $R2(k) = 1$ and the fractiles of a *joint* test statistic, $J2$, under the null hypothesis that *all* $R2(k)$ are equal unity have been calculated by Diebold (1989). The $R2$ and $J2$ tests statistics computed for all 108 overlapping observations on gold prices and exchange rates are reported in Panel A of Table A1. The null hypothesis of random walk with drift is rejected only in the case of the Deutsche mark, where the $J2$ joint test statistics rejects it.

The results of stationarity tests using maximum likelihood estimates of the order of differentiation, d , based on all 108 *overlapping* observations on spot and forward gold prices and the original US dollar exchange rate data are reported in Panel B of Table A1. As none of the estimates of d differ significantly from unity, all series, both spot and forward, appear to be non-stationary; moreover, a model selection procedure based on the Akaike and the Schwarz information criteria indicates that the series have no ARMA component and hence can be treated as random walks.

Notes

1. Of the currencies corresponding to the 150 members of the International Monetary Fund as of mid-1990, 12 were major currencies [10 of which comprised the European Monetary System (the EMS)], leaving 138 minor currencies. Fifty five minor currencies were tied to a single currency, 42 to a currency basket, and 41 were floating
2. To the best of the authors' knowledge, Ridler and Yandle (1972) were the first to use this approach to analyze the effect of exchange rate adjustments on commodity prices. The model presented here first appeared in Sjaastad (1985); a similar approach was developed by Dornbusch (1987).

3. *Global fundamentals* are defined as all factors *other than exchange rates and price levels* that influence the *global* demand for and supply of the commodity in question, including expectations.
4. See Frenkel (1981) concerning the large fluctuations in PPP, and hence in real exchange rates, experienced by the major currencies during the 1970s and Edwards (1989) for a massive compilation of real exchange rate data for smaller countries.
5. Lack of serial correlation in forecast errors also requires the absence of time-dependent risk premia; in what follows, that property is assumed to hold—and is subsequently tested—on forecast errors for exchange rates and the price of gold.
6. For the case at hand, the critical values for the *t*-statistics (two of which are reported in Table 1) are not the standard ones; for the correct critical values, see Hamilton (1994), page 592 and his Appendix tables.
7. While some *t*-lag statistics were significant for the pound/US dollar exchange rate, those forecast errors were never used in the actual estimation of Eq. (4). The rather voluminous details of these stationarity tests are available from the authors upon request.
8. The first subset contains all observations for the first month of each quarter, the second subset contains those on the second month, etc.
9. The co-integration tests were made using PCGIVE, version 8.0. The maximum likelihood estimates were made with a program kindly supplied by F. Sowell. Remaining estimation was by ESTIMA RATS 386 version 4.10c.
10. If Eq. (5) were estimated with a single lag on the independent variable, the expected value of the estimate of δ would be roughly 0.67 since two-thirds of the innovations in any observation on forecast errors tend to be common to adjoining observations. To avoid that bias, the independent variable was lagged three periods (i.e. one prediction period).
11. Since subsequent estimates of Eq. (5) using lags on the dependent variable reduces the regressions to 1983:01–90:12 period, all estimates were made with data over that range to facilitate comparisons.
12. Because the observations are overlapping, the residuals are serially correlated and hence it is not possible to conduct the usual LM tests for ARCH or normality tests, as those tests assume the series to be i.i.d.
13. The European price level was computed as a weighted average of German, UK and Italian producer prices, and French consumer prices (producer prices are unavailable for France). The weights, 0.3142, 0.2594, 0.2109, and 0.2125, respectively, were based on relative real GDPs for the 1982–1990 period.
14. An iterative procedure was used to set the weights in the world price level equal to the estimates of the thetas. Equation (4') was not parameterized to provide direct estimates of $\theta_j \equiv \Theta_j/\alpha(1)$ and $\Gamma \equiv \gamma(1)/\alpha(1)$ since the resulting non-linear equation would preclude using the Hansen–Hodrick (1980) method to estimate standard errors; instead, estimation was by OLS and the standard errors of θ_j and Γ were obtained by Taylor expansions.
15. Prior to calculating the standard deviations, the transformed residuals and spot prices were first differenced to remove negative trends, which are quite pronounced in the Deutsche mark and yen series. The sample average spot price of gold was used to compute coefficients of variation.

References

- Box, G. E. and Jenkins, G. M. (1976) *Time Series Analysis Forecasting and Control*, 2nd edn. Holden Day, San Francisco.
- Clements, K. and Sjaastad, L. (1984) *How Protection Taxes Exporters*. Thames essay, Trade Policy Research Center, Macmillan for the Trade Policy Research Center, London.
- Dickey, D. A. and Fuller, W. A. (1981) Likelihood ratio statistics for auto-regressive time series with a unit root, *Econometrica* **49**, 1057–1072.

- Diebold, F. X. (1989) Random walks versus fractional integration. In *Advances in Econometrics and Modeling*, ed. B. Raj. Kluwer Academic Publishers.
- Dornbusch, R. (1987) Exchange rate economics. *Economic Journal* **97**, 1–18.
- Edwards, S. (1989) *Real Exchange Rates, Devaluation, and Adjustment: Exchange Rate Policy in Developing Countries*. The MIT Press, Cambridge.
- Engle, R. F. and Granger, C. W. J. (1987) Co-integration and error correction: representation, estimation and testing. *Econometrica* **55**, 251–276.
- Frenkel, J. (1981) The collapse of purchasing power parity during the 1970s. *European Economic Review* **16**, 145–65.
- Granger, C. W. J. (1966) The typical spectral shape of an economic variable. *Econometrica* **34**, 151–161.
- Granger, C. W. J. and Joyeux, R. (1980) An introduction of long memory time series models. *The Journal of Time Series Analysis* **4**, 221–228.
- Hamilton, J. D. (1994) *Time Series Analysis*. Princeton University Press, Princeton.
- Hansen, L. P. and Hodrick, R. J. (1980) Forward exchange rates as optimal predictors of future spot rates: an econometric analysis. *The Journal of Political Economy* **88**, 829–53.
- Hosking, J.R.M (1981) Fractional differencing. *Biometrika* **68**, 165–176.
- Mussa, M. (1986) Nominal exchange rate regimes and the behavior of real exchange rates: evidence and implications. *Carnegie-Rochester Conference Series on Public Policy*, North-Holland Publishing Company, Amsterdam, 25, 117–213.
- Ridder, D. and Yandle, C. A. (1972) A simplified method for analyzing the effects of exchange rate changes on exports of a primary commodity. *IMF Staff Papers* 19.
- Sjaastad, L. A. (1985) Exchange rate regimes and the real rate of interest. In *The Economics of the Caribbean Basin*, eds M. Connolly and J. McDermott. Praeger, New York.
- Sowell, F. (1991) *Maximum Likelihood Estimation of Stationary Univariate Fractionally Integrated Time Series Models*, (mimeo), Graduate School of Industrial Administration, Carnegie Mellon University, Pittsburgh.
- White, H. (1980) A heteroscedasticity-consistent covariance matrix estimator and a direct test for heteroscedasticity. *Econometrica* **48**, 817–38.