Summary | Generalized Linear Models

# The need for more general linear models

* In many practical situations, we encounter data where the response variable Y is not approximately Normal or does not have constant variance given the predictors.
* Examples: binary outcome variables (e.g., Bernoulli), bounded counts (e.g., Binomial), unbounded counts (e.g., Poisson or Negative Binomial), strictly positive and continuous (e.g., Gamma, log-Normal), …
* We can generalize the linear models that we studied previously – i.e., linear regression – to handle these situations as well.
* The technical requirements are that the distribution for Y is chosen within an *exponential family* and that we can choose an appropriate function (i.e., the *link function*) that makes the conditional expectation of Y given X linear in the parameters.
* We showed that linear regression fits this exact framework when we assume that Y is Normally distributed and the link function is chosen to be the identity function.

# Maximum likelihood estimation

* We learned about the method of *maximum likelihood estimation*, which is the method used to fit these more general linear models.
* We also learned that the maximum likelihood estimator of the model parameters and the least squares estimator of the model parameters for linear regression are the same.
* In this section of the course, we learned about the general theory of maximum likelihood estimators, their properties, and the tests that can be derived for the parameters that they estimate (e.g., Wald tests, score tests, and likelihood ratio tests).

# Essential GLM theory

* We saw that the concept of Residual Sum of Squares (RSS) is generalized by the concept of *deviance* in the context of Generalized Linear Models.
* We learned that certain transformations of the response variable made in the context of linear regression approximate certain Generalized Linear Models (e.g., we looked at the relationship between log transformations of the response variable and Gamma regression).
* We discussed the ways in which Generalized Linear Models provide advantages over variable transformations (e.g., interpretability, decoupling between finding an appropriate distribution for the response variable and selecting a link function that helps the model achieve good fit to the data).
* We studied the maximum likelihood estimator of the parameters of a Generalized Linear Models and applied what we knew about maximum likelihood estimators in general to derive important properties for the construction of confidence intervals and tests.
* We talked about the existence of goodness of fit tests for Generalied Linear Models with known dispersion parameter.

# Model selection

* We discussed how some of the model selection techniques that we learned for linear regression carry over to Generalized Linear Models.

# Model diagnostics

* We studied how to translate what we already knew about the analysis of residuals for linear regression carries over to the analysis of residuals for Generalized Linear Models.

# Influential observations and outliers

* We studied how to translate what we already knew about the analysis of influential observations for linear regression carries over to the analysis of influential observations for Generalized Linear Models.

# Case study: logistic regression

* We discussed an application of logistic regression in some depth. In that example, we used logistic regression to model Binomial count data.
* We learned how to interpret the parameters of a logistic regression model.

# Case study: Poisson regression

* We discussed an application of Poisson regression in some depth. In that example, we used Poisson regression to model Poisson count data.
* We learned what *offset* terms are and how/why to include them in a Generalized Linear Model.