46-888 – Optimization for Prescriptive Analytics
Module 1 - Quiz

This quiz contains two problems. This offline part contains four questions in total. For each problem, the online quiz will ask you to upload your mathematical model and your Python implementation.

Note: Your linear programming formulation should be a *mathematical* model, and not Python code. Below are some examples (both correct and incorrect), based on the diet problem:

- ☐ Let F be the set of foods, let c_i be the cost of food $i \in F$, and let x_i be the decision variable representing the amount of food $i \in F$ in the diet. We define the objective function as follows:

$$\text{minimize } \sum_{i \in F} c_i x_i.$$

  This is a **correct** *mathematical formulation.*

- ☐ minimize $\sum_i c[i]x[i]$.

  This is **incorrect***: The use of brackets suggests that c and x are arrays, i.e., we multiply the i-th element from the c and the x array. However, in our mathematical models c and x are* vectors*, and the subscript i represents the i-th dimension of the vector space. Vectors have properties that arrays do not have (e.g., vector addition and scalar multiplication) and which are essential for the theory of linear programming.*

- ☐ Let c and x be given as above. We define the objective as

$$\text{minimize } c^\top x.$$

  This is a **correct** *mathematical formulation. We can multiply the transpose of the cost vector with the variable vector, which results in the expression under the first box. In most cases, however, summations (as under the first box) are recommended as they can be more flexibly defined for constraints.*

- ☐ $\sum a_{ij} x_i$ for i in $F \geq 700$

  This is an **incorrect** *formulation. It mixes Python syntax with mathematical expressions. Also, index j is not defined.*

- ☐ Let N be the set of nutrients and let a_{ij} represent the content of nutrient $j \in N$ in food $i \in F$. We define the requirement constraint as:

$$\sum_{i \in F} a_{ij} x_i \geq 700 \text{ for all } j \in N.$$

  This is a **correct** *mathematical formulation.*

- ☐ $60x_1 + 80x_2 + 25x_3 + 60x_4 >= 700$

  This is an **incorrect** *formulation: The sign $\geq$ should be used. Also, formulations using explicit numbers instead of generic input only work for (very) small problems.*

- ☐ $\sum_f amount_f * Nutrition_{f,n} \geq 700$, for each $n \in N$

  Assuming that 'amount' and 'Nutrition' are properly defined, one might argue that this is a correct formulation. It is not particularly elegant, however, and I do **not recommend** *this notation.*

Problem 1

Ohadi Foods is a distributor of consumer potatoes. Their market is partitioned into three product groups (large, medium, and small). Ohadi has determined the demand for each group for the upcoming planning period:

	Large	Medium	Small
Demand (tonne)	120	180	150

Demand per product group, in tonne.

Ohadi uses five large suppliers, each of which provides potatoes in bulk and without consideration of size (the potatoes in bulk contain all sizes). When the potatoes are received from the suppliers, Ohadi sorts them into groups (large, medium, and small) for further distribution.

Ohadi has visited the supplier warehouses and took samples to determine the percentages of large, small, and medium potatoes that are provided by each supplier. The results are indicated in the table below. The table also shows the price per tonne of potatoes for each supplier. For example, if we were to purchase 100 tonne of potatoes from supplier 1, it would contain 28 tonne Large, 45 tonne Medium, and 27 tonne Small potatoes. Ohadi does not impose any upper bound on the amount each supplier can provide.

	Cost ($/tonne)	Large	Medium	Small
Supplier 1	310	0.28	0.45	0.27
Supplier 2	270	0.25	0.40	0.35
Supplier 3	320	0.30	0.41	0.29
Supplier 4	240	0.17	0.48	0.35
Supplier 5	250	0.21	0.42	0.37

Cost and percentage content per product group for each supplier, in tonne.

This data can also be found in the file `ohadi.ipynb`.

Ohadi needs to determine how many potatoes to purchase from each supplier in order to meet the demand, with minimum total cost.

1. Formulate a linear programming model for this problem. Write down a version that is suitable for uploading (can be a digital document or a scan of a handwritten model).

2. Implement your model in Python and solve it to optimality.

Problem 2

A manufacturer of appliances, CleanCo, has three warehouses that supply washing machines to four retail outlets. The warehouses have respectively 6000, 9000, and 4000 machines available, and the demands at the retail outlets are projected to be at least 3900, 5200, 2700, and 6400 machines. The per unit costs (in dollars) of shipping from each warehouse to each retail outlet are given below in the table. The manufacturer needs to determine the minimum-cost shipping schedule that satisfies all demands.

	Retail Outlet 1	Retail Outlet 2	Retail Outlet 3	Retail Outlet 4
Warehouse 1	7	3	8	4
Warehouse 2	9	5	6	3
Warehouse 3	4	6	9	6

The data can also be found in the file `CleanCo.ipynb`.

3. Formulate a linear programming model for this problem. Write down a version that is suitable for uploading (can be a digital document or a scan of a handwritten model).

4. Implement your linear programming model in Python and solve it to optimality.