46-888 – Optimization for Prescriptive Analytics
Module 3 - Quiz

This quiz contains two problems. This offline part contains 8 questions in total. For each problem, the online quiz will ask you to upload your Python implementation and a description of your mathematical model.

Problem 1

Fogdish is a small-sized beer brewery in Western Pennsylvania. Their beer is produced in two factories (F1 and F2), and distributed via four retail points (R1, R2, R3, and R4). The factories can ship directly to only some retail points; the other retail points can be reached by shipping through two warehouses (W1 and W2). For logistic reasons, factory F1 can ship directly to retail point R1, but with a maximum of 400 gallons per week. Factory F2 can ship directly to retail point R4, with a maximum of 350 gallons per week. Furthermore, warehouse W1 can ship to all retail points except R4, while warehouse W2 can ship to all retail points except R1. There is no shipping limit on any of the other routes.

For the coming week, Fogdish needs to decide the amount of beer to be shipped from its factories to the retail points. The weekly production capacity of factory F1 is 3,200 gallons, while the weekly production capacity of factory F2 is 2,500 gallons. The demand for the coming week at each retail point is given in the following table. Note that our plan uses a weekly granularity and thus neglects individual weekdays; we may ignore the precise day on which the beer is produced or shipped.

Retail point	R1	R2	R3	R4
Demand (gallons)	700	1,800	1,100	900

The shipment costs (in dollars per gallon) between the factories, warehouses, and retail points are given in the table below. For example, shipping 100 gallon from factory F2 to retail point R4 costs $117. A dash (-) means that no shipment is possible on that route.

	W1	W2	R1	R2	R3	R4
F1	0.50	0.64	0.80	-	-	-
F2	0.40	0.55	-	-	-	1.17
W1	-	-	0.23	0.45	0.49	-
W2	-	-	-	0.85	0.25	0.64

Shipment costs in dollars per gallon

The goal is to find the minimum-cost distribution plan that satisfies all demand exactly.

1. Create a network representation of the problem, using the locations F1, F2, W1, W2, R1, R2, R3, and R4 as your nodes. Where applicable, define arcs, supply, demand, costs, and capacities, to correctly represent the above problem of finding a minimum-cost distribution plan.

2. What type of network flow model(s) does this problem resemble most (i.e., transportation problem, assignment problem, transshipment problem, shortest path problem, maximum flow problem, multi-commodity flow problem)?

3. Formulate the linear programming model associated with this network flow problem.

4. The data file for this problem is posted on the quiz Canvas site. Implement the linear programming model in Python, and solve it to optimality. Interpret the optimal solution to ensure its correctness.

Problem 2

Amin O. Acide owns a chemical company that needs to transport two large shipments from two different locations to a production plant. The shipments have equal volume and contain the same chemical (i.e., except for the origin location they can be considered indistinguishable). The transport takes place through an urban area, and for safety reasons the routes should be as short as possible. Also, both shipments are delivered in a large container, and *cannot* be split into multiple smaller shipments.

A schematic map of the area is given below, indicating the possible routes that can be taken. Nodes 1 and 2 correspond to the two points of origin. Node P corresponds to the destination plant. All other nodes correspond to an intersection of two roads. An arc between two nodes corresponds to a road segment, and has an associated length in miles, as indicated in the figure.

The area also contains a river (indicated by the light grey curve) that can be crossed by bridge or by a tunnel. The road segments $(5, 8)$ and $(6, 9)$ contain a bridge crossing the river, while the road segment $(7, 10)$ contains a tunnel. For safety reasons, the city allows each bridge and tunnel to be crossed by at most one of the shipments. For example, the shipments cannot both traverse the road segment $(5, 8)$.

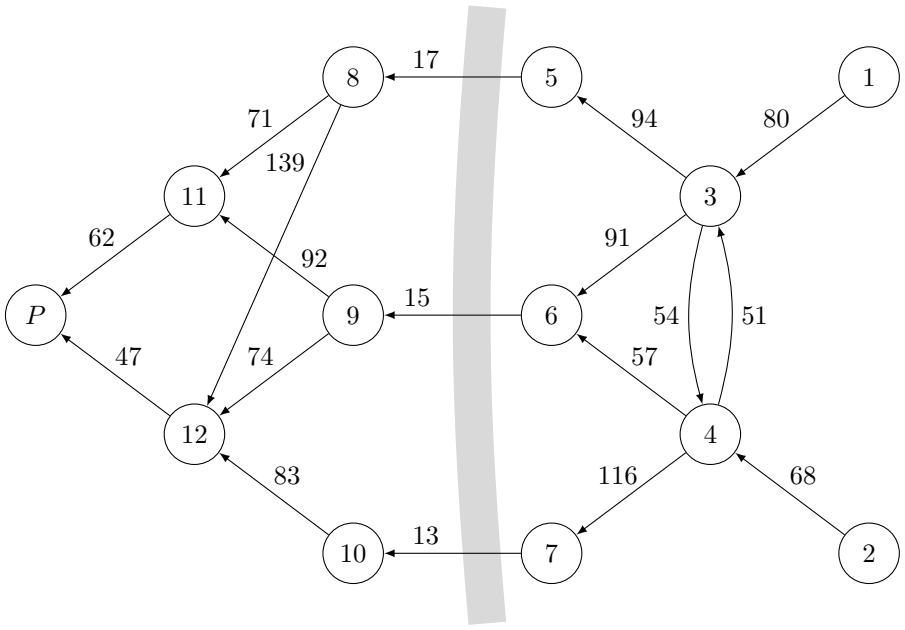
Amin O. Acide wishes to determine a delivery route for both shipments with minimum total distance. We will approach this problem as a network flow problem, and formulate it using linear programming.

5. What type of network flow model(s) does this problem resemble most (i.e., transportation problem, assignment problem, transshipment problem, shortest path problem, maximum flow problem, multi-commodity flow problem)?

6. Complete the network by adding supply, demand, and any non-trivial lower or upper bounds. (The trivial bounds $[0, \infty]$ are already assumed to be present and do not have to be added.)

7. Formulate the problem as a linear program.

8. The data file for this problem is posted on the quiz Canvas site. Implement the linear programming model in Python and solve it to optimality. Interpret the optimal solution to ensure its correctness.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