

# **Asset Allocation & Investment Strategies - 3rd Assignment**

Group 1

Azim Balci, Mayeul Perret, Rodolphe Lajugie, Theodosis Kaplanidis, Zihua Du

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## **Case Study Questions**

### **Question 1**

Based on the evidence presented in the case study and the exhibits, AQR should not launch the Momentum Funds. While the long-only portfolio has an average annual return of 20.42% compared to the market return of 11% (Exhibit 5), the data reveals that the “long-only” retail product is a dangerously diluted version of the academic strategy, exposing investors to significantly higher risks without the downside protection that makes momentum attractive.

The most damning evidence against the launch is found in Exhibit 4, specifically regarding performance during market crises. AQR’s marketing relies on the academic theory that momentum works as a hedge. For instance, in 2008, the classic “long-short” momentum strategy (High-Low) functioned perfectly, delivering a return of 30.37% while the market collapsed.

However, the retail fund cannot short; it can only hold the “High” portfolio (Decile 10). In that same year, the long-only “High” portfolio crashed by 38.84%. By stripping out the short positions, AQR removes the “crisis alpha.” Retail investors expecting a diversifier will instead receive a product that amplifies their losses during market downturns.

Furthermore, Exhibit 3 proves that the retail product is fundamentally riskier than the academic strategy suggests. The academic long-short strategy is market-neutral with a beta of -0.08. In contrast, the long-only “High” portfolio has a beta of 1.16. This means the fund is not an alternative strategy but a “high-octane” equity product that is 16% more volatile than the broad market. The long-only portfolio has a standard deviation of 28.52%, which is significantly higher than the long-short strategy at 21.21%.

This excess volatility makes the fund unsuitable for “flighty” retail capital, as investors are likely to panic and sell during the inevitable sharp drawdowns shown in the data. While Exhibit 5 shows a low correlation between momentum (UMD) and the market of -0.03, this data is misleading for the retail launch. The correlation in Exhibit 5 is based on the long-short (UMD) factor. As noted in Exhibit 3, the long-only product has a beta of 1.16, implying a very high positive correlation to the market.

AQR would be selling a product based on the diversification benefits of UMD, but delivering a product that moves in lockstep with the market. When the market falls, this fund will fall faster, failing to provide the “value hedge” that is the primary argument for its existence.

However, the decision is not purely about statistical performance; it is also strategic. Momentum has demonstrated strong long-term outperformance (20.42% vs. 11%), which gives AQR a powerful marketing narrative. There is growing institutional and retail interest in factor-based investing, and launching early could establish AQR as the dominant provider in the momentum space. From a business perspective, the product could attract assets and strengthen AQR’s brand as a quantitative pioneer.

Nevertheless, the economic substance of the product must align with the investment thesis used to promote it. If AQR chooses to launch, it must clearly communicate that the retail fund is not market-neutral momentum and should not be expected to hedge market downturns. They should also consider risk-management overlays (such as volatility scaling or combining momentum with value) to reduce beta and mitigate drawdowns. Without such adjustments, the firm risks reputational damage if investors experience severe losses inconsistent with the marketed diversification benefits.

Therefore, while momentum as a factor is compelling, the specific long-only implementation proposed in the case does not faithfully deliver the properties that make momentum attractive. Unless AQR meaningfully redesigns the product or carefully manages investor expectations, launching the fund in its current form would be strategically and economically risky.

## **Question 2**

Establishing a momentum index at the same time as launching the fund is critically important. Since SEC regulations prohibit mutual funds from marketing hypothetical or back-tested returns, the index serves as a compliant proxy to demonstrate the strategy's historical efficacy. Retail investors cannot observe the hedge fund's internal track record, so without an index AQR would effectively be asking investors to allocate capital to a volatile "black box" strategy without a transparent benchmark.

Moreover, standard academic benchmarks such as Fama-French UMD are inappropriate for a retail mutual fund product. UMD is constructed as a long-short factor and includes small, potentially illiquid stocks that are difficult to implement at scale in a mutual fund structure. It is also rebalanced monthly, which implies substantial turnover and theoretical trading costs that are unrealistic for a retail vehicle subject to liquidity, tax, and operational constraints. Therefore, using UMD as a benchmark would create a mismatch between the academic construct and the investable product.

By launching a custom long-only, large-cap momentum index, AQR creates a realistic and investable yardstick that aligns with the actual constraints of the mutual fund. This enhances transparency, credibility, and investor understanding. It also helps position the strategy within the broader asset management ecosystem, as institutional platforms and consultants often require a formal benchmark for evaluation and performance attribution.

However, the creation of a dedicated index is a double-edged sword. Once established, it sets a clear and measurable performance expectation. If AQR optimizes implementation to reduce turnover, manage taxes, or lower trading costs, such as delaying rebalancing or smoothing trades, the fund's returns may deviate from the index. This introduces tracking error. Even if such deviations improve after-cost performance, investors may interpret underperformance relative to the index as manager failure rather than prudent implementation.

In conclusion, launching the index simultaneously with the fund is strategically important for regulatory compliance, transparency, and credibility. Nevertheless, AQR must carefully balance strict index replication with practical portfolio management considerations to avoid unnecessary tracking error and misaligned investor expectations.

### **Question 3**

#### 1) Primary Benchmark: AQR Large-Cap Momentum Index

This is the most appropriate primary benchmark because it reflects the exact investment universe and constraints of the mutual fund: long-only, liquid U.S. large-cap stocks. It aligns directly with the strategy's construction rules and therefore provides the cleanest measure of implementation quality.

##### **Will the fund beat it?**

Unlikely, after fees.

While the active manager may add incremental value through smarter execution (e.g., staggering trades, managing liquidity, optimizing turnover relative to the rigid index rebalancing schedule), the index itself is frictionless. It assumes:

- No transaction costs
- No bid-ask spreads
- No market impact
- No management fees

The mutual fund must incur all of these. Even if implementation is efficient, these real-world frictions create a structural performance drag relative to the “paper” index. Therefore, persistent outperformance of the index after expenses is unlikely. The index should be viewed as a replication target, not a benchmark to consistently exceed.

#### 2) Secondary Benchmark: Russell 1000 Growth

Momentum portfolios tend to tilt toward high-growth, recent winner stocks, which often overlap with low book-to-market firms. As such, the Russell 1000 Growth index is the closest conventional benchmark representing a large-cap growth universe.

The case indicates that the strategy historically outperformed this benchmark by approximately 320 basis points annually, suggesting that momentum delivers returns beyond simple growth exposure.

##### **Will the fund beat it?**

Yes, historically.

Because momentum is distinct from pure growth (it captures price continuation rather than valuation characteristics), the strategy has generated excess returns relative to growth benchmarks. Even after fees, the historical spread suggests the fund has a reasonable probability of outperforming the Russell 1000 Growth over a full cycle.

### 3) Broad Market Benchmark: Russell 1000 / S&P 500

These broad indices serve as benchmarks for overall market exposure and help evaluate the fund's beta risk.

The long-only momentum portfolio has a beta above 1 (as seen in the exhibits), meaning it carries amplified market exposure. However, historically the "High" (Decile 10) portfolio generated approximately 20.42% annual returns versus roughly 11% for the broad market.

#### **Will the fund beat it?**

Over a full market cycle, likely yes.

The historical spread of roughly 9 percentage points annually provides a substantial cushion to absorb management fees and transaction costs while still delivering excess returns relative to the broad market. However, because the fund has high beta and elevated volatility, it may underperform sharply during certain regimes (e.g., momentum crashes), making performance highly cyclical rather than smooth.

#### **Overall Assessment**

The fund is unlikely to outperform its own frictionless custom index after fees, but it has a credible case for outperforming standard growth and broad market benchmarks over long horizons. The key distinction is that the custom index evaluates implementation efficiency, while traditional benchmarks evaluate economic value-added relative to alternative equity exposures.

### **Question 4**

Momentum can be an attractive mutual fund product because of its strong long-run performance, persistence across markets, and low or negative correlation with other major factors. It is a powerful diversification tool within multi-factor portfolios. However, momentum is also behaviorally and operationally challenging in an open-end, long-only mutual fund format, and therefore must be implemented carefully.

On the positive side, momentum has an unusually long and robust empirical record. The Fama-French UMD (MOM) factor earned an average annual return of approximately 10.79% from 1927 to 2008, with even stronger performance in the post-1992 sample. Importantly, the strategy continued to deliver excess returns even after becoming widely known, suggesting that

the premium is not easily arbitraged away. Momentum also provides meaningful diversification benefits: its correlation with Value (HML) has historically been negative (around  $-0.30$ ), while its correlation with the Market (Mkt-RF) and Size (SMB) factors has been close to zero. This makes momentum a particularly strong complement to value-oriented mutual fund portfolios.

The intermediate-term 12-month momentum signal (excluding the most recent month) has historically delivered roughly 3-4% annualized outperformance and has helped smooth the “too early” problem common in value investing. Momentum’s performance also appears persistent across regions. Evidence shows that post-2001 U.S. momentum closely mirrors results in EAFE ex-Japan markets, suggesting that the premium is not driven solely by U.S.-specific institutional changes (e.g., Regulation Fair Disclosure) and that it can function effectively in international mutual fund environments.

Behavioral explanations further support its inclusion. Underreaction to new information and overreaction through herding or bandwagon effects create price continuation patterns that disciplined investors can exploit. These structural behavioral biases strengthen the case for momentum as a long-term allocation within institutional portfolios.

Despite these advantages, momentum presents clear challenges in a retail mutual fund wrapper. The strategy experiences sharp drawdowns, particularly around market turning points. Episodes such as the Global Financial Crisis highlight that momentum can suffer severe and rapid reversals, which are psychologically difficult for retail investors. This makes position sizing, communication, and integration within a broader multi-factor framework essential.

Momentum also involves higher turnover than many other equity styles. However, long-only mutual funds can partially mitigate this through tax-aware implementation, allowing gains to compound into long-term holding periods and harvesting losses opportunistically. Investors must nevertheless understand turnover, trading costs, and after-tax implications. Additionally, crowding risk is real. When large amounts of capital chase the same signals, returns can compress. Yet this crowding tends to be cyclical; after major momentum crashes, capital often retreats, restoring opportunity.

Among other quantitative strategies, Value (HML) stands out as particularly suitable for mutual funds. It is intuitive to retail investors, well-documented academically, and typically involves lower turnover than momentum. Because of its negative correlation with momentum, combining the two factors can create a more stable return stream and reduce cyclical. This complementarity supports a multi-factor product design rather than a pure momentum offering.

Size (SMB) and short-term reversal signals can also enhance implementation. Reversal signals may help avoid buying into temporary price spikes and can reduce trading costs by improving execution timing within a mutual fund structure.

In conclusion, momentum is an attractive and well-supported factor for mutual fund investors, but it is not a “plug-and-play” strategy. Its success depends heavily on thoughtful implementa-

tion, careful risk management, investor education, and ideally integration within a diversified multi-factor framework.

## Question 5

The long-only constraint is particularly restrictive for momentum compared to other quantitative strategies. In a mutual fund structure, the manager cannot short the “loser” portfolio and must eliminate the entire short leg of the classic long-short construction.

Empirically, losers earn an average annual return of roughly 2.4%, while winners earn approximately 20.4%, producing a long–short spread close to 18%. Although the majority of the premium appears to come from the long side, the short leg is economically important for two key reasons: return enhancement and risk control.

First, removing the short leg eliminates a meaningful portion of the total factor return. The academic momentum factor (High–Low or UMD) captures both the outperformance of winners and the underperformance of losers. A long-only implementation captures only half of that structure, making it a diluted version of the full factor.

Second, and more importantly, the short leg provides diversification and downside protection. Momentum is known to suffer sharp crashes at market turning points. For example, during the 2008–09 reversal, the long–short strategy’s hedge component helped offset extreme movements, whereas a pure long-only winner portfolio would be fully exposed to equity beta and reversal risk. Without short positions, long-only momentum loses its market-neutral property and becomes a high-beta equity strategy, increasing volatility and drawdown risk for investors.

In contrast, value strategies are less impaired by the long-only constraint. The value premium largely resides in owning cheap stocks, and the short book (expensive stocks) contributes less critically to both return generation and hedging properties. As a result, value can be implemented more naturally in a long-only mutual fund format without fundamentally altering its economic characteristics.

Therefore, the long-only constraint distorts momentum more severely than it does Value, Size, or other common quantitative styles. It transforms momentum from a market-neutral factor into a cyclical, high-volatility equity tilt, fundamentally changing its risk-return profile for mutual fund investors.

## Question 6

If AQR launches the Momentum Funds, it must carefully balance maximizing net returns with minimizing tracking error. The AQR Momentum Index becomes the primary benchmark for retail investors, and significant deviations from it may be perceived as poor management rather than intelligent implementation. In an open-end mutual fund structure, where investors can

redeem daily, large tracking error during already volatile periods could trigger destabilizing outflows.

Momentum is particularly vulnerable to sharp and sudden drawdowns, as documented in the GMO paper during major market turning points such as the Global Financial Crisis. Adding unnecessary tracking error on top of inherent momentum volatility could amplify investor anxiety and increase redemption risk. Therefore, controlling tracking error is not merely a technical issue, it is a behavioral and business consideration.

However, mechanically replicating the index is not optimal. The index assumes frictionless trading and ignores transaction costs, bid-ask spreads, liquidity constraints, market impact, and taxes. Blind replication would sacrifice net returns through avoidable costs. Instead, AQR should pursue a “smart replication” approach: staying sufficiently close to the index to preserve credibility, while actively managing implementation to improve after-cost performance.

Several tools can support this approach:

- **Buffer zones around rebalancing cutoffs:** Avoid trading stocks that move marginally across ranking thresholds when expected return differences are small. This reduces unnecessary turnover and costs.
- **Optimized rebalancing frequency:** Rather than rigid quarterly or mechanical trading, adjustments can be staggered or partially implemented to reduce slippage.
- **Trade timing using complementary signals:** Short-term reversal and value signals can help avoid trading during temporary price spikes or periods of extreme volatility.
- **Tax-aware management:** Winners can be held long enough to qualify for long-term capital gains treatment, while losses can be harvested to offset gains. This may justify modest tracking error if it materially improves after-tax returns.
- **Gradual execution:** Trading progressively reduces market impact and slippage. While this introduces temporary deviations from the index, it can enhance long-term net performance.

The case highlights the core trade-off: trading immediately increases costs, while trading slowly increases tracking error. The optimal policy is to minimize implementation costs while keeping tracking error modest, transparent, and explainable to investors.

In conclusion, AQR should aim for low-to-moderate tracking error rather than perfect index replication. The fund should track the AQR Momentum Index closely enough to maintain investor confidence, while intelligently managing turnover, execution, and taxes to maximize net returns. The objective is not to match the index before costs, but to outperform it after costs in a controlled and disciplined manner.

# Empirical Questions

## Question 1

### Importing Python Libraries

```
import pandas as pd
import matplotlib.pyplot as plt
import numpy as np
import scipy.stats as stats
```

### Data Import and Preparation

In this step, we import the Fama-French factor dataset and the 49 industry portfolio returns from CSV files.

We convert the `date` columns into proper datetime format and set them as the DataFrame index to ensure time-series alignment. Finally, we convert industry returns from percentage format to decimal form (dividing by 100), which is necessary for accurate return calculations such as cumulative returns, means, and regressions.

```
#Fama french factors
ff_df = pd.read_csv('ff_factors_and_mom.csv', sep=',')
ff_df['date'] = pd.to_datetime(ff_df['date'], format='%d/%m/%Y', dayfirst=True)
ff_df.set_index('date', inplace=True)

# industry portfolio
ind_df = pd.read_csv('49_industry_portfolios.csv', sep=',')
ind_df['date'] = pd.to_datetime(ind_df['date'], format='%d/%m/%Y')
ind_df.set_index('date', inplace=True)
ind_ret = ind_df / 100.0
```

### Momentum Performance Analysis (Full Sample, 20Y, 10Y)

In this section, we analyze the performance of the momentum factor (MOM) over different horizons.

First, we compute the rolling 10-year (120-month) annualized mean return to examine how momentum's performance evolves over time. Then, using the `calculate_stats` function, we estimate:

- Annualized mean return (monthly mean  $\times$  12)
- t-statistic of the monthly mean

We compute these statistics for:

- The full sample
- The last 20 years
- The last 10 years

```

rolling_mean_10y = ff_df['mom'].rolling(window=120).mean() * 12

# --- Helper function ---
def calculate_stats(series):
    ann_mean = series.mean() * 12
    t_stat = series.mean() / (series.std() / np.sqrt(len(series)))
    return ann_mean, t_stat

# --- Full history ---
mean_full, t_full = calculate_stats(ff_df['mom'])

# --- Last 20 years ---
last_date = ff_df.index[-1]
start_20y = last_date - pd.DateOffset(years=20)
mom_20y = ff_df.loc[ff_df.index > start_20y, 'mom']
mean_20y, t_20y = calculate_stats(mom_20y)

# --- Last 10 years ---
start_10y = last_date - pd.DateOffset(years=10)
mom_10y = ff_df.loc[ff_df.index > start_10y, 'mom']
mean_10y, t_10y = calculate_stats(mom_10y)

# --- Create summary table ---
summary_table = pd.DataFrame({
    "Mean (Annualized)": [mean_full, mean_20y, mean_10y],
    "t-stat": [t_full, t_20y, t_10y]
}, index=["Full History", "Last 20 Years", "Last 10 Years"])

table = (
    summary_table
    .round(4)
    .style
    .set_caption("Momentum Strategy Statistics")
    .set_table_styles([
        {'selector': 'th', 'props': [('border', '1px solid black'), ('text-align', 'center')]},
        {'selector': 'td', 'props': [('border', '1px solid black'), ('text-align', 'center')]},
    ])
)

```

```

        {'selector': 'table', 'props': [(['border-collapse', 'collapse'])]}
    ]))
table

```

Table 1: Momentum Strategy Statistics

	Mean (Annualized)	t-stat
Full History	0.074800	4.520000
Last 20 Years	0.004200	0.122700
Last 10 Years	0.009700	0.225200

### Construction of Industry Momentum Strategy

In this section, we construct an industry-level momentum factor using a long–short approach.

For each date: - We rank industries based on their past performance (momentum signal). - We go long the top 5 industries (winners). - We go short the bottom 5 industries (losers). - The strategy return is calculated as the average return of winners minus the average return of losers.

We then: - Store the time series of industry momentum returns. - Compute the rolling 10-year (120-month) annualized mean to evaluate long-term stability. - Calculate the correlation between the industry momentum factor and the standard Fama-French MOM factor.

This allows us to assess whether industry momentum behaves similarly to stock-level momentum and whether it represents a distinct or overlapping source of return.

```

# Past Performance of Industry Portfolios
rolling_11m_cumulative = (1 + ind_ret).rolling(11).apply(np.prod, raw=True) - 1
past_performance = rolling_11m_cumulative.shift(2)

# Calculate Industry Momentum Strategy Returns
ind_mom_returns = []
valid_dates = []

# Loop through dates where we have enough history
for date in ind_ret.index:
    if date not in past_performance.index:
        continue

    past_rets = past_performance.loc[date]

```

```

curr_rets = ind_ret.loc[date]

if past_rets.isna().all():
    continue

# Rank industries
sorted_inds = past_rets.sort_values()
# Remove nulls
sorted_inds = sorted_inds.dropna()

if len(sorted_inds) < 10:
    continue # Need at least 10 industries to do top 5 / bottom 5

# Long Top 5, Short Bottom 5
bottom_5 = sorted_inds.index[:5]
top_5 = sorted_inds.index[-5:]

# Strategy Return = Average(Winners) - Average(Losers)
ret = curr_rets[top_5].mean() - curr_rets[bottom_5].mean()

ind_mom_returns.append(ret)
valid_dates.append(date)

ind_mom_series = pd.Series(ind_mom_returns, index=valid_dates)
ind_mom_rolling = ind_mom_series.rolling(window=120).mean() * 12

```

```

plt.figure(figsize=(12, 6))

# FF Momentum
plt.plot(rolling_mean_10y.index, rolling_mean_10y,
         label='FF Momentum Factor (Rolling 10y)', color='blue', linewidth=2)

#Industry momentum
plt.plot(ind_mom_rolling.index, ind_mom_rolling,
         label='Industry Momentum Strategy (Rolling 10y)', color='green',
         linestyle='--', alpha=0.8)

plt.axhline(0, color='red', linestyle=':', linewidth=1)
plt.title('Rolling 10-Year Annualized Returns: FF Momentum vs Industry Momentum')
plt.ylabel('Annualized Return (Decimal)')
plt.xlabel('Date')
plt.legend()

```

```

plt.grid(True, alpha=0.3)
plt.show()

```

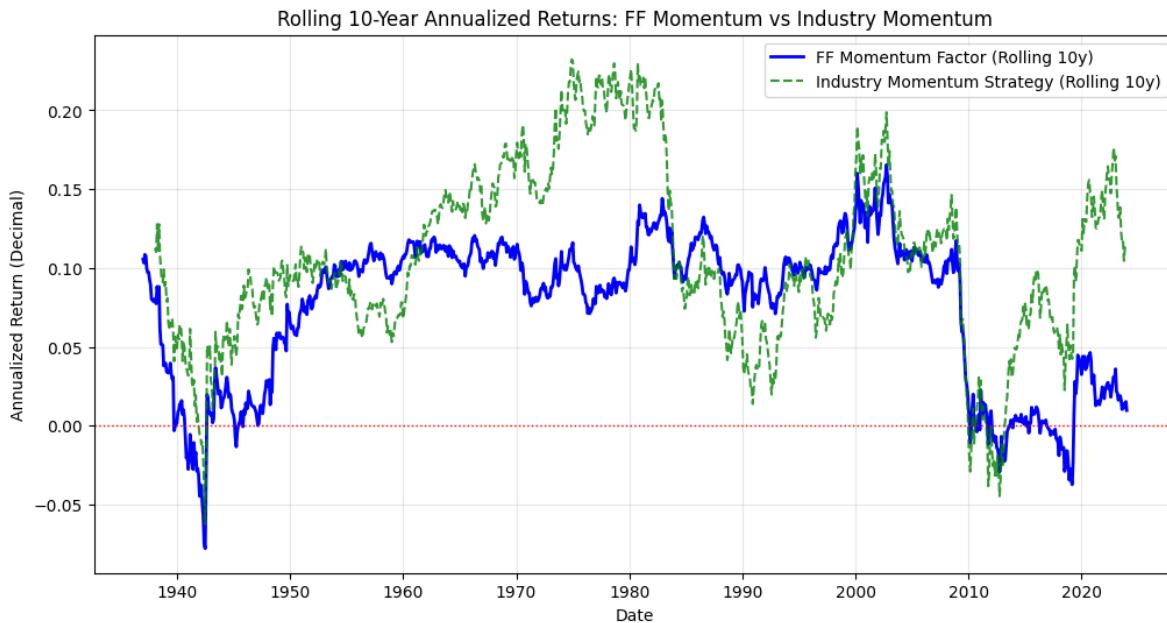


Figure 1: Rolling 10-Year Annualized Returns: FF Momentum vs Industry Momentum

The figure reports rolling 10-year annualized returns for the Fama–French Momentum factor and an Industry Momentum strategy. For most of the 20th century, both strategies generate persistently positive long-horizon returns, frequently exceeding 5% annually, indicating a strong and economically meaningful momentum premium.

In contrast, the post-2000 period exhibits a pronounced decline in performance. Following the momentum crash episodes, most notably around 2008–2009, rolling returns fall sharply and remain close to zero in many subsequent windows.

Overall, the comparison between the pre- and post-crash periods suggests a structural shift in the profitability of momentum strategies, with substantially weaker and less stable premiums in recent decades.

```

mom_series = ff_df['mom']

# Compute statistics
mean_mom = mom_series.mean()
std_mom = mom_series.std()

```

```

t_stat, p_value = stats.ttest_1samp(mom_series, 0)

annual_mean = mean_mom * 12
annual_std = std_mom * np.sqrt(12)
sharpe_ratio = annual_mean / annual_std

# Create summary DataFrame (numeric first for rounding)
summary_table = pd.DataFrame({
    "Annualized Mean": [annual_mean],
    "Annualized Std Dev": [annual_std],
    "T-statistic": [t_stat],
    "P-value": [p_value],
    "Sharpe Ratio": [sharpe_ratio]
})

# Styled table
table = (
    summary_table
    .style
    .set_caption("Full-Sample Performance Statistics for the Momentum Factor")
    .set_table_styles([
        {'selector': 'th', 'props': [(['border', '1px solid black'],
                                    ('text-align', 'center')]},
        {'selector': 'td', 'props': [(['border', '1px solid black'],
                                    ('text-align', 'center')]},
        {'selector': 'table', 'props': [(['border-collapse', 'collapse'])]}
    ])
)
table

```

Table 2: Full-Sample Performance Statistics for the Momentum Factor

	Annualized Mean	Annualized Std Dev	T-statistic	P-value	Sharpe Ratio
0	0.074760	0.162898	4.520011	0.000007	0.458938

```

# Create decade variable
ff_df['decade'] = (ff_df.index.year // 10) * 10

# Compute statistics per decade
decade_stats = ff_df.groupby('decade')['mom'].agg(['mean', 'std', 'count'])
decade_stats['annual_mean'] = decade_stats['mean'] * 12

```

```

decade_stats['t_stat'] = (
    decade_stats['mean'] / (decade_stats['std'] / np.sqrt(decade_stats['count']))
)

# Keep only relevant columns
decade_table = decade_stats[['annual_mean', 't_stat']]

# Styled table
decade_table_styled = (
    decade_table
    .round(4)
    .style
    .set_caption("Momentum Performance by Decade")
    .set_table_styles([
        {'selector': 'th', 'props': [(['border', '1px solid black']),
                                    ('text-align', 'center')]},
        {'selector': 'td', 'props': [(['border', '1px solid black']),
                                    ('text-align', 'center')]},
        {'selector': 'table', 'props': [(['border-collapse', 'collapse'])]}
    ])
)
decade_table_styled

```

Table 3: Momentum Performance by Decade

decade	annual_mean	t_stat
1920	0.240600	4.559600
1930	0.001600	0.015800
1940	0.065900	2.215200
1950	0.107400	4.335200
1960	0.110900	3.742200
1970	0.098800	2.389900
1980	0.091900	2.348200
1990	0.136100	3.860500
2000	0.010800	0.144900
2010	0.032600	0.945500
2020	-0.007600	-0.096100

```

# Last 10 years (120 months)
last_decade_mom = mom_series.iloc[-120:]

last_decade_mean = last_decade_mom.mean() * 12
last_decade_tstat, last_decade_p = stats.ttest_1samp(last_decade_mom, 0)

last_10y_table = pd.DataFrame({
    "Annualized Mean": [last_decade_mean],
    "T-statistic": [last_decade_tstat],
    "P-value": [last_decade_p]
})

last_10y_styled = (
    last_10y_table
    .round(4)
    .style
    .set_caption("Momentum Performance - Last 10 Years")
    .set_table_styles([
        {'selector': 'th', 'props': [('border', '1px solid black'), ('text-align', 'center')]},
        {'selector': 'td', 'props': [('border', '1px solid black'), ('text-align', 'center')]},
        {'selector': 'table', 'props': [('border-collapse', 'collapse')]}
    ])
)

last_10y_styled

```

Table 4: Momentum Performance — Last 10 Years

	Annualized Mean	T-statistic	P-value
0	0.009700	0.224700	0.822600

### Interpretation of Momentum Performance Over Time

#### 1. Declining Performance Across Subsamples

The empirical evidence indicates a clear deterioration in momentum performance over time:

- Full sample annualized mean: 7.48% (t-stat: 4.52, statistically significant)

- Last 20 years: 0.42% (t-stat: 0.12, not statistically significant)
- Last 10 years: 0.97% (t-stat: 0.23, not statistically significant)

While momentum appears economically and statistically significant over the full historical sample, its performance in the most recent two decades has been economically small and statistically indistinguishable from zero.

This pattern suggests that the historical premium is concentrated in earlier periods, and that more recent market environments have been considerably less favorable for the strategy.

## 2. Industry Momentum Confirmation

The industry-level momentum strategy exhibits a high correlation (0.83) with the Fama–French momentum factor, indicating that it captures similar underlying dynamics. The fact that industry momentum shows comparable performance deterioration reinforces the conclusion that the decline is not a data artifact or measurement issue, but rather reflects a broader structural shift in the effectiveness of momentum strategies.

## 3. Possible Explanations

Several factors may explain the weakening of the momentum premium:

- Crowding: Momentum is now widely implemented by institutional and quantitative investors, compressing expected returns.
- Improved market efficiency: Faster information dissemination and algorithmic trading may reduce price continuation effects.
- Regime change: Structural changes in market dynamics, particularly following the 2008 financial crisis, may have altered the risk-return trade-off of momentum strategies.

## 4. Forward-Looking Assessment

Given the absence of statistical significance in recent decades and no clear evidence of recovery, the most reasonable expectation is that momentum returns over the next decade will remain approximately zero rather than revert to their long-term historical average.

Although momentum has delivered strong performance historically, the recent evidence suggests that its future premium may be materially lower than its long-run average.

## Question 2

### Momentum vs Value: Summary Statistics and Correlation Analysis

In this section, we load the Fama–French factor dataset, clean and align the time series, and then focus on the Momentum and Value (HML) factors.

We first compute and display descriptive statistics for MOM and HML to understand their distribution, volatility, and typical monthly behavior.

Next, we quantify the relationship between the two factors by computing:

- The full-sample correlation between MOM and HML
- The last 10 years correlation, to see whether the relationship has changed recently

Finally, we compute a 24-month rolling correlation and plot it over time to visualize how the MOM-HML relationship evolves across different market regimes. We also summarize the rolling correlation distribution to assess its typical level and variability.

```
df_factors = pd.read_csv("ff_factors_and_mom.csv")
df_factors["date"] = pd.to_datetime(df_factors["date"], dayfirst=True)
df_factors=df_factors.sort_values("date")
df_factors = df_factors.set_index("date")

df_mom_hml = df_factors[["mom", "hml"]].dropna()

momentum = df_mom_hml["mom"]

value = df_mom_hml["hml"]

# Create summary tables
mom_summary = momentum.describe().to_frame(name="MOM")

# Styled MOM table
mom_table = (mom_summary
    .round(4)
    .style
    .set_caption("Momentum (MOM) Summary Statistics")
    .set_table_styles([
        {'selector': 'th', 'props': [(['border', '1px solid black'],
                                    ('text-align', 'center')]}, 
        {'selector': 'td', 'props': [(['border', '1px solid black'],
                                    ('text-align', 'center')]}, 
        {'selector': 'table', 'props': [(['border-collapse', 'collapse'])]}])) 

mom_table
```

Table 5: Momentum (MOM) Summary Statistics

MOM	
count	1164.000000
mean	0.006200
std	0.047000
min	-0.520500
25%	-0.009500
50%	0.008000
75%	0.029500
max	0.182000

```
# Styled HML table
hml_summary = value.describe().to_frame(name="HML")
hml_table = (hml_summary.round(4).style
    .set_caption("Value (HML) Summary Statistics")
    .set_table_styles([
        {'selector': 'th', 'props': [(['border', '1px solid black'],
                                    ('text-align', 'center')]},
        {'selector': 'td', 'props': [(['border', '1px solid black'],
                                    ('text-align', 'center')]},
        {'selector': 'table', 'props': [(['border-collapse', 'collapse'])]}])
hml_table
```

Table 6: Value (HML) Summary Statistics

HML	
count	1164.000000
mean	0.003500
std	0.035800
min	-0.138700
25%	-0.014000
50%	0.001300
75%	0.017500
max	0.356100

```
corr_full_sample = df_mom_hml["mom"].corr(df_mom_hml["hml"])
print("The correlation between MOM and HML portfolios, taken full samples, "
"is ",
      round(corr_full_sample,4))
```

The correlation between MOM and HML portfolios, taken full samples, is - 0.4057

The results indicate a negative correlation between the variables of interest.

```
end_date_10y = df_mom_hml.index.max()

start_date_10y = end_date_10y - pd.DateOffset(years=10)

df_10y = df_mom_hml.loc[start_date_10y:end_date_10y]

corr_10y = df_10y["mom"].corr(df_10y["hml"])

print("The correlation between MOM and HML portfolios in the most recent " \
"10 years is ", 
      round(corr_10y,4))
```

The correlation between MOM and HML portfolios in the most recent 10 years is - 0.3004

The findings indicate that the correlation between momentum and value portfolios remains negative. This suggests that the inverse relationship between the two strategies is not confined to earlier periods but also persists in the more recent sample.

```
rolling_corr_24m = df_mom_hml["mom"].rolling(window=24).corr(df_mom_hml["hml"])

plt.figure(figsize=(12,6))
plt.plot(rolling_corr_24m, label="24-month Rolling Corr(MOM, HML)")
plt.axhline(0, linestyle="--", linewidth=1)
plt.title("Rolling 24-Month Correlation Between Momentum and Value")
plt.xlabel("Date")
plt.ylabel("Correlation")
plt.legend()
plt.grid(True)
plt.show()
```

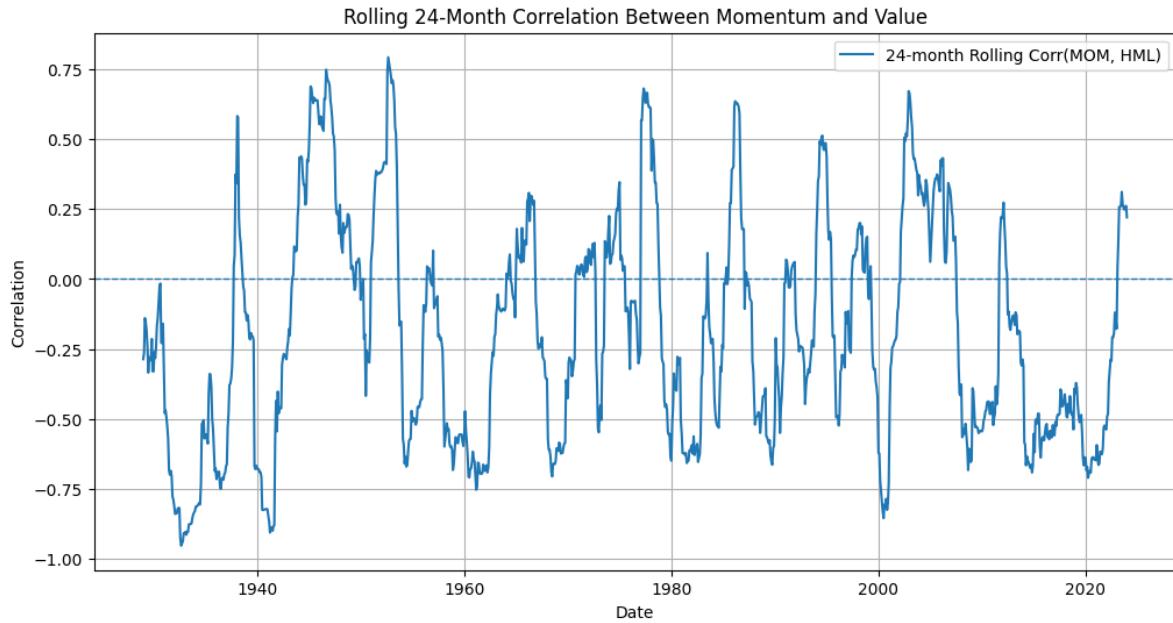


Figure 2: Rolling 24-Month Correlation Between FF Momentum and Industry Momentum

```
# Create summary table for rolling correlation
rolling_corr_summary = rolling_corr_24m.describe() \
    .to_frame(name="24M Rolling Corr (MOM, HML)")

rolling_corr_table = (
    rolling_corr_summary
    .style
    .set_caption("Rolling 24-Month Correlation Summary Statistics")
    .set_table_styles([
        {'selector': 'th', 'props': [(['border', '1px solid black'],
                                    ('text-align', 'center')])),
        {'selector': 'td', 'props': [(['border', '1px solid black'],
                                    ('text-align', 'center')])),
        {'selector': 'table', 'props': [(['border-collapse', 'collapse'])]}
    ])
)
rolling_corr_table
```

Table 7: Rolling 24-Month Correlation Summary Statistics

24M Rolling Corr (MOM, HML)	
count	1141.000000
mean	-0.196593
std	0.404951
min	-0.952903
25%	-0.542226
50%	-0.243549
75%	0.094360
max	0.792801

The figure presents the correlation between momentum and value portfolios computed over 24-month rolling windows. The rolling correlation exhibits substantial time variation and occasionally turns positive; however, it remains predominantly below zero throughout the sample period.

Summary statistics from the rolling-window analysis further confirm this pattern: the average correlation is negative ( $-0.1966$ ). Overall, the evidence indicates a persistent inverse relationship between momentum and value strategies.

### Rolling 24-Month Cumulative Returns for Value and Momentum

In this section, we analyze Value (HML) and Momentum (MOM) over a 24-month rolling horizon by converting monthly returns into rolling cumulative returns.

For each month, we compute the compounded return over the previous 24 months and store these rolling cumulative returns for both factors in a single DataFrame. We then calculate the correlation between the 24-month cumulative Value and Momentum returns, which captures how the two strategies co-move over medium-term investment horizons.

Finally, we visualize the rolling 24-month cumulative returns using a side-by-side bar chart. This makes it easy to identify periods where Value and Momentum move in opposite directions (diversification) versus periods where they move together (reduced diversification), while also displaying the overall correlation across the rolling windows.

```
window = 24
def rolling_cumulative_return(r, window):
    # compute the cumulative return for each 24 month window.
    cumulative_return = (1.0 + r).rolling(window=window) \
        .apply(np.prod, raw=True) - 1.0
    return cumulative_return
```

```

roll_hml = rolling_cumulative_return(df_mom_hml["hml"], window)
roll_mom = rolling_cumulative_return(df_mom_hml["mom"], window)
# create a data frame for the rolling window data
roll = pd.DataFrame({"Value(HML)": roll_hml, "Momentum(MOM)": roll_mom}).dropna()
# the x-axis shows the end year of each window
years = roll.index.year.astype(int)
# compute the correlation
corr_24m_roll = roll["Value(HML)"].corr(roll["Momentum(MOM)"])
# plot
x = np.arange(len(roll))
bar_w = 0.42
plt.figure(figsize=(16, 7))
plt.bar(x - bar_w/2, roll["Value(HML)"] * 100, width=bar_w, color="green",
        alpha=0.7, label="Value (24m cumulative, HML)")
plt.bar(x + bar_w/2, roll["Momentum(MOM)"] * 100, width=bar_w, color="blue",
        alpha=0.7, label="Momentum (24m cumulative, MOM)")
plt.axhline(0, linestyle="--", linewidth=1)
plt.title("Value and Momentum: 24-Month Rolling Cumulative Returns")
plt.ylabel("24-Month Cumulative Return")
plt.xlabel("Window end date (year shown)")
# Make x-axis more readable: show every Nth year label
N = max(1, len(years) // 30) # ~30 labels across
tick_idx = np.arange(0, len(years), N)
plt.xticks(tick_idx, years[tick_idx], rotation=90)
plt.legend()
# Correlation value box
plt.text(
    0.02, 0.05,
    f"Correlation (Value, Momentum) = {corr_24m_roll:.2f}",
    transform=plt.gca().transAxes,
    fontsize=12,
    bbox=dict(boxstyle="round,pad=0.5", facecolor="white", edgecolor="purple",
              linewidth=2))
plt.grid(True, axis="y", alpha=0.25)
plt.tight_layout()
plt.show()

```

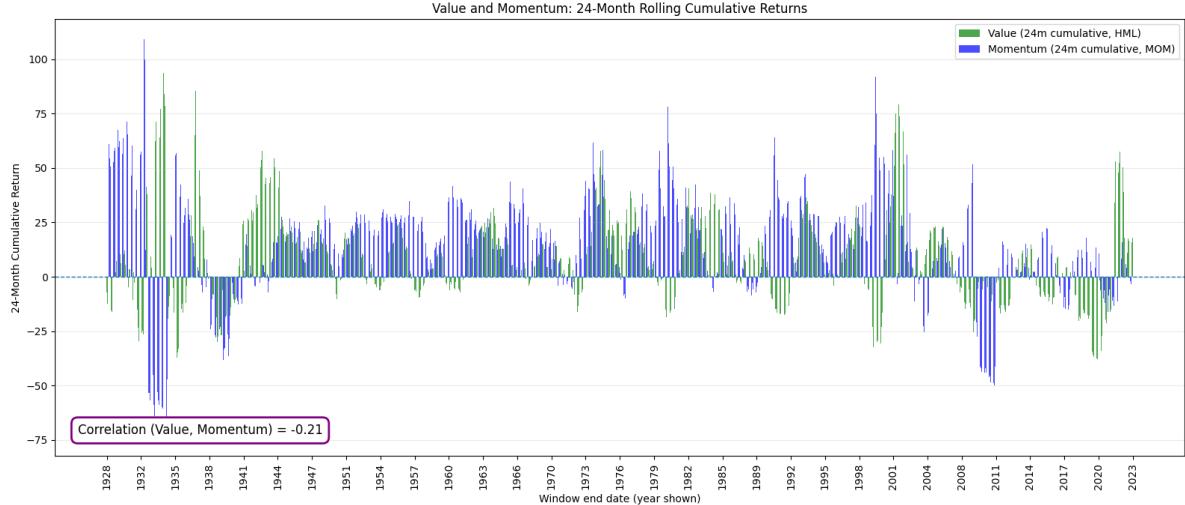


Figure 3: Rolling 24-Month Cumulative Returns: FF Momentum vs Value

The figure above plots the cumulative return of 24-month rolling windows of value and momentum portfolios over time. The green bars represent the cumulative returns of the value portfolio over each 24-month window. The blue bars represent the cumulative returns of the momentum portfolio. Each bar shows the performance of the portfolio in a 2-year (24 months) period, and the x-axis shows the ending year of that period.

This figure is designed to investigate whether the returns of momentum and value portfolios tend to move together or in opposite directions. The result shows that both portfolios experience periods with strong positive cumulative returns as well as negative periods. Momentum does not always bring positive returns, with unsatisfactory performance in years like 1933 and 2003, and 2009. This is likely due to the over exposure of momentum strategy in high-return sectors, resulting in a less diversification across industries and making it vulnerable in economic crisis or major regime shifts. However, we also notice that a negative performance of momentum is often accompanied by a relatively better or even positive performance of value portfolios, and vice versa. The figure therefore suggest a possible reduction of risks by combining momentum with value portfolios.

We compute the correlation between momentum and value portfolios using three approaches: the full sample, the most recent 10-year subsample, and 24-month rolling windows. The full-sample correlation is  $-0.4057$ , while the correlation over the most recent 10 years is  $-0.3004$ . The rolling-window analysis further confirms this pattern, with an average correlation of  $-0.1966$ .

Taken together, the results indicate a persistently negative relationship between momentum and value returns. On average, when momentum performs well, value tends to underperform, and vice versa.

This inverse relationship has important investment implications. Because the two strategies tend to perform well in different market environments, combining them can generate meaningful diversification benefits. Periods of weakness in one portfolio are often offset by relative strength in the other, potentially reducing overall portfolio volatility and improving risk-adjusted performance.

Building on this insight, the next step is to consider how AQR could design a combined Value + Momentum strategy that effectively exploits these diversification gains while delivering stable performance to mutual fund investors.

### **Momentum in Combination with Value**

To evaluate the return-generating effectiveness of a combined Momentum–Value approach, we compare the following strategies:

1. Value-only

$$r_t^V = HML_t$$

2. Momentum-only

$$r_t^M = MOM_t$$

3. Equal-weighted (50/50) Momentum & Value

$$r_t^{50/50} = 0.5 HML_t + 0.5 MOM_t$$

4. Volatility-scaled Momentum & Value

$$r_t^{\text{scaled}} = w_V(t) HML_t + w_M(t) MOM_t$$

subject to

$$w_V(t) + w_M(t) = 1$$

In the 50/50 strategy, capital is allocated equally between the value and momentum portfolios. This static allocation captures diversification benefits arising from their negative correlation while maintaining simplicity and transparency.

In contrast, the volatility-scaled strategy adjusts portfolio weights dynamically. A larger weight is assigned to the factor exhibiting lower recent volatility, while the higher-volatility factor receives a smaller allocation. This approach aims to stabilize portfolio risk over time and potentially improve risk-adjusted returns by exploiting time variation in factor volatility.

Below, we implement each of the four strategies in code and compute their corresponding return series.

```

df_strategy = pd.DataFrame(index=df_mom_hml.index)
# below we have thr Value only, Momentum only, and 50/50 strategies.
df_strategy["Value (HML)"] = df_mom_hml["hml"]
df_strategy["Momentum (MOM)"] = df_mom_hml["mom"]
df_strategy["50/50 Combo"] = (
    0.5 * df_strategy["Value (HML)"]
    + 0.5 * df_strategy["Momentum (MOM)"])

# we construct the volatility-adjusted strategies.
window = 24
volatility_value = df_strategy["Value (HML)"].rolling(window=window).std()
volatility_mom = df_strategy["Momentum (MOM)"].rolling(window=window).std()
weight_value = 1 / volatility_value
weight_mom = 1 / volatility_mom

weight_sum = weight_value + weight_mom
weight_value = (weight_value / weight_sum).shift(1)
weight_mom = (weight_mom / weight_sum).shift(1)

# add the volatility-scaled strategy into the dataframe
df_strategy["Volatility-Scaled Combo"] = (
    weight_value * df_strategy["Value (HML)"]
    + weight_mom * df_strategy["Momentum (MOM)"])

def annualized_return(r):
    return r.mean() * 12

def annualized_volatility(r):
    return r.std() * np.sqrt(12)

def sharpe_ratio(r):
    return annualized_return(r) / annualized_volatility(r)

def max_drawdown(r):
    wealth = (1 + r).cumprod()
    peak = wealth.cummax()
    drawdown = (wealth - peak) / peak
    return drawdown.min()

df_common = df_strategy.dropna()

performance = pd.DataFrame(index=df_common.columns)

```

```

for col in df_common.columns:
    r = df_common[col].dropna()
    performance.loc[col, "Ann. Mean"] = annualized_return(r)
    performance.loc[col, "Ann. Vol"] = annualized_volatility(r)
    performance.loc[col, "Sharpe"] = sharpe_ratio(r)
    performance.loc[col, "Max Drawdown"] = max_drawdown(r)

performance = performance.astype(float).round(4)

# Styled table
performance_table = (
    performance
    .style
    .set_caption("Strategy Performance Metrics")
    .set_table_styles([
        {'selector': 'th', 'props': [('border', '1px solid black'),
                                    ('text-align', 'center')]},
        {'selector': 'td', 'props': [('border', '1px solid black'),
                                    ('text-align', 'center')]},
        {'selector': 'table', 'props': [('border-collapse', 'collapse')]})
    ])
)

performance_table

```

Table 8: Strategy Performance Metrics

	Ann. Mean	Ann. Vol	Sharpe	Max Drawdown
Value (HML)	0.043400	0.124500	0.348600	-0.584100
Momentum (MOM)	0.072000	0.164100	0.438700	-0.772400
50/50 Combo	0.057700	0.080400	0.717900	-0.404500
Volatility-Scaled Combo	0.056900	0.077700	0.732300	-0.387900

The results indicate that the 50/50 Value + Momentum strategy achieves substantially lower volatility (0.0804) compared to holding value (0.1245) or momentum alone (0.1641).

In terms of risk-adjusted performance, both the 50/50 and the volatility-scaled combinations deliver markedly higher Sharpe ratios (0.7179 and 0.7323, respectively) relative to the standalone value (0.3486) and momentum (0.4387) strategies.

The combined strategies also exhibit improved downside risk characteristics. The maximum drawdowns of the 50/50 and volatility-scaled portfolios ( $-0.4045$  and  $-0.3879$ ) are significantly smaller than those of value ( $-0.5841$ ) and especially momentum ( $-0.7724$ ).

Overall, these findings suggest that the severe drawdowns associated with momentum during adverse market conditions can be substantially mitigated by combining it with the value factor, leading to more stable performance and superior risk-adjusted outcomes.

```
df_wealth = (1 + df_strategy.dropna()).cumprod()
df_cumret = (df_wealth - 1) * 100
plt.figure(figsize=(14, 7))

df_cumret_1980 = df_cumret.loc["1980-01-01":]

for col in df_cumret_1980.columns:
    plt.plot(df_cumret_1980.index, df_cumret_1980[col], label=col)

plt.title("Cumulative Returns (%) over Time (Start from 1980)")
plt.xlabel("Date")
plt.ylabel("Cumulative Returns (%)")
plt.legend()
plt.grid(True, alpha=0.3)
plt.show()
```

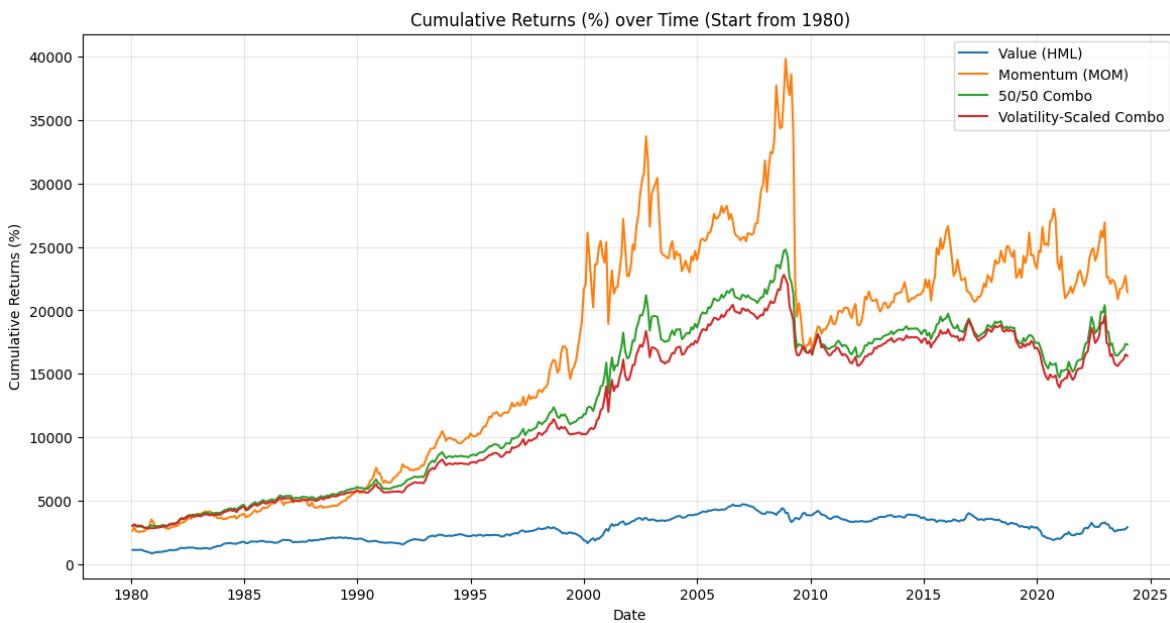


Figure 4: Cumulative Returns (%) over Time (Start from 1980)

The cumulative return plot further illustrates that both combined strategies produce a smoother and more stable performance trajectory over time. In particular, during major market downturns, most notably the 2008–2009 period, when the standalone momentum strategy experienced a severe drawdown, the diversified portfolios exhibit significantly milder declines.

This visual evidence reinforces the risk-mitigating benefits of combining value and momentum, as the joint strategies reduce the magnitude of losses during stress episodes while maintaining steady long-term growth.

Overall, the evidence suggests that the value factor can serve as an effective hedge against momentum drawdowns. Given the persistently negative correlation and the improved risk characteristics of the combined strategies, value meaningfully offsets the tail risk inherent in standalone momentum exposures.

Accordingly, we recommend that AQR position momentum as a complementary return driver within a broader value-oriented framework. A straightforward implementation would involve allocating capital equally between momentum and value (50/50). Alternatively, AQR could adopt a volatility-scaled approach, dynamically adjusting factor weights based on recent realized volatility to enhance risk control.

A blended “Momentum + Value” fund structure would allow investors to capture diversification benefits, improve risk-adjusted performance, and mitigate severe drawdowns, thereby delivering a more stable return profile over time.

Although the empirical evidence supports combining value and momentum, AQR must account for practical implementation constraints. Momentum strategies are typically associated with higher portfolio turnover, leading to increased transaction costs and potential tax inefficiencies, particularly in a mutual fund structure.

Moreover, unlike hedge funds, many mutual funds operate under long-only mandates. In such cases, the fund can take long positions in recent winners but cannot short recent losers, limiting its ability to fully replicate the standard long–short momentum factor.

Given these constraints, AQR should adopt a portfolio construction framework that preserves meaningful momentum exposure while reducing trading frequency and implementation costs. Techniques such as turnover-aware optimization, signal smoothing, staggered rebalancing, or partial signal integration could help maintain the diversification benefits of momentum while enhancing real-world feasibility within a long-only mutual fund structure.

### **Question 3**

#### **Construction of the Industry Momentum Factor (12–1 Strategy)**

In this section, we construct an industry-level momentum factor using the standard 12–1 momentum definition.

First, we compute the momentum signal for each industry by:

- Looking back 12 months,
- Skipping the most recent month (to avoid short-term reversal effects),
- Calculating cumulative returns over months t-12 to t-2.

Next, for each month:

- Industries are ranked based on their momentum signal.
- We go long the top 20% (winners) and
- Short the bottom 20% (losers) using equal weights.

The resulting time series (`ind_mom`) represents a long-short industry momentum factor. This allows us to test whether momentum exists at the industry level and compare it with the traditional Fama–French momentum factor.

```
def compute_momentum_signal(ret_df, lookback_months=12, skip_months=1):
    """
    Compute 12-1 style momentum signal per column.
    Signal at time t uses returns from t-lookback_months to t-skip_months-1.

    Parameters
    -----
    ret_df : pd.DataFrame
        Monthly returns in decimal form (0.01 = 1%).
    lookback_months : int
        Total lookback window length (e.g., 12).
    skip_months : int
        Number of most recent months to skip (e.g., 1).

    Returns
    -----
    signal : pd.DataFrame
        Momentum signals aligned to ret_df index.
    """
    # We want window length = lookback_months - skip_months
    window = lookback_months - skip_months

    # Shift returns forward by skip_months to align with signal timing
    shifted = ret_df.shift(skip_months)

    # Cumulative return over the window: product(1+r) - 1
    signal = (1.0 + shifted).rolling(window=window, min_periods=window).apply(
        lambda x: np.prod(x) - 1.0,
        raw=True
    )
    return signal
```

```

def build_industry_momentum_factor(
    industry_ret, top_frac=0.2, lookback_months=12, skip_months=1
):
    """
    Construct an industry momentum factor (long winners, short losers)
    using equal weighting.

    Parameters
    -----
    industry_ret : pd.DataFrame
        Industry returns in decimal form, indexed by date.
    top_frac : float
        Fraction of industries in winner/loser baskets (e.g., 0.2 for quintiles).
    lookback_months : int
        Lookback length (e.g., 12).
    skip_months : int
        Skip most recent months (e.g., 1).

    Returns
    -----
    ind_mom : pd.Series
        Monthly IND_MOM factor returns (long-short), aligned to dates.
    signal : pd.DataFrame
        Momentum signals used for ranking.
    """
    signal = compute_momentum_signal(
        industry_ret, lookback_months=lookback_months, skip_months=skip_months
    )

    ind_mom = []
    dates = industry_ret.index

    for dt in dates:
        sig_t = signal.loc[dt]
        ret_t = industry_ret.loc[dt]

        # Use only industries with both signal and return available at time t
        valid = sig_t.dropna().index.intersection(ret_t.dropna().index)
        if len(valid) < 10:
            ind_mom.append(np.nan)
            continue

        # Compute long-short returns
        sig_t = sig_t.reindex(valid)
        ret_t = ret_t.reindex(valid)
        ind_mom.append((ret_t - sig_t).mean())

```

```

sig_valid = sig_t.loc[valid]
ret_valid = ret_t.loc[valid]

# Determine basket sizes
n = len(valid)
k = max(1, int(np.floor(n * top_frac)))

# Rank by signal
winners = sig_valid.sort_values(ascending=False).head(k).index
losers = sig_valid.sort_values(ascending=True).head(k).index

# Equal-weighted long-short return
long_ret = ret_valid.loc[winners].mean()
short_ret = ret_valid.loc[losers].mean()
ind_mom.append(long_ret - short_ret)

ind_mom = pd.Series(ind_mom, index=dates, name="ind_mom")
return ind_mom, signal

# Build IND_MOM using your already-defined industry return DataFrame: ind_ret
ind_mom, ind_mom_signal = build_industry_momentum_factor(
    ind_ret,
    top_frac=0.2,           # quintiles
    lookback_months=12,     # 12-month lookback
    skip_months=1           # skip the most recent month => 12-1 momentum
)

```

```

def annualized_mean_and_tstat(monthly_series):
    """
    Annualized mean (12x) and t-stat of monthly mean.
    """
    s = monthly_series.dropna()
    ann_mean = s.mean() * 12
    t_stat = s.mean() / (s.std(ddof=1) / np.sqrt(len(s)))
    return ann_mean, t_stat, len(s)

# Align IND_MOM with FF factors
df_q3 = pd.concat([
    ff_df[['mom', 'hml']],      # standard MOM and HML
    ind_mom                      # new IND_MOM
], axis=1).dropna()

```

```

# Define last decade window from the end of the sample
last_date = df_q3.index[-1]
start_10y = last_date - pd.DateOffset(years=10)
df_10y = df_q3[df_q3.index > start_10y]

# Stats: full sample and last decade
stats = {}

for label, dfx in [("Full sample", df_q3), ("Last 10Y", df_10y)]:
    stats[(label, "FF_MOM")] = annualized_mean_and_tstat(dfx["mom"])
    stats[(label, "IND_MOM")] = annualized_mean_and_tstat(dfx["ind_mom"])

# Correlations (full sample)
corr_full_mom = df_q3["ind_mom"].corr(df_q3["mom"])
corr_full_hml = df_q3["ind_mom"].corr(df_q3["hml"])

# Correlations (last decade)
corr_10y_mom = df_10y["ind_mom"].corr(df_10y["mom"])
corr_10y_hml = df_10y["ind_mom"].corr(df_10y["hml"])

def cumulative_growth(series):
    """
    Growth of $1 from monthly returns in decimal form.
    """
    s = series.dropna()
    return (1.0 + s).cumprod()

# Plot last decade cumulative performance: IND_MOM vs FF MOM
plt.figure(figsize=(10, 5))
plt.plot(cumulative_growth(df_10y["mom"]), label="FF MOM")
plt.plot(cumulative_growth(df_10y["ind_mom"]), label="IND_MOM")
plt.title("Cumulative Growth (Last 10 Years): FF MOM vs IND_MOM")
plt.xlabel("Date")
plt.ylabel("Growth of $1")
plt.legend()
plt.grid(True)
plt.show()

```

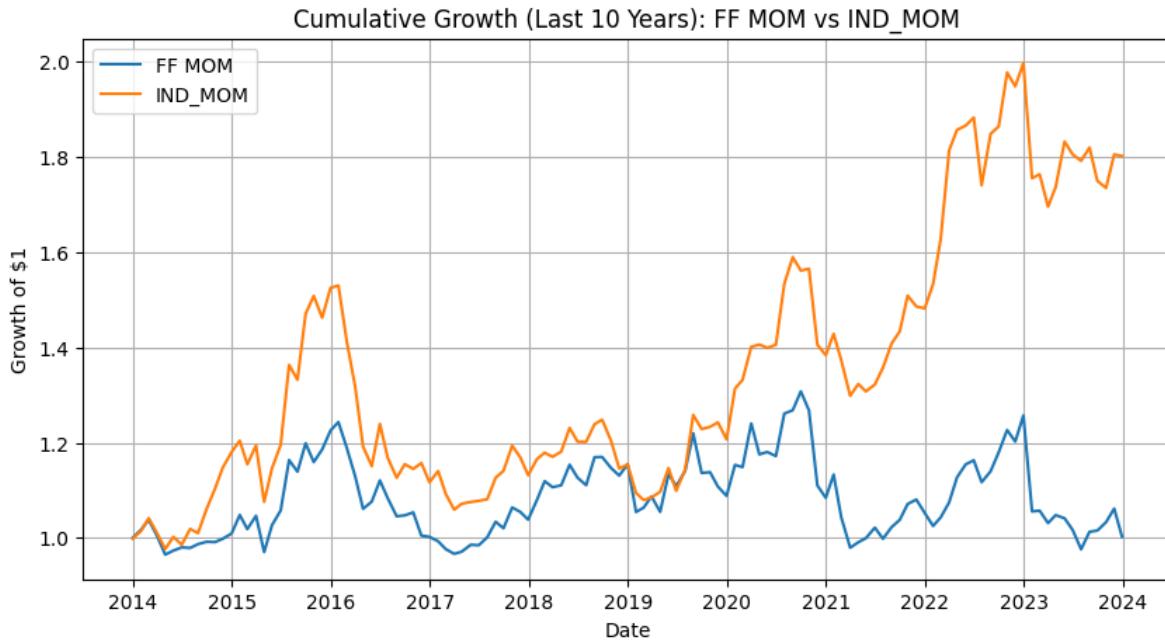


Figure 5: Cumulative Growth of \$1: FF Momentum vs Industry Momentum (Last 10 Years)

```

summary_rows = []
for label, dfx in [("Full sample", df_q3), ("Last 10Y", df_10y)]:
    for fac in ["mom", "ind_mom"]:
        ann_mean, t_stat, n = annualized_mean_and_tstat(dfx[fac])
        summary_rows.append({
            "Window": label,
            "Factor": "FF_MOM" if fac == "mom" else "IND_MOM",
            "Ann. Mean": ann_mean,
            "t-stat": t_stat,
            "N months": n})
summary_df = pd.DataFrame(summary_rows)
# Add correlations in a readable way
corr_df = pd.DataFrame({
    "Window": ["Full sample", "Last 10Y"],
    "corr(IND_MOM, FF_MOM)": [corr_full_mom, corr_10y_mom],
    "corr(IND_MOM, HML)": [corr_full_hml, corr_10y_hml],})

```

```

# Styled summary table
summary_table = (
    summary_df
    .round(4)

```

```

.style
.set_caption("Momentum Strategy Statistics: FF MOM vs Industry MOM")
.set_table_styles([
    {'selector': 'th', 'props': [(['border', '1px solid black'],
                                ('text-align', 'center')])),
    {'selector': 'td', 'props': [(['border', '1px solid black'],
                                ('text-align', 'center')])),
    {'selector': 'table', 'props': [(['border-collapse', 'collapse'])]}
])
)

summary_table

```

Table 9: Momentum Strategy Statistics: FF MOM vs Industry MOM

	Window	Factor	Ann. Mean	t-stat	N months
0	Full sample	FF_MOM	0.073800	4.425000	1153
1	Full sample	IND_MOM	0.081700	4.557400	1153
2	Last 10Y	FF_MOM	0.009700	0.225200	121
3	Last 10Y	IND_MOM	0.069900	1.462800	121

```

# Styled correlation table
corr_table = (
    corr_df
    .round(4)
    .style
    .set_caption("Correlation Analysis (Full Sample and Last 10 Years)")
    .set_table_styles([
        {'selector': 'th', 'props': [(['border', '1px solid black'],
                                    ('text-align', 'center')])),
        {'selector': 'td', 'props': [(['border', '1px solid black'],
                                    ('text-align', 'center')])),
        {'selector': 'table', 'props': [(['border-collapse', 'collapse'])]}
    ])
)

corr_table

```

Table 10: Correlation Analysis (Full Sample and Last 10 Years)

	Window	corr(IND_MOM, FF_MOM)	corr(IND_MOM, HML)
0	Full sample	0.773100	-0.291700
1	Last 10Y	0.782300	-0.186400

### Interpretation of Results

#### 1) Is There Evidence of Momentum at the Industry Level?

Evidence in favor of industry-level momentum exists if:

- IND\_MOM exhibits a positive annualized mean return,
- The t-statistic is economically and statistically meaningful (particularly over the most recent 10-year period), and
- The cumulative return plot shows persistent and relatively stable outperformance.

If IND\_MOM delivers positive and economically significant risk-adjusted returns—especially in the last decade, this suggests that momentum effects extend beyond individual securities and are also present at the industry level.

Conversely, weak or statistically insignificant recent performance would imply that industry momentum may not constitute a reliable standalone factor in the current market environment.

#### 2) Comparison: IND\_MOM vs. Standard Fama–French Momentum (Last Decade)

Using the results reported in `summary_df`:

- Compare the annualized mean returns of FF\_MOM and IND\_MOM over the last 10 years.
- If IND\_MOM delivers comparable or superior performance, this strengthens the case for a dedicated industry momentum strategy.
- If IND\_MOM materially underperforms, it may offer limited incremental value relative to traditional stock-level momentum.

This comparison helps determine whether industry momentum represents:

- A distinct and economically meaningful signal, or
- A weaker proxy for standard momentum exposure.

#### 3) Correlation Analysis

##### *Correlation Between IND\_MOM and FF\_MOM*

- High positive correlation → substantial overlap with standard momentum (redundancy risk).
- Moderate or low correlation → partial independence and potential diversification benefits.
- Low or unstable correlation → stronger argument for combining both signals.

A high correlation would suggest that IND\_MOM largely repackages existing momentum exposure rather than introducing a distinct return source.

#### *Correlation Between IND\_MOM and HML*

- Negative or low correlation → diversification benefits relative to value.
- Strong positive correlation → limited incremental diversification.

A negative relationship with HML would enhance the attractiveness of IND\_MOM within a broader multi-factor framework that already includes value.

#### 4) Should AQR Consider a VALUE + MOM + IND\_MOM Strategy?

#### **Potential Advantages**

- Diversification across different manifestations of momentum.
- Smoother multi-factor performance.
- Exposure to cross-industry rotation effects not fully captured by stock-level momentum.

#### **Key Considerations**

- Factor redundancy versus incremental alpha.
- Turnover and transaction costs.
- Implementation scalability within fund constraints.
- Sector concentration risk.
- Sensitivity to different market regimes.

The final decision should be guided by:

1. Risk-adjusted performance (annualized mean and t-statistic),
2. Stability across subperiods,
3. Correlation structure relative to existing factors,
4. Net performance after accounting for realistic implementation costs.