Empirical Finance: Week 3

Final Consolidated Exam Study Guide

Economic Evaluation of Asset Returns & OOS Predictability

Compiled from Lecture Captions & Handout 3

CRITICAL EXAM INSTRUCTIONS FROM PROFESSOR

Exam Format:

- **Pen and paper exam** with numerical exercises you can do by hand
- Multiple choice questions included — especially for Stambaugh bias implications
- *"Tutorials are mostly for the coursework. The slides are mostly for the exam."*
- *"If you understand the basic concept, then if I give you something slightly different, you should be able to get there."*

What You MUST Memorise:

- Henriksson-Merton Market Timing Test formula and Z-statistic
- Bootstrap algorithm steps **in words** (not mathematical details)
- Stambaugh bias **implications** (not the formula itself)
- R^2_{OOS} formula and interpretation

What Will Be Given:

- Stambaugh bias formula (understand direction of bias only)
- Clark-West f_{t+1} construction
- Portfolio optimisation closed-form solutions

1 Predictive Regressions & Stambaugh Bias (Pages 2–14)

Professor's Key Teaching Points

Why Predictive Regressions Are Challenging:

- *"In finance we have these problems... Beta estimates are distorted because we have tons of problems working together"*
- *"X is very persistent [e.g., interest rates with $\rho \approx 0.99$], shocks to Y and X are correlated (often negatively), and we have small samples"*
- *"When you put all of them together, you get these distortions"*

The Three Problems Combined:

1. Predictors are highly persistent (nearly non-stationary, $\rho \approx 0.99$)
2. Returns are noisy with fast mean-reversion
3. Shocks to returns (u_t) and predictors (v_t) are contemporaneously correlated

Why In-Sample Analysis Fails:

- *"Bottom line: we don't work with in-sample regressions. In-sample regressions are useless to assess predictability of X on Y"*
- *"We need to combine with an out-of-sample assessment"*

EXAM INSTRUCTIONS: Stambaugh Bias

"For the exam you don't have to remember the formula. However, I can ask a multiple choice question where I say: look, under what conditions do you have a bias? And what is the direction of the bias?"

"So in the exam, there could be a multiple choice question where I say: this is the setup, pick the right answer. For example: Does the distortion increase or decrease when T decreases? Is the bias upward or downward if the covariance between u and v is positive or negative?"

"You don't remember the formula, but please try to remember how to interpret these results."

Exam Learning Objectives — You Must Be Able To:

1. Identify when Stambaugh bias occurs (persistent predictor + correlated shocks + small sample)
2. **State the direction of bias given the sign of Cov**(u_t, v_t)
3. Explain how bias changes with sample size T and persistence ρ
4. Justify why out-of-sample analysis is necessary in finance

Stambaugh Bias Implications — MUST UNDERSTAND

Key relationships (likely MCQ topics):

- Higher ρ (more persistent predictor) $\Rightarrow$ **larger bias**
- Larger T (more data) $\Rightarrow$ **smaller bias**
- $\text{Cov}(u_t, v_t) < 0 \Rightarrow$ **upward bias** (typical in finance!)
- $\text{Cov}(u_t, v_t) > 0 \Rightarrow$ **downward bias**

Intuition: The minus sign in the formula means negative covariance produces positive (upward) bias.

Formulas Given on Exam:

Stambaugh (1999) bias approximation:

$$\text{Bias}(\hat{\beta}) \approx -\frac{\text{Cov}(u_t, v_t)}{\text{Var}(v_t)} \times \frac{1 + 3\rho}{T}$$

2 Out-of-Sample Predictability (Pages 15–38)

Professor's Key Teaching Points

Three Key Ingredients for OOS Analysis:

1. **Real-time information:** *"I can only use information that was available to an investor at the time"*
2. **Benchmark model:** Compare against a reference (typically random walk)
3. **Performance evaluation:** Statistical or economic criteria

Why Random Walk as Benchmark:

- *"There is plenty of empirical evidence suggesting that it's very hard to beat the random walk"*
- *"Sometimes you can beat it, but there is no paper showing you can systematically beat the random walk in all states"*

Macro Data Challenges:

- *"Inflation for December 2025 is not available in December 2025 — it's made available six weeks after"*
- *"Macro data are often revised, so you don't know if the data today will be the same six months after"*

Exam Learning Objectives — You Must Be Able To:

1. Distinguish between **expanding window** (keeps all past data) and **rolling window** (fixed sample, drops oldest)
2. Set up a proper out-of-sample forecasting exercise
3. Explain real-time information constraints
4. Apply Campbell & Thompson (2008) sign restrictions

Formulas Given on Exam:

OLS Estimators:

$$\hat{\beta} = \frac{\text{Cov}(y_t, x_{t-1})}{\text{Var}(x_{t-1})}, \quad \hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}$$

One-step ahead forecast:

$$\hat{y}_{t+1|t} = \hat{\alpha}_t + \hat{\beta}_t x_t$$

Forecast error:

$$\hat{\varepsilon}_{t+1} = y_{t+1} - \hat{y}_{t+1|t}$$

3 Statistical Evaluation (Pages 39–53)

Professor's Key Teaching Points

Interpreting R^2_{OOS}:

- *"1% is wow. Sometimes you have less than 1%"*
- *"If someone shows me an out-of-sample R-squared of 2-3%, it's amazing — you should bet everything on that model"*
- *"When I mark coursework and find R-squared of 20-30%, clearly something is wrong"*
- *"You need to have a sense of the data"*

Why Simple MSE Comparison Fails (Clark-West):

- *"The null hypothesis is incorrectly formulated"*
- *"Under the null that the model and benchmark are equally bad, the model has an extra parameter"*
- *"Adding an extra parameter means estimation noise"*
- *"So under the null of equal predictability, the MSE of the model is always higher than the MSE of the benchmark"*
- *"Your out-of-sample R-squared could be negative, but maybe your model is still better than the benchmark"*

Interpreting OOS R-squared Values

R^2_{OOS} **Value**	**Interpretation**
< 0	Model performs **worse** than benchmark
0.1% – 0.5%	Typical, potentially useful
0.5% – 1%	Good performance
1% – 2%	**"Wow"** — excellent
2% – 3%	**"Amazing — bet everything"**
$> 10\%$	**Something is wrong!** Check your code

Exam Learning Objectives — You Must Be Able To:

1. Calculate and interpret MSE, MAE, and R^2_{OOS}
2. Explain why R^2_{OOS} can be negative
3. Understand why Clark-West test is needed (extra parameters add noise under null)
4. Apply the Henriksson-Merton market timing test (see below)

R^2_{OOS} Formula — MUST MEMORISE

$$R^2_{OOS} = 1 - \frac{MSE_{MOD}}{MSE_{BEN}}$$

Interpretation:

- $R^2_{OOS} > 0$: Model beats benchmark (lower MSE)
- $R^2_{OOS} < 0$: Benchmark beats model
- $R^2_{OOS} = 0$: Equal performance

Formulas Given on Exam:

Mean Squared Error:

$$MSE = \frac{1}{T-t} \sum_{i=t+1}^{T} \hat{\varepsilon}_i^2$$

Mean Absolute Error:

$$MAE = \frac{1}{T-t} \sum_{i=t+1}^{T} |\hat{\varepsilon}_i|$$

Clark-West statistic:

$$f_{t+1} = \hat{\varepsilon}^2_{t+1,BEN} - \left[\hat{\varepsilon}^2_{t+1,MOD} - (\hat{y}_{t+1,BEN} - \hat{y}_{t+1,MOD})^2\right]$$

Regress f_i on a constant, compute t-statistic with Newey-West standard errors.

4 Henriksson-Merton Market Timing Test (Pages 48–49)

CRITICAL: Professor's Exam Instructions

"For the exam, I might give you a small table with 5 or 6 data points, and I will ask you to calculate the test."

"This formula must be remembered. If I tell you to remember something, it means you have to remember it."

"This one could be in the exam. In the exam, I can ask you exactly to apply the formula."

Professor's Key Teaching Points

What Market Timing Measures:

- *"You want to make sure that the model gives you the right side of the market"*
- *"You don't care about the magnitude. You care about the sign"*
- *"If you flip a coin, you are right 50% of the time, wrong 50% of the time — that's not skill, just luck"*

What's a Good p Value:

- *"If your model is good, you are slightly better than 50%"*
- *"51-52% — wow, that's very good"*
- *"Most of the times it's just a little bit above 50%"*

Exam Learning Objectives — You Must Be Able To:

1. **Calculate** $p = c/n$ from a table of forecasts and actual returns
2. **Compute the Z-statistic**
3. **Compare to critical value 1.65** and state conclusion
4. Interpret market timing ability

Henriksson-Merton Formulas — MUST MEMORISE

Directional accuracy:

$$p = \frac{c}{n}$$

where c = number of correct forecasts (same sign as actual return), n = total forecasts.
Hypotheses:

- $H_0 : p = 0.5$ (no market timing ability — just luck)
- $H_A : p > 0.5$ (positive market timing ability)

Test statistic (for $n > 30$, binomial $\approx$ Normal):

$$Z = \frac{p - 0.5}{\sqrt{\frac{p(1-p)}{n}}}$$

Decision rule: If $Z > 1.65$, reject H_0 at 5% level (one-sided test).

5 Bootstrap Algorithm (Pages 50–53)

EXAM INSTRUCTIONS: Bootstrap

"I might ask a question about the bootstrap in the exam. However, I'm not asking you the maths. I'm asking you to explain in words what the steps are."

"You don't need to remember the details, but you have to remember the general idea."

"When I say describe the bootstrap algorithm, it means you don't remember the formula but in words you can describe how you run the recipe."

Professor's Key Teaching Points

Why Bootstrap vs Monte Carlo:

- *"In bootstrap, you don't randomly draw from a distribution — you resample from the actual errors"*
- *"This preserves the linear dependence that exists in the data"*
- *"Monte Carlo samples from a parametric distribution, which may miss higher-moment dependencies"*

Key Technical Points:

- *"You resample from a uniform between 1 and 100 [or n], then use those draws to pick rows"*
- *"You have a matrix with two columns — u1 and u2 — you sample both at the same time"*
- *"How many errors do you sample? Exactly the same number as the original data"*

Exam Learning Objectives — You Must Be Able To:

1. **Describe the bootstrap algorithm in words** (6 steps below)
2. Explain why bootstrap preserves dependence structure
3. Calculate an empirical p-value

Bootstrap Algorithm Steps — DESCRIBE IN WORDS

1. **Compute test statistic on actual data:** Calculate R^2_{OOS}, CW, or other statistic. Save this number $\hat{\tau}$.
2. **Estimate model under null of no predictability:**
   - $y_t = \alpha + u_{1,t}$ (returns are unpredictable)
   - $x_t = c + \rho_1 x_{t-1} + \ldots + \rho_p x_{t-p} + u_{2,t}$ (predictor follows AR process)
   - Extract residuals $\hat{u}_t = (\hat{u}_{1,t}, \hat{u}_{2,t})'$
3. **Generate synthetic data by resampling residuals:**
   - Draw indices from uniform distribution (with replacement)
   - Resample residual **pairs** (preserve correlation!)
   - Construct synthetic y^* and x^* using estimated parameters
4. **Compute test statistic on simulated data:** Run the same OOS procedure on synthetic data, get $\hat{\tau}^*$.
5. **Repeat steps 3–4 many times:** $B = 1000$ or more iterations.
6. **Calculate p-value:**

$$\text{p-value} = \frac{1}{B}\sum_{j=1}^{B} \mathbf{1}(\hat{\tau}^*_j > \hat{\tau})$$

6 Economic Evaluation (Pages 54–65)

Professor's Key Teaching Points

Why Statistical Measures Aren't Enough:

- *"There's a paper by Leitch and Tanner (1992) showing that statistical measures like MSE have no consistent relationship with profits"*
- *"You can have low MSE but low profit, or high MSE but high profit"*
- *"That's why we complement statistical criteria with economic criteria"*

Sharpe vs Sortino Ratio:

- *"The Sharpe ratio only cares about mean and variance"*
- *"But market participants are very much concerned about downside risk — negative skewness"*
- *"If things go wrong, you go bankrupt"*
- *"That's why we also calculate the Sortino ratio, which uses only downside volatility"*

Sortino Calculation:

- *"You only take the subset of negative portfolio returns"*
- *"Calculate the standard deviation on those negative returns"*
- *"The numerator is the same as Sharpe; only the denominator changes"*

Exam Learning Objectives — You Must Be Able To:

1. Explain why economic evaluation complements statistical evaluation
2. Define and interpret Sharpe ratio, Sortino ratio, Certainty Equivalent Return
3. Understand mean-variance optimisation setup
4. Explain the break-even transaction cost concept

Formulas Given on Exam:

Sharpe Ratio:

$$SR = \frac{E(r_{p,t+1} - r_{f,t})}{\sqrt{\text{Var}(r_{p,t+1} - r_{f,t})}}$$

Sortino Ratio:

$$SO = \frac{E(r_{p,t+1} - r_{f,t})}{\sqrt{\text{Var}(r_{p,t+1} - r_{f,t} \mid r_{p,t+1} - r_{f,t} < 0)}}$$

7 Transaction Costs & Break-Even (Pages 63–65)

Professor's Key Teaching Points

The Problem:

- *"You can have a great strategy, but then this strategy rebalances your portfolio so often, so quickly"*
- *"If you rebalance often and have large swings in your portfolio weights, you're going to pay more transaction costs"*

Why Break-Even Approach:

- *"You don't know what transaction costs you'll actually pay — it depends on how much you trade"*
- *"If you're a small fund you pay one cost; if you're Blackrock you pay another cost; if you're a pension fund you pay another cost"*
- *"So we calculate the break-even transaction cost: the cost that would kill the excess performance of your model relative to the benchmark"*
- *"If you pay less than break-even, there's still value in your model"*

Exam Learning Objectives — You Must Be Able To:

1. Explain why transaction costs matter for backtesting
2. Define break-even transaction cost conceptually
3. Interpret: if actual cost < break-even cost $\Rightarrow$ model still has value

8 Model Combination (Pages 66–72)

Professor's Key Teaching Points

Why Combine Forecasts:

- *"We've evaluated individual models relative to the benchmark, but ex-ante we don't know which model is true"*
- *"This generates model uncertainty"*
- *"The superior performance of combined forecasts is known since Bates and Granger (1969)"*

Exam Learning Objectives — You Must Be Able To:

1. Explain why model combination is used (model uncertainty)
2. Describe the three types of forecast combination
3. Write the combined forecast formula

Model Combination — UNDERSTAND CONCEPTS

Combined forecast:

$$\tilde{y}_{t+1} = \sum_{m=1}^{M} \kappa_t^m \hat{y}_{t+1}^m, \quad \text{where } \sum_{m=1}^{M} \kappa_t^m = 1$$

Three Weighting Schemes:

1. **Simple Averaging:**
   - Mean rule: $\kappa_t^m = 1/M$ (equal weights)
   - Median rule: Select median forecast
2. **Statistical Averaging** (inverse MSE weighting):

$$\kappa_t^m = \frac{1/MSE_t^m}{\sum_{j=1}^{M} 1/MSE_t^j}$$

   Lower MSE $\Rightarrow$ higher weight
3. **Economic Averaging** (e.g., Sharpe ratio weighting):

$$\kappa_t^m = \frac{SR_t^m}{\sum_{j=1}^{M} SR_t^j}$$

   Higher Sharpe ratio $\Rightarrow$ higher weight

9 Final Exam Checklist

Formulas You MUST Memorise

1. Henriksson-Merton Market Timing Test:

- $p = c/n$ (directional accuracy)
- $Z = \frac{p-0.5}{\sqrt{p(1-p)/n}}$
- Reject H_0 if $Z > 1.65$ (5% one-sided)

2. Out-of-Sample R-squared:

$$R^2_{OOS} = 1 - \frac{MSE_{MOD}}{MSE_{BEN}}$$

3. Bootstrap Algorithm (in words):

1. Compute statistic on actual data
2. Estimate model under null (no predictability)
3. Resample residual pairs with replacement
4. Recompute statistic on synthetic data
5. Repeat 1000+ times
6. p-value = fraction of simulated statistics > actual

Conceptual Understanding Required (Likely MCQ)

Stambaugh Bias:

- Higher $\rho \Rightarrow$ larger bias
- Larger $T \Rightarrow$ smaller bias
- $\text{Cov}(u, v) < 0 \Rightarrow$ **upward** bias
- $\text{Cov}(u, v) > 0 \Rightarrow$ **downward** bias

Clark-West Test:

- Why simple null is "incorrectly formulated"
- Extra parameters add estimation noise under null
- R^2_{OOS} can be negative but model still better

R^2_{OOS} **Interpretation:**

- 1% = "wow", 2-3% = "amazing", 10%+ = suspicious

Economic Evaluation:

- Sharpe ratio: total volatility
- Sortino ratio: downside volatility only
- Break-even cost: cost that kills model's advantage

Model Combination:

- Simple (equal weights), Statistical (inverse MSE), Economic (Sharpe)

Professor's Final Warnings

- *"If I tell you to remember something, it means you have to remember it"* — H-M test!
- *"You need to have a sense of the data"* — know typical R^2_{OOS} values
- *"You're not just a scientist, you're also an artist"* — understand the data
- Bootstrap: describe in **words**, not formulas
- Stambaugh: understand **implications**, not formula