

# tuto\_week3

Rodolphe Lajugie

2026-01-21

## Contents

Setup . . . . .	2
<b>Excercise 1</b>	<b>3</b>
$y_t = c + \epsilon_t$ . . . . .	3
$y_t = c + \beta t + \delta t^2 + \epsilon_t$ . . . . .	5
$y_t = c + y_{t-1} + \epsilon_t$ . . . . .	7
$y_t = c + \phi_1 y_{t-1} + \epsilon_t$ . . . . .	9
$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \phi_2 y_{t-3} + \epsilon_t$ . . . . .	11
$y_t = c + \epsilon_t + \theta_1 \epsilon_{t-1}$ . . . . .	13
$y_t = c + \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2}$ . . . . .	15
$y_t = c + \phi_1 y_{t-1} + \epsilon_t + \theta_1 \epsilon_{t-1}$ . . . . .	17
<b>Excercise 2</b>	<b>19</b>
Loading data . . . . .	19
S&P500 . . . . .	19
GBPUSD . . . . .	19
TBILL . . . . .	19
VIX . . . . .	19
Transformation . . . . .	19
Plot ACF and PACF for each stock . . . . .	20

## Setup

```
packages <- c(
  "quantmod",      # download time-series
  "tidyverse",     # data manipulation and visualization
  "stargazer",    # publication-ready tables
  "conflicted",   # management of function name conflicts across packages
  "moments",       # statistical moments, like skewness and kurtosis
  "lubridate",     # manipulation of data
  "knitr",         # integrate R code with text (e.g. LaTeX, HTML, Markdown)
  "sn",            # simulation of skewed distributions
  "patchwork",    # composition of graphs
  "latex2exp"     # latex
)

# --- from the ones needed extract the ones not installed
to_install <- packages[!packages %in% installed.packages()[, "Package"]]

# --- install the packages
if (length(to_install) > 0) {
  install.packages(to_install)
}

# --- load the packages in our session
invisible(lapply(packages, library, character.only = TRUE))

# --- defining `lag()` from the dplyr package as the preference
conflict_prefer("lag", "dplyr")
```

## Excercise 1

```
set.seed(111)

T <- 1000 #nb of obs
c <- 0.2
beta <- 0.1
delta <- 0.05
sigma <- 1 # std normal law
phi1 <- 0.4
phi2 <- 0.3
phi3 <- 0.2
theta1 <- 0.4
theta2 <- 0.3

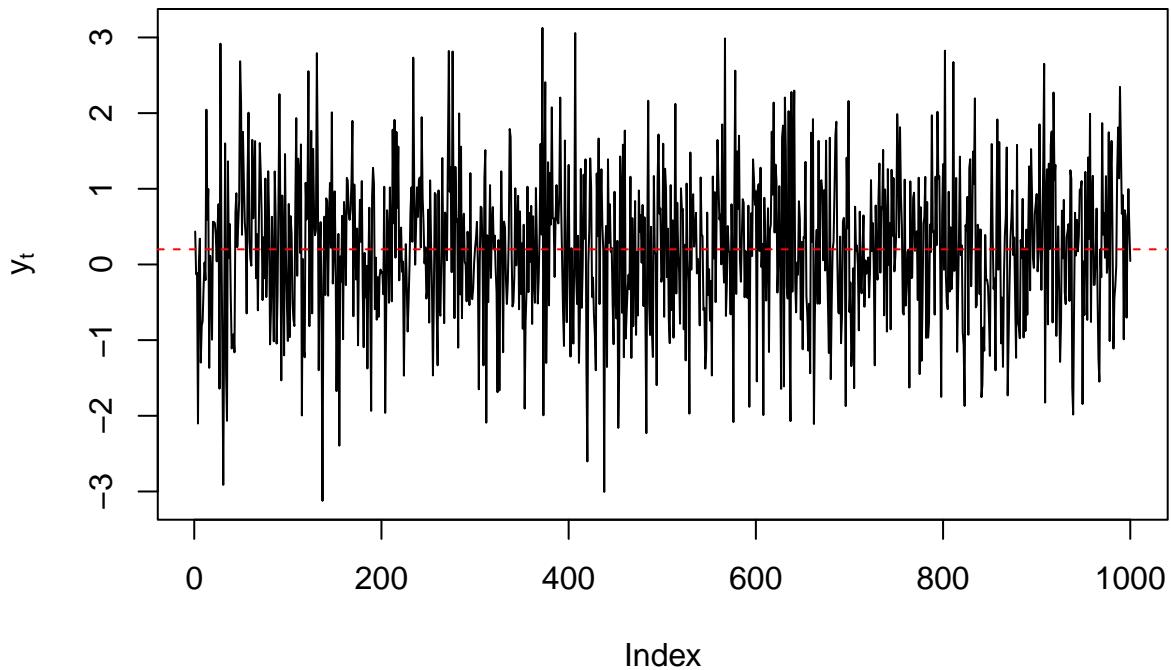
epsilon <- rnorm(T, mean = 0, sd = sigma)
t <- 1:T
```

$$y_t = c + \epsilon_t$$

```
y1 <- c + epsilon

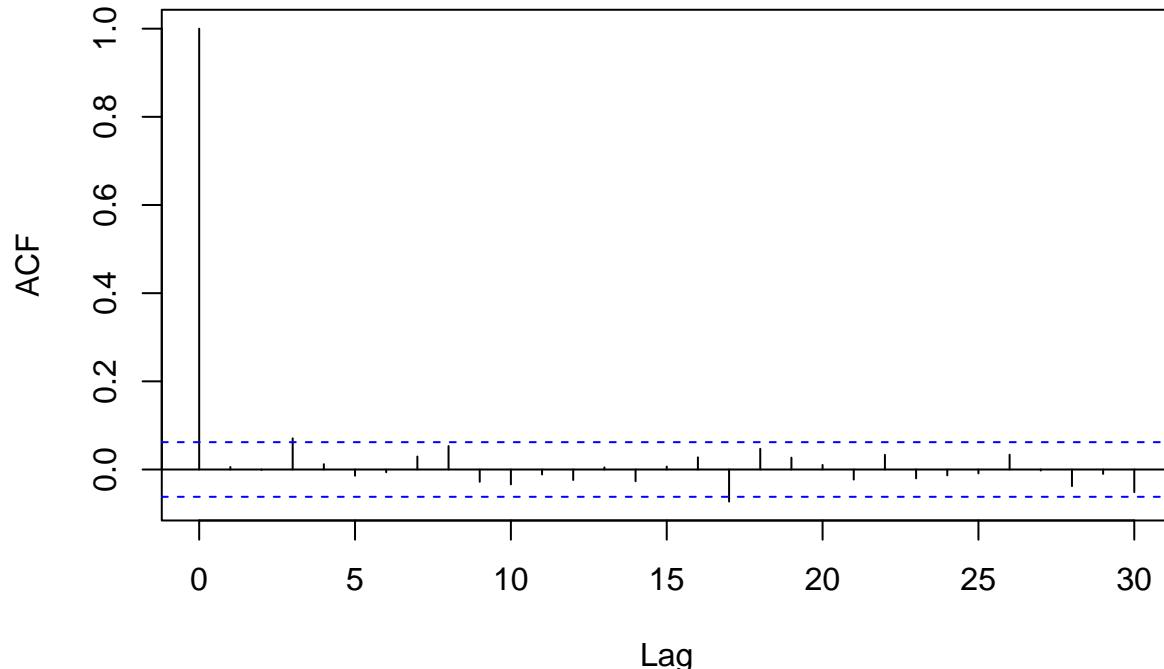
plot(y1, type = "l", main = TeX("$y_t = c + \epsilon_t$"), ylab=TeX("$y_t$"))
abline(h = c, col = "red", lty = 2)
```

$$y_t = c + \epsilon_t$$



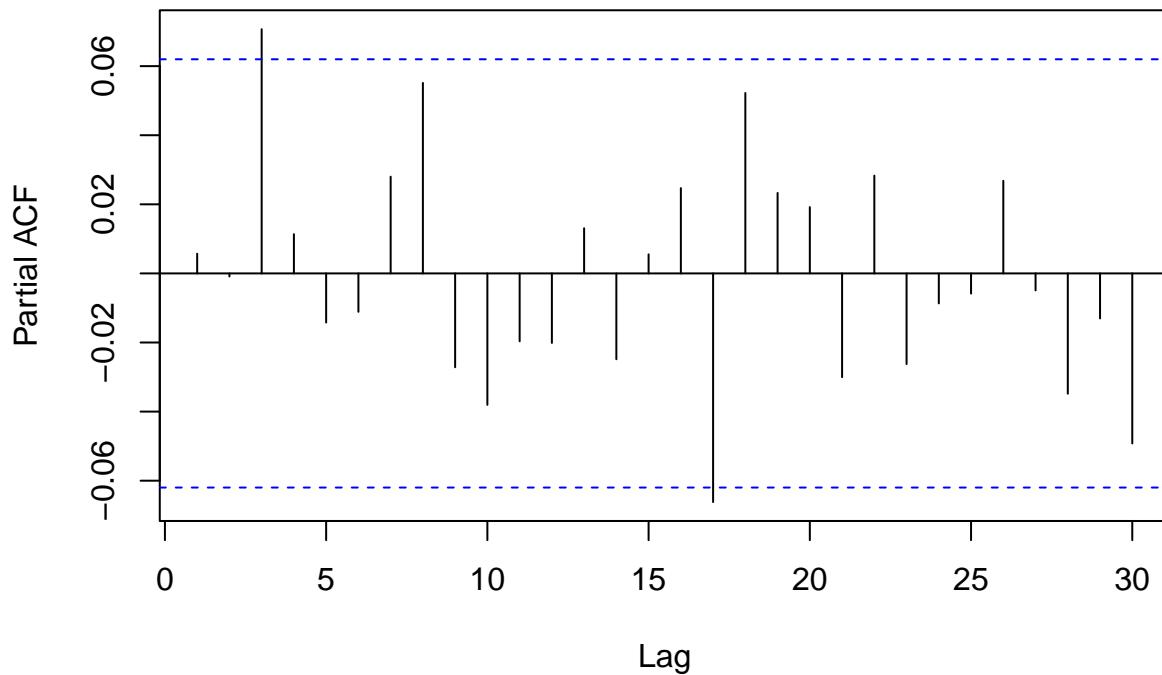
```
acf(y1, main = "ACF of y")
```

### ACF of y



```
pacf(y1, main = "PACF of y")
```

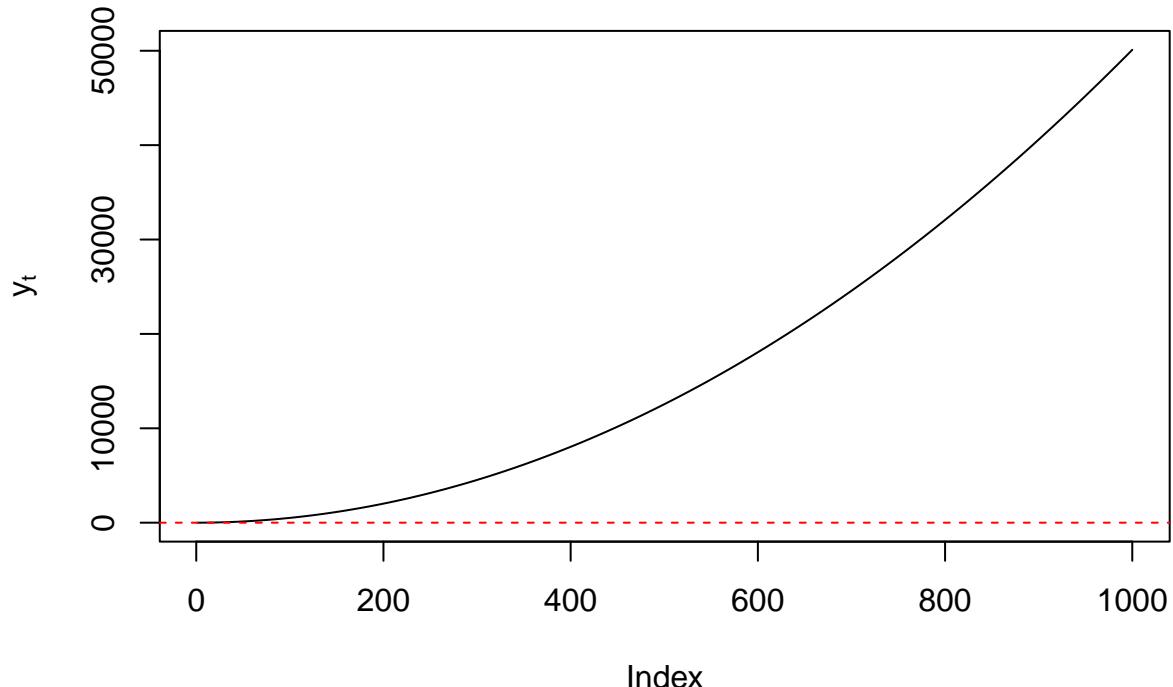
### PACF of y



$$y_t = c + \beta t + \delta t^2 + \epsilon_t$$

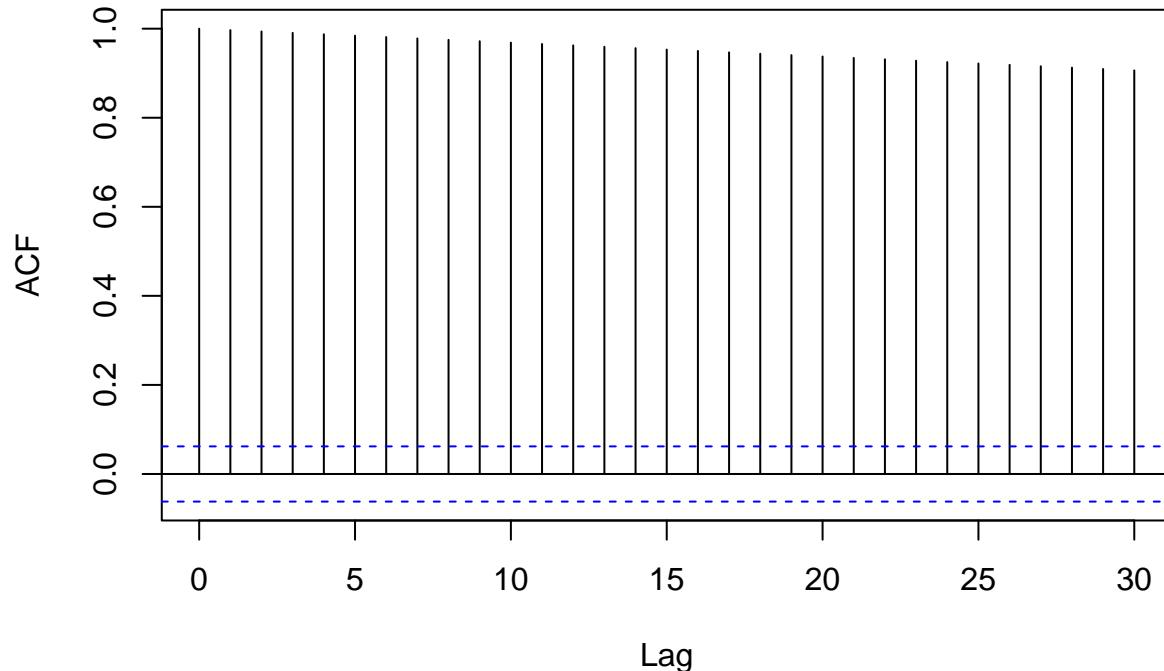
```
y2 <- c + beta*t + delta*t^2 + epsilon
plot(y2, type = "l", main = TeX("$y_t = c + \beta t + \delta t^2 + \epsilon_t$"),
      abline(h = c, col = "red", lty = 2)
```

$$y_t = c + \beta t + \delta t^2 + \epsilon_t$$



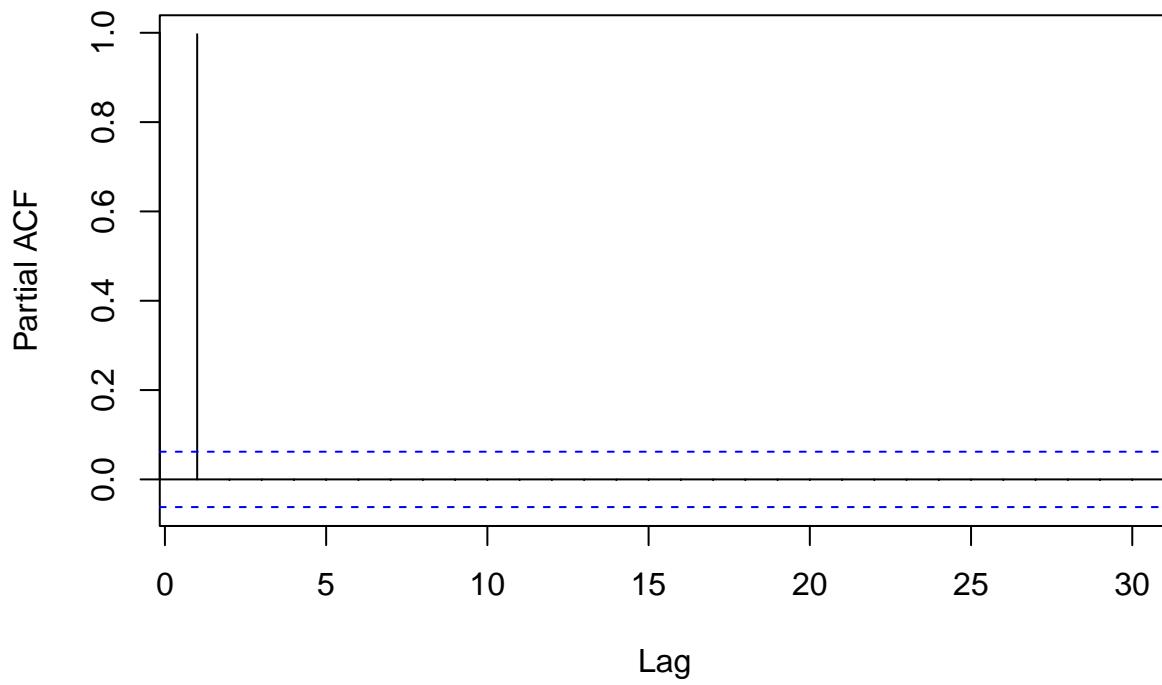
```
acf(y2, main = "ACF of y")
```

### ACF of y



```
pacf(y2, main = "PACF of y")
```

### PACF of y

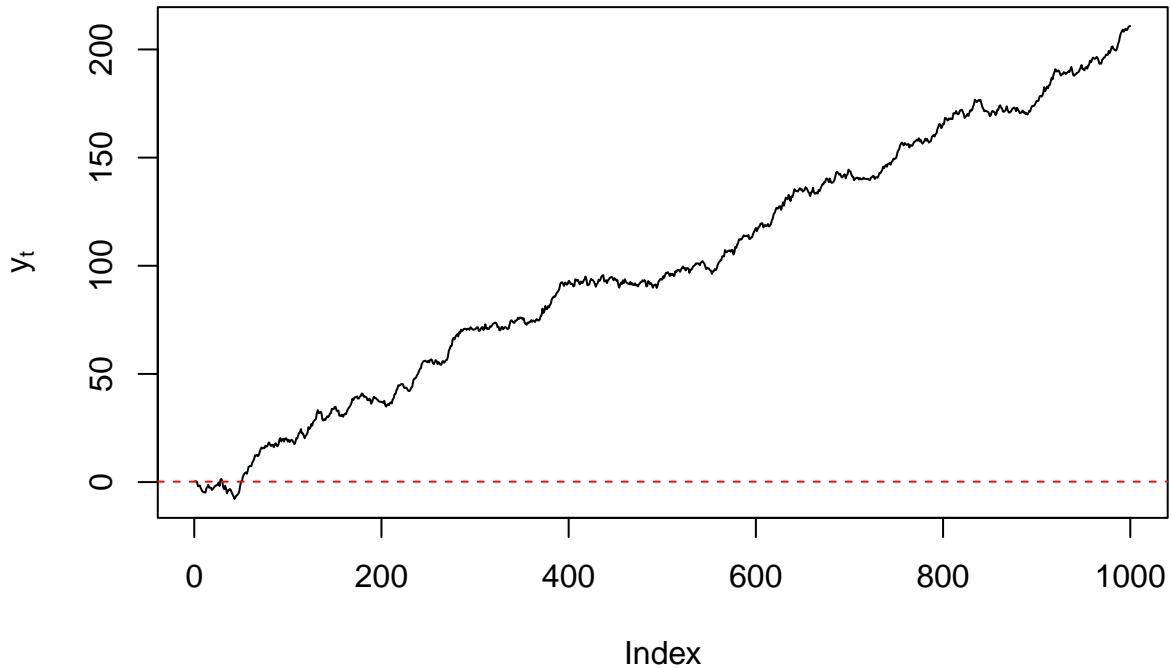


$$y_t = c + y_{t-1} + \epsilon_t$$

```
y3 = numeric(T)
y3[1] <- c + epsilon[1] # value init

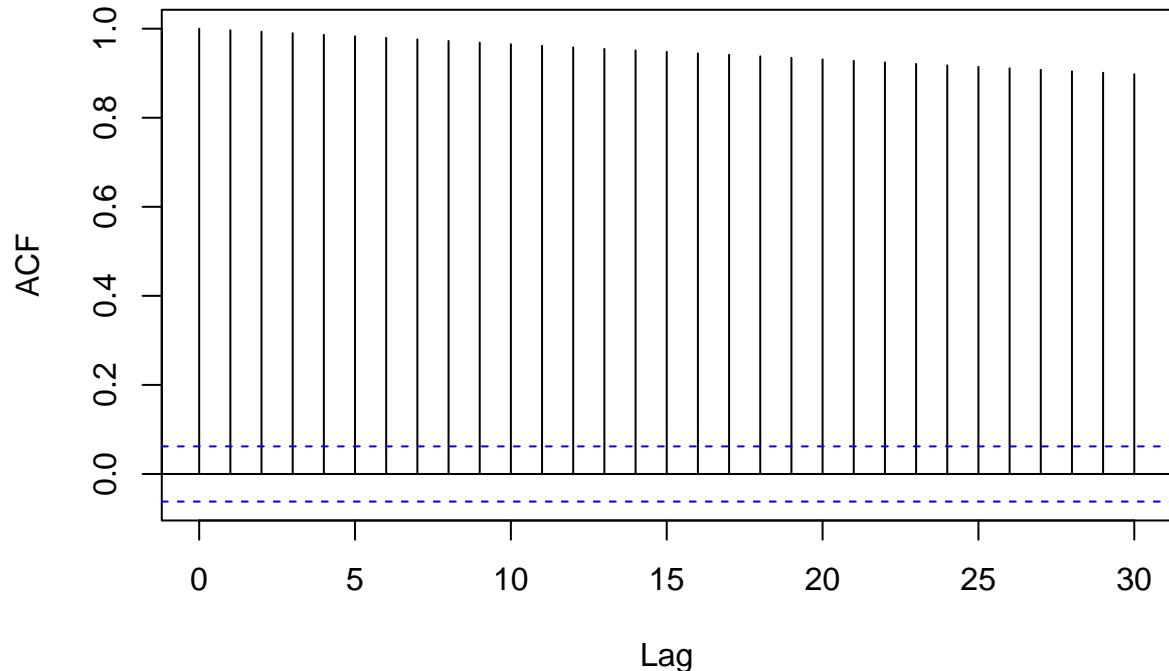
for (i in 2:T) {
  y3[i] <- c + y3[i-1] + epsilon[i]
}
plot(y3, type = "l", main = TeX("$y_t = c + y_{t-1} + \epsilon_t$"), ylab=TeX("$y_t$"))
abline(h = c, col = "red", lty = 2)
```

$$y_t = c + y_{t-1} + \epsilon_t$$



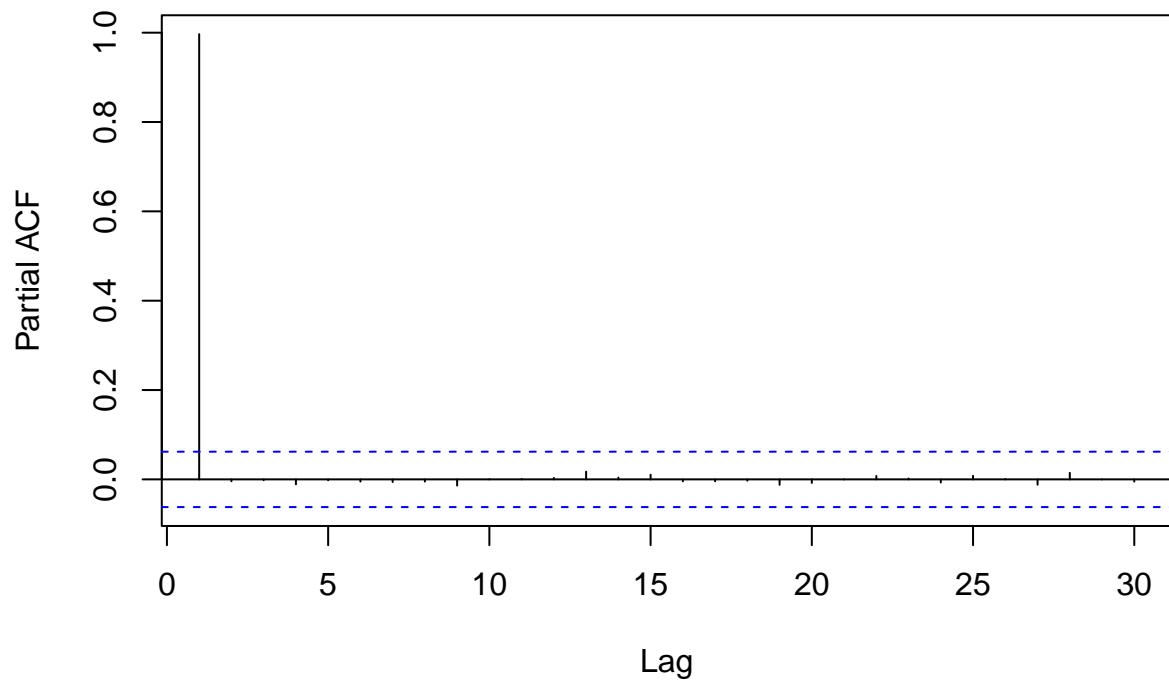
```
acf(y3, main = "ACF of y")
```

### ACF of y



```
pacf(y3, main = "PACF of y")
```

### PACF of y



$$y_t = c + \phi_1 y_{t-1} + \epsilon_t$$

```

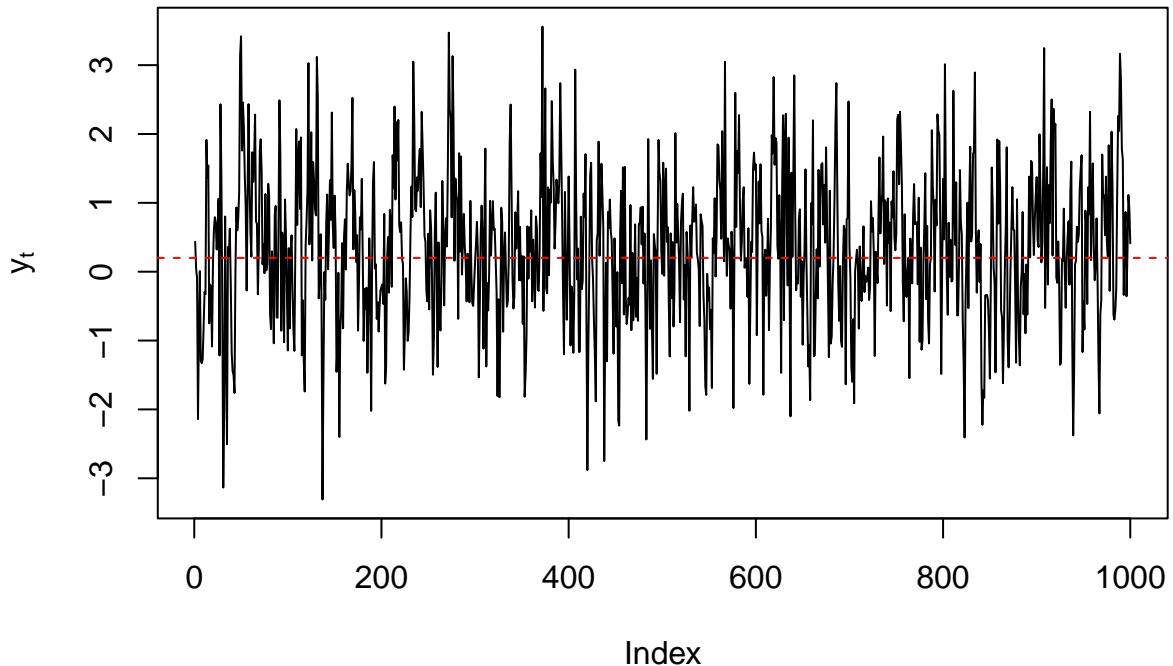
y4 <- numeric(T)
y4[1] <- c + epsilon[1]

for (i in 2:T){
  y4[i] <- c + phi1*y4[i-1] + epsilon[i]
}

plot(y4, type = "l", main = TeX("$y_t = c + \phi_1 y_{t-1} + \epsilon_t$"), ylab=TeX("$y_t$"))
abline(h = c, col = "red", lty = 2)

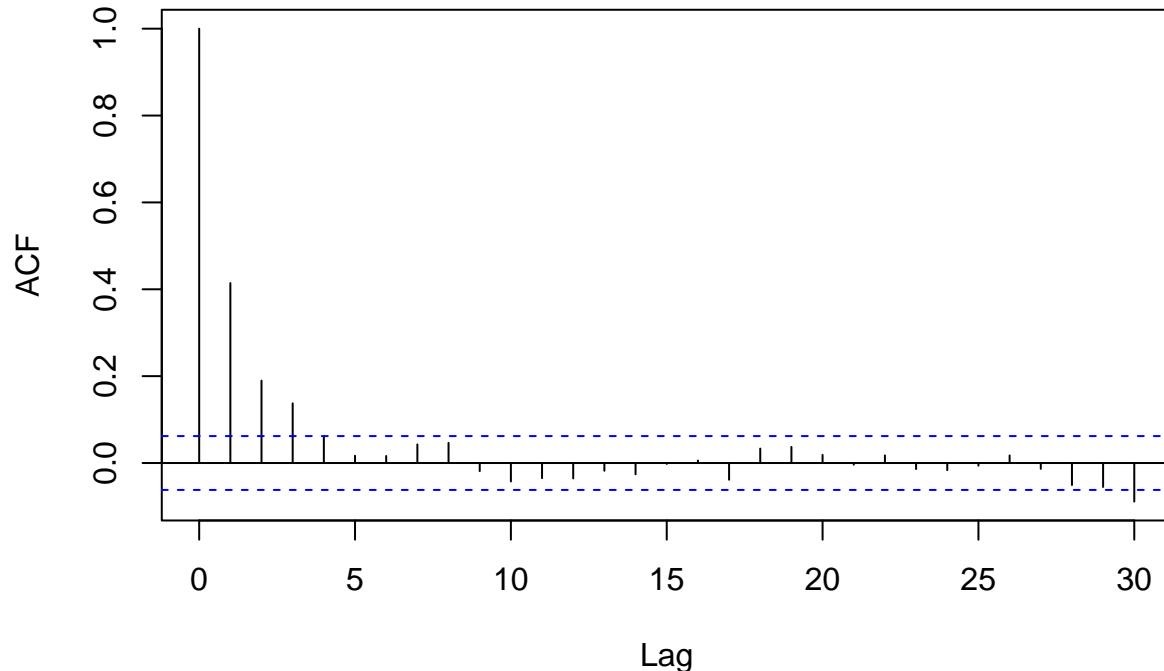
```

$$y_t = C + \phi_1 y_{t-1} + \epsilon_t$$



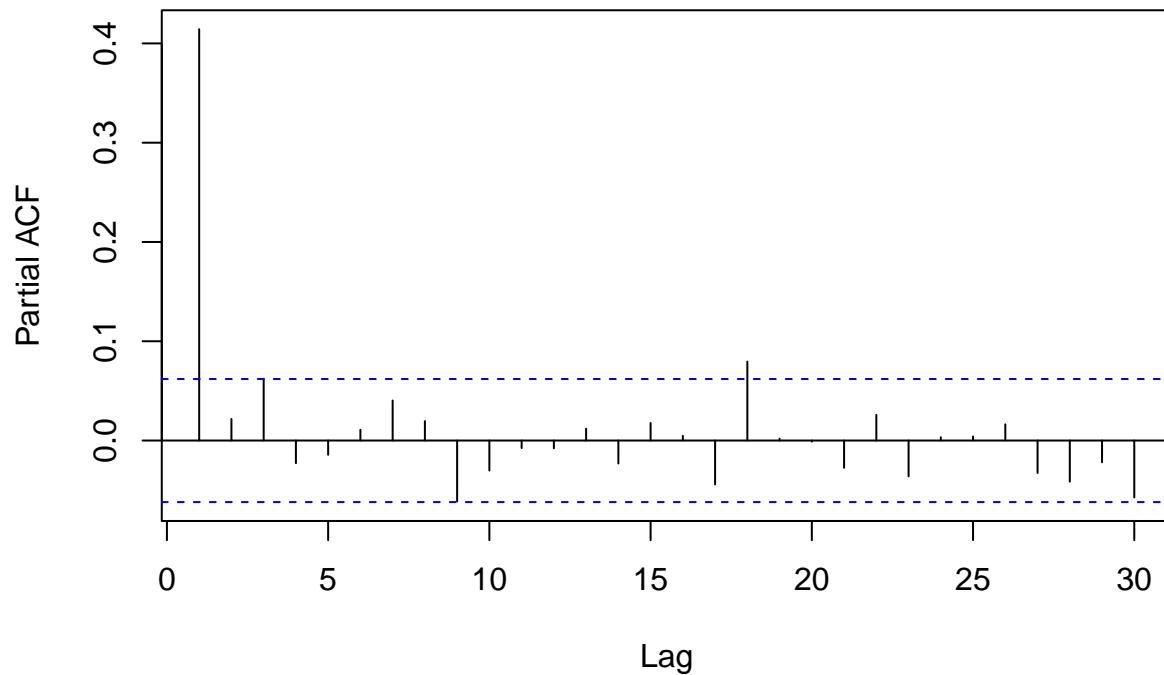
```
acf(y4, main = "ACF of y")
```

### ACF of y



```
pacf(y4, main = "PACF of y")
```

### PACF of y



$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \phi_3 y_{t-3} + \epsilon_t$$

```

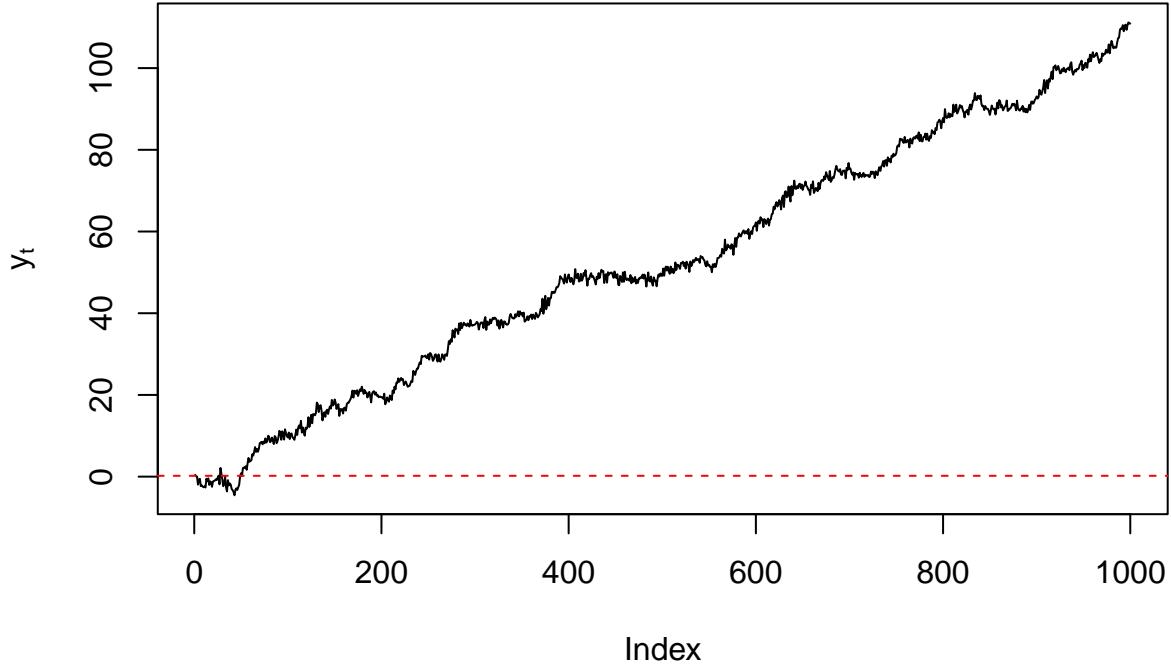
y5 <- numeric(T)
y5[1] <- c + epsilon[1]
y5[2] <- c + phi1*y5[1] + epsilon[2]
y5[3] <- c + phi1*y5[2] + phi2*y5[1] + epsilon[3]

for (i in 4:T){
  y5[i] <- c + phi1*y5[i-1] + phi2*y5[i-2] + phi3*y5[i-3] + epsilon[i]
}

plot(y5, type = "l", main = TeX("$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \phi_3 y_{t-3} + \epsilon_t$"))
abline(h = c, col = "red", lty = 2)

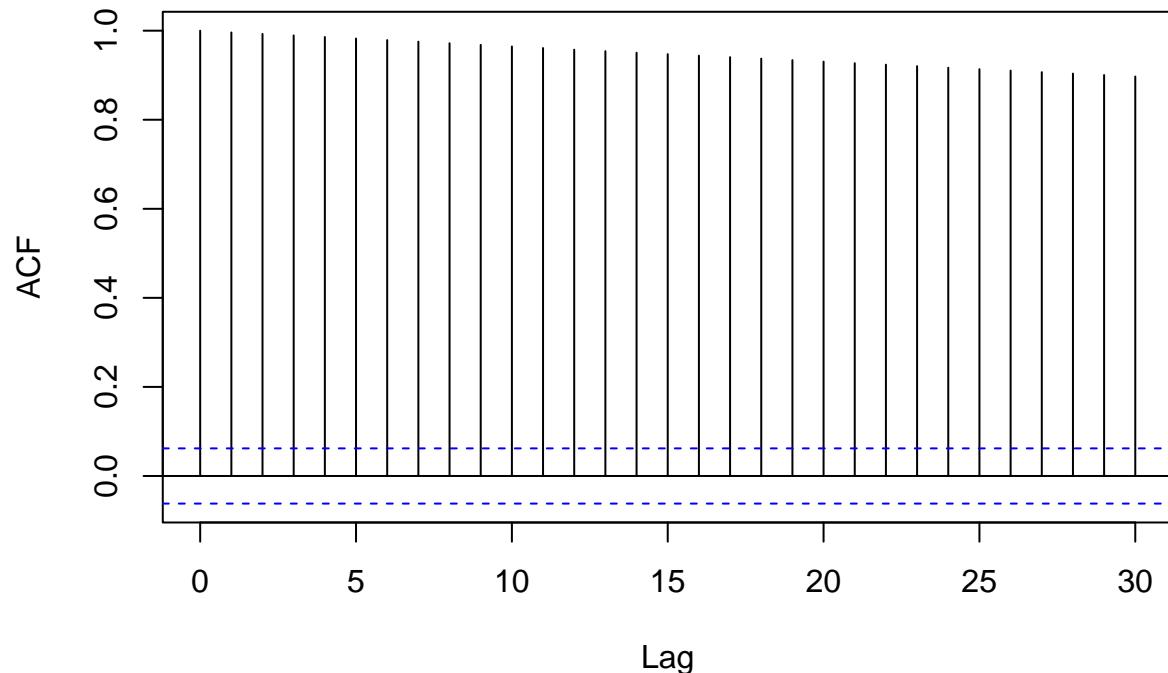
```

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \phi_3 y_{t-3} + \epsilon_t$$



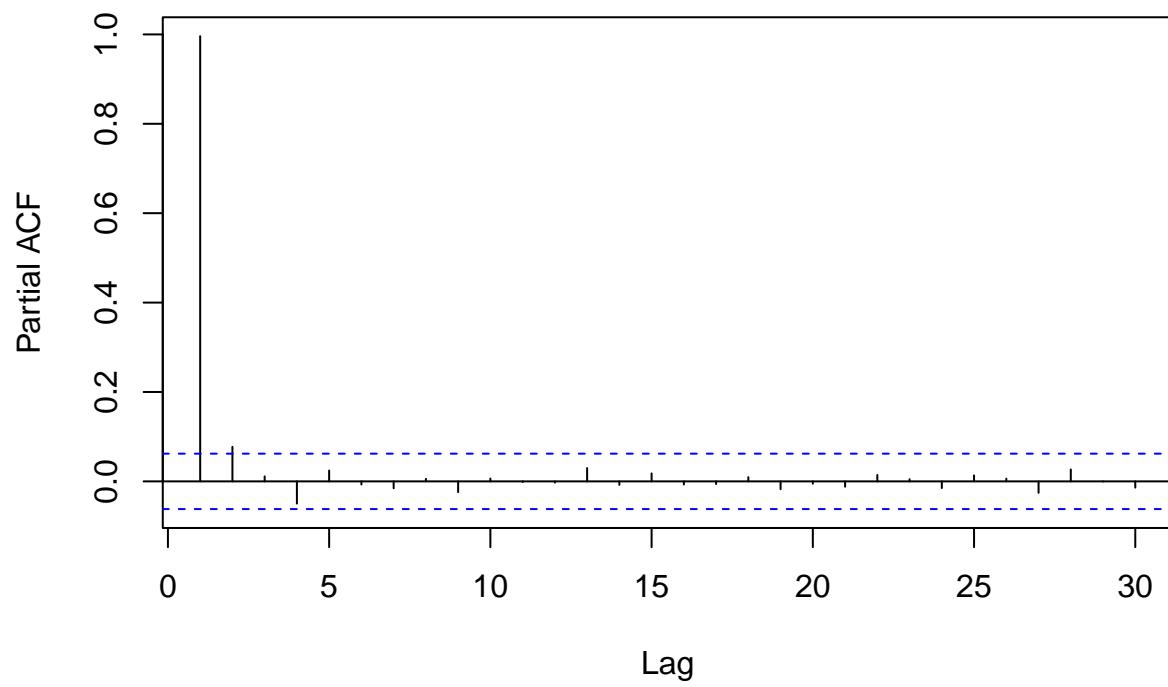
```
acf(y5, main = "ACF of y")
```

### ACF of y



```
pacf(y5, main = "PACF of y")
```

### PACF of y



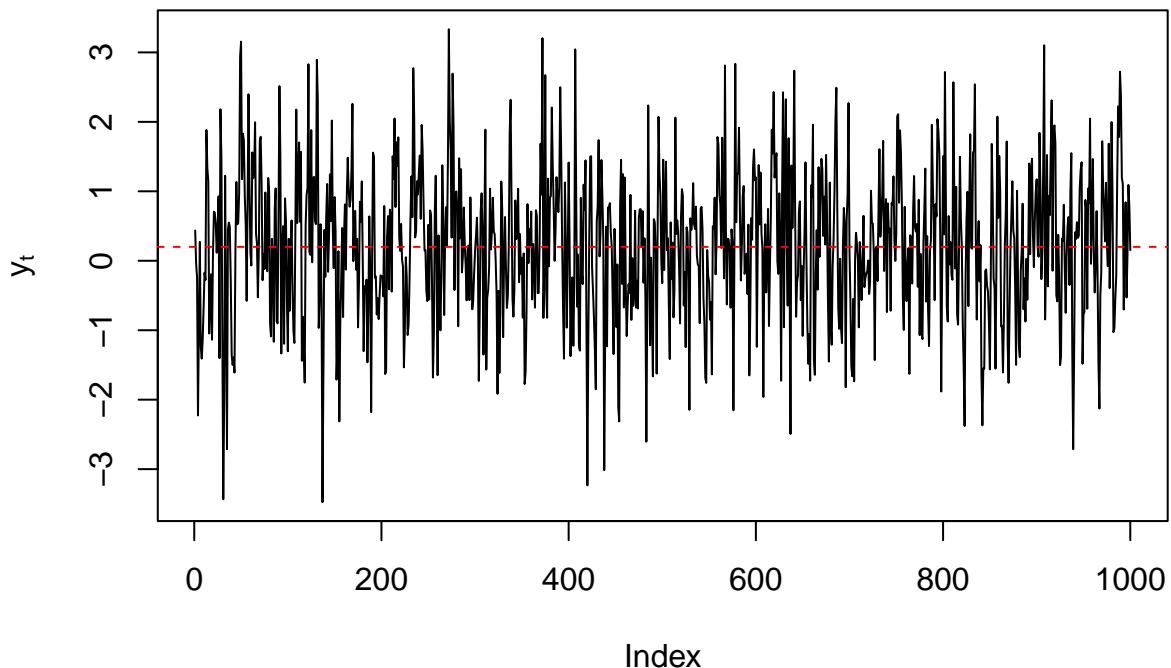
$$y_t = c + \epsilon_t + \theta_1 \epsilon_{t-1}$$

```
y6 <- numeric(T)
y6[1] <- c + epsilon[1]

for (i in 2:T){
  y6[i] <- c + epsilon[i] + theta1*epsilon[i-1]
}

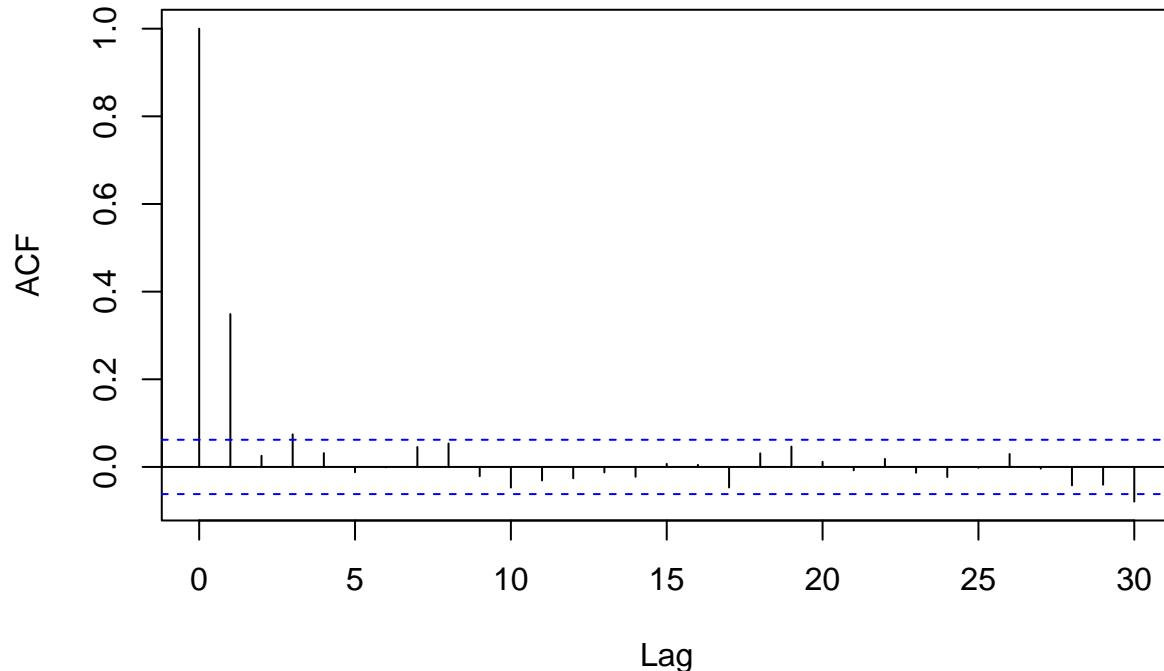
plot(y6, type = "l",
      main = expression(y[t] == c + epsilon[t] + theta[1]*epsilon[t-1]),
      ylab = expression(y[t]))
abline(h = c, col = "red", lty = 2)
```

$$y_t = C + \varepsilon_t + \theta_1 \varepsilon_{t-1}$$



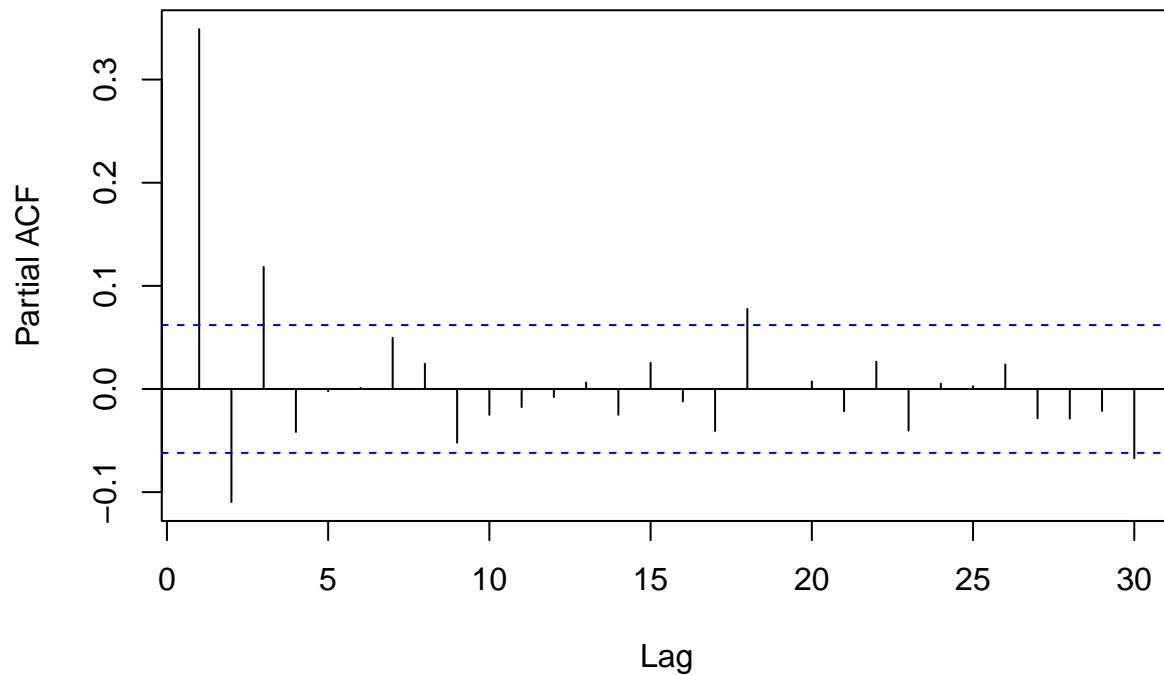
```
acf(y6, main = "ACF of y")
```

### ACF of y



```
pacf(y6, main = "PACF of y")
```

### PACF of y



$$y_t = c + \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2}$$

```

y7 <- numeric(T)

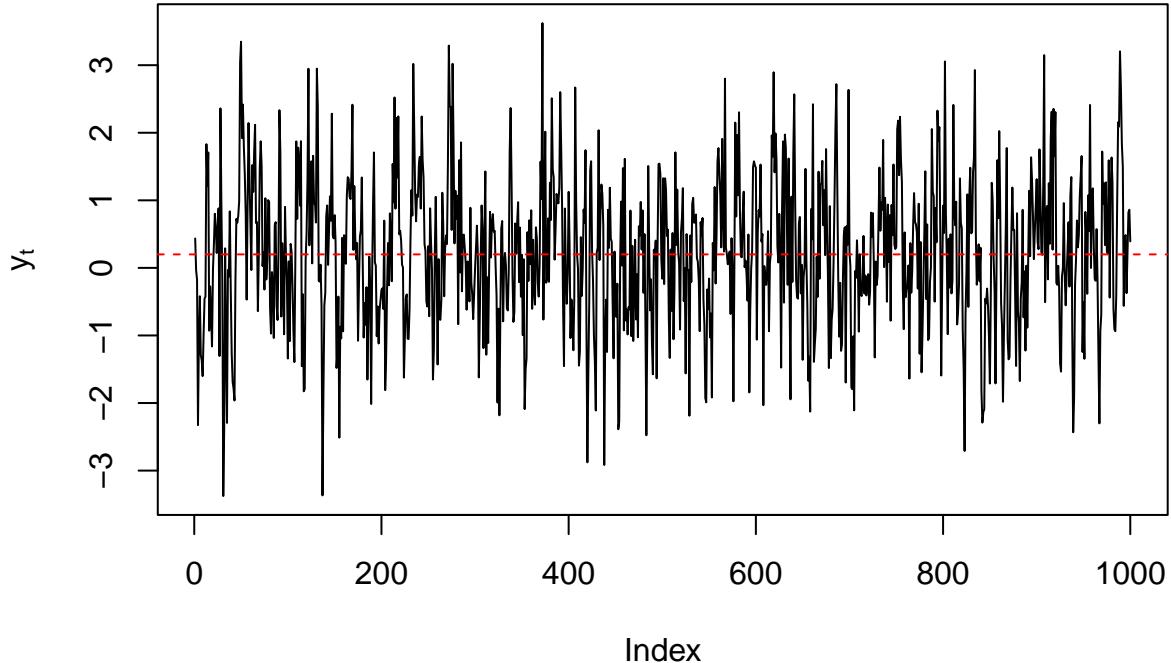
y7[1] <- c + epsilon[1]
y7[2] <- c + epsilon[2] + theta1*epsilon[1]

for (i in 3:T){
  y7[i] <- c + epsilon[i] + theta1*epsilon[i-1] + theta2*epsilon[i-2]
}

plot(y7, type = "l",
      main = expression(y[t] == c + epsilon[t] + theta[1]*epsilon[t-1] + theta[2]*epsilon[t-2]),
      ylab = expression(y[t]))
abline(h = c, col = "red", lty = 2)

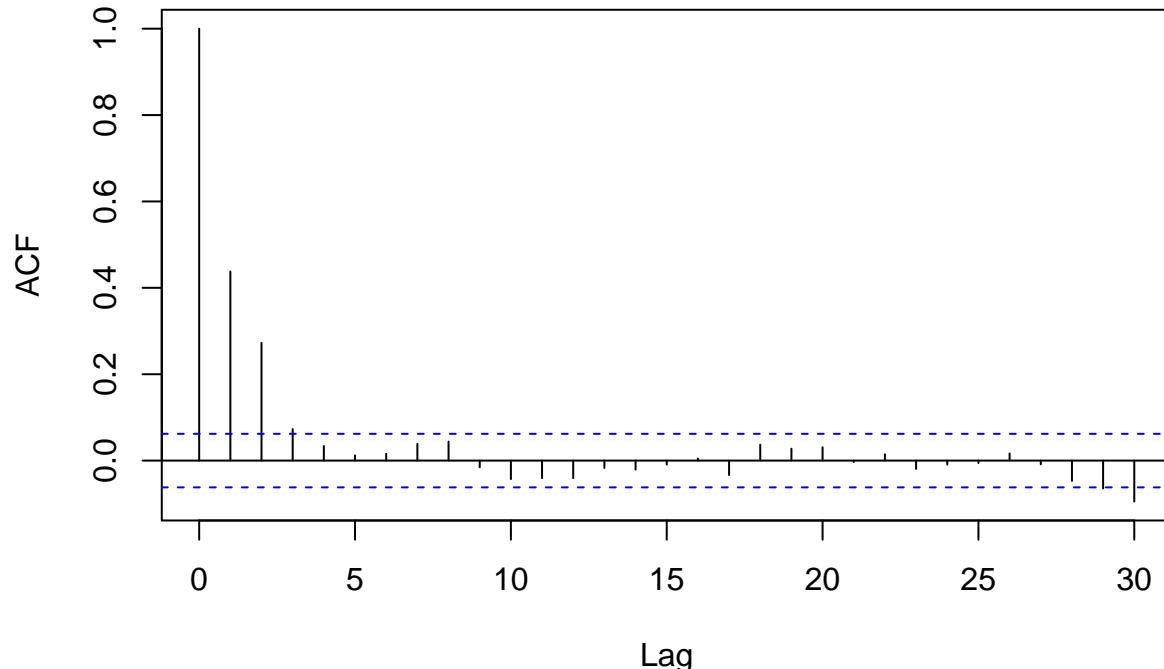
```

$$y_t = c + \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2}$$



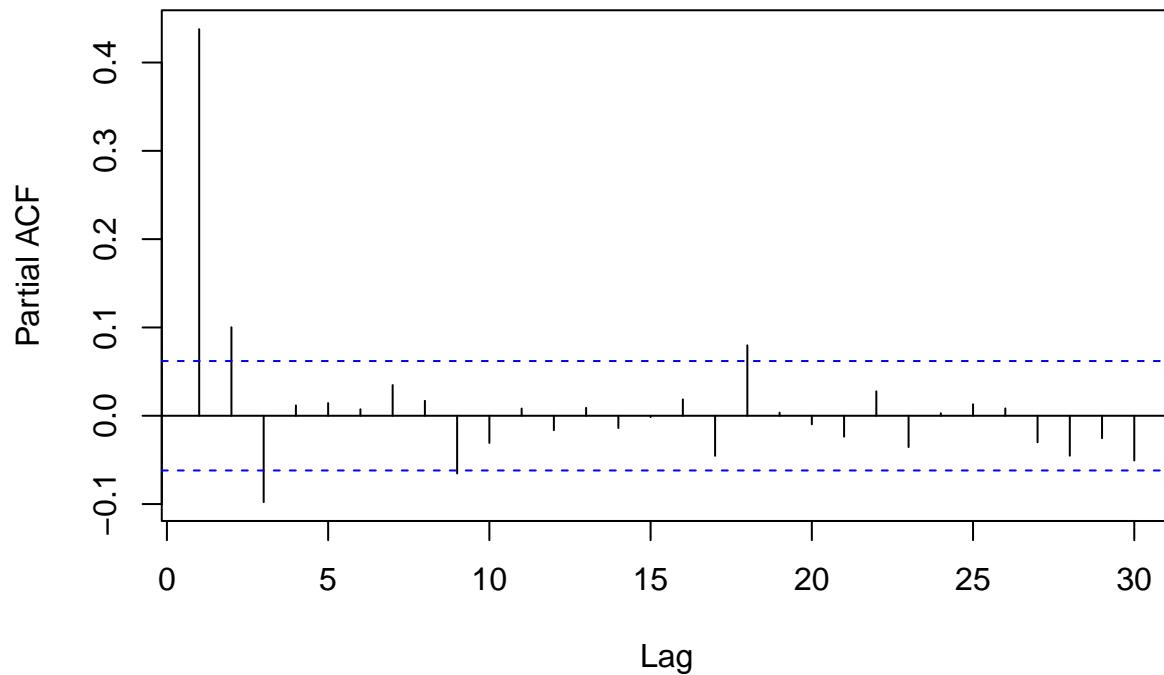
```
acf(y7, main = "ACF of y")
```

### ACF of y



```
pacf(y7, main = "PACF of y")
```

### PACF of y



$$y_t = c + \phi_1 y_{t-1} + \epsilon_t + \theta_1 \epsilon_{t-1}$$

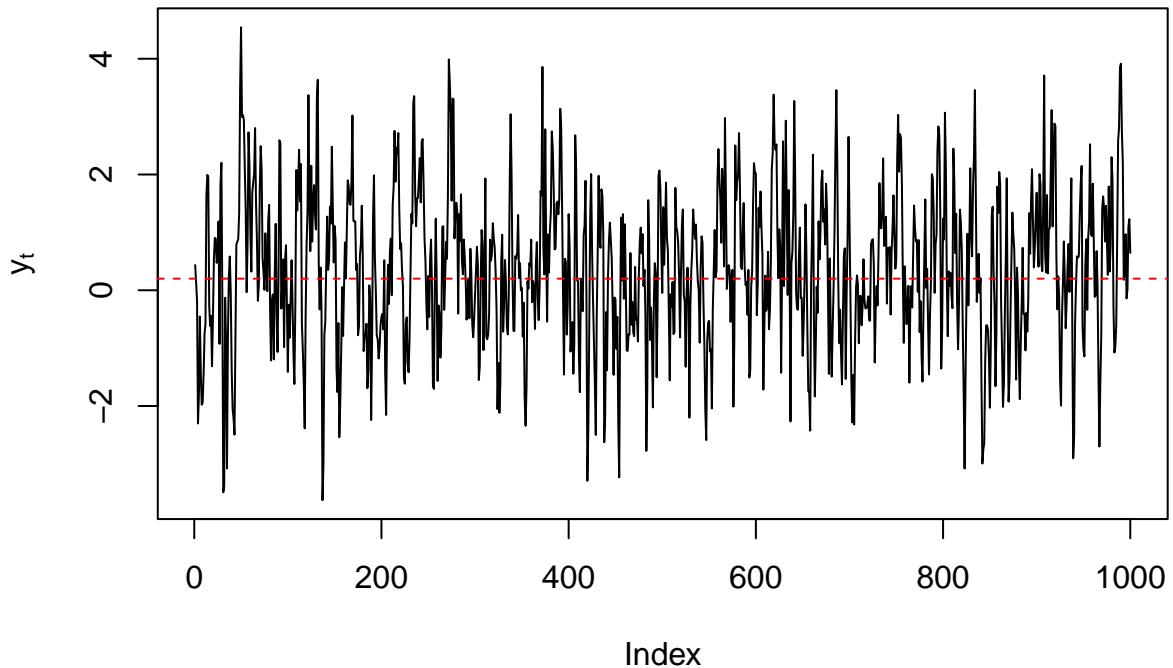
```
y8 <- numeric(T)

y8[1] <- c + epsilon[1]

for (i in 2:T){
  y8[i] <- c + phi1*y8[i-1] + epsilon[i] + theta1*epsilon[i-1]
}

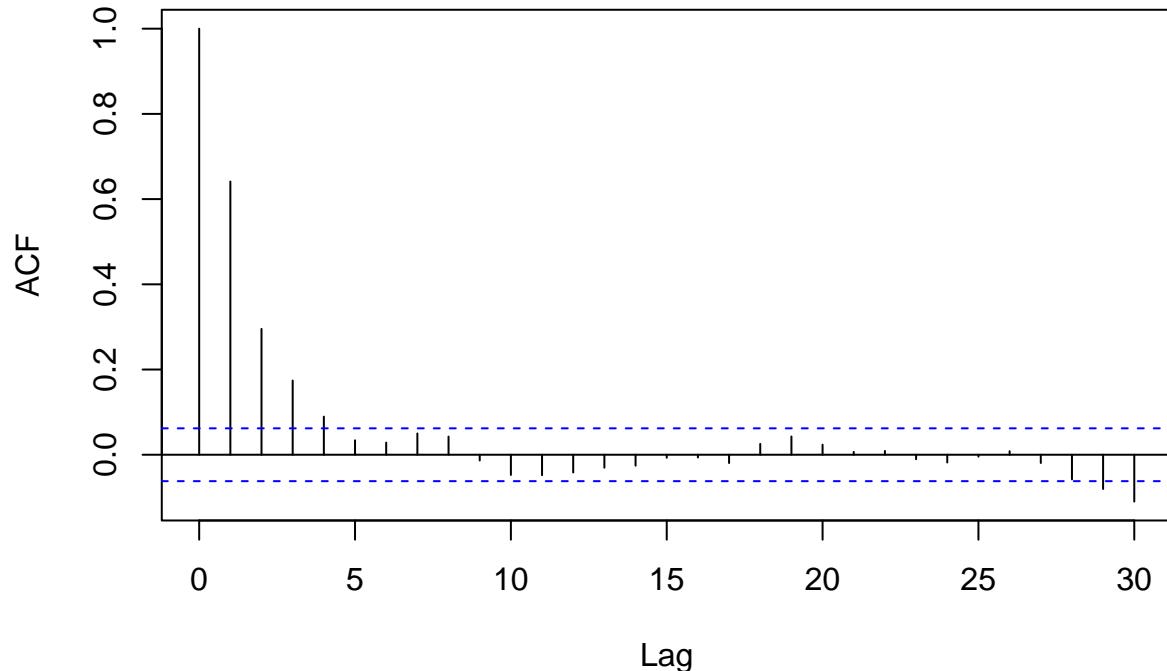
plot(y8, type = "l",
      main = expression(y[t] == c + phi[1]*y[t-1]+ epsilon[t] + theta[1]*epsilon[t-1]),
      ylab = expression(y[t]))
abline(h = c, col = "red", lty = 2)
```

$$y_t = C + \phi_1 y_{t-1} + \epsilon_t + \theta_1 \epsilon_{t-1}$$



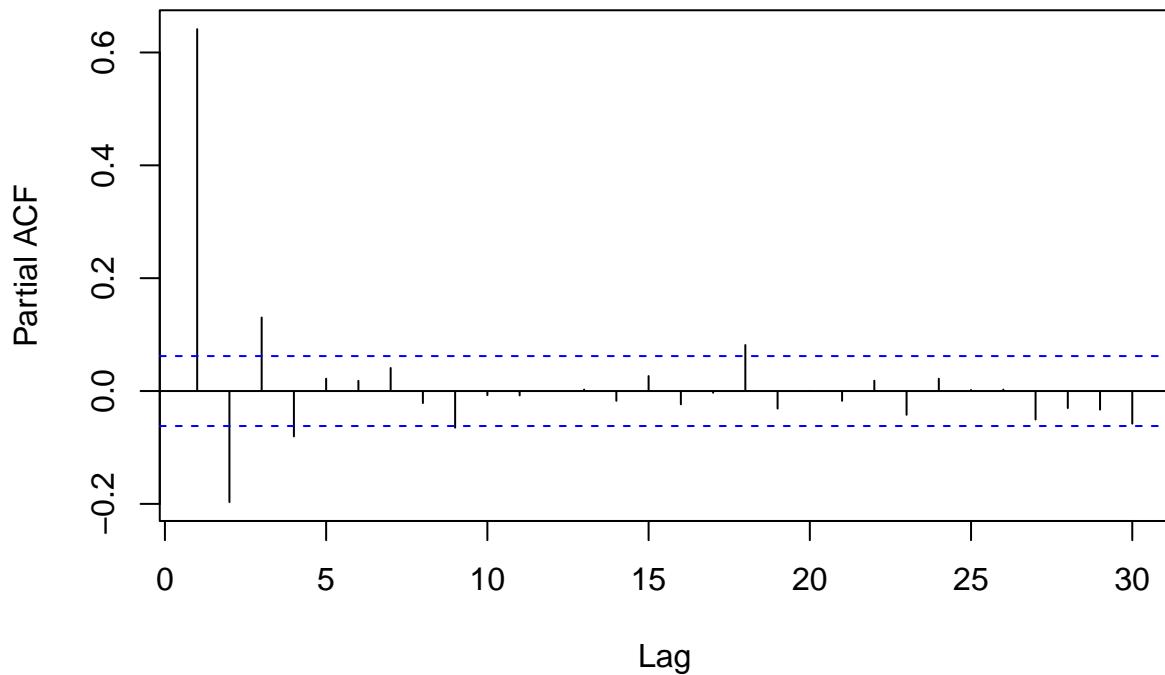
```
acf(y8, main = "ACF of y")
```

### ACF of y



```
pacf(y8, main = "PACF of y")
```

### PACF of y



## Exercice 2

### Loading data

#### S&P500

```
getSymbols(Symbols = '^GSPC', src="yahoo")

## [1] "GSPC"

sp500 <- data.frame(
  date = index(GSPC),
  SP500 = as.numeric(Ad(GSPC))
)
```

#### GBPUSD

```
getSymbols(Symbols = 'GBPUSD=X', src="yahoo")

## Warning: GBPUSD=X contains missing values. Some functions will not work if
## objects contain missing values in the middle of the series. Consider using
## na.omit(), na.approx(), na.fill(), etc to remove or replace them.

## [1] "GBPUSD=X"

gbpusd <- data.frame(
  date = index(`GBPUSD=X`),
  GBPUSD = as.numeric(Ad(`GBPUSD=X`))
)
```

#### TBILL

```
getSymbols(Symbols = 'DTB3', src="FRED")

## [1] "DTB3"

dtb3 <- data.frame(
  date = index(`DTB3`),
  DTB3 = as.numeric(`DTB3`)
)
```

#### VIX

```
getSymbols(Symbols = '^VIX', src="yahoo")

## [1] "VIX"

vix <- data.frame(
  date = index(`VIX`),
  VIX = as.numeric(Ad(`VIX`))
)
```

### Transformation

```
sp500_d <- sp500 %>%
  mutate(
```

```

lr = log(SP500) - log(lag(SP500))
) %>%
na.omit()

gbpusd_d <- gbpusd %>%
  mutate(
    lr = log(GBPUSD) - log(lag(GBPUSD))
  ) %>%
na.omit()

vix_d <- vix %>%
  mutate(
    lr = log(VIX) - log(lag(VIX))
  ) %>%
na.omit()

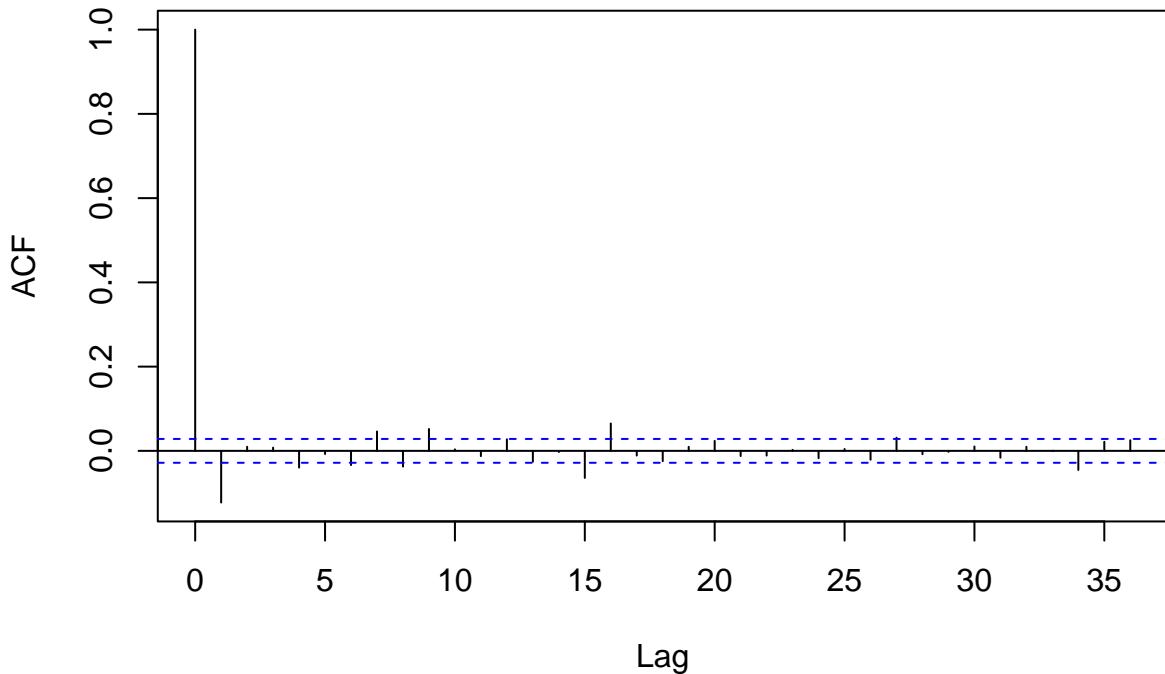
dtb3_d <- dtb3 %>%
  na.omit() %>%
  mutate(
    d = DTB3 - lag(DTB3)
  ) %>%
na.omit()

```

Plot ACF and PACF for each stock

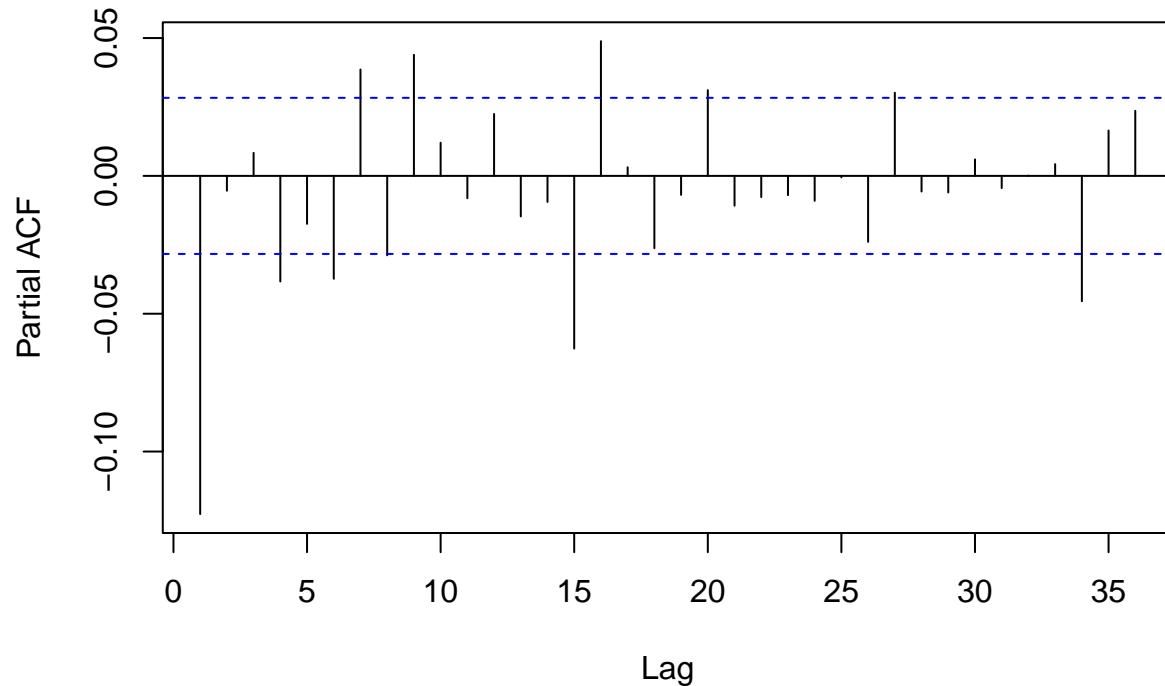
```
acf(sp500_d$lr, main = "ACF S&P500")
```

**ACF S&P500**



```
pacf(sp500_d$lr, main = "PACF S&P500")
```

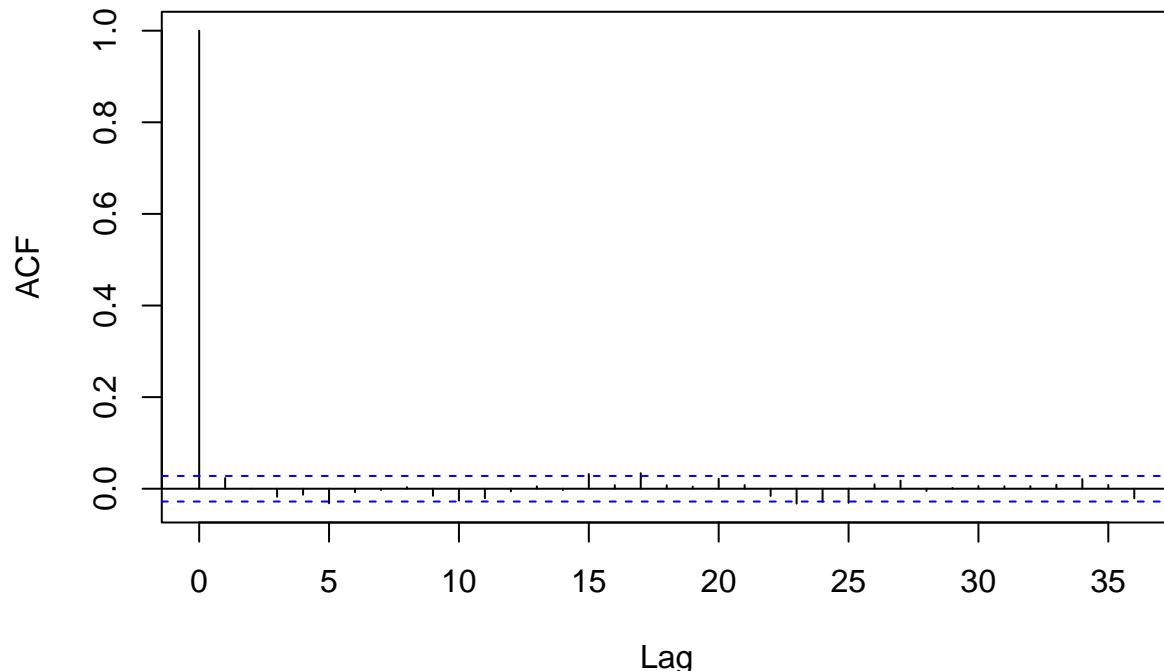
## PACF S&P500



Conclusion ARMA(1) car pic (négatif) significatif au lag 1 ACF et PACF

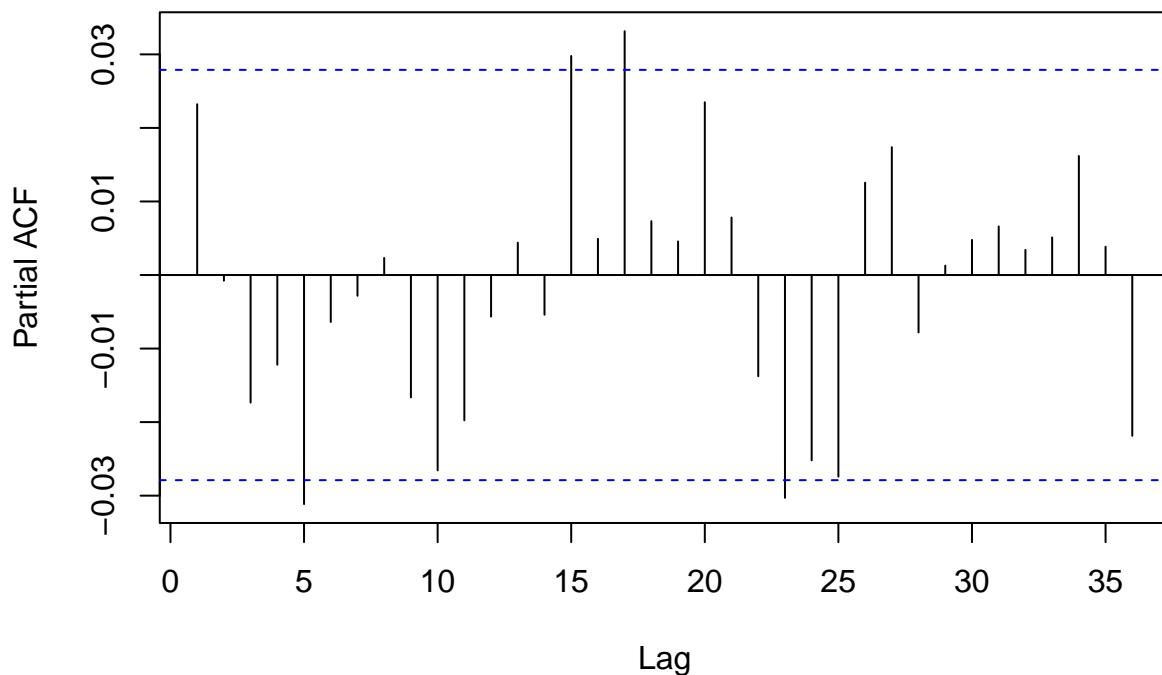
```
acf(gbpusd_d$lr, main = "ACF GBPUSD")
```

### ACF GBPUSD



```
pacf(gbpusd_d$lr, main = "PACF GBPUSD")
```

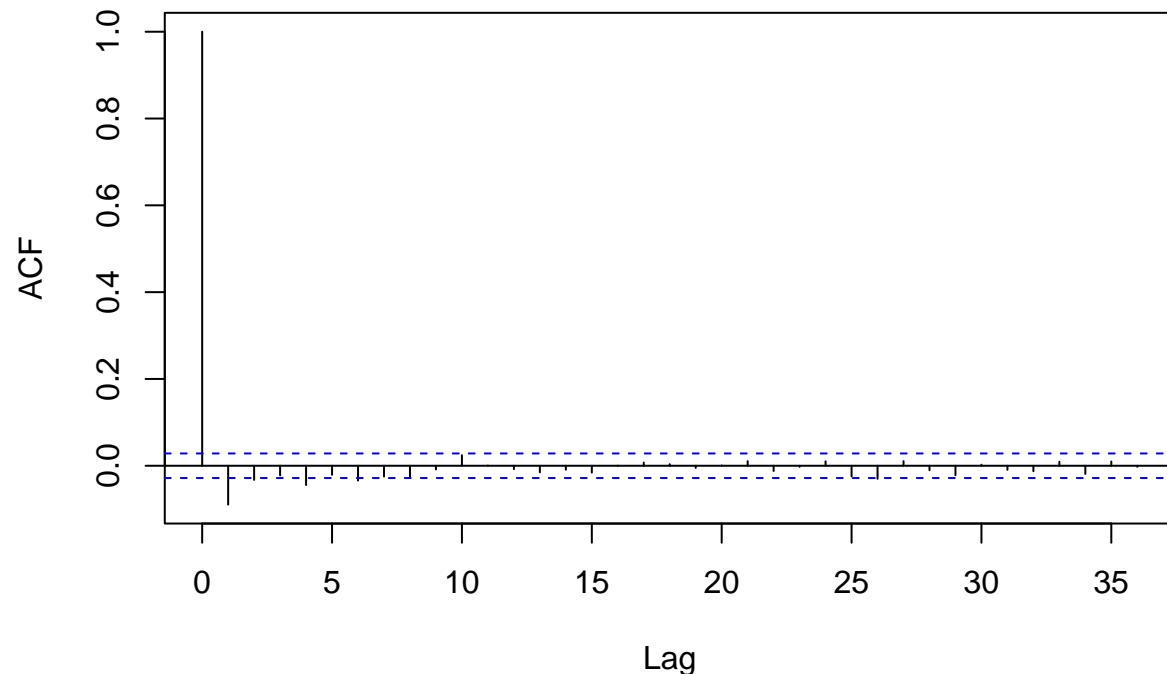
### PACF GBPUSD



Conclusion rien de significatif, que ce soit dans le ACF ou PACF. ARMA(0,0)

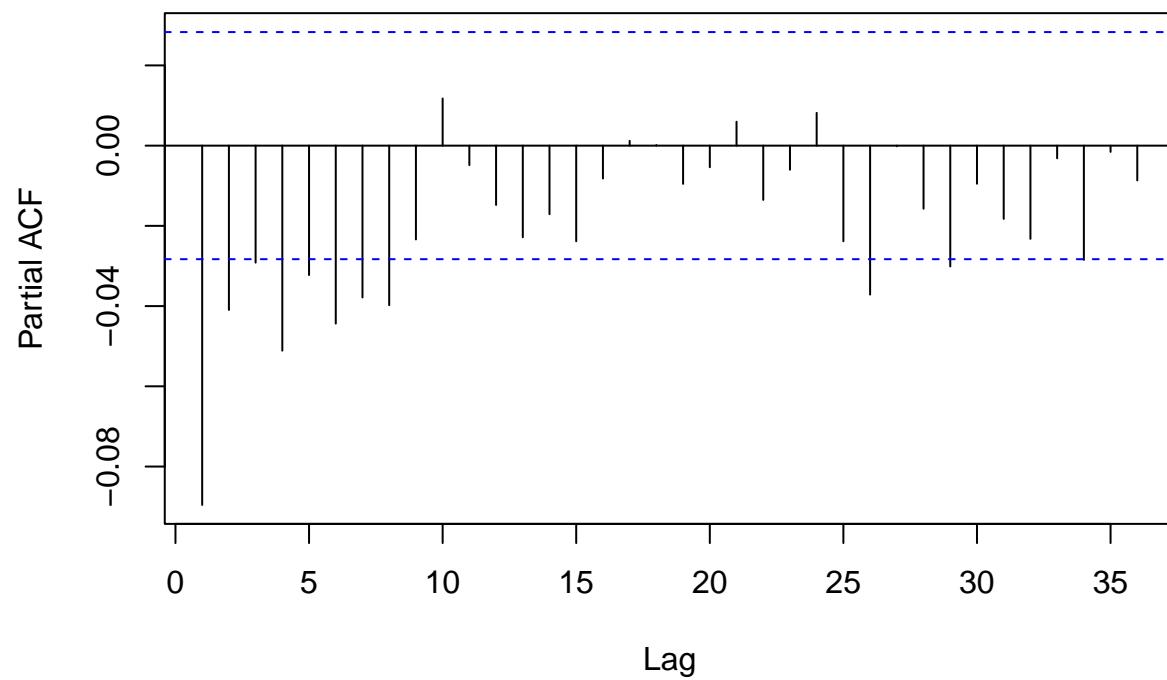
```
acf(vix_d$lr, main = "ACF VIX")
```

**ACF VIX**



```
pacf(vix_d$lr, main = "PACF VIX")
```

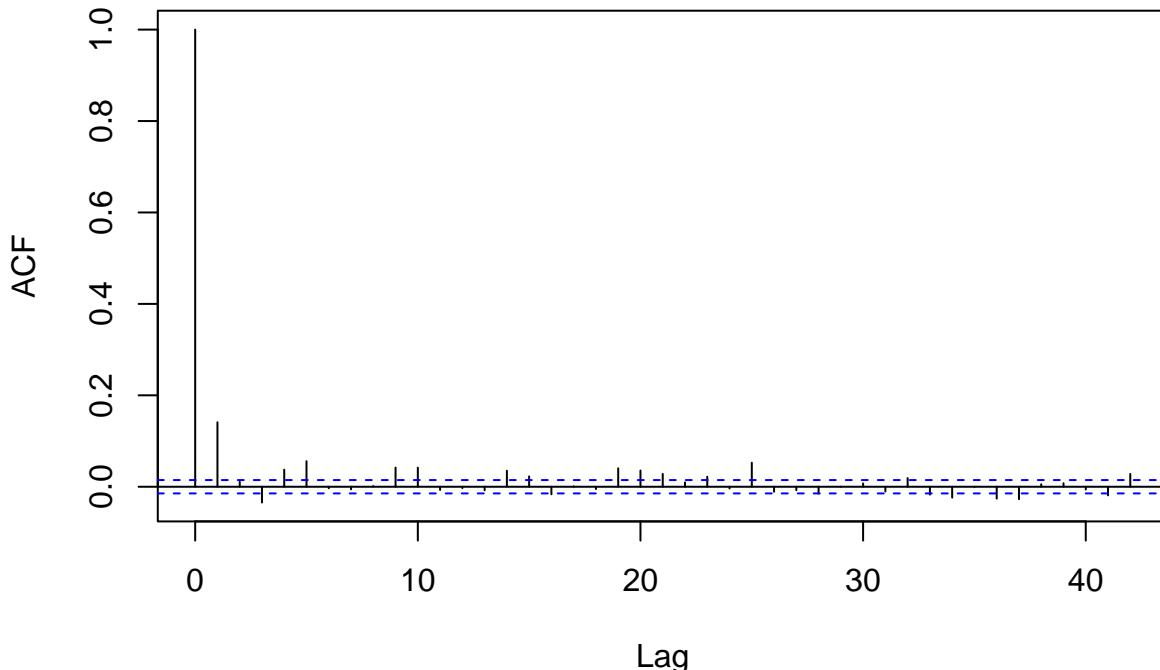
**PACF VIX**



Conclusion AR(1) car pic au ACF - Décroissance lente PACF

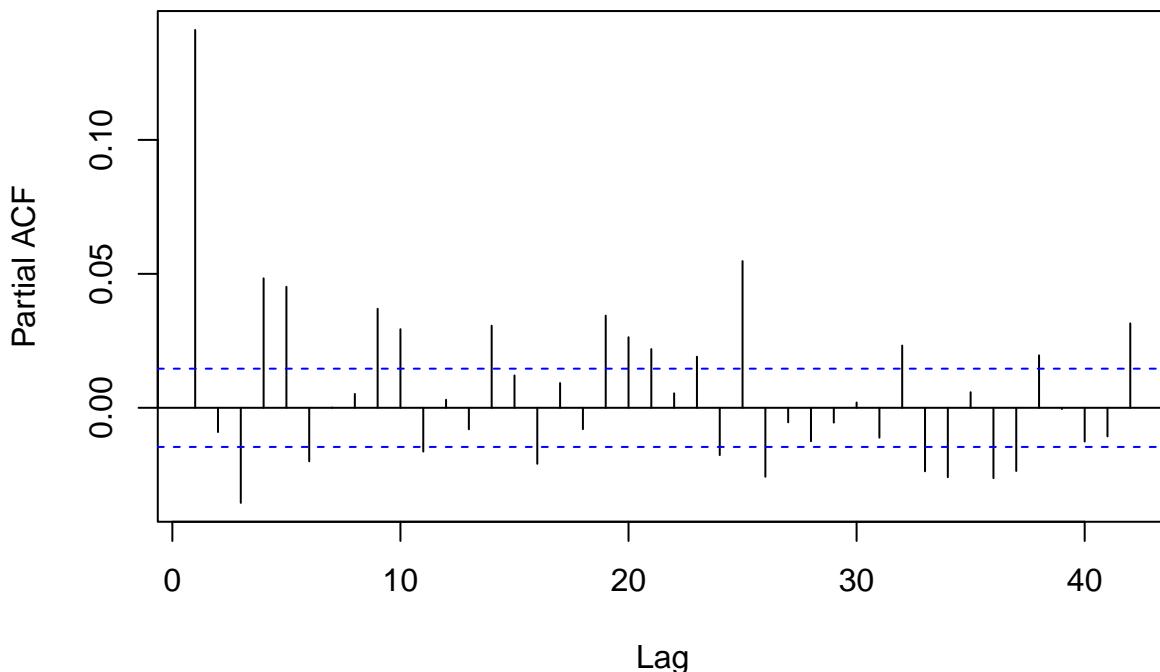
```
acf(dtb3_d$d, main = "ACF DTB3")
```

**ACF DTB3**



```
pacf(dtb3_d$d, main = "PACF DTB3")
```

**PACF DTB3**



## **Conclusion**