



# Intro to Evolutionary Computing

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Slides adapted from a guest  
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# CSIRO Autonomous Systems

CSIRO is Australia's largest science/research agency.

AS lab facts: ~100 people, mainly in Brisbane and Sydney.

Highly collaborative, strong ties to industry.



**Lab focus is Field Robotics:** engineering, control, vision, planning...

**My role:** apply evolutionary algorithms to increase the long term autonomy of real-world robots doing useful tasks.



# MOTIVATION

3.8 to 2.5 billion years ago



# Mantis Shrimp



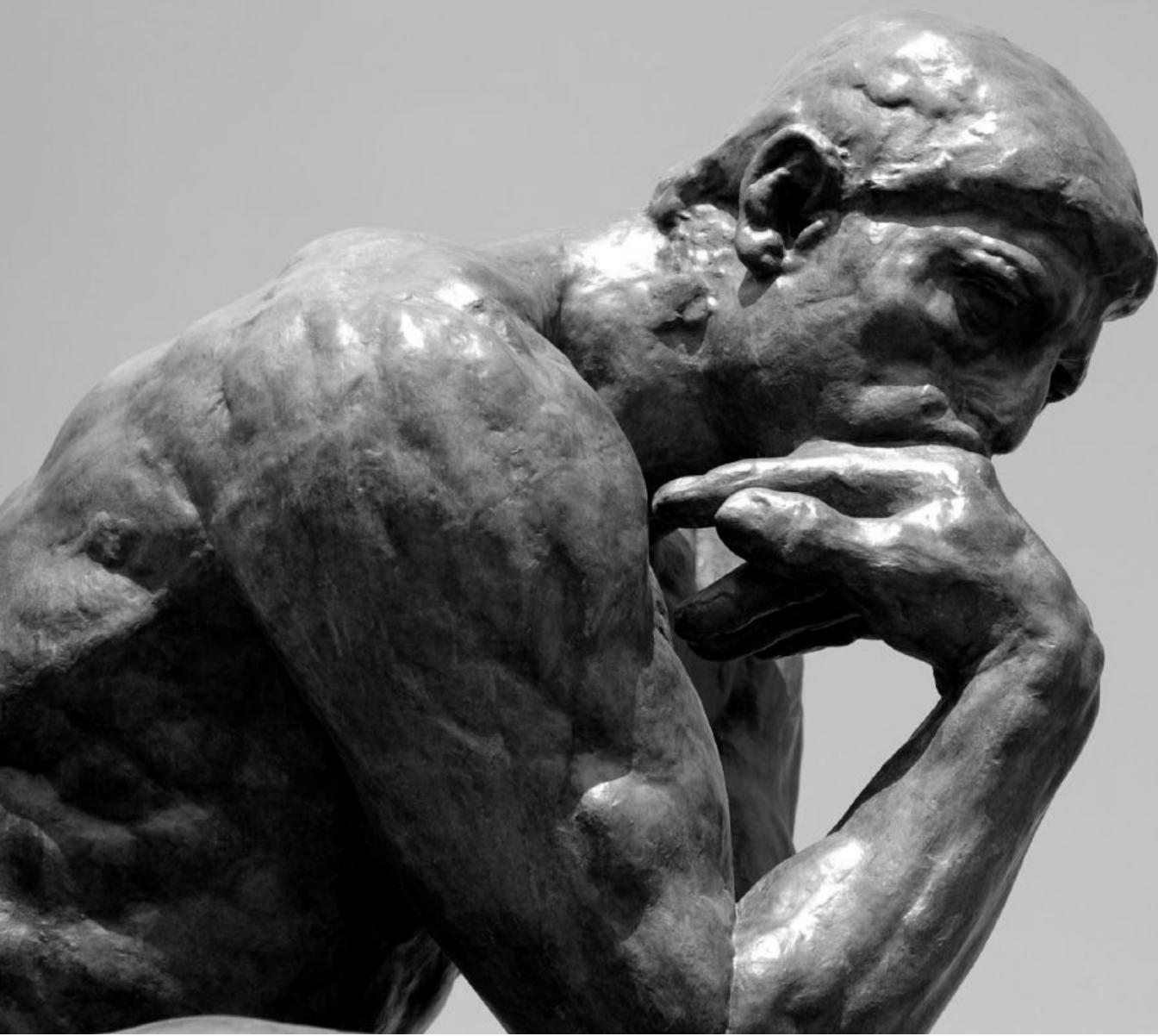


# Zombie ant fungus



# Tardigrade





# Evolution in the real world

1. A creature is defined by a genome
2. Genome encodes traits (blonde hair, brown eyes, running speed, etc.) to appear in a phenotype
3. Genotype *expresses* a phenotype
4. The environment creates a selection pressure which gives us “survival of the fittest”
5. Highly fit creatures become parents and can pass on beneficial traits to children
6. Transfer is messy = random variations



# The benefits of evolution

**Creativity  
Unsupervised  
Complexity  
Adaptive  
Specialised  
High performing  
Robust**

Incrementalism

Cooperation

Altruism

Evolution lets us study the effects that natural processes may have, and can harness them if beneficial!

Creatures vs robots

Co-evolution

# **History of Evolutionary Algorithms**

# Evolutionary Computing: the Origins

- Historical perspective
- Biological inspiration:
  - Darwinian evolution theory (simplified!)
  - Genetics (simplified!)
- Motivation for EC

# Historical perspective (1/3)

- 1948, Turing:  
proposes “genetical or evolutionary search”
- 1962, Bremermann:  
optimization through evolution and recombination
- 1964, Rechenberg:  
introduces evolution strategies
- 1965, L. Fogel, Owens and Walsh:  
introduce evolutionary programming
- 1975, Holland:  
introduces genetic algorithms
- 1992, Koza:  
introduces genetic programming

# Historical perspective (2/3)

- 1985: first international conference (ICGA)
- 1990: first international conference in Europe (PPSN)
- 1993: first scientific EC journal (MIT Press)
- 1997: launch of European EC Research Network EvoNet

# Historical perspective (3/3)

EC in the early 21<sup>st</sup> Century:

- 4 major EC conferences, about 10 small related ones
- Many scientific core EC journals (ECJ, TEC, GPEM, EI)
- 1000+ EC-related papers published last year(estimate)
- uncountable (meaning: many) applications
- uncountable (meaning: ?) consultancy and R&D firms
- part of many university curricula

# Darwinian Evolution (1/3): Survival of the fittest

- All environments have finite resources  
(i.e., can only support a limited number of individuals)
- Life forms have basic instinct/lifecycles geared towards reproduction
- Therefore some kind of selection is inevitable
- Those individuals that compete for the resources most effectively have increased chance of reproduction
- Note: fitness in natural evolution is a derived, secondary measure, i.e., we (humans) assign a high fitness to individuals with many offspring

# Darwinian Evolution (2/3): Diversity drives change

- Phenotypic traits:
  - Behaviour / physical differences that affect response to environment
  - Partly determined by inheritance, partly by factors during development (nature vs. nurture)
  - Unique to each individual, partly as a result of random changes
- If phenotypic traits:
  - Lead to higher chances of reproduction
  - Can be inherited
    - then they will tend to increase in subsequent generations, leading to new combinations of traits ...

# Darwinian Evolution (3/3): Summary

- Population consists of diverse set of individuals
- Combinations of traits that are better adapted tend to increase representation in population
  - Individuals are “units of selection”
- Variations occur through random changes yielding constant source of diversity, coupled with selection means that:
  - Population is the “unit of evolution”
- Note the absence of “guiding force”

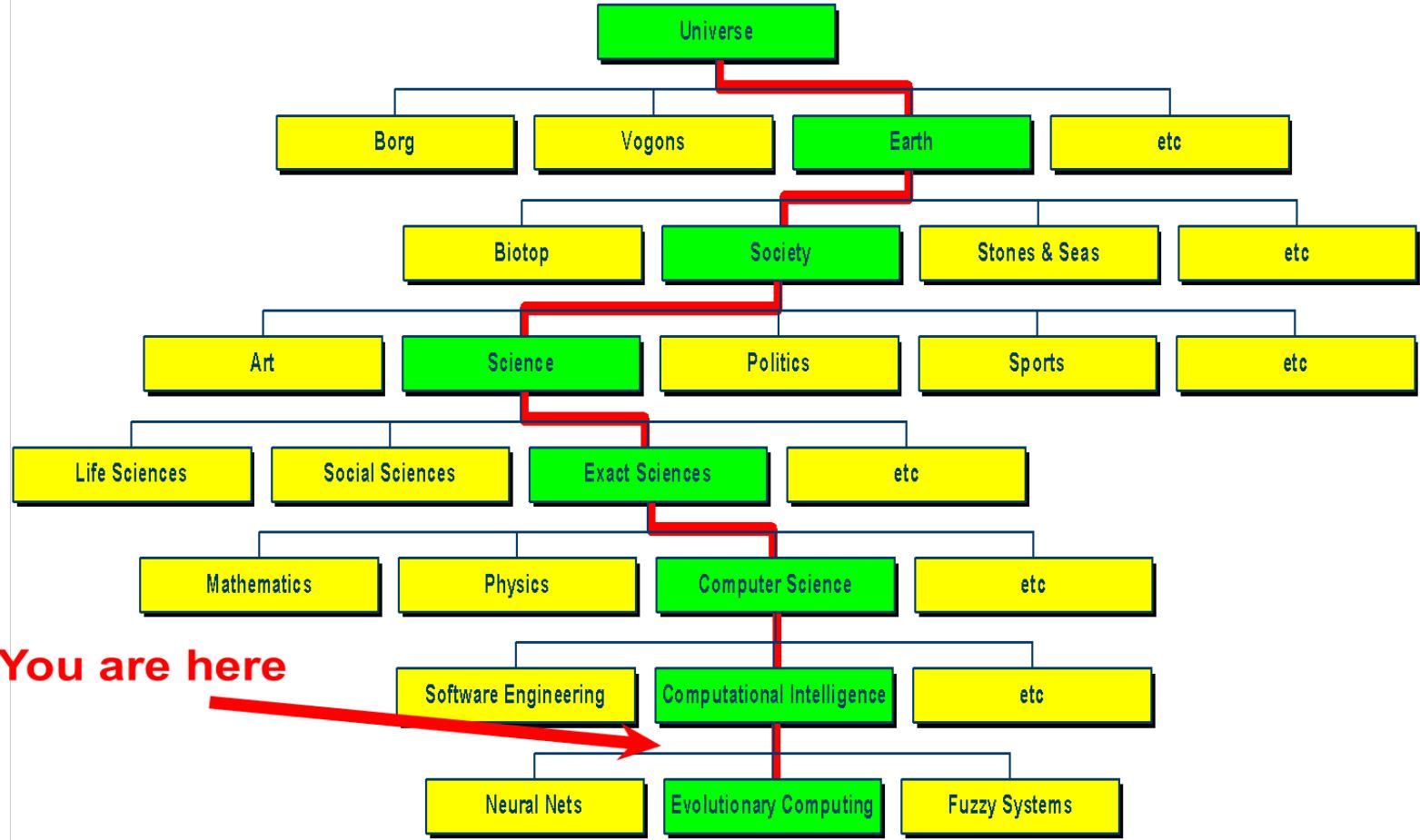
# Quote (D. Dennett)

“If you have variation, heredity, and selection, then you must get evolution”

# Quote (John Maynard Smith)

“So far, we have been able to study only one evolving system and we cannot wait for interstellar flight to provide us with a second. If we want to discover generalizations about evolving systems, we have to look at artificial ones.”

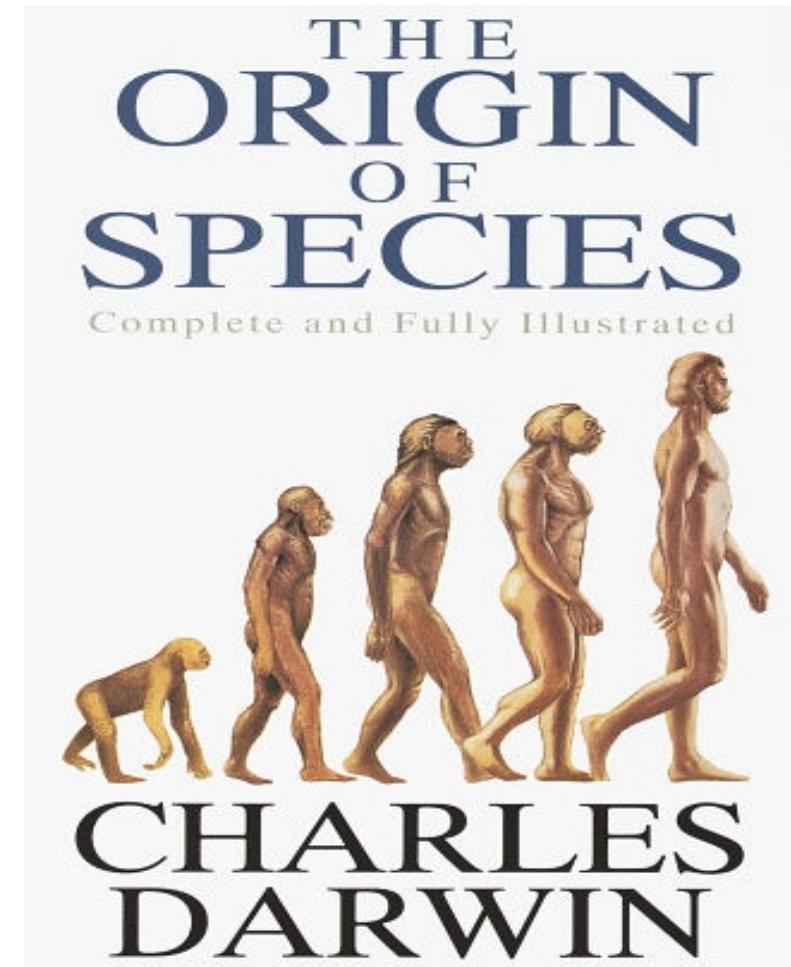
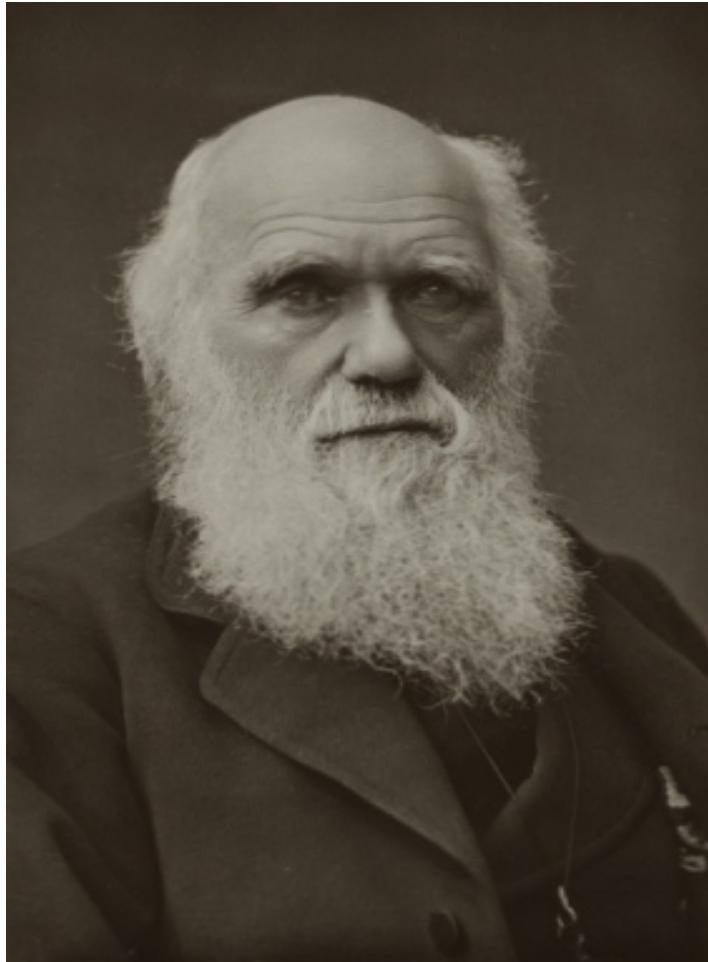
# Positioning of EC (1/2)



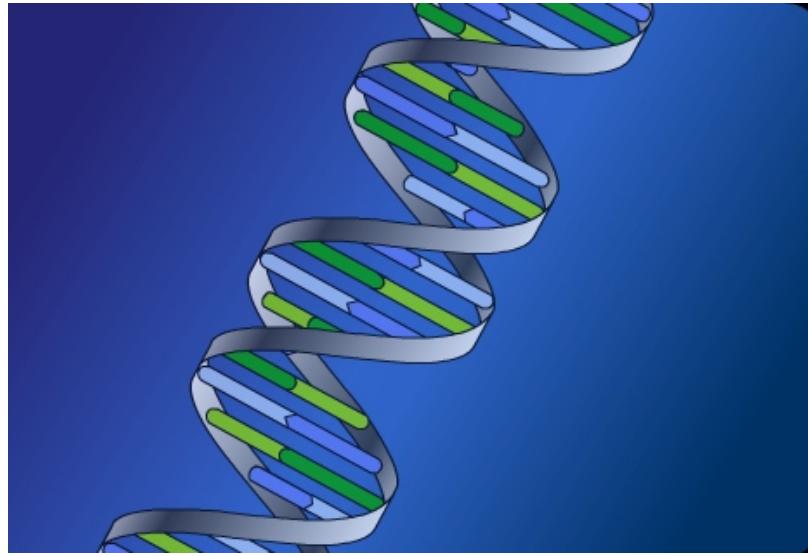
# Positioning of EC (2/2)

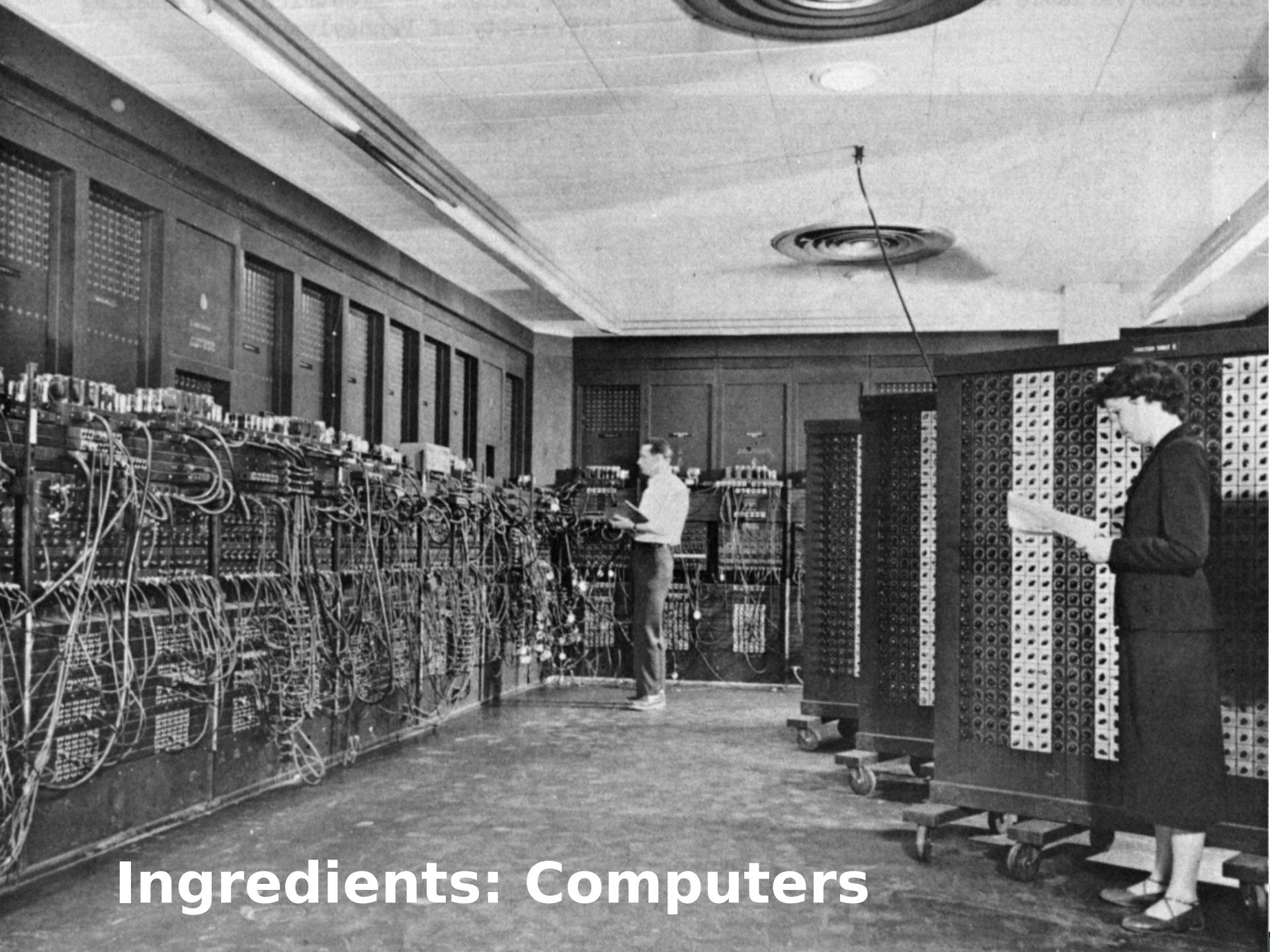
- EC is part of computer science
- EC is not part of life sciences/biology
- Biology delivered inspiration and terminology
- EC can be applied in biological research

# Ingredients: Evolution



# Ingredients: Genetics



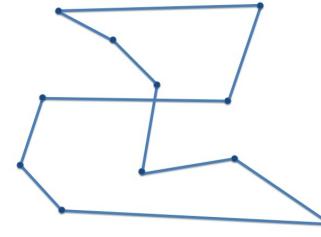


# Ingredients: Computers

# The EC Metaphor (1/2)



Individuals



Natural  
selection



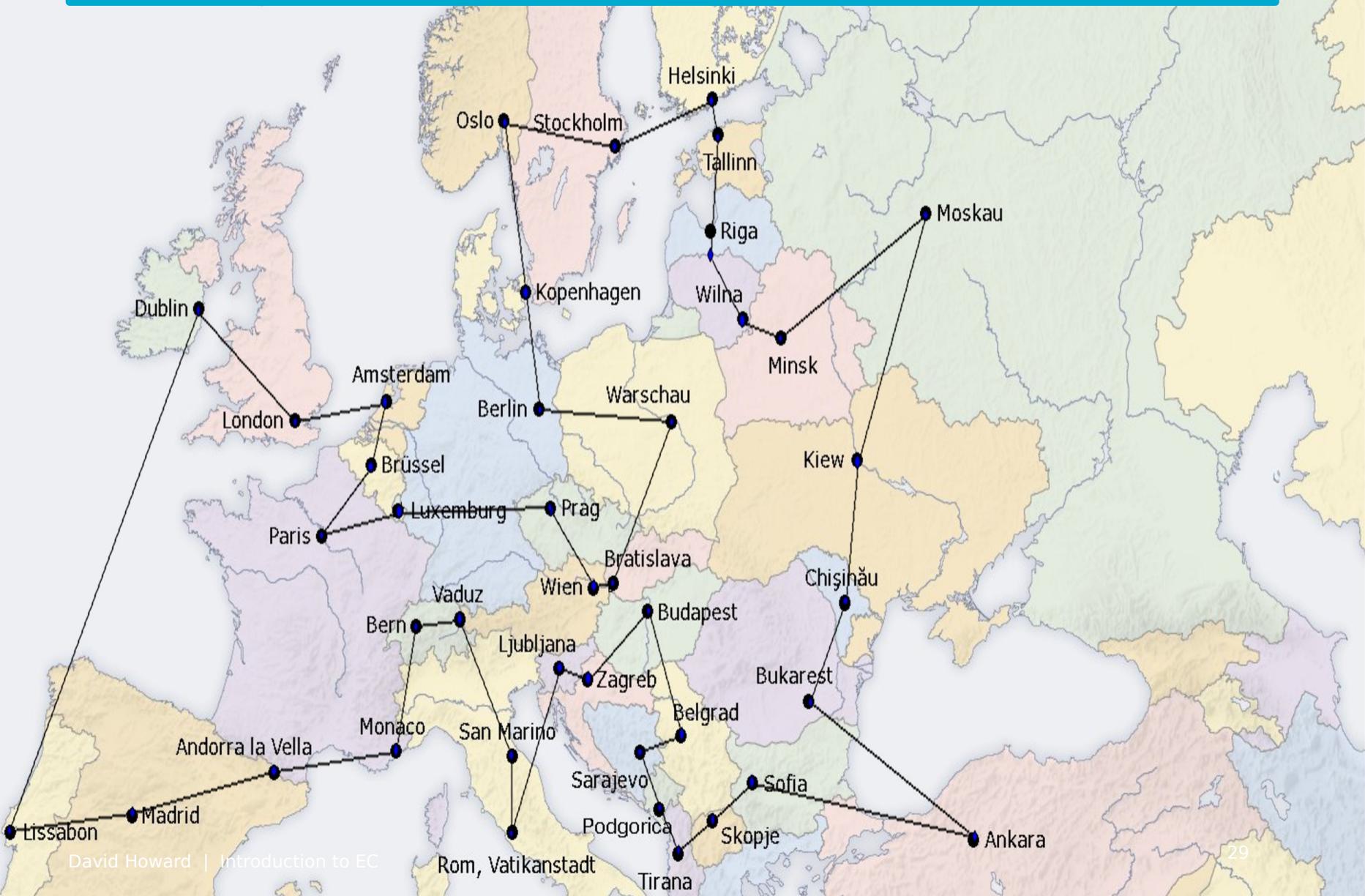
Reproduction

digital  
sex

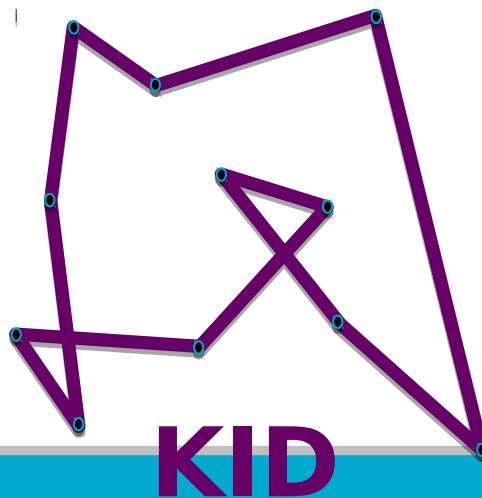
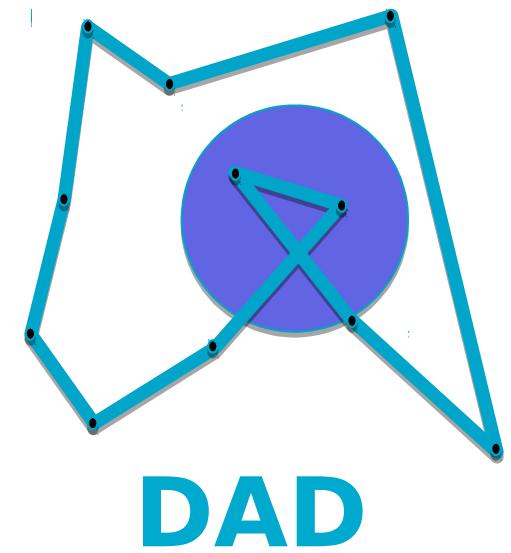
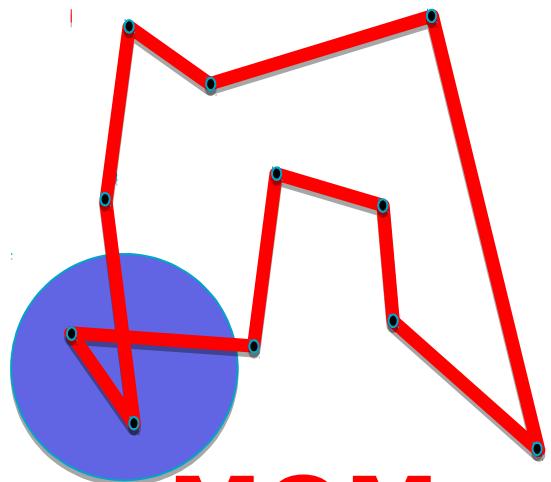
# TSP example :

- Problem:
  - Given n cities
  - Find a complete tour with minimal length
- Encoding?



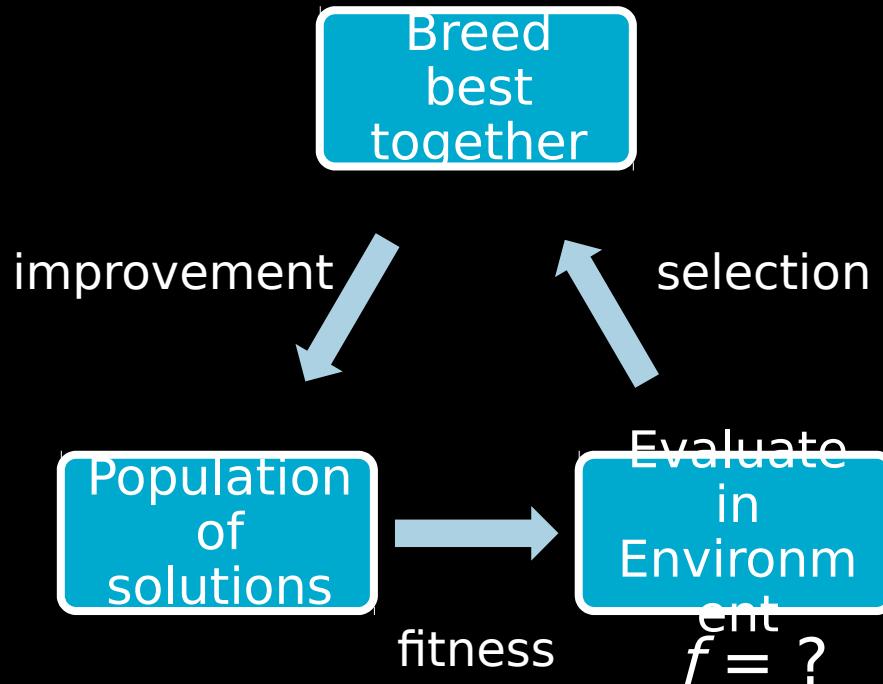


# The EC Metaphor (2/2)



# Evolutionary algorithms 101

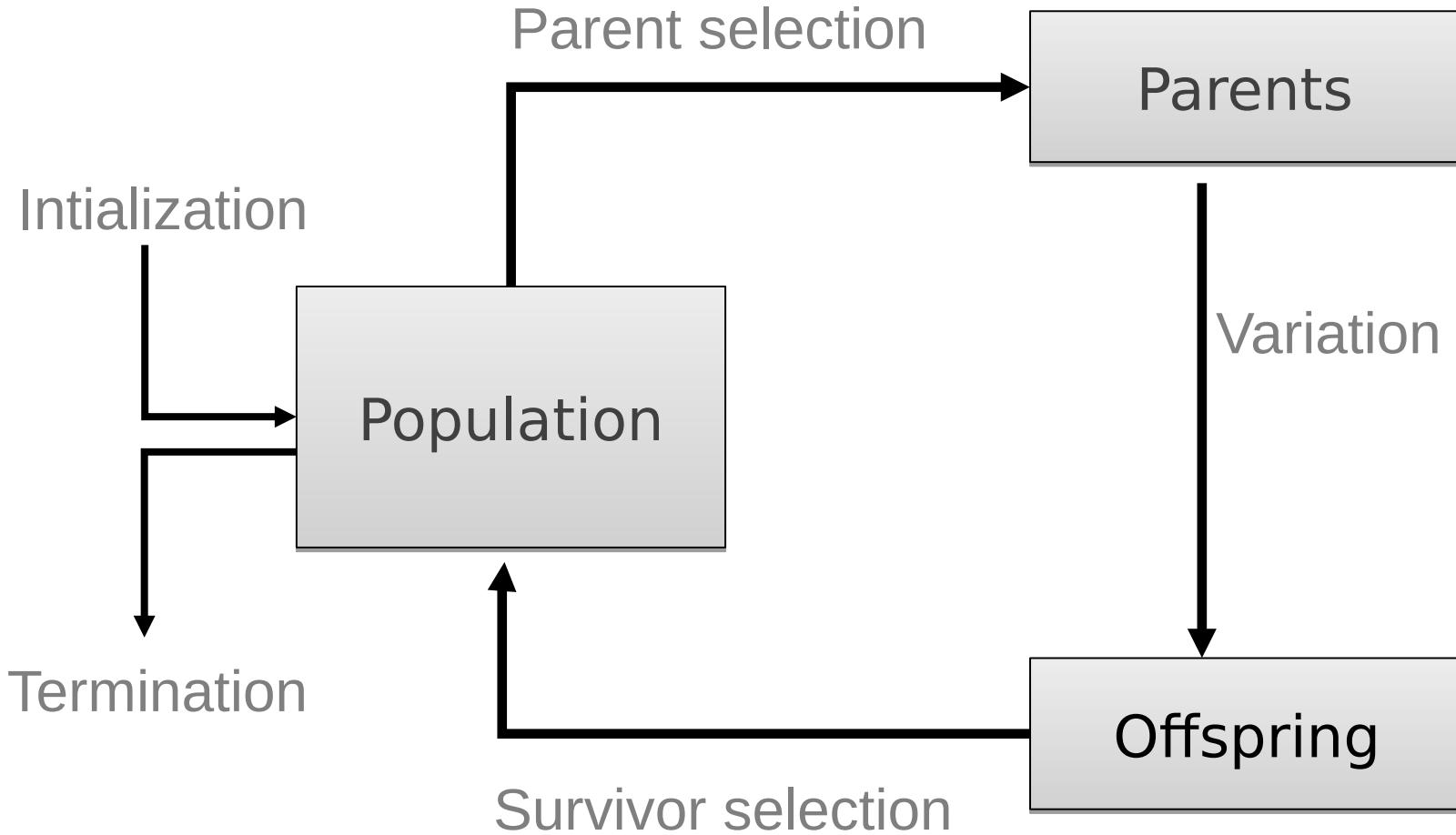
# Evolutionary algorithms 101



# What is an Evolutionary Algorithm?

- Scheme of an EA
- Main EA components:
  - Representation / evaluation / population
  - Parent selection / survivor selection
  - Recombination / mutation
- Typical EA behaviour

# Scheme of an EA: General scheme of EAs



# Scheme of an EA: EA scheme in pseudo-code

BEGIN

*INITIALISE* population with random candidate solutions;  
  *EVALUATE* each candidate;  
  REPEAT UNTIL ( *TERMINATION CONDITION* is satisfied ) DO  
    1 *SELECT* parents;  
    2 *RECOMBINE* pairs of parents;  
    3 *MUTATE* the resulting offspring;  
    4 *EVALUATE* new candidates;  
    5 *SELECT* individuals for the next generation;  
  OD  
END

# Scheme of an EA: Common model of evolutionary processes

- Population of individuals
- Individuals have a fitness
- Variation operators: crossover, mutation
- Selection towards higher fitness
  - “survival of the fittest” and
  - “mating of the fittest”

# Scheme of an EA: Two pillars of evolution

There are two competing forces

**Increasing** population **diversity**  
by genetic operators

- mutation
- recombination

Push towards **novelty**

**Decreasing** population **diversity** by  
selection

- of parents
- of survivors

Push towards **quality**

# Natural evolution vs. EC

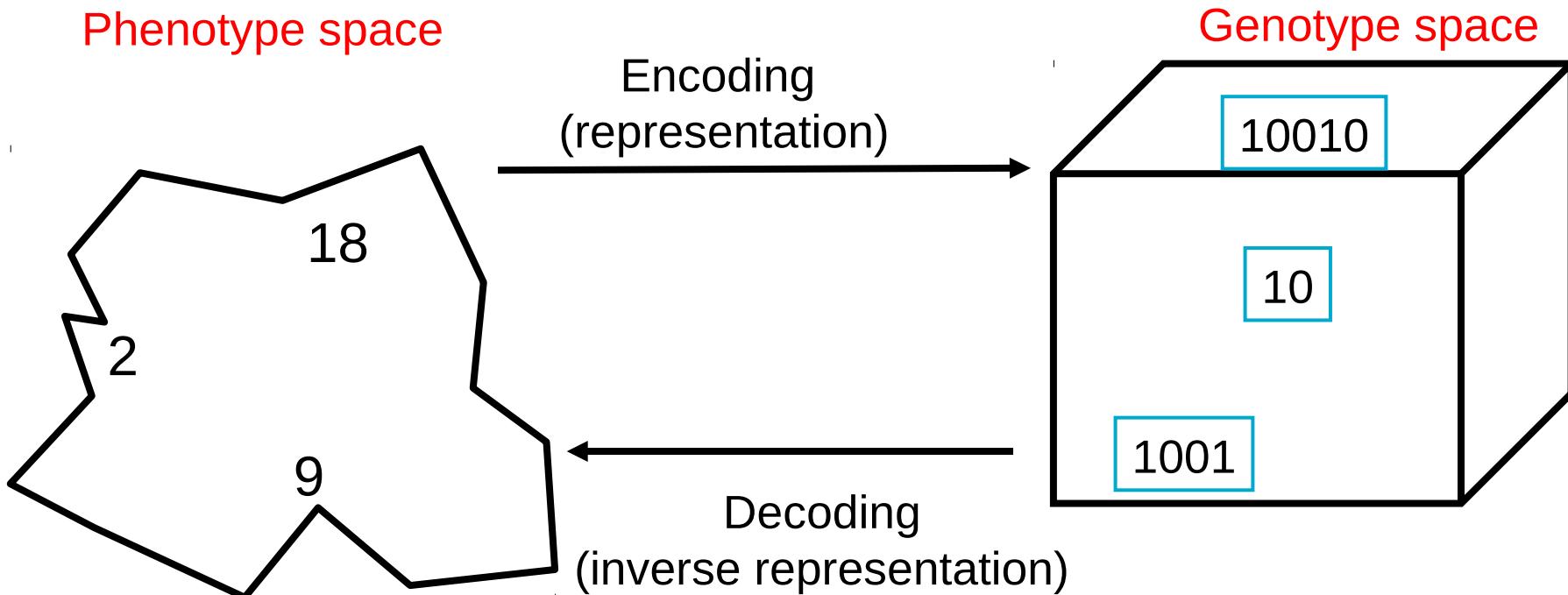
- **Fitness**
  - Natural: observed, 2ndary, “in the eye of the observer” (many offspring  $\square$  fit)
  - EC: predefined, primary (fit  $\square$  many offspring)
- **Execution**
  - Natural: decentralized, asynchronous birth & death, no formal generations
  - EC: centralized, synchronized birth & death, generation cycles
- **Genotype-phenotype mapping**
  - Natural: extremely complex, influenced by environment
  - EC: simple mathematical mapping or (parameterized) procedure
- **Population**
  - Natural: divided in species, spatial niches, population size can vary
  - EC: panmictic (all individuals are potential partners), population size is constant

# Main EA components: Representation (1/2)

- Role: provides code for candidate solutions that can be manipulated by variation operators
- Leads to two levels of existence
  - **phenotype: object** in original problem context
  - **genotype: code** to denote that object, the inside (chromosome, “digital DNA”)
- Implies two mappings:
  - Encoding : phenotype=> genotype (not necessarily one to one)
  - Decoding : genotype=> phenotype (must be one to one)

# Main EA components: Representation (2/2)

Example: represent integer values by their binary code



In order to find the global optimum, every feasible solution must be represented in genotype space

# Main EA components: Evaluation (fitness) function

- Role:
  - Represents the task to solve, the requirements to adapt to (can be seen as “the environment”)
  - Enables selection (provides basis for comparison)
  - e.g., some phenotypic traits are advantageous, desirable, e.g. big ears cool better, these traits are rewarded by more offspring that will expectedly carry the same trait
- A.k.a. *quality* function or *objective* function
- Assigns a single real-valued fitness to each phenotype which forms the basis for selection
  - So the more discrimination (different values) the better
- Typically we talk about fitness being maximised
  - Some problems may be best posed as minimisation problems, but conversion is trivial

# Main EA components: Population (1/2)

- Role: holds the candidate solutions of the problem as individuals (genotypes)
- Formally, a population is a multiset of individuals, i.e. repetitions are possible
- Population is the basic unit of evolution, i.e., the population is evolving, not the individuals
- Selection operators act on population level
- Variation operators act on individual level

# Main EA components: Population (2/2)

- Some sophisticated EAs also assert a spatial structure on the population e.g., a grid
- Selection operators usually take whole population into account i.e., reproductive probabilities are *relative* to *current* generation
- **Diversity** of a population refers to the number of different fitnesses / phenotypes / genotypes present (note: not the same thing)

# Main EA components: Selection mechanism (1/3)

Role:

- Identifies individuals
  - to become parents
  - to survive
- Pushes population towards higher fitness
- Usually probabilistic
  - high quality solutions more likely to be selected than low quality
  - but not guaranteed
  - even worst in current population usually has non-zero probability of being selected
- This *stochastic* nature can aid escape from local optima

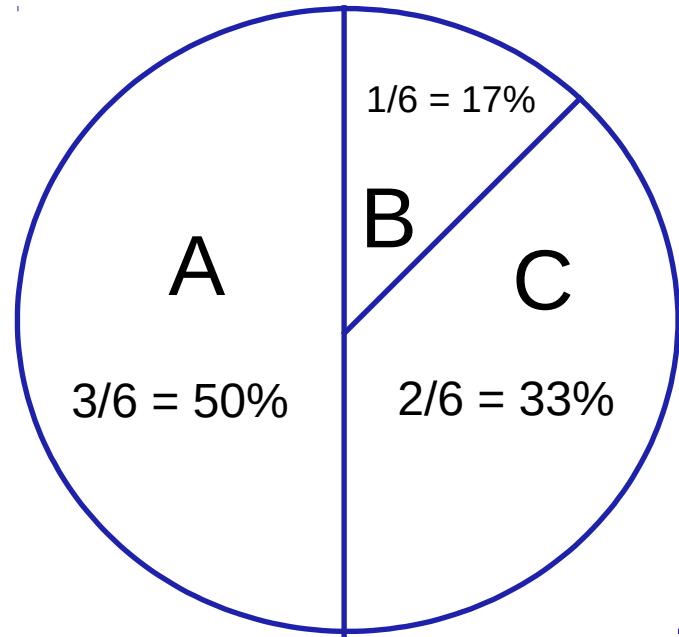
# Main EA components: Selection mechanism (2/3)

Example: roulette wheel selection

$$\text{fitness}(A) = 3$$

$$\text{fitness}(B) = 1$$

$$\text{fitness}(C) = 2$$



In principle, any selection mechanism can be used for parent selection as well as for survivor selection

# Main EA components: Selection mechanism (3/3)

- Survivor selection A.k.a. ***replacement***
- Most EAs use fixed population size so need a way of going from (parents + offspring) to next generation
- Often deterministic (while parent selection is usually stochastic)
  - Fitness based : e.g., rank parents + offspring and take best
  - Age based: make as many offspring as parents and delete all parents
- Sometimes a combination of stochastic and deterministic

# Main EA components: Variation operators

- Role: to generate new candidate solutions
- Usually divided into two types according to their **arity** (number of inputs):
  - Arity 1 : mutation operators
  - Arity  $>1$  : recombination operators
  - Arity = 2 typically called **crossover**
  - Arity  $> 2$  is formally possible, seldom used in EC
- There has been much debate about relative importance of recombination and mutation
  - Nowadays most EAs use both
  - Variation operators must match the given representation

# Main EA components:

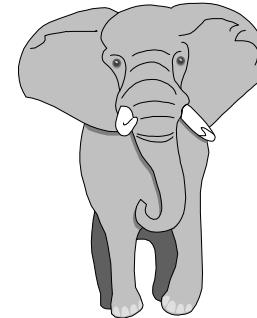
## Mutation (1/2)

- Role: causes small, random variance
- Acts on one genotype and delivers another
- Element of randomness is essential and differentiates it from other heuristic operators
- Importance ascribed depends on representation and historical dialect:
  - Binary GAs – background operator responsible for preserving and introducing diversity
  - Evolutionary Programming for continuous variables – only search operator
  - Genetic Programming – hardly used

# Main EA components: Mutation (2/2)

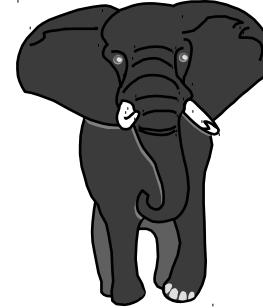
**before**

```
1 1 1 1 1 1 1
```



**after**

```
1 1 1 0 1 1 1
```

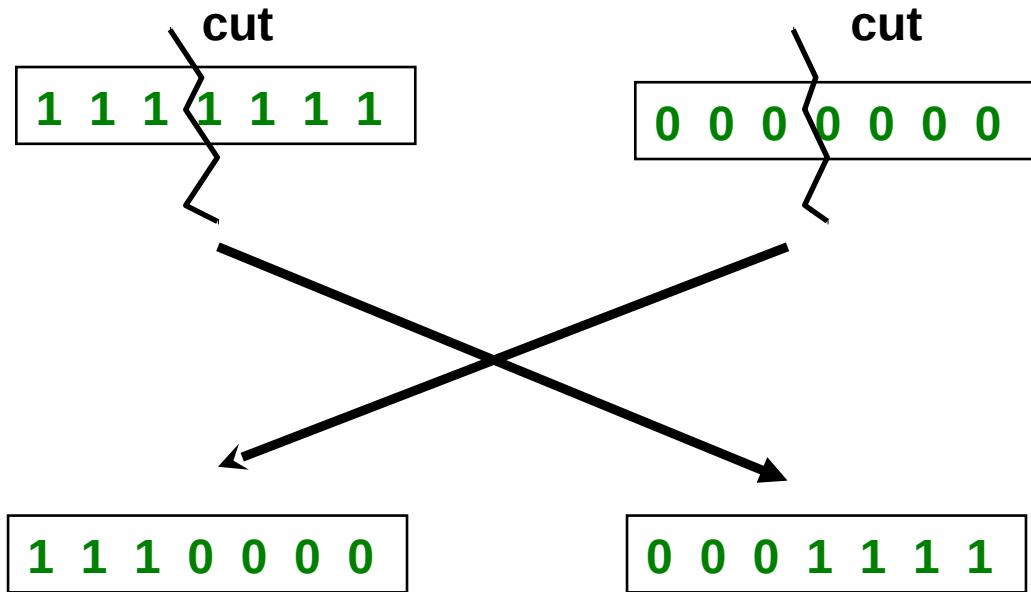


# Main EA components: Recombination (1/2)

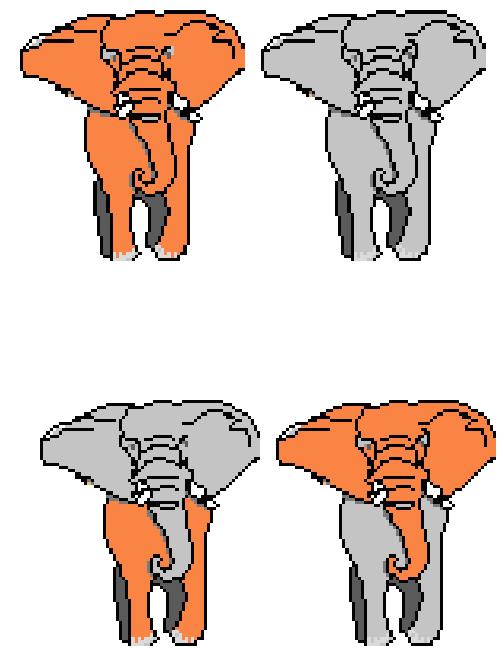
- Role: merges information from parents into offspring
- Choice of what information to merge is stochastic
- Most offspring may be worse, or the same as the parents
- Hope is that some are better by combining elements of genotypes that lead to good traits
- Principle has been used for millennia by breeders of plants and livestock

# Main EA components: Recombination (2/2)

## Parents



## Offspring



# Main EA components: Initialisation / Termination

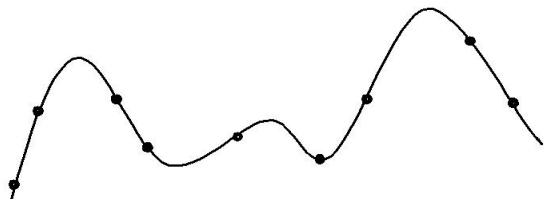
- Initialisation usually done at random,
  - Need to ensure even spread and mixture of possible allele values
  - Can include existing solutions, or use problem-specific heuristics, to “seed” the population
- Termination condition checked every generation
  - Reaching some (known/hoped for) fitness
  - Reaching some maximum allowed number of generations
  - Reaching some minimum level of diversity
  - Reaching some specified number of generations without fitness improvement

# Main EA components: What are the different types of EAs

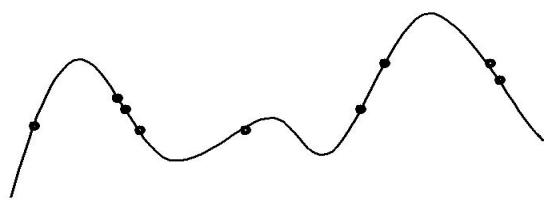
- Historically different flavours of EAs have been associated with different data types to represent solutions
  - Binary strings : Genetic Algorithms
  - Real-valued vectors : Evolution Strategies
  - Finite state Machines: Evolutionary Programming
  - Trees: Genetic Programming
- These differences are largely irrelevant, best strategy
  - choose representation to suit problem
  - choose variation operators to suit representation
- Selection operators only use fitness and so are independent of representation

# Typical EA behaviour: Stages

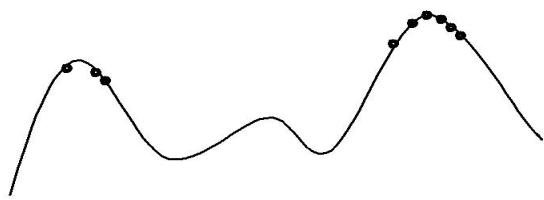
Stages in optimising on a 1-dimensional fitness landscape



Early stage:  
quasi-random population distribution



Mid-stage:  
population arranged around/on hills



Late stage:  
population concentrated on high hills

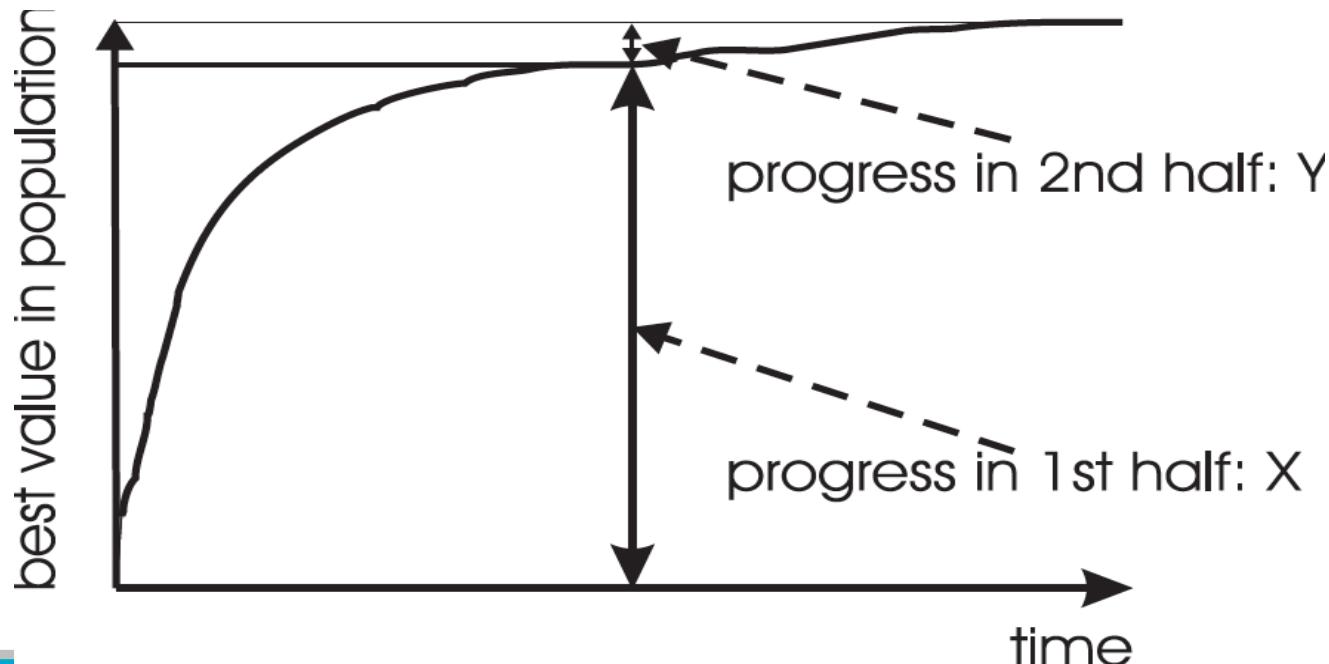
# Typical EA behaviour: Typical run: progression of fitness



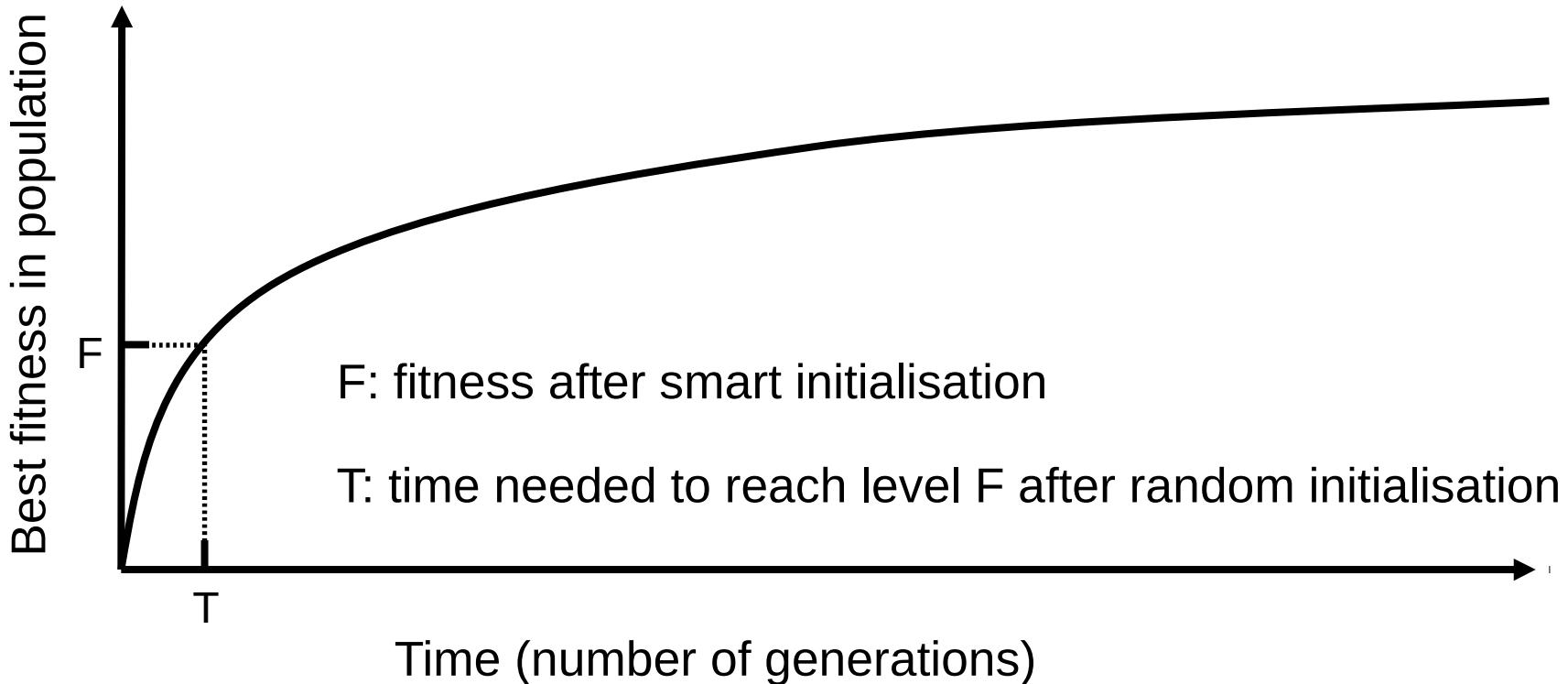
Typical run of an EA shows so-called “anytime behavior”

# Typical EA behaviour: Are long runs beneficial?

- Answer:
  - It depends on how much you want the last bit of progress
  - May be better to do more short runs



# Typical EA behaviour: Is it worth expending effort on smart initialisation?

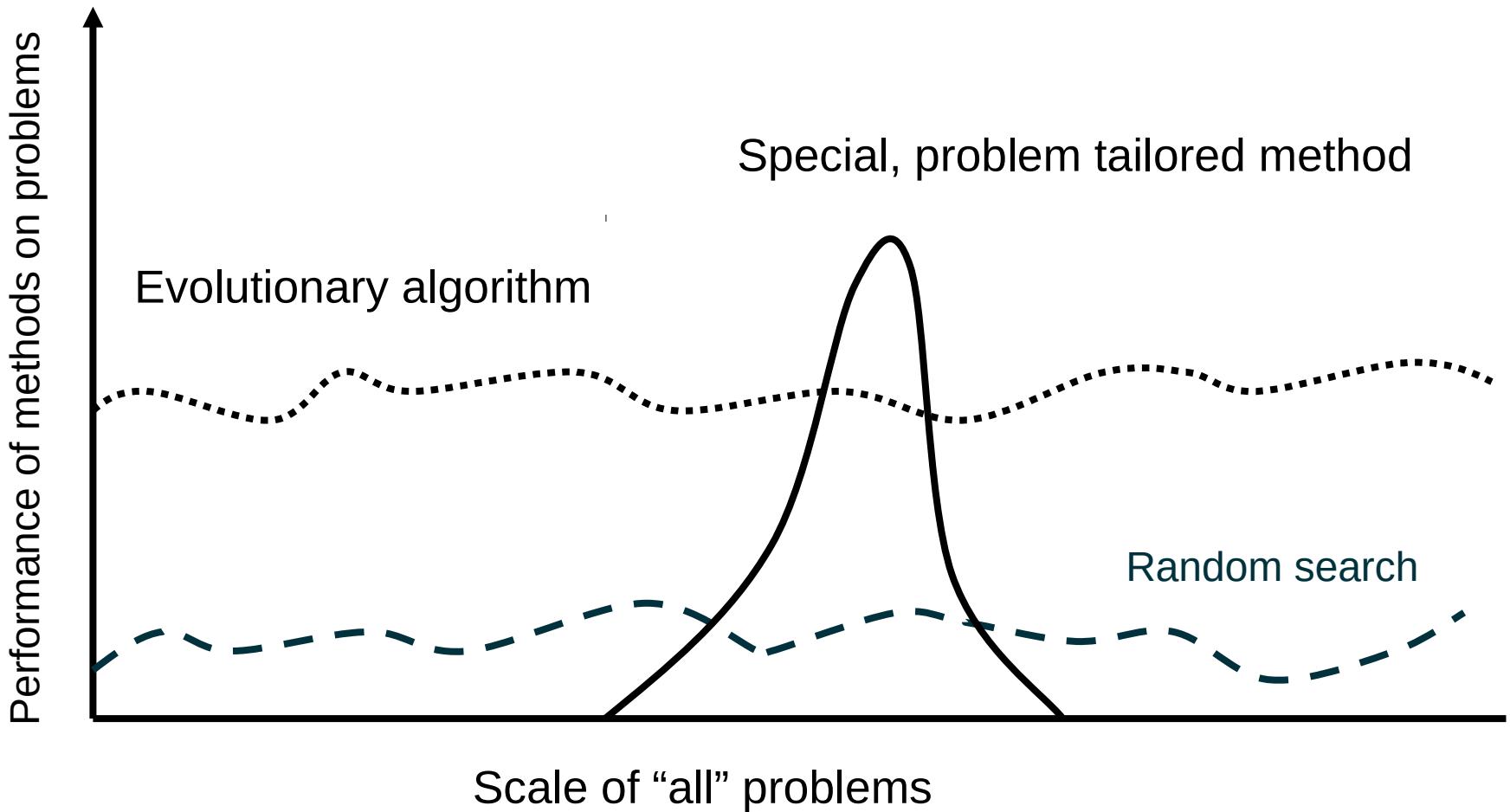


- Answer: it depends.
  - Possibly good, if good solutions/methods exist.
  - Care is needed, we don't want to bias/constrain!

# Typical EA behaviour: Evolutionary Algorithms in context

- There are many views on the use of EAs as robust problem solving tools
- For most problems a problem-specific tool may:
  - perform better than a generic search algorithm on most instances,
  - have limited utility,
  - not do well on all instances
- Goal is to provide robust tools that provide:
  - evenly good performance
  - over a range of problems and instances

# Typical EA behaviour: EAs as problem solvers: Goldberg view (1989)



# Important points:

- There are different ways to visualise an EA scheme
- Variation operators push towards novelty and selection operators push towards quality
- Selection operators are independent from problem but variation operators need to suit need to match the representation
- Selection operators act on the population level and variation operators on the individual level

# Representations and search

# Representation, Mutation, and Recombination

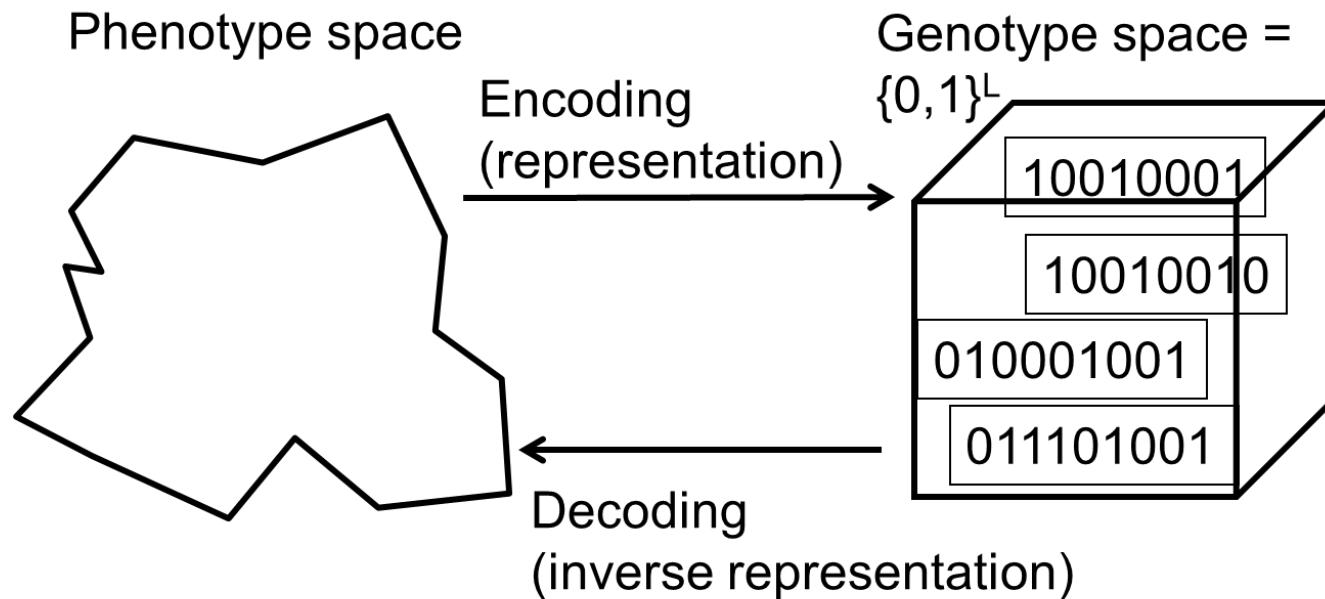
- Role of representation and variation operators
- Most common representation of genomes:
  - Binary
  - Integer
  - Real-Valued or Floating-Point
  - Permutation
  - Tree

# Role of representation and variation operators

- First stage of building an EA and most difficult one: choose *right* representation for the problem
- Variation operators: mutation and crossover
- Type of variation operators needed depends on chosen representation

# Binary Representation

- One of the earliest representations
- Genotype consists of a string of binary digits



# Binary Representation: Mutation

- Alter each gene independently with a probability  $p_m$
- $p_m$  is called the mutation rate
  - Typically between 1/pop\_size and 1/ chromosome\_length

parent      

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

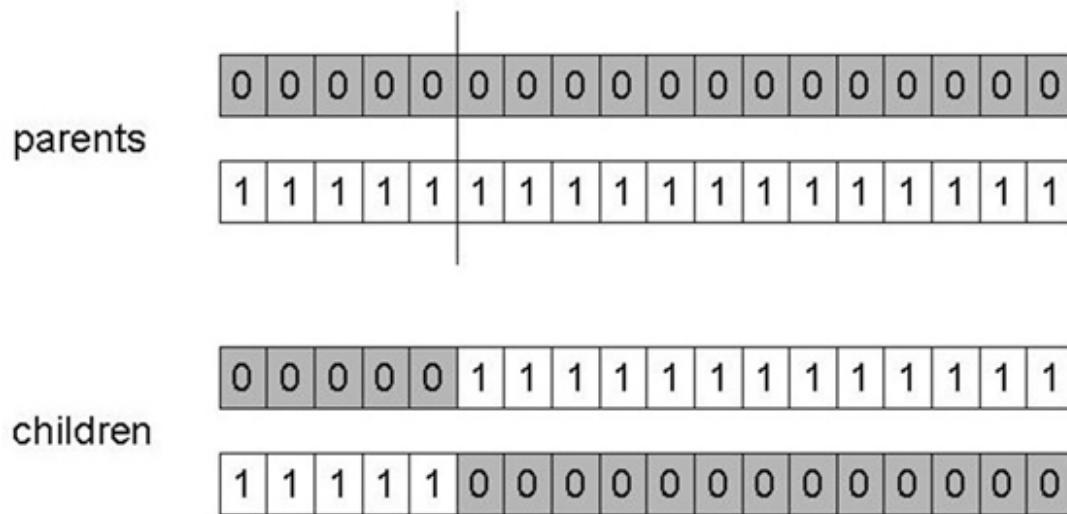
child      

0	1	0	0	1	0	1	1	0	0	0	1	0	1	1	0	0	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

- Mutation can cause variable effect (use gray coding)

# Binary Representation: 1-point crossover

- Choose a random point on the two parents
- Split parents at this crossover point
- Create children by exchanging tails
- $P_c$  typically in range (0.6, 0.9)

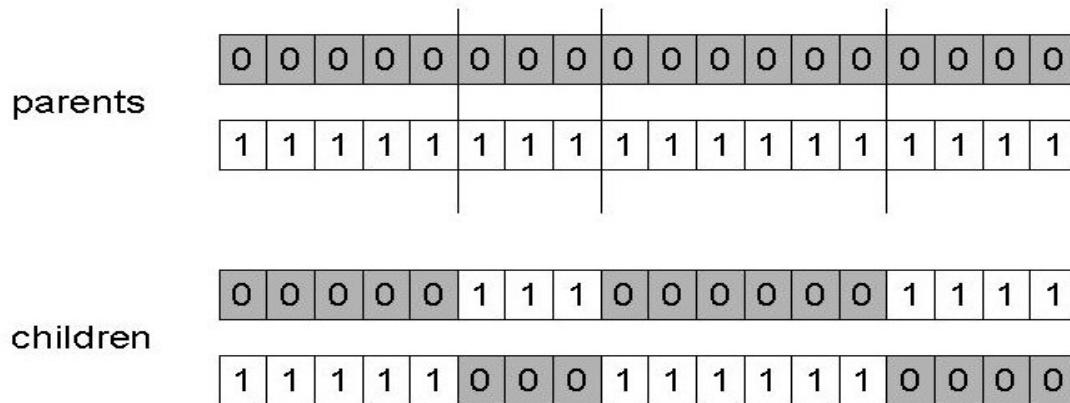


# Binary Representation: Alternative Crossover Operators

- Why do we need other crossover(s)?
- Performance with 1-point crossover depends on the order that variables occur in the representation
  - More likely to keep together genes that are near each other
  - Can never keep together genes from opposite ends of string
  - This is known as Positional Bias
  - Can be exploited if we know about the structure of our problem, but this is not usually the case

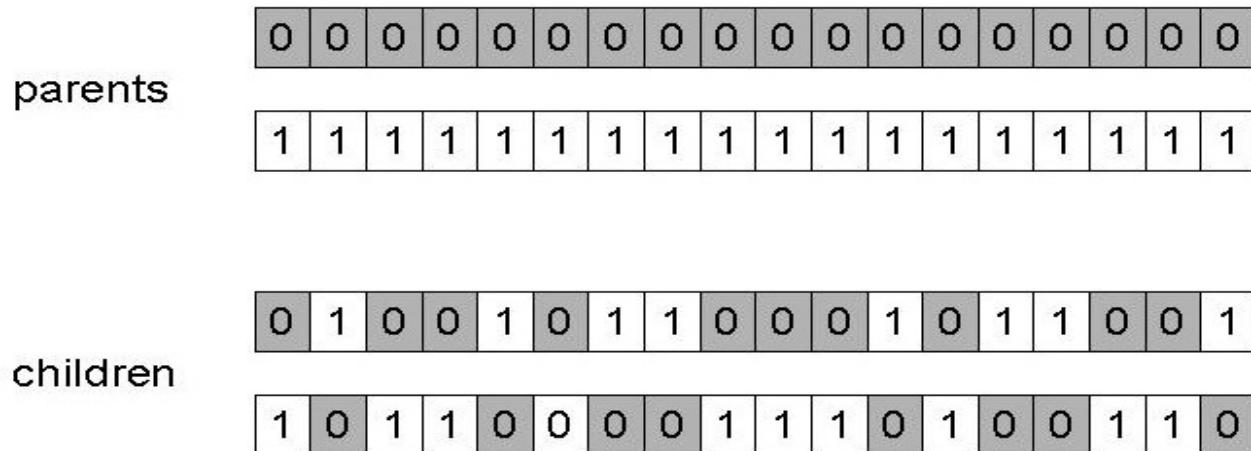
# Binary Representation: n-point crossover

- Choose n random crossover points
  - Split along those points
  - Glue parts, alternating between parents
  - Generalisation of 1-point (still some positional bias)



# Binary Representation: Uniform crossover

- Assign 'heads' to one parent, 'tails' to the other
- Flip a coin for each gene of the first child
- Make an inverse copy of the gene for the second child
- Inheritance is independent of position



# Binary Representation: Crossover OR mutation? (1/3)

- Decade long debate: which one is better / necessary / main-background
- Answer (at least, rather wide agreement):
  - it depends on the problem, but
  - in general, it is good to have both
  - both have another role
  - mutation-only-EA is possible, xover-only-EA would not work

# Binary Representation: Crossover OR mutation? (2/3)

**Exploration:** Discovering promising areas in the search space, i.e. gaining information on the problem

**Exploitation:** Optimising within a promising area, i.e. using information

There is co-operation AND competition between them

- Crossover is explorative, it makes a *big* jump to an area somewhere “in between” two (parent) areas
- Mutation is exploitative, it creates random *small* diversions, thereby staying near (in the area of ) the parent

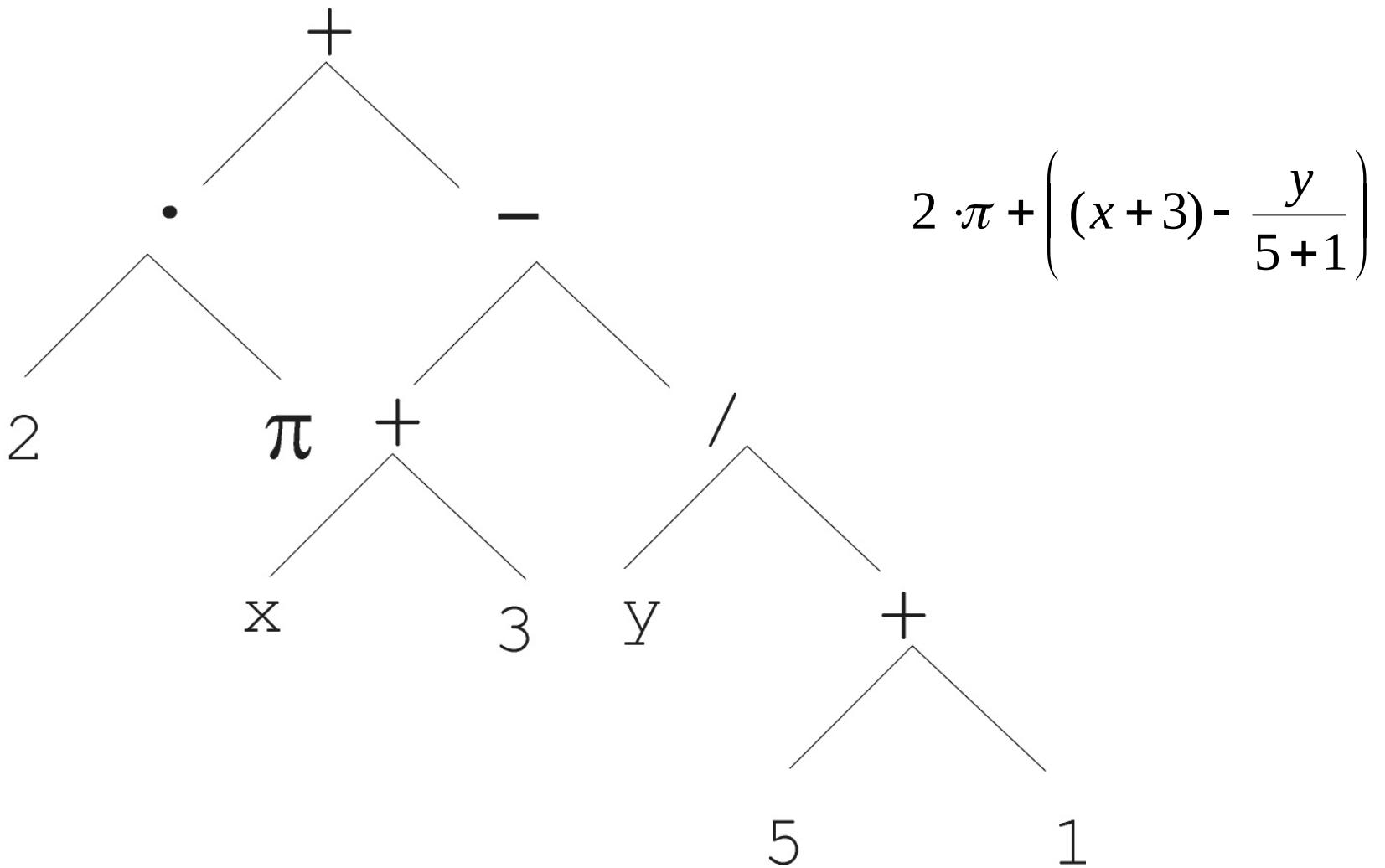
# Binary Representation: Crossover OR mutation? (3/3)

- Only crossover can combine information from two parents
- Only mutation can introduce new information (alleles)
- Crossover does not change the allele frequencies of the population (thought experiment: 50% 0's on first bit in the population, ?% after performing n crossovers)
- To hit the optimum you often need a 'lucky' mutation

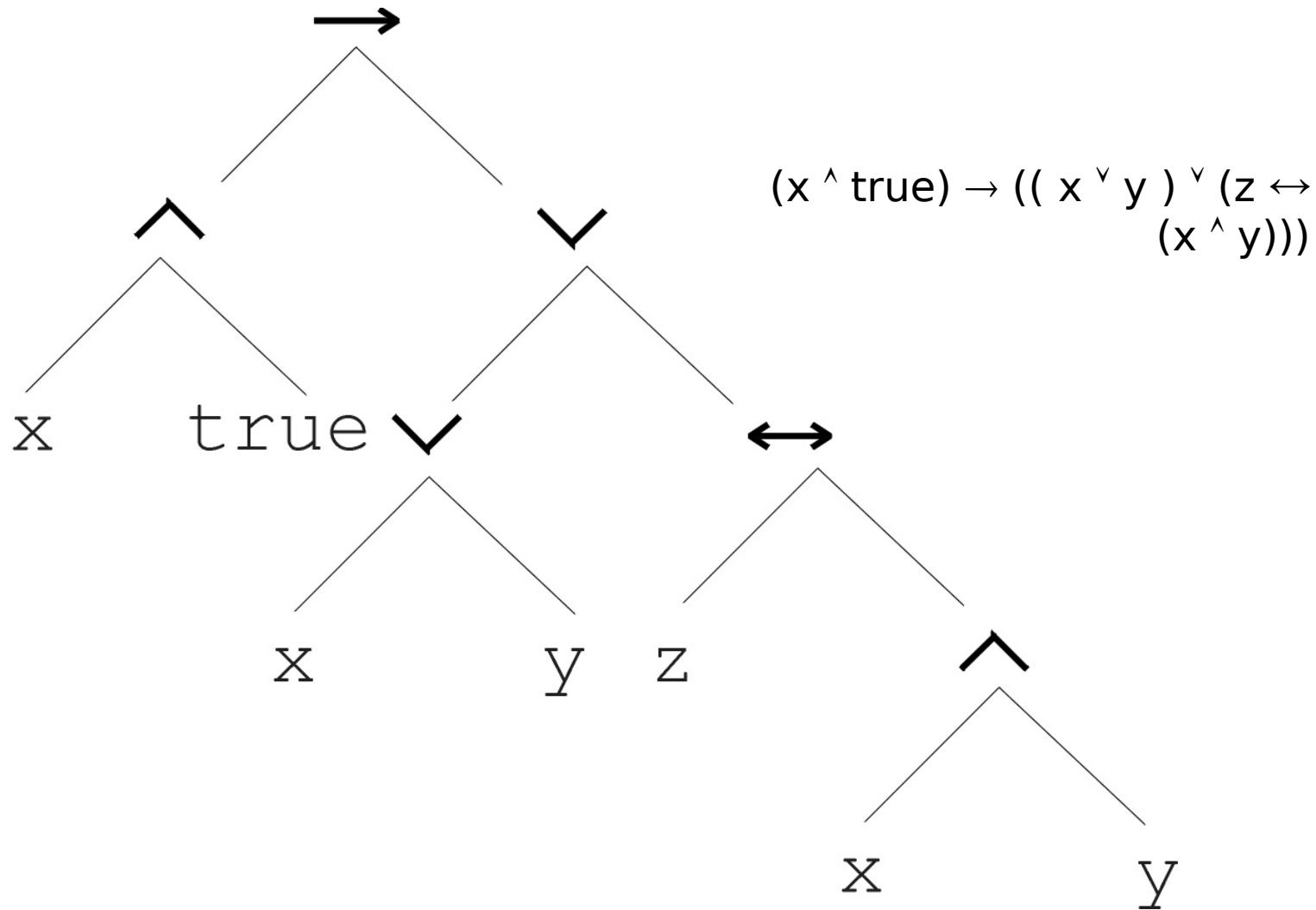
# Integer & real representations

- Nowadays it is generally accepted that it is better to encode numerical variables directly (integers, floating point variables)
- Some problems naturally have integer variables, e.g. image processing parameters
- Others take categorical values from a fixed set e.g. {blue, green, yellow, pink}
- Many real-world problems are real valued
- N-point / uniform crossover operators work
- Mutation must search a range, rather than bit-flipping (integer selection, upper-lower, centre-spread, etc...)
- Same recombination as for binary representation

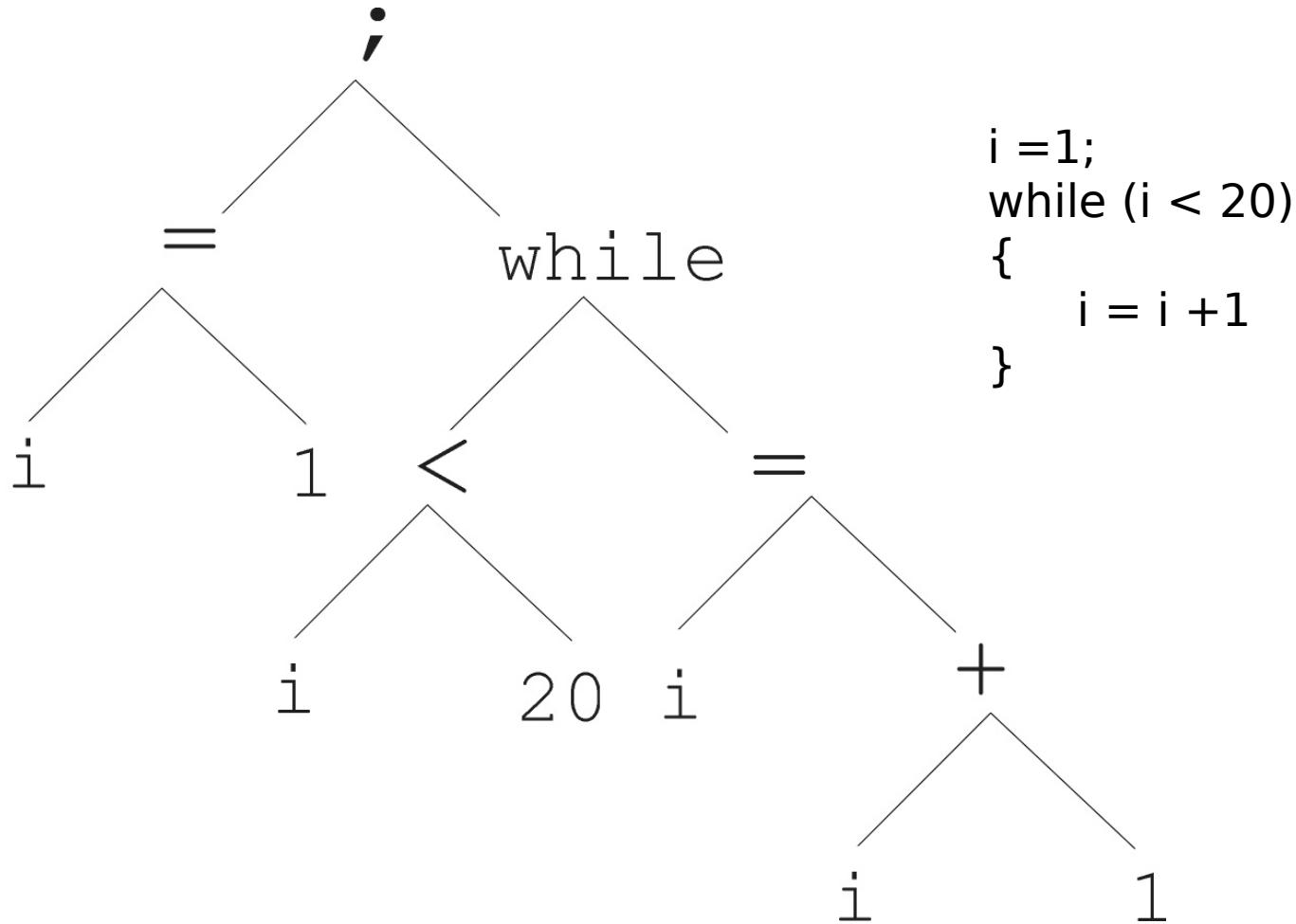
# Tree Representation



# Tree Representation



# Tree Representation



# Tree Representation

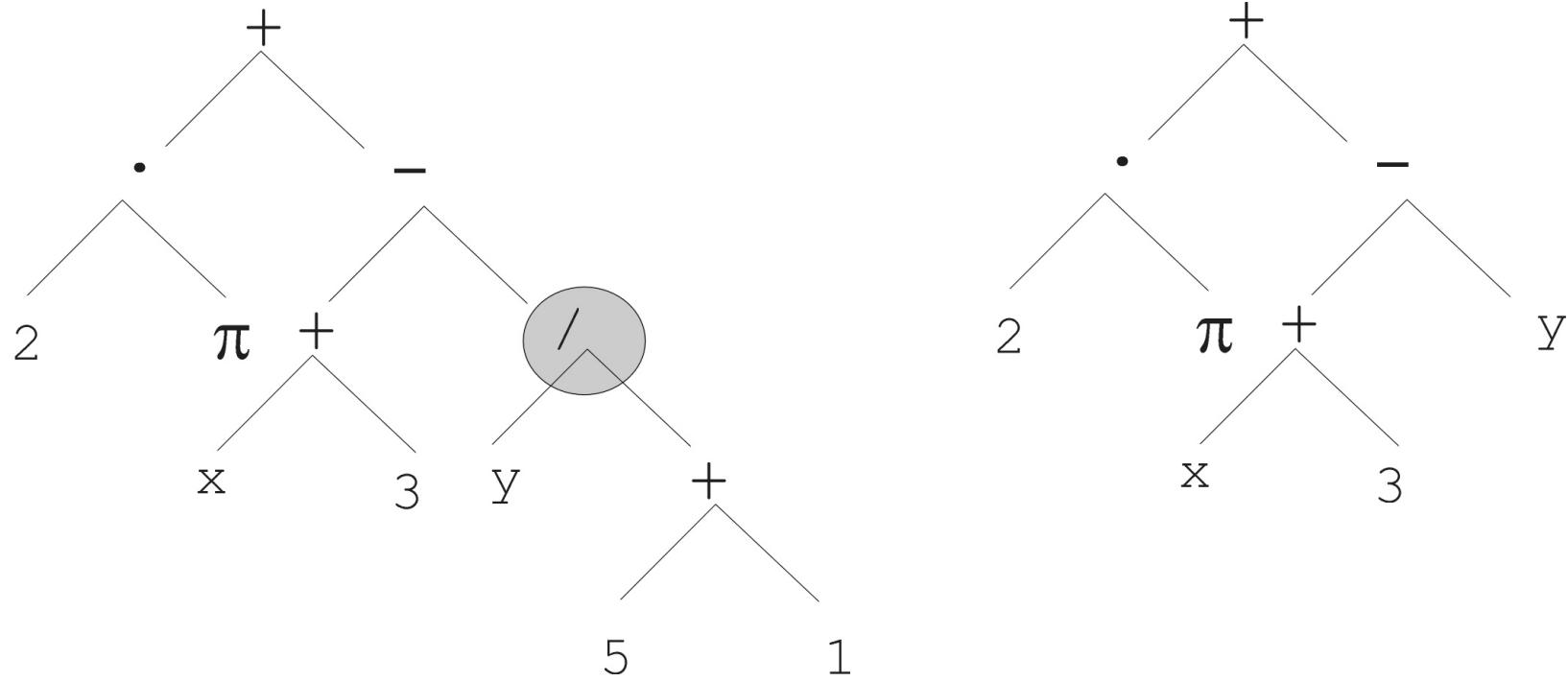
- In GA, ES, EP chromosomes are linear structures (bit strings, integer string, real-valued vectors, permutations)
- Tree shaped chromosomes are non-linear structures
- In GA, ES, EP the size of the chromosomes is fixed
- Trees in GP may vary in depth and width

# Tree Representation

- Symbolic expressions can be defined by
  - Terminal set  $T$
  - Function set  $F$  (with the arities of function symbols)
- Adopting the following general recursive definition:
  - Every  $t \in T$  is a correct expression
  - $f(e_1, \dots, e_n)$  is a correct expression if  $f \in F$ ,  $\text{arity}(f)=n$  and  $e_1, \dots, e_n$  are correct expressions
  - There are no other forms of correct expressions

# Tree Representation: Mutation (1/2)

- Most common mutation: replace randomly chosen subtree by randomly generated tree



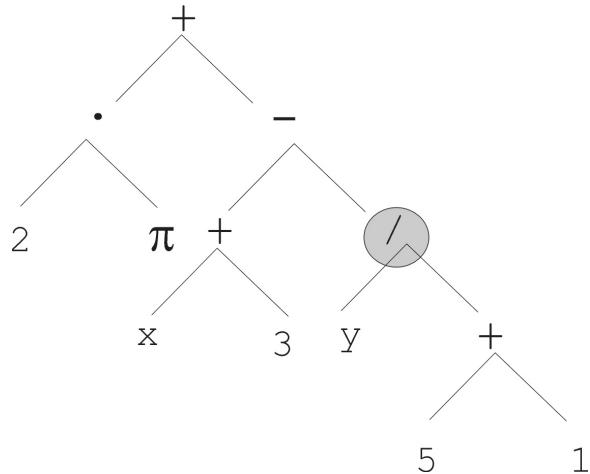
# Tree Representation: Mutation (2/2)

- Mutation has two parameters:
  - Probability  $p_m$  to choose mutation
  - Probability to chose an internal point as the root of the subtree to be replaced
- The size of the child can exceed the size of the parent

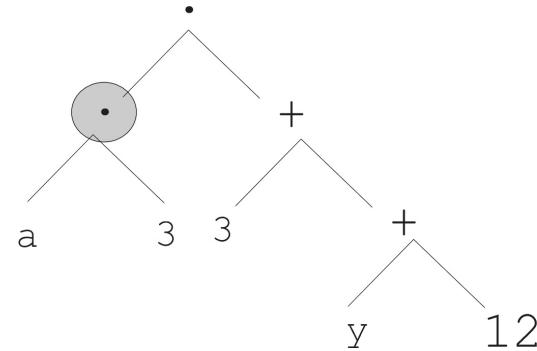
# Tree Representation: Recombination (1/2)

- Most common recombination: exchange two randomly chosen subtrees among the parents
- Recombination has two parameters:
  - Probability  $p_c$  to choose recombination
  - Probability to chose an internal point within each parent as crossover point
- The size of offspring can exceed that of the parents

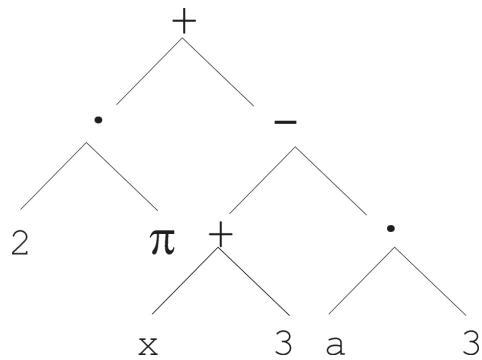
# Tree Representation: Recombination (2/2)



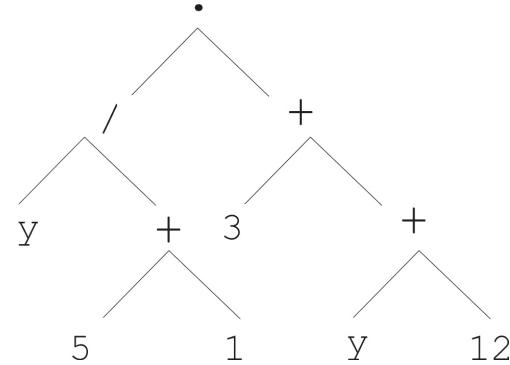
Parent 1



Parent 2



Child 1

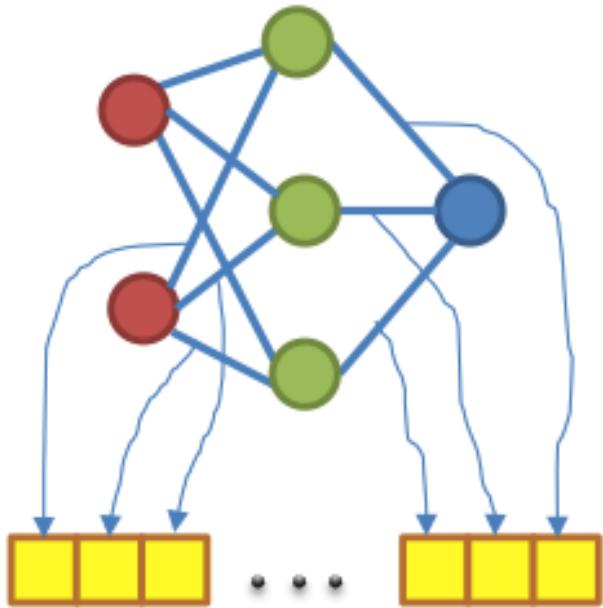


Child 2

# Important points

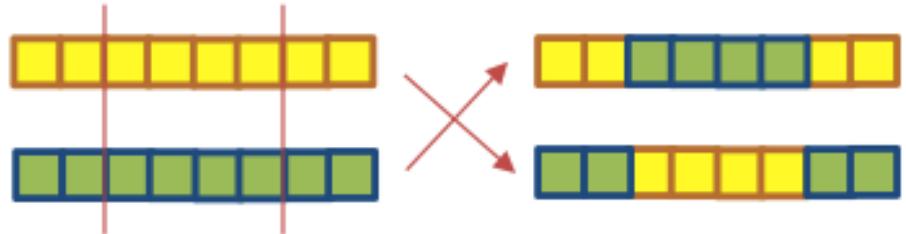
- Representation is key when designing an EA.
- A handful of data structures is enough to represent many (all?) problems.
- For any given problem there can be more suitable representations. Some are better than others.
- Self-adaptive mutation is a powerful mechanism. But remember to change the sigma first!

# Advanced encodings



Transforming neural network into linear chromosome

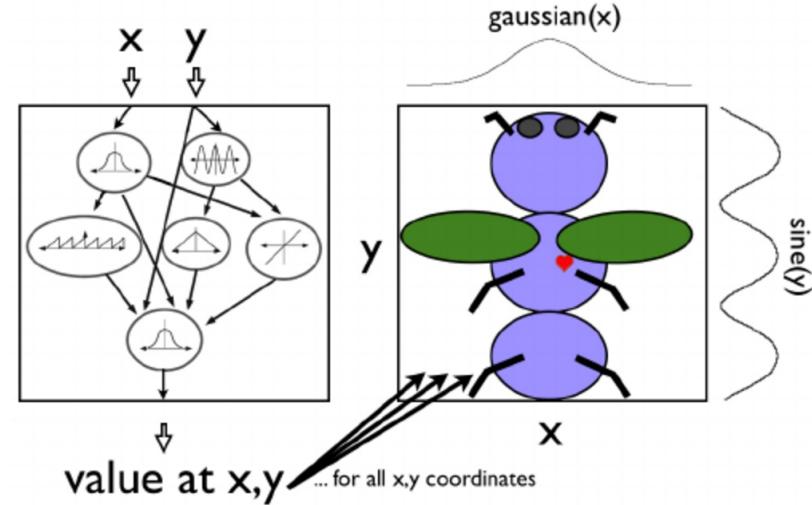
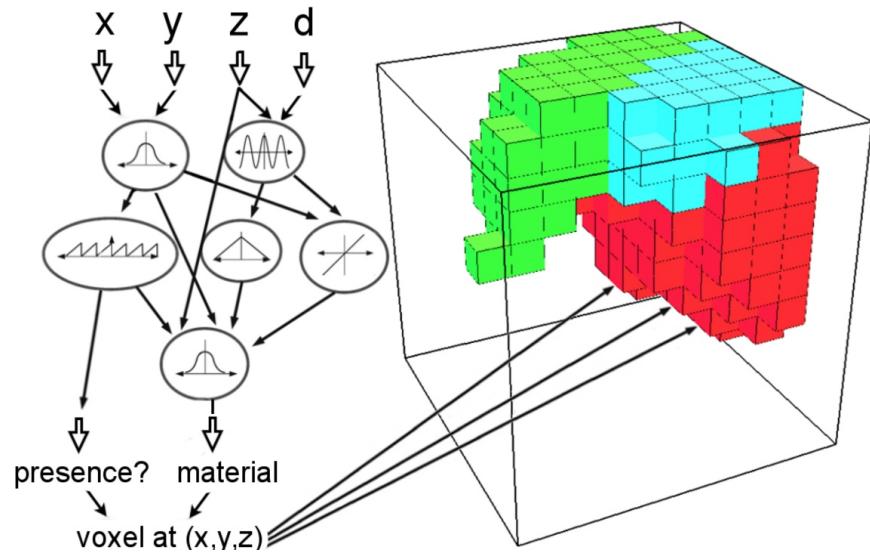
Crossover



Mutation

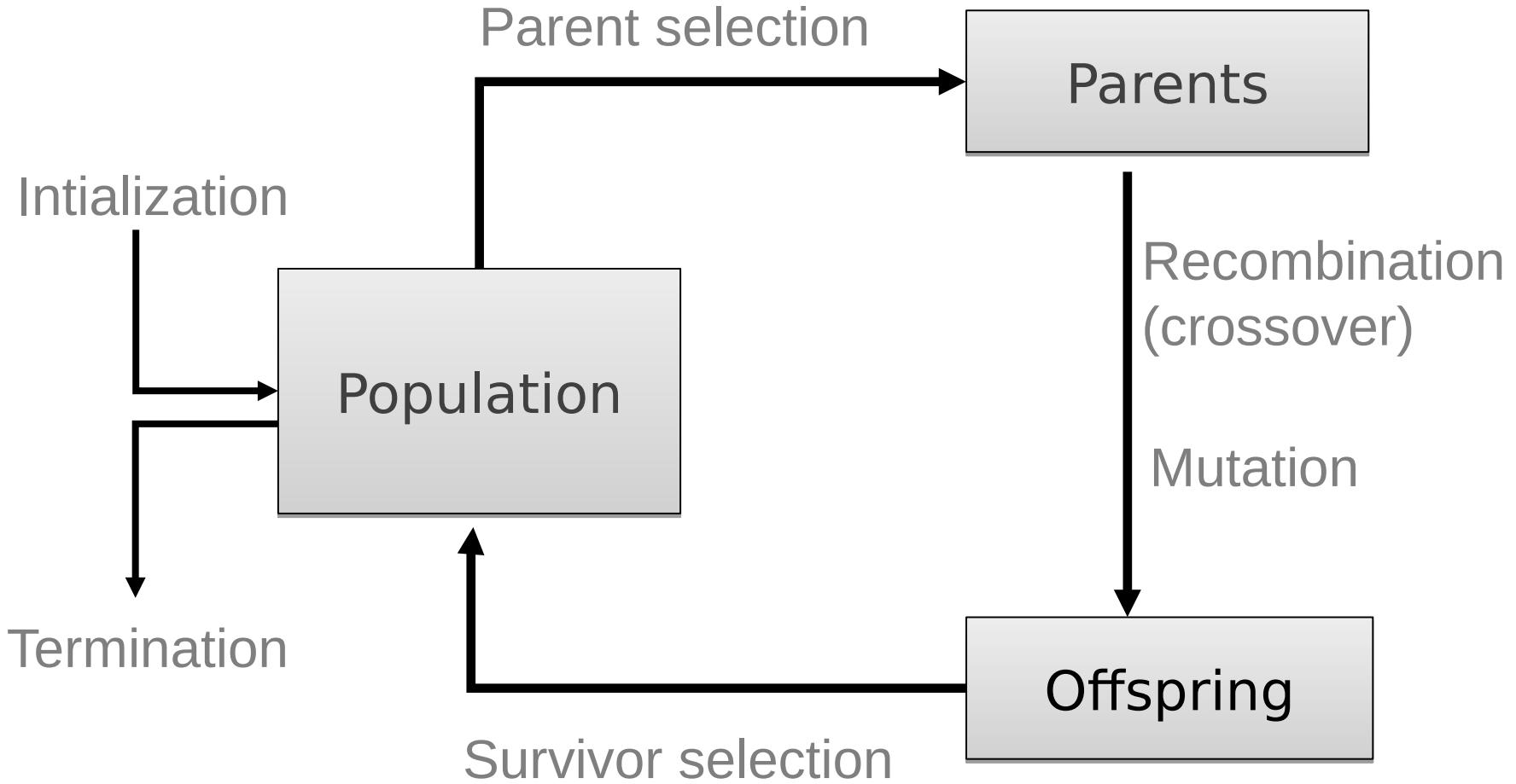


# Compositional pattern-producing networks



# **Fitness, selection, population management**

# Scheme of an EA: General scheme of EAs



# How to design a fitness function

- What might good fitness functions be for:
- **EASY:**
- Classification of a series of characters
- **HARDER:**
- Exploration of an environment? (reward a high level behaviour, not easily defined)
- Stable hover of a helicopter (compound weighting)
- **IMPOSSIBLE (?)**:
- Human behaviour?

# Population Management Models: Introduction

- Two different population management models exist:
  - Generational model
    - each individual survives for exactly one generation
    - the entire set of parents is replaced by the offspring
  - Steady-state model
    - a few offspring are generated per generation
    - a few members of the population are replaced
- Generation Gap
  - The proportion of the population replaced

# Population Management Models: Fitness based competition

- Selection can occur in two places:
  - Selection from current generation to take part in mating (**parent selection**)
  - Selection from parents + offspring to go into next generation (**survivor selection**)
- Selection operators work on whole individuals
  - i.e. they are representation-independent !
- Distinction between selection
  - Operators: define selection probabilities
  - Algorithms: define how probabilities are implemented

# Parent Selection: Fitness-Proportionate Selection

- Probability for individual  $i$  to be selected for mating in a population size  $\mu$  with FPS is

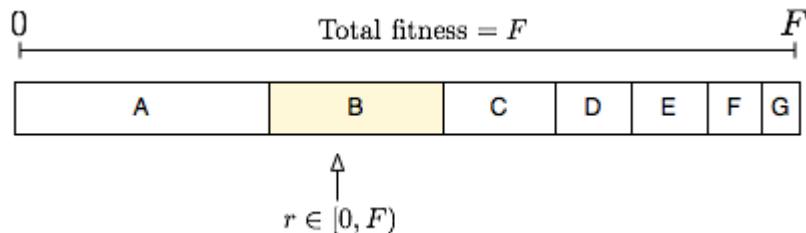
$$P_{FPS}(i) = f_i / \sum_{j=1}^{\mu} f_j$$

- Problems include

- One highly fit member can rapidly take over if rest of population is much less fit: **Premature Convergence**

- At the end of runs when fitnesses are similar, loss of selection pressure

- Highly susceptible to function transposition



FPS is also known as  
*roulette wheel selection*

# Parent Selection: Fitness-Proportionate Selection 2

individual	Fitness for f	Sel. Prob. for f	Fitness for f+10	Sel. Prob. for f + 10	Fitness for f+100	Sel. Prob. for f + 100
A	1	0.1	11	0.275	101	0.326
B	4	0.4	14	0.35	104	0.335
C	5	0.5	15	0.375	105	0.339
SUM	10	1	40	1	310	1

# Parent Selection: Rank-based Selection

- Attempt to remove problems of FPS by basing selection probabilities on *relative* rather than *absolute* fitness
- Rank population according to fitness and then base selection probabilities on rank
- In a population of size  $\mu$  the fittest has rank  $\mu-1$  and worst rank 0
- This imposes a sorting overhead on the algorithm, but this is usually negligible compared to the fitness evaluation time

# Rank-based Selection: Linear Ranking

$$P_{lin\text{-}rank}(i) = \frac{(2 - s)}{\mu} + \frac{2i(s - 1)}{\mu(\mu - 1)}$$

- Parameterised by factor  $s: 1 < s \leq 2$ 
  - measures advantage of best individual
- Simple 3 member example

Individual	Fitness	Rank	$P_{selFP}$	$P_{selLR} \ (s = 2)$	$P_{selLR} \ (s = 1.5)$
A	1	0	0.1	0	0.167
B	4	1	0.4	0.33	0.33
C	5	2	0.5	0.67	0.5
Sum	10		1.0	1.0	1.0

# Parent Selection: Tournament Selection (1/2)

- All methods above rely on global population statistics
  - Could be a bottleneck especially on structured or very large populations
  - Relies on external fitness function which might not exist: e.g. evolving game players
- Idea for a procedure using only local fitness information:
  - Pick  $k$  members at random then select the best of these
  - Repeat to select more individuals

# Parent Selection: Tournament Selection (2/2)

- Probability of selecting  $i$  will depend on:
  - Rank of  $i$
  - Size of sample  $k$ 
    - higher  $k$  increases selection pressure
  - Whether contestants are picked with replacement
    - Picking without replacement increases selection pressure
  - Whether fittest contestant always wins (deterministic) or this happens with probability  $p$

# Survivor Selection

- Select  $\mu$  individuals to form the next generation from  $\mu$  parents and  $\lambda$  offspring
- The parent selection mechanisms can also be used for selecting survivors
- Survivor selection can be divided into two approaches:
  - Age-based selection
    - Fitness is not taken into account
    - In SSGA can implement as “delete-random” (not recommended) or as first-in-first-out (a.k.a. delete-oldest)
  - Fitness-based selection

# Fitness-based selection (1/2)

- Elitism
  - Always keep at least one copy of the fittest solution so far
  - Widely used in both population models (GGA, SSGA)
  - Also with keep best N individuals or keep best  $x\%$
- Delete worst (a.k.a. GENITOR)
  - From Whitley's original Steady-State algorithm (he also used linear ranking for parent selection)
  - Rapid takeover: use with large populations or "no duplicates" policy
- Round-robin tournament
  - $P: \mu$  parents,  $O: \lambda$  offspring
  - Pairwise competitions in round-robin format:
    - Each solution  $x$  from  $P \cup O$  is evaluated against  $q$  other randomly chosen solutions
    - For each comparison, a "win" is assigned if  $x$  is better than its opponent
    - The  $\mu$  solutions with the greatest number of wins are retained to be parents of the next generation
  - Parameter  $q$  allows tuning selection pressure
  - Typically  $q = 10$

# Fitness-based replacement (2/2)

- $(\mu, \lambda)$ -selection a.k.a. “comma strategy”
  - based on the set of children only ( $\lambda > \mu$ )
  - choose best  $\mu$
- $(\mu + \lambda)$ -selection a.k.a. “plus strategy”
  - based on the set of parents and children
  - choose best  $\mu$
- Often  $(\mu, \lambda)$ -selection is preferred for:
  - Better in leaving local optima
  - Better in following moving optima
  - Using the + strategy bad  $\sigma$  values can survive in  $\langle x, \sigma \rangle$  too long if their host  $x$  is very fit
- $\lambda \approx 7 \cdot \mu$  is a traditionally good setting (decreasing over the last couple of years,  $\lambda \approx 3 \cdot \mu$  seems more popular lately)

# Selection Pressure

- Takeover time  $\tau^*$  is a measure to quantify the selection pressure
- The number of generations it takes until the application of selection completely fills the population with copies of the best individual
- Goldberg and Deb showed:

$$\tau^* = \frac{\ln \lambda}{\ln(\lambda / \mu)}$$

- Specific values for
  - an evolution strategy with  $\mu = 15$  and  $\lambda = 100$ :  $\tau^* \approx 2$
  - a GA with proportional selection:  $\tau^* = 460$

# Summary

**Evaluate population of individuals** vs. some fitness function.

**Select** some parents, copy them, mutate and crossover, put them in the population

*Children are a bit like parents, randomly better or worse but some will eventually be better!*

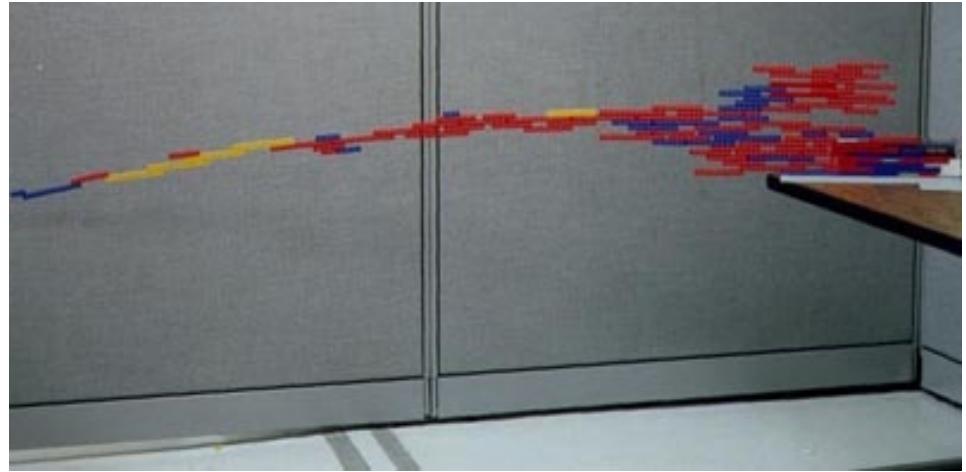
**Delete** unfit individuals

*Population gradually improves!*



# **Applications of Evolutionary Algorithms**

# Creative designs



# Interactive evolutionary art



