Zebra Puzzle
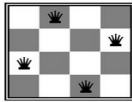# A search tree example

Pedagogical objectives

- Gentle introduction to

    – Constraint Satisfaction Problems
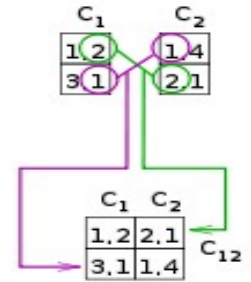
    – Search Trees

Puzzle description

There are five houses. The Englishman lives in the red house. The Spaniard owns the dog. Coffee is drunk in the green house. The Ukrainian drinks tea. The green house is immediately to the right of the ivory house. The Old Gold smoker owns snails. Kools are smoked in the yellow house. Milk is drunk in the middle house. The Norwegian lives in the first house. The man who smokes Chesterfields lives in the house next to the man with the fox. Kools are smoked in the house next to the house where the horse is kept. The Lucky Strike smoker drinks orange juice. The Japanese smokes Parliaments. The Norwegian lives next to the blue house.

Who drinks water? Who owns the zebra?

This is an old puzzle written at a time when people were still chain smoking in planes and restaurants...

QUT

Concept inventory for this puzzle

- Let's make sure we all interpret the puzzle the same way

- Each **house** is painted a different **color**, and their occupants are of different **nationalities**, own different **pets**, drink different **beverages** and smoke different brands of **cigarettes**.

- How many houses?

- How are the houses organized?

- What are the properties considered?

Further assumptions

- There are **5 houses** each with a **different colour**

- Their **owners**,

  - each with a unique **heritage/nationality**,

  - drinks a certain type of **beverage**,

  - smokes a certain brand of **cigarette**,

  - and keep a certain variety of **pet**.

- None of the owners have the same pet, smoke the same brand of cigarette or drink the same beverage.

QUT

- Five houses in a row
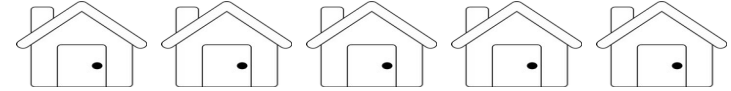
- Colour of the house ?
- Nationality ?
- Drink ?
- Pet ?
- Smoke ?

How to approach this puzzle?

- Make smart deductions?
  - *need to be inspired!*

- Enumerate candidate solutions (hypotheses), and see if any satisfies the constraints?
  - and make the computer do the work!

Ballpark figure of the computation time

- There are 5 house properties in total (colour, drink, nationality, pet, smoke)

- Let's call a *candidate solution* a 5-tuple of assignments "*property to house number*"

- How many different *candidate solutions* are there?

Counting the candidate solutions

- Each candidate solution can be viewed as a table

House	1	2	3	4	5
Color	Yellow	Blue	Red	Ivory	Green
Nationality	Norwegian	Ukrainian	Englishman	Spaniard	Japanese
Drink	**Water**	Tea	Milk	Orange juice	Coffee
Smoke	Kools	Chesterfield	Old Gold	Lucky Strike	Parliament
Pet	Fox	Horse	Snails	Dog	**Zebra**

- Each property assignment (red, green, ivory, yellow, blue) = (3,0,2,1,4) corresponds to a permutation of the first 5 integers

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- How many different ways can we assign a given property class (like colour) to the 5 houses?!

$$5, \; 5^2, \; 2^5, \; 5\,!$$

```
>>> import math
>>> math.factorial(5)**5
24883200000
>>>
```

- There are about 25 billions candidate solutions

- Assume that your computer can test 1,000,000 candidates per second. How long will it takes to enumerate all candidates?

Running time estimate of a full enumeration

```
File Edit View Search Terminal Help
>>>
>>> import math
>>> math.factorial(5)**5
24883200000
>>> num_cand = math.factorial(5)**5
>>> s = num_cand / 1000000
>>> s
24883.2
>>> h=s/3600
>>> h
6.912
>>>
```

Enumerating all the candidate solutions could take almost 7 hours!
In the first practical (Week 02), you will investigate how the search time can be dramatically sped up

QUT

Toy search problem on 3 variables

- Suppose that we have 3 variables A, B and C
- The domain of A is [false, true]
- The domain of B is [-1, 0, +1]
- The domain of C is [1, 2, 3, 4]

- Can you represent the collection of possible assignments with a tree?
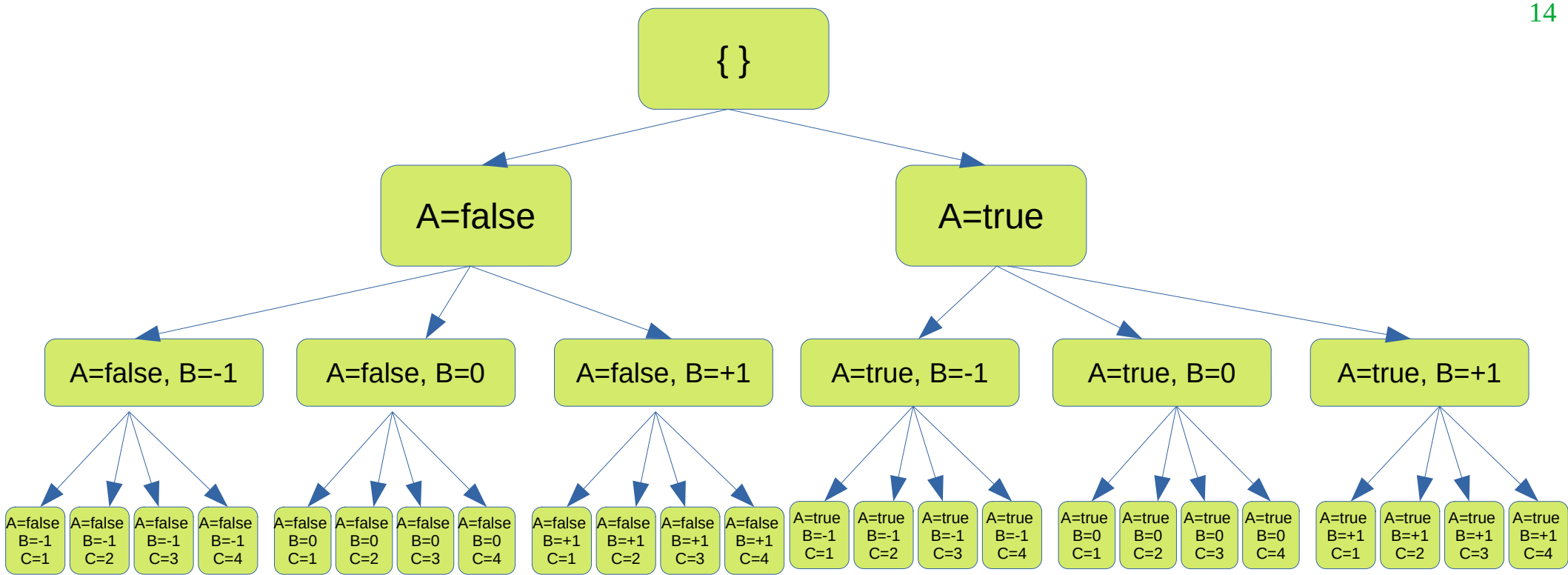
Each leaf represents a candidate assignment

Home Work

- Write Python code to enumerate all the candidate solutions made of triplet (A,B,C)

- Hint: use nested loops

- Can you generalize this example to the Zebra Puzzle?