

Solutions to Prac Sheet - Intro to ML

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Exercise 5

- Recall Bayes rule, $P(A | B) = P(B | A) P(A) / P(B)$
- Show that we have also $P(A | B, C) = P(B | A, C) P(A | C) / P(B | C)$

We have

$$(1) \quad P(A | B, C) = P(A, B, C) / P(B, C) \quad \text{by the product rule}$$

and

$$(2) \quad P(A, B, C) = P(B | A, C) P(A, C) \quad \text{by the same product rule}$$

Substituting (2) in (1), we get

$$(3) \quad P(A | B, C) = P(B | A, C) P(A, C) / P(B, C)$$

Using

$$P(B, C) = P(B|C) P(C) \quad \text{and} \quad P(A, C) = P(A|C) P(C)$$

and replacing in (3), we finally get

$$P(A | B, C) = P(B | A, C) P(A | C) / P(B | C)$$

Exercise 6

- Suppose we want to fit a linear function $f_w(x) = wx$ where x is scalar to a data set $\{(x_1, y_1), \dots, (x_n, y_n)\}$.
- Compute the derivative of the error $E = \sum_{i=1}^n (y_i - wx_i)^2$ with respect to w .

Let compute the derivative of E with respect to w .

$$\frac{\partial E}{\partial w} = \sum_n -2 x_i (y_i - wx_i)$$

The derivative becomes zero when

$$w = \frac{\sum_n x_i y_i}{\sum_n x_i^2}$$

- This is the most probable w .